Scaling laws for saturation at running coupling

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Outline

- Outline
- Introduction
- New scaling solution
- Scalings in DIS data

- Introduction: The BK equation and geometric scaling

- A second asymptotic scaling solution for high energy QCD saturation with running coupling

  G. B., arXiv:0803.2167

- Phenomenological scaling properties in DIS

  G. B., R. Peschanski, C. Royon, D. Šálek, arXiv:0803.2186

Dipole amplitude and BK equation

Introduction
- BK equation
- Universality
- Traveling wave
- RC solution?

New scaling solution

Scalings in DIS data

Dipole factorization of the DIS:

$$\sigma^{\gamma^* p}(Y, Q^2) = |\psi(Q^2, \boldsymbol{r})|^2 \otimes T(\boldsymbol{r}, Y)$$

Fourier transform: $\boldsymbol{r} \mapsto \boldsymbol{k}$

$$T(\boldsymbol{r}, Y) \mapsto N(L, Y) \text{ , with } L \equiv \log(\boldsymbol{k}^2/\Lambda^2_{QCD})$$

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$T(\boldsymbol{r}, Y) \mapsto N(L, Y)$, with $L \equiv \log(\boldsymbol{k}^2/\Lambda^2_{QCD})$

Balitsky-Kovchegov equation:

$$\partial_Y N(L, Y) = \bar{\alpha}\chi(-\partial_L)N(L, Y) - \bar{\alpha}N(L, Y)^2$$

Dipole amplitude and BK equation

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$$\partial_Y N(L, Y) = \bar{\alpha}\chi(-\partial_L)N(L, Y) - \bar{\alpha}N(L, Y)^2$$

Running coupling prescription: $\bar{\alpha} \mapsto \bar{\alpha}(L) = 1/bL$

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Balitsky-Kovchegov equation with running coupling:

$$bL\,\partial_Y N(L,Y) = \chi(-\partial_L)N(L,Y) - N(L,Y)^2$$

Universality

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Solution of the BFKL equation (or linearized BK):

$$N(L,Y) = \int \frac{d\gamma}{2\pi i}\, e^{-(\gamma L - \chi(\gamma)\bar{\alpha} Y)}\, N_0(\gamma)$$

Sum of scaling solutions with different parameters: no scaling in general.

For BK, at Y large enough, the existence of the nonlinear damping selects dynamically the wave solution with $\gamma = \gamma_c$ (defined by $\chi(\gamma_c) = \gamma_c\, \chi'(\gamma_c)$):

$$N(L,Y) \propto e^{-(\gamma_c L - \bar{\alpha}\chi(\gamma_c) Y)}$$

$\Rightarrow$ Geometric scaling.

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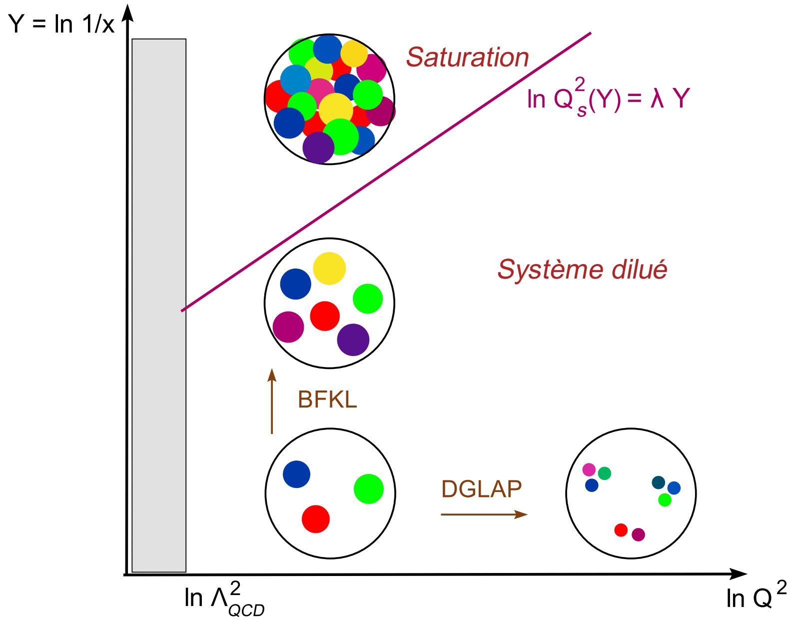

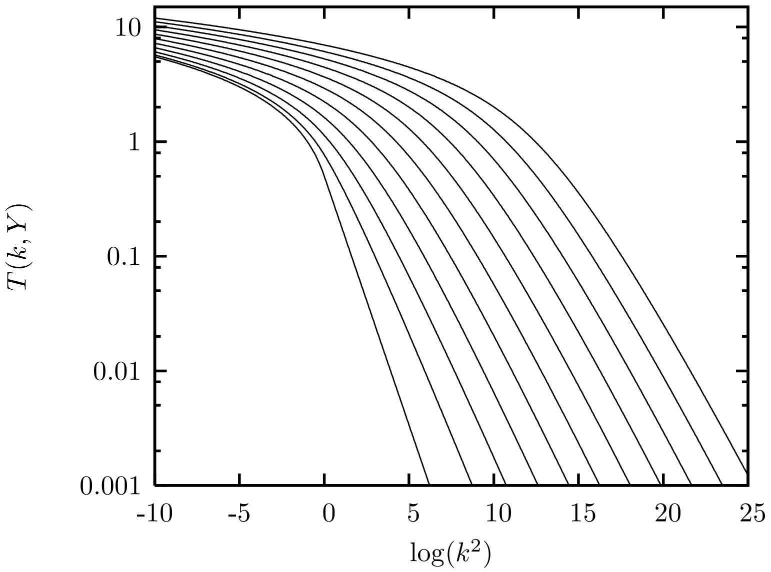

Geometric scaling $\leftrightarrow$ traveling wave solution of the BK (or B-JIMWLK) equation at fixed coupling.

Running coupling solution?

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Within some approximations, a large Y and large L solution of the BK equation with running coupling has been found:

$$N(L,Y) \propto e^{-\gamma_c \bar{s}}\ \mathrm{Ai}\left(\xi_1 + \frac{\bar{s}}{D_g Y^{1/6}}\right)$$

$$\bar{s} = L - v_g\sqrt{\frac{Y}{b}} - \frac{3\xi_1}{4} D_g Y^{1/6}$$

Mueller, Triantafyllopoulos (2002)
Munier, Peschanski (2004)

To what extent is the traveling wave formation mechanism different at running coupling from the one at fixed coupling?

Scaling laws

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- New TW solution
- Shape of the front
- Saturation scale

Scalings in DIS data

$$bL\,\partial_Y N(L,Y) = \chi(-\partial_L)N(L,Y) - N(L,Y)^2$$

Exact scaling solution: $N(L,Y) \equiv N_s(s(L,Y))$

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Sufficient conditions:

$$\begin{aligned} bL\,\partial_Y s(L,Y) &= f_1(s) \\ \partial_L s(L,Y) &= f_2(s) \end{aligned}$$

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$\rightarrow$ Incompatible conditions, as they imply

$$\partial_L\partial_Y s(L,Y) \neq \partial_Y\partial_L s(L,Y)\,.$$

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$$bL\, \partial_Y N(L,Y) = \chi(-\partial_L) N(L,Y) - N(L,Y)^2$$

Approximate scaling solution: $N(L,Y) \simeq N_s(s(L,Y))$

First choice:

$$\begin{aligned} bL\, \partial_Y s(L,Y) &\simeq f_1(s) \\ \partial_L s(L,Y) &= f_2(s) \end{aligned}$$

$\longrightarrow$ RC geometric scaling: $s(L,Y) = L - \sqrt{v\, \frac{Y-Y_0}{b}}$

Valid scaling law when $L \gg 1$ and $|s(L,Y)| \ll L$.

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$$bL\,\partial_Y N(L,Y) = \chi(-\partial_L)N(L,Y) - N(L,Y)^2$$

Approximate scaling solution: $N(L,Y)\simeq N_s(s(L,Y))$

Second choice:

$$\begin{aligned} bL\,\partial_Y s(L,Y) &= f_1(s) \\ \partial_L s(L,Y) &\simeq f_2(s) \end{aligned}$$

$\longrightarrow$ New RC scaling: $s(L,Y) = \frac{L}{2} - v\,\frac{Y-Y_0}{2bL}$

Valid scaling law when $L \gg 1$ and $|s(L,Y)| \ll L$.

New traveling wave solution

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Approximate solution in the range $L \gg 1$ and $1 \lesssim \bar{s} \ll \sqrt{L}$:

$$N(L,Y) \propto e^{-\gamma_c \bar{s} + \mathcal{O}(\log L)} \left[\text{Ai}\left(\xi_1 + \frac{\bar{s}}{(DL)^{1/3}}\right) + \mathcal{O}\left(L^{-1/3}\right)\right]$$

$$\bar{s} = \frac{L}{2} - \frac{v_c Y}{2bL} - \frac{3\xi_1}{4}(DL)^{1/3}$$

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Leading behavior: new RC scaling law.

Saturation critical exponent: γ_c, solution of $\chi(\gamma_c) = \gamma_c \, \chi'(\gamma_c)$.

Critical velocity: $v_c = \frac{2\chi(\gamma_c)}{\gamma_c}$.

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Universal scaling violations:

- Partly absorbed by a redefinition of the scaling law.
- Scaling violations from BFKL diffusion remains.

Shape of the front

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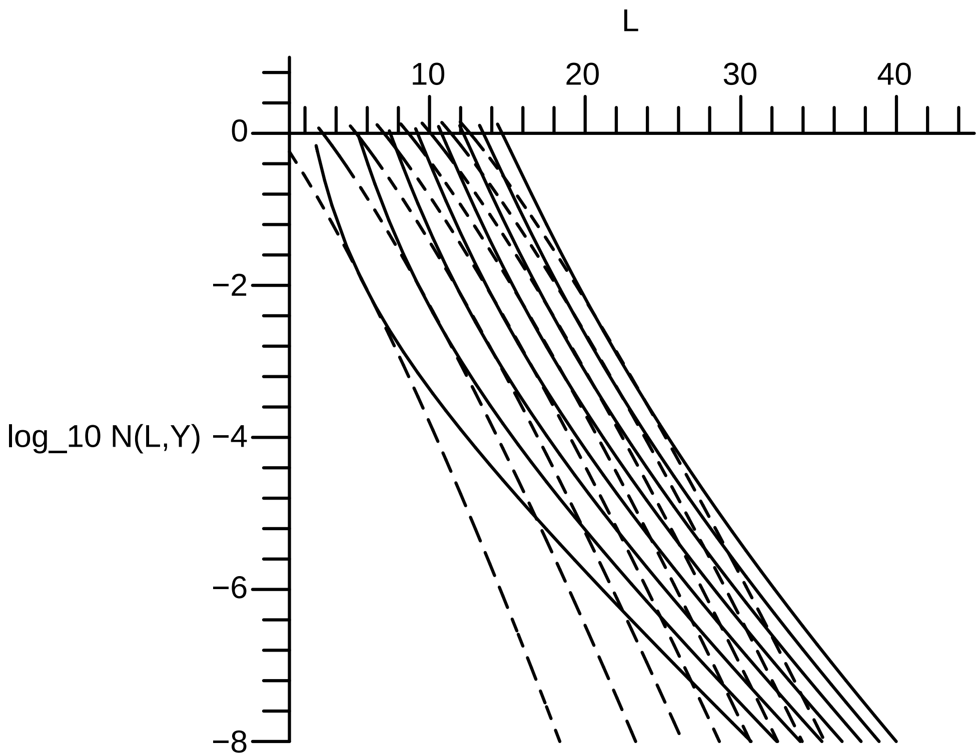
Scalings in DIS data



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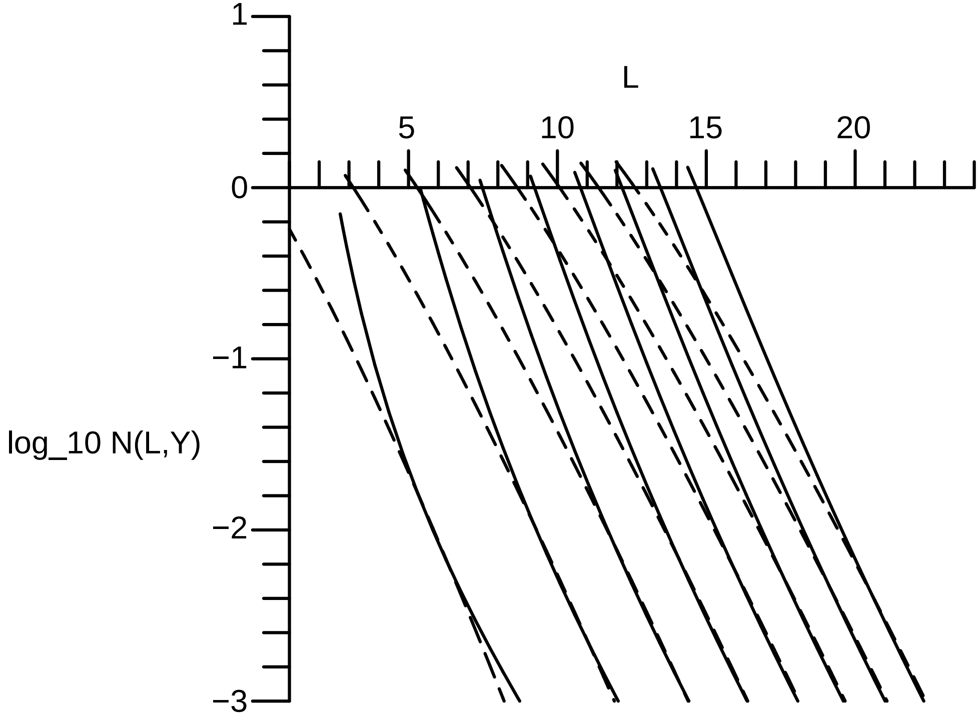

Saturation scale

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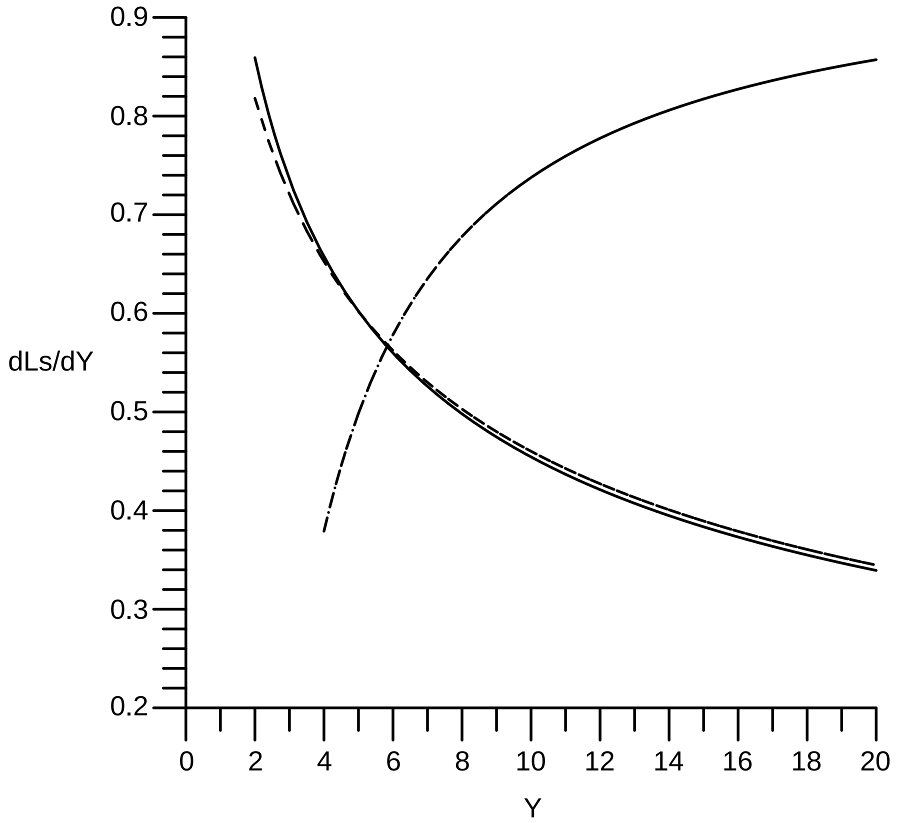

$$\frac{\mathrm{d}\ L_s(Y)}{\mathrm{d}Y} = \frac{\mathrm{d}}{\mathrm{d}Y}\ \log\left(\frac{Q_s^2(Y)}{\Lambda_{QCD}^2}\right)$$

QF method

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Quality factor (QF) method: model-independant test of a scaling law, in particular independant of the scaling function.

Gelis, Peschanski, Soyez, Schoeffel (2006)

- Data points i: $D_i \equiv \log(\sigma^{\gamma^* p}(Q_i^2, Y_i))$
- Scaling variable to be tested: $\tau_i \equiv \tau(Q_i^2, Y_i, \{\lambda, \dots\})$

$$QF(\{\lambda, \dots\}) \equiv \left[\sum_i \frac{(D_i - D_{i-1})^2}{(\tau_i - \tau_{i-1})^2 + \epsilon^2}\right]^{-1}$$

$\rightarrow$ Optimal values of the parameters $\{\lambda, \dots\}$.

Scaling of the DIS cross section

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- Any geometric scaling of the dipole amplitude $N(L, Y) = N_g(\log(k^2/Q_s^2(Y)))$ crosses automatically the dipole factorization, with only the replacement $k^2 \to Q^2$, to give

$$\sigma^{\gamma^* p}(Y, Q^2) = \sigma(\tau(Y, Q^2)),$$

with $\tau(Y, Q^2) = \log(Q^2/\Lambda^2) - \lambda Y$ (FC),

or $\tau(Y, Q^2) = \log(Q^2/\Lambda^2) - \lambda\sqrt{Y - Y_0}$ (RC1).

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or $\tau(Y, Q^2) = \log(Q^2/\Lambda^2) - \lambda\sqrt{Y - Y_0}$ (RC1).

- Property only approximate for the new running coupling scaling, and also for the diffusive scaling. Let us test nevertheless the scalings $\tau(Y, Q^2) = \log(Q^2/\Lambda^2) - \lambda \frac{Y - Y_0}{\log(Q^2/\Lambda^2)}$ (RC2),

or $\tau(Y, Q^2) = [\log(Q^2/\Lambda^2) - \lambda(Y - Y_0)]/\sqrt{(Y - Y_0)}$ (DS).

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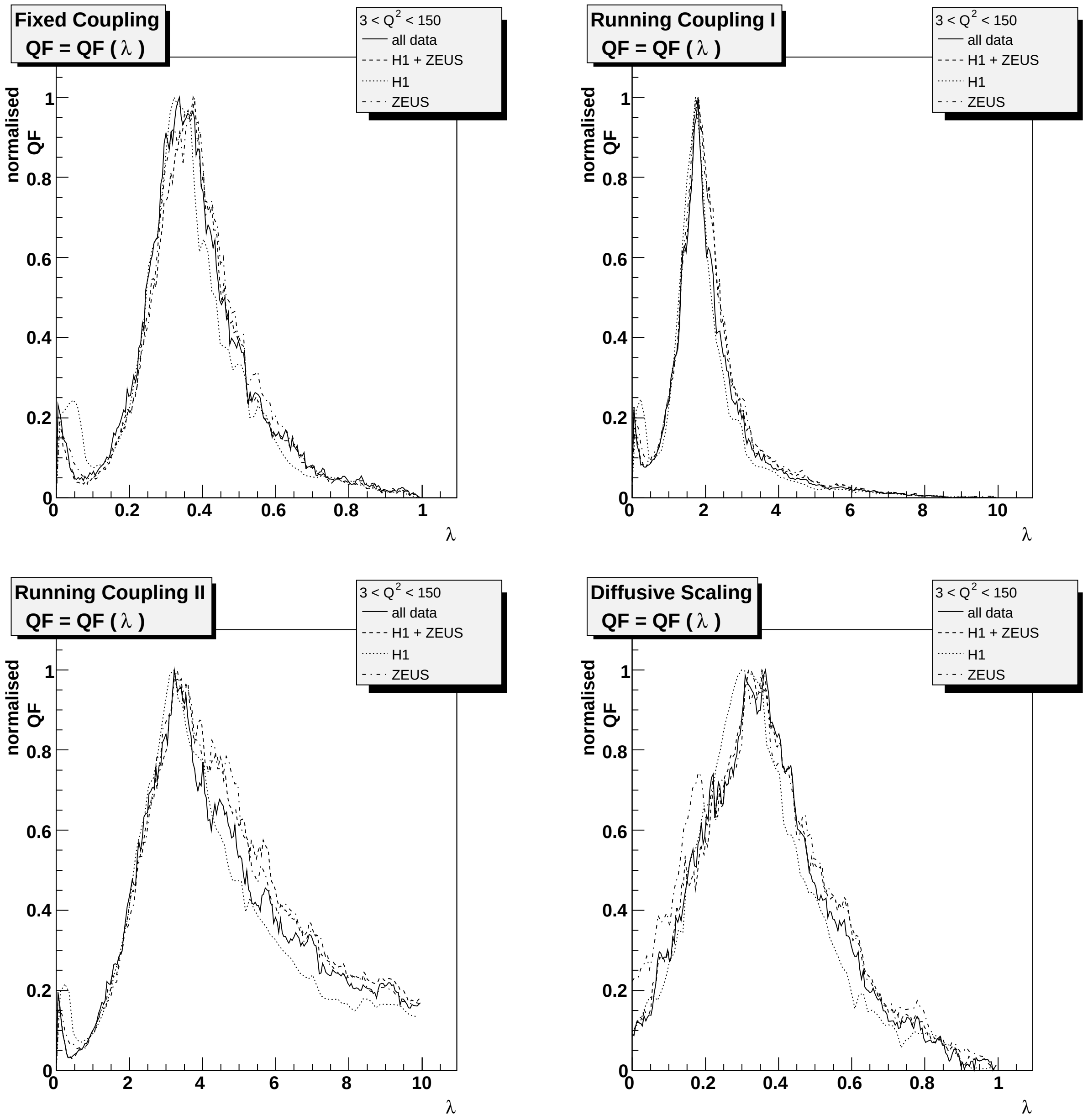
Inclusive DIS data used: from H1, ZEUS, NMC, E665,

in the range $x < 0.01$ and 3 GeV$^2 < Q^2 < 150$ GeV^2.

For RC1, RC2 and DS scalings, Y_0 either set to 0, or included as a parameter in the QF fit.

For RC2 and DS, Λ either fixed (Λ_{QCD} for RC2 and 1 GeV for DS), or included as a parameter in the QF fit.

Scaling of the DIS cross section



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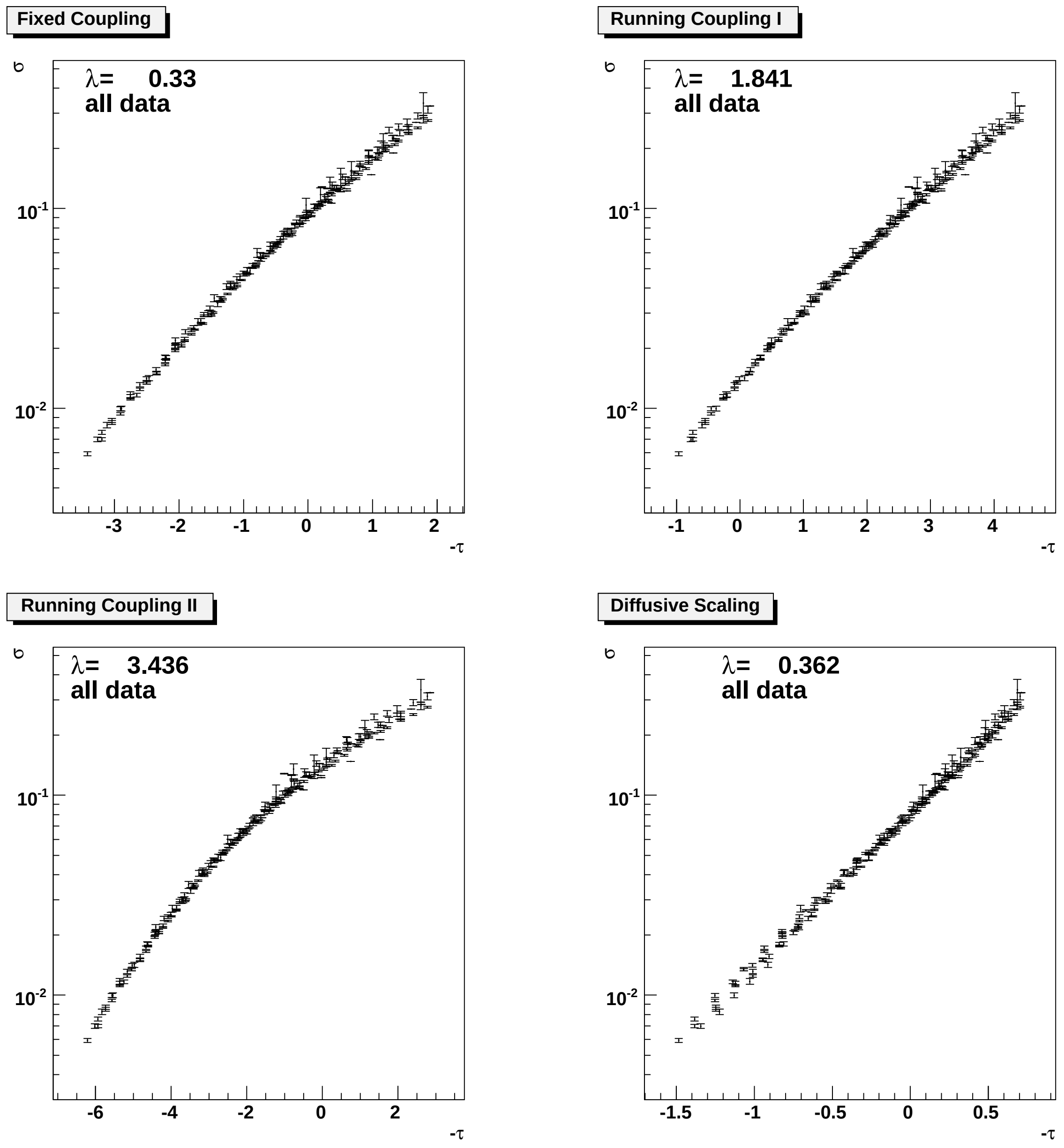

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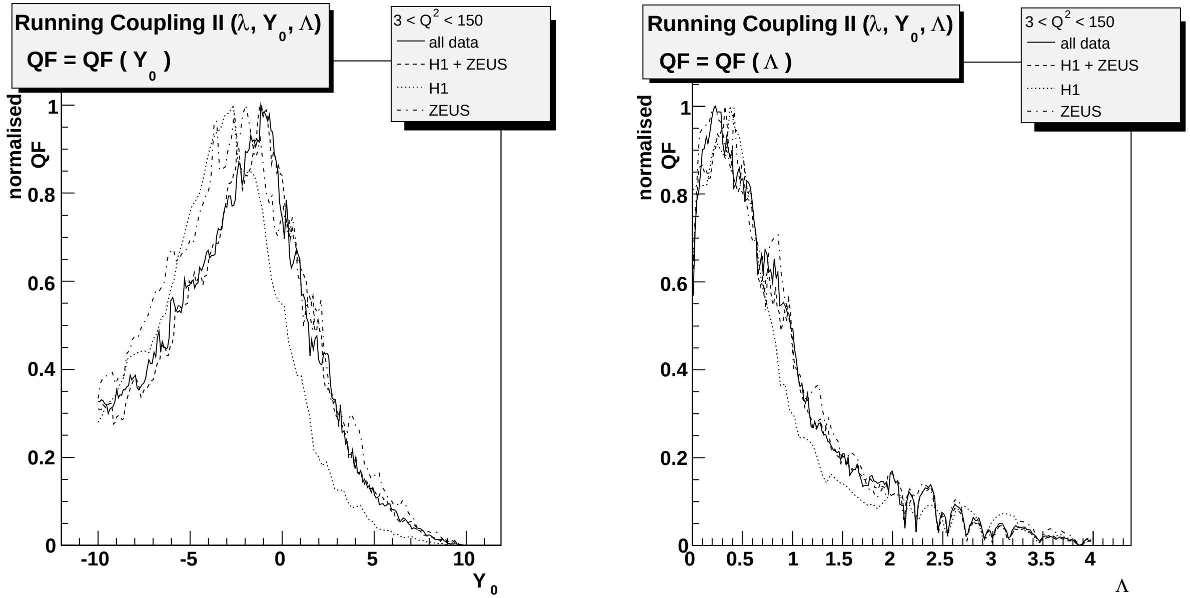

FC	RC I	RC II	RC II bis	DS
λ=0.330	λ=1.841	λ=3.436	λ=3.905	λ=0.362
			Y_0=-1.200	
			$\Lambda = 0.300$	
QF=1.63	QF=1.62	QF=1.69	QF=1.82	QF=1.44

Other observables

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Other observables admit a dipole factorization, and show (at fixed coupling) geometric scaling behavior.

Marquet, Schoeffel (2006)

- Deeply Virtual Compton Scattering
- Exclusive vector meson production
- Diffractive DIS

Let us test all the scaling variables with the QF method on these processes.

Scaling of the DVCS

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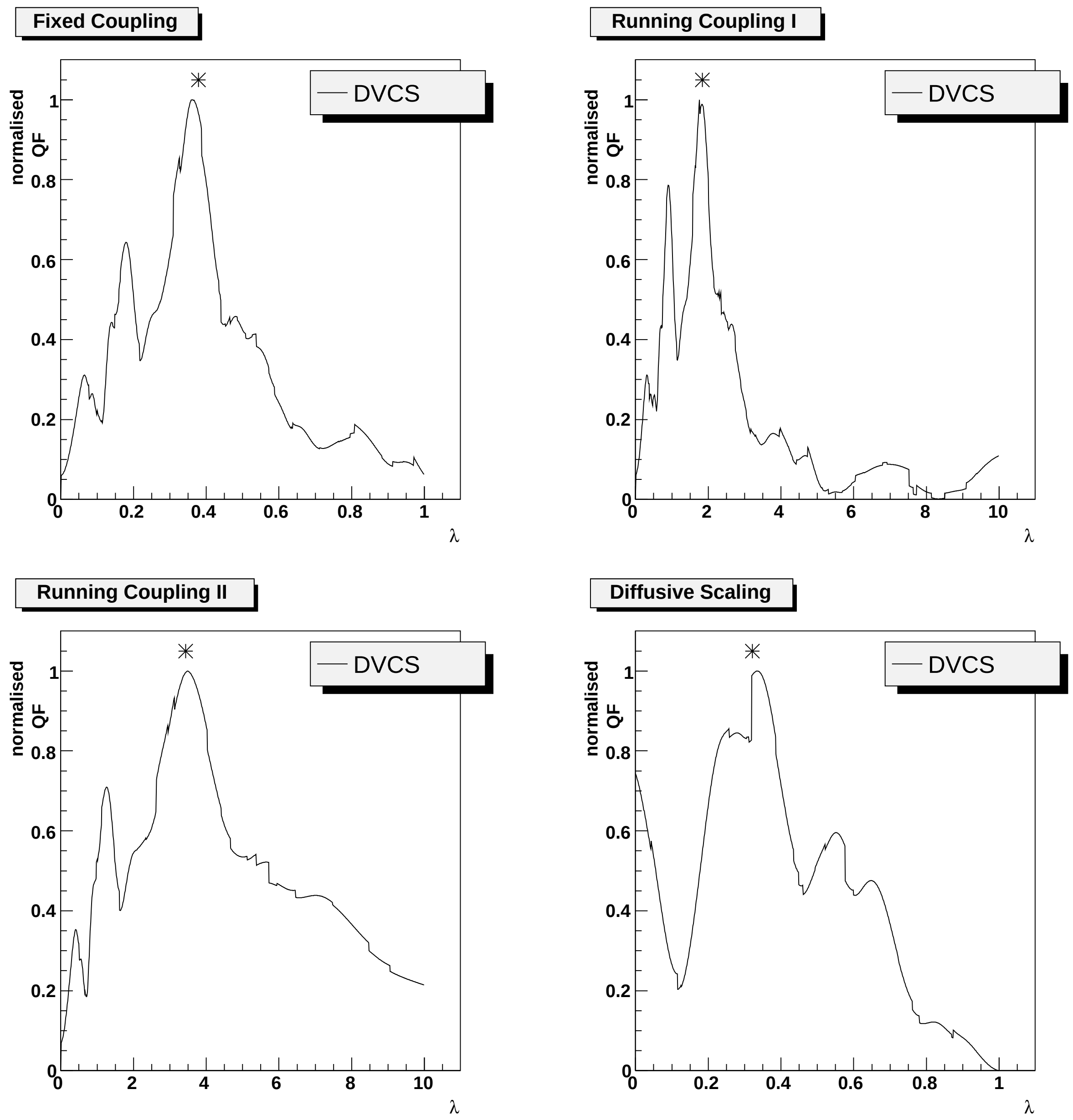

Scaling of the DVCS

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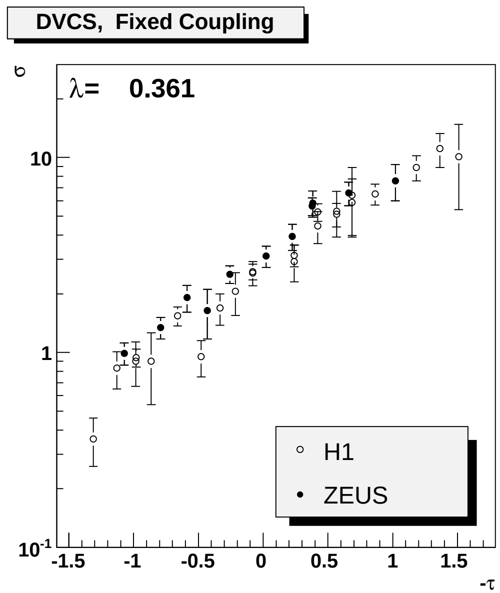

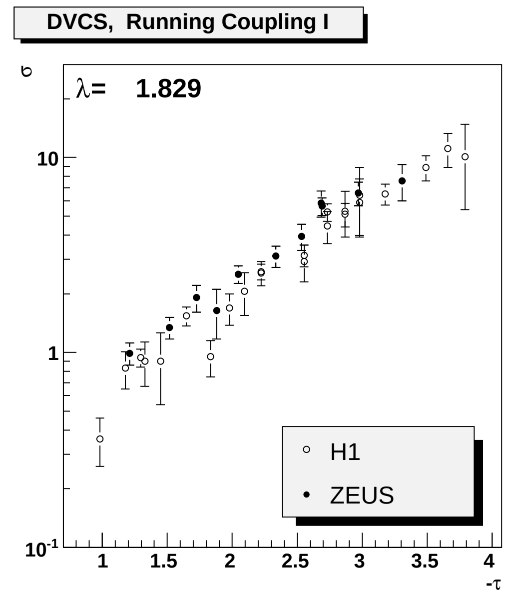

FC	RC I	RC II	RC II bis	DS
λ=0.361	λ=1.829	λ=3.481	λ=5.717	λ=0.335
			Y_0=-1.89	
			$\Lambda = 0.01$	
QF=3.75	QF=3.62	QF=3.24	QF=3.52	QF=3.38

Exclusive vector meson production

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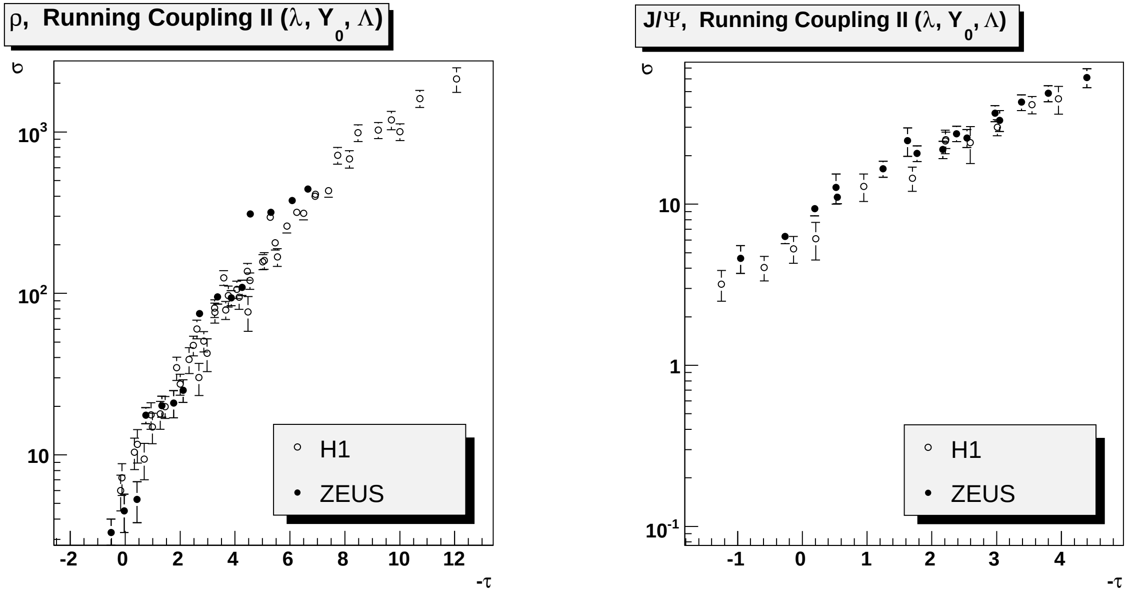

Non-perturbative assumption for the γ^* - vector meson wave functions overlap: hard scale taken to be $Q^2 + M_V^2$.

The fit of the QF gives different optimal values for the λ.
$\longrightarrow$ The assumption seems disfavored.

However: reasonable scaling behaviors with the λ obtained with the previous observables.

Diffraction at fixed β

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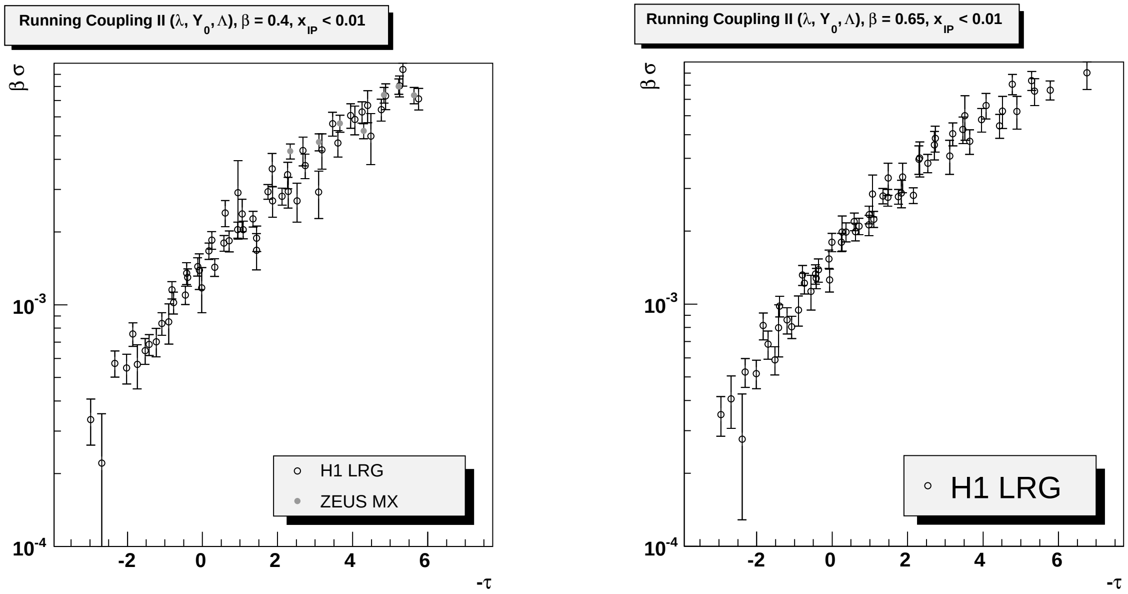

We test the scaling:

$$\frac{d\sigma^{\gamma^* p \to Xp}}{d\beta}(\beta, x_{pom}, Q^2) = \frac{d\sigma^{\gamma^* p \to Xp}}{d\beta}(\beta, \tau[\log 1/x_{pom}, Q^2])$$

The fit of the QF gives different optimal values for the λ.
However: reasonable scaling behaviors with the λ obtained with the previous observables.

Diffraction at fixed x_{pom}

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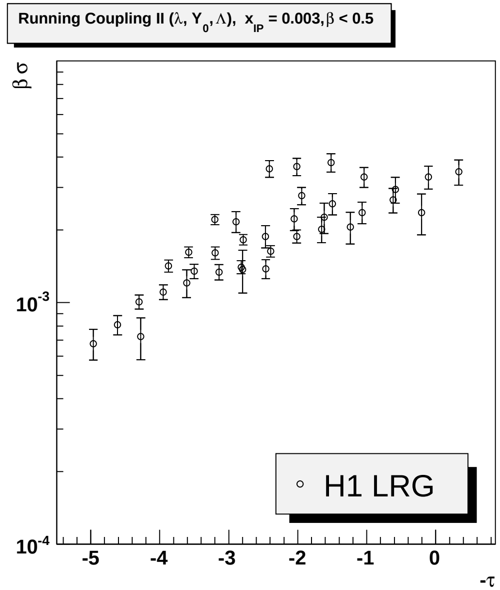

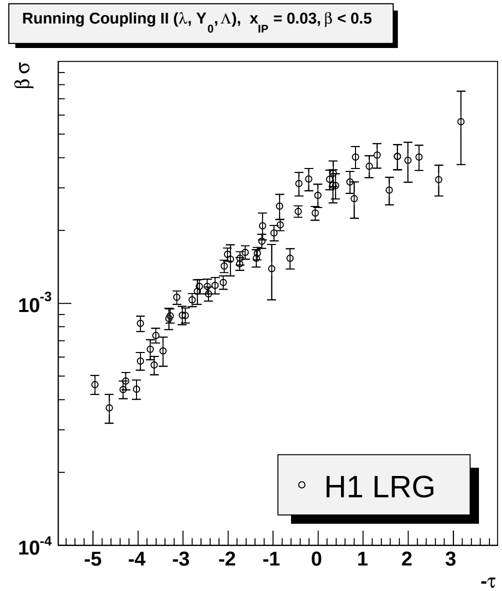

We test the scaling:

$$\frac{d\sigma^{\gamma^* p\to Xp}}{d\beta}(\beta, x_{pom}, Q^2) = \frac{d\sigma^{\gamma^* p\to Xp}}{d\beta}(x_{pom}, \tau[\log 1/\beta, Q^2])$$

Problem: no data for $\beta < 0.01$. Hence, we include all data with $\beta < 0.5$. $\Rightarrow$ Rough scaling for the λs fitted in inclusive DIS, for the higher values of x_{pom}.

Summary

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Conclusion

The solution of the BK equation with running coupling has *no* exact scaling, but can be approached simultaneously by two asymptotic expansions, featuring two *different* scalings. Each of them give the same saturation scale.

- The data for various DIS observables shows the fixed coupling and the two running coupling scalings.
- Inclusive DIS favors the RC2 scaling.
- Inclusive DIS and DVCS shows good scaling properties, with consistent parameters.
- The scaling properties of VM production and diffraction, relying on additional assumption, are not so good and less consistent.