Quarkonia propagation in a hot-dense medium

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Summary

- Quarkonium as a probe for the onset of deconfinement;
- Present understanding from lattice-QCD calculations and experimental data;
- In-medium $Q\overline{Q}$ propagator in the complex-time plane;
- Basic questions we want to answer;
- An explicit example: $Q\overline{Q}$ in a hot QED plasma;
- Some ideas for future work.

Quarkonium as a probe for the onset of deconfinement

Original idea by Matsui and Satz [a]

⇒Statement: the J/Ψ *anomalous suppression* in high energy AA collisions represents an unambiguous signature of deconfinement.

⇒Underlying assumptions:

•The J/Ψ are produced in the very early stage of the collision $\tau_{\rm form} \approx 0.3{\rm fm/c}$;

•The medium resulting from the HIC thermalizes in a time $\tau_{\rm therm} \approx 0.5 \div 1{\rm fm/c}$;

•Crossing a deconfined medium the $c\bar{c}$ bound states tend to melt (Debye screening):

$$V(r) \sim -\frac{\alpha}{r} \rightarrow -\frac{\alpha}{r} e^{-m_D r}$$

•The heavy quarks hadronize by combining with light quarks only.

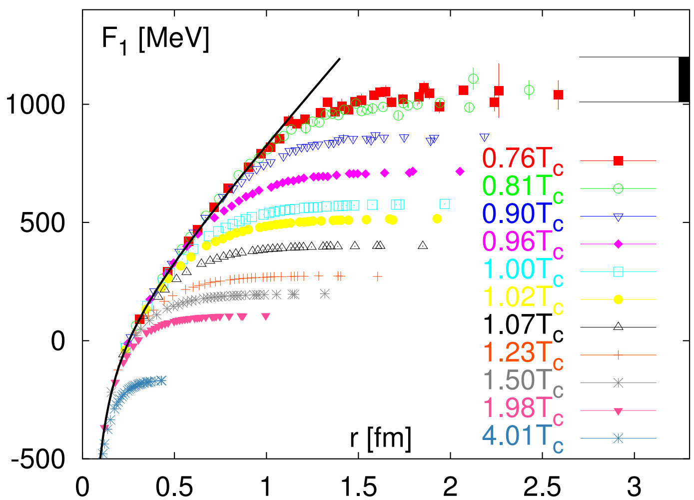
[a] T. Matsui and H. Satz, PLB 178 (1986).

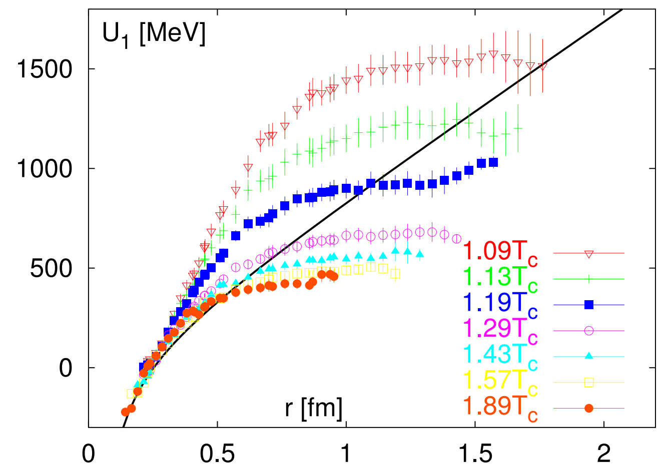
Quarkonium in hot-QCD: what can lattice simulations tell us?

- **Heavy-quark free-energy** calculations:
  evaluate ΔF *occurring once a* $Q\overline{Q}$ *pair is placed in a thermal bath of gluons and light quarks*;
- **Meson Spectral Function** reconstruction:
  look for resonance-peaks in the spectral densities extracted from in-medium quarkonium propagators.

$Q\overline{Q}$ free-energy [a]

$$e^{-\beta \Delta F_{Q\overline{Q}}(\boldsymbol{x}-\boldsymbol{y},T)} \sim \langle \chi(\beta,\boldsymbol{y})\psi(\beta,\boldsymbol{x})\psi^\dagger(0,\boldsymbol{x})\chi^\dagger(0,\boldsymbol{y})\rangle$$





Can one exploit this information to build an effective $Q\overline{Q}$ potential?

state	J/ψ	χ_c	ψ'
$T_d/T_c\,(V_{\rm eff} \equiv F_1)$	1.1	0.74	0.1-0.2
$T_d/T_c\,(V_{\rm eff} \equiv U_1)$	1.78-1.92	1.14-1.15	1.11-1.12

[a] O. Kaczmarek and F. Zantow, PoS LAT2005:192 (2006).

Meson Spectral Functions

⇒One usually *measures* the imaginary-time propagator of a meson

•at rest in the QGP

$$G_M(\tau) \equiv \int d\boldsymbol{x} \langle J_M(\tau, \boldsymbol{x}) J_M^\dagger(0, \mathbf{0}) \rangle$$

•produced by a local operator

$$J_M(\tau, \boldsymbol{x}) \equiv \overline{q}(\tau, \boldsymbol{x}) \Gamma_M q(\tau, \boldsymbol{x})$$

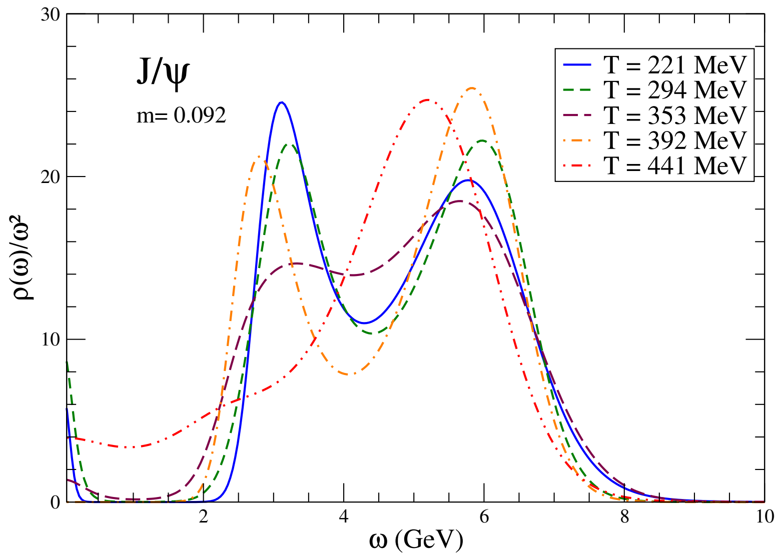
⇒From $G_M(\tau)$ the MSF has to be *reconstructed*:

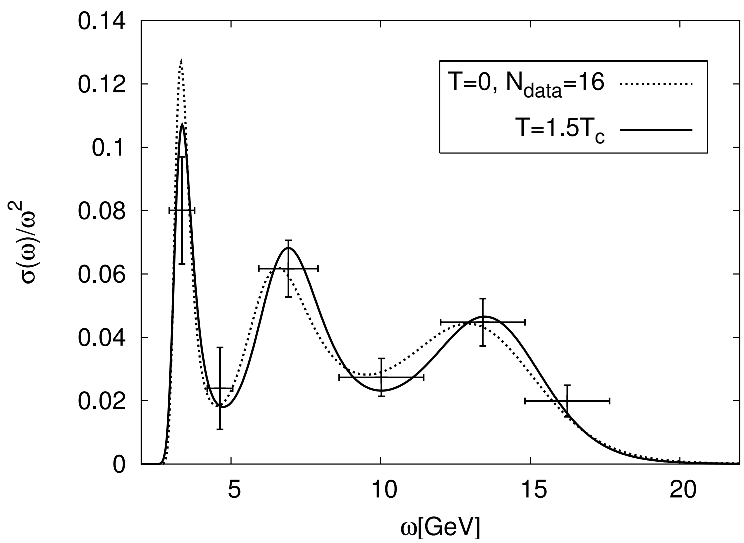
$$G_M(\tau) = \int_0^\infty d\omega \underbrace{\sigma_M(\omega)}_{MSF} \frac{\cosh(\omega(\tau - \beta/2))}{\sinh(\beta\omega/2)}$$

NB: Typically $G_M(\tau)$ is known for a quite limited set of points ($\lesssim$50)

→ *problems in inverting the above transform*.

⇒What is found[a,b]?





The vector (left) *and pseudoscalar* (right) *MSFs display well-defined peaks up to temperature* $T \sim 2T_c$.

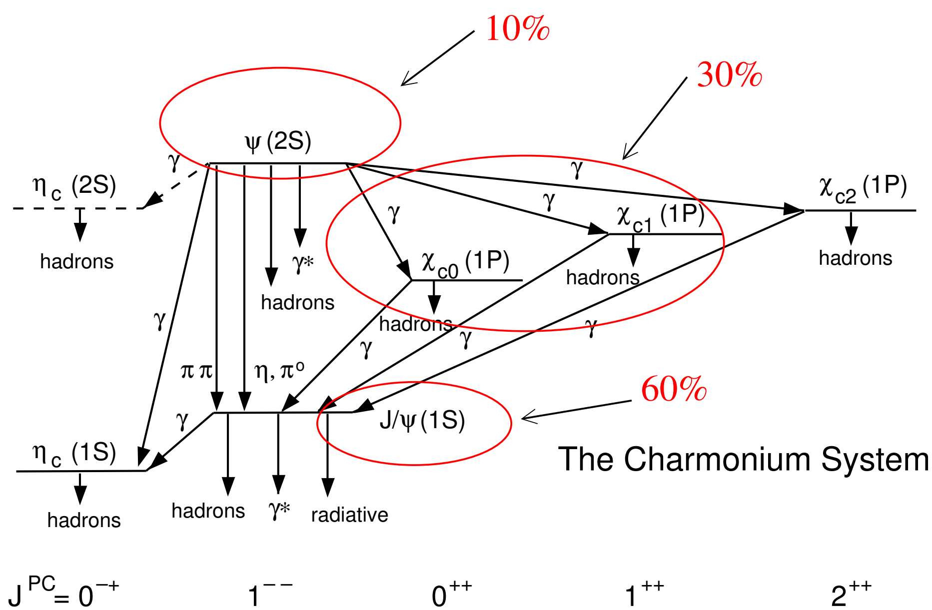
[a] G. Aarts *et al.*, arXiv:0705.2198 [hep-lat]

[b] A. Jakovac *et al.*, Phys.Rev. D75 (2007) 014506.

Experimental data

Charmonia production

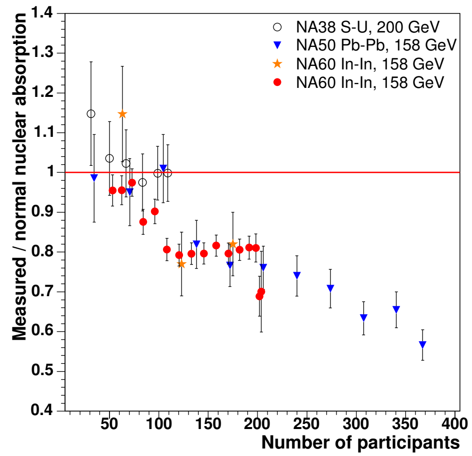
... in hadronic collisions



... in AA collisions: sequential suppression scenario

As the centrality of the collision increases one has first the suppression of the *feed-down* contribution (Ψ' and χ_c). Then also the melting of the direct J/Ψ sets in.

⇒... at SPS (Na50 and Na60) [a]

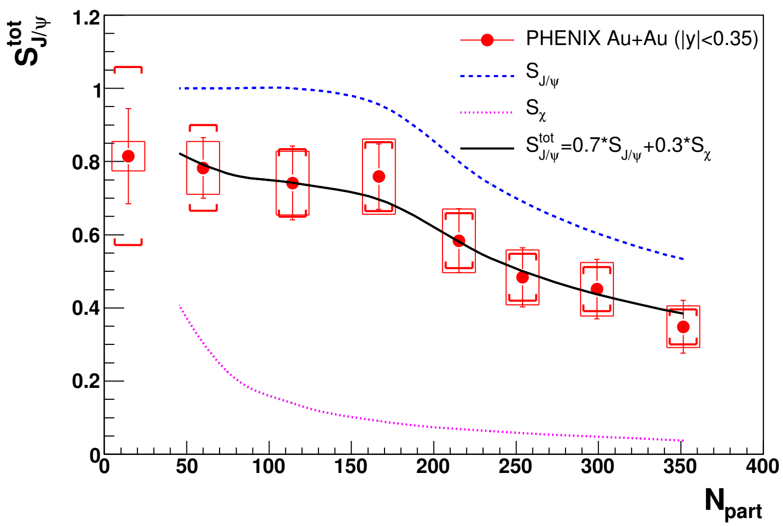


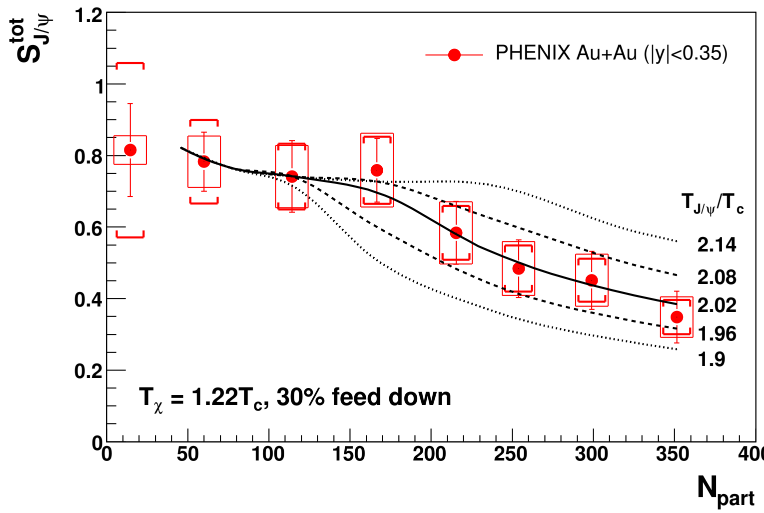
The direct contribution seems to survive!

[a] R. Arnaldi, NPA774:711-714,2006.

⇒... at RHIC (PHENIX)

From sequential suppression + hydro evolution one gets:





"It is noticeable that the RHIC data analyzed with the state-of-the-art hydrodynamics leads to a rather stable value for the melting temperature of the J/Ψ to be around $T/T_c \simeq 2$." [a]

[a] T. Gunji *et al.*, Phys. Rev. C **76**, 051901 (2007).

Some open problems: a brief summary

- Potential models: which effective potential from the $Q\overline{Q}$ free-energy data?
- MSF: in principle would contain the full information on the in-medium quarkonium properties, BUT large uncertainties from inverting the transform.
- Is it possible to establish a link between screened potential models and spectral studies?

The basic object of our study

$$G^{>}(t,\boldsymbol{r}_1;t,\boldsymbol{r}_2|0,\boldsymbol{r}_1';0,\boldsymbol{r}_2') \equiv \langle \underbrace{\chi(t,\boldsymbol{r}_2)\psi(t,\boldsymbol{r}_1)}_{J_M(t)} \underbrace{\psi^\dagger(0,\boldsymbol{r}_1')\chi^\dagger(0,\boldsymbol{r}_2')}_{J_M^\dagger(0)} \rangle$$

- Spectral decomposition:

$$
\begin{aligned}
G_M^>(t) &= Z^{-1}\sum_n e^{-\beta E_n}\langle n|J_M(t)J_M^\dagger(0)|n\rangle \\
&= Z^{-1}\sum_n e^{-\beta E_n}\sum_m e^{i(E_n-E_m)t}|\langle m|J_M^\dagger(0)|n\rangle|^2,
\end{aligned}
$$

  - $G^>(t)$ is an **analytic function** in the strip $-\beta < \mathrm{Im}t < 0 \Longrightarrow$ *unified description of real and imaginary-time propagation*;
  - $Q\overline{Q}$ pair: *external probe* placed *in a hot/dense medium* of light particles $\Longrightarrow \{|n\rangle\}$ do not contain heavy quarks.

- One gets the *excitations* (poles of the retarded propagator) *which propagate in the medium*

$$
\rho_M(\omega) = G^>(\omega) \quad \Longrightarrow \quad G_M^R(\omega) = -\int \frac{dq^0}{2\pi}\frac{\rho_M(q^0)}{\omega - q^0 + i\eta}.
$$

A recent approach[a]

⇒Evaluate perturbatively

$$G^{>}_{M=\infty}(t) = G^{(0)>}(t) + G^{(2)>}(t) + \dots$$

⇒**Ansatz**: $G^{>}_{M=\infty}(t)$ is solution of

$$(i\partial_t - V_{eff})G^{>}_{M=\infty}(t) = 0$$

⇒Identify the LO perturbative contribution to the effective potential:

$$V_{eff} = V^{(2)}_{eff} + \dots$$

⇒Get $G^{>}(t)$ from the solution of

$$(i\partial_t - T - V^{(2)}_{eff})G^{>}(t) = 0$$

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[a] M. Laine *et al.*

The basic questions we want to answer:

- Does *$G^>(t)$ obey a closed Schrödinger equation* at finite T? i.e. is it possible to define an effective potential?
- What's the *link* of the effective potential *with the $Q\overline{Q}$ free-energy*?
- Is it possible to include the *effect of collisions* in a consistent way?

QED toy-model

A $Q\overline{Q}$ pair in a plasma of

photons, electrons and positrons

$$\mathcal{L}_{\rm QED}^{M=\infty} = \mathcal{L}_{\rm em} + \mathcal{L}_{\rm light} + \underbrace{\psi^\dagger i(\partial_0 - igA_0)\psi}_{\text{heavy } Q} + \underbrace{\chi^\dagger i(\partial_0 + igA_0)\chi}_{\text{heavy } \overline{Q}}$$

The strategy

- Consider the $Q\overline{Q}$ propagation in a given background configuration of the gauge-field A_μ

$$G_A(t,\boldsymbol{r}_1;t,\boldsymbol{r}_2|0,\boldsymbol{r}'_1;0,\boldsymbol{r}'_2)=\delta(\boldsymbol{r}_1-\boldsymbol{r}'_1)\delta(\boldsymbol{r}_2-\boldsymbol{r}'_2)\times$$
$$\times\exp\left(ig\int_0^t dt'A_0(\boldsymbol{r}_1,t')\right)\exp\left(-ig\int_0^t dt'A_0(\boldsymbol{r}_2,t')\right)$$

- Average over the gauge-field configuration with an action accounting for thermal effects

$$G^>(t,\boldsymbol{r}_1;t,\boldsymbol{r}_2|0,\boldsymbol{r}'_1;0,\boldsymbol{r}'_2)=Z^{-1}\int[\mathcal{D}A]G_A(t,\boldsymbol{r}_1;t,\boldsymbol{r}_2|0,\boldsymbol{r}'_1;0,\boldsymbol{r}'_2)\,e^{iS[A]}$$

Which is the action to employ to weight the field configurations?

The HTL effective action I

$\Rightarrow$Relevant momentum scales in a ultra-relativistic plasma:

- **Hard** (*plasma particles*):

$$E \sim T^4 \quad N \sim T^3 \quad \Longrightarrow \quad K \sim T;$$

- **Soft** (*collective modes*): $K \sim gT$.

$\Rightarrow$Mean Free Path of a plasma particle:

- For hard momentum exchange: $\lambda_{mfp}^{hard} \sim 1/g^4 T$,
- For soft momentum exchange: $\lambda_{mfp}^{soft} \sim 1/g^2 T$.

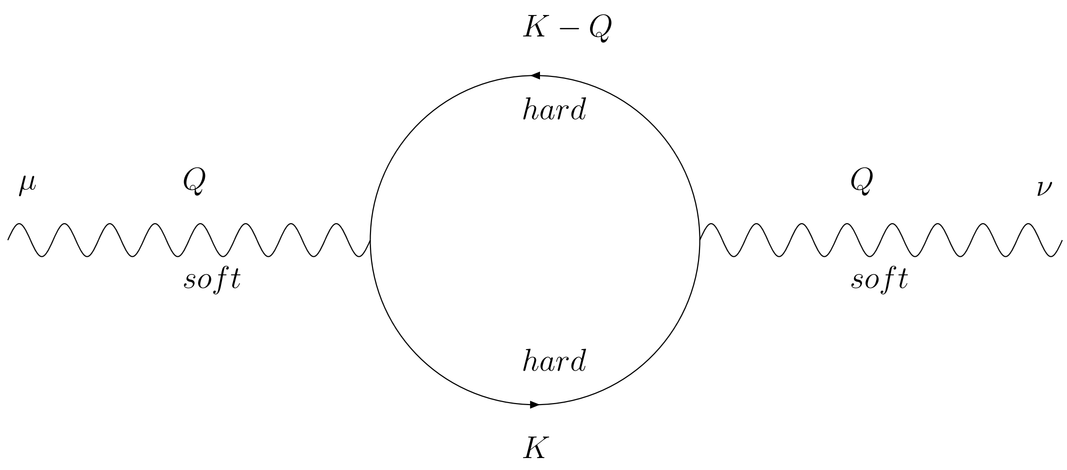
For weak coupling one has $\lambda_{mfp}^{soft} \ll \lambda_{mfp}^{hard}$, i.e.

most of the scattering processes involve small momentum transfer.

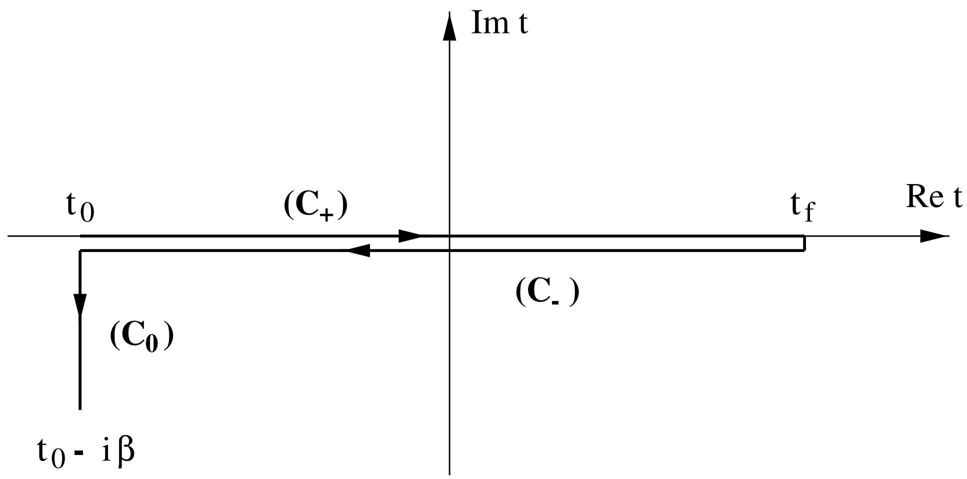
The HTL effective action II

- Assume the ***interaction*** being mostly ***carried by soft photons*** ($Q \sim gT \ll T$)

- The propagation of soft photons is ***dressed by the interactions with the light fermions of the thermal bath*** which are ***hard*** ($K \sim T$)



The HTL effective action III



⇒The photon propagator in the *complex-time plane*:

$$iD_{\mu\nu}(x-y) \equiv \theta_C(x^0 - y^0)\langle A_\mu(x)A_\nu(y)\rangle + \theta_C(y^0 - x^0)\langle A_\nu(y)A_\mu(x)\rangle$$

⇒The HTL effective action:

$$S_C^{HTL}[A] = \frac{1}{2}\int_C d^4x \int_C d^4y\, A^\mu(x)\left(D^{-1}\right)_{\mu\nu}^{HTL}(x-y)A^\nu(y).$$

It is gaussian!

Performing the functional integral

⇒Being the action gaussian...

$$G^{>}(t,\boldsymbol{r}_1;t,\boldsymbol{r}_2|0,\boldsymbol{r}'_1;0,\boldsymbol{r}'_2)=\delta(\boldsymbol{r}_1-\boldsymbol{r}'_1)\delta(\boldsymbol{r}_2-\boldsymbol{r}'_2)\overline{G}(t,\boldsymbol{r}_1-\boldsymbol{r}_2),$$

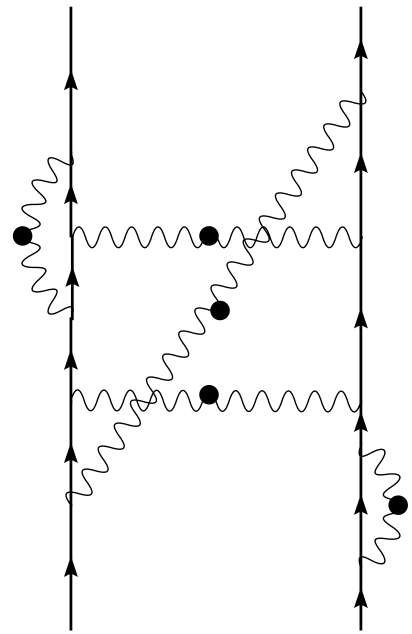
where

$$\overline{G}(t,\boldsymbol{r}_1-\boldsymbol{r}_2)=\exp\left[-\frac{i}{2}\int_C d^4x\int_C d^4y\, J^\mu(x)D_{\mu\nu}^{HTL}(x-y)J^\nu(y)\right]$$

with $J^\mu(x)$ the $Q\overline{Q}$ current.

Unified description of real and imaginary-time propagation!

In terms of Feynman diagrams...



Real-time $Q\overline{Q}$ propagator

$\Rightarrow Q\overline{Q}$ current non-vanishing along C_+:

$$\overline{G}(t, \boldsymbol{r}_1 - \boldsymbol{r}_2) = \exp\left[-\frac{i}{2}\int_{C_+} d^4x \int_{C_+} d^4y \, J^\mu(x) D_{\mu\nu}(x-y) J^\nu(y)\right]$$

with

$$J^\mu(z) = \delta^{\mu 0}\theta(z^0)\theta(t-z^0)[-g\delta(\boldsymbol{z}-\boldsymbol{r}_1) + g\delta(\boldsymbol{z}-\boldsymbol{r}_2)]$$

$\Rightarrow$One gets:

$$\overline{G}(t, \boldsymbol{r}_1 - \boldsymbol{r}_2) = \exp\left[-2ig^2 \int \frac{d\omega}{2\pi} \int \frac{d\boldsymbol{q}}{(2\pi)^3} \frac{1-\cos(\omega t)}{\omega^2} \left(1 - e^{i\boldsymbol{q}\cdot(\boldsymbol{r}_1 - \boldsymbol{r}_2)}\right) D_{00}(\omega, \boldsymbol{q})\right]$$

⇒Large time behavior

- $Q\overline{Q}$ propagator

$$\overline{G}(t,\boldsymbol{r}_1-\boldsymbol{r}_2) \underset{t\to\infty}{\sim} \exp[-iV_\infty(\boldsymbol{r}_1-\boldsymbol{r}_2)t]$$

- Temporal evolution equation (∼ Schrödinger!)

$$\lim_{t\to+\infty}[i\partial_t-V_\infty(\boldsymbol{r}_1-\boldsymbol{r}_2)]\overline{G}(t,\boldsymbol{r}_1-\boldsymbol{r}_2)=0$$

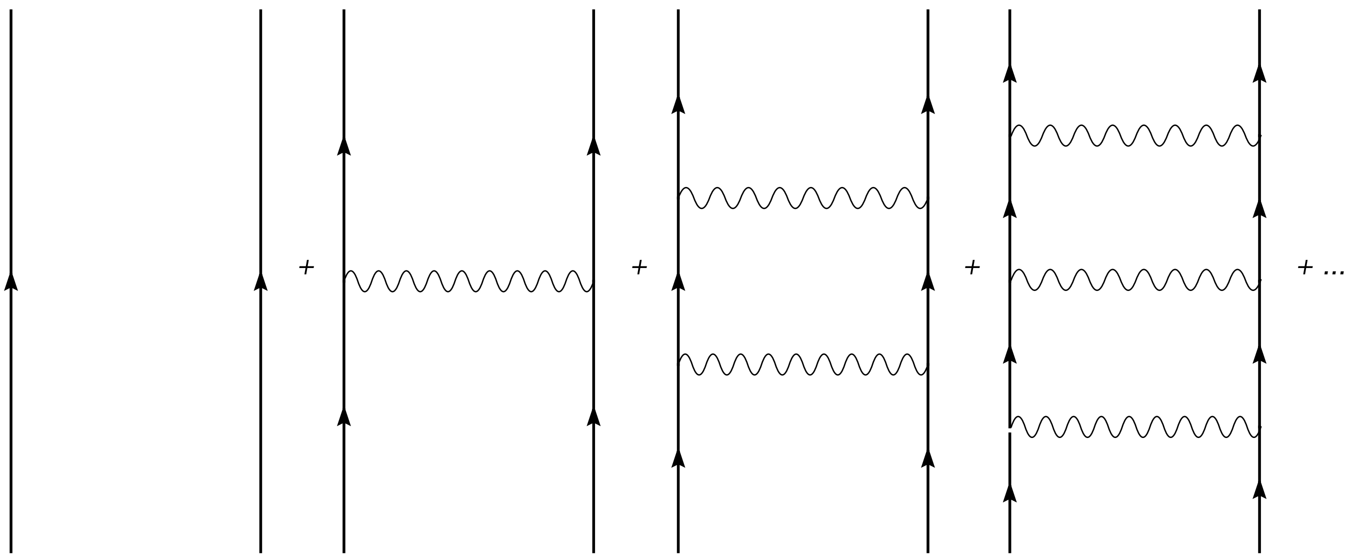
where:

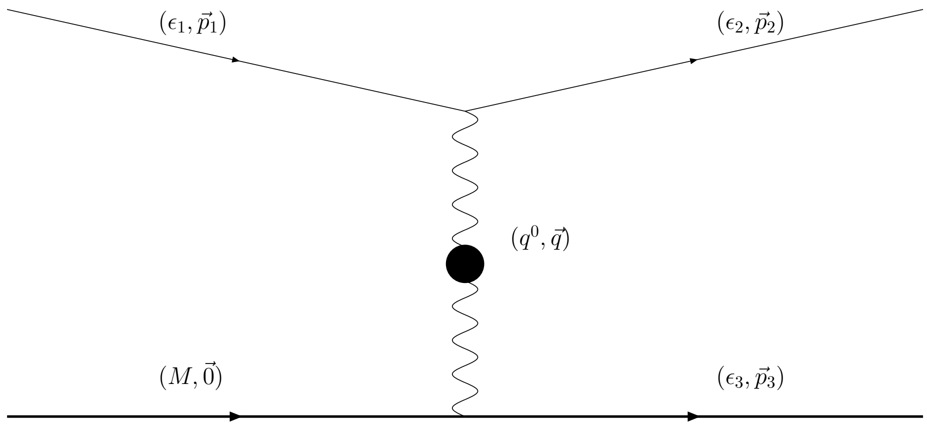
$$V_\infty(\boldsymbol{r}_1-\boldsymbol{r}_2)\equiv g^2\int\frac{d\boldsymbol{q}}{(2\pi)^3}\left(1-e^{i\boldsymbol{q}\cdot(\boldsymbol{r}_1-\boldsymbol{r}_2)}\right)D_{00}(\omega=0,\boldsymbol{q})$$

$$=g^2\int\frac{d\boldsymbol{q}}{(2\pi)^3}\left(1-e^{i\boldsymbol{q}\cdot(\boldsymbol{r}_1-\boldsymbol{r}_2)}\right)\Big[\underbrace{\frac{1}{\boldsymbol{q}^2+m_D^2}}_{\text{screening}}-i\underbrace{\frac{\pi m_D^2 T}{|\boldsymbol{q}|(\boldsymbol{q}^2+m_D^2)^2}}_{\text{collisions}}\Big]$$

$$=-\frac{g^2}{4\pi}\left[m_D+\frac{e^{-m_D r}}{r}\right]-i\frac{g^2T}{4\pi}\phi(m_D r)$$

⇒**In terms of Feynman diagrams** the large time behavior can be described as *a ladder of instantaneous effective interactions*:





⇒**Interpretation of the damping**: interaction rate of a heavy fermion in the thermal bath

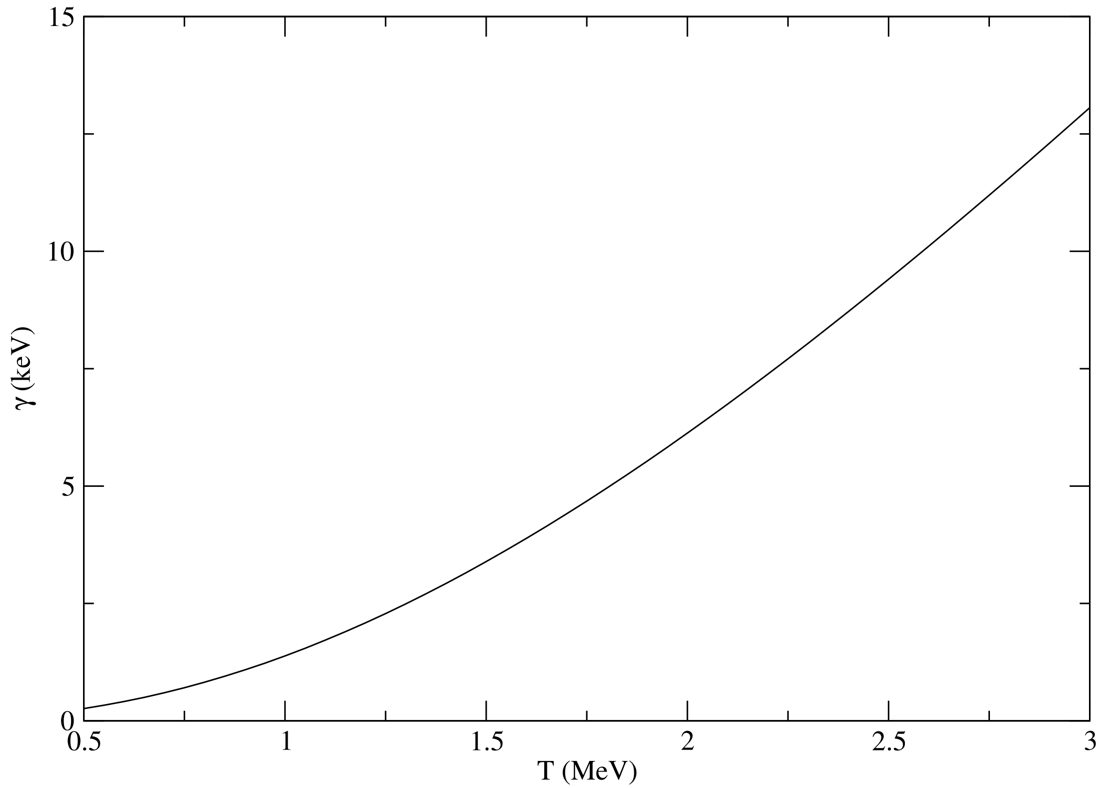
$$\Gamma(M) = 2\,\frac{1}{2M}\int_{p_1}\int_{p_2}\int_{p_3}(2\pi)^4\delta^{(4)}(P+P_1-P_2-P_3)\times$$
$$\times\,[n_1(1-n_2)(1-n_3)+(1-n_1)n_2n_3]\,\overline{|\mathcal{M}|^2}$$

In the $M\to\infty$ limit:

$$\Gamma(\infty) = g^2T\int\frac{d\boldsymbol{q}}{(2\pi)^3}\,\frac{\pi m_D^2}{(\boldsymbol{q}^2+m_D^2)^2 q}$$

⇒**An example**: a $\mu^+\mu^-$ pair in a hot QED plasma.

$$r = \langle r \rangle_{1S} = \frac{3}{2} a_{\rm Bohr} \equiv \frac{3}{2} \frac{1}{\mu \alpha_{\rm QED}} \approx 3.89\,{\rm MeV}^{-1}\,,$$



Real-time propagation: what we learnt

In the case of $M = \infty$ and soft-photon exchange:

- Exact expression for $G^>(t)$;
- Closed temporal evolution equation for $G^>(t)$;
- From the large-time behavior $\rightarrow$ **effective potential**
  - Real part: screening,
  - Imaginary part: collisional damping;
- Connection of the imaginary part with the interaction rate.

Imaginary-time $Q\overline{Q}$ propagator

$\Rightarrow$Analyticity of $G^{>}(t)$ $\rightarrow$ simply set $t=-i\tau$ with $\tau\in[0,\beta]$

$$\overline{G}(-i\tau,\boldsymbol{r}_1-\boldsymbol{r}_2)=\exp\left[g^2\int_0^\tau d\tau'\int_0^\tau d\tau''\int\frac{d\boldsymbol{q}}{(2\pi)^3}\left(1-e^{i\boldsymbol{q}\cdot(\boldsymbol{r}_1-\boldsymbol{r}_2)}\right)\Delta_{00}(\tau'-\tau'',\boldsymbol{q})\right]$$

$\Rightarrow$Propagation till $\tau=\beta$:

$$\overline{G}(-i\beta,\boldsymbol{r}_1-\boldsymbol{r}_2)=\exp\left\{-\beta g^2\int\frac{d\boldsymbol{q}}{(2\pi)^3}\left(1-e^{i\boldsymbol{q}\cdot(\boldsymbol{r}_1-\boldsymbol{r}_2)}\right)\frac{1}{\boldsymbol{q}^2+m_D^2}\right\}$$

Since:

$$\overline{G}(-i\beta,\boldsymbol{r}_1-\boldsymbol{r}_2)=\exp\left(-\beta\Delta F_{Q\overline{Q}}(r,T)\right)$$

One gets the $Q\overline{Q}$ **free-energy**:

$$\Delta F_{Q\overline{Q}}(r,T)=-\frac{g^2 m_D}{4\pi}-\frac{g^2}{4\pi}\frac{e^{-m_D r}}{r}\,,$$

It coincides with the ***real part of the effective potential***!

Imaginary-time propagation: what we learnt

- $G^>(t=-i\tau)$ follows simply from the analyticity;
- The **free-energy** coincides with the **real part of the effective potential**.
  This relies essentially on the analyticity properties of $G^>(t)$. *Hence we think the argument being very general, not specific of the model we investigated*;
- No information on the imaginary-part can be obtained from $G^>(t=-i\beta)$ (*i.e. what is usually evaluated on the lattice*).

Have we answered to the initial questions?

Let us summarize...

- Under some assumptions ($Q\overline{Q}$ external probes, effective interaction accounting for medium effects, $M = \infty$) $G^{>}(t)$ obeys a closed equation. Is it possible to relax the above constraints?
- Large-time behavior governed by the static limit of the effective interaction
- Analyticity of $G^{>}(t)$ allows a *unified treatment of real and imaginary-time propagation*;
- The **real part of the effective potential** has to be identified with the **free-energy**;
- Imaginary part of the effective potential arises naturally.

The finite mass case:
a possible strategy

The general idea

Treat the heavy fermion propagating in a thermal bath as a point-like particle in Quantum-Mechanics. Hence:

- Sum over all the possible trajectories in a given background field:

$$\langle \boldsymbol{x}_f \tau_f | \boldsymbol{x}_i \tau_i \rangle = \int_{\boldsymbol{x}(\tau_i)=\boldsymbol{x}_i}^{\boldsymbol{x}(\tau_f)=\boldsymbol{x}_f} [\mathcal{D}\boldsymbol{x}(\tau')] \exp\left[-\int_{\tau_i}^{\tau_f} d\tau' \left(\frac{1}{2}M\dot{\boldsymbol{x}}^2 + V(\boldsymbol{x})\right)\right],$$

where $V(\boldsymbol{x}) \equiv g\Phi(\boldsymbol{x})$ (scalar interaction) and $\dot{\boldsymbol{x}} \equiv d\boldsymbol{x}/d\tau'$.

- Average over all the possible field configurations (the action accounting for medium effects)

$$G^>(-i\tau, \boldsymbol{r}_1|0, \boldsymbol{r}'_1) = Z^{-1} \int_{\boldsymbol{z}_1(0)=\boldsymbol{r}'_1}^{\boldsymbol{z}_1(\tau)=\boldsymbol{r}_1} [\mathcal{D}\boldsymbol{z}_1] \int [\mathcal{D}\Phi] \exp\left[-\int_0^\tau d\tau' \frac{1}{2} M \dot{\boldsymbol{z}}_1{}^2\right] \times$$
$$\times \exp\left[-g\int_0^\tau d\tau' \Phi(t', \boldsymbol{z}_1(t'))\right] e^{-S_E^{\rm eff}[\Phi]}$$

For a gaussian effective action...

$\Rightarrow$Single particle propagator:

$$G^{>}(-i\tau, \boldsymbol{r}_1|0, \boldsymbol{r}_1') = \int_{\boldsymbol{z}(0)=\boldsymbol{r}_1'}^{\boldsymbol{z}(\tau)=\boldsymbol{r}_1} [\mathcal{D}\boldsymbol{z}] \exp\left[-\int_0^\tau d\tau' \frac{1}{2} M \dot{\boldsymbol{z}}^2\right] \times$$
$$\times \exp\left[\frac{g^2}{2}\int_0^\tau d\tau' \int_0^\tau d\tau'' \Delta(\tau'-\tau'', \boldsymbol{z}(\tau')-\boldsymbol{z}(\tau''))\right],$$

with $\Delta(\tau, \boldsymbol{z})$ the *Matsubara propagator of the exchanged meson.*

NB *Imaginary-time propagation in view of the numerical evaluation of the path-integral!*

⇒Two-particle propagator:

$$G^{>}(-i\tau,\boldsymbol{r}_1;-i\tau,\boldsymbol{r}_2|0,\boldsymbol{r}_1',0,\boldsymbol{r}_2')=\int_{\boldsymbol{r}_1'}^{\boldsymbol{r}_1}[\mathcal{D}\boldsymbol{z}_1]\int_{\boldsymbol{r}_2'}^{\boldsymbol{r}_2}[\mathcal{D}\boldsymbol{z}_2]\times$$
$$\times\exp\left[-\int_0^\tau d\tau'\left(\frac{1}{2}M\dot{\boldsymbol{z}}_1{}^2-\frac{g^2}{2}\int_0^\tau d\tau''\Delta(\tau'-\tau'',\boldsymbol{z}_1(\tau')-\boldsymbol{z}_1(\tau''))\right)\right]\times$$
$$\times\exp\left[-\int_0^\tau d\tau'\left(\frac{1}{2}M\dot{\boldsymbol{z}}_2{}^2-\frac{g^2}{2}\int_0^\tau d\tau''\Delta(\tau'-\tau'',\boldsymbol{z}_2(\tau')-\boldsymbol{z}_2(\tau''))\right)\right]\times$$
$$\times\exp\left[g^2\int_0^\tau d\tau'\int_0^\tau d\tau''\Delta(\tau'-\tau'',\boldsymbol{z}_1(\tau')-\boldsymbol{z}_2(\tau''))\right].$$