Mueller-Navelet Jets

Dionysis Triantafyllopoulos

ECT*, Trento, Italy

“Structure Functions, Geometric Scaling and Parton Saturation”
EMMI, Darmstadt, November 2010

With: E. Iancu, M.S. Kugeratski
Outline

- Mueller-Navelet Jets
  - Definition
  - BFKL resummation
  - Properties: $K$-factor, decorrelations
- $\gamma^* p$ Deep Inelastic Scattering
  - Geometric scaling at small Bjorken-$x$
  - Theoretical explanation (BFKL and saturation)
- Geometric Scaling in Mueller-Navelet Jets and large saturation momentum
Mueller-Navelet Jets

- Inclusive production of two jets separated by large rapidity interval in $p-p$ or $p-\bar{p}$ collisions

\[ p_{1}^{\mu} = (x_{1}\sqrt{s}, 0, 0_{\perp}) \]
\[ p_{2}^{\mu} = (0, x_{2}\sqrt{s}, 0_{\perp}) \]
\[ x_{1}\sqrt{s} = k_{1\perp}e^{\eta_{1}} \]
\[ x_{2}\sqrt{s} = k_{2\perp}e^{-\eta_{2}} \]
\[ Y = \ln \left( \frac{x_{1}x_{2}s}{k_{1}k_{2}} \right) \gg 1 \]
Cross Section

- Cross section

\[
\frac{d\sigma}{dx_1 dx_2 d^2k_1 d^2k_2} = f_{\text{eff}}(x_1, \mu^2) f_{\text{eff}}(x_2, \mu^2) \frac{d\hat{\sigma}}{d^2k_1 d^2k_2}
\]

with \( f_{\text{eff}} \) containing both quark and gluon.

- Choose \( x_1, x_2 \) to be “large”. Say \( \sim O(0.1) \)

Parton distributions are known

- Large logarithms of \( Y = \ln \hat{s}/k^2 \) in partonic cross section

- Rapidity strong ordering gives dominant contribution

\( \eta_1 \gg \eta_1' \gg \ldots \) and \( -\eta_2 \gg -\eta_{N'} \gg \ldots \)
Resummation and BFKL

- Born level (no minijet), back to back jets: $k_1 = -k_2$
  
  In terms of $Y$: $\hat{\sigma} \sim O(1)$

- One minijet $\leadsto$ decorrelation: $k_1 \neq -k_2$
  
  In terms of $Y$: $\hat{\sigma} \sim O(Y)$

- Two minijets $\leadsto$ more decorrelation
  
  In terms of $Y$: $\hat{\sigma} \sim O(Y^2)$

- Integrate over minijet phase space and sum
  
  $$\frac{d\hat{\sigma}}{d^2k_1 d^2k_2} \sim \frac{\alpha_s^2}{k_1^2 k_2^2} \Phi(Y, k_1, k_2)$$

- $\Phi$: all orders $K$-factor and satisfies BFKL equation

- At large $Y$
  
  $$\Phi \sim \frac{1}{k_1 k_2} \exp(\omega_P Y)$$
Properties

- Exponential in $Y K$-factor
- Momentum decorrelation
- Angular decorrelations

Status

- Proposal: Mueller, Navelet 87
- Angular decorrelation: Del Duca, Schmidt 94
- Running coupling and energy conservation: Orr, Stirling, 97
- Unitarity corrections: Marquet, Peschanski, Royon, ~ 05
- NLO Green’s function: Sabio Vera, Schwennsen, 07
- Full NLO: Colferai, Schwennsen, Szymanowski, Wallon, 10
- $D\phi$: 2 points. CMS?
Angular decorrelation

- Born level $\hat{\sigma} \propto \delta(\phi - \pi)$
- BFKL at $Y \rightarrow \infty \Rightarrow d\hat{\sigma}/d\phi = 0$
- BFKL $\Rightarrow \langle \cos(m\phi) \rangle$ decreases with $Y$
- LO BFKL too fast decrease
- NLO BFKL, ambiguous? Too slow?
Angular decorrelation

Generalities
BFKL
Signatures of BFKL
Gluon saturation

\[ \langle \cos \phi \rangle \text{ from NLO BFKL (Colferai et al, 2010)} \]

Colferai et al, 10
**Angular decorrelation**

- **Left:** Sabio Vera et al, 07
- **Right:** Colferai et al, 10
DIS - Scaling

- Cross section $\sigma(x, Q^2, \Lambda)$ in $\gamma^*-p$ DIS. Data for $x < 10^{-2}$

$$\sigma \sim \frac{1}{\Lambda^2} f(Q^2/Q_s^2)$$

Saturation momentum

$$Q_s^2 \sim \Lambda^2 x^{-\lambda}$$

In other DIS processes too.
Scaling above $Q_s$ (Iancu, Itakura, McLerran / Mueller, DNT)

- Eigenfunctions are pure powers
  A single one selected asymptotically

$$\sigma_{dp} \sim \exp[\chi(\gamma_s) \ln(1/x)](r^2 \Lambda^2)^{1-\gamma_s} \sim (r^2 \frac{\Lambda^2 x^{-\lambda}}{Q_s^2})^{1-\gamma_s}$$

- Approximate scaling with running coupling
  NLO: $\lambda \simeq 0.3$ with $Q_g^2 \sim 50\text{GeV}^2$ for $Q_s \sim 1\text{GeV}$ (DNT)

- Scaling in $Q^2$ after convoluting with $\gamma^*$ wavefunction

- Dynamically generated scale sets the scale for observables
MN Jets and Unitarity

- Inclusive dijet cross section should respect unitarity limits. That would also cut diffusion to IR.
- Not a total cross section but difficult to imagine otherwise.
- More established for single forward jet (...)

\[
\frac{d\sigma}{d\eta d^2 k} \sim \frac{1}{k^2} xG_1(x, k^2) \int d^2 r \exp[-i \mathbf{k} \cdot \mathbf{r}] \nabla_r^2 \sigma_{gg-B}(\mathbf{r})
\]

\(\sigma_{gg-B}(\mathbf{r})\) unitarizes too.
Not virtual gluonic dipole. From amplitude \(\times\) amplitude*

- Conjecture expression involving \(\sigma_{gg-gg}(\mathbf{r}_1, \mathbf{r}_2)\) (Marquet).
Not necessary for our purposes.
MN Jets and Scaling

- Integrate jet transverse momenta above $Q_1, Q_2$

$$\frac{d\sigma}{dx_1 dx_2} = F_{\text{eff}} \frac{\alpha_s^2}{Q_2^2} \int \frac{d\gamma}{2\pi i} \exp[\bar{\alpha}_s \chi(\gamma) Y] \left( \frac{Q_2^2}{Q_1^2} \right)^{1-\gamma}$$

- Saddle point and vanishing exponent ($Q_2 \ll Q_1$) $\Rightarrow$

$$\frac{d\sigma}{dx_1 dx_2} \sim F_{\text{eff}} \frac{1}{Q_2^2} \left( \frac{Q_2^2 e^{\lambda(Y-Y_0)}}{Q_1^2} \right)^{1-\gamma_s} \text{ with } \gamma_s = 0.372$$

Geometric scaling. Similar to DIS: $\Lambda \rightarrow Q_2$
MN Jets

- $Q_2$ will be large
  - $(+)$ Large initial saturation scale
  - $(-)$ Cross section $\sim 1/Q_2^2$

- Three points for total energy $s$
  - Keep kinematics of the softer (2) jet fixed
  - Vary kinematics of harder (1) jet so that $x_1 = Q_1 e^{\eta_1}/\sqrt{s} = \text{fixed}$

- Typical values
  - $Y \gtrsim Y_0 \sim 8$, $Q_2 \sim 20\text{GeV}$, $30\text{GeV} \lesssim Q_1 \lesssim 100\text{GeV}$
Conclusion

- Inclusive cross section for production of two jets very separated in rapidity should exhibit geometric scaling
- Particular case of strong momentum decorrelation
- Very hard saturation momentum

E. Iancu, M.S. Kugeratski, D.N. Triantafyllopoulos

*Geometric Scaling in Mueller-Navelet Jets*

arXiv:0802.0343 [NPA]
Logarithmic Plane

$Y = \ln \frac{1}{x}$

$\ln Q_s^2(Y) = \lambda Y$

Saturation

Dilute system

$\ln \Lambda_{QCD}^2$

$\ln Q^2$

BFKL

DGLAP