Small-x physics with e+A collisions

Cyrille Marquet

Theory Division - CERN
Outline

• Diffractive structure functions
• Diffractive vector meson production and DVCS
• Semi-inclusive DIS
• Di-hadron production in DIS
Diffractive structure functions
Inclusive diffraction in DIS

\begin{align*}
\gamma^* \rightarrow q + X \\
Q^2 & , \quad M_X^2 = (q + p - p')^2 \\
\beta & = \frac{Q^2}{Q^2 + M_X^2 - t} \\
(1 - y + \frac{y^2}{2}) & F_{2D,4}^D(x, Q^2, \beta, t) - \frac{y^2}{2} F_{L,4}^D(x, Q^2, \beta, t) \\
\end{align*}

- the measured cross-section

\[
\frac{d^4\sigma_{e^+e^-\rightarrow eXh}}{dx \, dQ^2 \, d\beta \, dt} = \frac{4\pi\alpha_{em}^2}{\beta^2 Q^4} \left[ (1 - y + \frac{y^2}{2}) F_{2D,4}^D(x, Q^2, \beta, t) - \frac{y^2}{2} F_{L,4}^D(x, Q^2, \beta, t) \right]
\]

almost only the t-integrated cross-section is discussed
The dipole picture for $F_2^D$

the diffractive final state is decomposed into $q\bar{q}$, $q\bar{q}g$, ... contributions

- the $q\bar{q}$ contribution

double differential cross-section
(proportional to the structure function)
for a given photon polarization:

$$\frac{d\sigma}{d\beta dt} = \frac{Q^2}{4\beta^2} \sum_f \int \frac{d^2r}{2\pi} \int \frac{d^2r'}{2\pi} \int_0^1 dz (1-z) \Theta(\kappa_f^2) e^{i\kappa_f \cdot (r'-r)}$$

Fourier transform to $M_X^2$
$$\kappa_f^2 = z(1-z)Q^2(1-\beta)/\beta - m_f^2$$

overlap of wavefunctions

Fourier transform to $t$
$$t = -\Delta^2$$

dipole amplitudes

- higher Fock states

contribute to $F_2^D$ but also to semi inclusive diffraction

also taken into account with dipoles
Diffraction in the dipole picture

contributions of the different final states to the diffractive structure function:

\[ x_{\perp} F_2^{D,3}(\beta, Q^2 = 5 \text{ GeV}^2, x_{\perp} = 0.001) \]

- at small \( \beta \): quark-antiquark-gluon
- at intermediate \( \beta \): quark-antiquark (T)
- at large \( \beta \): quark-antiquark (L)
Recent e+p data
From protons to nuclei

inclusive diffraction off nuclei has never been measured
we rely on model predictions

- **the dipole-nucleus cross-section** \( T_{qq}^p(r, b, x) = 1 - e^{-f(r, x, b)} \) \( \Rightarrow \) \( T_{qq}^A(r, b, x) = 1 - e^{-\sum_i f(r, x, b - b_i)} \)

averaged with the Woods-Saxon distribution \( T_A(\{b_i\}) \) position of the nucleons

\[
\langle O \rangle = \int \prod_i d^2b_i T_A(b_i) \ O(\{b_i\}) \quad T_A(b) = C \int dz \left\{ 1 + \exp \left[ \left( \sqrt{b^2 + z^2} - R_A \right) / d \right] \right\}^{-1}
\]

- **the Woods-Saxon averaging**

in diffraction, averaging at the level of the amplitude corresponds to a final state where the nucleus is intact

averaging at the cross-section level allows the breakup of the nucleus into nucleons
The ratio $F_{2}^{D,A} / F_{2}^{D,p}$

- for each contribution as a function of $\beta$:
  - quark-antiquark-gluon $< 1$ and $\sim$ const.
  - quark-antiquark (T) $> 1$ and $\sim$ const.
  - quark-antiquark (L) $> 1$ and decreases with $\beta$

- nuclear effects
  - enhancement at large $\beta$
    - the quark-antiquark contribution dominates
    - the ratio is almost constant and decreases with $A$
  - suppression at small $\beta$
    - the quark-antiquark-gluon contribution dominates

Kowalski, Lappi, CM and Venugopalan (2008)
Incoherent diffraction

in this study the breakup of the nucleus into nucleons is allowed

Kowalski, Lappi, CM and Venugopalan (2008)

• as a function of $Q^2$
  the quark-antiquark contributions
  for $\beta$ values at which they dominate

• as a function of $A$
  for a gold nucleus, the diffractive structure function
  is 15 % bigger when allowing breakup into nucleons
  the proportion of incoherent diffraction decreases with $A$
Diffractive VM production
VM production off the CGC

• the diffractive cross section

\[
\frac{d\sigma^{\gamma^* p \to VY}}{dt} = \frac{1}{4\pi} \int d^2rd^2r' \varphi(r, Q^2, M_V^2) \varphi^*(r', Q^2, M_V^2) \int d^2b d^2b' e^{iq_{\perp}(b-b')} \left< T_{q\bar{q}}(r, b) T_{q\bar{q}}(r', b') \right>_x
\]

overlapping functions

the conjugate amplitude

\( r \) : dipole size in the amplitude

\( r' \) : dipole size in the conjugate amplitude

target average at the cross-section level:
contains both broken-up and intact events

one needs to compute a 4-point function, that gives access to gluon correlations

• the exclusive part

obtained by averaging at the level of the amplitude:

\[ \left< T_{q\bar{q}}(r, b) T_{q\bar{q}}(r', b') \right>_x \rightarrow \left< T_{q\bar{q}}(r, b) \right>_x \left< T_{q\bar{q}}(r', b') \right>_x \]

probes b dependence:

\[
\frac{d\sigma^{\gamma^* p \to Vp}}{dt} = \frac{1}{4\pi} \left| \int d^2r \varphi(r, Q^2, M_V^2) \int d^2b e^{iq_{\perp}b} \left< T_{q\bar{q}}(r, b) \right>_x \right|^2
\]
Coherent part (proton case)

Munier, Stasto and Mueller (2001)
the no-scattering probability \( (S=1-T) \)
is extracted from the \( \rho \) data
\( S(1/r \approx 1 \text{Gev}, b \approx 0, x \approx 5.10^{-4}) = 0.6 \)

\[ r_Q = \frac{4}{(Q^2 + M_P^2)} \]

\[ r_Q=0.35 \text{fm} \ (Q^2=0.45 \text{GeV}^2) \]
\[ r_Q=0.21 \text{fm} \ (Q^2=3.5 \text{GeV}^2) \]
\[ r_Q=0.16 \text{fm} \ (Q^2=7.0 \text{GeV}^2) \]

\[ \exp(-\lambda b) \quad \frac{1}{b} \quad \frac{1}{b^2} \]

\( \Rightarrow \) HERA is entering the saturation regime

• success of the dipole models
  t-CGC \( \chi^2/\text{points} = 1.2 \)
  CM, Peschanski and Soyez (2007)
  b-CGC appears to work well
  also but no \( \chi^2 \) given
  Kowalski, Motyka and Watt (2006)

\begin{align*}
\text{rho} & \quad \text{J/Psi} \\
\text{predictions for DVCS have been verified} & \quad \text{predictions for DVCS have been verified}
\end{align*}
Expectations with nuclei

Caldwell and Kowalski (2010)

- using the same model as before

\[ T^{A}_{qq}(r, b, x) = 1 - e^{-\sum_i f(r, x, b-b_i)} \]

with nucleons distributed according to the Woods-Saxon distribution

sat vs non-sat: differences are seen and they are big enough to be measured

estimation of the incoherent part

data is crucial to see if at small x the Woods-Saxon distribution is still valid
Incoherent part (proton case)

Dominguez, CM and Wu (2009)

• as a function of $t$
  exclusive production:
    the proton undergoes elastic scattering
    dominates at small $|t|$
  diffractive production:
    the proton undergoes inelastic scattering
    dominates at large $|t|$

• two distinct regimes
  exclusive
    $\to$ exp. fall at $-t < 0.7$ GeV$^2$
  diffractive
    $\to$ power-law tail at large $|t|$

the transition point is where the data on exclusive production stop
From protons to nuclei

• qualitatively, one expects three contributions

  exclusive production is called coherent diffraction
  the nucleus undergoes elastic scattering, dominates at small $|t|$
  intermediate regime (absent with protons)
  the nucleus breaks up into its constituents nucleons, intermediate $|t|$
  then there is fully incoherent diffraction
  the nucleons undergo inelastic scattering, dominates at large $|t|$

• three regimes as a function of $t$:

  coherent diffraction
  → steep exp. fall at small $|t|$
  breakup into nucleons
  → slower exp. fall at $0.05 < -t < 0.7$ GeV$^2$
  incoherent diffraction
  → power-law tail at large $|t|$

Lappi and Mantysaari (2010)
Semi-inclusive DIS
The dipole factorization in SIDIS

- the cross section at small $x$

$$\Phi(\xi, x, y; Q^2) = \psi(\xi, x; Q^2)\psi^*(\xi, y; Q^2)$$

dipoles in amplitude / conj. amplitude

$$\frac{d\sigma^e_{\gamma^* p \to h X}}{dz_h d^2 P} = \frac{d\sigma^e_{T,L}}{d\xi d^2 k} \left( k_{\perp} = \frac{\xi}{z_h} P_{\perp} \right) \otimes D_{h/q}(z_h/\xi)$$

$$\frac{d\sigma^e_{\gamma^* p \to q X}}{dz_h d^2 P} = \int d^2 x d^2 y \frac{e^{-ik_{\perp} \cdot (x-y)}}{2\pi} \Phi_{T,L}(\xi, x, y; Q^2) \int d^2 b [T_{q\bar{q}}(x, x_B)+T_{q\bar{q}}(y, y_B)-T_{q\bar{q}}(x-y, x_B)]$$

McLerran and Venugopalan, Mueller, Kovchegov and McLerran (1999)
**k_T-dependent small-x gluons**

- at small $x$ and large $Q^2$

  the cross section is proportional to the TMD quark distribution

\[
xq(x, k_\perp) = \frac{N_c}{4\pi^4} \int d^2b d^2q_\perp F(q_\perp, x) \left[1 - \frac{k_\perp \cdot (k_\perp - q_\perp)}{k_\perp^2 - (k_\perp - q_\perp)^2} \ln \left(\frac{k_\perp^2}{(k_\perp - q_\perp)^2}\right)\right]
\]

- CM, Xiao and Yuan (2009)

obtained from non-linear QCD evolution

for saturation physics the relevant regime is low $P_T (\sim Q_s)$

the transition with collinear factorization at large $P_T$ is interesting also
TMD-pdf / u-pdf relation

- at small $x$ and large $Q^2$

  in the overlapping domain of validity, TMD & $k_T$ factorization are consistent

- the saturation regime

  the TMD factorization can be used in the saturation regime, when $P_{\perp}^2 \sim Q_s^2$

  there $xq(x, k_{\perp}) \to \text{const.}$

  even if $Q^2$ is much bigger than $Q_s^2$, the saturation regime will be important when

  in fact, thanks to the existence of $Q_s$, the limit $|P_{\perp}| \to 0$ is finite, and computable with weak-coupling techniques ($Q_s \gg \Lambda_{QCD}$)

  eventually true at small $x$
x evolution of the TMD-pdf

- from small $x$ to smaller $x$

\[
\frac{xq(x, k_\perp)}{x_0q(x_0, k_\perp)}
\]

$x_0 = 10^{-2}$

at small $k_t$

$xq(x, 0) = c$

at large $k_t$

\[
\frac{Q_s^2(x)}{Q_s^2(x_0)}
\]

not full BK evolution here, but GBW parametrization

\[
F(q_\perp, x) = e^{-q_\perp^2/Q_s^2(x)/Q_s^2(x)}
\]

\[
Q_s^2(x) = (3.10^{-4}/x)^{0.28} \text{ GeV}^2
\]
HERA data probe saturation

- ratio of SIDIS cross sections at two different values of $x$

$$\frac{d\sigma(ep \rightarrow e'hX)}{dp} \bigg|_{x_B \ll 1, \frac{Q^2}{P^2}} \approx q(x_B, P)$$

our (crude) calculation

one can do much better with actual BK evolution and quark fragmentation

the data show the expected trend

- at future EIC’s

the SIDIS measurement provides direct access to the transverse momentum distribution of partons in the proton/nucleus, and the saturation regime can be easily investigated

H1 collaboration (1997)
Di-hadron production in DIS
Di-hadron correlations

• the di-jet cross section in the dipole picture

\[
\frac{d\sigma_{T,L}^{\gamma^* p \to q\bar{q} X}}{d^2 k_\perp d^2 k'_\perp} = \int \frac{d^2 x}{2\pi} \frac{d^2 y}{2\pi} \frac{d^2 x'}{2\pi} \frac{d^2 y'}{2\pi} e^{-ik_\perp \cdot (x-y)} e^{-ik'_\perp \cdot (x'-y')} \int d\xi \Phi_{T,L}(\xi, x-x', y-y'; Q^2) \times [T_{q\bar{q}}(x-x', x_B) + T_{q\bar{q}}(y-y', x_B) - T_{q\bar{q}q\bar{q}}(x, x', y', y, x_B)]
\]

because of the 4-point function \(T_{q\bar{q}q\bar{q}}\), there is no \(k_T\) factorization (unless saturation and multiple scatterings can be safely neglected)

• SIDIS was a special case

in SIDIS, the \(k'_\perp\) integration sets \(x'=y'\), and then \(T_{q\bar{q}q\bar{q}}(x, x', x', y, x_B) = T_{q\bar{q}}(x - y, x_B)\)

this cancellation of the interactions involving the spectator antiquark in SIDIS is what led to \(k_T\) factorization

with dijets, this does not happen, and as expected, the cross section is a non-linear function of the u-pdf
Constraining the 4-point function

Unlike most observables considered in DIS, di-hadrons probe more than the dipole scattering amplitude, it probes the 4-point function

\[ T_{q\bar{q}q\bar{q}}(x, x', y, y', x_B) \]

only in special limits it can be simplified, such as \( |k_\perp + k'_\perp| \ll |k_\perp|, |k'_\perp| \)

Dominguez, Xiao and Yuan (2010)

We expect to see the same effect in e+A vs e+p than the one discovered in d+Au vs p+p collisions at RHIC.

The same 4-point function is involved in the d+Au case.

But the e+A measurement could help constrain it better.

The background will be much smaller than in d+Au for instance.

The evolution of higher point functions (~ multi-gluon distribution) is different from that of the 2-point function (single gluon distribution).

It is equally important to understand it.

Dumitru and Jalilian-Marian (2010)
Di-hadron $p_T$ imbalance in $d+Au$

$$x_A = \frac{k_1 e^{-\gamma_1} + k_2 e^{-\gamma_2}}{\sqrt{s}} \ll 1$$

Albacete and CM (2010)

$p+p$ approach

$p+p \rightarrow \pi^0 \pi^0 + X$, $\sqrt{s} = 200$ GeV

$p_{\pi^0} > 2$ GeV/c, $1$ GeV/c $< p_{\pi^0} < p_{\pi^0}$

$\langle \eta_L \rangle = 3.2$, $\langle \eta_S \rangle = 3.1$

Uncorrected Coincidence Probability (radon) vs. $\Delta \phi$ (rad)

$d+Au$ central approach

$d+Au \rightarrow \pi^0 \pi^0 + X$, $\sqrt{s} = 2000$, $2000 < p_{\pi^0} < 4000$

$p_{\pi^0} > 2$ GeV/c, $1$ GeV/c $< p_{\pi^0} < p_{\pi^0}$

$\langle \eta_L \rangle = 3.1$, $\langle \eta_S \rangle = 3.2$

Uncorrected Coincidence Probability (radon) vs. $\Delta \phi$ (rad)

This happens at forward rapidities, but at central rapidities, the $p+p$ and $d+Au$ signal are almost identical.
Di-hadron $p_T$ imbalance in e+A

- the di-hadron cross section in the small momentum imbalance limit

\[ |k_{\perp} + k'_{\perp}| \ll |k_{\perp}|, |k'_{\perp}| \]

PRELIMINARY

\[ \sqrt{s} = 100 \text{ GeV} \]

\[ \sqrt{s} = 200 \text{ GeV} \]

not e+A vs e+p but rather e+A at two different energies

z and $k_T$ dependent fragmentation included

Dominguez, CM, Xiao and Yuan, in preparation
Conclusions

- very little is known about the structure of heavy nuclei at small-x
  - only for inclusive structure functions we have data at moderate x
  - SIDIS and exclusive VM production data are at high x or for light nuclei
  - diffractive structure functions have never been measured

- e+A collisions are ideal to learn about this
  - so far we have expectations using small-x QCD evolution
  - the CGC initial conditions for HIC are based on these expectations
  - but this needs to be checked with data

- what are crucial measurements?
  - SIDIS & di-hadrons: the $k_T$ dependence of the gluon distribution
  - coherent diffraction: the impact-parameter dependence
  - incoherent diffraction: correlations between small-x gluons