Gluon saturation, Factorization, and Parton Distributions

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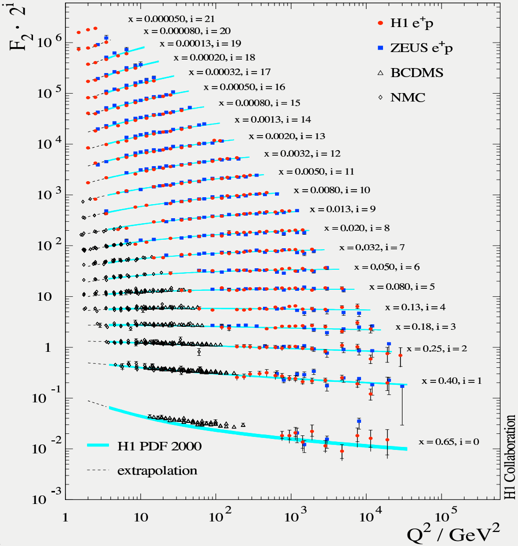
- Gluon saturation
- Factorization in the dense regime
- From dense to dilute
- When can we use standard PDFs ?

Gluon Saturation

What do we know from QCD?

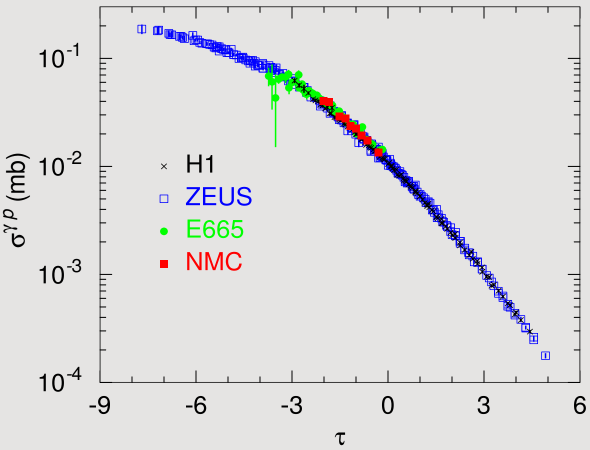
- Asymptotic freedom + time dilation in a high energy hadron explain why the partons appear as almost free at large Q^2
- QCD loop corrections lead to violations of Bjorken scaling, that are visible as a Q^2 dependence of the structure functions ($1/Q$ is the spatial resolution at which the hadron is probed)
- Parton distributions are non-perturbative in QCD, but their Q^2 and x dependence are governed by equations that are perturbative (DGLAP, BFKL)
- One can prove that the parton distributions are universal, i.e. are the same in all inclusive processes

DIS RESULTS FOR F_2 AND DGLAP FIT AT NLO :

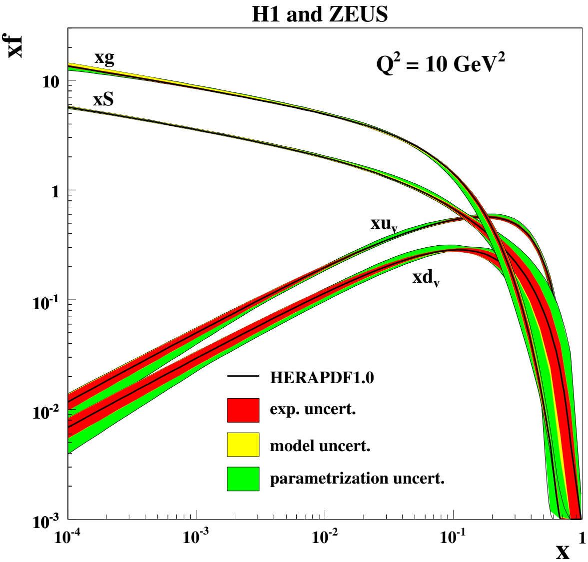


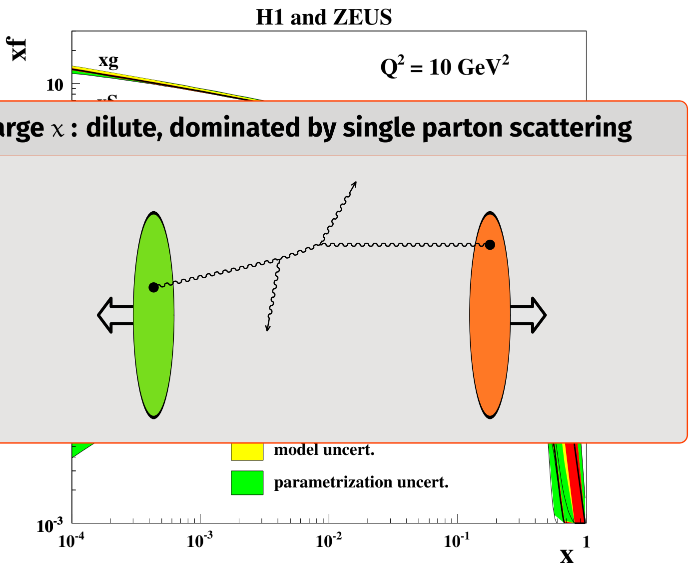
SMALL x DATA DISPLAYED DIFFERENTLY... (GEOMETRICAL SCALING)

Small x data ($x \leq 10^{-2}$) displayed against $\tau \equiv \log(x^{0.32}\,Q^2)$:



NNLO PARTON DISTRIBUTIONS – AND POSSIBLE CAVEATS





Large x : dilute, dominated by single parton scattering

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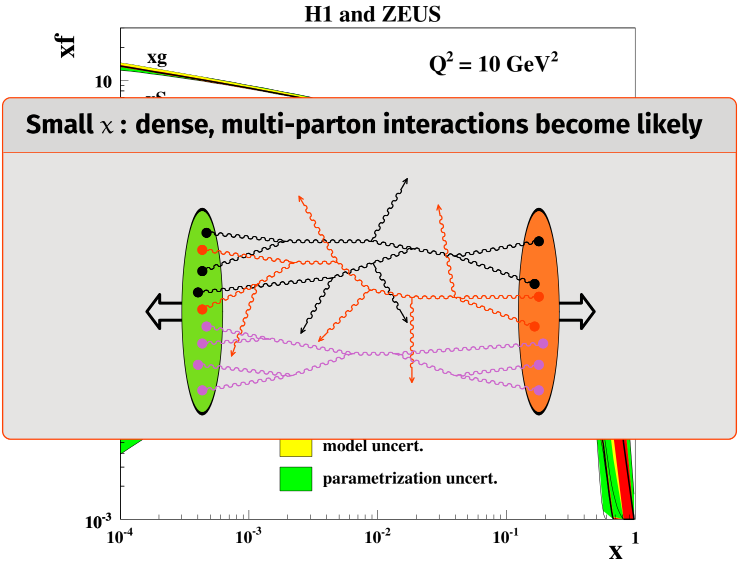


Small x : dense, multi-parton interactions become likely

- When their occupation number becomes large, gluons can recombine :

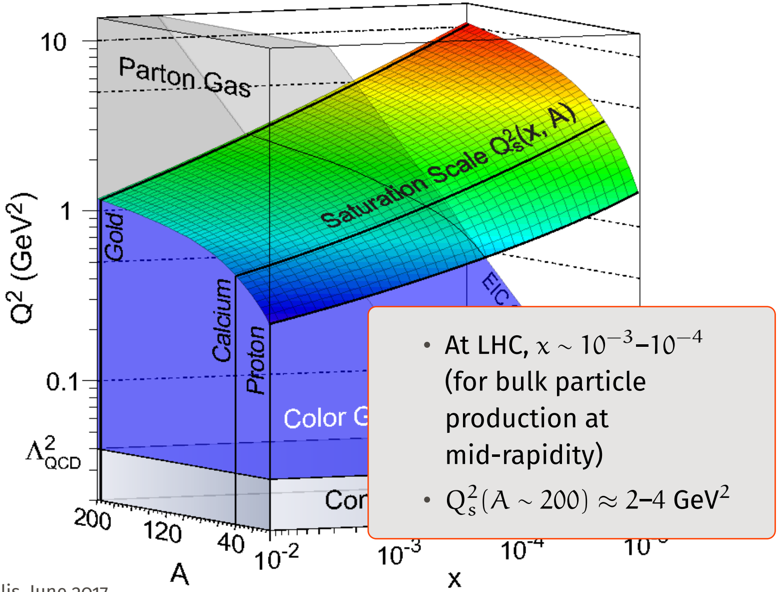
Gluon Saturation

Saturation criterion [Gribov, Levin, Ryskin (1983)]

$$\underbrace{\alpha_s Q^{-2}}_{\sigma_{gg\to g}} \times \underbrace{A^{-2/3} xG(x, Q^2)}_{\text{surface density}} \geq 1$$

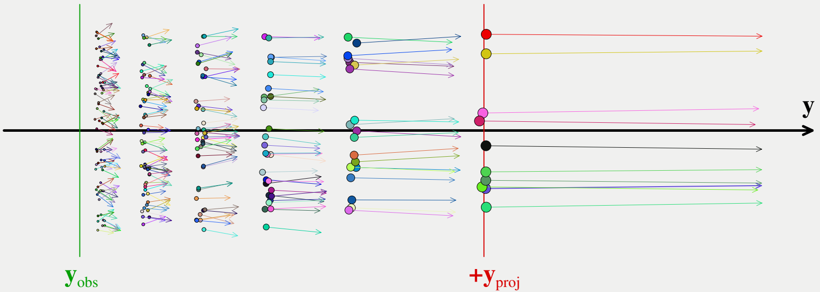
$$Q^2 \leq \underbrace{Q_s^2 \equiv \frac{\alpha_s xG(x, Q_s^2)}{A^{2/3}}}_{\text{(saturation momentum)}^2} \sim A^{1/3} x^{-0.3}$$

Saturation domain



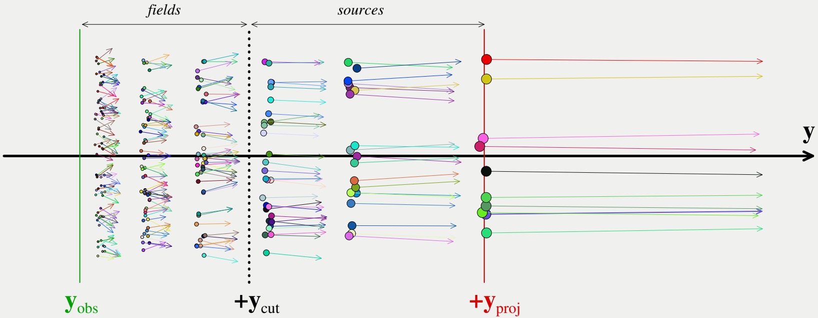
- At LHC, $x \sim 10^{-3}$–10^{-4} (for bulk particle production at mid-rapidity)
- $Q_s^2(A \sim 200) \approx 2$–$4$ GeV^2

Degrees of freedom



- $p_\perp^2 \sim Q_s^2 \sim \Lambda_{QCD}\, e^{\lambda(y_{proj}-y)}$, $p_z \sim Q_s\, e^{y-y_{obs}}$
- Fast partons : frozen dynamics, negligible $p_\perp$ $\Rightarrow$ classical sources
- Slow partons : evolve with time $\Rightarrow$ gauge fields

Degrees of freedom



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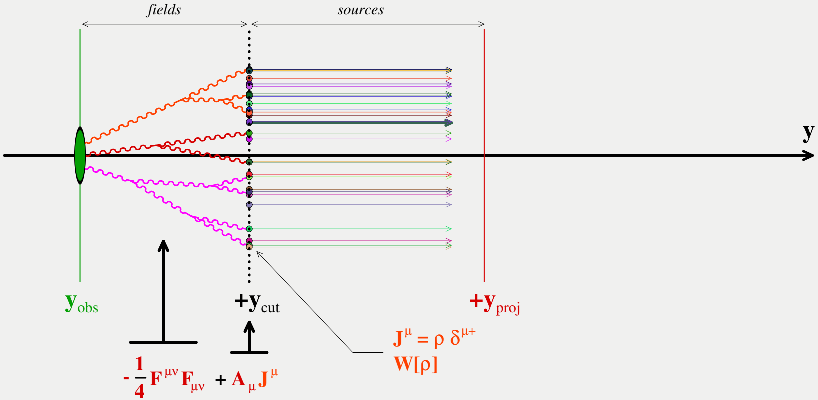

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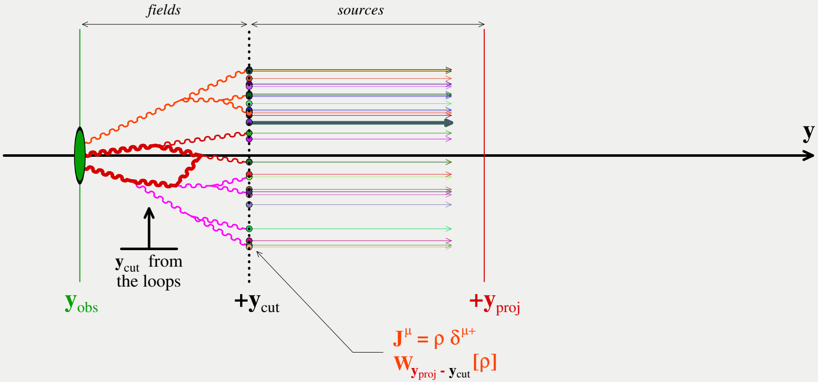

- The cutoff $y_{\rm cut}$ is arbitrary and should not affect the result
- The probability density $W[\rho]$ changes with the cutoff
- Loop corrections cancel the cutoff dependence from $W[\rho]$

B-JIMWLK evolution equation

Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner

$$\frac{\partial W_Y[\rho]}{\partial Y} = \underbrace{\frac{1}{2}\int_{\vec{x}_\perp,\vec{y}_\perp} \frac{\delta}{\delta\rho_a(\vec{x}_\perp)}\chi_{ab}(\vec{x}_\perp,\vec{y}_\perp)\frac{\delta}{\delta\rho_b(\vec{y}_\perp)}}_{\mathcal{H}\quad \text{(JIMWLK Hamiltonian)}} \; W_Y[\rho]$$

- Mean field approx. (BK equation) : [Kovchegov (1999)]
- Langevin form of B-JIMWLK : [Blaizot, Iancu, Weigert (2003)]
- First numerical solution : [Rummukainen, Weigert (2004)]

Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner

Recent developments :

Running coupling correction

[Lappi, Mäntysaari (2012)]

B-JIMWLK equation at Next to Leading Log

[Kovner, Lublinsky, Mulian (2013)]

[Caron-Huot (2013)] [Balitsky, Chirilli (2013)]

- Me
- Lan 03)]
- First numerical solution : [Rummukainen, Weigert (2004)]

Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner

Recent developments :

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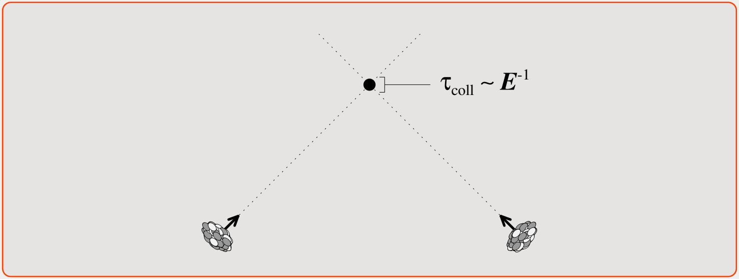
Open questions for practical uses :

- Does the NLO evolution preserve the positivity of $W[\rho]$? (non trivial if the JIMWLK Hamiltonian contains higher derivatives at NLO)
- Can the NLO B-JIMWLK equation still be mapped into a Langevin equation?

Factorization in the dense regime

- Deep inelastic scattering
- Nucleus-nucleus collisions

Handwaving argument for factorization



- The duration of the collision is very short: $\tau_{\rm coll} \sim E^{-1}$

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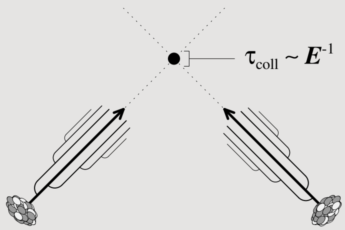

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  ▷ it must happen (long) before the collision

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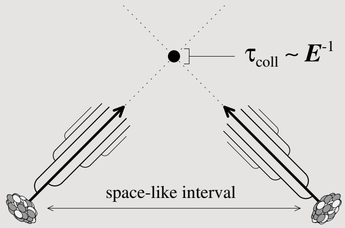
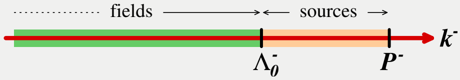


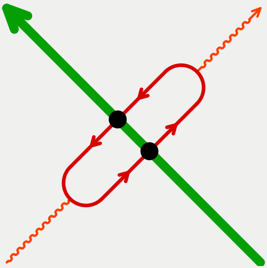
- The duration of the collision is very short: $\tau_{\rm coll} \sim E^{-1}$
- The logarithms we want to resum are due to the radiation of soft gluons, which takes a long time
  ▷ it must happen (long) before the collision
- The projectiles are not in causal contact before the impact
  ▷ the logarithms are intrinsic properties of the projectiles, independent of the measured observable

Deep Inelastic Scattering

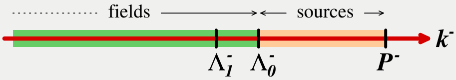
- CGC effective theory with cutoff at the scale Λ_0^- :



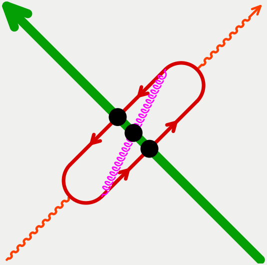
- At Leading Order, DIS can be seen as the interaction between the target and a $q\bar{q}$ fluctuation of the virtual photon :

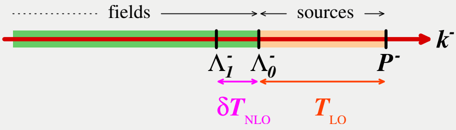


- Consider now quantum corrections to the previous result, restricted to modes with $\Lambda_1^- < k^- < \Lambda_0^-$ (the upper bound prevents double-counting with the sources):



- At NLO, the $q\bar{q}$ dipole must be corrected by a gluon, e.g. :





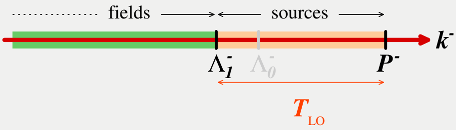
- At leading log accuracy, the contribution of the quantum modes in that strip is :

$$\delta\mathbf{T}_{\rm NLO}(\vec{x}_\perp, \vec{y}_\perp) = \ln\left(\frac{\Lambda_0^-}{\Lambda_1^-}\right)\ \mathcal{H}\ \mathbf{T}_{\rm LO}(\vec{x}_\perp, \vec{y}_\perp)$$

- These NLO corrections can be absorbed in the LO result,

$$\left\langle \mathsf{T}_{\mathrm{LO}} + \delta\mathsf{T}_{\mathrm{NLO}} \right\rangle_{\Lambda_0^-} = \left\langle \mathsf{T}_{\mathrm{LO}} \right\rangle_{\Lambda_1^-}$$

provided one defines a new effective theory with a lower cutoff Λ_1^- and an extended distribution of sources $W_{\Lambda_1^-}[\rho]$:



$$W_{\Lambda_1^-} \equiv \left[1 + \ln\left(\frac{\Lambda_0^-}{\Lambda_1^-}\right)\ \mathcal{H}\right] W_{\Lambda_0^-}$$

Nucleus-Nucleus collisions

Leading Log corrections in AA collisions

- By keeping only the terms that contain logarithms of the cutoff, the NLO result can be written as :

$$\mathcal{O}_{\text{NLO}} \underset{\text{Leading Log}}{=} \Big[\log\left(\Lambda^+\right) \mathcal{H}_1 + \log\left(\Lambda^-\right) \mathcal{H}_2\Big] \,\mathcal{O}_{\text{LO}}$$

$\mathcal{H}_{1,2}$: JIMWLK Hamiltonians for the two nuclei

- Note : the logs do not mix the two nuclei $\Rightarrow$ Factorization

- By integrating over $\rho_{1,2}$'s, one can absorb the logarithms into universal distributions $W_{1,2}[\rho_{1,2}]$

Inclusive observables at Leading Log accuracy

$$\mathcal{O}_{\text{leading log}} = \int \big[D\rho_1 \; D\rho_2\big] \; W_1\big[\rho_1\big] \; W_2\big[\rho_2\big] \quad \underbrace{\mathcal{O}_{\text{LO}}}_{\text{fixed } \rho_{1,2}}$$

- Logs absorbed into the evolution of $W_{1,2}$ with the scales

$$\Lambda \frac{\partial W}{\partial \Lambda} = \mathcal{H}\, W \qquad \text{(JIMWLK equation)}$$

From dense to dilute

Dense → dilute limit

- Factorization in the saturated regime:

$$\langle \mathcal{O} \rangle_{\text{LLog}} = \int \big[D\rho_1\, D\rho_2\big]\; W_1\big[\rho_1\big]\; W_2\big[\rho_2\big]\; \mathcal{O}[\rho_{1,2}]$$

($\mathcal{O}[\rho_{1,2}]$ can only be calculated numerically)

- When ρ_1 is a weak source (projectile 1 is dilute):

$$\mathcal{O}[\rho_{1,2}] = \int_{\vec{k}_{1\perp}} \rho_1^2(\vec{k}_{1\perp})\; \mathcal{O}_2[\vec{k}_{1\perp}, \rho_2] + \rho_1^4(\vec{k}_{1\perp})\; \mathcal{O}_4[\vec{k}_{1\perp}, \rho_2] + \cdots$$

and $\mathcal{O}_2[\vec{k}_{1\perp}, \rho_2]$ has a compact analytical expression

- One gets the non-integrated gluon distribution:

$$\int [D\rho_1]\; W_1[\rho_1]\; \rho_1^2(\vec{k}_{1\perp}) \equiv \varphi_1(\vec{k}_{1\perp})$$

- The expectation value of $\mathcal{O}$ can be rewritten as

$$\left\langle \mathcal{O} \right\rangle_{\rm LLog} = \int_{\vec{k}_{1\perp}} \varphi_1(\vec{k}_{1\perp}) \int \left[D\rho_2\right]\; W_2\left[\rho_2\right]\; \mathcal{O}_2[\vec{k}_{1\perp}, \rho_2]$$

- $\mathcal{O}_2[\vec{k}_{1\perp}, \rho_2]$ is made of correlators of Wilson lines

Example : heavy quarks production in pA collisions

Pair production cross-section:

$$\frac{d\sigma_{q\bar{q}}}{d^2\vec{p}_\perp d^2\vec{q}_\perp dy_p dy_q} = \frac{\alpha_s^2 N}{8\pi^4 d_A} \int_{\vec{k}_{1\perp},\vec{k}_{2\perp}} \frac{\delta(\vec{p}_\perp + \vec{q}_\perp - \vec{k}_{1\perp} - \vec{k}_{2\perp})}{k_{1\perp}^2 k_{2\perp}^2}$$

$$\times \Big\{ \int_{\vec{k}_\perp,\vec{k}'_\perp} \mathrm{tr}\Big[(\not{q}+m) T_{q\bar{q}}(\vec{k}_\perp)(\not{p}-m) T^*_{q\bar{q}}(\vec{k}'_\perp)\Big] \phi_A^{(4)}(\vec{k}_{2\perp}|\vec{k}_\perp,\vec{k}'_\perp)$$

$$+ \int_{\vec{k}_\perp} \mathrm{tr}\Big[(\not{q}+m) T_{q\bar{q}}(\vec{k}_\perp)(\not{p}-m) \not{L}^* + \mathrm{h.c.}\Big] \phi_A^{(3)}(\vec{k}_{2\perp}|\vec{k}_\perp)$$

$$+ \mathrm{tr}\Big[(\not{q}+m)\not{L}(\not{p}-m)\not{L}^*\Big] \phi_A^{(2)}(\vec{k}_{2\perp}) \Big\} \varphi_1(\vec{k}_{1\perp})$$

▷ standard factorization schemes broken for the nucleus: one needs three different “distributions” in order to describe the target

Target correlators

$$\phi_A^{(2)}(\vec{k}_{2\perp}) \propto \int_{\vec{x}_\perp,\vec{y}_\perp} e^{i\vec{k}_{2\perp}\cdot(\vec{x}_\perp-\vec{y}_\perp)}\ \mathrm{tr}\Big\langle U(\vec{x}_\perp)U^\dagger(\vec{y}_\perp)\Big\rangle$$

$$\phi_A^{(3)}(\vec{k}_{2\perp}|\vec{k}_\perp) \propto \int_{\vec{x}_\perp,\vec{y}_\perp,\vec{z}_\perp} e^{i\left[\vec{k}_\perp\cdot\vec{x}_\perp+(\vec{k}_{2\perp}-\vec{k}_\perp)\cdot\vec{y}_\perp-\vec{k}_{2\perp}\cdot\vec{z}_\perp\right]} \times\ \mathrm{tr}\Big\langle \widetilde{U}(\vec{x}_\perp)t^a\widetilde{U}^\dagger(\vec{y}_\perp)t^bU_{ba}(\vec{z}_\perp)\Big\rangle$$

$$\phi_A^{(4)}(\vec{k}_{2\perp}|\vec{k}_\perp,\vec{k}'_\perp) \propto \int_{\vec{x}_\perp,\vec{y}_\perp,\vec{x}'_\perp,\vec{y}'_\perp} e^{i\left[\vec{k}_\perp\cdot\vec{x}_\perp-\vec{k}'_\perp\cdot\vec{x}'_\perp+(\vec{k}_{2\perp}-\vec{k}_\perp)\cdot\vec{y}_\perp-(\vec{k}_{2\perp}-\vec{k}'_\perp)\cdot\vec{y}'_\perp\right]} \times\ \mathrm{tr}\Big\langle \widetilde{U}(\vec{x}_\perp)t^a\widetilde{U}^\dagger(\vec{y}_\perp)\widetilde{U}(\vec{y}'_\perp)t^a\widetilde{U}^\dagger(\vec{x}'_\perp)\Big\rangle$$

Limit of Kt factorization

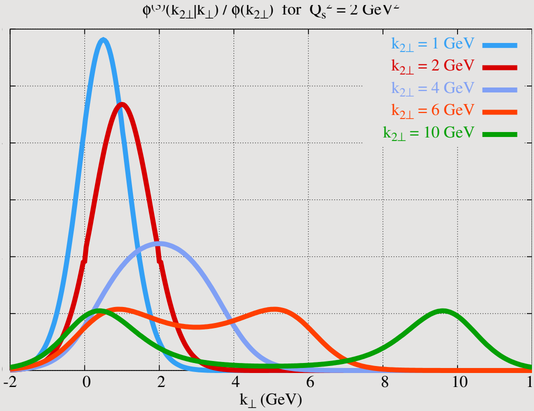
- In the *single quark cross-section*, the integration over the $k_\perp$ of the antiquark simplifies $\phi_A^{(4)}$ into a 2-point function
- The quark cross-section factorizes in terms of transverse momentum dependent distributions provided that the the 3-point and 2-point functions are related by:

$$\phi_A^{(3)}(\vec{k}_{2\perp}|\vec{k}_\perp) = (2\pi)^2 \frac{1}{2}\left[\delta(\vec{k}_\perp) + \delta(\vec{k}_\perp - \vec{k}_{2\perp})\right]\phi_A^{(2)}(\vec{k}_{2\perp})$$

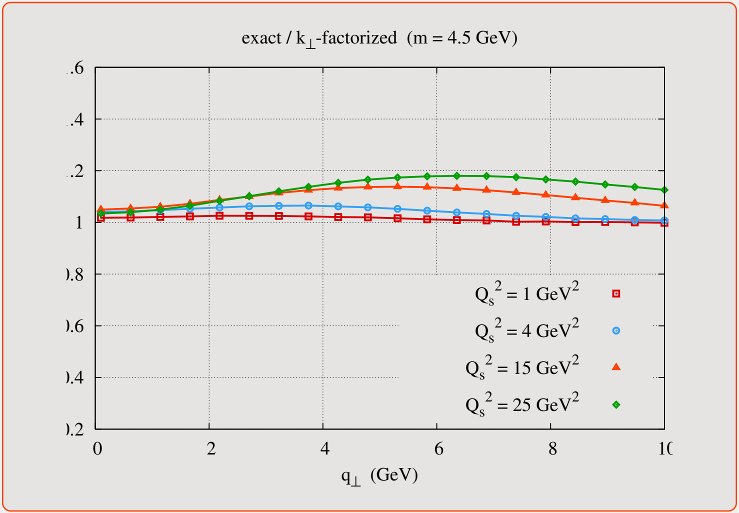
  - This relation is satisfied if the $Q\overline{Q}$ pair interacts with the target in such a way that all the momentum exchanged goes to the quark or to the antiquark

- The ratio $\phi_A^{(3)}(\vec{k}_{2\perp}|\vec{k}_\perp)/\phi_A(\vec{k}_{2\perp})$ should be close to the sum of two delta functions for factorization to be approximately valid

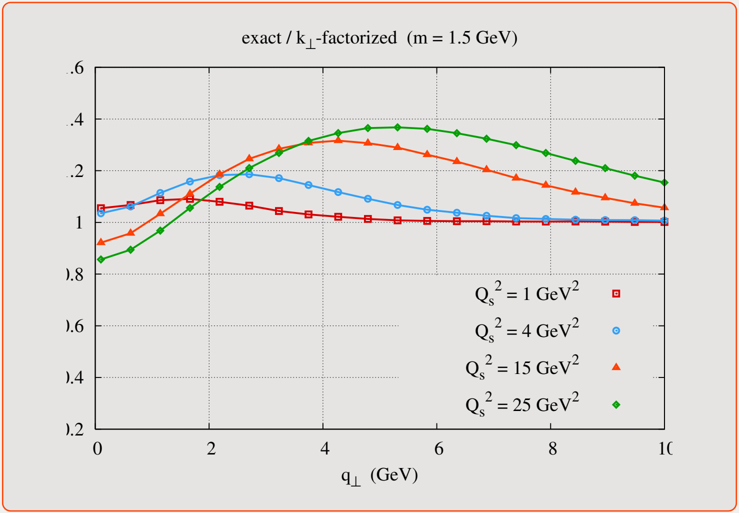
3-POINT / 2-POINT RATIO



FACTORIZATION VIOLATION FOR B QUARKS



Factorization violation for c quarks



When can we use standard PDFs?

Definition

$$q(x, Q^2) \sim \int d^4y \; e^{iq\cdot y} \; \left\langle \overline{\Psi}(0) \cdots \Psi(y) \right\rangle$$

$$G(x, Q^2) \sim \int d^4y \; e^{iq\cdot y} \; \left\langle \mathcal{F}(0) \cdots \mathcal{F}(y) \right\rangle$$

- In the OPE classification, these are *leading twist* operators
- OPE evolution : form a closed set that mix only within itself
- Universality : the same PDFs appear in all observables

Collinear factorization

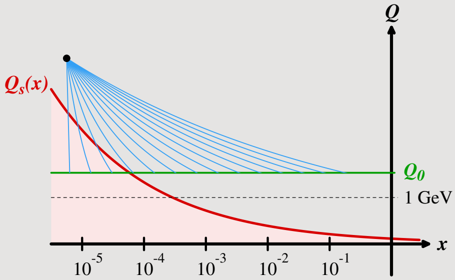
- From their operator definition, it is possible to calculate the PDFs in the dense regime
- Nevertheless, their use would be dubious in this regime because collinear factorization is broken by power corrections that become large when $k_{\mathrm{T}} \lesssim Q_s$

$$\mathcal{O}_{\text{hadrons}} = f \otimes \mathcal{O}_{\text{partons}} \ \oplus \ \underbrace{\sum_{n \geq 1} \left(\frac{Q_s^2}{k_{\mathrm{T}}^2}\right)^n}_{\text{power corrections}}$$

Note : some nuclear effects (e.g. leading twist shadowing) may be included in standard PDFs

Collinear factorization

- Even when used in the non-saturated domain, PDFs may have been contaminated by using DGLAP evolution at too low Q. The initial scale Q_0 should be large enough to mitigate this effect



Summary and Conclusions

- Gluon saturation enhanced in nuclei, reached earlier than in nucleons
- A form of factorization exists in the dense regime (established at Next-to-Leading Log for DIS, at Leading Log for nucleus-nucleus collisions)
  - The universal object is a functional distribution of sources
  - Complicated to use in practice (evolution hard to solve, initial condition poorly constrained)
- When one of the projectiles is dilute, the observables depend only on a few correlators of Wilson lines in the field of the dense projectile. These correlators are universal but more of them are needed for more complicated final states
- Collinear factorization in terms of nuclear PDFs valid when $k_T \gg Q_s$. But beware of possible contamination by DGLAP evolution in unsafe region