

The First Stages of a Heavy Ion Collision

François Gelis

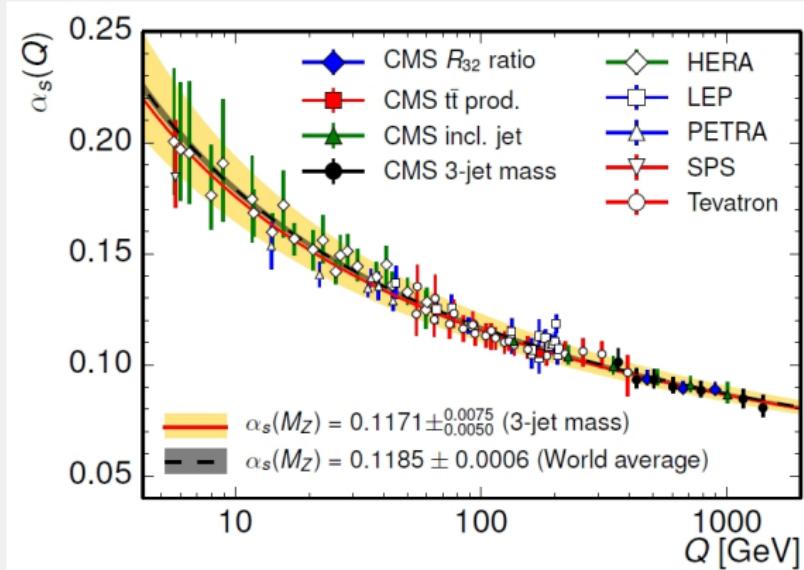
IPhT, March 27th, 2018



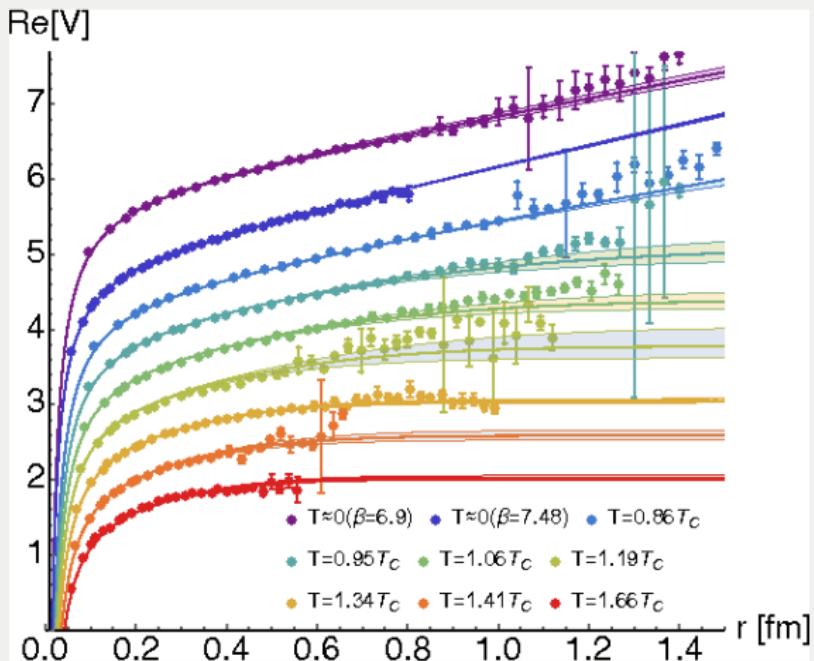
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Heavy Ion Collisions

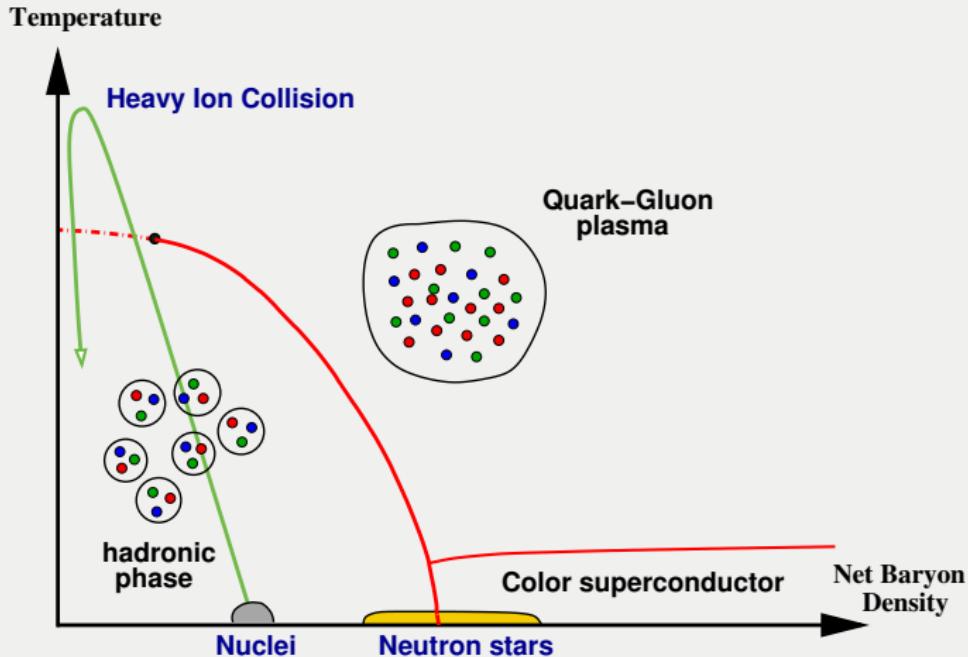
ASYMPTOTIC FREEDOM



QUARK-ANTIQUARK POTENTIAL AT FINITE TEMPERATURE



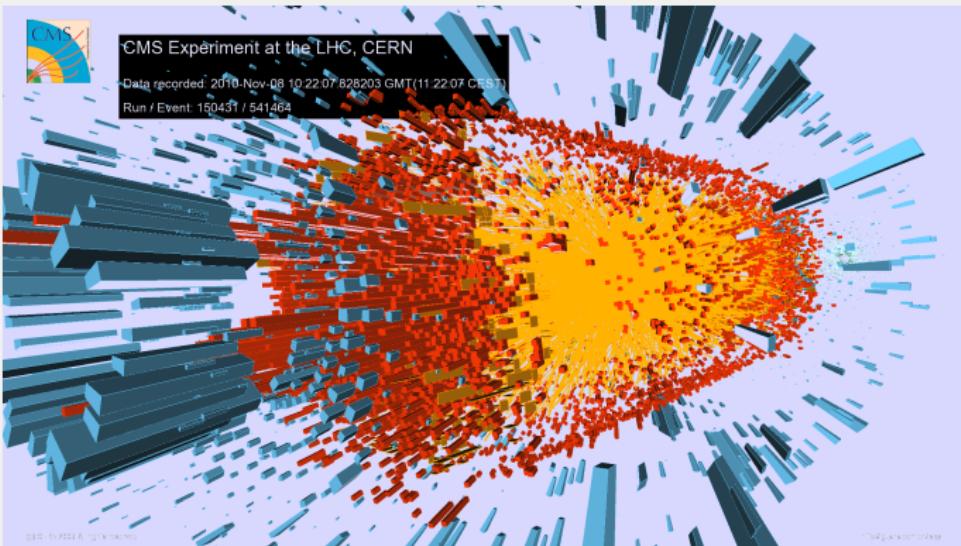
HEAVY ION COLLISIONS



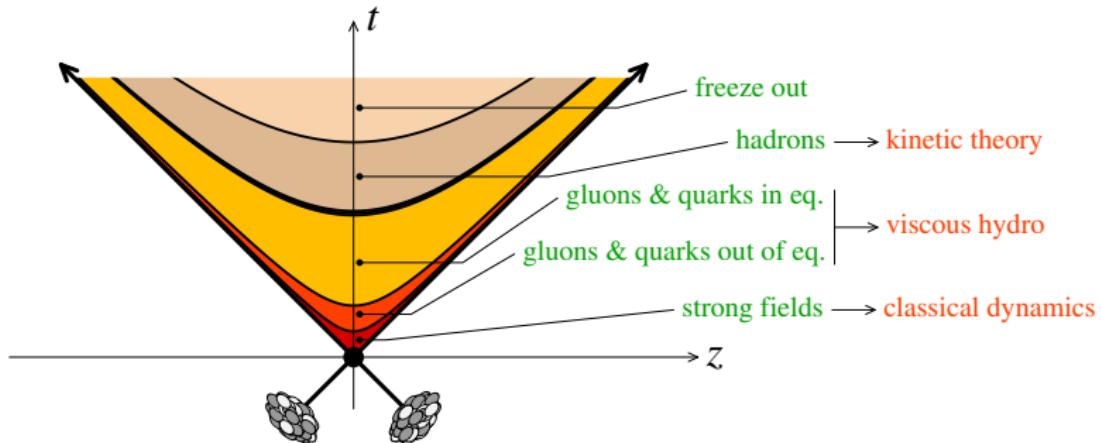
EXPERIMENTAL FACILITIES : RHIC AND LHC



HEAVY ION COLLISION AT THE LHC

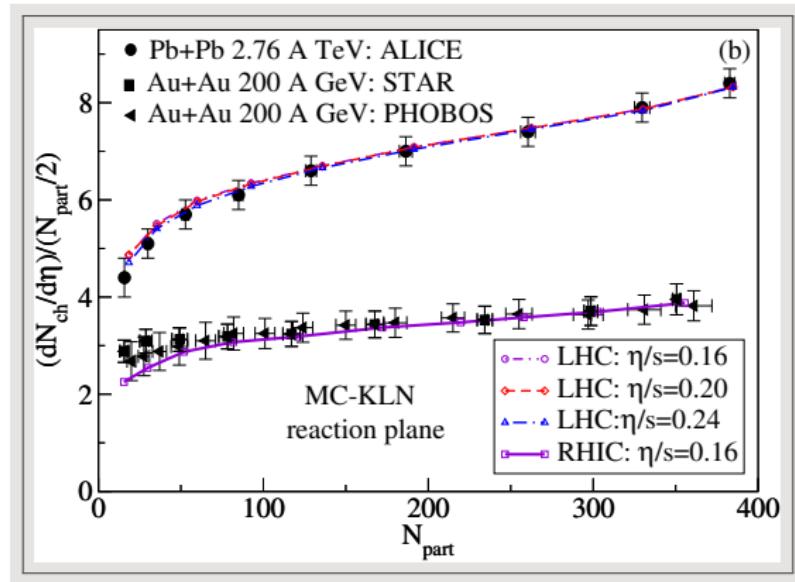


STAGES OF A NUCLEUS-NUCLEUS COLLISION

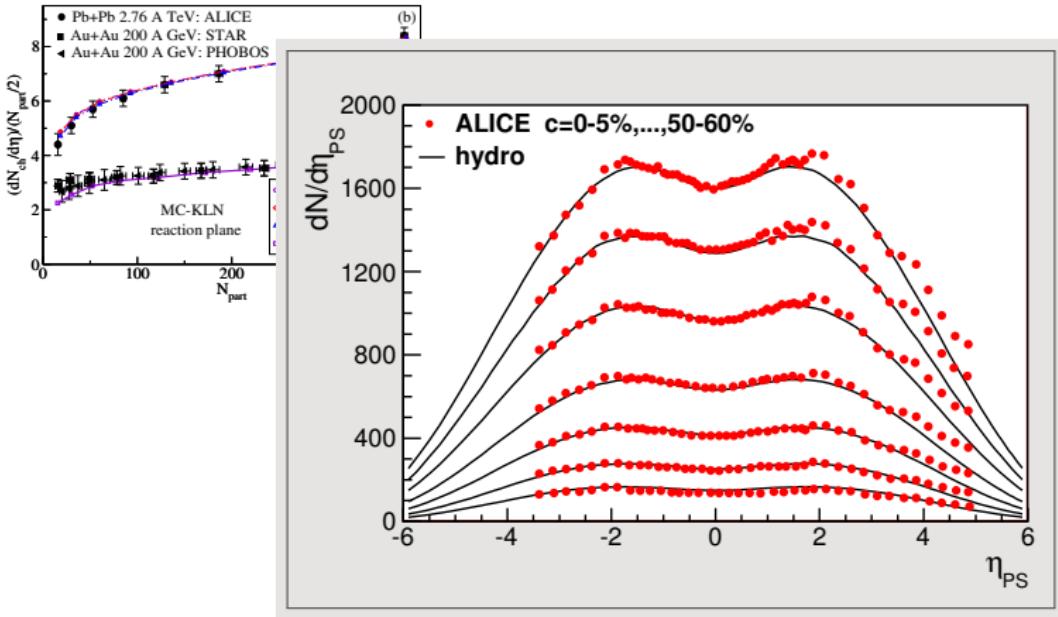


- Hydrodynamics successful at describing the bulk evolution
- In this talk : **Pre-hydrodynamical evolution**

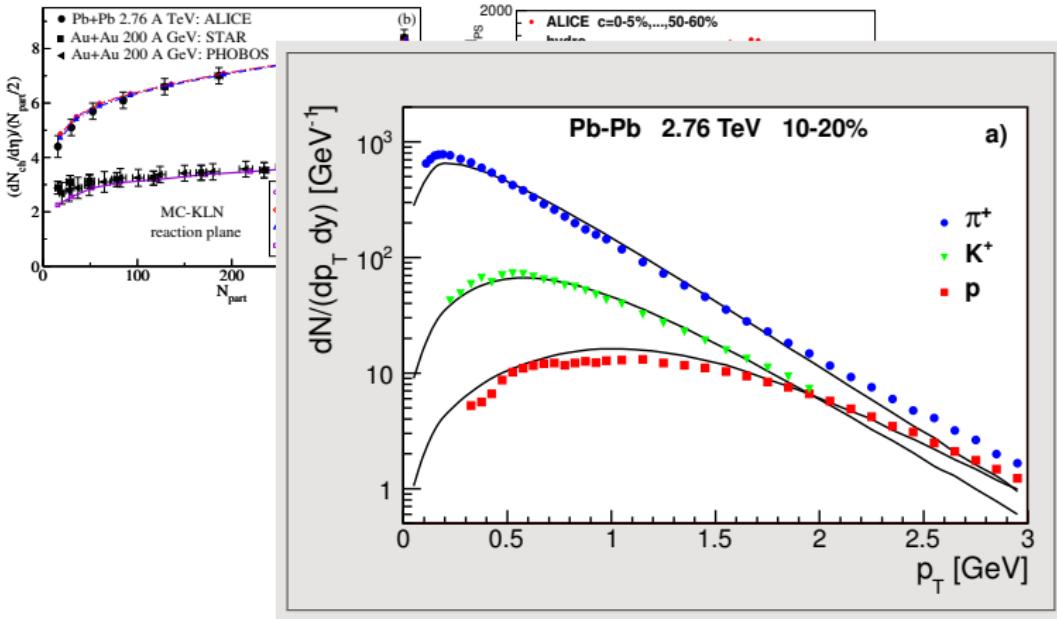
EVIDENCE FOR HYDRODYNAMICAL EXPANSION



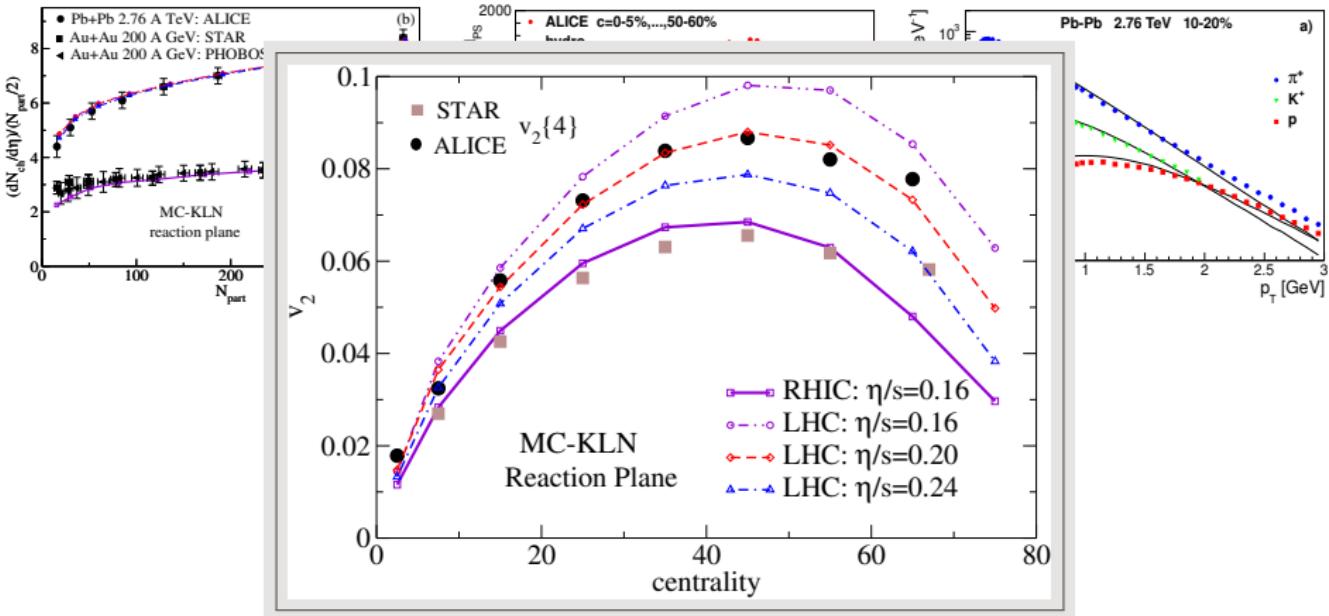
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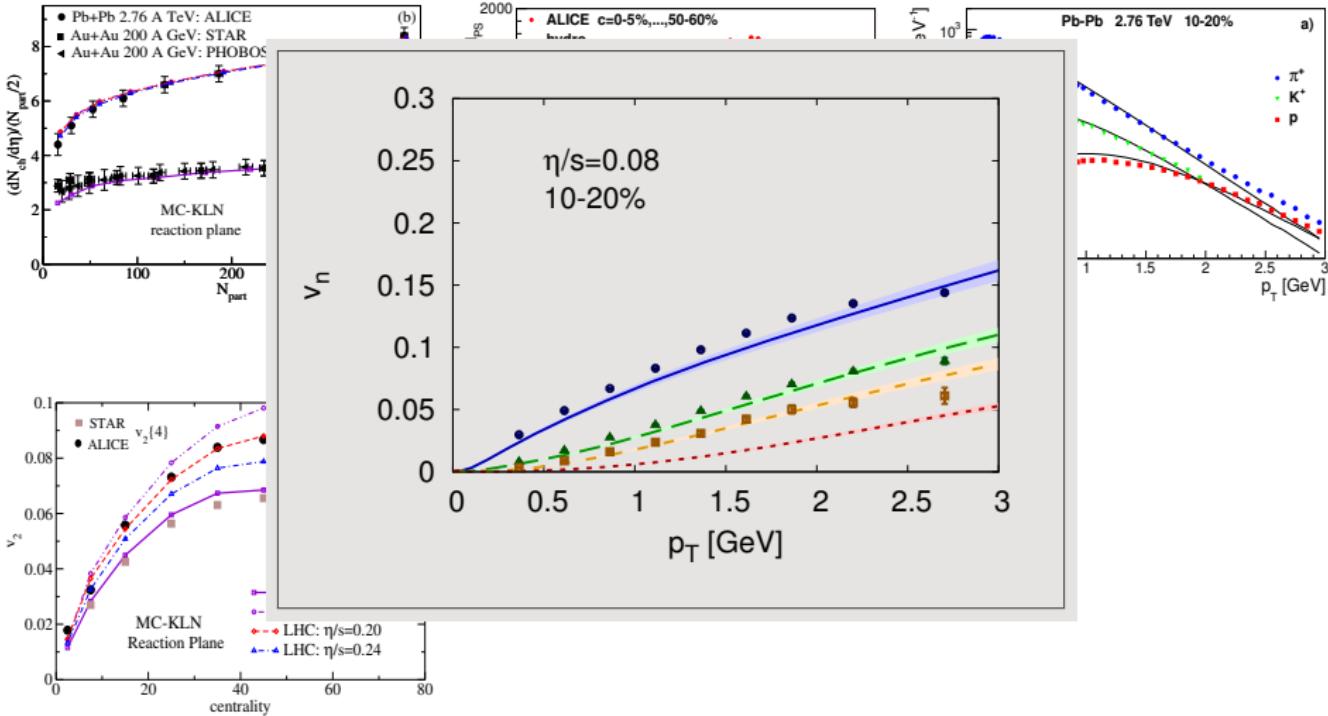
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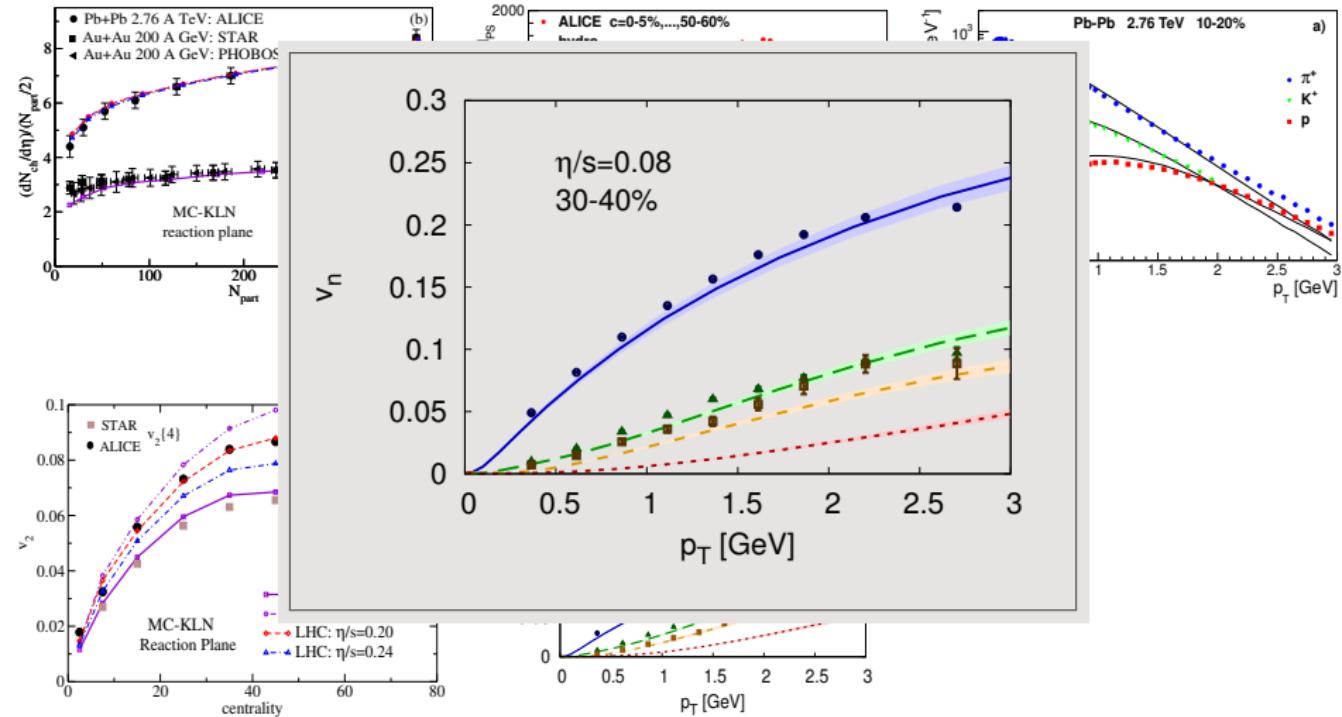
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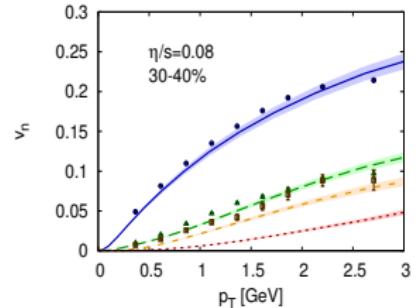
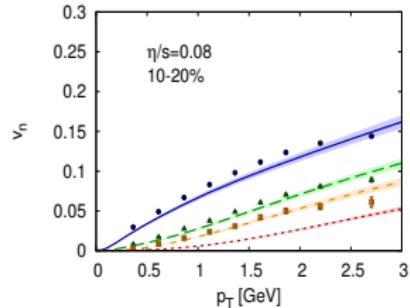
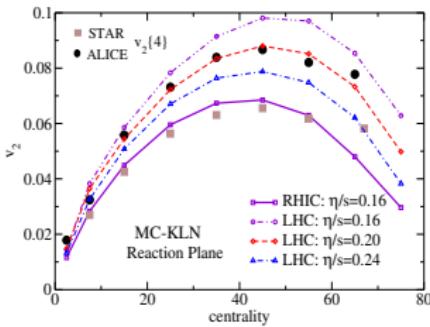
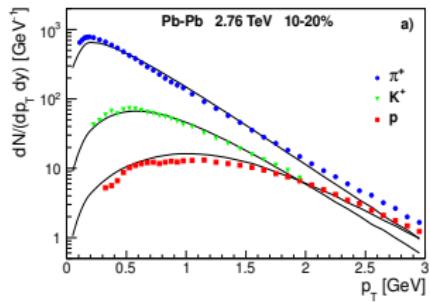
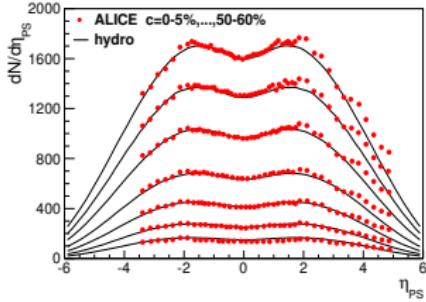
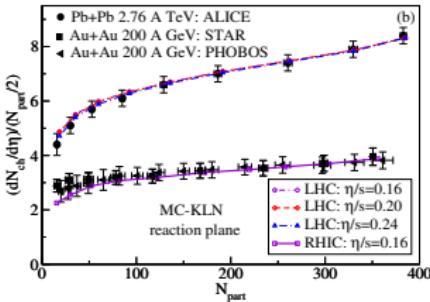
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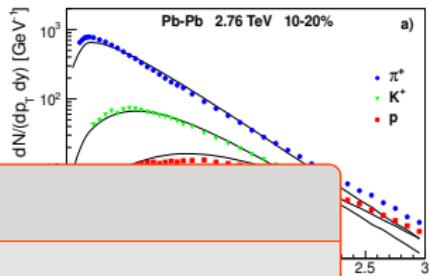
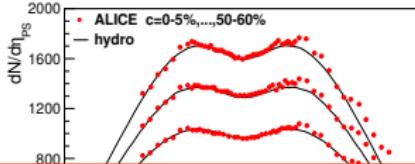
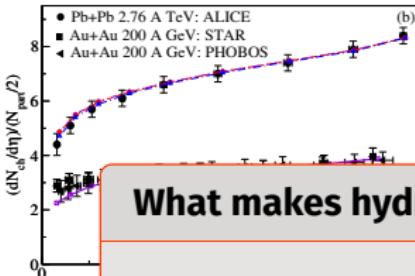
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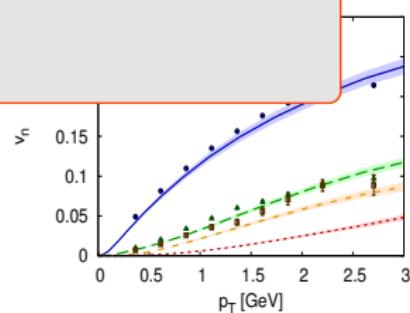
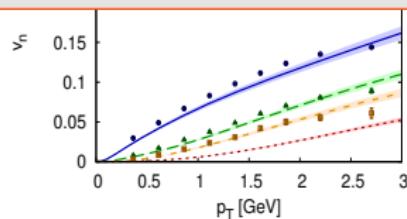
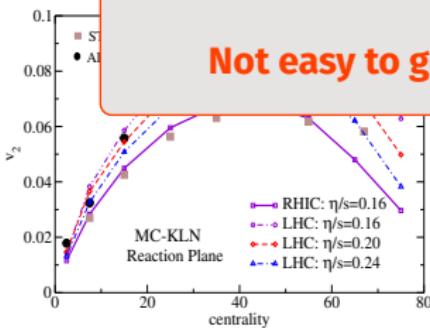
EVIDENCE FOR HYDRODYNAMICAL EXPANSION



What makes hydrodynamics work so well?

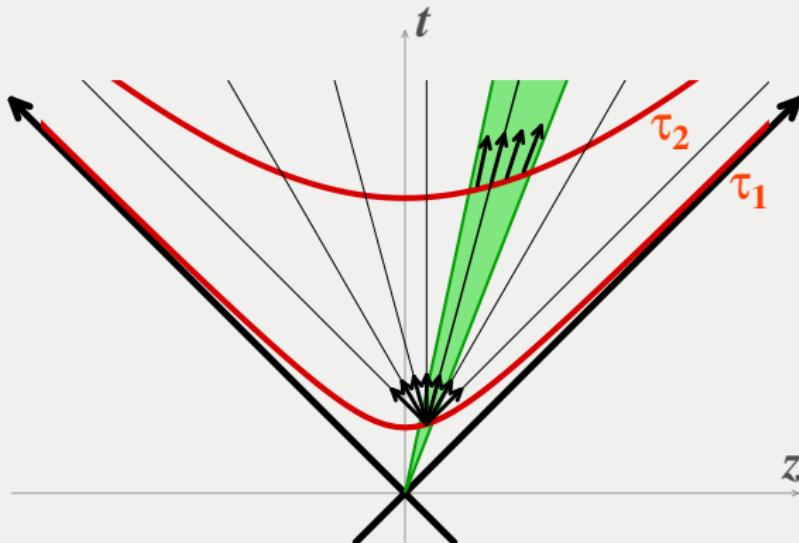
- Pressure tensor not too far from isotropy
- Low viscosity (compared to entropy density)

Not easy to get in QCD...



COMPETITION BETWEEN EXPANSION AND INTERACTIONS

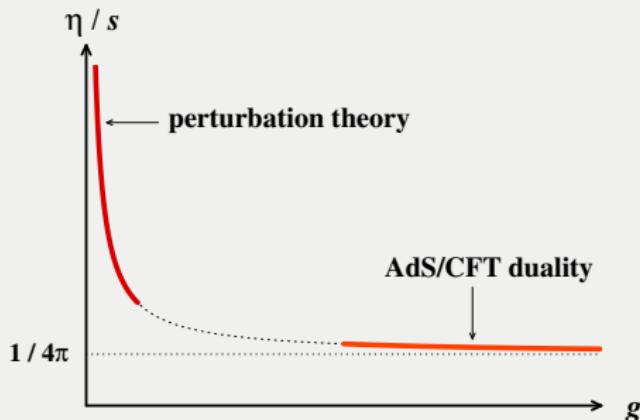
- Very different from isotropization in a box
- Sustained interactions are needed for isotropy to persist despite the expansion

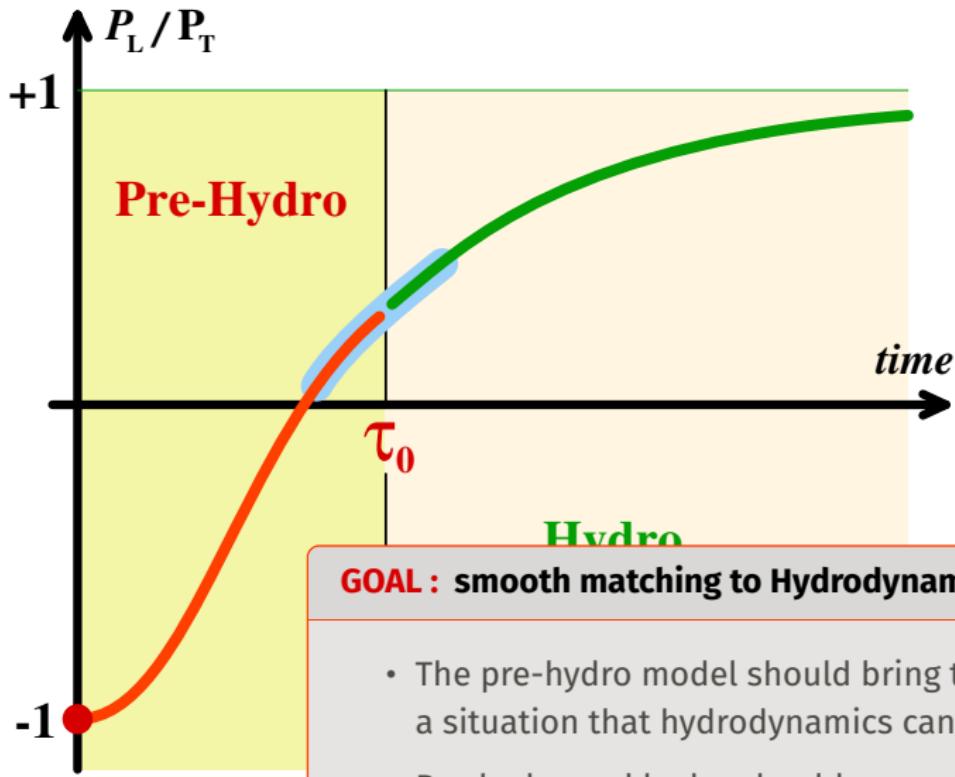


SHEAR VISCOSITY AT WEAK AND STRONG COUPLING (IN EQUILIBRIUM)

Weak coupling result [Arnold, Moore, Yaffe (2000)]

$$\frac{\eta}{s} \approx \frac{5.1}{g^4 \ln\left(\frac{2.4}{g}\right)}$$



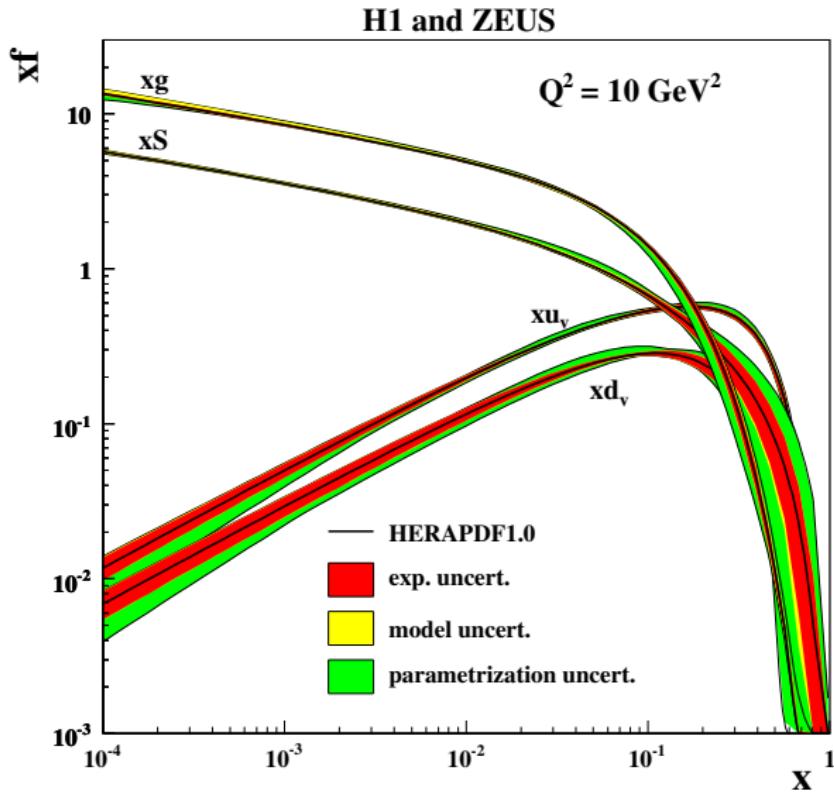


GOAL : smooth matching to Hydrodynamics

- The pre-hydro model should bring the system to a situation that hydrodynamics can handle
- Pre-hydro and hydro should agree over some range of time \Rightarrow no τ_0 dependence
- Description as close as possible to QCD

QCD description up to $\tau = 0^+$

PARTON DISTRIBUTIONS IN A NUCLEON

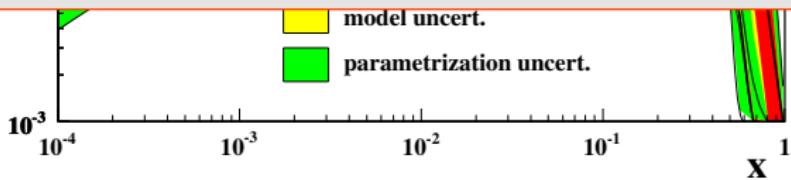
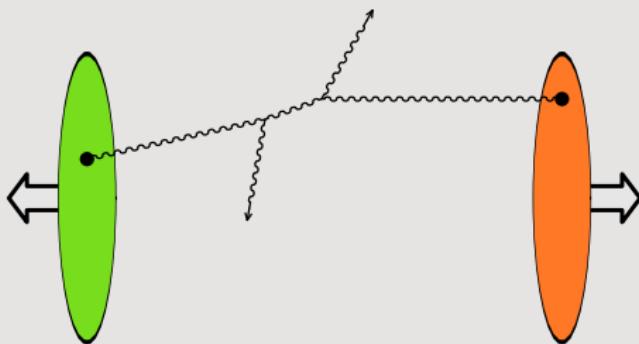


PARTON DISTRIBUTIONS IN A NUCLEON

H1 and ZEUS

$Q^2 = 10 \text{ GeV}^2$

Large x : dilute, dominated by single parton scattering



PARTON DISTRIBUTIONS IN A NUCLEON

H1 and ZEUS

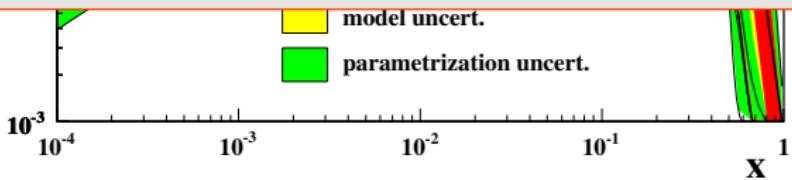
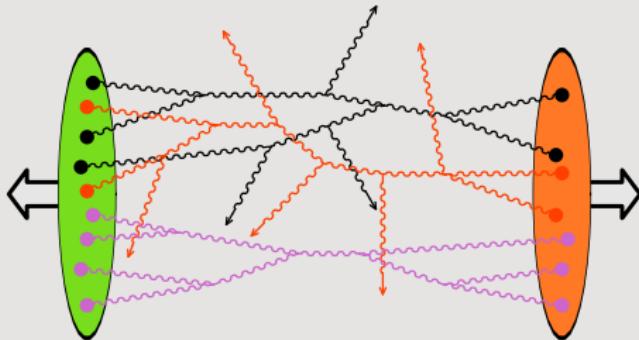
x_f

10

x_g
 x_S

$Q^2 = 10 \text{ GeV}^2$

Small x : dense, multi-parton interactions become likely



- When their occupation number becomes large, gluons can recombine :

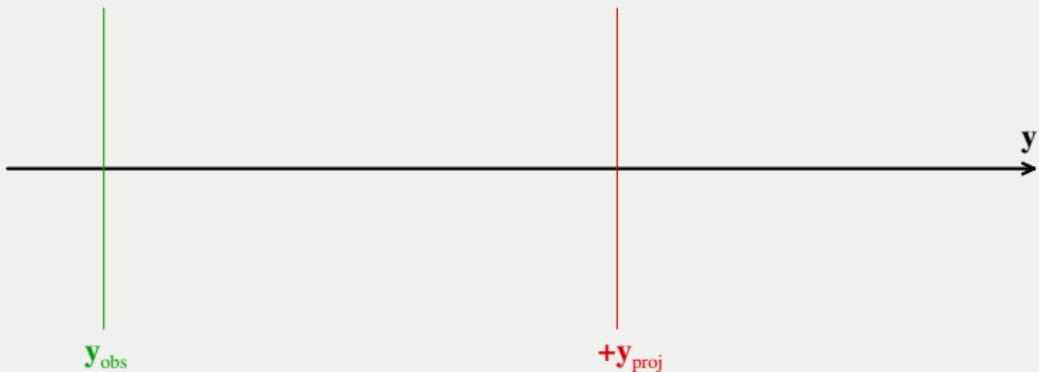
Gluon Saturation

Saturation criterion [Gribov, Levin, Ryskin (1983)]

$$\underbrace{\alpha_s Q^{-2}}_{\sigma_{gg \rightarrow g}} \times \underbrace{A^{-2/3} x G(x, Q^2)}_{\text{surface density}} \geq 1$$

$$Q^2 \leq Q_s^2 \equiv \underbrace{\frac{\alpha_s x G(x, Q_s^2)}{A^{2/3}}}_{(\text{saturation momentum})^2} \sim A^{1/3} x^{-\lambda} \quad (\lambda \approx 0.25)$$

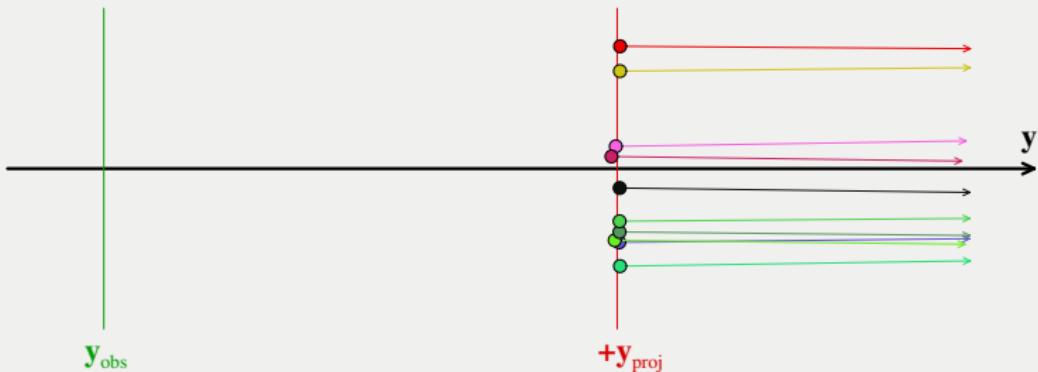
DEGREES OF FREEDOM AT VARIOUS RAPIDITIES ($y \sim \ln(p_z)$)



McLerran-Venugopalan model :

- Fast partons : frozen dynamics, negligible $p_\perp \Rightarrow$ classical current
- Slow partons : evolve with time \Rightarrow gauge fields

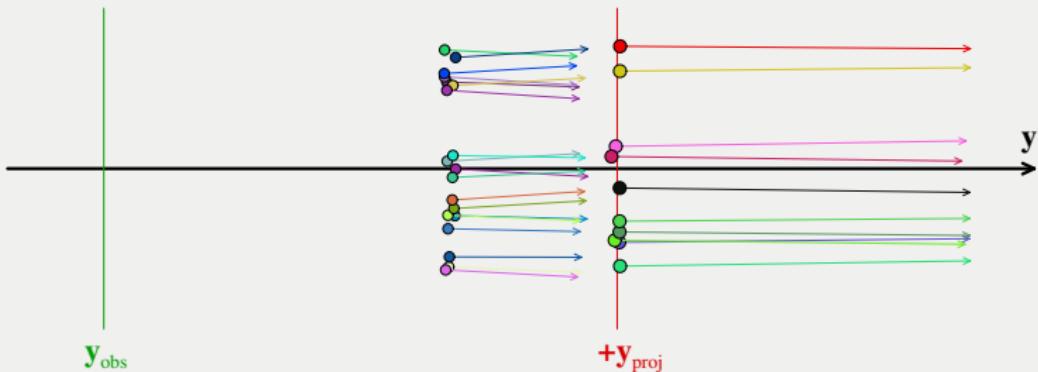
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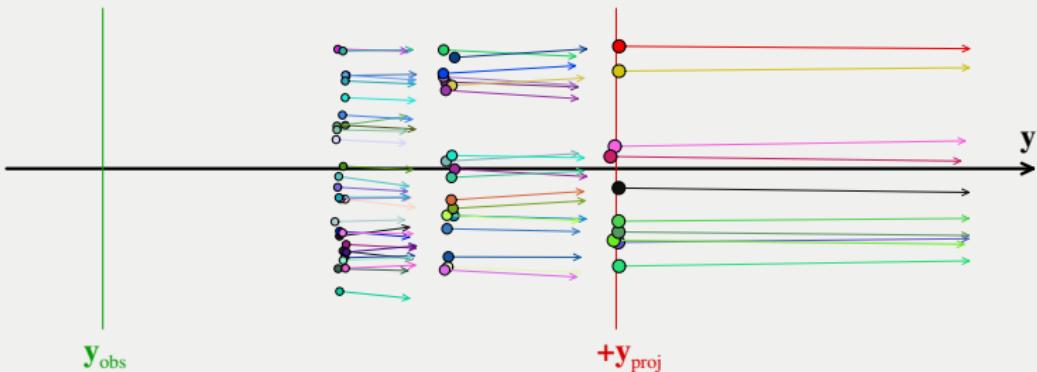
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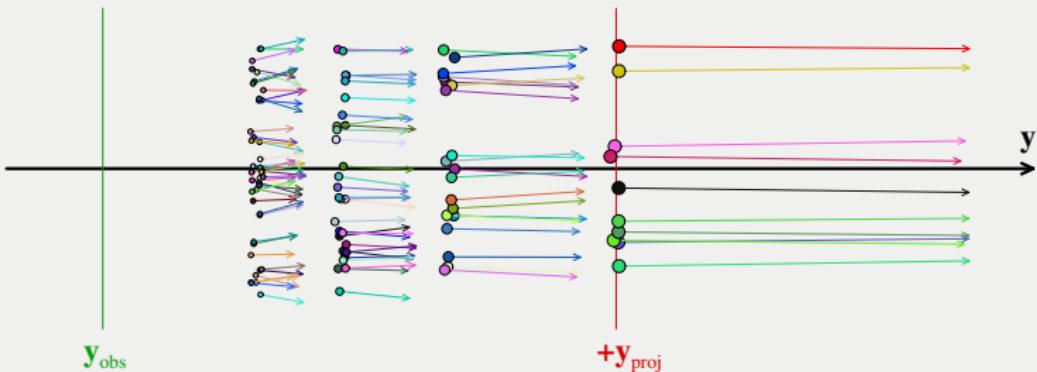
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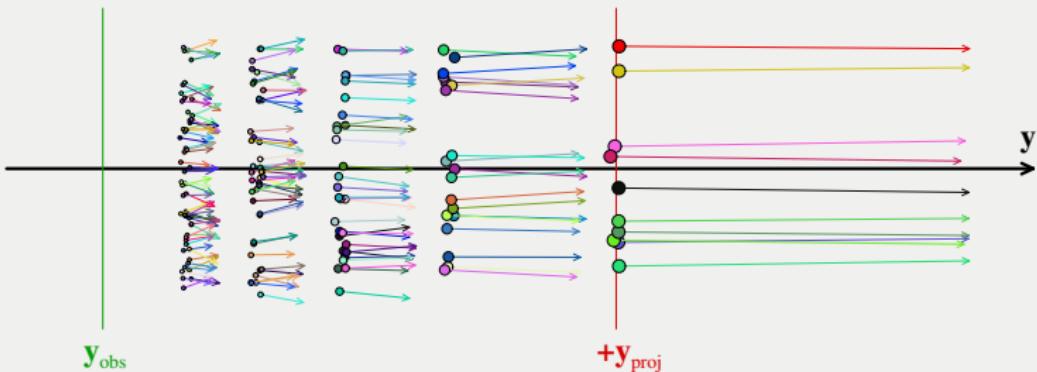
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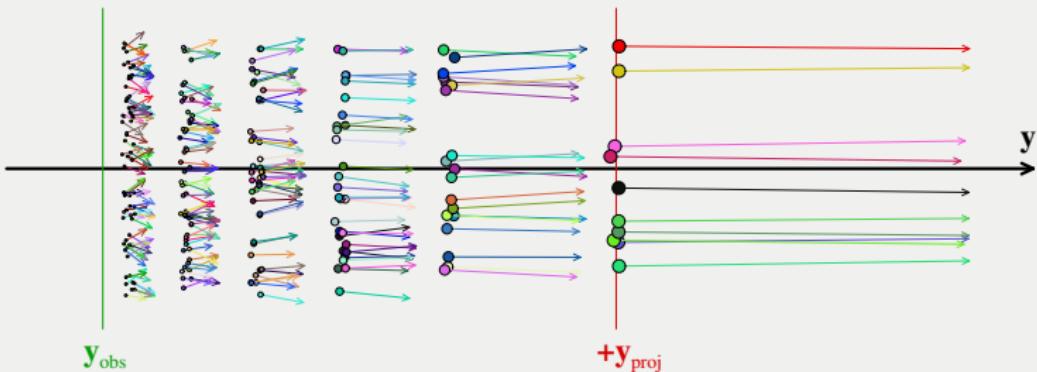
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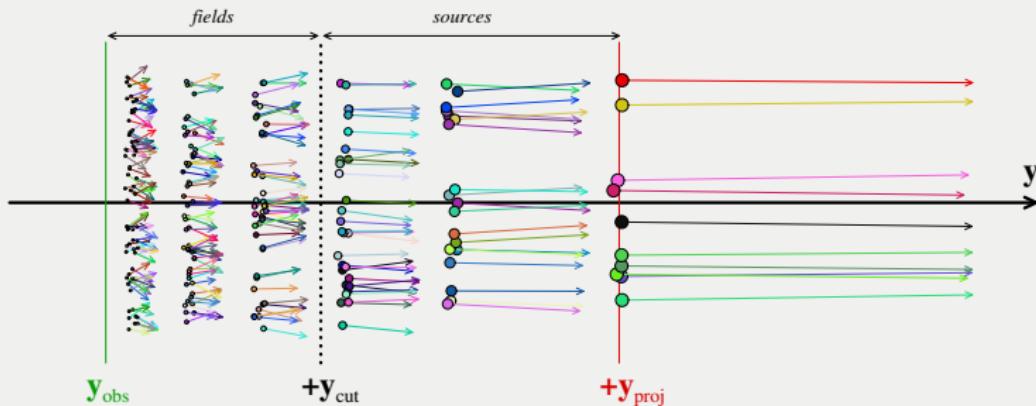
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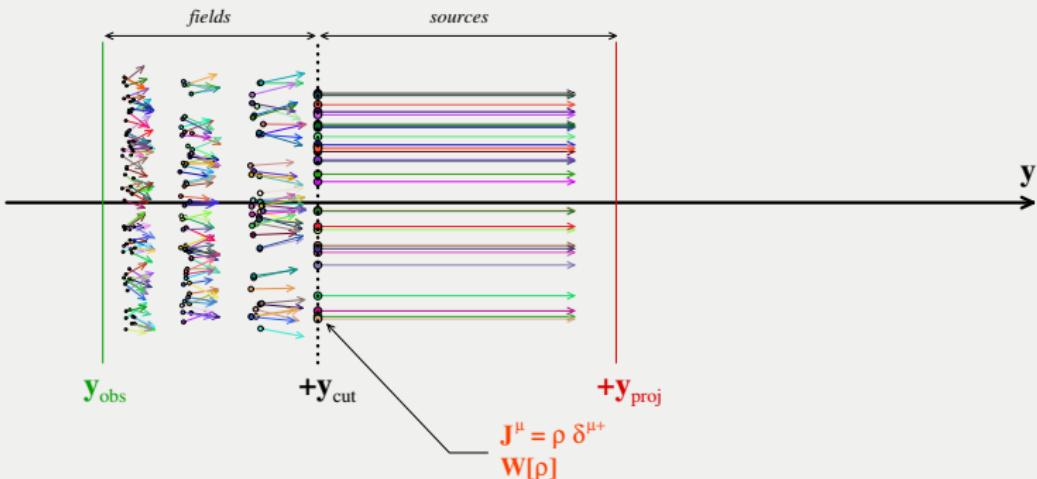
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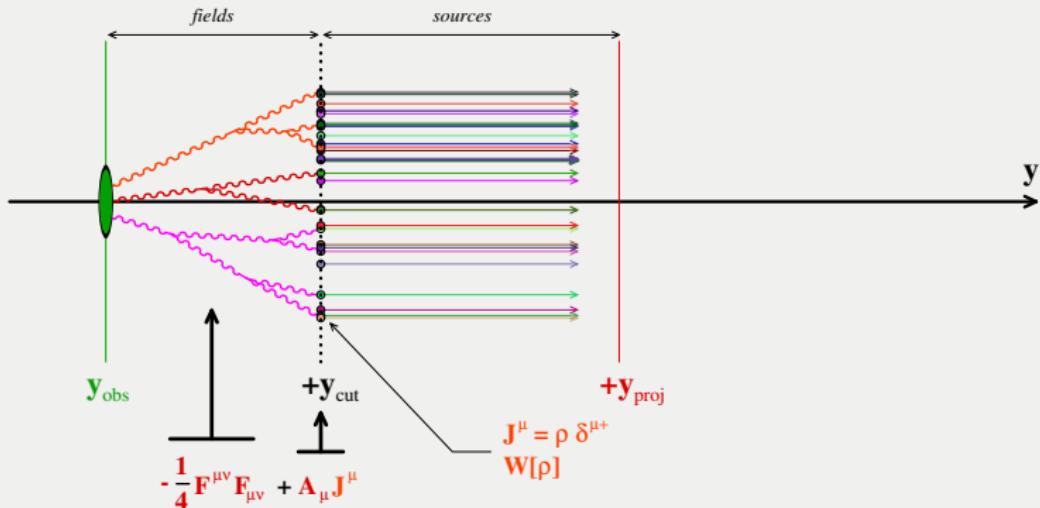
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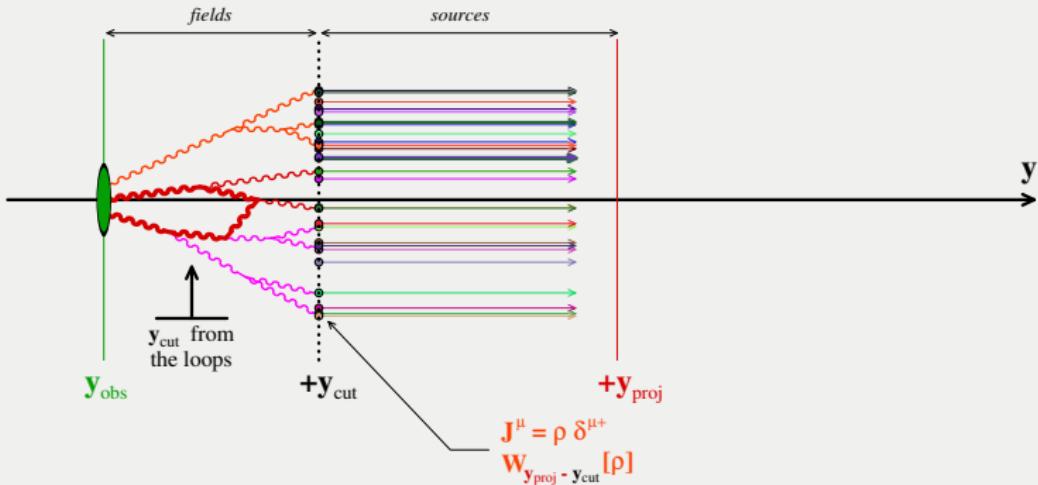
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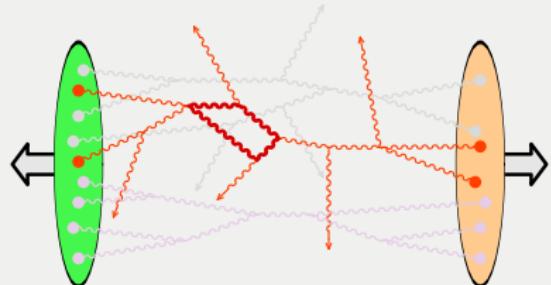
CANCELLATION OF THE CUTOFF DEPENDENCE



- The probability density $W[\rho]$ changes with the cutoff
- Loop corrections cancel the cutoff dependence from $W[\rho]$

POWER COUNTING IN THE SATURATED REGIME

$$\mathcal{S} = \underbrace{-\frac{1}{4} \int F_{\mu\nu} F^{\mu\nu}}_{\text{slow}} + \underbrace{\int (J_1^\mu + J_2^\mu) A_\mu}_{\text{fast}}$$



In the saturated regime: $J^\mu \sim g^{-1}$, $A^\mu \sim g^{-1}$, $f_k \sim g^{-2}$

$$T^{\mu\nu} \sim \frac{1}{g^2} [c_0 + c_1 g^2 + c_2 g^4 + \dots]$$

$c_0 \equiv$ tree level, $c_1 \equiv$ one loop, etc...

SHEAR VISCOSITY IN THE SATURATED REGIME

From kinetic theory :

$$\frac{\eta}{s} \sim \frac{\text{mean free path}}{\text{de Broglie wavelength}}$$

- (de Broglie wavelength) $^{-1} \sim Q$
- (mean free path) $^{-1} \sim \underbrace{g^4 Q^{-2}}_{\text{cross section}} \times \int_k f_k \underbrace{(1 + f_k)}_{\text{density}} \underbrace{\text{Bose enhancement}}$

If $g \ll 1$ but $f_k \sim g^{-2}$ (weakly coupled, but strongly interacting)

$$\frac{\eta}{s} \sim g^0$$

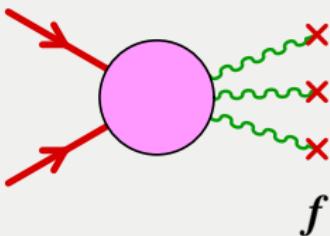
INCLUSIVE QUANTITIES

- Average particle multiplicity $\sim 1/g^2 \gg 1$
- Probability of a given final state $\sim \exp(-\frac{\#}{g^2}) \ll 1$
 \implies not very useful
- Inclusive observables :
average of some quantity over all possible final states

$$\langle \mathcal{O} \rangle \equiv \sum_{\text{all final states } f} \mathcal{P}(AA \rightarrow f) \mathcal{O}(f)$$

Schwinger-Keldysh formalism : technique to perform the sum over final states without computing the individual transition probabilities $\mathcal{P}(AA \rightarrow f)$

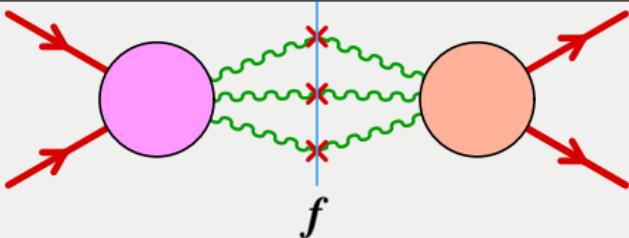
SCHWINGER-KELDYSH FORMALISM



**Time-ordered
perturbation theory :**

$$G_{++}(p) = \frac{i}{p^2 + i\epsilon}$$

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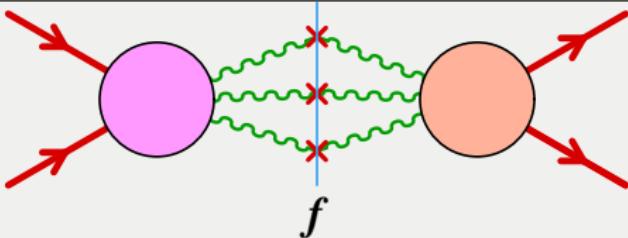
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**Anti time-ordered
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$$G_{--}(p) = \frac{-i}{p^2 - i\epsilon}$$

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Schwinger-Keldysh formalism :

- Across the cut : $G_{+-}(p) \equiv 2\pi\theta(-p^0)\delta(p^2)$
- Final state sum : sum over all the assignments of the labels + and - to vertices and sources

LEADING ORDER

- Leading Order = sum of all tree diagrams

Expressible in terms of **classical solutions of Yang-Mills equations** :

$$[\mathcal{D}_\mu, \mathcal{F}^{\mu\nu}] = J_1^\nu + J_2^\nu$$

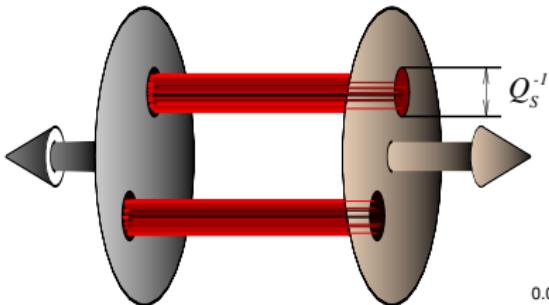
- Initial condition : $\lim_{x^0 \rightarrow -\infty} \mathcal{A}^\mu(x) = 0$

Components of the energy-momentum tensor

$$T_{\text{LO}}^{00} = \frac{1}{2} \left[\underbrace{\mathbf{E}^2 + \mathbf{B}^2}_{\text{class. fields}} \right] \quad T_{\text{LO}}^{0i} = [\mathbf{E} \times \mathbf{B}]^i$$

$$T_{\text{LO}}^{ij} = \frac{\delta^{ij}}{2} [\mathbf{E}^2 + \mathbf{B}^2] - [\mathbf{E}^i \mathbf{E}^j + \mathbf{B}^i \mathbf{B}^j]$$

LO : STRONG PRESSURE ANISOTROPY AT ALL TIMES

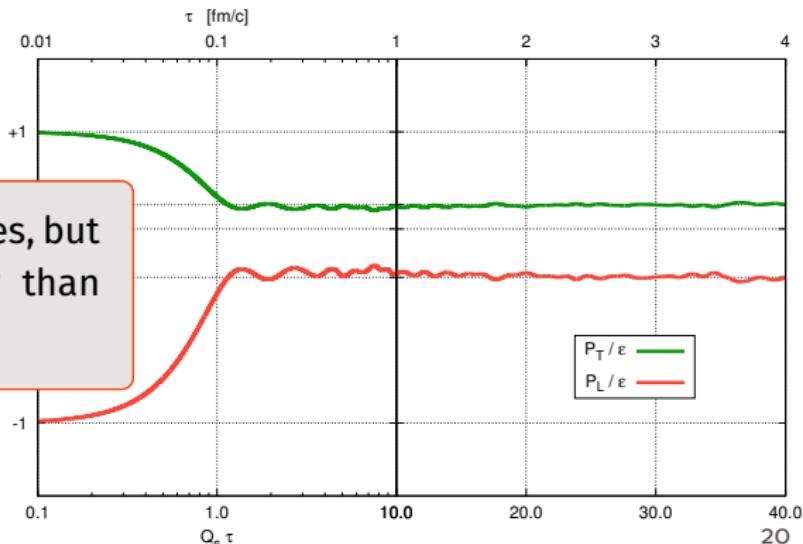


At $\tau = 0^+$

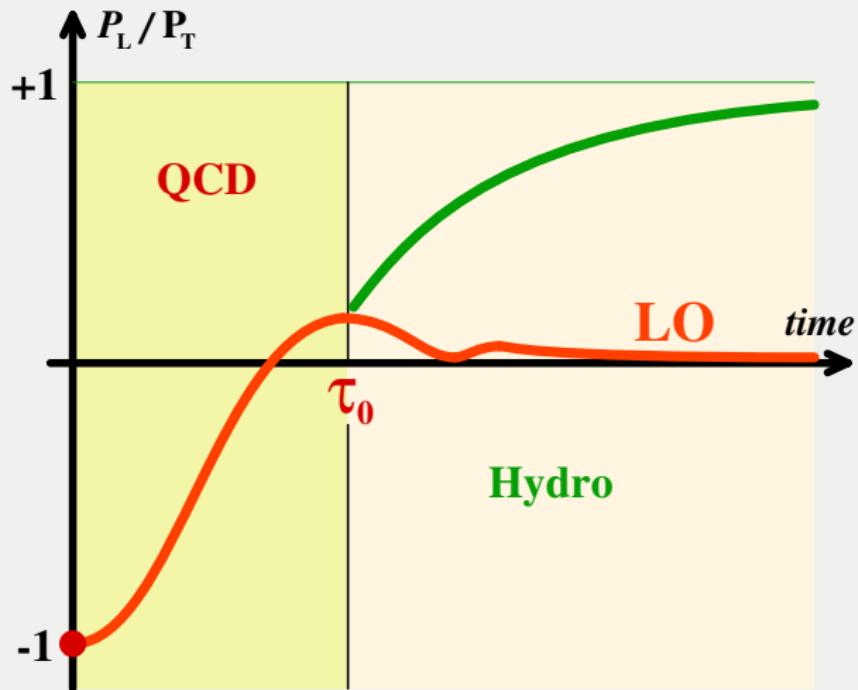
$$\mathbf{E} \parallel \mathbf{B} \parallel \hat{\mathbf{z}}$$

$$P_T = \epsilon, P_L = -\epsilon$$

P_L rises to positive values, but remains much smaller than P_T (free streaming)



LO : UNSATISFACTORY MATCHING TO HYDRODYNAMICS



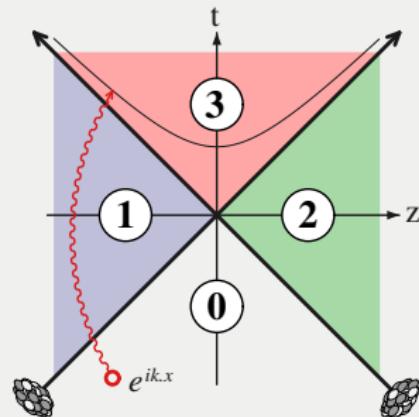
NEXT-TO-LEADING ORDER

- LO : classical field $\mathcal{A}_{\text{LO}}^{\mu} \sim Q_s/g$
- NLO : one-loop in a non-trivial background
 - Gaussian fluctuations $\alpha^{\mu} \sim Q_s$ on top of $\mathcal{A}_{\text{LO}}^{\mu}$
 - Variance $\sigma(u, v)$ of the fluctuations known analytically at $\tau = 0^+$

$$\sigma(u, v) = \frac{1}{2} \int_{\text{modes } k} \alpha_k(u) \alpha_k^*(v)$$

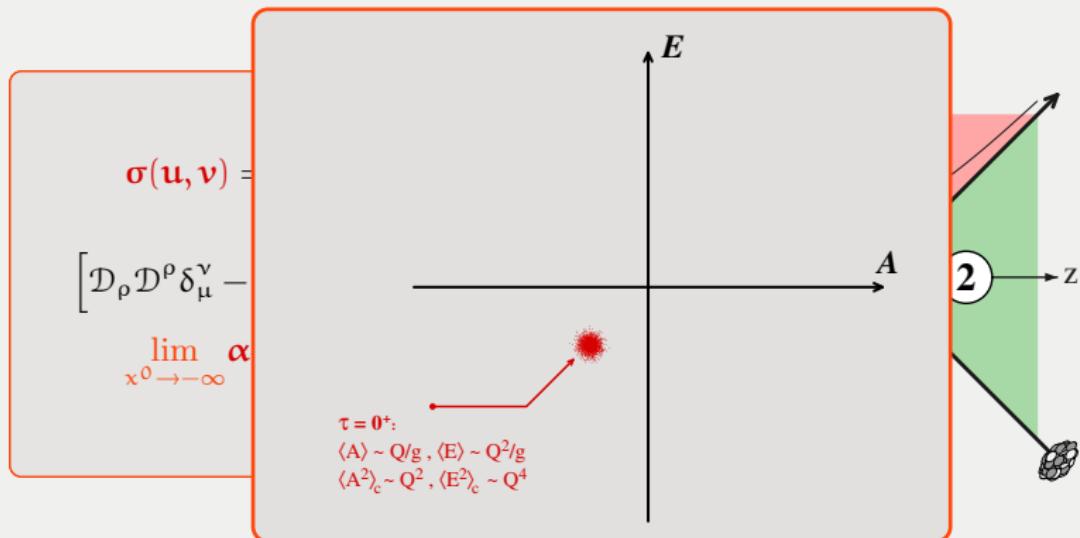
$$[\mathcal{D}_\rho \mathcal{D}^\rho \delta_\mu^\nu - \mathcal{D}_\mu \mathcal{D}^\nu + ig \mathcal{F}_\mu^\nu] \alpha_k^\mu = 0$$

$$\lim_{x^0 \rightarrow -\infty} \alpha_k^\mu(x) = \epsilon^\mu e^{ik \cdot x}$$



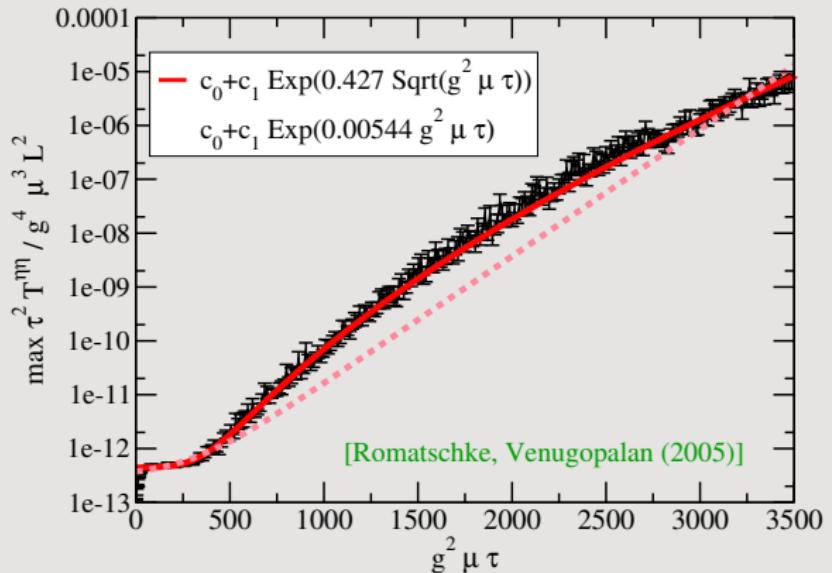
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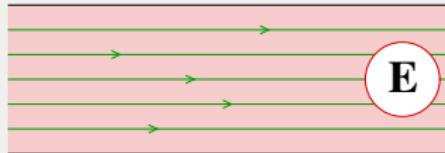


Instabilities and Resummation

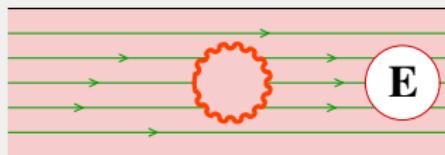
INSTABILITY OF CLASSICAL SOLUTIONS



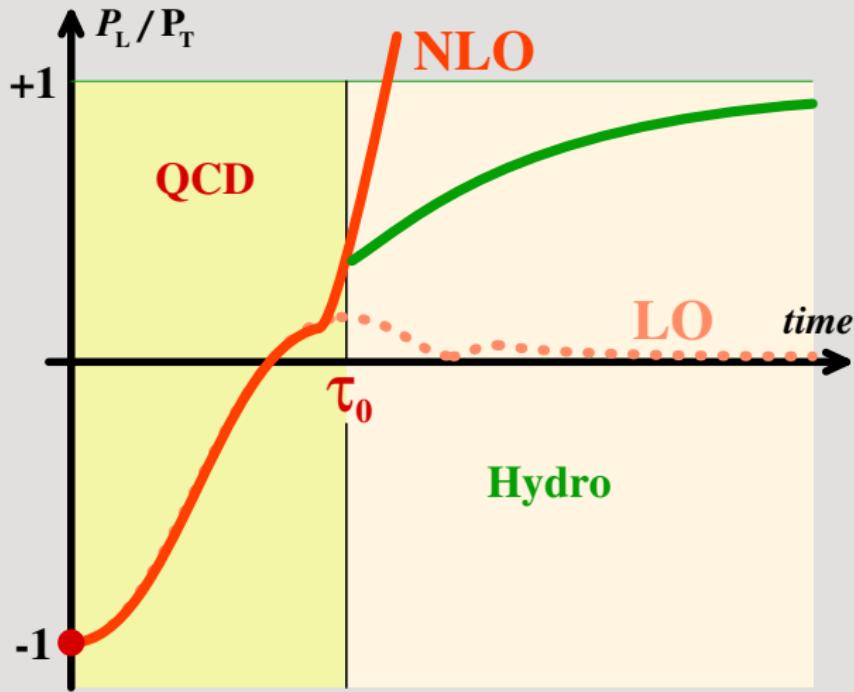
- LO = longitudinal chromo-E and chromo-B fields



- NLO = gluon loop embedded in this field



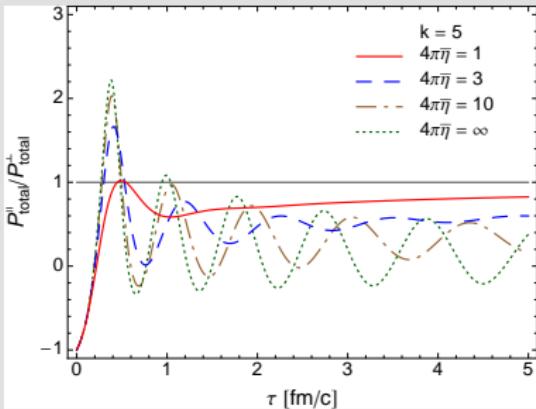
- instability \sim imaginary part of the loop \sim gluon pair production
- BUT : at NLO, no feedback of the produced gluons on the LO field!



Color flux tube model : [Ryblewski, Florkowski (2013)]

$$(p^\mu \partial_\mu + \underbrace{g F^{\mu\nu} p_\nu}_{\text{Lorentz force}} \partial_p^\mu) G = \underbrace{\frac{dN}{d\Gamma}}_{\text{Schwinger}} + C_p[G] + \underbrace{C_c[G]}_{\text{collisions}}$$

$$\partial_\mu F^{\mu\nu} = j^\nu \quad (\text{feedback})$$



- Field converted into particles by instability
- Nearly constant P_L/P_T
- Ratio depends on $\bar{\eta} \equiv \eta/s$

INCLUSIVE OBSERVABLES AT NLO

[FG, Lappi, Venugopalan (2007–2008)]

- Observables at NLO can be obtained from the LO by “fiddling” with the initial condition of the classical field :

$$\mathcal{O}_{\text{NLO}} = \frac{\hbar}{2} \int_{\mathbf{u}, \mathbf{v}} \sigma(\mathbf{u}, \mathbf{v}) \frac{\delta}{\delta \mathcal{A}_{\text{ini}}(\mathbf{u})} \frac{\delta}{\delta \mathcal{A}_{\text{ini}}(\mathbf{v})} \mathcal{O}_{\text{LO}}$$

- NLO : the time evolution remains classical;
 \hbar only enters in the initial condition
(NNLO : \hbar starts appearing in the time evolution itself)

ANALOGUE IN QUANTUM MECHANICS

- Consider the Liouville–von Neumann equation :

$$i\hbar \frac{\partial \hat{\rho}_\tau}{\partial \tau} = [\hat{H}, \hat{\rho}_\tau]$$

- Introduce the Wigner transforms :

$$\begin{aligned} W_\tau(x, p) &\equiv \int ds e^{ip \cdot s} \langle x + \frac{s}{2} | \hat{\rho}_\tau | x - \frac{s}{2} \rangle \\ \mathcal{H}(x, p) &\equiv \int ds e^{ip \cdot s} \langle x + \frac{s}{2} | \hat{H} | x - \frac{s}{2} \rangle \end{aligned}$$

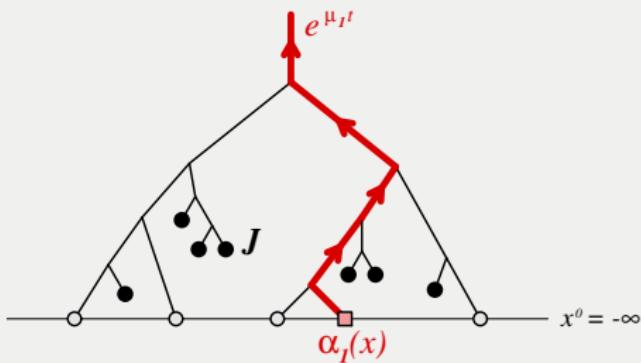
- LvN equation is equivalent to Moyal-Groenewold equation

$$\begin{aligned} \frac{\partial W_\tau}{\partial \tau} &= \mathcal{H}(x, p) \frac{2}{i\hbar} \sin \left(\frac{i\hbar}{2} (\overleftarrow{\partial}_p \overrightarrow{\partial}_x - \overleftarrow{\partial}_x \overrightarrow{\partial}_p) \right) W_\tau(x, p) \\ &= \underbrace{\{\mathcal{H}, W_\tau\}}_{\text{Poisson bracket}} + \underbrace{\mathcal{O}(\hbar^2)}_{\text{deviation from classical dynamics}} \end{aligned}$$

IMPROVED POWER COUNTING

- For an unstable mode:

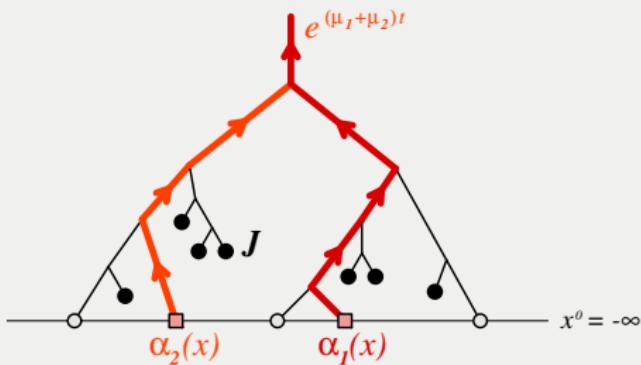
$$\alpha_k(x) \underset{x^0 \rightarrow +\infty}{\sim} e^{\mu_k x^0} \quad (\mu_k = \text{Lyapunov exponent})$$



IMPROVED POWER COUNTING

- For an unstable mode:

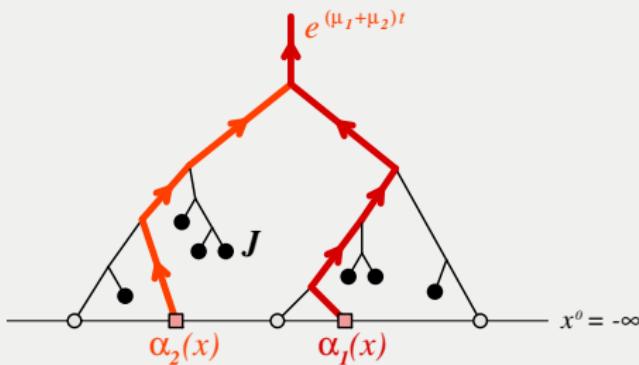
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IMPROVED POWER COUNTING

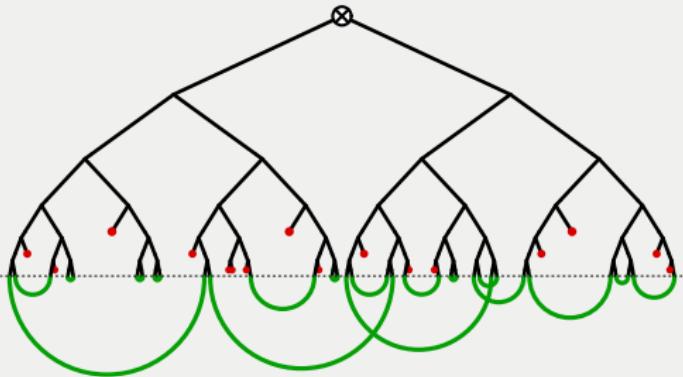
- For an unstable mode:

$$\alpha_k(x) \underset{x^0 \rightarrow +\infty}{\sim} e^{\mu_k x^0} \quad (\mu_k = \text{Lyapunov exponent})$$



- 1 loop : $g^2 \hbar e^{2\mu_k t}$
- n loops : $(g^2 \hbar e^{2\mu_k t})^n$

RESUMMATION OF THE LEADING TERMS



$$\mathcal{O}_{\text{resummed}} \equiv \exp \left[\frac{\hbar}{2} \int_{\mathbf{u}, \mathbf{v}} \boldsymbol{\sigma}(\mathbf{u}, \mathbf{v}) \frac{\delta}{\delta \mathcal{A}_{\text{ini}}(\mathbf{u})} \frac{\delta}{\delta \mathcal{A}_{\text{ini}}(\mathbf{v})} \right] \mathcal{O}_{\text{LO}}$$

$$\mathcal{O}_{\text{resummed}} = \mathcal{O}_{\text{LO}} + \mathcal{O}_{\text{NLO}} + \text{subset of all higher orders}$$

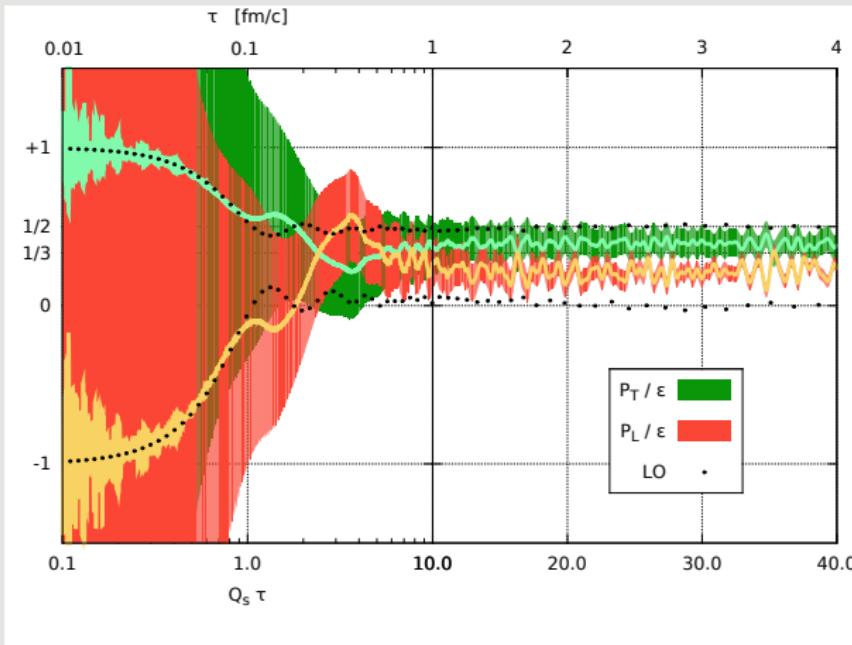
RESUMMATION : CLASSICAL STATISTICAL APPROXIMATION

$$\begin{aligned} & \exp \left[\frac{\hbar}{2} \int_{\mathbf{u}, \mathbf{v}} \underbrace{\sigma(\mathbf{u}, \mathbf{v}) \frac{\delta}{\delta \mathcal{A}_{\text{ini}}(\mathbf{u})} \frac{\delta}{\delta \mathcal{A}_{\text{ini}}(\mathbf{v})}}_{\sim \text{Laplacian}} \right] \mathcal{O}_{\text{LO}} [\mathcal{A}_{\text{ini}}] \\ &= \int [D\alpha(\mathbf{u})] \exp \left[-\frac{1}{2\hbar} \int_{\mathbf{u}, \mathbf{v}} \alpha(\mathbf{u}) \sigma^{-1}(\mathbf{u}, \mathbf{v}) \alpha(\mathbf{v}) \right] \mathcal{O}_{\text{LO}} [\mathcal{A}_{\text{ini}} + \alpha] \end{aligned}$$

- In this resummation, the observable is obtained as an average over classical fields with fluctuating initial conditions
- The variance of the fluctuations ($\hbar \sigma$) is prescribed by the NLO

Evolution at small coupling : $g = 0.5$ ($\lambda \equiv g^2 N_c = 0.5$)

[Epelbaum, FG (2013)]



IS IT POSSIBLE TO START FROM A DECOHERED STATE ?

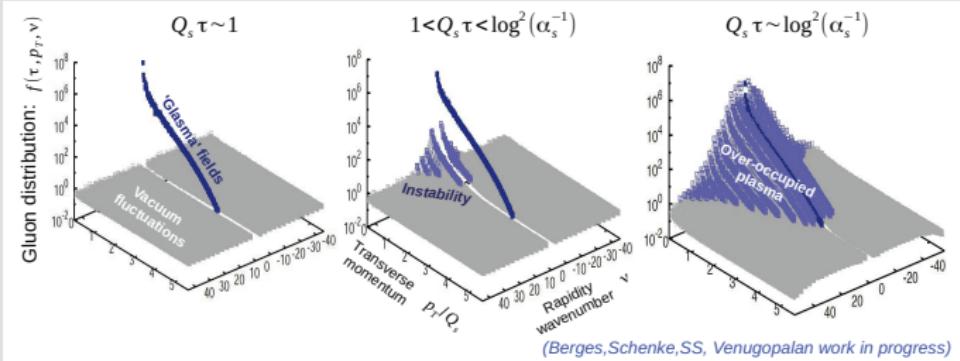
QCD at $\tau = 0^+$: coherent initial state

$$A = \mathcal{A}_{\text{LO}} + \int_{\mathbf{p}} c_{\mathbf{p}} \alpha_{\mathbf{p}} \quad \langle c_{\mathbf{p}} c_{\mathbf{p}'} \rangle \sim \frac{1}{2} \delta_{\mathbf{p} \mathbf{p}'}$$

Occupation number :

$$\langle \tilde{A} \tilde{A}^* \rangle_{\tau=0^+} = \underbrace{\tilde{\mathcal{A}}_{\text{LO}} \tilde{\mathcal{A}}_{\text{LO}}^*}_{\sim \delta(\mathbf{p}_z) f(\mathbf{p}_\perp)} + \frac{1}{2}$$

IS IT POSSIBLE TO START FROM A DECOHERED STATE ?



- At $\tau \gtrsim Q_s^{-1}$, large occupation in a broad range of p_z, p_\perp

IS IT POSSIBLE TO START FROM A DECOHERED STATE ?

Incoherent distribution of particles :

$$A = \int_{\mathbf{p}} c_{\mathbf{p}} \alpha_{\mathbf{p}} \quad \langle c_{\mathbf{p}} c_{\mathbf{p}'} \rangle \sim \delta_{\mathbf{p}\mathbf{p}'} \left[\frac{1}{2} + f_0(\mathbf{p}) \right]$$

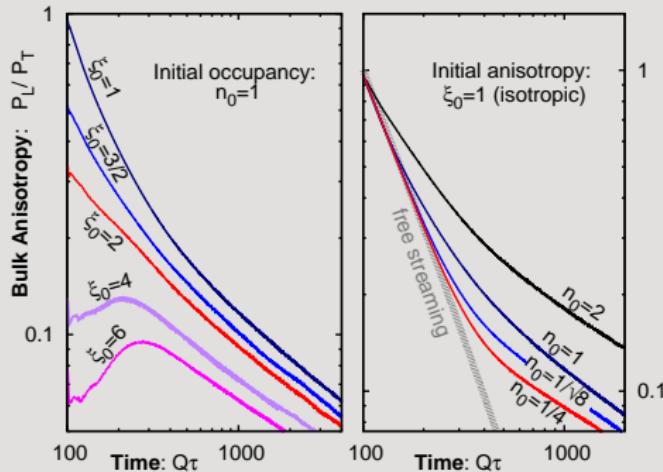
$\frac{1}{2}$ \iff zero point fluctuations

$f_0(\mathbf{p})$ \iff initial particle distribution ($\sim g^{-2}$)

If $f_0(\mathbf{p}) \gg 1$, approximate $\frac{1}{2} + f_0 \rightarrow f_0$?

IS IT POSSIBLE TO START FROM A DECOHERED STATE ?

[Berges, Boguslavski, Schlichting, Venugopalan (2013)]



- No dependence on the coupling (can be scaled out)
- $P_T/P_L \sim \tau^{-2/3}$ (approx breaks at $Q\tau \sim \alpha_s^{-3/2}$)

Classical Statistical Approximation

FROM THE SCHWINGER-KELDYSH PATH INTEGRAL

$$\langle \mathcal{O} \rangle = \int [D\phi_+ D\phi_-] \mathcal{O}[\phi] e^{i(S[\phi_+] - S[\phi_-])}$$

- ϕ_+ = amplitude ϕ_- = conjugate amplitude
- $\phi_+ - \phi_-$ = quantum interference
- Introduce : $\phi_1 \equiv \phi_+ - \phi_-$, $\phi_2 \equiv \frac{1}{2}(\phi_+ + \phi_-)$
$$\underbrace{S[\phi_+] - S[\phi_-]}_{\text{odd in } \phi_1} = \phi_1 \cdot \frac{\delta S[\phi_2]}{\delta \phi_2} + \text{terms cubic in } \phi_1$$
- Strong field regime : ϕ_\pm large, but $\phi_+ - \phi_-$ small
→ Neglect the terms cubic in ϕ_1
 $D\phi_1 \rightarrow$ classical Euler-Lagrange equation for ϕ_2
- Remaining fluctuations in the initial condition for ϕ_2

Schwinger-Keldysh perturbation theory

$$G_{++}(p) = \frac{i}{p^2 + i\epsilon} + 2\pi f_0(p) \delta(p^2) \quad G_{--}(p) = [G_{++}^*(p)]^*$$

$$G_{-+}(p) = 2\pi (\theta(p^0) + f_0(p)) \delta(p^2) \quad G_{+-}(p) = G_{-+}(-p)$$

$$\Gamma_{++++} = -ig^2 \quad \Gamma_{----} = +ig^2$$

IN PERTURBATION THEORY

After rotation $\phi_{\pm} \rightarrow \phi_{1,2}$:

$$G_{21}(p) = \frac{i}{p^2 + ip^0\epsilon} \quad G_{12}(p) = \frac{i}{p^2 - ip^0\epsilon}$$
$$G_{22}(p) = 2\pi\left(\frac{1}{2} + f_0(p)\right)\delta(p^2) \quad G_{11}(p) = 0$$

$$\Gamma_{1222} = -ig^2 \quad \Gamma_{1112} = -\frac{i}{4}g^2$$

- **Weak CSA** : drop Γ_{1112}
- **Strong CSA** : drop Γ_{1112} AND the $1/2$ in $\frac{1}{2} + f_0(p)$

ULTRAVIOLET SENSITIVITY

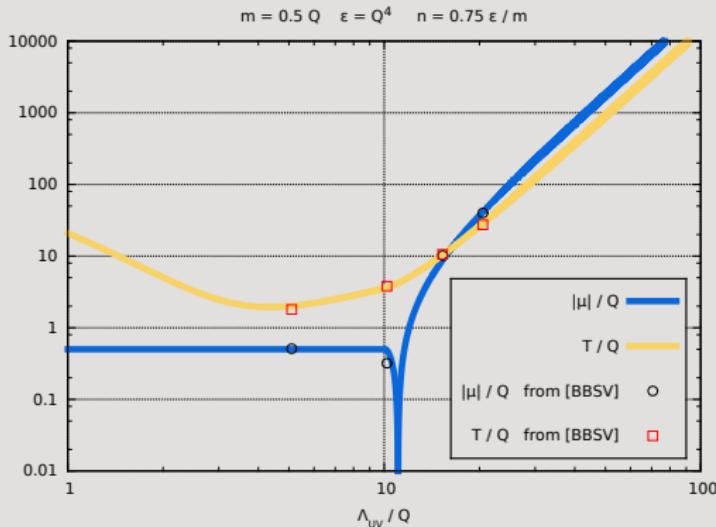
- CSA \neq underlying theory at 2-loops and beyond
- Vacuum fluctuations make the **Weak CSA** non-renormalizable
Example of problematic graph :

$$\text{Im } \frac{1}{\text{---}} \begin{array}{c} \textcolor{red}{2} \\ \diagup \quad \diagdown \\ \textcolor{magenta}{\text{---}} \end{array} \textcolor{red}{2} = -\frac{g^4}{1024\pi^3} \left(\Lambda_{\text{UV}}^2 - \frac{2}{3} p^2 \right)$$

\implies divergence in an operator not present in the Lagrangian

- Strong CSA has no such problem of UV sensitivity

ULTRAVIOLET SENSITIVITY



- Weak cutoff dependence if $\Lambda_{\text{UV}} \sim (3 - 6) \times (\text{physical scales})$

Classical approximations in Kinetic Theory

FROM QFT TO KINETIC THEORY

Dyson-Schwinger equations → Kadanoff-Baym equations → Boltzmann : $p^\mu \partial_\mu f = C_p[f]$

- Collision term in the $\phi_{1,2}$ basis:

$$C_p[f] = \frac{i}{2} \left[\Sigma_{11}(p) + \left(\frac{1}{2} + f(p) \right) \left(\Sigma_{21}(p) - \Sigma_{12}(p) \right) \right]$$



$$\begin{aligned} \Rightarrow C_p[f] = & \frac{g^4}{4E_p} \int_k \int_{p'} \int_{k'} (2\pi)^4 \delta(P + K - P' - K') \\ & \times \left[f(p')f(k') (1 + f(p))(1 + f(k)) \right. \\ & \left. - f(p)f(k) (1 + f(p'))(1 + f(k')) \right] \end{aligned}$$

FROM QFT TO KINETIC THEORY

D

Weak CSA collision term :

$$\begin{aligned} C_p[f] = & \frac{g^4}{4E_p} \int_k \int_{p'} \int_{k'} (2\pi)^4 \delta(P + K - P' - K') \\ & \times \left[\left(\frac{1}{2} + f(p') \right) \left(\frac{1}{2} + f(k') \right) \left(\frac{1}{2} + f(p) \right) \left(\frac{1}{2} + f(k) \right) \right. \\ & \left. - \left(\frac{1}{2} + f(p) \right) \left(\frac{1}{2} + f(k) \right) \left(\frac{1}{2} + f(p') \right) \left(\frac{1}{2} + f(k') \right) \right] \end{aligned}$$

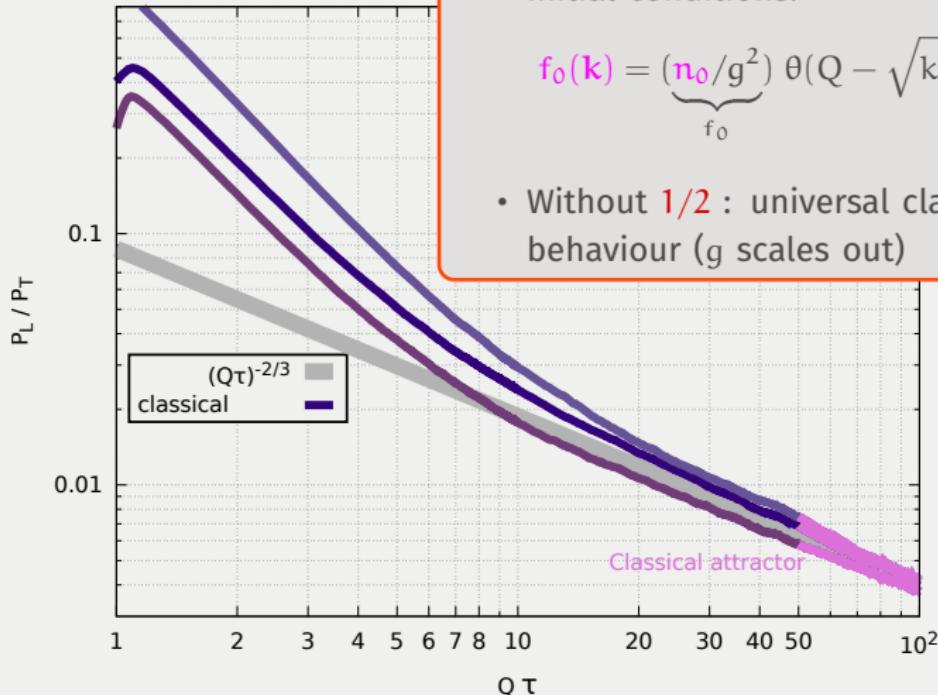
(Terms in f^3 and f^2 correct, but spurious f^1 terms)

– **Strong CSA** : drop also all the $\frac{1}{2}$ (Terms in f^3 correct) K'

$$\begin{aligned} & \times \left[f(p')f(k') \left(1 + f(p) \right) \left(1 + f(k) \right) \right. \\ & \left. - f(p)f(k) \left(1 + f(p') \right) \left(1 + f(k') \right) \right] \end{aligned}$$

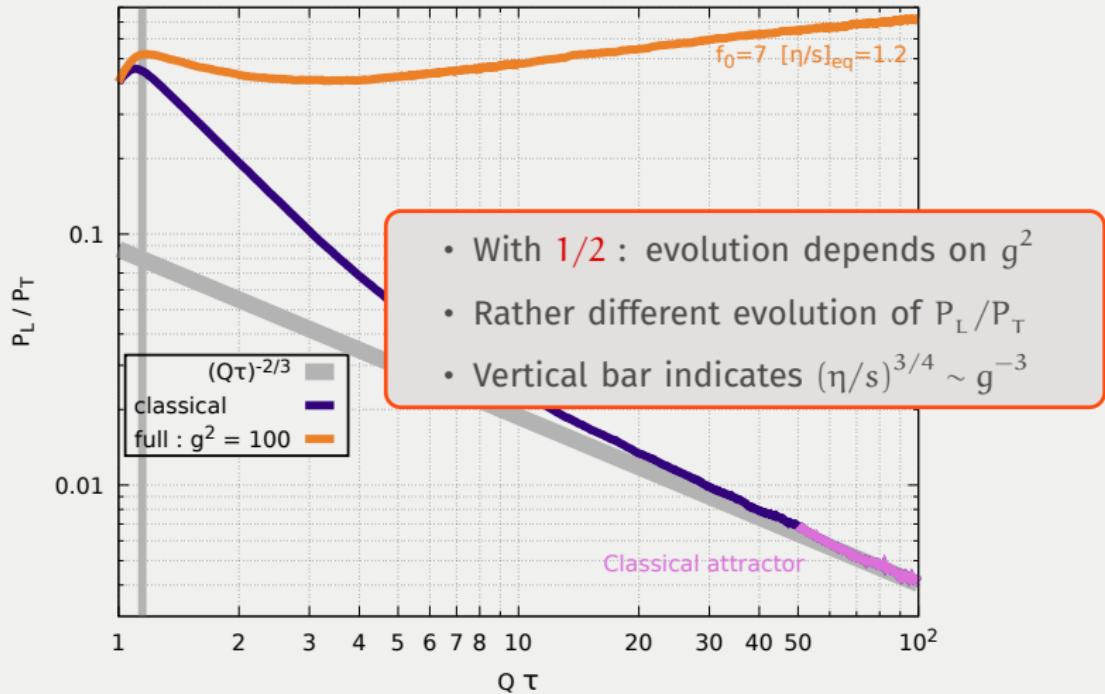
ISOTROPIZATION IN A LONGITUDINALLY EXPANDING SYSTEM

[Epelbaum, FG, Jeon, Moore, Wu (2015)]



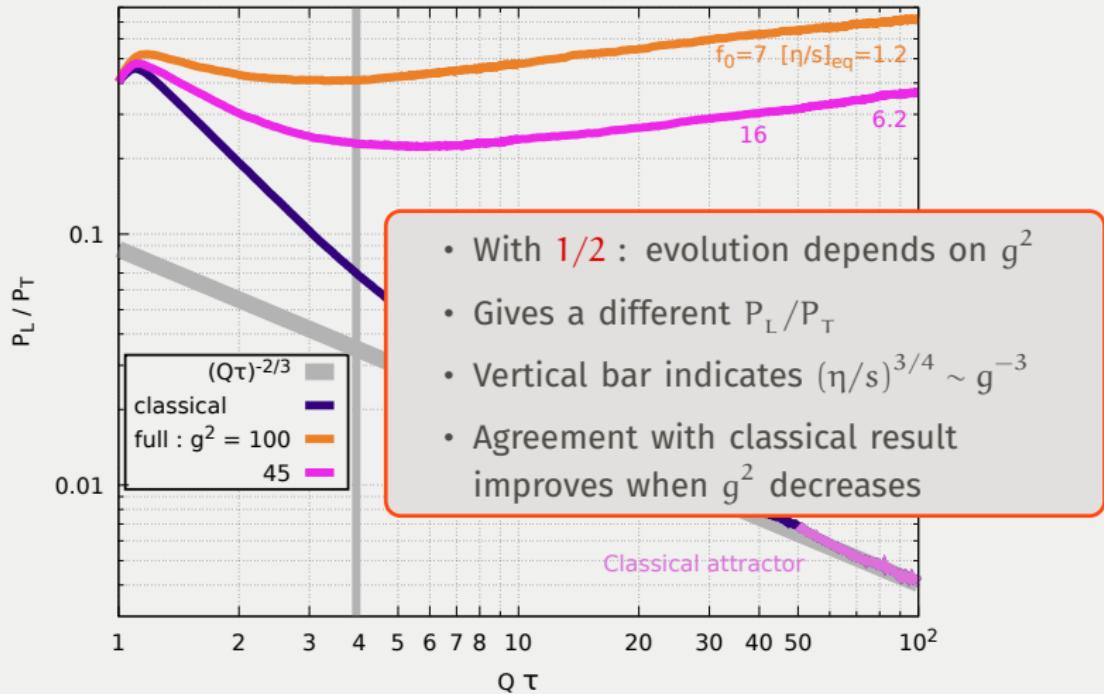
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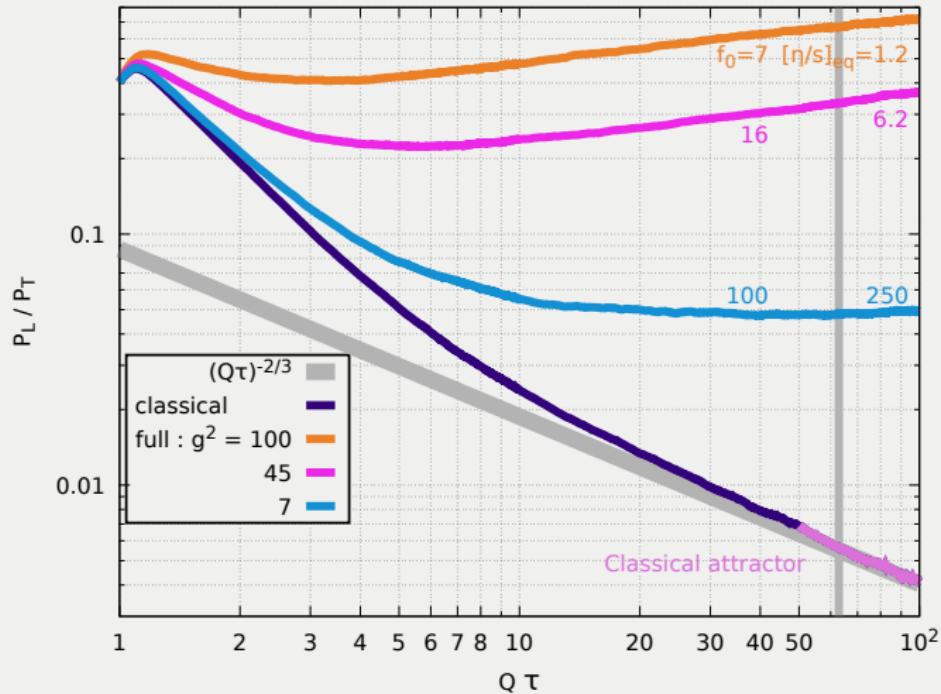
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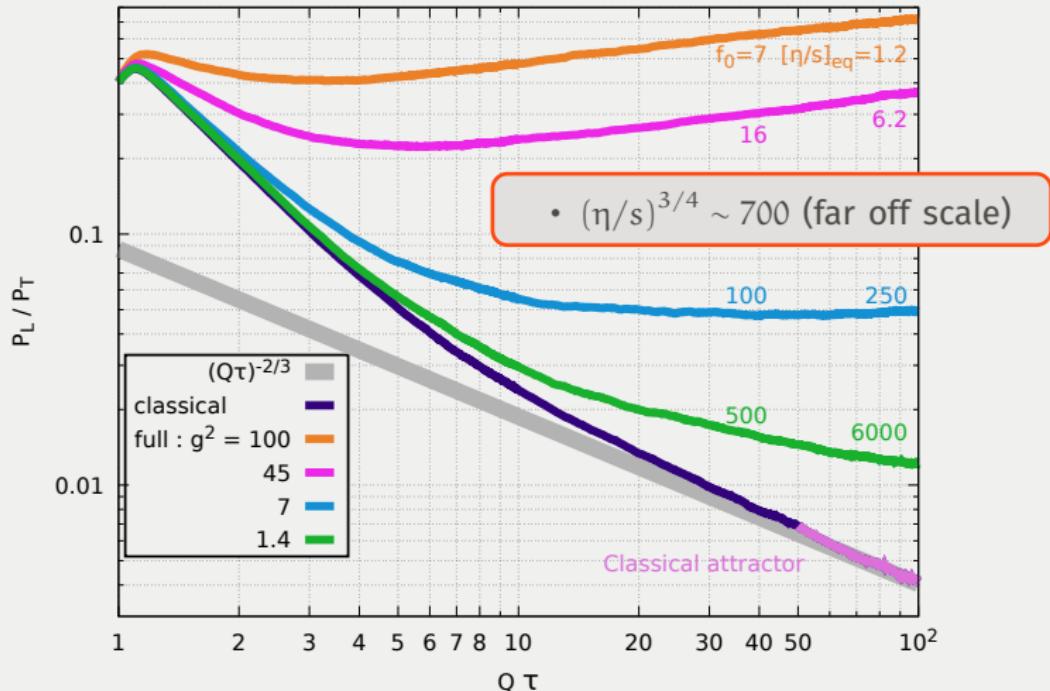
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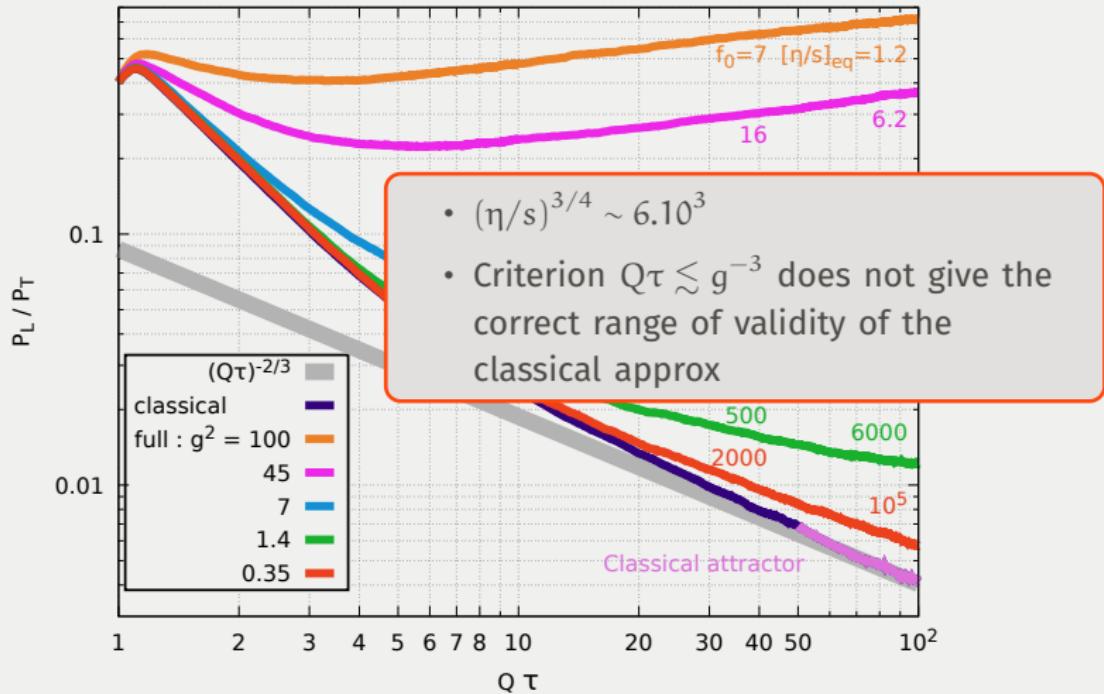
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ISOTROPIZATION IN A LONGITUDINALLY EXPANDING SYSTEM

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WHY IS THE VACUUM $1/2$ IMPORTANT ?

- The $1/2$'s ensure that the terms f^3 and f^2 are correct
- The quadratic terms are important in anisotropic systems

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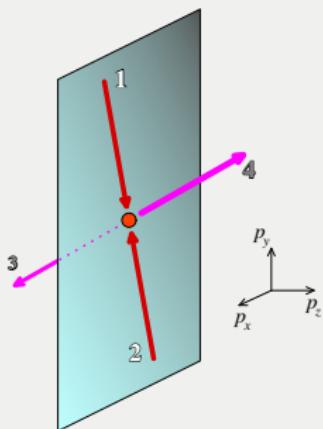
- The $1/2$'s ensure that the terms f^3 and f^2 are correct
- The quadratic terms are important in anisotropic systems

- No $1/2 \implies$ no f^2 terms in Boltzmann eq. :

$$\partial_t f_4 \sim g^4 \int_{123} \dots [f_1 f_2 (f_3 + f_4) - f_3 f_4 (f_1 + f_2)] \\ + \dots [\cancel{f_1 f_2} - \cancel{f_3 f_4}]$$

WHY IS THE VACUUM $1/2$ IMPORTANT ?

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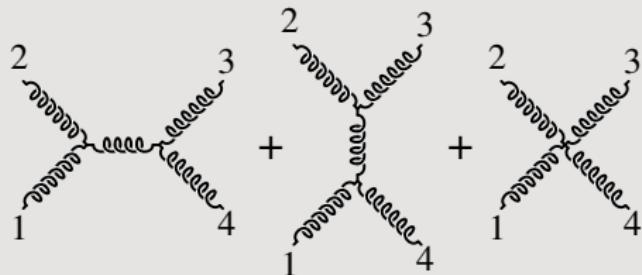
$$\partial_t f_4 \sim g^4 \int_{123} \cdots [f_1 f_2 (f_3 + f_4) - f_3 f_4 (f_1 + f_2)] + \cdots [f_1 f_2 - f_3 f_4]$$

- When the distribution is very anisotropic, trying to produce the particle 4 at large angle results in $f_3 \approx f_4 \approx 0 \Rightarrow$ nothing left
- Cubic terms \Leftrightarrow stimulated emission : ineffective to produce particles in empty regions of phase-space

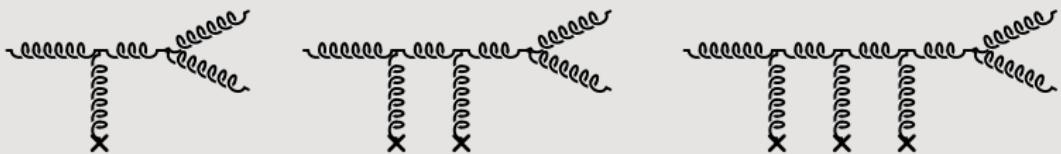
More insights from kinetic theory

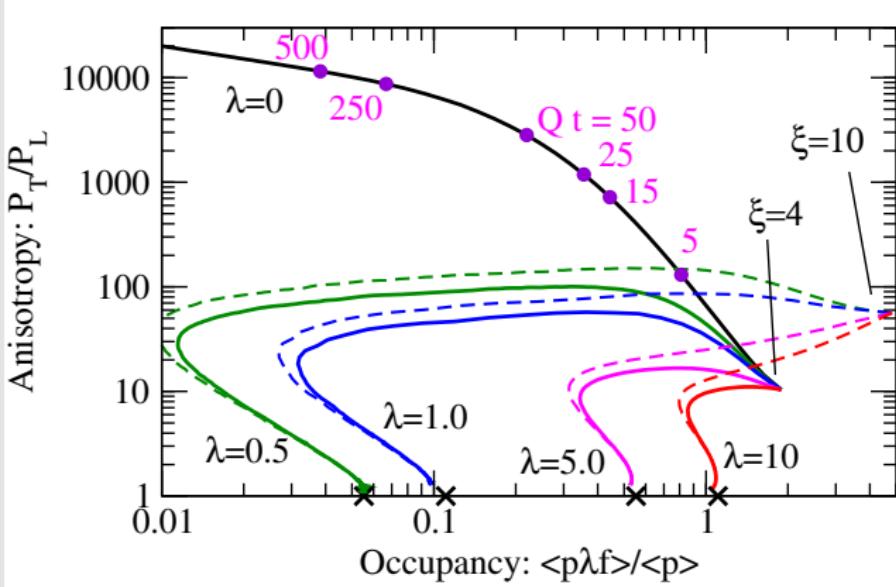
KINETIC THEORY FOR GLUONS [Kurkela, Zhu (2015)]

$2 \rightarrow 2$



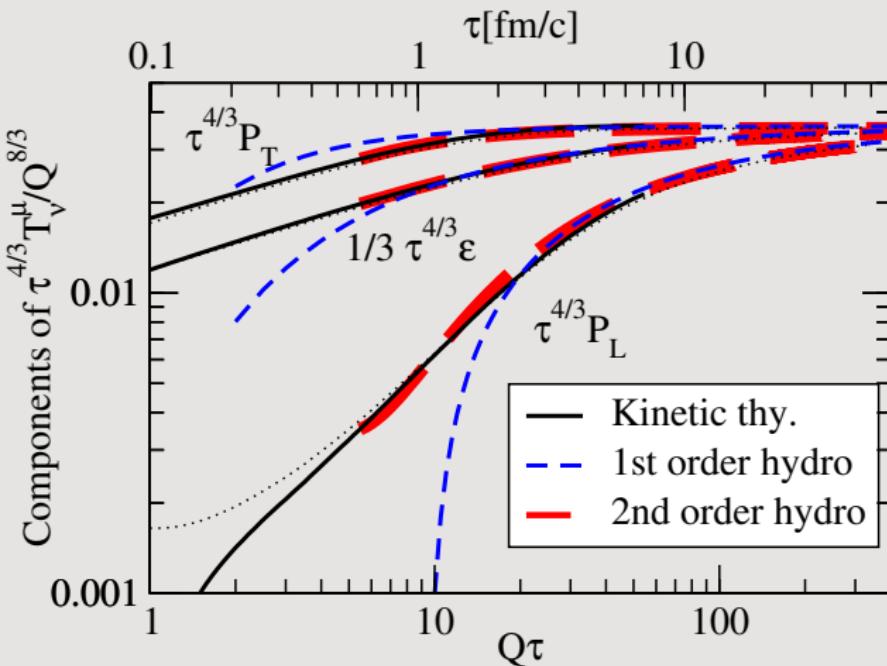
$1 \rightarrow 2, 2 \rightarrow 1 + \text{Landau-Pomeranchuk-Migdal resummation}$





For $\lambda = 0.5$, the **Strong CSA** breaks down at $Q\tau \approx 2$, while simple estimates suggested that it would be valid up to $Q\tau \approx \alpha_s^{-3/2} \approx 350$

Consistent with hydrodynamics before full isotropization



Boltzmann in the relaxation time approximation

$$\left(\partial_\tau - \frac{p_z}{\tau} \right) f(\tau, p) = - \frac{f - f_{\text{eq}}}{\tau_R}$$

τ_R \equiv relaxation time

f_{eq} \equiv local equilibrium dist

- $\tau_R = \infty$: no collisions
- $\tau_R \sim \epsilon^{-1/4}$: conformal; rate scales as inverse temperature
- $\tau_R = \text{const}$: fixed collision rate (not realistic with expansion)

- Define moments :

$$L_n \equiv \int_{\mathbf{p}} \mathbf{p}^2 P_{2n}(p_z/p) f(\tau, \mathbf{p}) , \quad g_n \equiv \tau \partial_\tau \ln L_n$$

$$L_0 = \epsilon = P_L + 2P_T, \quad L_1 = P_L - P_T$$

Boltzmann \Leftrightarrow coupled equations for L_n

$$\begin{aligned}\partial_\tau L_0 &= -\frac{a_0 L_0 + c_0 L_1}{\tau} \\ \partial_\tau L_n &= -\frac{a_n L_n + c_n L_{n+1} + b_n L_{n-1}}{\tau} - \frac{L_n}{\tau_R} \quad (n \geq 1)\end{aligned}$$

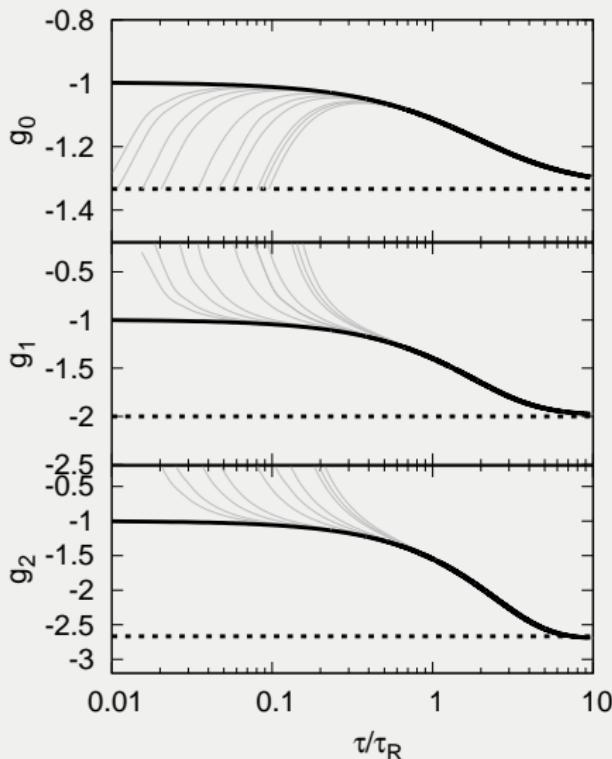
a_n, b_n, c_n = pure numbers, known explicitly (depend only on the free streaming part of Boltzmann eq.)

Free streaming fixed point ($\tau_R = \infty$)

- All the g_n behave as τ^{-1} , with fixed ratios
- $L_1/L_0 \rightarrow -\frac{1}{2}$, i.e. $P_L/P_T \rightarrow 0$

Interacting fixed point ($\tau_R \sim \epsilon^{-1/4}$)

- $g_0 \rightarrow -4/3$, $g_1 \rightarrow -2$
- Locally isotropic distribution



- Universal attractor
- $\tau \lesssim \tau_R$: trajectories first approach free streaming fixed point
- $\tau \gtrsim \tau_R$: trajectories go to the local equilibrium fixed point

Two-Particle Irreducible framework

IS SOMETHING MISSING ?

	Weak CSA	Strong CSA	Kinetic th.
Ultraviolet	✗	✓	✓
f^2 terms	✓	✗	✓

Is SOMETHING MISSING ?

	Weak CSA	Strong CSA	Kinetic th.
Ultraviolet	✗	✓	✓
f^2 terms	✓	✗	✓
Screening	✓	✓	✗

Is SOMETHING MISSING ?

	Weak CSA	Strong CSA	Kinetic th.	2-PI
Ultraviolet	✗	✓	✓	✓
f^2 terms	✓	✗	✓	✓
Screening	✓	✓	✗	✓

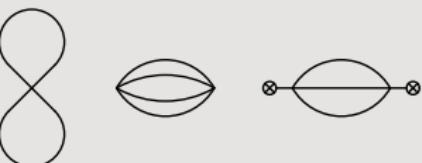
TWO-PARTICLE IRREDUCIBLE FORMALISM

Quantum effective action

$$\Gamma[\varphi, G] = S[\varphi] - \frac{i}{2} \text{tr} \log G + \frac{i}{2} \text{tr} \left((G_0^{-1} - G^{-1}) G \right) + \Phi[\varphi, G]$$

$\Phi[\varphi, G]$ = sum of vacuum 2PI graphs

Truncation at order g^4

$$\Phi[\varphi, G] =$$


TWO-PARTICLE IRREDUCIBLE FORMALISM

Equations of motion

$$\frac{\delta \Gamma}{\delta \varphi_x} = 0$$

$$\frac{\delta \Gamma}{\delta G_{xy}} = 0$$

COMPUTATIONAL COST PER TIME STEP

Kinetic theory (deterministic algorithm)

3d-isotropic :

$$N^3$$

Long. expansion + azimuthal symm. :

$$N^3 N_z^3$$

2PI with N_{mem} -deep time memory integral

3d-isotropic :

$$N \log(N) \times N_{\text{mem}}^2$$

Long. expansion + azimuthal symm. : $NN_z \log(NN_z) \times N_{\text{mem}}^2$

[So far, one implementation : **Hatta, Nishiyama (2013)**]

Summary

- LO : no pressure isotropization, NLO : instabilities
- Beyond NLO : **Classical statistical approximation**
 - **Weak CSA :**
non-renormalizable, sensitive to UV cutoff
 - **Strong CSA :**
underestimates large angle scatterings
breaks rapidly unless η/s very large
- **Kinetic theory :** avoids all these difficulties, but does not cope well with screening effects in the soft region
- **Two-PI for longitudinally expanding systems :**
Important to properly treat screening effects

TWO-PARTICLE IRREDUCIBLE FORMALISM

Energy-momentum tensor

$$\begin{aligned} T^{\mu\nu} = & \nabla^\mu \varphi \nabla^\nu \varphi - g^{\mu\nu} \mathcal{L} + \left[\nabla_x^\mu \nabla_y^\nu G_{xy} \right]_{x=y} \\ & + \frac{1}{2} g^{\mu\nu} \left\{ V''(\varphi_x) G_{xx} - \left[\nabla_\alpha^x \nabla_y^\alpha G_{xy} \right]_{x=y} \right\} \\ & - g^{\mu\nu} \frac{\delta \Phi}{\delta \sqrt{-g}} \end{aligned}$$

ISOTROPIZATION IN A FIXED BOX

