Next-to-leading order evolution of color dipole

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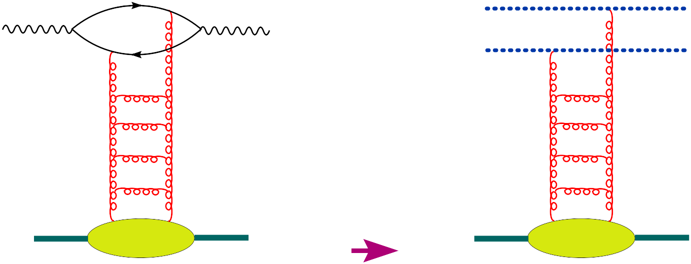
JLAB & ODU

Les Houches April 3, 2008

Otuline

- DIS from nucleus at high energy and Wilson line.
- Evolution equation.
- Leading order: BK equation.
- Non linear evolution equation in the NLO.
- NLO kernel.
- NLO kernel in $\mathcal{N} = 4$.
- Conclusions.

- At high energies, the amplitude of $\gamma^* A \to \gamma^* A$ scattering reduces to the matrix element of a two-Wilson-line operator (color dipole):



$$A(s) = \int \frac{d^2k_\perp}{4\pi^2} I^A(k_\perp)\langle B|\mathrm{Tr}\{U(k_\perp)U^\dagger(-k_\perp)\}|B\rangle \quad (1)$$

The structure function of a hadron is proportional to a matrix element of this color dipole operator

$$\hat{\mathcal{U}}^{\eta}(x_\perp, y_\perp) = 1 - \frac{1}{N_c}\mathrm{Tr}\{\hat{U}^{\eta}(x_\perp)\hat{U}^{\dagger\eta}(y_\perp)\}$$

switched between the target's states ($N_c = 3$ for QCD). The gluon parton density is approximately:

$$x_B G(x_B, \mu^2 = Q^2) \ \simeq \ \langle p|\ \hat{\mathcal{U}}^{\eta}(x_\perp, 0)|p\rangle\Big|_{x_\perp^2 = Q^{-2}}$$

where $\eta = \ln\frac{1}{x_B}$

Propagation in the shock wave: Wilson line

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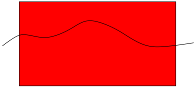

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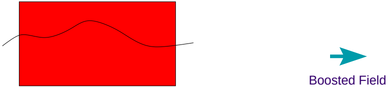

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Quarks and gluons do not have time to deviate in the transverse space. Each path is weighted with the gauge factor $Pe^{ig\int dx_\mu A^\mu}$. Since the external field exists only within the infinitely thin wall, we can replace the gauge factor along the actual path with the one along the straight-line path.

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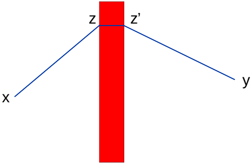

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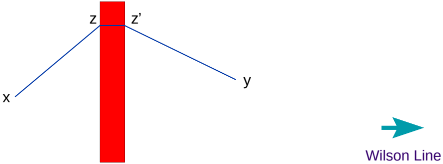

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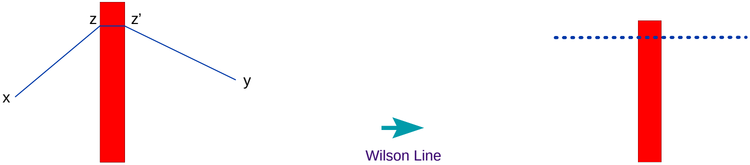

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$$[x, y] = Pe^{ig\int_0^1 du(x-y)^\mu A_\mu(ux+(1-u)y)}$$

Evolution Equation

To get the evolution equation, consider the dipole with the slope $|| \eta_1$ and integrate over the gluons with ripidities $\eta_1 > \eta > \eta_2$. This integral gives the kernel of the evolution equation (multiplied by the dipole(s) with the slope corresponding to η_2).

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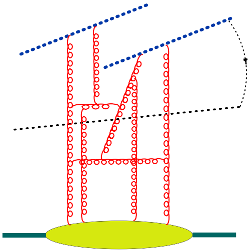

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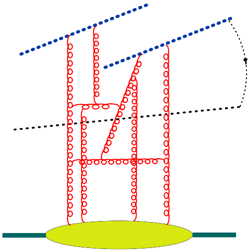

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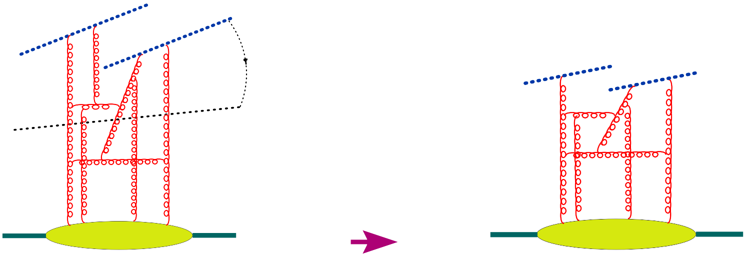

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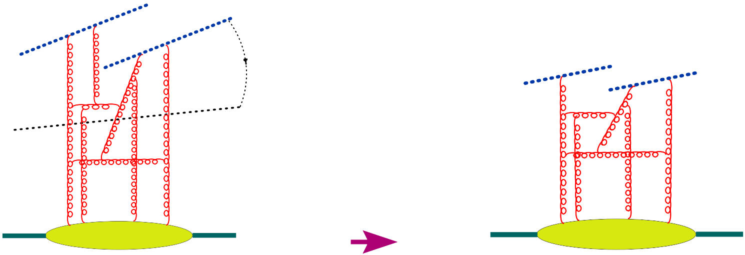

$$\alpha_s(\eta_1 - \eta_2)\otimes$$

Leading order: BK equation

$$\frac{d}{d\eta}\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_y\} = K_{\mathrm{LO}}\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_y\} + \ldots \quad \Rightarrow$$

$$\frac{d}{d\eta}\langle\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_y\}\rangle_{\mathrm{shockwave}} = \langle K_{\mathrm{LO}}\mathrm{Tr}\{\hat{U}_x\hat{U}^\dagger_y\}\rangle_{\mathrm{shockwave}}$$

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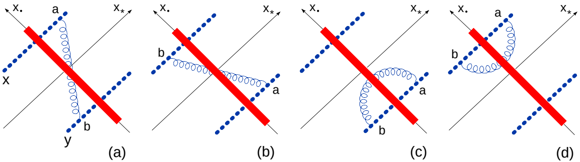

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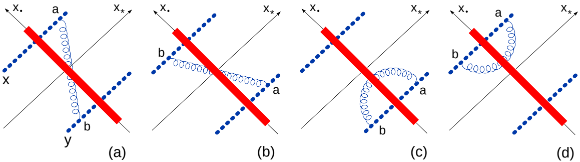

[$x \to z$: free propagation]$\times$

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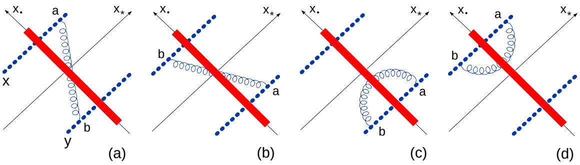

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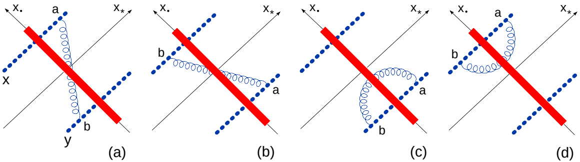

[$x \longrightarrow z$: free propagation]$\times$
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[$z \longrightarrow y$: free propagation]

Non linear evolution equation

$$U_z^{ab} = \mathrm{Tr}\{t^a U_z t^b U_z^\dagger\} \quad \Rightarrow (U_x U_y^\dagger)^{\eta_1} \to (U_x U_y^\dagger)^{\eta_1} + \alpha_s(\eta_1 - \eta_2)(U_x U_z^\dagger U_z U_y^\dagger)^{\eta_2}$$

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I. Balitsky (1996)

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$$\frac{\alpha_s N_c}{2\pi^2}\int dz_\perp \frac{(\vec{x}-\vec{y})^2_\perp}{(\vec{x}_\perp - \vec{z}_\perp)^2(\vec{z}_\perp - \vec{y}_\perp)^2}\Big\{\hat{\mathcal{U}}(x,z) + \hat{\mathcal{U}}(z,y) - \hat{\mathcal{U}}(x,y) - \hat{\mathcal{U}}(x,z)\hat{\mathcal{U}}(z,y)\Big\}$$

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LLA for DIS in sQCD $\Rightarrow$ BK eqn

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  - Experimental view point: The cross section is proportional to some power of the coupling constant, so the argument of the coupling constant determined how big or how small the cross section is.

Non linear evolution equation in the NLO

$$\frac{d}{d\eta}Tr\{U_xU_y^\dagger\} =$$
$$\int \frac{d^2z}{2\pi^2}\left(\alpha_s\frac{(x-y)^2}{(x-z)^2(z-y)^2} + \alpha_s^2 K_{NLO}(x,y,z)\right)[Tr\{U_xU_z^\dagger\}Tr\{U_zU_y^\dagger\} - N_c Tr\{U_zU_y^\dagger\}] +$$
$$\alpha_s^2\int d^2zd^2z'\left(K_4(x,y,z,z')\{U_x,U_{z'}^\dagger,U_z,U_y^\dagger\} + K_6(x,y,z,z')\{U_x,U_{z'}^\dagger,U_{z'},U_z,U_z^\dagger,U_y^\dagger\}\right)$$

K_{NLO} is the next-to-leading order correction to the dipole kernel and K4 and K6 are the coefficients in front of the (tree) four- and six-Wilson line operators with arbitrary white arrangements of color indices.

Definition of the NLO kernel

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Subtraction of BK^2 contribution $\Rightarrow$ $\left[\frac{1}{u}\right]_+$ prescription

$$\int_0^1 du\, f(u)\left[\frac{1}{u}\right]_+ \equiv \int_0^1 du\,\frac{f(u)-f(0)}{u}, \qquad \int_0^1 du\, f(u)\left[\frac{1}{\bar{u}}\right]_+ \equiv \int_0^1 du\,\frac{f(u)-f(1)}{\bar{u}}$$

Regularizing the infinities

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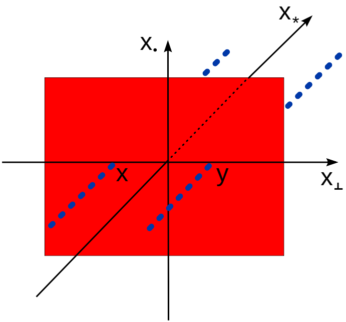

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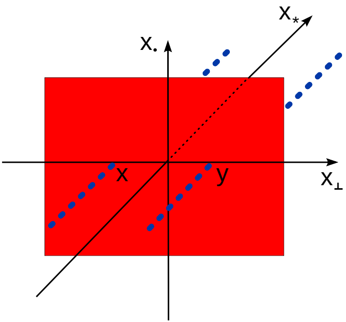

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Regularization by: slope

$$U^\eta(x_\perp) = \text{Pexp}\Big\{ig\int_{-\infty}^{\infty} du\ n_\mu\ A^\mu(un + x_\perp)\Big\}$$

$$n^\mu = p_1^\mu + e^{-2\eta} p_2^\mu$$

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Regularization by:Rigid cut-off

$$U_x^{\eta} = \text{Pexp}\Big[ig\int_{-\infty}^{\infty} du\; p_1^{\mu} A_{\mu}^{\eta}(up_1 + x_{\perp})\Big]$$

$$A_{\mu}^{\eta}(x) = \int \frac{d^4k}{(2\pi)^4}\theta(e^{\eta} - |\alpha_k|)e^{-ik\cdot x}A_{\mu}(k)$$

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Regularization by:Rigid cut-off

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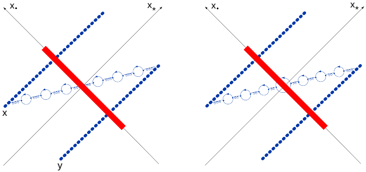
The rigid cut-off leads to the conformal result

Argument of coupling constant

$$\frac{d}{d\eta}\hat{\mathcal{U}}(x,y) =$$
$$\frac{\alpha_s(?_\perp)N_c}{2\pi^2}\int dz_\perp \frac{(\vec{x}-\vec{y})^2_\perp}{(\vec{x}_\perp-\vec{z}_\perp)^2(\vec{z}_\perp-\vec{y}_\perp)^2}\Big\{\hat{\mathcal{U}}(x,z)+\hat{\mathcal{U}}(z,y)-\hat{\mathcal{U}}(x,y)-\hat{\mathcal{U}}(x,z)\hat{\mathcal{U}}(z,y)\Big\}$$

I. Balitsky (2006)

NLO result does not lead automatically to the argument of coupling constant in front of the leading term. In order to get this argument, we can use the renormalon-based approach: first we get the quark part of the running coupling constant coming from the bubble chain of quark loops and then make a conjecture that the gluon part of the ß-function will follow that pattern



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$$\text{Running coupling} \ = \ \alpha_s(\mu)[1 + \frac{b_0\alpha_s}{4\pi}\ln(x-y)^2\mu^2] \ \rightarrow \ \alpha_s(|x-y|)$$

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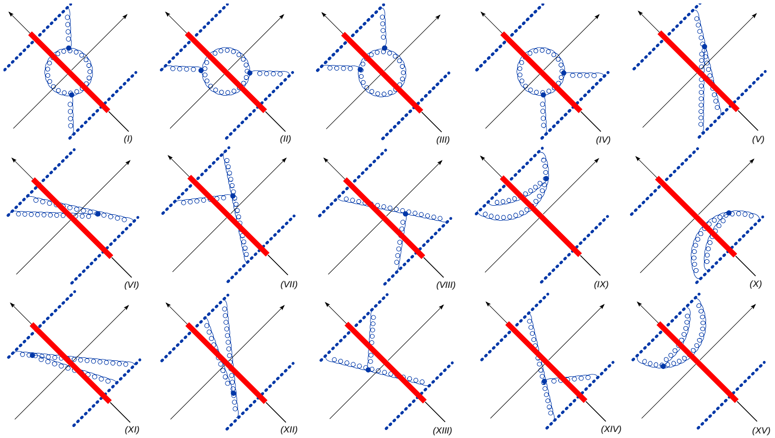
However... When the sizes of the dipoles are very different the kernel reduces to:

$$\begin{array}{cc}
\frac{\alpha_s((x-y)^2)}{2\pi^2}\frac{(x-y)^2}{X^2Y^2} & |x-y| \ll |x-z|, |y-z| \\
\frac{\alpha_s(X)^2)}{2\pi^2X^2} & |x-z| \ll |x-y|, |y-z| \\
\frac{\alpha_s(Y)^2)}{2\pi^2Y^2} & |y-z| \ll |x-y|, |x-z|
\end{array}$$

Therefore the argument of the coupling constant is given by the size of the smallest dipole.

Diagrams with 2 gluons interaction



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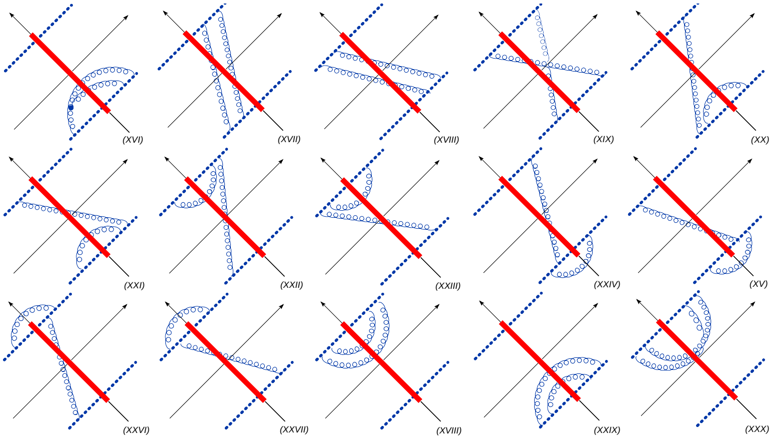
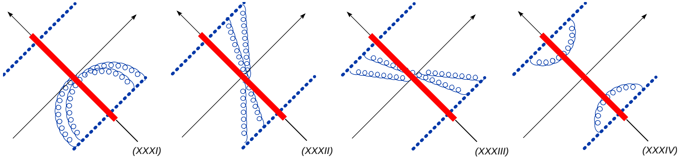
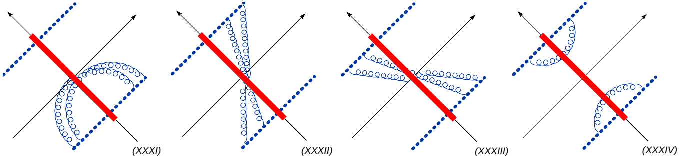


Diagrams with 2 gluons interaction



Diagrams with 2 gluons interaction

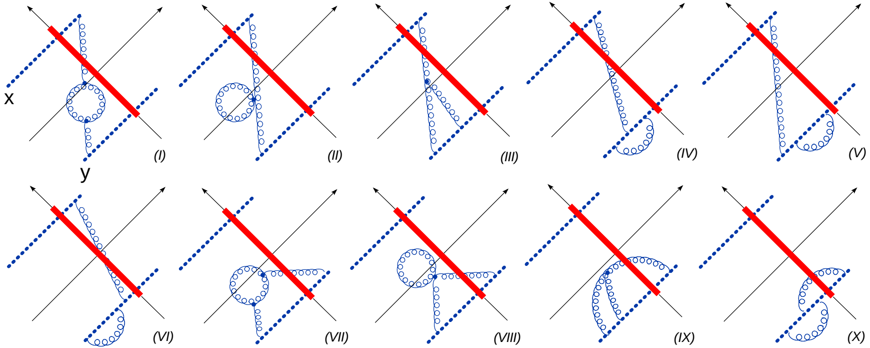


Extracting the UV divergencies

$$\mathrm{Tr}\{t^a U_z t^b U^\dagger_{z'}\} = \mathrm{Tr}\{t^a U_z t^b U^\dagger_{z'} - t^a U_z t^b U^\dagger_z\} + \mathrm{Tr}\{t^a U_z t^b U^\dagger_z\}$$

I.Balitsky 2006;   Y. Kovchegov, H. Weigert 2006

"Running coupling" diagrams

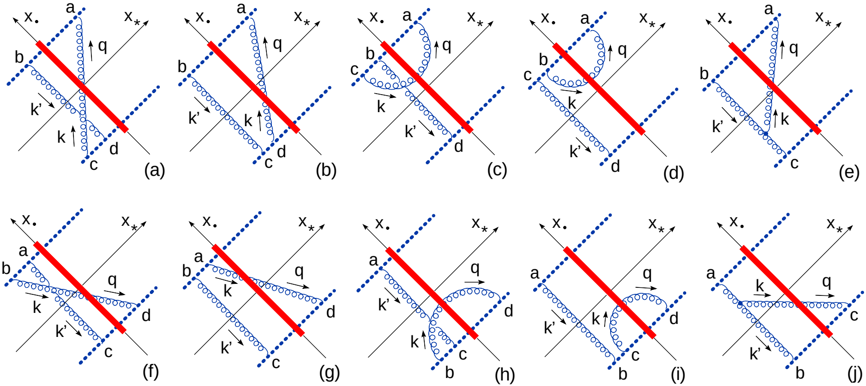


Balitsky 2006

A way to get a potential extra term is to find the light-cone expansion of $U_x U_y^\dagger$ at $x_\perp \to y_\perp$ up to twist-4 terms and compare it with the expansion of the sum of the diagrams with the quark loop cut by the shock wave and quark loop ouside the shock wave.

The expansion coincide $\Rightarrow$ There are no contribution coming from quark loop in the shock wave.

1 → 2 dipole transition diagrams



Gluon contribution to the NLO kernel

$$
\begin{aligned}
\frac{d}{d\eta}\mathrm{Tr}\{U_x U^\dagger_y\} \;=\; & \frac{\alpha_s}{2\pi^2}\int d^2z\,\Big([\mathrm{Tr}\{U_x U^\dagger_z\}\mathrm{Tr}\{U_z U^\dagger_y\} - N_c\mathrm{Tr}\{U_x U^\dagger_y\}] \\
& \times\Big\{\frac{(x-y)^2}{X^2Y^2}\Big[1+\frac{\alpha_s N_c}{4\pi}\big(\frac{11}{3}\ln(x-y)^2\mu^2+\frac{67}{9}-\frac{\pi^2}{3}\big)\Big] \\
& -\frac{11}{3}\frac{\alpha_s N_c}{4\pi}\frac{X^2-Y^2}{X^2Y^2}\ln\frac{X^2}{Y^2} - \frac{\alpha_s N_c}{2\pi}\frac{(x-y)^2}{X^2Y^2}\ln\frac{X^2}{(x-y)^2}\ln\frac{Y^2}{(x-y)^2}\Big\} \\
& +\frac{\alpha_s}{4\pi^2}\int d^2z'\,\Big\{[\mathrm{Tr}\{U_x U^\dagger_z\}\mathrm{Tr}\{U_z U^\dagger_{z'}\}\{U_{z'} U^\dagger_y\} - \mathrm{Tr}\{U_x U^\dagger_z U_{z'} U^\dagger_y U_z U^\dagger_{z'}\} \\
& -(z'\to z)]\frac{1}{(z-z')^4}\Big[-2+\frac{X'^2Y^2+Y'^2X^2-4(x-y)^2(z-z')^2}{2(X'^2Y^2-Y'^2X^2)}\ln\frac{X'^2Y^2}{Y'^2X^2}\Big] \\
& +[\mathrm{Tr}\{U_x U^\dagger_z\}\mathrm{Tr}\{U_z U^\dagger_{z'}\}\{U_{z'} U^\dagger_y\} - \mathrm{Tr}\{U_x U^\dagger_{z'} U_z U^\dagger_y U_{z'} U^\dagger_z\} - (z'\to z)] \\
& \times\Big[\frac{(x-y)^4}{X^2Y'^2(X^2Y'^2-X'^2Y^2)}+\frac{(x-y)^2}{(z-z')^2X^2Y'^2}\Big]\ln\frac{X^2Y'^2}{X'^2Y^2}\Big\}\Big)
\end{aligned}
$$

Gluon contribution to the NLO kernel

$$
\begin{aligned}
\frac{d}{d\eta}\mathrm{Tr}\{U_xU_y^\dagger\} \;=\; & \frac{\alpha_s}{2\pi^2}\int d^2z\,\Big([\mathrm{Tr}\{U_xU_z^\dagger\}\mathrm{Tr}\{U_zU_y^\dagger\} - N_c\mathrm{Tr}\{U_xU_y^\dagger\}] \\
& \times\Big\{\frac{(x-y)^2}{X^2Y^2}\Big[1+\frac{\alpha_sN_c}{4\pi}\big(\frac{11}{3}\ln(x-y)^2\mu^2+\frac{67}{9}-\frac{\pi^2}{3}\big)\Big] \\
& -\frac{11}{3}\frac{\alpha_sN_c}{4\pi}\frac{X^2-Y^2}{X^2Y^2}\ln\frac{X^2}{Y^2} - \frac{\alpha_sN_c}{2\pi}\frac{(x-y)^2}{X^2Y^2}\ln\frac{X^2}{(x-y)^2}\ln\frac{Y^2}{(x-y)^2}\Big\} \\
& +\frac{\alpha_s}{4\pi^2}\int d^2z'\,\Big\{[\mathrm{Tr}\{U_xU_z^\dagger\}\mathrm{Tr}\{U_zU_{z'}^\dagger\}\{U_{z'}U_y^\dagger\} - \mathrm{Tr}\{U_xU_z^\dagger U_{z'}U_y^\dagger U_zU_{z'}^\dagger\} \\
& -(z'\to z)]\frac{1}{(z-z')^4}\Big[-2+\frac{X'^2Y^2+Y'^2X^2-4(x-y)^2(z-z')^2}{2(X'^2Y^2-Y'^2X^2)}\ln\frac{X'^2Y^2}{Y'^2X^2}\Big] \\
& +[\mathrm{Tr}\{U_xU_z^\dagger\}\mathrm{Tr}\{U_zU_{z'}^\dagger\}\{U_{z'}U_y^\dagger\} - \mathrm{Tr}\{U_xU_{z'}^\dagger U_zU_y^\dagger U_{z'}U_z^\dagger\} - (z'\to z)] \\
& \times\Big[\frac{(x-y)^4}{X^2Y'^2(X^2Y'^2-X'^2Y^2)}+\frac{(x-y)^2}{(z-z')^2X^2Y'^2}\Big]\ln\frac{X^2Y'^2}{X'^2Y^2}\Big\}\Big) =
\end{aligned}
$$

Running coupling part

Gluon contribution to the NLO kernel

$$\frac{d}{d\eta}\mathrm{Tr}\{U_xU_y^\dagger\} = \frac{\alpha_s}{2\pi^2}\int d^2z\,\Big([\mathrm{Tr}\{U_xU_z^\dagger\}\mathrm{Tr}\{U_zU_y^\dagger\} - N_c\mathrm{Tr}\{U_xU_y^\dagger\}]$$

$$\times\Big\{\frac{(x-y)^2}{X^2Y^2}\Big[1+\frac{\alpha_sN_c}{4\pi}(\frac{11}{3}\ln(x-y)^2\mu^2+\frac{67}{9}-\frac{\pi^2}{3})\Big]$$

$$-\frac{11}{3}\frac{\alpha_sN_c}{4\pi}\frac{X^2-Y^2}{X^2Y^2}\ln\frac{X^2}{Y^2} - \frac{\alpha_sN_c}{2\pi}\frac{(x-y)^2}{X^2Y^2}\ln\frac{X^2}{(x-y)^2}\ln\frac{Y^2}{(x-y)^2}\Big\}$$

$$+\frac{\alpha_s}{4\pi^2}\int d^2z'\,\Big\{[\mathrm{Tr}\{U_xU_z^\dagger\}\mathrm{Tr}\{U_zU_{z'}^\dagger\}\{U_{z'}U_y^\dagger\} - \mathrm{Tr}\{U_xU_z^\dagger U_{z'}U_y^\dagger U_zU_{z'}^\dagger\}$$

$$-(z'\to z)]\frac{1}{(z-z')^4}\Big[-2+\frac{X'^2Y^2+Y'^2X^2-4(x-y)^2(z-z')^2}{2(X'^2Y^2-Y'^2X^2)}\ln\frac{X'^2Y^2}{Y'^2X^2}\Big]$$

$$+[\mathrm{Tr}\{U_xU_z^\dagger\}\mathrm{Tr}\{U_zU_{z'}^\dagger\}\{U_{z'}U_y^\dagger\} - \mathrm{Tr}\{U_xU_{z'}^\dagger U_zU_y^\dagger U_{z'}U_z^\dagger\} - (z'\to z)]$$

$$\times\Big[\frac{(x-y)^4}{X^2Y'^2(X^2Y'^2-X'^2Y^2)}+\frac{(x-y)^2}{(z-z')^2X^2Y'^2}\Big]\ln\frac{X^2Y'^2}{X'^2Y^2}\Big\}\Big) =$$

Running coupling part + Non-conformal part

$$\frac{d}{d\eta}\mathrm{Tr}\{U_xU_y^\dagger\} = \frac{\alpha_s}{2\pi^2}\int d^2z\,\Big([\mathrm{Tr}\{U_xU_z^\dagger\}\mathrm{Tr}\{U_zU_y^\dagger\} - N_c\mathrm{Tr}\{U_xU_y^\dagger\}]$$
$$\times\Big\{\frac{(x-y)^2}{X^2Y^2}\Big[1+\frac{\alpha_sN_c}{4\pi}(\frac{11}{3}\ln(x-y)^2\mu^2+\frac{67}{9}-\frac{\pi^2}{3})\Big]$$
$$-\frac{11}{3}\frac{\alpha_sN_c}{4\pi}\frac{X^2-Y^2}{X^2Y^2}\ln\frac{X^2}{Y^2} - \frac{\alpha_sN_c}{2\pi}\frac{(x-y)^2}{X^2Y^2}\ln\frac{X^2}{(x-y)^2}\ln\frac{Y^2}{(x-y)^2}\Big\}$$
$$+\frac{\alpha_s}{4\pi^2}\int d^2z'\,\Big\{[\mathrm{Tr}\{U_xU_z^\dagger\}\mathrm{Tr}\{U_zU_{z'}^\dagger\}\{U_{z'}U_y^\dagger\} - \mathrm{Tr}\{U_xU_z^\dagger U_{z'}U_y^\dagger U_zU_{z'}^\dagger\}$$
$$-(z'\to z)]\frac{1}{(z-z')^4}\Big[-2+\frac{X'^2Y^2+Y'^2X^2-4(x-y)^2(z-z')^2}{2(X'^2Y^2-Y'^2X^2)}\ln\frac{X'^2Y^2}{Y'^2X^2}\Big]$$
$$+[\mathrm{Tr}\{U_xU_z^\dagger\}\mathrm{Tr}\{U_zU_{z'}^\dagger\}\{U_{z'}U_y^\dagger\} - \mathrm{Tr}\{U_xU_{z'}^\dagger U_zU_y^\dagger U_{z'}U_z^\dagger\} - (z'\to z)]$$
$$\times\Big[\frac{(x-y)^4}{X^2Y'^2(X^2Y'^2-X'^2Y^2)}+\frac{(x-y)^2}{(z-z')^2X^2Y'^2}\Big]\ln\frac{X^2Y'^2}{X'^2Y^2}\Big\}\Big) =$$

Running coupling part + Non-conformal part + Conformal "non-analytic" part

Gluon contribution to the NLO kernel

$$\frac{d}{d\eta}\mathrm{Tr}\{U_xU_y^\dagger\} = \frac{\alpha_s}{2\pi^2}\int d^2z\,\Big([\mathrm{Tr}\{U_xU_z^\dagger\}\mathrm{Tr}\{U_zU_y^\dagger\}-N_c\mathrm{Tr}\{U_xU_y^\dagger\}]$$

$$\times\Big\{\frac{(x-y)^2}{X^2Y^2}\Big[1+\frac{\alpha_sN_c}{4\pi}(\frac{11}{3}\ln(x-y)^2\mu^2+\frac{67}{9}-\frac{\pi^2}{3})\Big]$$

$$-\frac{11}{3}\frac{\alpha_sN_c}{4\pi}\frac{X^2-Y^2}{X^2Y^2}\ln\frac{X^2}{Y^2}-\frac{\alpha_sN_c}{2\pi}\frac{(x-y)^2}{X^2Y^2}\ln\frac{X^2}{(x-y)^2}\ln\frac{Y^2}{(x-y)^2}\Big\}$$

$$+\frac{\alpha_s}{4\pi^2}\int d^2z'\,\Big\{[\mathrm{Tr}\{U_xU_z^\dagger\}\mathrm{Tr}\{U_zU_{z'}^\dagger\}\{U_{z'}U_y^\dagger\}-\mathrm{Tr}\{U_xU_z^\dagger U_{z'}U_y^\dagger U_zU_{z'}^\dagger\}]$$

$$-(z'\to z)\frac{1}{(z-z')^4}\Big[-2+\frac{X'^2Y^2+Y'^2X^2-4(x-y)^2(z-z')^2}{2(X'^2Y^2-Y'^2X^2)}\ln\frac{X'^2Y^2}{Y'^2X^2}\Big]$$

$$+[\mathrm{Tr}\{U_xU_z^\dagger\}\mathrm{Tr}\{U_zU_{z'}^\dagger\}\{U_{z'}U_y^\dagger\}-\mathrm{Tr}\{U_xU_{z'}^\dagger U_zU_y^\dagger U_{z'}U_z^\dagger\}-(z'\to z)]$$

$$\times\Big[\frac{(x-y)^4}{X^2Y'^2(X^2Y'^2-X'^2Y^2)}+\frac{(x-y)^2}{(z-z')^2X^2Y'^2}\Big]\ln\frac{X^2Y'^2}{X'^2Y^2}\Big\}\Big) =$$

Running coupling part $+$ Non-conformal part $+$ Conformal "non-analytic" part $+$ "conformal-analytic" ($\mathcal{N}=4$) part

Gluon contribution to the NLO kernel

$$
\begin{aligned}
\frac{d}{d\eta}\mathrm{Tr}\{U_xU^\dagger_y\} \;=\; & \frac{\alpha_s}{2\pi^2}\int d^2z\,\Big([\mathrm{Tr}\{U_xU^\dagger_z\}\mathrm{Tr}\{U_zU^\dagger_y\} - N_c\mathrm{Tr}\{U_xU^\dagger_y\}]\\
&\times\Big\{\frac{(x-y)^2}{X^2Y^2}\Big[1+\frac{\alpha_sN_c}{4\pi}\big(\frac{11}{3}\ln(x-y)^2\mu^2+\frac{67}{9}-\frac{\pi^2}{3}\big)\Big]\\
&-\frac{11}{3}\frac{\alpha_sN_c}{4\pi}\frac{X^2-Y^2}{X^2Y^2}\ln\frac{X^2}{Y^2}-\frac{\alpha_sN_c}{2\pi}\frac{(x-y)^2}{X^2Y^2}\ln\frac{X^2}{(x-y)^2}\ln\frac{Y^2}{(x-y)^2}\Big\}\\
&+\frac{\alpha_s}{4\pi^2}\int d^2z'\,\Big\{[\mathrm{Tr}\{U_xU^\dagger_z\}\mathrm{Tr}\{U_zU^\dagger_{z'}\}\{U_{z'}U^\dagger_y\} - \mathrm{Tr}\{U_xU^\dagger_zU_{z'}U^\dagger_yU_zU^\dagger_{z'}\}\\
&-(z'\to z)]\frac{1}{(z-z')^4}\Big[-2+\frac{X'^2Y^2+Y'^2X^2-4(x-y)^2(z-z')^2}{2(X'^2Y^2-Y'^2X^2)}\ln\frac{X'^2Y^2}{Y'^2X^2}\Big]\\
&+[\mathrm{Tr}\{U_xU^\dagger_z\}\mathrm{Tr}\{U_zU^\dagger_{z'}\}\{U_{z'}U^\dagger_y\} - \mathrm{Tr}\{U_xU^\dagger_{z'}U_zU^\dagger_yU_{z'}U^\dagger_z\} - (z'\to z)]\\
&\times\Big[\frac{(x-y)^4}{X^2Y'^2(X^2Y'^2-X'^2Y^2)}+\frac{(x-y)^2}{(z-z')^2X^2Y'^2}\Big]\ln\frac{X^2Y'^2}{X'^2Y^2}\Big\}\Big) + \frac{\alpha_s^2N_c^2}{4\pi^2}\zeta(3)\mathrm{Tr}\{U_xU^\dagger_y\} =
\end{aligned}
$$

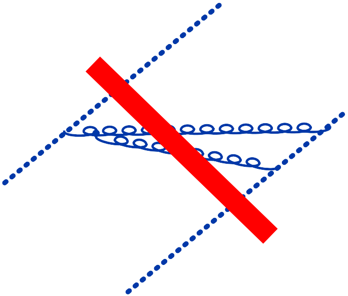
Our result + Extra term $\Rightarrow$ Agrees with NLO BFKL

(Comparing the eigenvalue of the forward kernel)

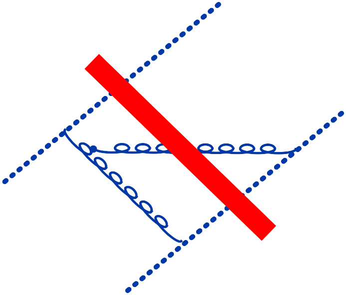
$$
\begin{aligned}
\frac{d}{d\eta}\mathrm{Tr}\{U_xU_y^\dagger\} \;=\;& \frac{\alpha_s}{2\pi^2}\int d^2z\,\Big([\mathrm{Tr}\{U_xU_z^\dagger\}\mathrm{Tr}\{U_zU_y^\dagger\} - N_c\mathrm{Tr}\{U_xU_y^\dagger\}]\\
&\times\Big\{\frac{(x-y)^2}{X^2Y^2}\Big[1+\frac{\alpha_sN_c}{4\pi}\big(\frac{11}{3}\ln(x-y)^2\mu^2+\frac{67}{9}-\frac{\pi^2}{3}\big)\Big]\\
&-\frac{11}{3}\frac{\alpha_sN_c}{4\pi}\frac{X^2-Y^2}{X^2Y^2}\ln\frac{X^2}{Y^2}-\frac{\alpha_sN_c}{2\pi}\frac{(x-y)^2}{X^2Y^2}\ln\frac{X^2}{(x-y)^2}\ln\frac{Y^2}{(x-y)^2}\Big\}\\
&+\frac{\alpha_s}{4\pi^2}\int d^2z'\,\Big\{[\mathrm{Tr}\{U_xU_z^\dagger\}\mathrm{Tr}\{U_zU_{z'}^\dagger\}\{U_{z'}U_y^\dagger\}-\mathrm{Tr}\{U_xU_z^\dagger U_{z'}U_y^\dagger U_zU_{z'}^\dagger\}\\
&-(z'\to z)]\frac{1}{(z-z')^4}\Big[-2+\frac{X'^2Y^2+Y'^2X^2-4(x-y)^2(z-z')^2}{2(X'^2Y^2-Y'^2X^2)}\ln\frac{X'^2Y^2}{Y'^2X^2}\Big]\\
&+[\mathrm{Tr}\{U_xU_z^\dagger\}\mathrm{Tr}\{U_zU_{z'}^\dagger\}\{U_{z'}U_y^\dagger\}-\mathrm{Tr}\{U_xU_{z'}^\dagger U_zU_y^\dagger U_{z'}U_z^\dagger\}-(z'\to z)]\\
&\times\Big[\frac{(x-y)^4}{X^2Y'^2(X^2Y'^2-X'^2Y^2)}+\frac{(x-y)^2}{(z-z')^2X^2Y'^2}\Big]\ln\frac{X^2Y'^2}{X'^2Y^2}\Big\}\Big)+\frac{\alpha_s^2N_c^2}{4\pi^2}\zeta(3)\mathrm{Tr}\{U_xU_y^\dagger\}
\end{aligned}
$$

However the term $\frac{\alpha_s^2N_c^2}{4\pi^2}\zeta(3)\mathrm{Tr}\mathrm{U_x}\mathrm{U_y^\dagger}$ contradicts the requirement $\frac{d}{d\eta}U_xU_Y^\dagger = 0$ at $x=y$.

Conformal and non-conformal diagrams



Conformal



Non−Conformal

Quark-contribution to the NLO

$$\frac{d}{d\eta}\mathrm{Tr}\{U_xU^\dagger_y\} = \frac{\alpha_s}{2\pi^2}\int d^2z\ [\mathrm{Tr}\{U_xU^\dagger_z\}\mathrm{Tr}\{U_zU^\dagger_y\} - N_c\mathrm{Tr}\{U_xU^\dagger_y\}]$$
$$\times\Big[\frac{(x-y)^2}{X^2Y^2}\Big(1-\frac{\alpha_s n_f}{6\pi}[\ln(x-y)^2\mu^2+\frac{5}{3}]\Big)+\frac{\alpha_s n_f}{6\pi}\frac{X^2-Y^2}{X^2Y^2}\ln\frac{X^2}{Y^2}\Big]$$
$$+\frac{\alpha_s^2}{\pi^4}n_f\mathrm{Tr}\{t^aU_xt^bU^\dagger_y\}\int d^2zd^2z'\ \mathrm{Tr}\{t^aU_zt^bU^\dagger_{z'}-t^aU_zt^bU^\dagger_z\}\ \frac{1}{(z-z')^4}$$
$$\times\Big\{1-\frac{X'^2Y^2+Y'^2X^2-(x-y)^2(z-z')^2}{2(X'^2Y^2-Y'^2X^2)}\ln\frac{X'^2Y^2}{Y'^2X^2}\Big\}$$

$$X=x-z,\ \ X'=x-z',\ \ Y=y-z,\ \ Y'=y-z'$$

Quark-contribution to the NLO

$$\frac{d}{d\eta}\mathrm{Tr}\{U_xU_y^\dagger\} = \frac{\alpha_s}{2\pi^2}\int d^2z\,[\mathrm{Tr}\{U_xU_z^\dagger\}\mathrm{Tr}\{U_zU_y^\dagger\} - N_c\mathrm{Tr}\{U_xU_y^\dagger\}]$$
$$\times\Big[\frac{(x-y)^2}{X^2Y^2}\Big(1-\frac{\alpha_s n_f}{6\pi}[\ln(x-y)^2\mu^2+\frac{5}{3}]\Big)+\frac{\alpha_s n_f}{6\pi}\frac{X^2-Y^2}{X^2Y^2}\ln\frac{X^2}{Y^2}\Big]$$
$$+\frac{\alpha_s^2}{\pi^4}n_f\mathrm{Tr}\{t^aU_xt^bU_y^\dagger\}\int d^2zd^2z'\,\mathrm{Tr}\{t^aU_zt^bU_{z'}^\dagger - t^aU_zt^bU_z^\dagger\}\,\frac{1}{(z-z')^4}$$
$$\times\Big\{1-\frac{X'^2Y^2+Y'^2X^2-(x-y)^2(z-z')^2}{2(X'^2Y^2-Y'^2X^2)}\ln\frac{X'^2Y^2}{Y'^2X^2}\Big\}$$

$$X=x-z,\quad X'=x-z',\quad Y=y-z,\quad Y'=y-z'$$

Running coupling

$$
\begin{aligned}
\frac{d}{d\eta}\mathrm{Tr}\{U_x U^\dagger_y\} \;&=\; \frac{\alpha_s}{2\pi^2}\int d^2z\;[\mathrm{Tr}\{U_x U^\dagger_z\}\mathrm{Tr}\{U_z U^\dagger_y\} - N_c\mathrm{Tr}\{U_x U^\dagger_y\}] \\
&\times\Big[\frac{(x-y)^2}{X^2Y^2}\Big(1-\frac{\alpha_s n_f}{6\pi}[\ln(x-y)^2\mu^2+\frac{5}{3}]\Big)+\frac{\alpha_s n_f}{6\pi}\frac{X^2-Y^2}{X^2Y^2}\ln\frac{X^2}{Y^2}\Big] \\
&+\frac{\alpha_s^2}{\pi^4}n_f\mathrm{Tr}\{t^aU_xt^bU^\dagger_y\}\int d^2zd^2z'\;\mathrm{Tr}\{t^aU_zt^bU^\dagger_{z'}-t^aU_zt^bU^\dagger_z\}\;\frac{1}{(z-z')^4} \\
&\times\Big\{1-\frac{X'^2Y^2+Y'^2X^2-(x-y)^2(z-z')^2}{2(X'^2Y^2-Y'^2X^2)}\ln\frac{X'^2Y^2}{Y'^2X^2}\Big\}
\end{aligned}
$$

$X=x-z,\;\; X'=x-z',\;\; Y=y-z,\;\; Y'=y-z'$

Running coupling $+$ Conformal part

NLO kernel

$$\frac{d}{d\eta}\mathrm{Tr}\{\hat{U}_x\hat{U}_y^\dagger\}$$

$$= \frac{\alpha_s}{2\pi^2}\int d^2z\, \frac{(x-y)^2}{X^2Y^2}\Big\{1+\frac{\alpha_s}{4\pi}\Big[b\ln(x-y)^2\mu^2 - b\frac{X^2-Y^2}{(x-y)^2}\ln\frac{X^2}{Y^2} + (\frac{67}{9}-\frac{\pi^2}{3})N_c - \frac{10}{9}n_f$$

$$- 2N_c\ln\frac{X^2}{(x-y)^2}\ln\frac{Y^2}{(x-y)^2}\Big]\Big\}\ [\mathrm{Tr}\{\hat{U}_x\hat{U}_z^\dagger\}\mathrm{Tr}\{\hat{U}_z\hat{U}_y^\dagger\} - N_c\mathrm{Tr}\{\hat{U}_x\hat{U}_y^\dagger\}]$$

$$+ \frac{\alpha_s^2}{16\pi^4}\int d^2zd^2z'\Bigg[\Big(-\frac{4}{(z-z')^4} + \Big\{2\frac{X^2Y'^2+X'^2Y^2-4(x-y)^2(z-z')^2}{(z-z')^4[X^2Y'^2-X'^2Y^2]}$$

$$+ \frac{(x-y)^4}{X^2Y'^2-X'^2Y^2}\Big[\frac{1}{X^2Y'^2}+\frac{1}{Y^2X'^2}\Big] + \frac{(x-y)^2}{(z-z')^2}\Big[\frac{1}{X^2Y'^2}-\frac{1}{X'^2Y^2}\Big]\Big\}\ln\frac{X^2Y'^2}{X'^2Y^2}\Big)$$

$$\times\ [\mathrm{Tr}\{\hat{U}_x\hat{U}_z^\dagger\}\mathrm{Tr}\{\hat{U}_z\hat{U}_{z'}^\dagger\}\mathrm{Tr}\{\hat{U}_{z'}\hat{U}_y^\dagger\} - \mathrm{Tr}\{\hat{U}_x\hat{U}_z^\dagger\hat{U}_{z'}U_y^\dagger\hat{U}_z\hat{U}_{z'}^\dagger\} - (z'\to z)]$$

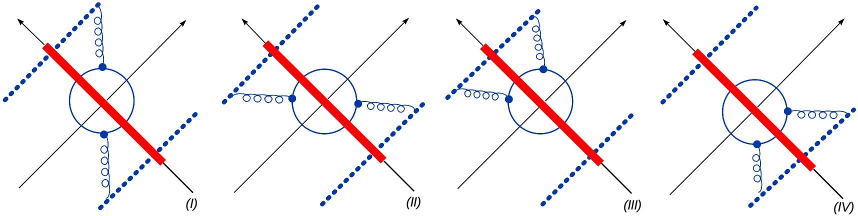
$$+ \Big\{\frac{(x-y)^2}{(z-z')^2}\Big[\frac{1}{X^2Y'^2}+\frac{1}{Y^2X'^2}\Big] - \frac{(x-y)^4}{X^2Y'^2X'^2Y^2}\Big\}\ln\frac{X^2Y'^2}{X'^2Y^2}\ \mathrm{Tr}\{\hat{U}_x\hat{U}_z^\dagger\}\mathrm{Tr}\{\hat{U}_z\hat{U}_{z'}^\dagger\}\mathrm{Tr}\{\hat{U}_{z'}\hat{U}_y^\dagger\}$$

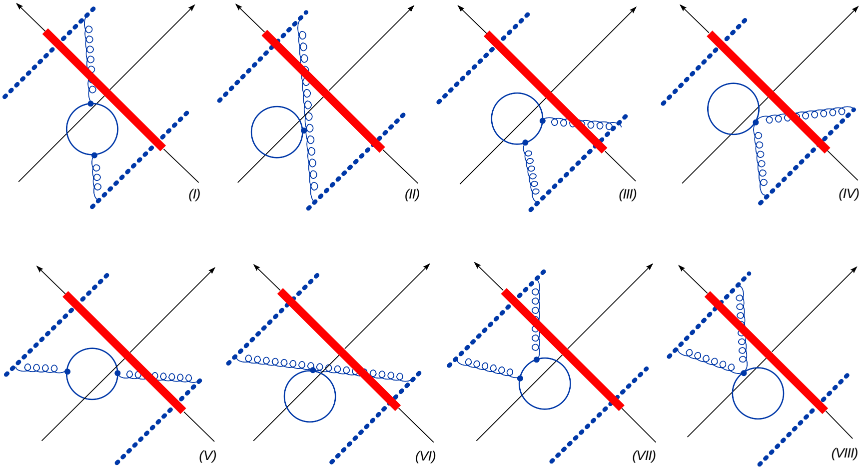
$$+ 4n_f\Big\{\frac{4}{(z-z')^4} - 2\frac{X'^2Y^2+Y'^2X^2-(x-y)^2(z-z')^2}{(z-z')^4(X^2Y'^2-X'^2Y^2)}\ln\frac{X^2Y'^2}{X'^2Y^2}\Big\}\mathrm{Tr}\{t^a\hat{U}_xt^b\hat{U}_y^\dagger\}[\mathrm{Tr}\{t^a\hat{U}_zt^b\hat{U}_{z'}^\dagger\} - (z'\to z)]\Bigg]$$

Quark contribution: Balitsky (2006); Kovchegov, Weigert (2006)

Gluon contribution: Balitsky, G.A.C. (2007)

$\mathcal{N} = 4$




(I)
(II)
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(VIII)

Evolution equation in $\mathcal{N} = 4$

$$
\begin{aligned}
\frac{d}{d\eta}\mathrm{Tr}\{U_x U_y^\dagger\} \;=\; & \frac{\alpha_s}{2\pi^2}\int d^2z\, \Big([\mathrm{Tr}\{U_x U_z^\dagger\}\mathrm{Tr}\{U_z U_y^\dagger\} - N_c \mathrm{Tr}\{U_x U_y^\dagger\}] \\
& \times \Big\{ \frac{(x-y)^2}{X^2Y^2}\Big[1 + \frac{\alpha_s N_c}{12\pi}(1-\pi^2)\Big] - \frac{\alpha_s N_c}{2\pi}\frac{(x-y)^2}{X^2Y^2}\ln\frac{X^2}{(x-y)^2}\ln\frac{Y^2}{(x-y)^2}\Big\} \\
& + \frac{\alpha_s}{16\pi^2}\int d^2z' \;[\mathrm{Tr}\{U_x U_z^\dagger\}\mathrm{Tr}\{U_z U_{z'}^\dagger\}\{U_{z'} U_y^\dagger\} - \mathrm{Tr}\{U_x U_z^\dagger U_{z'} U_y^\dagger U_z U_{z'}^\dagger\} \\
& \qquad - \mathrm{Tr}\{U_x U_{z'}^\dagger U_z U_y^\dagger U_{z'} U_z^\dagger\} - (z' \to z)] \\
& \times \Big[\frac{(x-y)^4}{X^2Y'^2(X^2Y'^2 - X'^2Y^2)} + \frac{(x-y)^2}{(z-z')^2X^2Y'^2}\Big]\ln\frac{X^2Y'^2}{X'^2Y^2}\Big)
\end{aligned}
$$

Evolution equation in $\mathcal{N} = 4$

$$
\begin{aligned}
\frac{d}{d\eta}\mathrm{Tr}\{U_x U_y^\dagger\} \;=\; & \frac{\alpha_s}{2\pi^2}\int d^2z\,\Big([\mathrm{Tr}\{U_x U_z^\dagger\}\mathrm{Tr}\{U_z U_y^\dagger\} - N_c\mathrm{Tr}\{U_x U_y^\dagger\}] \\
& \times\Big\{\frac{(x-y)^2}{X^2Y^2}\Big[1+\frac{\alpha_s N_c}{12\pi}(1-\pi^2)\Big] - \frac{\alpha_s N_c}{2\pi}\frac{(x-y)^2}{X^2Y^2}\ln\frac{X^2}{(x-y)^2}\ln\frac{Y^2}{(x-y)^2}\Big\} \\
& + \frac{\alpha_s}{16\pi^2}\int d^2z'\;[\mathrm{Tr}\{U_x U_z^\dagger\}\mathrm{Tr}\{U_z U_{z'}^\dagger\}\{U_{z'} U_y^\dagger\} - \mathrm{Tr}\{U_x U_z^\dagger U_{z'} U_y^\dagger U_z U_{z'}^\dagger\} \\
& \qquad - \mathrm{Tr}\{U_x U_{z'}^\dagger U_z U_y^\dagger U_{z'} U_z^\dagger\} - (z' \to z)] \\
& \times\Big[\frac{(x-y)^4}{X^2Y'^2(X^2Y'^2 - X'^2Y^2)} + \frac{(x-y)^2}{(z-z')^2X^2Y'^2}\Big]\ln\frac{X^2Y'^2}{X'^2Y^2}\Big)
\end{aligned}
$$

Conformal scheme-dependent part

Evolution equation in $\mathcal{N}=4$

$$
\begin{aligned}
\frac{d}{d\eta}\mathrm{Tr}\{U_xU_y^\dagger\} \;=\; & \frac{\alpha_s}{2\pi^2}\int d^2z\,\Big([\mathrm{Tr}\{U_xU_z^\dagger\}\mathrm{Tr}\{U_zU_y^\dagger\}-N_c\mathrm{Tr}\{U_xU_y^\dagger\}] \\
& \times\Big\{\frac{(x-y)^2}{X^2Y^2}\Big[1+\frac{\alpha_sN_c}{12\pi}(1-\pi^2)\Big]-\frac{\alpha_sN_c}{2\pi}\frac{(x-y)^2}{X^2Y^2}\ln\frac{X^2}{(x-y)^2}\ln\frac{Y^2}{(x-y)^2}\Big\} \\
& +\frac{\alpha_s}{16\pi^2}\int d^2z'\;[\mathrm{Tr}\{U_xU_z^\dagger\}\mathrm{Tr}\{U_zU_{z'}^\dagger\}\{U_{z'}U_y^\dagger\}-\mathrm{Tr}\{U_xU_z^\dagger U_{z'}U_y^\dagger U_zU_{z'}^\dagger\} \\
& -\mathrm{Tr}\{U_xU_{z'}^\dagger U_zU_y^\dagger U_{z'}U_z^\dagger\}-(z'\to z)] \\
& \times\Big[\frac{(x-y)^4}{X^2Y'^2(X^2Y'^2-X'^2Y^2)}+\frac{(x-y)^2}{(z-z')^2X^2Y'^2}\Big]\ln\frac{X^2Y'^2}{X'^2Y^2}\Big)
\end{aligned}
$$

Conformal scheme-dependent part $+$ Non-conformal part

Evolution equation in $\mathcal{N}=4$

$$\frac{d}{d\eta}\mathrm{Tr}\{U_xU_y^\dagger\} = \frac{\alpha_s}{2\pi^2}\int d^2z\,\Big([\mathrm{Tr}\{U_xU_z^\dagger\}\mathrm{Tr}\{U_zU_y^\dagger\}-N_c\mathrm{Tr}\{U_xU_y^\dagger\}]$$

$$\times\Big\{\frac{(x-y)^2}{X^2Y^2}\Big[1+\frac{\alpha_sN_c}{12\pi}(1-\pi^2)\Big]-\frac{\alpha_sN_c}{2\pi}\frac{(x-y)^2}{X^2Y^2}\ln\frac{X^2}{(x-y)^2}\ln\frac{Y^2}{(x-y)^2}\Big\}$$

$$+\frac{\alpha_s}{16\pi^2}\int d^2z'\ \ [\mathrm{Tr}\{U_xU_z^\dagger\}\mathrm{Tr}\{U_zU_{z'}^\dagger\}\{U_{z'}U_y^\dagger\}-\mathrm{Tr}\{U_xU_z^\dagger U_{z'}U_y^\dagger U_zU_{z'}^\dagger\}$$

$$-\mathrm{Tr}\{U_xU_{z'}^\dagger U_zU_y^\dagger U_{z'}U_z^\dagger\}-(z'\to z)]$$

$$\times\Big[\frac{(x-y)^4}{X^2Y'^2(X^2Y'^2-X'^2Y^2)}+\frac{(x-y)^2}{(z-z')^2X^2Y'^2}\Big]\ln\frac{X^2Y'^2}{X'^2Y^2}\Big)$$

Conformal scheme-dependent part $+$ Non-conformal part
$+$ Conformal analitic part

Evolution equation in $\mathcal{N}=4$

$$
\begin{aligned}
\frac{d}{d\eta}\mathrm{Tr}\{U_x U_y^\dagger\} \;=\; & \frac{\alpha_s}{2\pi^2}\int d^2z\,\Big([\mathrm{Tr}\{U_xU_z^\dagger\}\mathrm{Tr}\{U_zU_y^\dagger\} - N_c\mathrm{Tr}\{U_xU_y^\dagger\}] \\
& \times\Big\{\frac{(x-y)^2}{X^2Y^2}\Big[1+\frac{\alpha_sN_c}{12\pi}(1-\pi^2)\Big] - \frac{\alpha_sN_c}{2\pi}\frac{(x-y)^2}{X^2Y^2}\ln\frac{X^2}{(x-y)^2}\ln\frac{Y^2}{(x-y)^2}\Big\} \\
& + \frac{\alpha_s}{16\pi^2}\int d^2z'\;[\mathrm{Tr}\{U_xU_z^\dagger\}\mathrm{Tr}\{U_zU_{z'}^\dagger\}\{U_{z'}U_y^\dagger\} - \mathrm{Tr}\{U_xU_z^\dagger U_{z'}U_y^\dagger U_zU_{z'}^\dagger\} \\
& \qquad - \mathrm{Tr}\{U_xU_{z'}^\dagger U_zU_y^\dagger U_{z'}U_z^\dagger\} - (z'\to z)] \\
& \times\Big[\frac{(x-y)^4}{X^2Y'^2(X^2Y'^2-X'^2Y^2)} + \frac{(x-y)^2}{(z-z')^2X^2Y'^2}\Big]\ln\frac{X^2Y'^2}{X'^2Y^2}\Big) + \frac{\alpha_s^2N_c^2}{4\pi^2}\zeta(3)\mathrm{Tr}\{U_xU_y^\dagger\} =
\end{aligned}
$$

Our result + Extra term $\Rightarrow$ Agrees with NLO BFKL in $\mathcal{N}=4$
Lipatov and Kotikov, 2004

(Comparing the eigenvalue of the forward kernel)

Conclusions

- The NLO kernel for the evolution of the color dipole has been calculated. It consists of three parts: the running-coupling part proportional to β-function, the conformal part describing 1 $\rightarrow$ 3 dipoles transition and the non-conformal term.
- The result agrees with the forward NLO BFKL kernel up to a term proportional $\alpha_s^2\zeta(3)$ times the original dipole.
- For the creation of dipoles in the small-x evolution, the coupling constant is determined by the size of the smallest dipole.
- The NLO evolution kernel depends on the precise definition of the cutoff in the longitudinal momenta.
- With $|\alpha| < \sigma$ cutoff, the NLO-BK and the NLO-BFKL for $\mathcal{N} = 4$ is not conformally invariant in the transverse plane.

Back up Slides 1: Gluon loop in the shock wave

From NLO BFKL kernel

$$s\frac{d}{ds}\langle\hat{\mathcal{U}}(n,\gamma)\rangle = \frac{\alpha_s N_c}{\pi}\Big\{\Big[1-\frac{b\alpha_s}{4\pi}\frac{d}{d\gamma}+\Big(\frac{67}{9}-\frac{\pi^2}{3}\Big)N_c-\frac{10}{9}\frac{n_f}{N_c^2}\Big]\chi(n,\gamma)$$
$$+\frac{\alpha_s b}{4\pi}\Big[\frac{1}{2}\chi^2(n,\gamma)-\frac{1}{2}\chi'(n,\gamma)-\frac{2\gamma}{\gamma^2-\frac{n^2}{4}}\chi(\gamma)\Big]$$
$$+\frac{\alpha_s N_c}{4\pi}\Big[-\chi"(n,\gamma)-2\chi(n,\gamma)\chi'(n,\gamma)+6\zeta(3)+F(n,\gamma)-2\Phi(n,\gamma)-2\Phi(n,1-\gamma)\Big]\Big\}\langle\hat{\mathcal{U}}(n,\gamma)\rangle$$

Back up Slides 1: Gluon loop in the shock wave

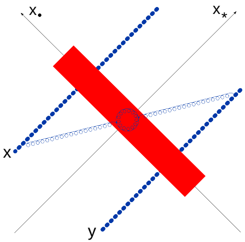
From NLO BK kernl

$$
s\frac{d}{ds}\langle\hat{\mathcal{U}}(n,\gamma)\rangle = \frac{\alpha_s N_c}{\pi}\Big\{\Big[1-\frac{b\alpha_s}{4\pi}\frac{d}{d\gamma}+\Big(\frac{67}{9}-\frac{\pi^2}{3}\Big)N_c-\frac{10}{9}\frac{n_f}{N_c^2}\Big]\chi(n,\gamma)
+\frac{\alpha_s b}{4\pi}\Big[\frac{1}{2}\chi^2(n,\gamma)-\frac{1}{2}\chi'(n,\gamma)-\frac{2\gamma}{\gamma^2-\frac{n^2}{4}}\chi(\gamma)\Big]
+\frac{\alpha_s N_c}{4\pi}\Big[-\chi"(n,\gamma)-2\chi(n,\gamma)\chi'(n,\gamma)+4\zeta(3)+F(n,\gamma)-2\Phi(n,\gamma)-2\Phi(n,1-\gamma)\Big]\Big\}\langle\hat{\mathcal{U}}(n,\gamma)\rangle
$$

Back up Slides 1: Gluon loop in the shock wave

From NLO BK kernl

$$s\frac{d}{ds}\langle\hat{\mathcal{U}}(n,\gamma)\rangle = \frac{\alpha_s N_c}{\pi}\Big\{\Big[1-\frac{b\alpha_s}{4\pi}\frac{d}{d\gamma}+\Big(\frac{67}{9}-\frac{\pi^2}{3}\Big)N_c-\frac{10}{9}\frac{n_f}{N_c^2}\Big]\chi(n,\gamma)$$
$$+\frac{\alpha_s b}{4\pi}\Big[\frac{1}{2}\chi^2(n,\gamma)-\frac{1}{2}\chi'(n,\gamma)-\frac{2\gamma}{\gamma^2-\frac{n^2}{4}}\chi(\gamma)\Big]$$
$$+\frac{\alpha_s N_c}{4\pi}\Big[-\chi''(n,\gamma)-2\chi(n,\gamma)\chi'(n,\gamma)+4\zeta(3)+F(n,\gamma)-2\Phi(n,\gamma)-2\Phi(n,1-\gamma)\Big]\Big\}\langle\hat{\mathcal{U}}(n,\gamma)\rangle$$



It should be emphasized that the coincidence of terms with the nontrivial γ dependence proves that there is no additional $O(\alpha_s)$ correction to the vertex of the gluon - shock wave interaction coming from the small loop inside the shock wave

The coefficient $6\zeta(3)$ agrees with the $j \to 1$ asymptotics of the three-loop anomalous dimensions of leading-twist gluon operators

Are you a wolf?
No I'm not...Yes, of cource I am...

.. And I like Falcao.

Ta tu ti ta tu ti ta tu ti ta tu tu