The Early Stages of Heavy Ion Collisions

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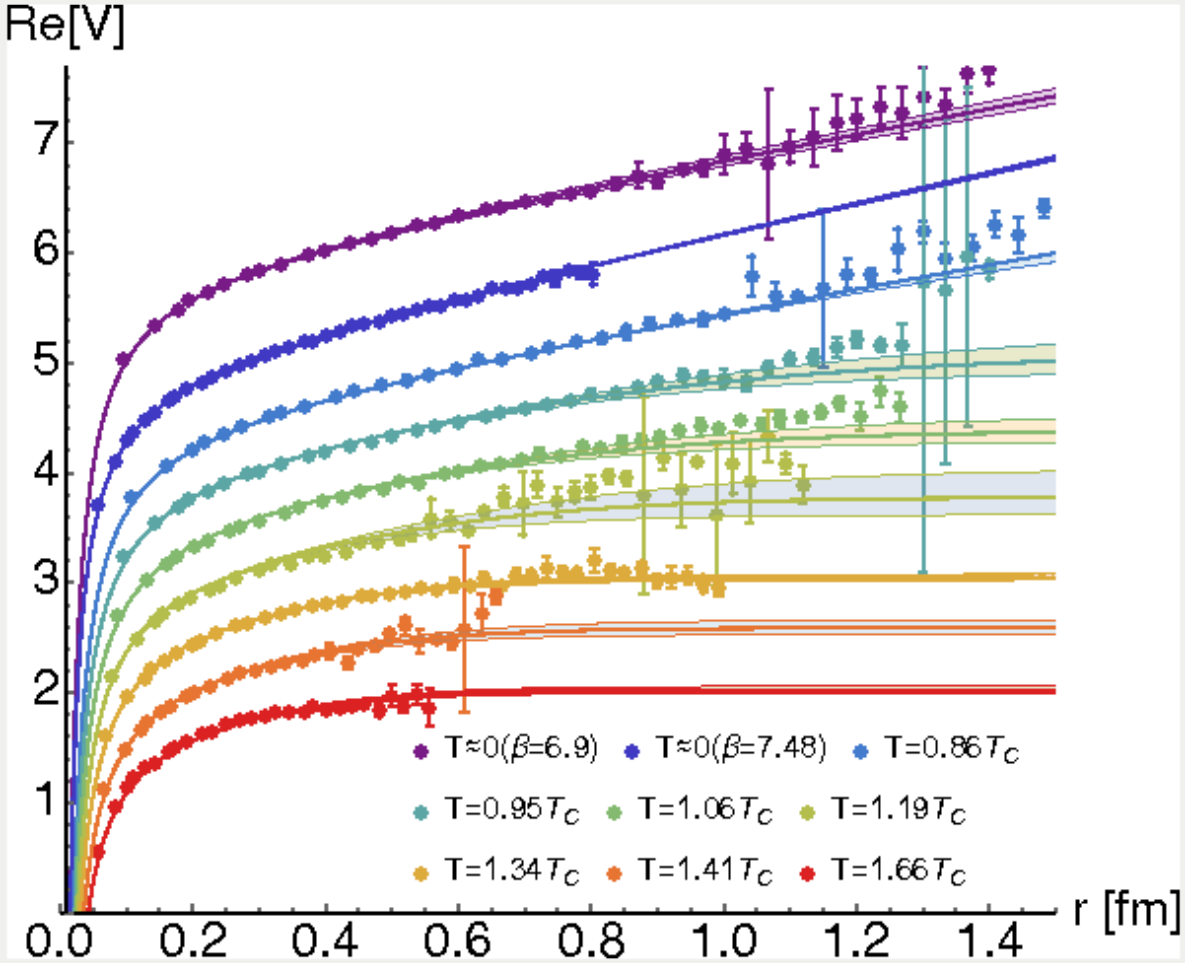
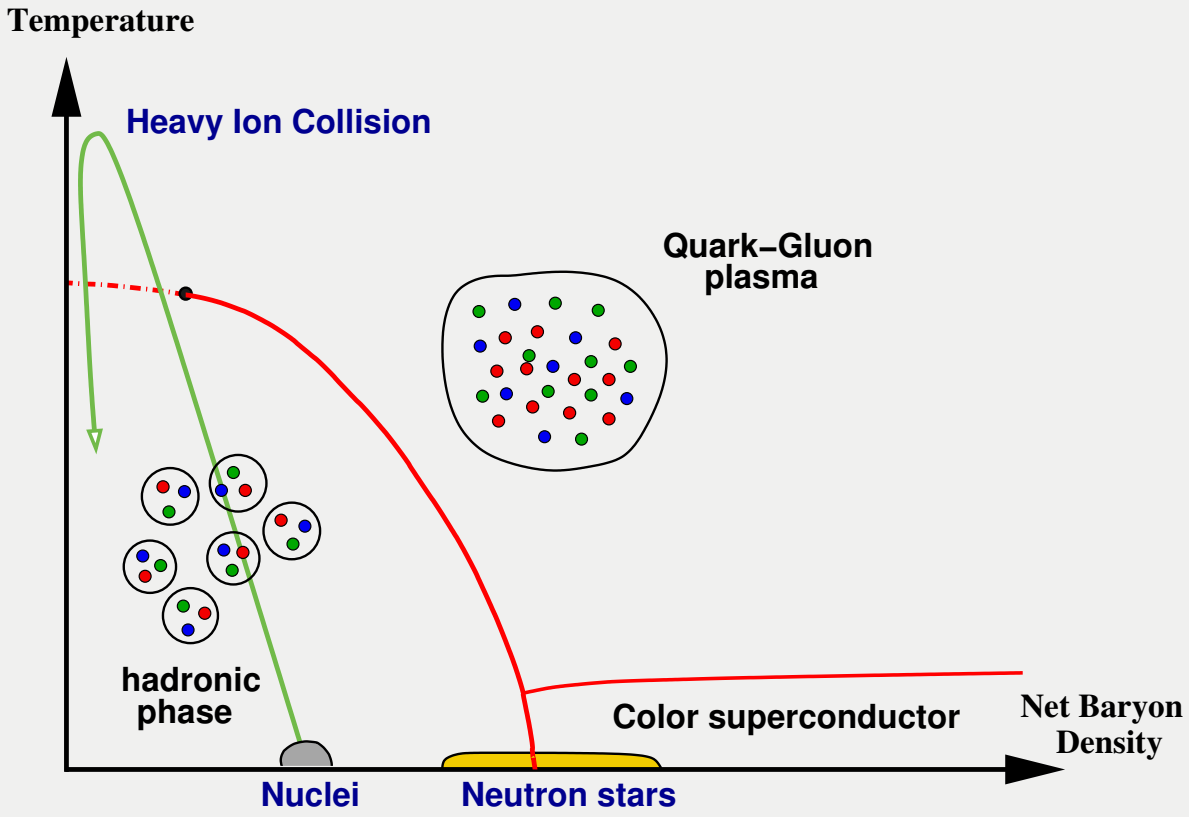
Heavy Ion Collisions

Quark-antiquark potential at high temperature

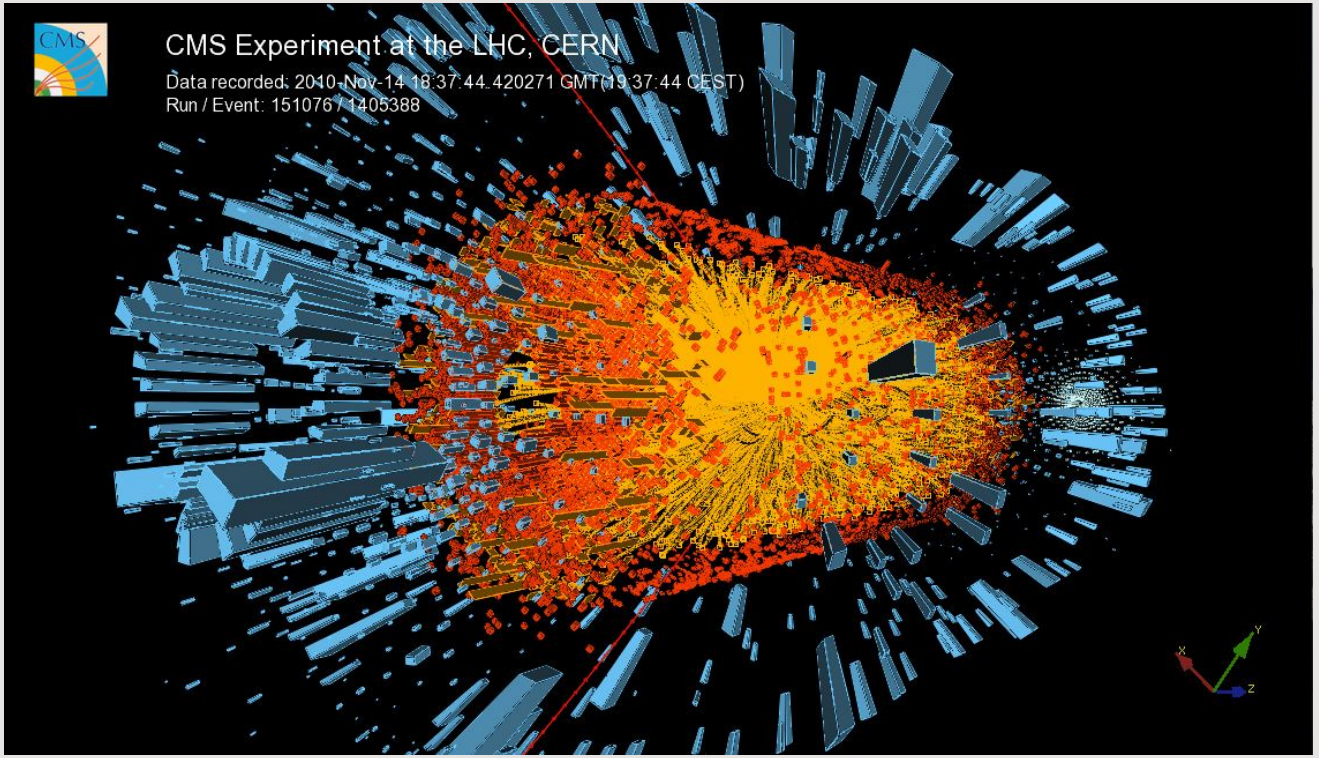


Heavy ion collisions



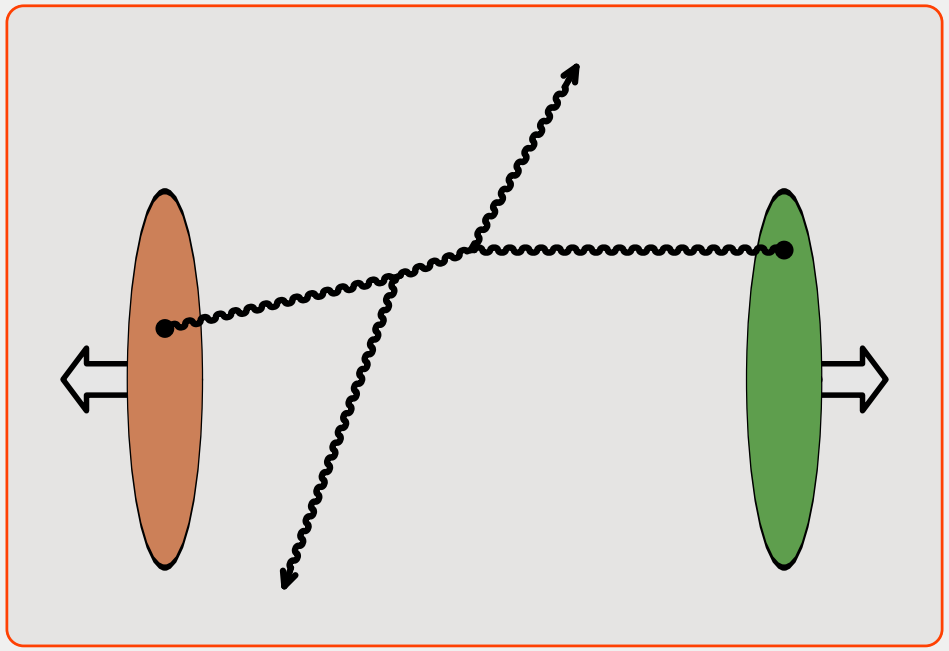
Experimental facilities : RHIC and LHC

Heavy Ion Collisions



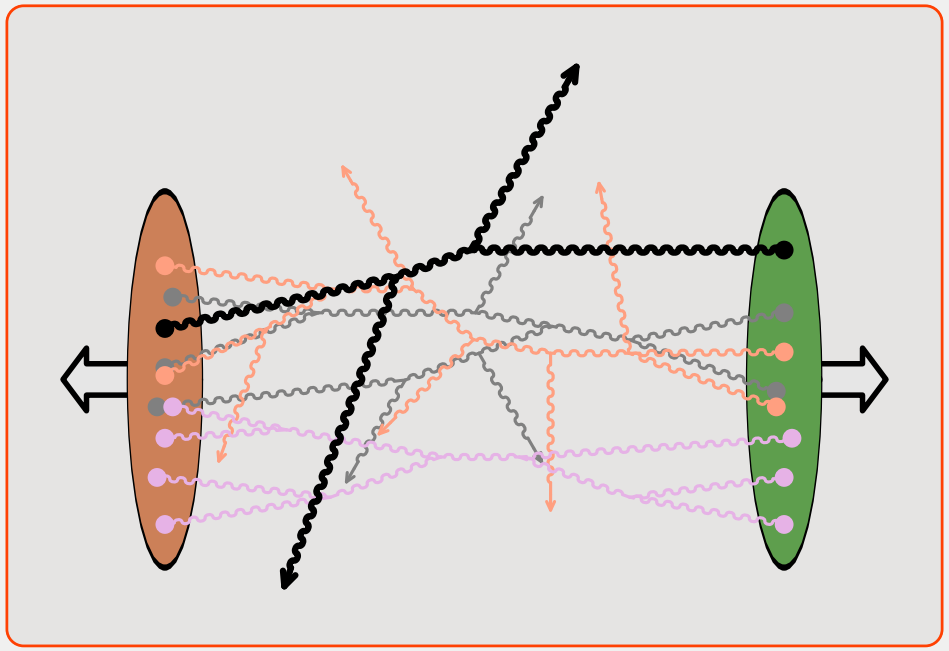
- Very high multiplicity (~ 20000 produced particles)
- Most of them rather soft ($P \lesssim 2$ GeV)

Initial state and Parton distributions



- **Factorization :** (partonic cross-section) $\otimes$ (parton distribution)
  Applicable to rare high momentum processes

Initial state and Parton distributions



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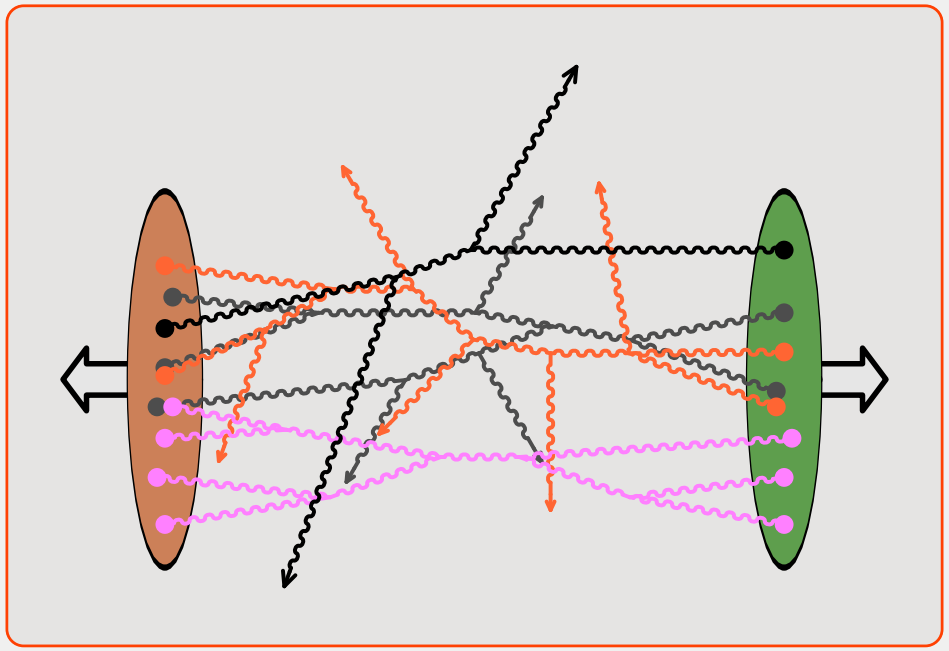
- **Underlying event :** cannot be calculated in this framework

Initial state and Parton distributions



- **Factorization :** (partonic cross-section) $\otimes$ (parton distribution)
  Applicable to rare high momentum processes
- **Underlying event :** cannot be calculated in this framework
- **In a Heavy Ion Collision**, this is the most interesting part...

Gluon Saturation

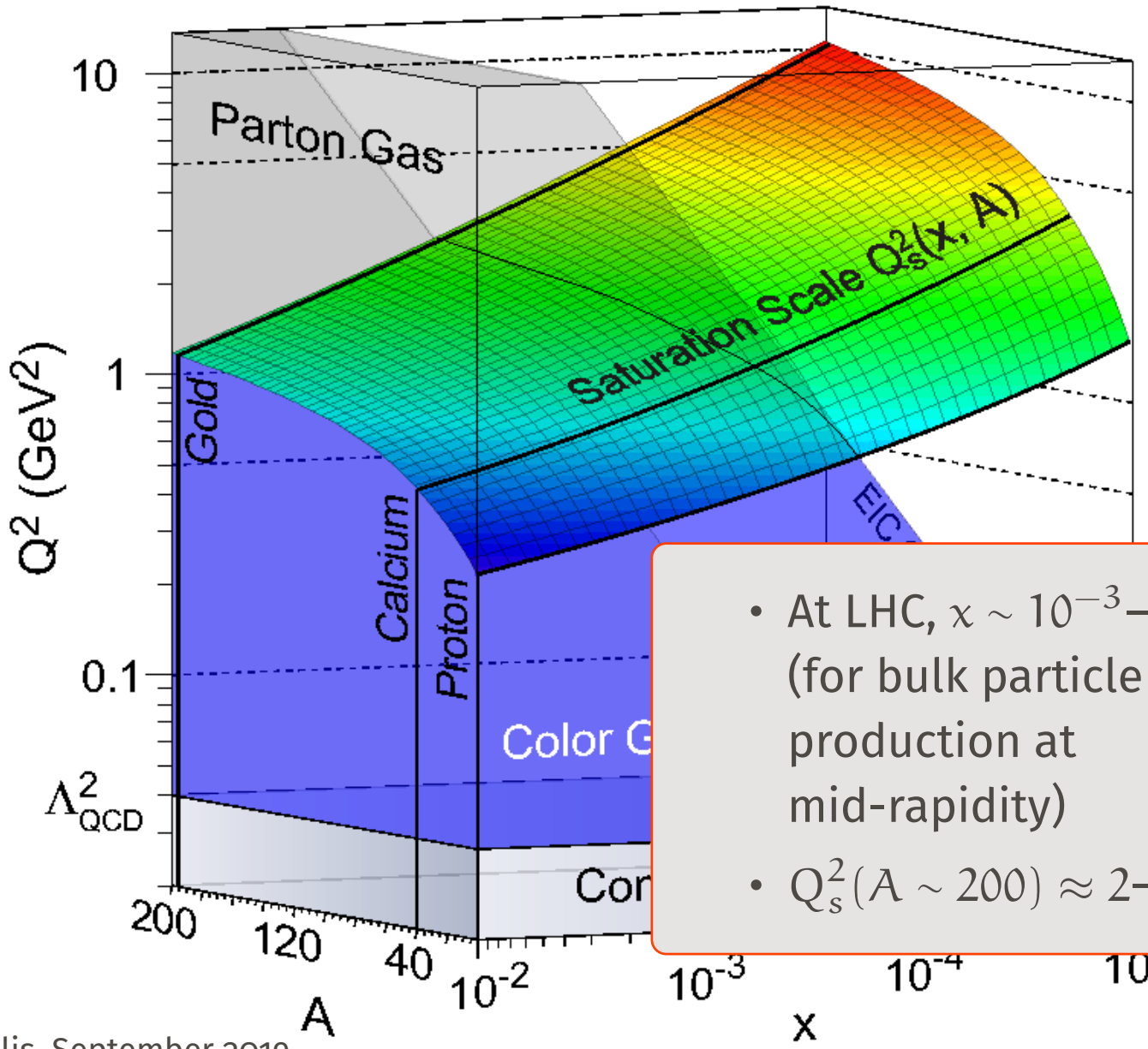
- Gluons recombine at large density

Saturation criterion **[Gribov, Levin, Ryskin (1983)]**

$$\underbrace{\alpha_s Q^{-2}}_{\sigma_{gg\to g}} \times \underbrace{A^{-2/3} x G(x, Q^2)}_{\text{surface density}} \geq 1$$

$$Q^2 \leq \underbrace{Q_s^2 \equiv \frac{\alpha_s x G(x, Q_s^2)}{A^{2/3}}}_{(\text{saturation momentum})^2} \sim A^{1/3} x^{-\lambda} \quad (\lambda \approx 0.25)$$

Saturation domain

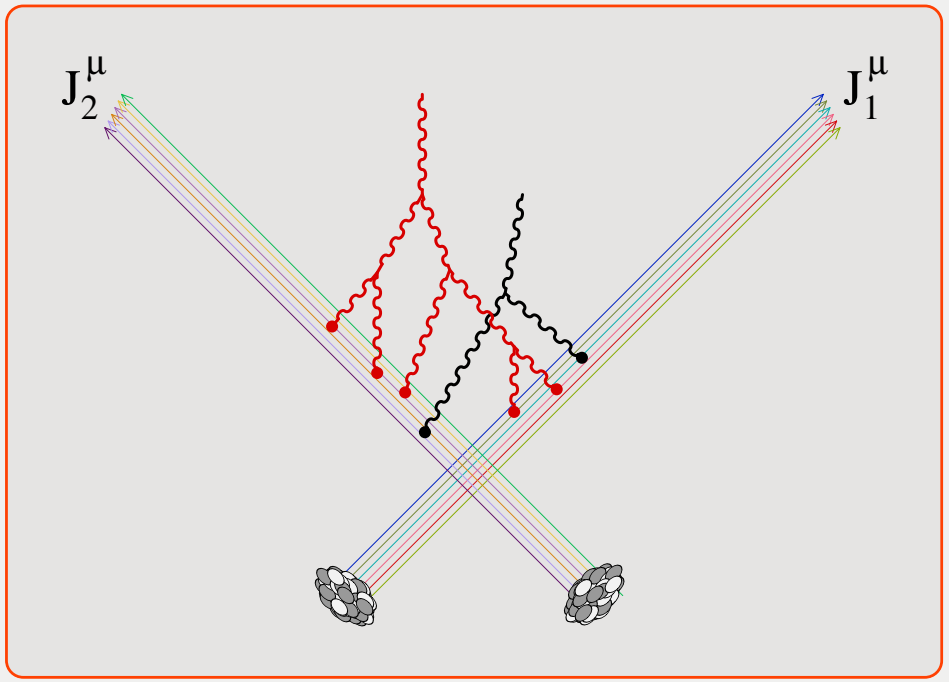


- At LHC, $x \sim 10^{-3}$–10^{-4} (for bulk particle production at mid-rapidity)
- $Q_s^2(A \sim 200) \approx 2$–$4$ GeV^2

Effective description: Color Glass Condensate

Snapshot of the constituents by color currents :

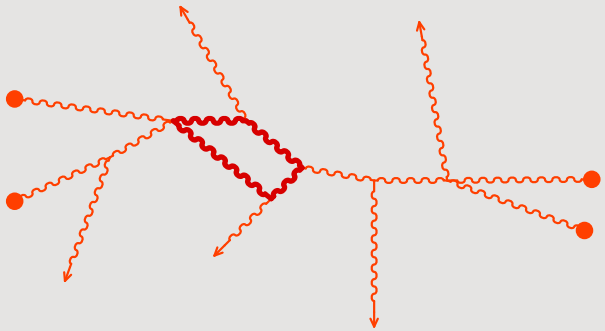
$$\mathcal{S} \equiv \int d^4x \left(-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + J^\mu(x) A_\mu(x) \right)$$



- Time dilation: static current
- Many constituents: J^μ large
- Current conservation: $[\mathcal{D}_\mu, J^\mu] = 0$

Quantum Field Theories with (Strong) Sources

Order of magnitude of connected graphs



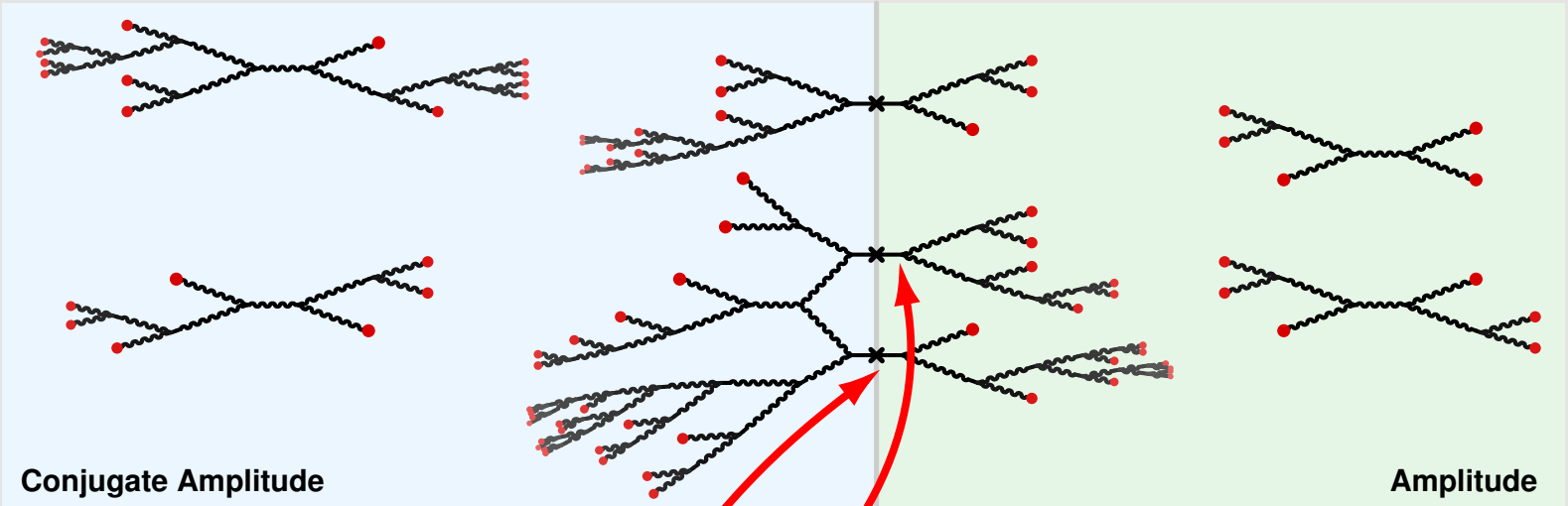
$$\sim \underbrace{g^{n_E-2}}_{\text{ext. lines}} \underbrace{(\hbar g^2)^{n_L}}_{\text{loops}} \underbrace{(g\,J)^{n_j}}_{\text{sources}}$$

- $g\,J \gtrsim 1$: strong source regime

  $\Rightarrow$ Non-perturbative dependence on $g\,J$

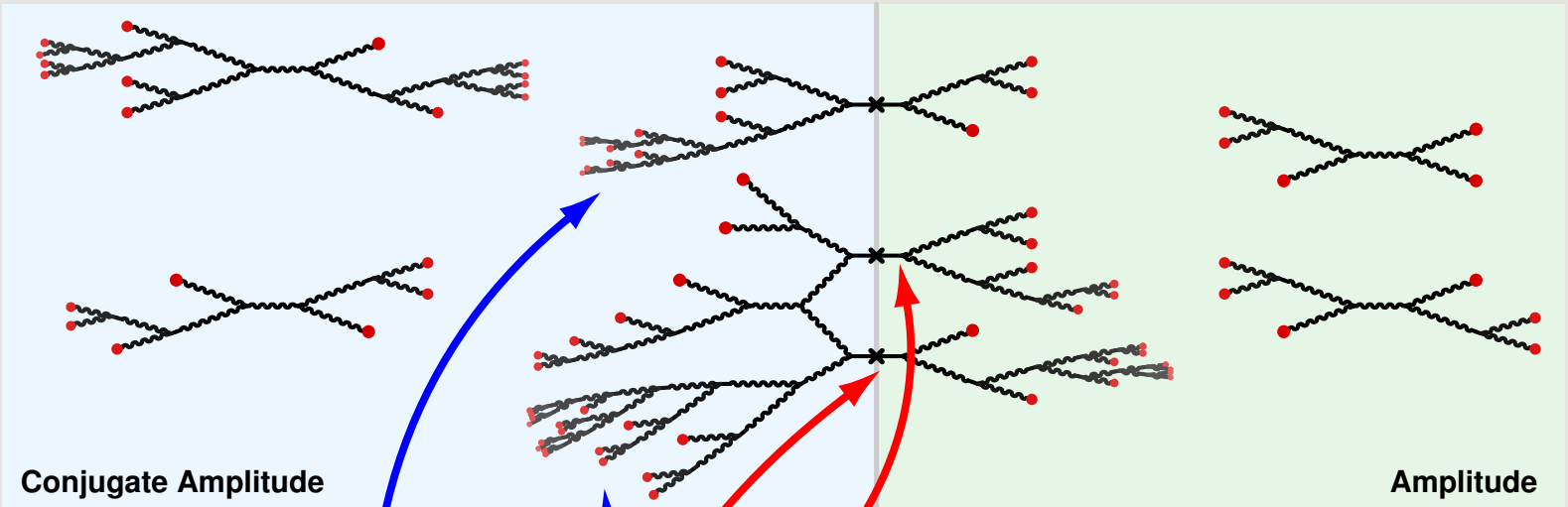
- **What happens when $g\,J \gtrsim 1$?**
  - Short mean free path
  - Thermalization

Exclusive final states : there be dragons...



- Correlations among the produced particles

Exclusive final states : there be dragons...



- Correlations among the produced particles
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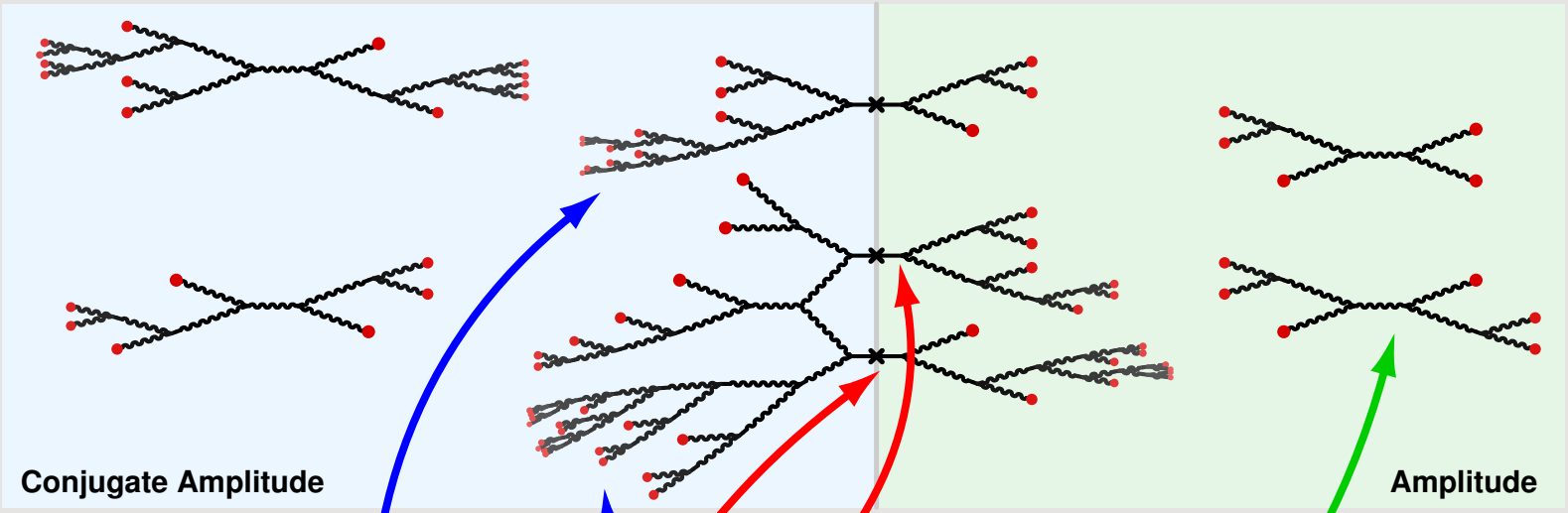

- Correlations among the produced particles
- Many disconnected graphs
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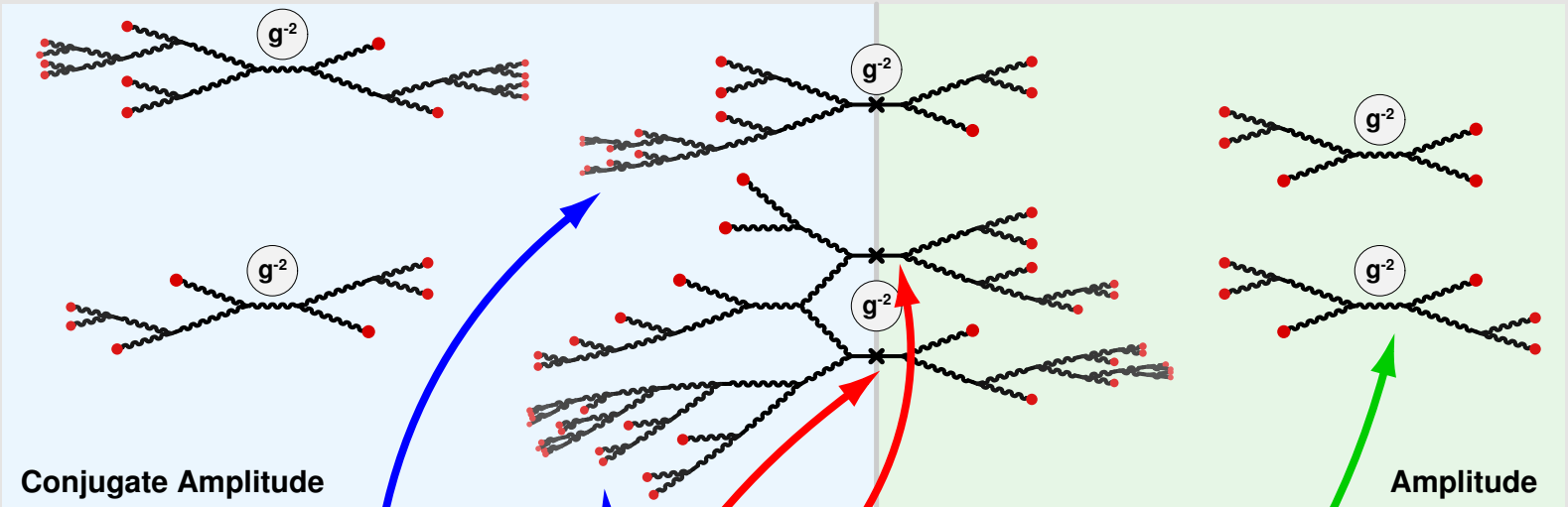
Exclusive final states : there be dragons...



- Correlations among the produced particles
- Many disconnected graphs
- Vacuum graphs do not cancel
- Pathological Taylor expansion : $g^{-2(\#\text{ connected components})}$

More modest goal : inclusive observables

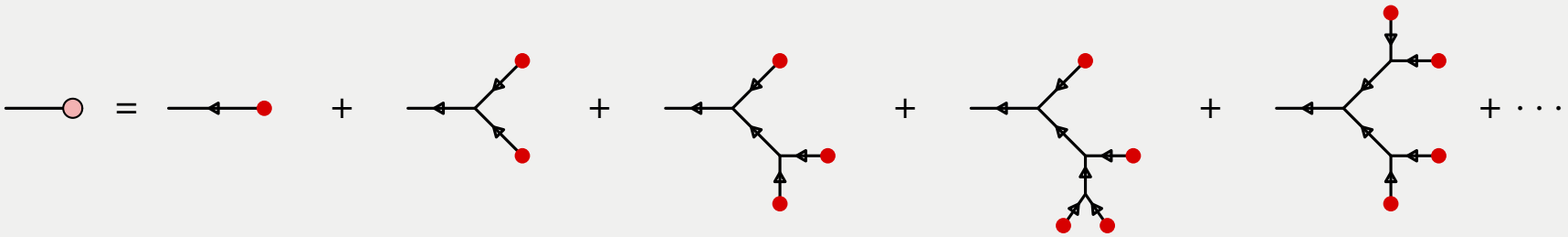
Example: gluon multiplicity

$$\left.\frac{d\overline{N}}{d^3\mathbf{p}}\right|_{LO} \sim \left|\widetilde{\mathcal{A}}(\mathbf{p})\right|^2$$

$$\underbrace{\left[\mathcal{D}_\mu, \mathcal{F}^{\mu\nu}\right] = J^\nu}_{\text{Yang-Mills eq.}} \qquad \lim_{t\to-\infty} \mathcal{A} = 0$$

- Sum of *connected graphs* (vacuum graphs cancel)
- Expressible in terms of the classical field with *retarded boundary conditions*

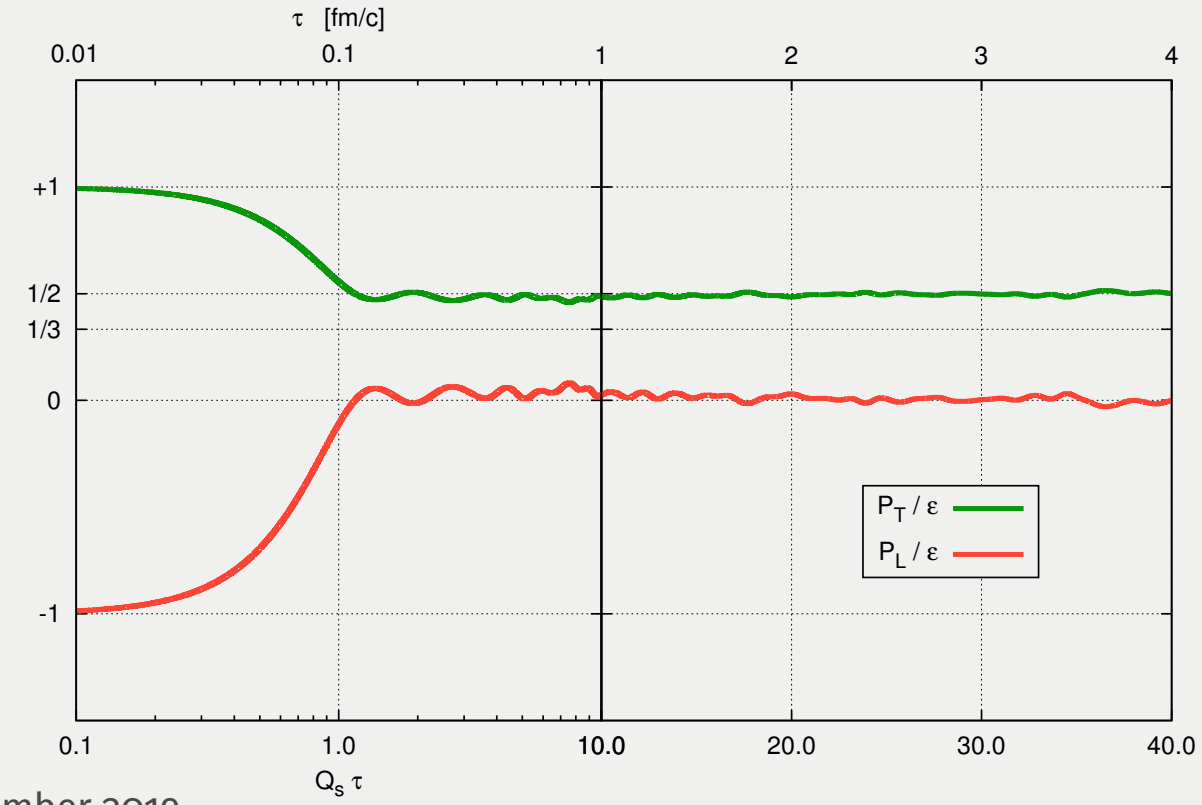
$\overline{N}_{LO} =$ 



Energy-Momentum Tensor at LO

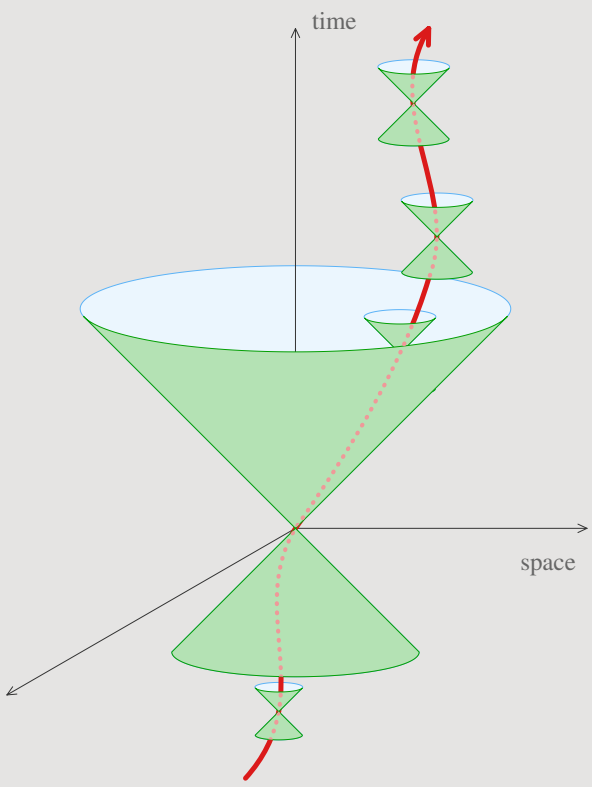
$$T^{00}_{\rm LO} = \frac{1}{2}\big[\underbrace{\mathbf{E}^2 + \mathbf{B}^2}_{\text{class. fields}}\big] \qquad T^{0i}_{\rm LO} = \big[\mathbf{E}\times\mathbf{B}\big]^i$$

$$T^{ij}_{\rm LO} = \frac{\delta^{ij}}{2}\big[\mathbf{E}^2 + \mathbf{B}^2\big] - \big[\mathbf{E}^i\mathbf{E}^j + \mathbf{B}^i\mathbf{B}^j\big]$$



Inclusive observables : generic features

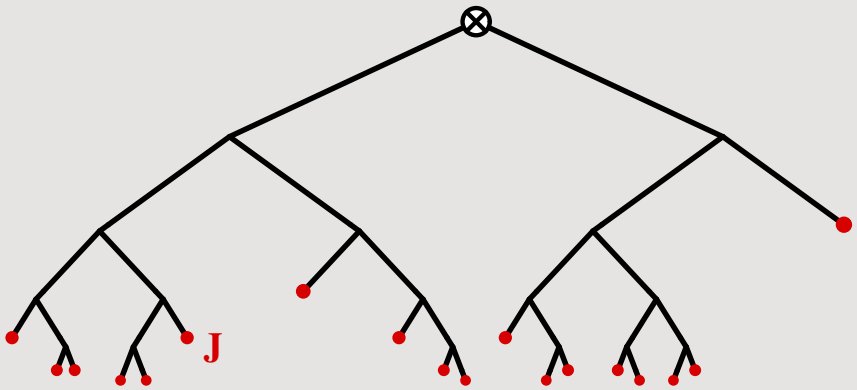
Retarded propagation



- **Inclusive** measurement :
  - Average of an observable over *all* final states
  - No constraint on the final state
  - No boundary condition for the fields at $t = +\infty$
- **Retarded = Causal evolution**
- Numerically straightforward

Inclusive observable at order $\hbar^0$

$\mathcal{O}_{\mathrm{LO}}[\mathrm{J}] =$



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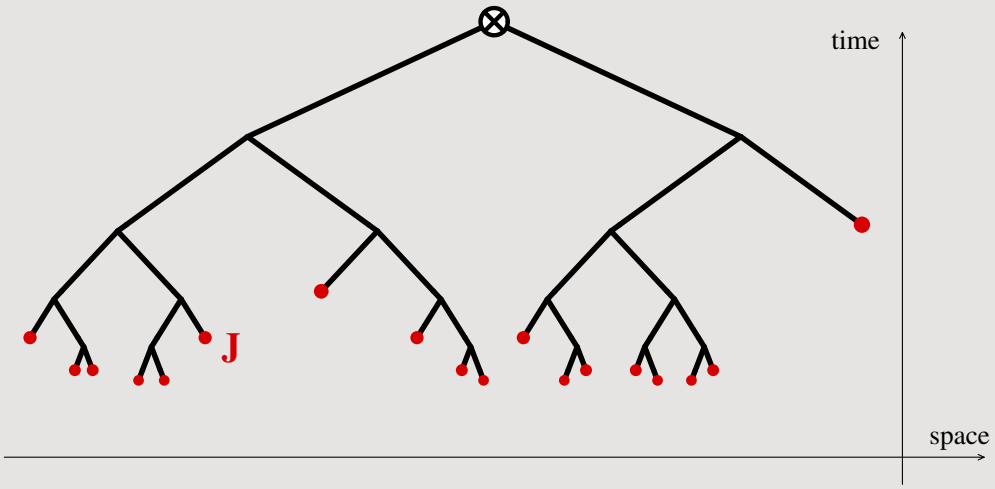

How to calculate the Next to Leading Order ?

Inclusive observable at order $\hbar^1$

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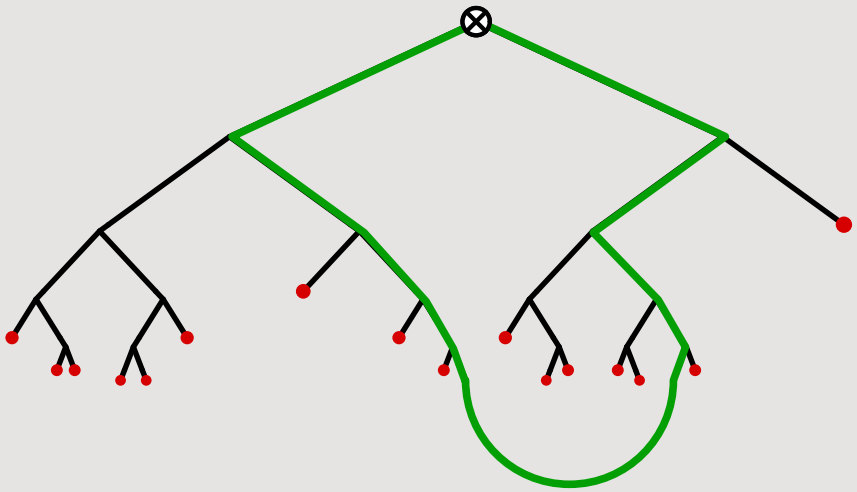
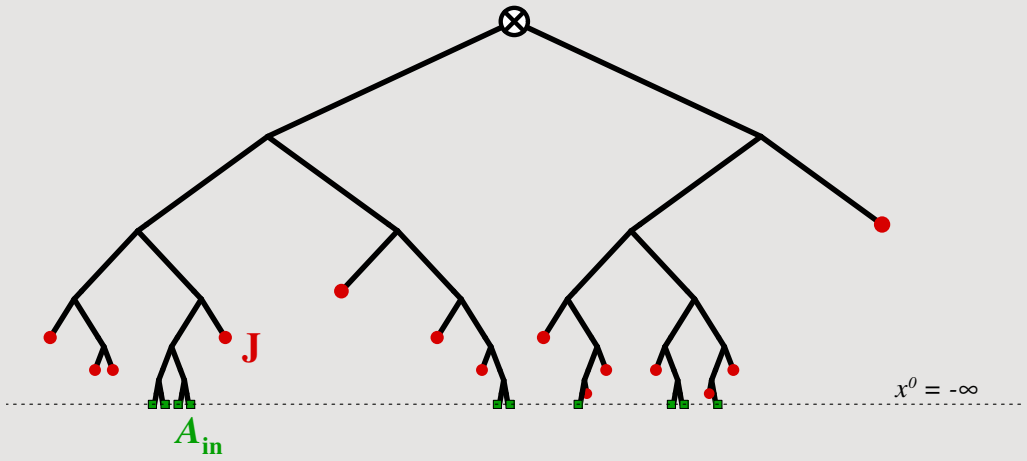


How to calculate the Next to Leading Order ?

Step 1 : generalize to an arbitrary initial field at $\mathrm{t} = -\infty$

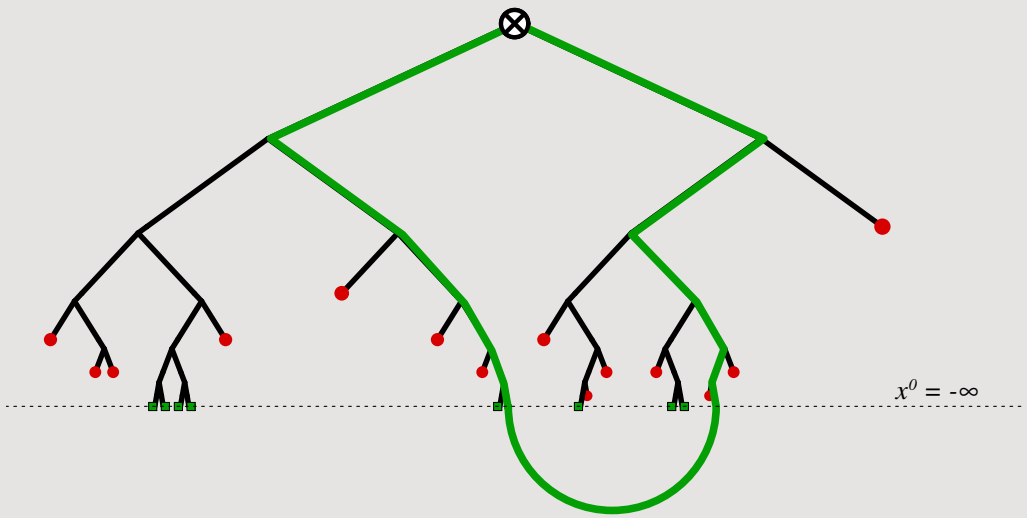
$$\mathcal{O}_{\mathrm{LO}}[\mathrm{J}, \mathcal{A}_{\mathrm{in}}] =$$



How to calculate the Next to Leading Order ?

Step 2 : add one loop

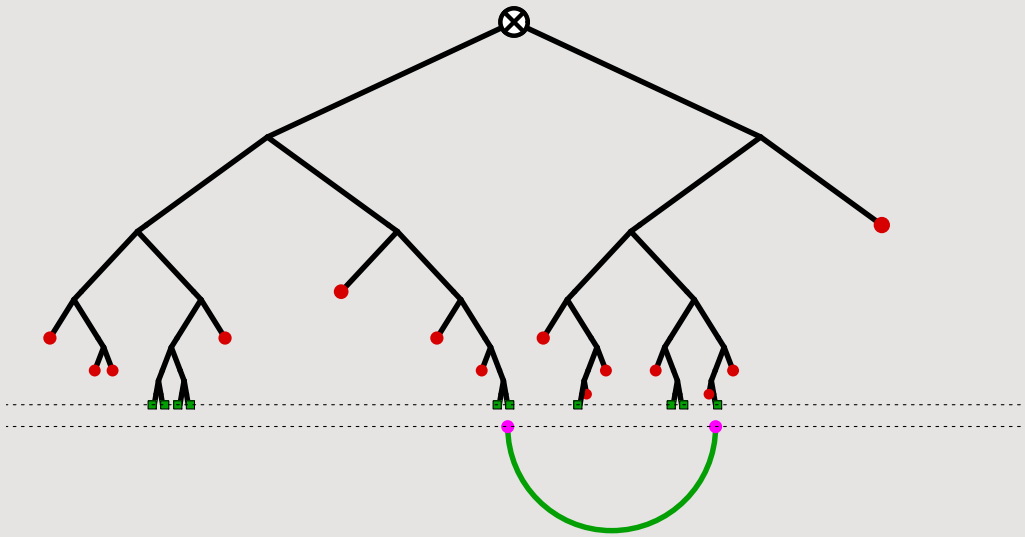
$$\mathcal{O}_{\mathrm{NLO}}[\mathrm{J}, \mathcal{A}_{\mathrm{in}}] =$$



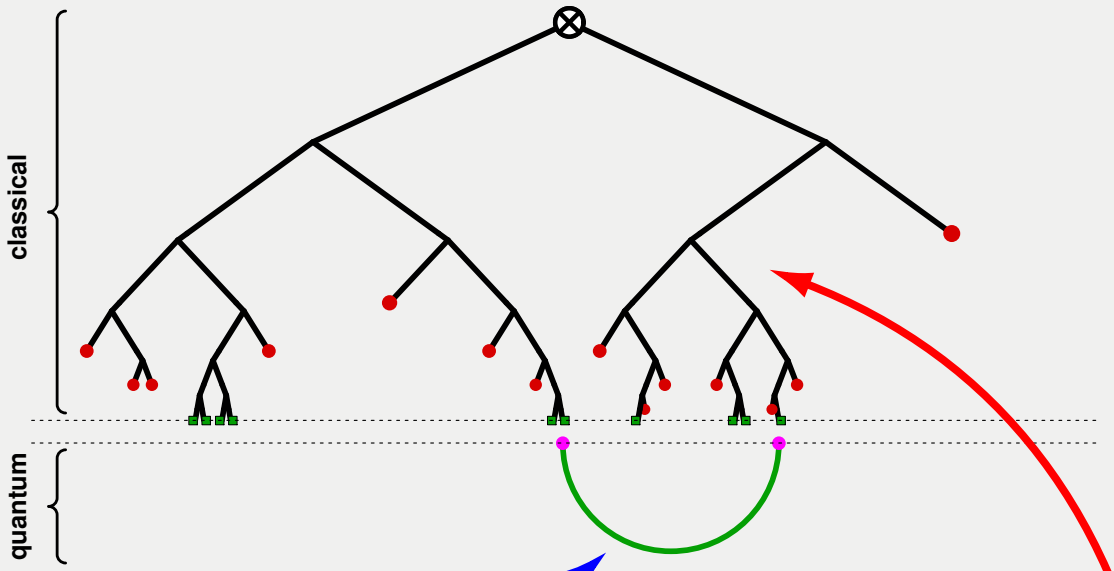
How to calculate the Next to Leading Order ?

Step 3 : view the loop as an operator acting on $\mathcal{O}_{\mathrm{LO}}$

$\mathcal{O}_{\mathrm{NLO}}[\mathrm{J}, \mathcal{A}_{\mathrm{in}}] =$



Next to Leading Order



$$\mathcal{O}_{\underline{\mathrm{NLO}}}[J, \mathcal{A}_{\mathrm{in}}] = \left[\frac{\hbar}{2} \int d^3x d^3y \; \Gamma(x, y) \; \frac{\delta}{\delta \mathcal{A}_{\mathrm{in}}(x)} \frac{\delta}{\delta \mathcal{A}_{\mathrm{in}}(y)} \right] \mathcal{O}_{\mathrm{LO}}[J, \mathcal{A}_{\mathrm{in}}]$$

Remarks

- $\Gamma(x, y)$ is *universal*, and known analytically :

$$\Gamma(x, y) = \int \frac{d^3 p}{(2\pi)^3 2E_p} \, e^{ip\cdot(x-y)}$$

- *LO contains NLO* (in a somewhat obfuscated way...)
- Applications :
  - JIMWLK factorization of large logs of $p^{\pm}$
  - Study of isotropization/thermalization

What happens if the

classical dynamics is unstable ?

Instabilities

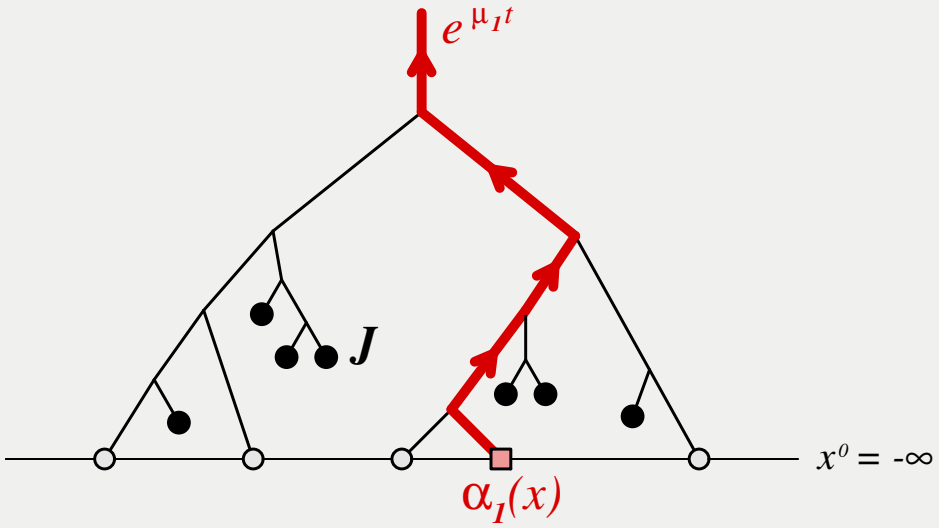
- The derivatives $\delta\mathcal{O}_{\rm LO}/\delta\mathcal{A}_{\rm in}$ are large if the classical solutions have instabilities (they measure the sensitivity to the initial condition)
- This behaviour is ubiquitous in field theory:
  - Scalar field with a ϕ^4 interaction : parametric resonance
  - Yang-Mills theory : Weibel instability
- Consequence : $\mathcal{O}_{\rm NLO}$ growths exponentially with time, and eventually becomes larger than $\mathcal{O}_{\rm LO}$

  $\Longrightarrow$ breakdown of the perturbative expansion

Improved power counting

- For an unstable mode:

$$\alpha(x) \underset{x^0 \to +\infty}{\sim} e^{\mu x^0} \qquad (\mu = \text{Lyapunov exponent})$$



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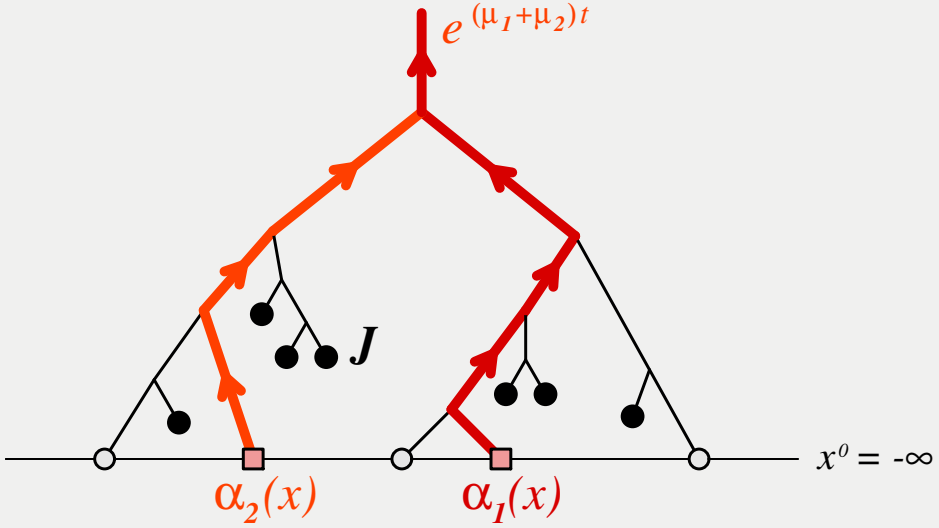

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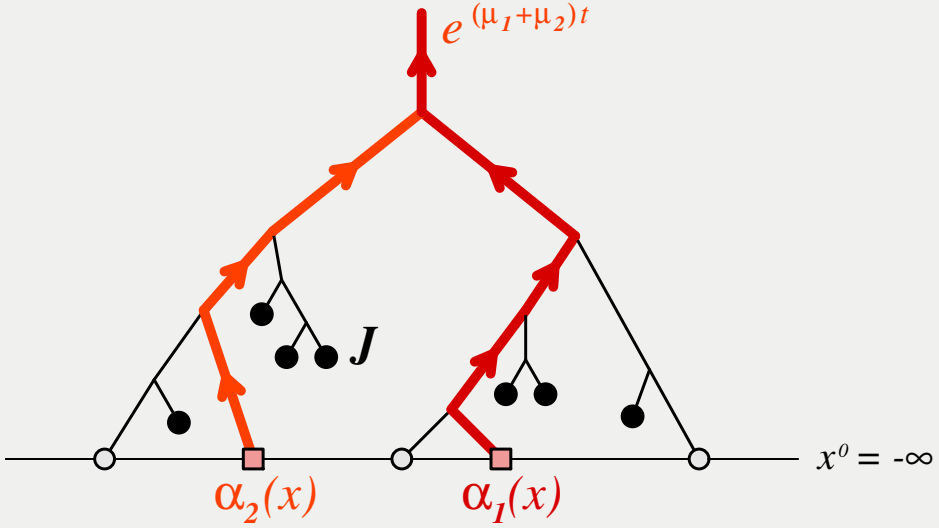
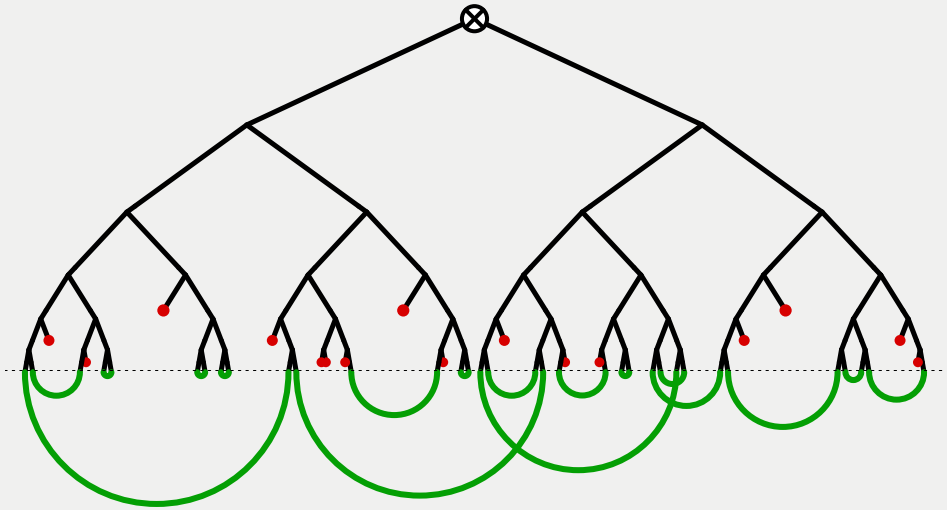


- $\mathcal{O}_{\text{NLO}} \sim e^{2\mu t}$
- At order n, there are terms $\sim e^{2n\mu t}$

Resummation of the leading terms

Resummation

$$\mathcal{O}_{\text{RESUM}} \equiv \exp\left[\frac{\hbar}{2}\int d^3x d^3y \;\; \Gamma(x,y) \;\; \frac{\delta}{\delta\mathcal{A}_{\text{in}}(x)}\frac{\delta}{\delta\mathcal{A}_{\text{in}}(y)}\right]\mathcal{O}_{\text{LO}}$$



$\mathcal{O}_{\text{RESUM}} = \mathcal{O}_{\text{LO}} + \mathcal{O}_{\text{NLO}} +$ subset of all higher orders

Leading terms : Classical Statistical Approximation

$$\underbrace{\exp\left[\frac{\hbar}{2}\int_{x,y}\underbrace{\Gamma_2(x,y)\,\frac{\delta}{\delta\mathcal{A}_{\rm in}(x)}\frac{\delta}{\delta\mathcal{A}_{\rm in}(y)}}_{\text{"Laplacian"}}\right]}_{\text{Diffusion operator on the classical phase-space}}\mathcal{O}_{\rm LO}[\mathcal{A}_{\rm in}]$$

$$=\int[\mathrm{D}a]\,\exp\left[-\frac{1}{2\hbar}\int_{x,y}a(x)\Gamma_2^{-1}(x,y)a(y)\right]\mathcal{O}_{\rm LO}[\mathcal{A}_{\rm in}+a]$$

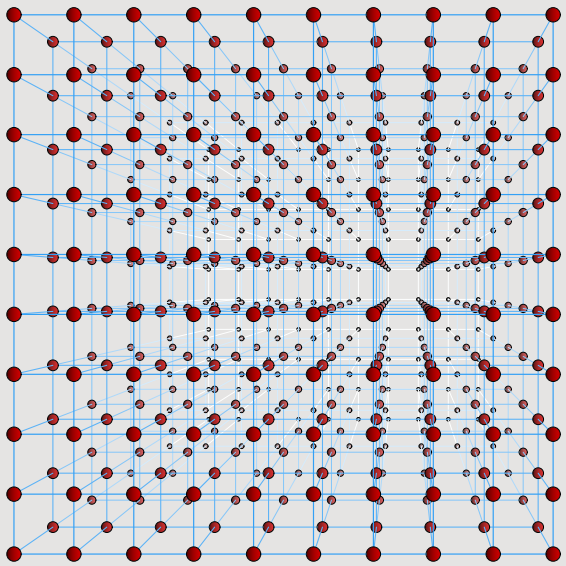
- In this resummation, the observable is obtained as an average over classical fields with fluctuating initial conditions
- The exponentiation of the 1-loop result promotes the classical vacuum $\mathcal{A}_{\rm in}\equiv 0$ into the coherent quantum state $\left|0_{\rm in}\right\rangle$

Numerical implementation

Hamiltonian lattice formalism

Space

$\Rightarrow$ **3D cubic lattice**

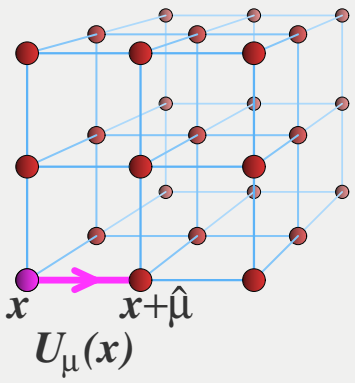


- Discrete space, continuous time
- Hamilton equations :

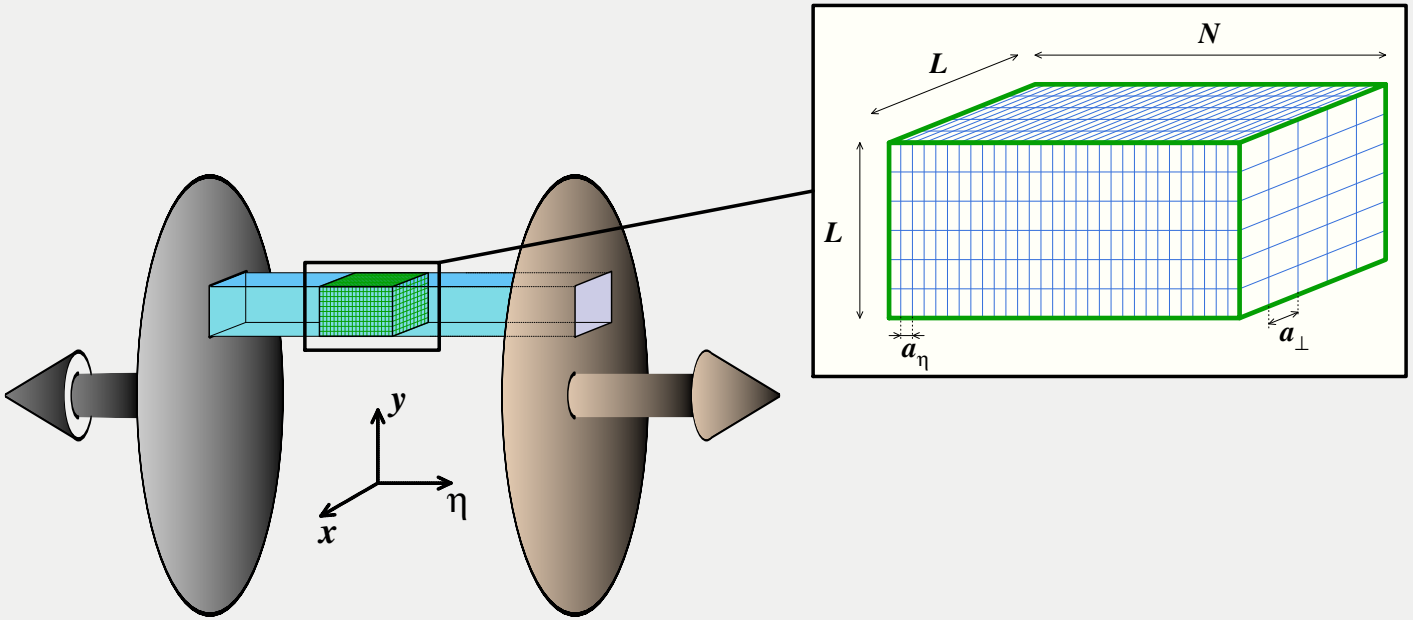
$$\partial_t \mathcal{A} = \mathcal{E}$$
$$\partial_t \mathcal{E} = F(\mathcal{A})$$

- **Yang-Mills case :**

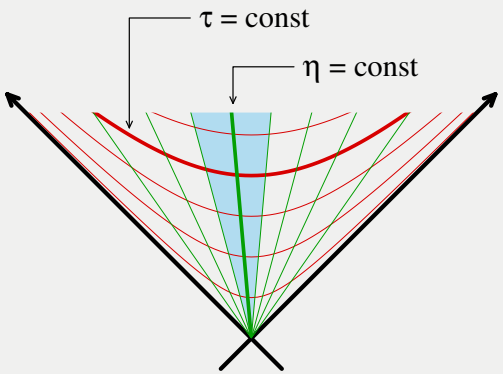
  Use link variables instead of $\mathcal{A}$ to preserve residual gauge symmetry



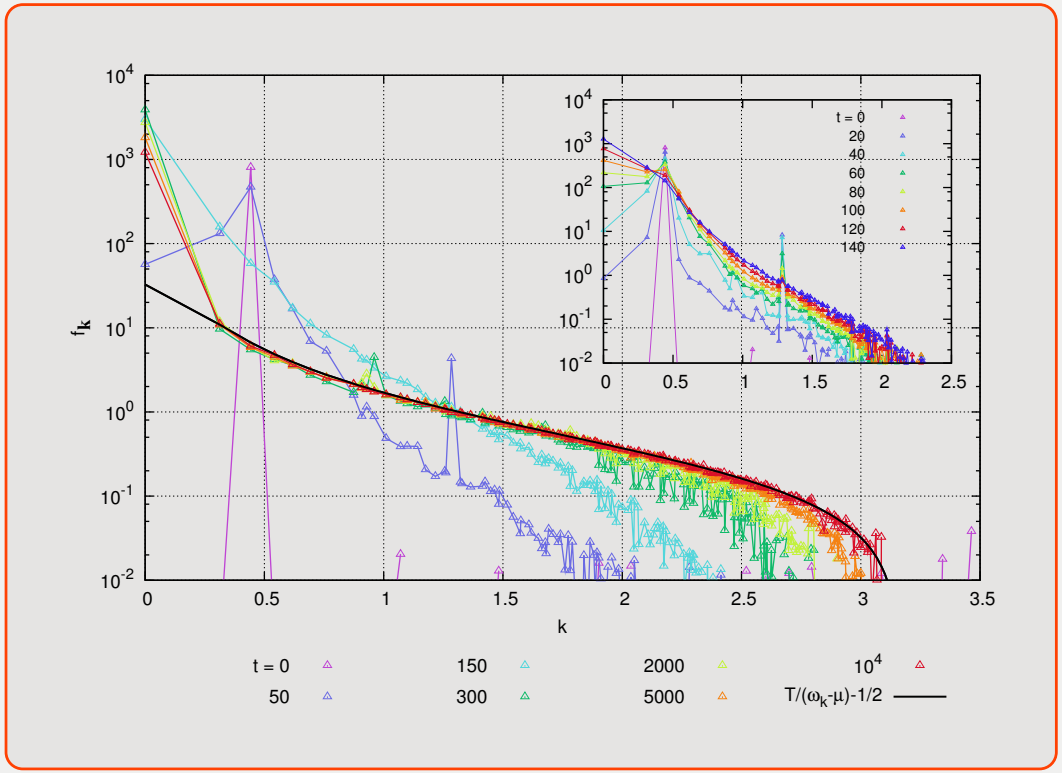
Discretization of the expanding volume



- Comoving coordinates : $\tau, \eta, x_\perp$
- Only a small volume is simulated
  + periodic boundary conditions

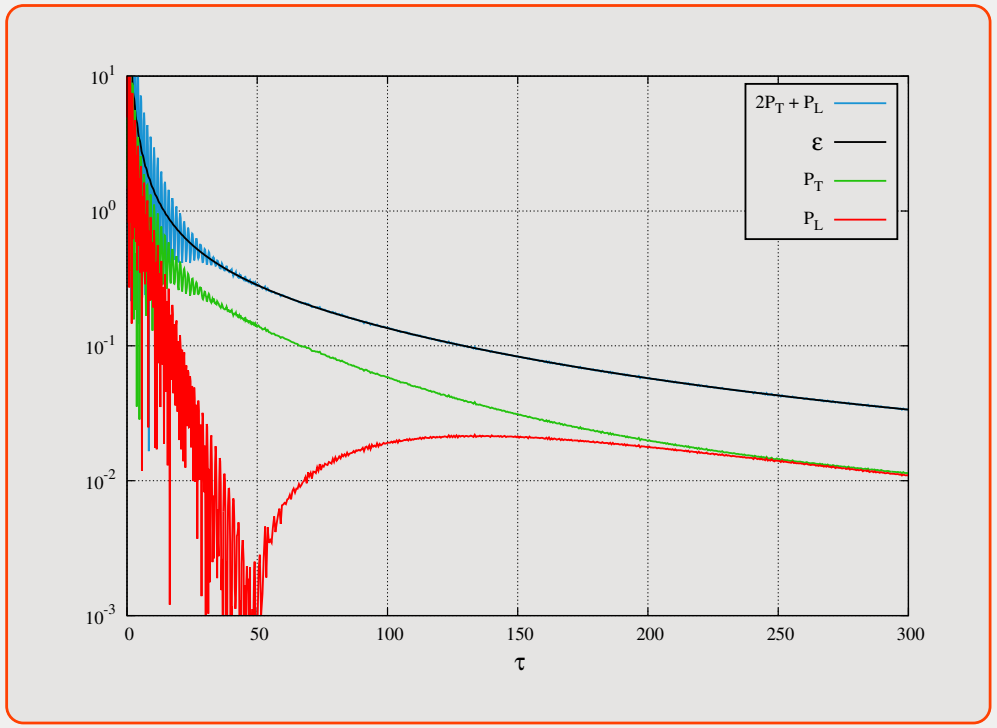


Thermalization



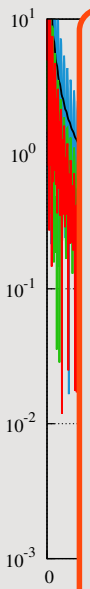
- Unstable modes grow very quickly
- Other modes are filled later
- Asymptotic distribution: classical equilibrium $T(\omega-\mu)^{-1}-\frac{1}{2}$

Pressure isotropization



- At early times, P_L drops much faster than P_T (redshifting of the longitudinal momenta due to the expansion)
- Drastic change of behavior when the expansion rate becomes smaller than the growth rate of the unstability
- Eventually, isotropic pressure tensor : $P_L \approx P_T$

Pressure isotropization



Pros :

- Straightforward implementation
- Manifest residual gauge symmetry

Caveats :

- Non renormalizable approximation
- Sensitive to the UV cutoff

- At early times, P_L drops much faster than P_T (rodshifting of the dinal momenta he expansion)

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lly, isotropic e tensor :

$P_L \approx P_T$

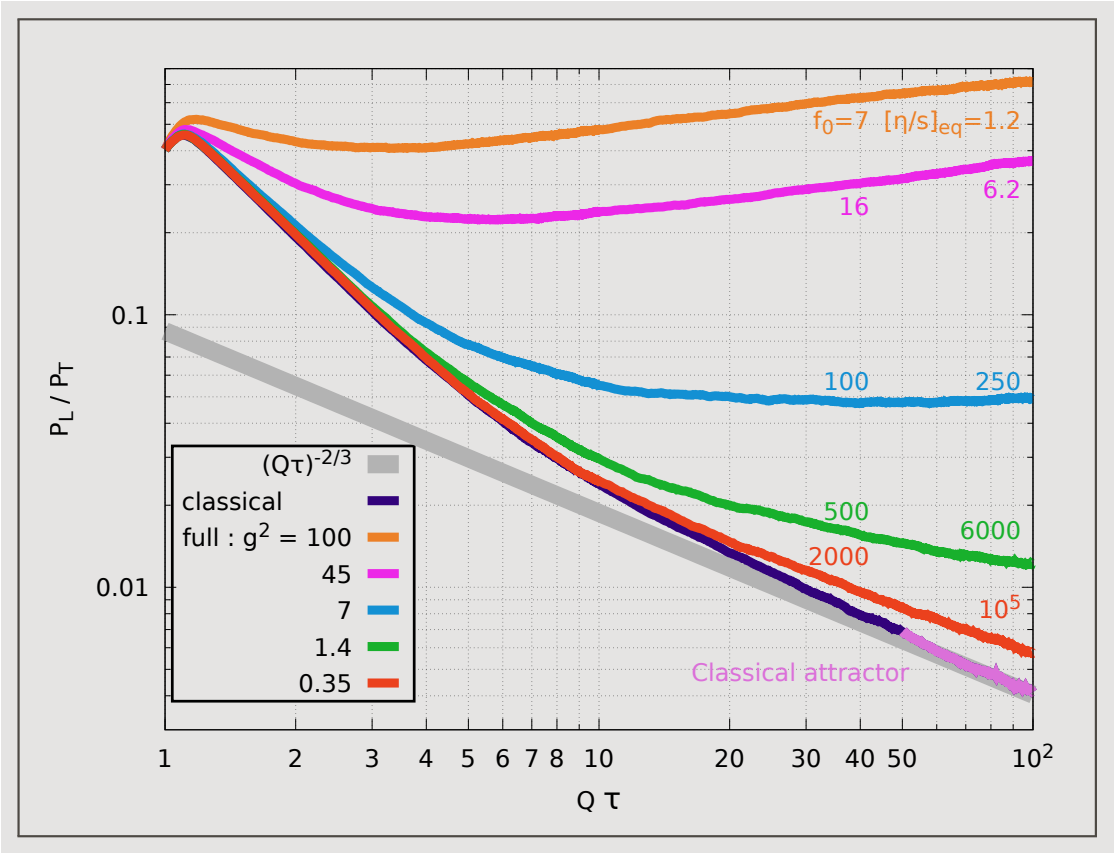
Alternatives

all-orders QFT $\rightarrow$ Kadanoff-Baym equations $\rightarrow$ Kinetic theory

	Semi-Classical	**Kinetic theory**	**Kadanoff-Baym**
Cont. limit	✗	✓	✓
Screening	✓	✗	✓

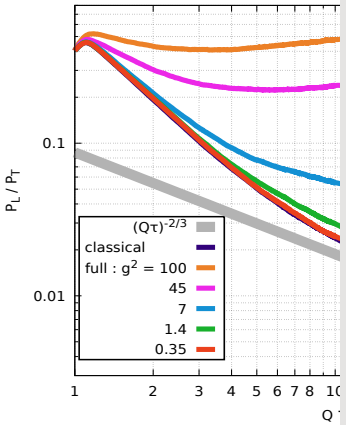
- BUT: Kadanoff-Baym equations are hard to implement and very heavy to solve numerically..

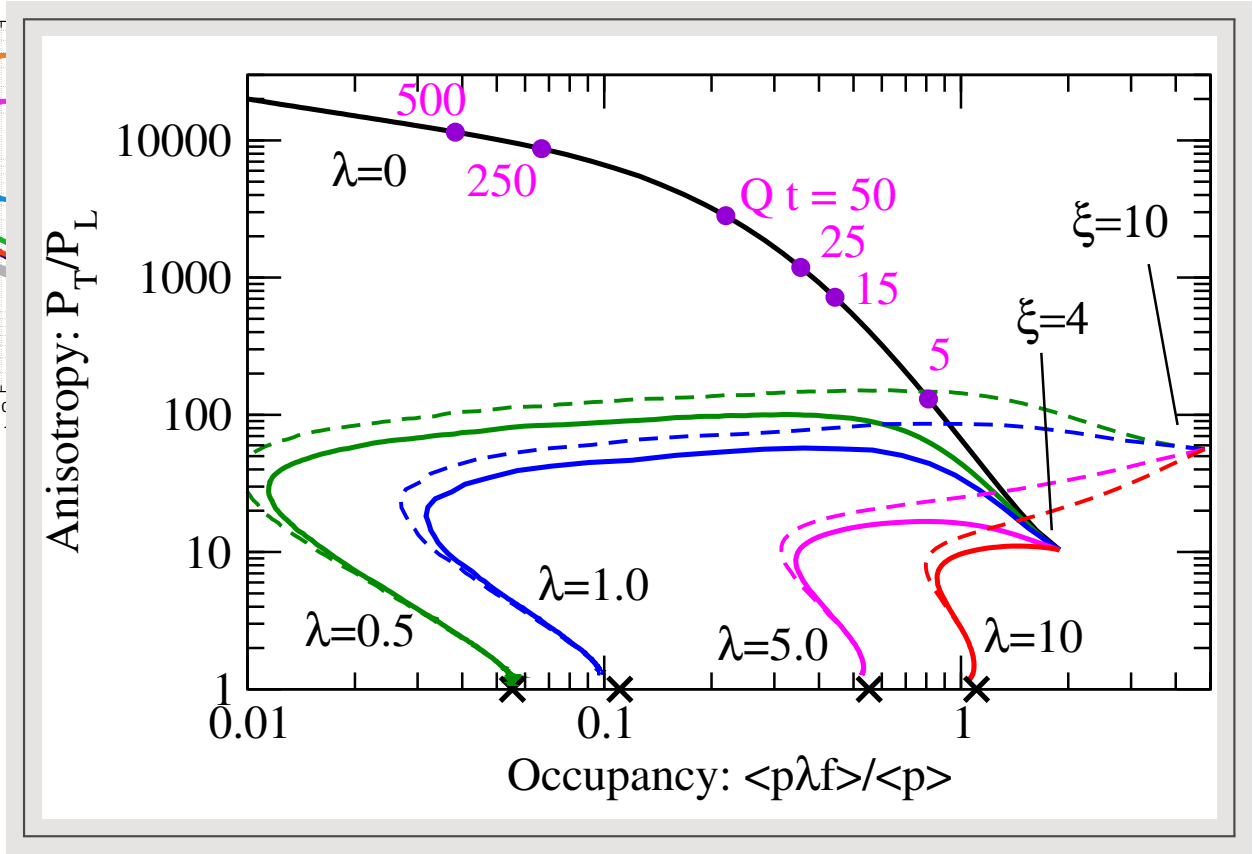
Insights from kinetic theory



- Zero point fluctuations matter

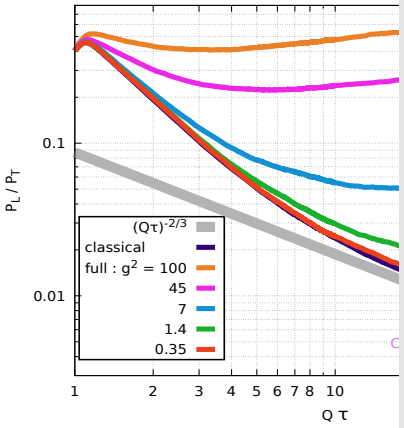
Insights from kinetic theory

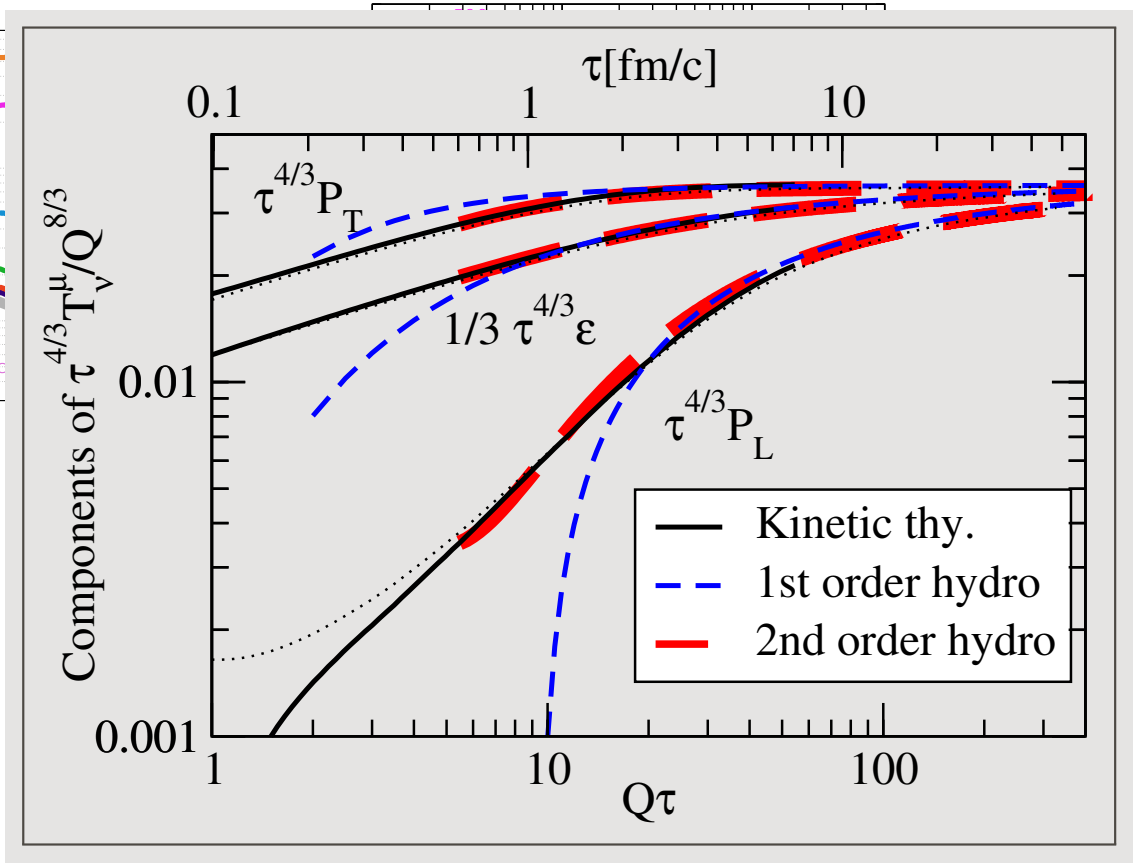




- Isotropization is rather fast

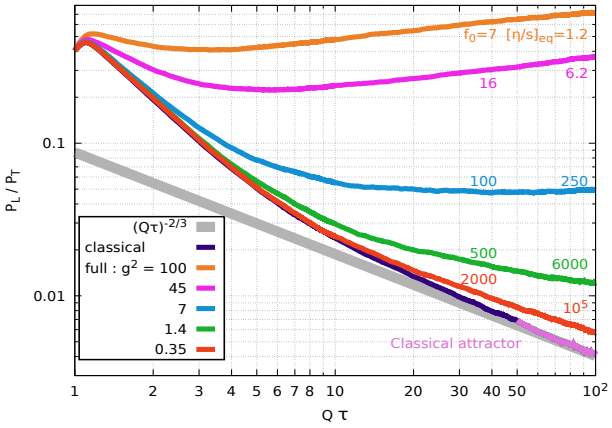
Insights from kinetic theory

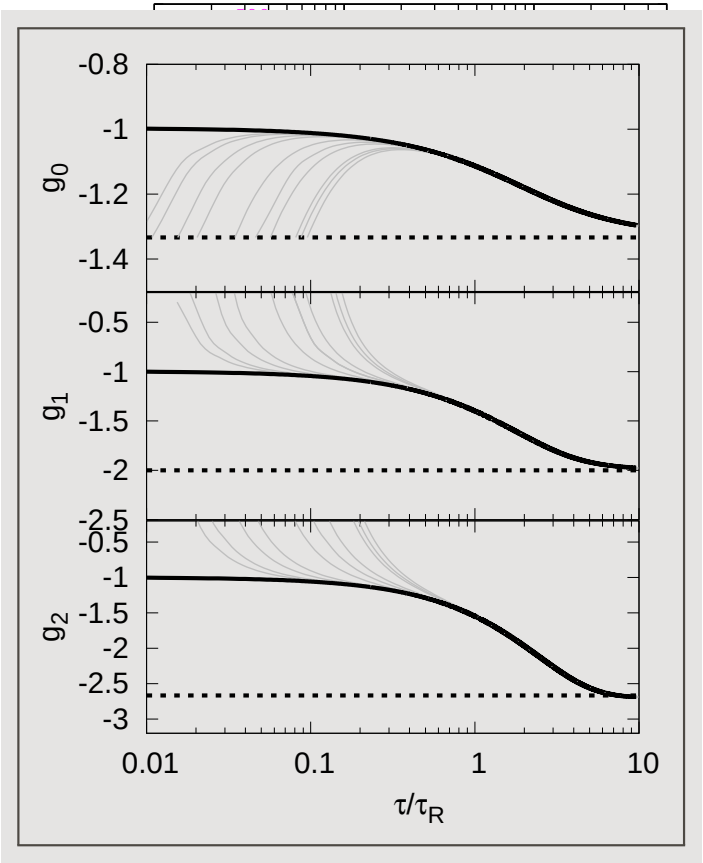


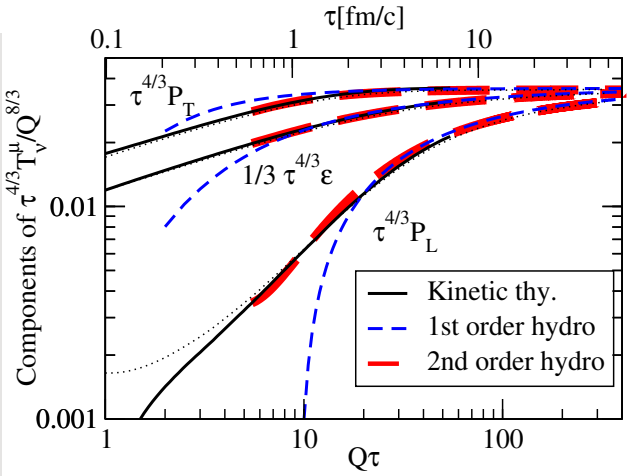


- Hydrodynamical behavior in less than 1 fm/c

Insights from kinetic theory





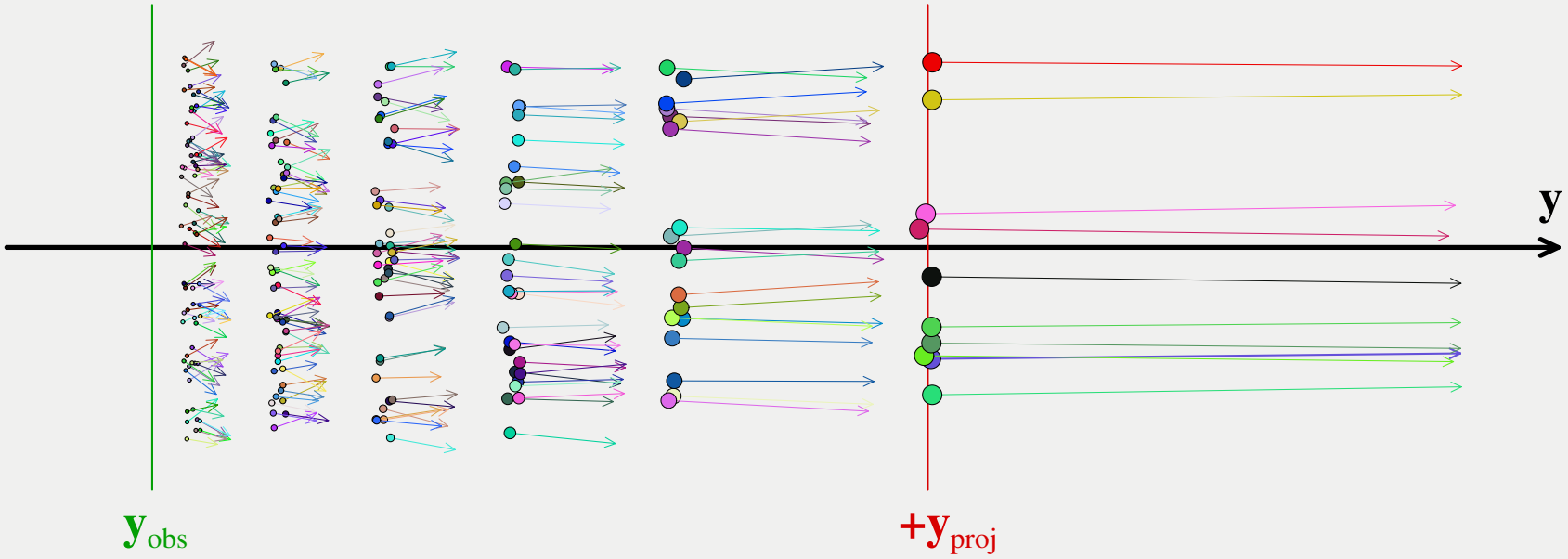


- Only two attractors: free streaming and isotropic

Thank you !!

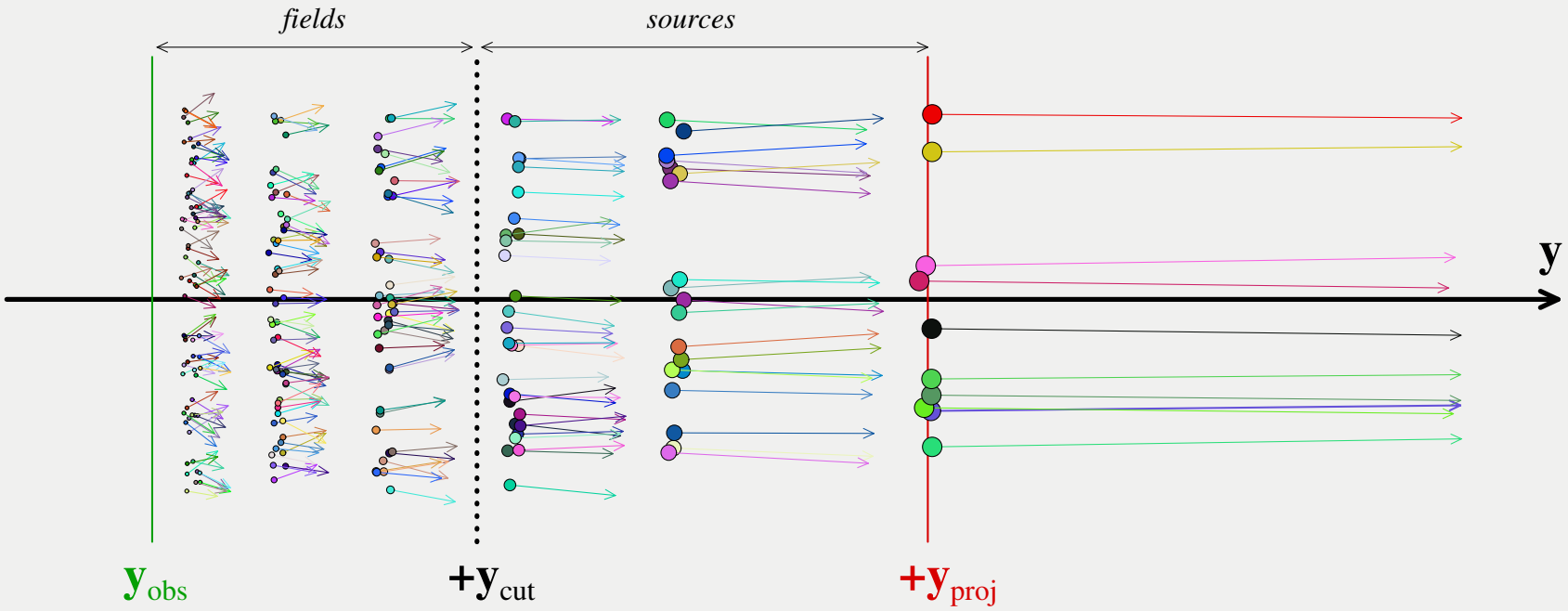
Backup

Degrees of freedom



- $p_\perp^2 \sim Q_s^2 \sim \Lambda_{QCD}\, e^{\lambda(y_{proj}-y)}$, $p_z \sim Q_s\, e^{y-y_{obs}}$
- Fast partons : frozen dynamics, negligible $p_\perp$ $\Rightarrow$ classical sources
- Slow partons : evolve with time $\Rightarrow$ gauge fields

Degrees of freedom



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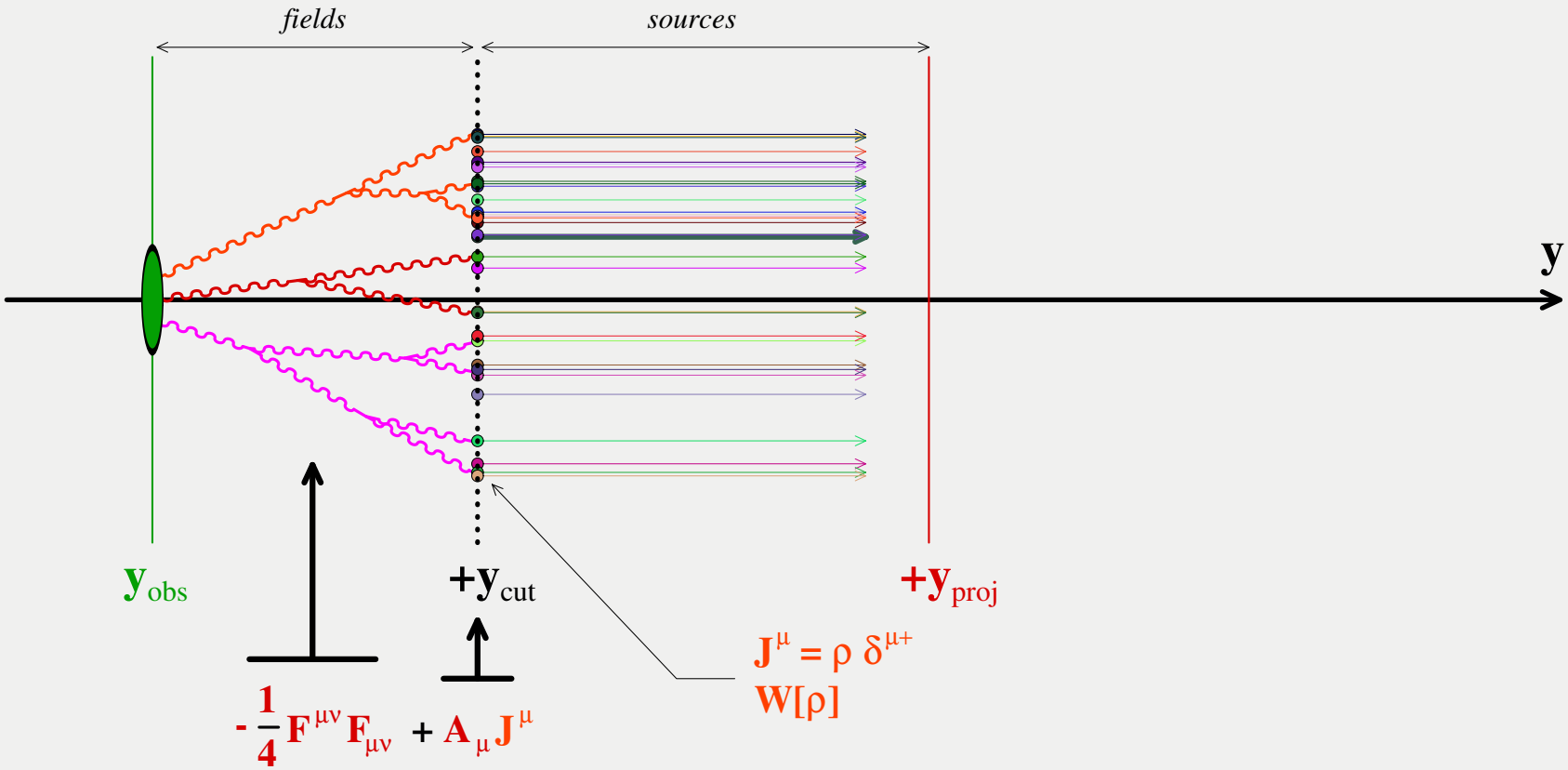

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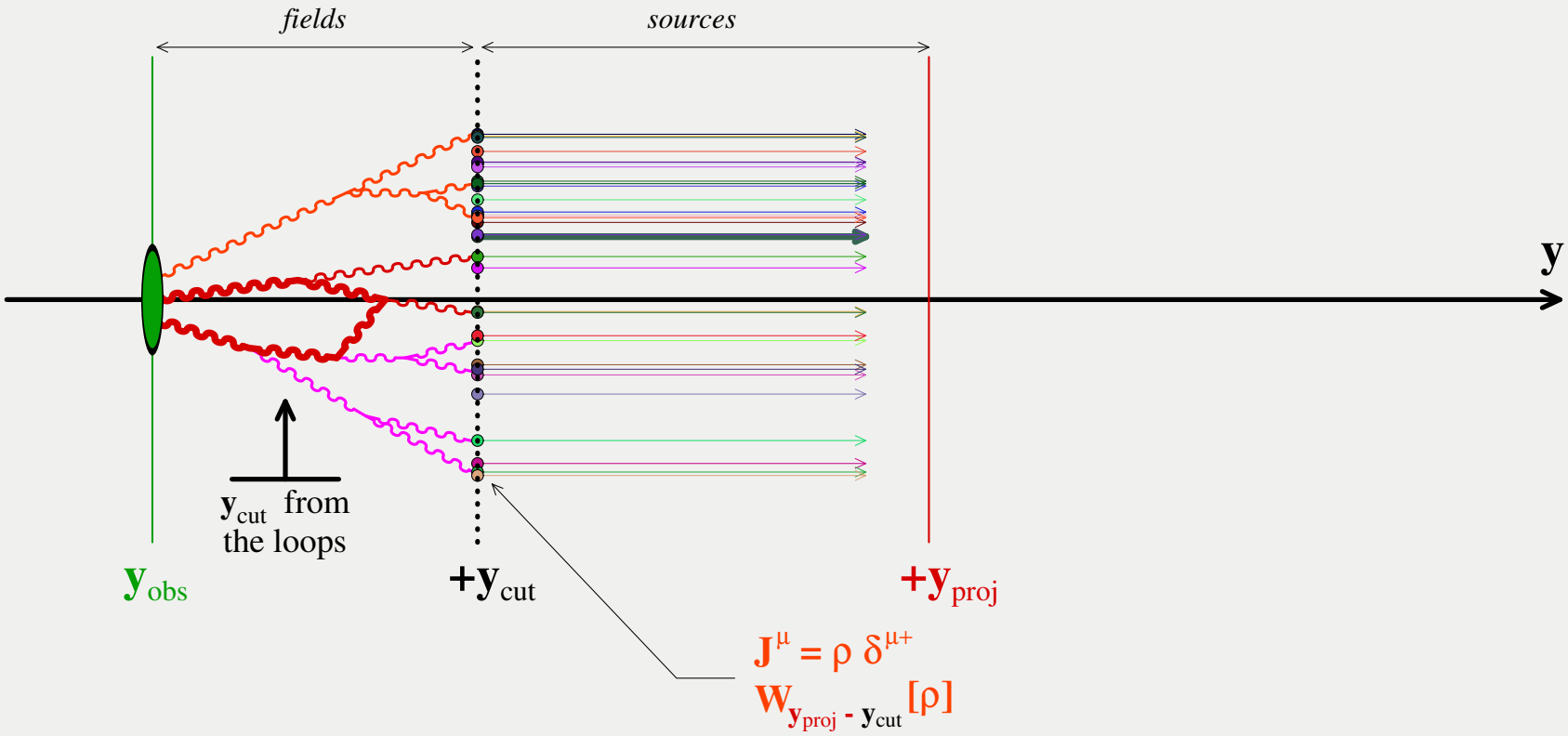
Cancellation of the cutoff dependence



- The cutoff y_{cut} is arbitrary and should not affect physical results
- Loop corrections depend on the cutoff in a universal way
- $W[\rho]$ must depend on y_{cut} to cancel this dependence

B-JIMWLK EVOLUTION EQUATION

Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner

$$\frac{\partial W_Y[\rho]}{\partial Y} = \underbrace{\frac{1}{2}\int_{\vec{x}_\perp,\vec{y}_\perp} \frac{\delta}{\delta\rho_a(\vec{x}_\perp)}\chi_{ab}(\vec{x}_\perp,\vec{y}_\perp)\frac{\delta}{\delta\rho_b(\vec{y}_\perp)}}_{\mathcal{H}\quad\text{(JIMWLK Hamiltonian)}} W_Y[\rho]$$

- Mean field approx. (BK equation) : [Kovchegov (1999)]
- Langevin form of B-JIMWLK : [Blaizot, Iancu, Weigert (2003)]
- First numerical solution : [Rummukainen, Weigert (2004)]
- Note: $Y \sim$ logarithm of longitudinal momentum

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$$\partial W_Y[\rho] = \frac{1}{\ldots}\int \ldots \frac{\delta}{\ldots} \ldots \frac{\delta}{\ldots}$$

More recent developments :

Running coupling correction

[Lappi, Mäntysaari (2012)]

B-JIMWLK equation at Next to Leading Log

[Kovner, Lublinsky, Mulian (2013)]

[Caron-Huot (2013)] [Balitsky, Chirilli (2013)]

- Me
- Lar ... 03)]
- Firs ... 4)]
- Note: $Y \sim$ logarithm of longitudinal momentum

B-JIMWLK EVOLUTION EQUATION

Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner

$\partial W_{\ldots}[\rho] \quad 1 \int \quad \delta \quad \delta$

More recent developments :

R

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B

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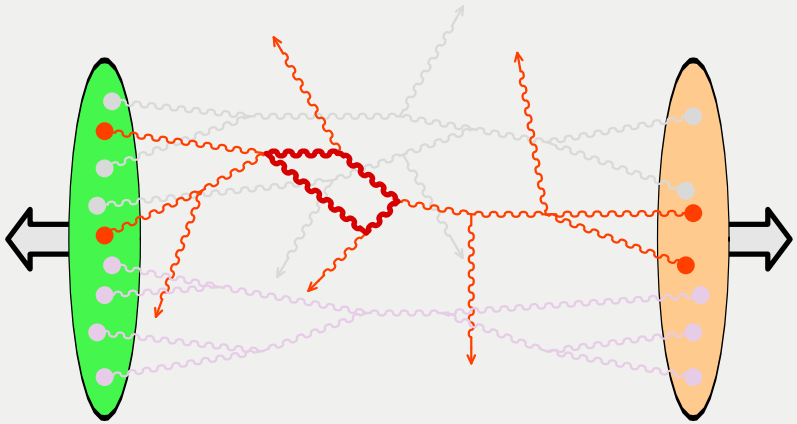
Open questions for practical uses :

- Does the NLO evolution preserve the positivity of $W[\rho]$? (non trivial if the JIMWLK Hamiltonian contains higher derivatives at NLO)
- Can the NLO B-JIMWLK equation still be mapped into a Langevin equation?

- Me
- Lar
- Firs
- Note:

Power counting

$$\mathcal{S} = \underbrace{-\frac{1}{4}\int F_{\mu\nu}F^{\mu\nu}}_{\text{slow}} + \int \underbrace{(J_1^\mu + J_2^\mu)}_{\text{fast}} A_\mu$$



In the saturated regime: $J^\mu \sim g^{-1}$, $A^\mu \sim g^{-1}$, $f_k \sim g^{-2}$

$$T^{\mu\nu} \sim \frac{1}{g^2}\left[c_0 + c_1\, g^2 + c_2\, g^4 + \cdots\right]$$

$c_0 \equiv$ tree level, $c_1 \equiv$ one loop, etc...

Inclusive observables at NLO

[FG, Lappi, Venugopalan (2007–2008)]

- Observables at NLO can be obtained from the LO by differentiation with respect to the initial condition of the classical field :

$$\mathcal{O}_{\rm NLO} = \frac{\hbar}{2} \int_{\boldsymbol{u},\boldsymbol{v}} \boldsymbol{\sigma}(\boldsymbol{u},\boldsymbol{v}) \, \frac{\partial}{\partial \mathcal{A}_{\rm init}(\boldsymbol{u})} \frac{\partial}{\partial \mathcal{A}_{\rm init}(\boldsymbol{v})} \quad \mathcal{O}_{\rm LO}$$

  - NLO : the time evolution remains classical;

    $\hbar$ only enters in the initial condition
  - NNLO : $\hbar$ starts appearing in the time evolution itself
  - $\boldsymbol{\sigma}(\boldsymbol{u},\boldsymbol{v})$ is universal; calculable analytically; depends on $y_{\rm cut}$
    Factorization: $y_{\rm cut}$ can be absorbed in $W[\rho_{1,2}]$

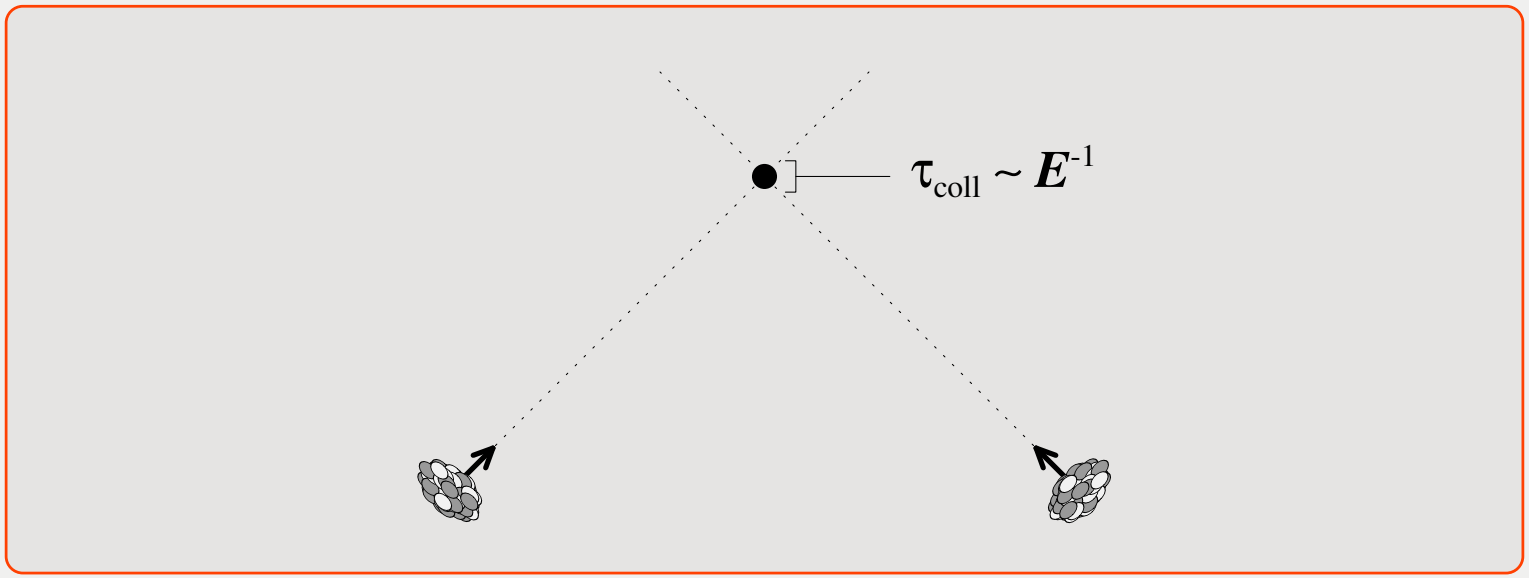
Another take on *LO contains NLO* : Moyal equation

- Liouville-von Neumann equation : $i\,\hbar\,\dfrac{\partial\widehat{\rho}_\tau}{\partial\tau} = [\widehat{H},\widehat{\rho}_\tau]$
- Wigner transform : $W_\tau(\mathbf{x},\mathbf{p}) \equiv \displaystyle\int \mathrm{d}\mathbf{s}\; e^{i\mathbf{p}\cdot\mathbf{s}}\,\langle \mathbf{x}+\frac{\mathbf{s}}{2}|\widehat{\rho}_\tau|\mathbf{x}-\frac{\mathbf{s}}{2}\rangle$
- LvN equation is equivalent to Moyal equation

$$\begin{aligned}\frac{\partial W_\tau}{\partial\tau} &= \mathcal{H}(\mathbf{x},\mathbf{p})\,\frac{2}{i\,\hbar}\,\sin\left(\frac{i\,\hbar}{2}\left(\overleftarrow{\partial}_{\mathbf{p}}\overrightarrow{\partial}_{\mathbf{x}}-\overleftarrow{\partial}_{\mathbf{x}}\overrightarrow{\partial}_{\mathbf{p}}\right)\right)\,W_\tau(\mathbf{x},\mathbf{p})\\ &= \underbrace{\{\mathcal{H},W_\tau\}}_{\text{Poisson bracket}} + \mathcal{O}(\hbar^2)\end{aligned}$$

- At $\mathcal{O}(\hbar)$, the evolution is still classical (the $\hbar^1$ corrections come from the quantum nature of the *initial state*)

Handwaving argument for factorization



- The duration of the collision is very short: $\tau_{\rm coll} \sim E^{-1}$

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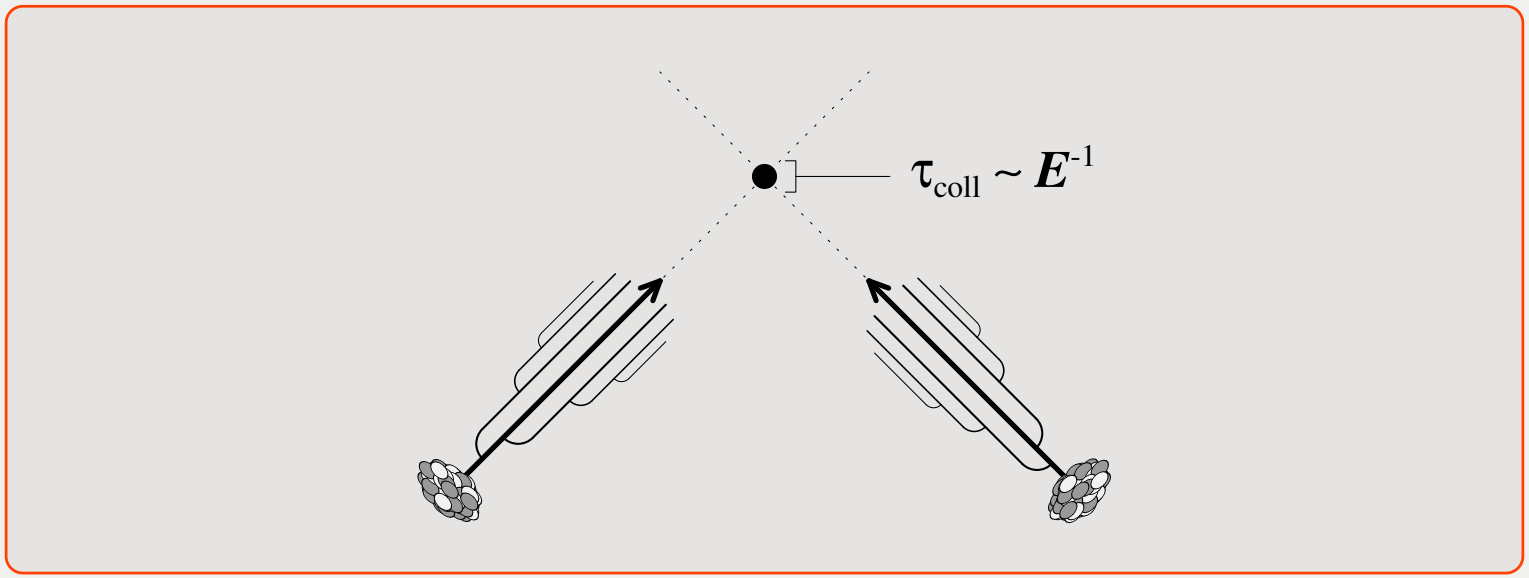

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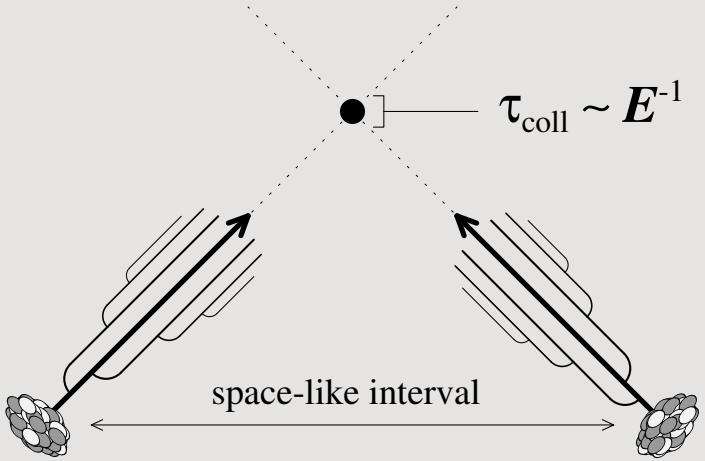


- The duration of the collision is very short: $\tau_{\rm coll} \sim E^{-1}$
- The logarithms we want to resum are due to the radiation of soft gluons, which takes a long time
  ▷ it must happen (long) before the collision
- The projectiles are not in causal contact before the impact
  ▷ the logarithms are intrinsic properties of the projectiles, independent of the measured observable

Leading Log corrections in AA collisions

- The $y_{\rm cut}$-dependent part of the NLO correction can be shown to be:

$$\mathcal{O}_{\rm NLO} = y_{\rm cut} \left[\mathcal{H}_1 - \mathcal{H}_2 \right] \mathcal{O}_{\rm LO} + \cdots$$

$\mathcal{H}_{1,2}$: JIMWLK Hamiltonians for the two nuclei

- Notes :
  - does not mix the two nuclei $\Rightarrow$ Factorization
  - does not work for exclusive quantities

Factorization

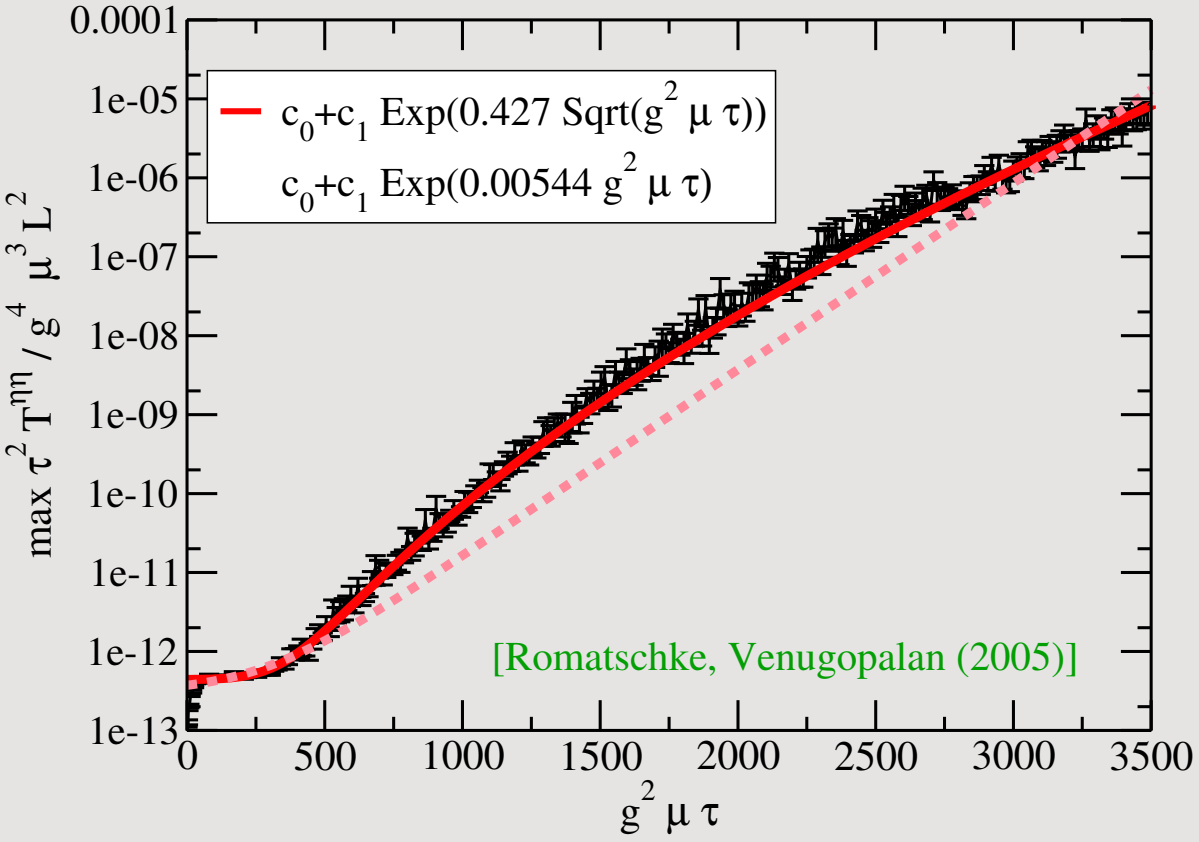
- By integrating over $\rho_{1,2}$'s, one can transfer the $y_{\rm cut}$ dependence into universal distributions $W_{1,2}[\rho_{1,2}]$

Resummation of $(\alpha_s y_{\rm cut})^n$ to all orders

$$\mathcal{O}_{\text{leading log}} = \int [D\rho_1\, D\rho_2]\; W_1[\rho_1]\; W_2[\rho_2] \quad \underbrace{\mathcal{O}_{\rm LO}}_{\text{fixed } \rho_{1,2}}$$

$$\frac{\partial W}{\partial Y} = \mathcal{H}\, W \qquad \text{(JIMWLK equation)}$$

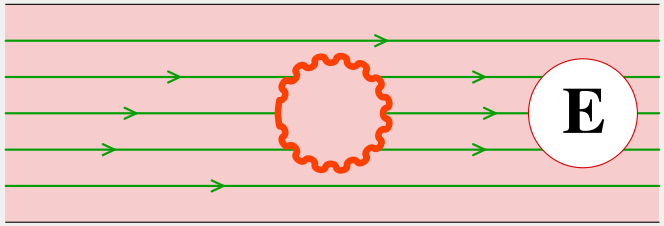
Instability of classical solutions



- Consequence: NLO corrections grow exponentially with time

- LO = classical chromo-$\mathbf{E}$ and chromo-$\mathbf{B}$ fields

- NLO = gluon loop embedded in this field



  - instability $\sim$ imaginary part of the loop $\sim$ gluon pair production
  - BUT : at NLO, no feedback of the produced gluons on the LO field!