

# Glasma Dynamics at Early Times

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François Gelis

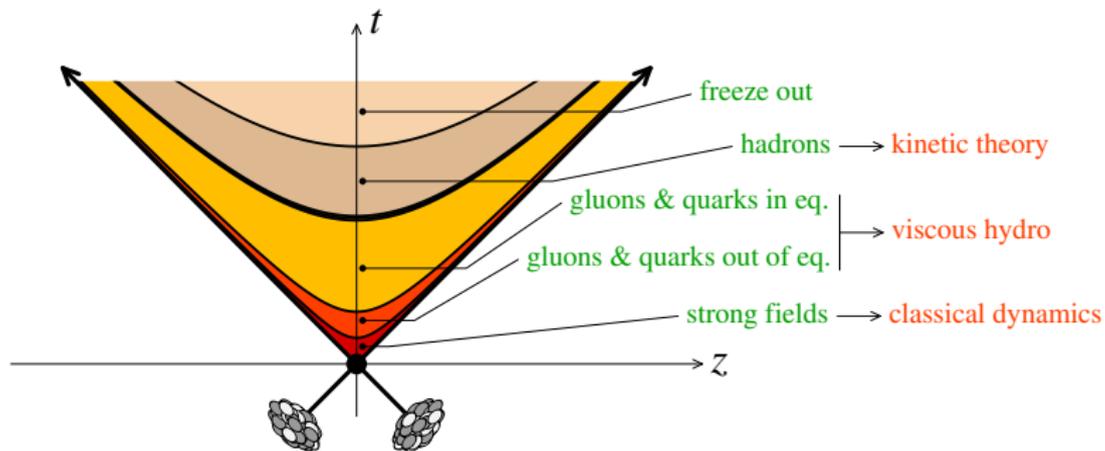
Initial Stages 2017

*Krakow, September 18-22*

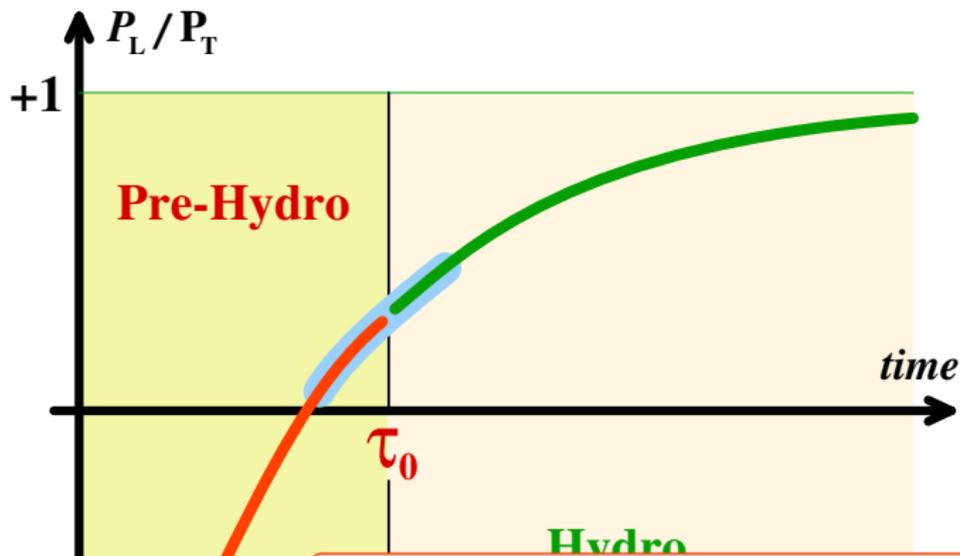


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# STAGES OF A NUCLEUS-NUCLEUS COLLISION



- Hydrodynamics successful at describing the bulk evolution
- In this talk : **Pre-hydro evolution**

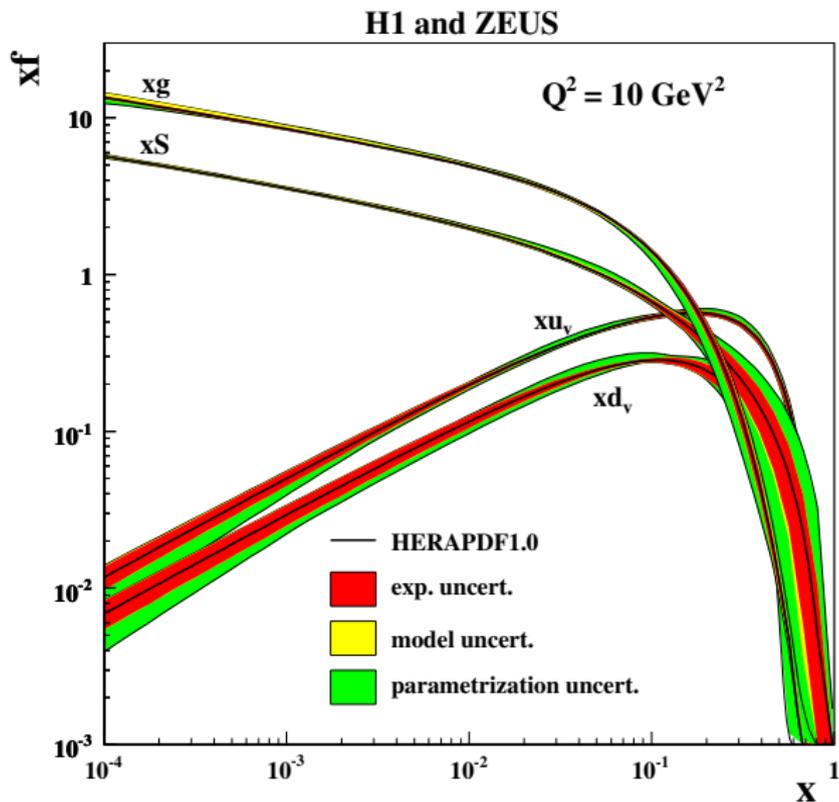


**GOAL : smooth matching to Hydrodynamics**

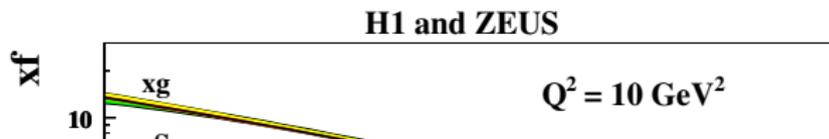
- The pre-hydro model should bring the system to a situation that hydrodynamics can handle
- Pre-hydro and hydro should agree over some range of time  $\Rightarrow$  no  $\tau_0$  dependence
- Description as close as possible to QCD

**Glasma at  $\tau = 0^+$**

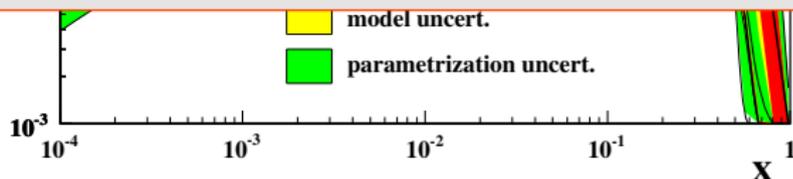
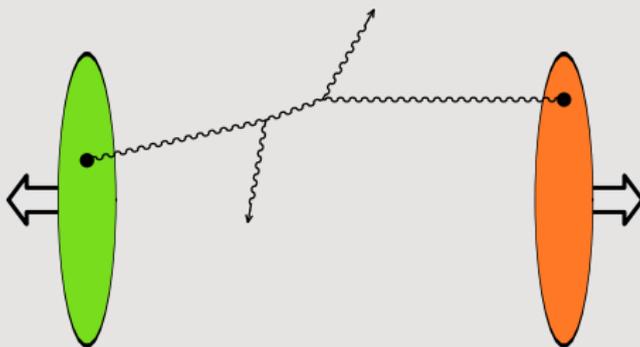
# PARTON DISTRIBUTIONS IN A NUCLEON



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**Large  $x$  : dilute, dominated by single parton scattering**

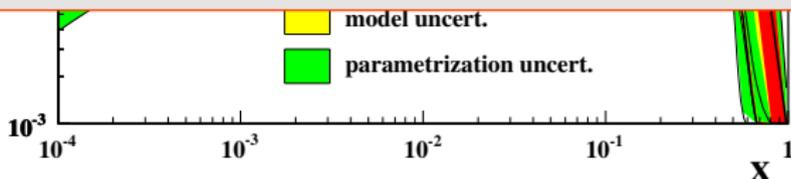
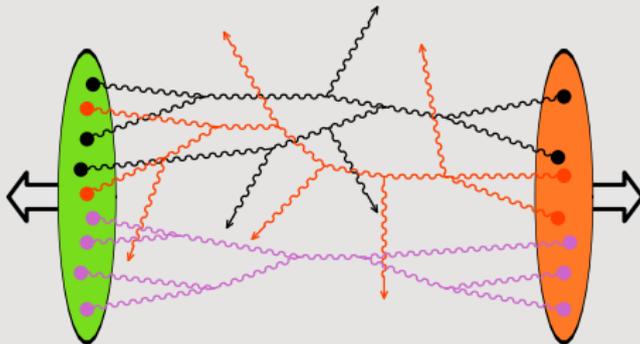


# PARTON DISTRIBUTIONS IN A NUCLEON

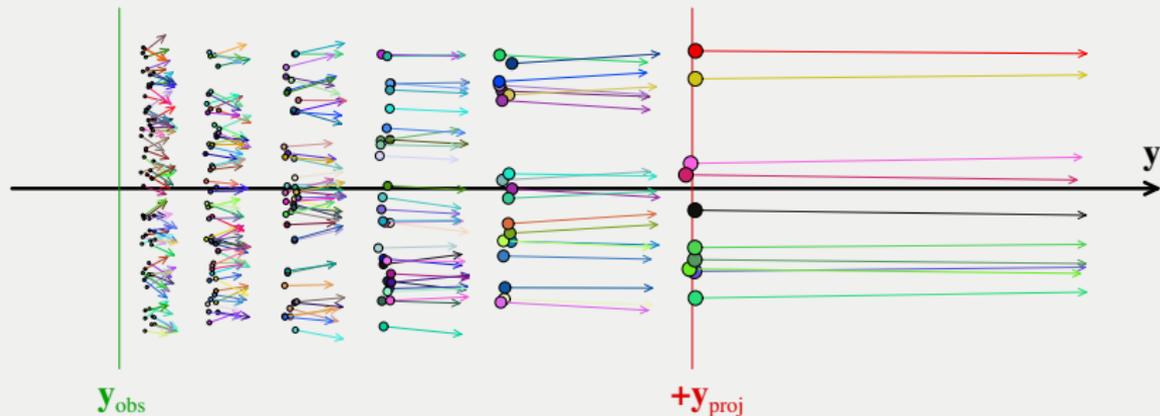
H1 and ZEUS



**Small  $x$  : dense, multi-parton interactions become likely**

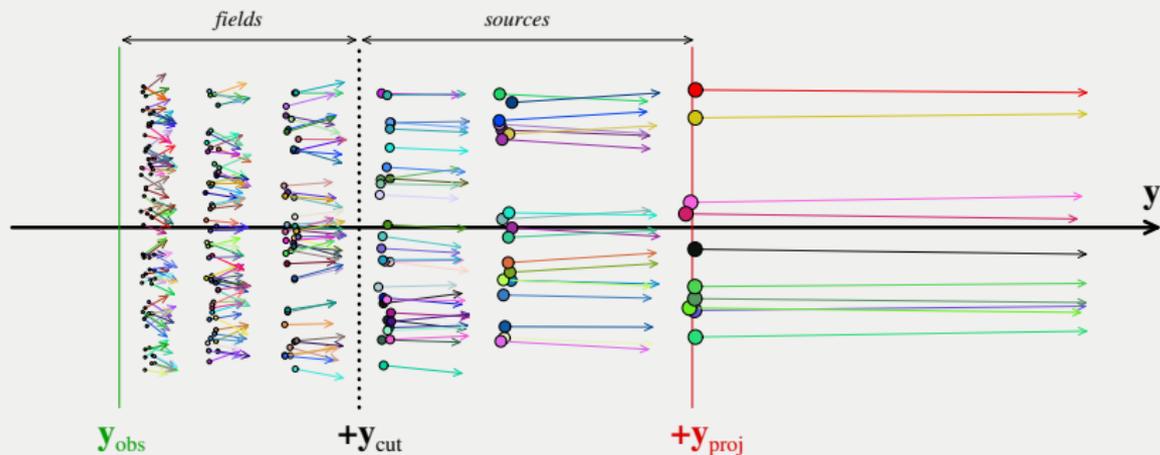


# COLOR GLASS CONDENSATE



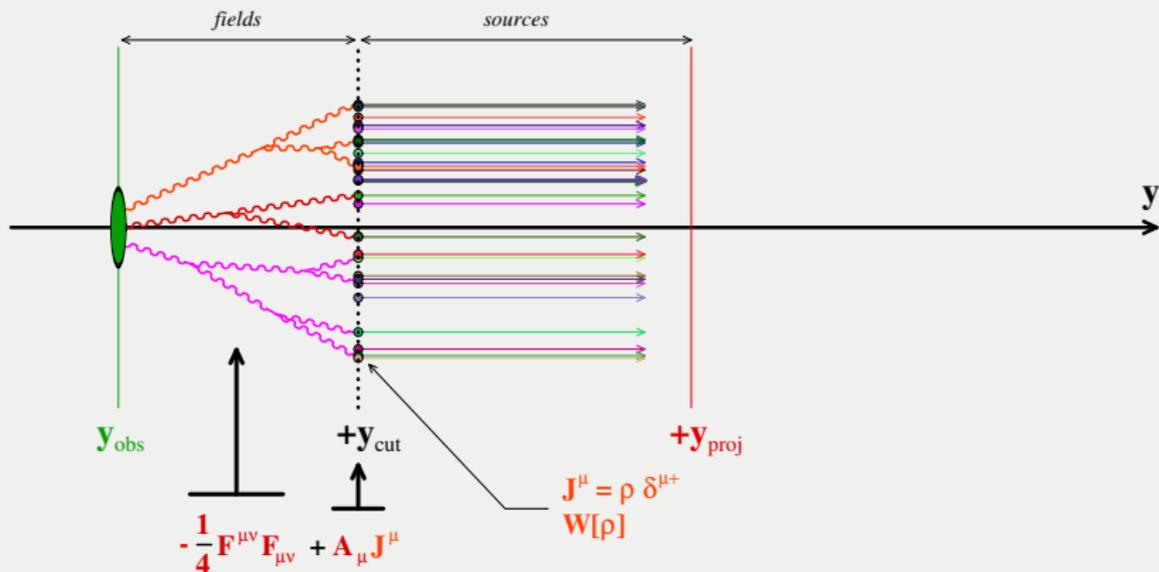
- $p_{\perp}^2 \sim Q_s^2 \sim \Lambda_{\text{QCD}} e^{\lambda(y_{\text{proj}} - y)}$  ,  $p_z \sim Q_s e^{y - y_{\text{obs}}}$
- Fast partons : frozen dynamics, negligible  $p_{\perp} \Rightarrow$  **classical sources**
- Slow partons : evolve with time  $\Rightarrow$  **gauge fields**

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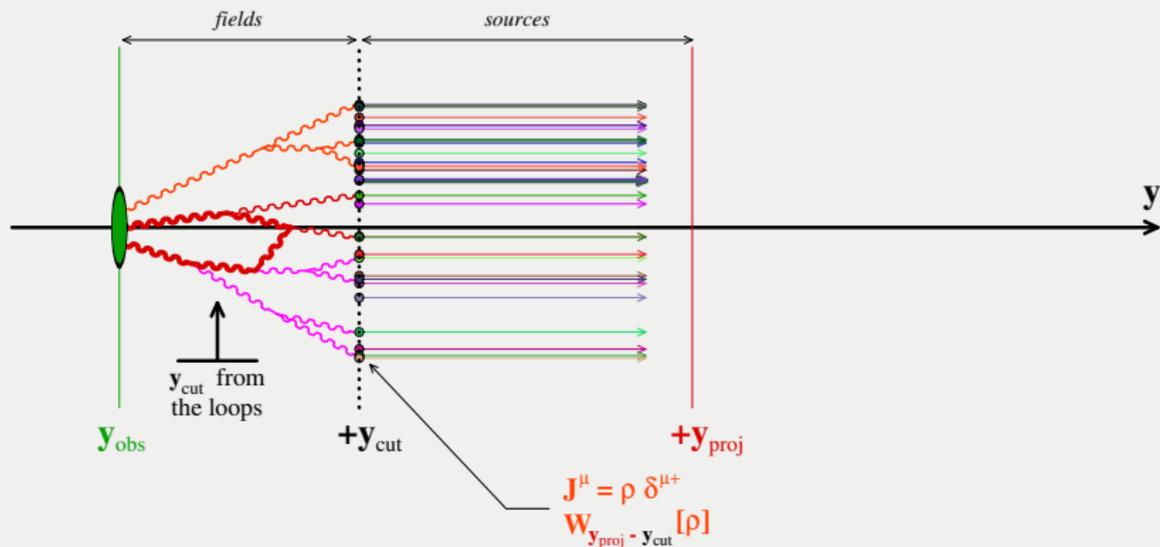
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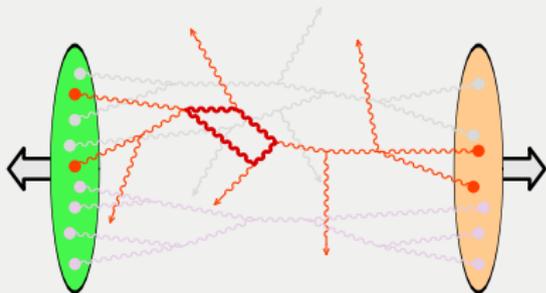
# CANCELLATION OF THE CUTOFF DEPENDENCE



- The cutoff  $y_{\text{cut}}$  is arbitrary and should not affect the result
- The probability density  $W[\rho]$  changes with the cutoff
- Loop corrections cancel the cutoff dependence from  $W[\rho]$

# POWER COUNTING IN THE SATURATED REGIME

$$\mathcal{S} = \underbrace{-\frac{1}{4} \int F_{\mu\nu} F^{\mu\nu}}_{\text{slow gluons}} + \underbrace{\int (J_1^\mu + J_2^\mu) A_\mu}_{\text{fast partons}}$$



**In the saturated regime:**  $J^\mu \sim g^{-1}$

$$T^{\mu\nu} \sim \frac{1}{g^2} \left[ c_0 + c_1 g^2 + c_2 g^4 + \dots \right]$$

# LEADING ORDER

- The Leading Order is the sum of all the tree diagrams  
Expressible in terms of **classical solutions of Yang-Mills equations** :

$$\mathcal{D}_\mu \mathcal{F}^{\mu\nu} = J_1^\nu + J_2^\nu$$

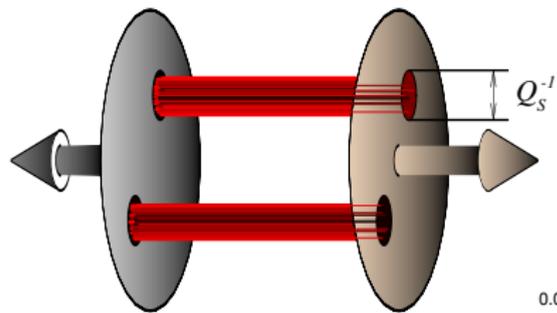
- Initial condition :  $\lim_{x^0 \rightarrow -\infty} \mathcal{A}^\mu(x) = 0$

## Components of the energy-momentum tensor at **LO** :

$$T_{LO}^{00} = \frac{1}{2} \left[ \underbrace{\mathbf{E}^2 + \mathbf{B}^2}_{\text{class. fields}} \right] \quad T_{LO}^{0i} = [\mathbf{E} \times \mathbf{B}]^i$$

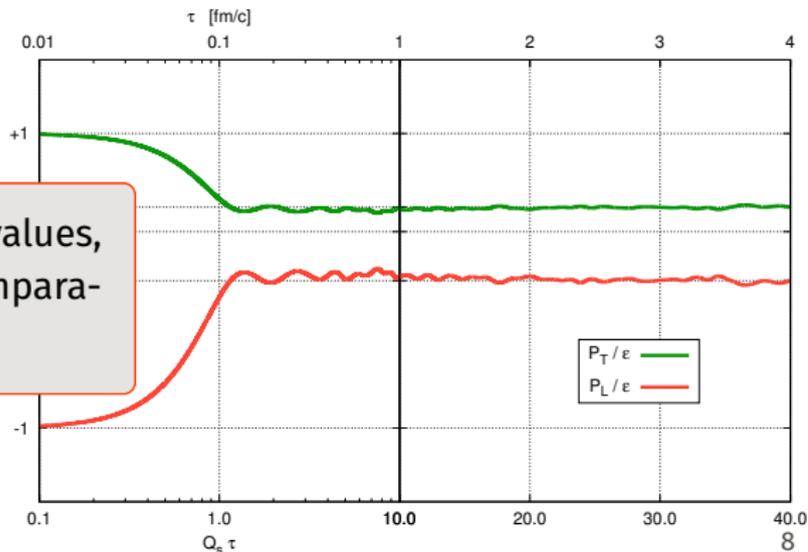
$$T_{LO}^{ij} = \frac{\delta^{ij}}{2} [\mathbf{E}^2 + \mathbf{B}^2] - [\mathbf{E}^i \mathbf{E}^j + \mathbf{B}^i \mathbf{B}^j]$$

# LO : STRONG PRESSURE ANISOTROPY AT ALL TIMES

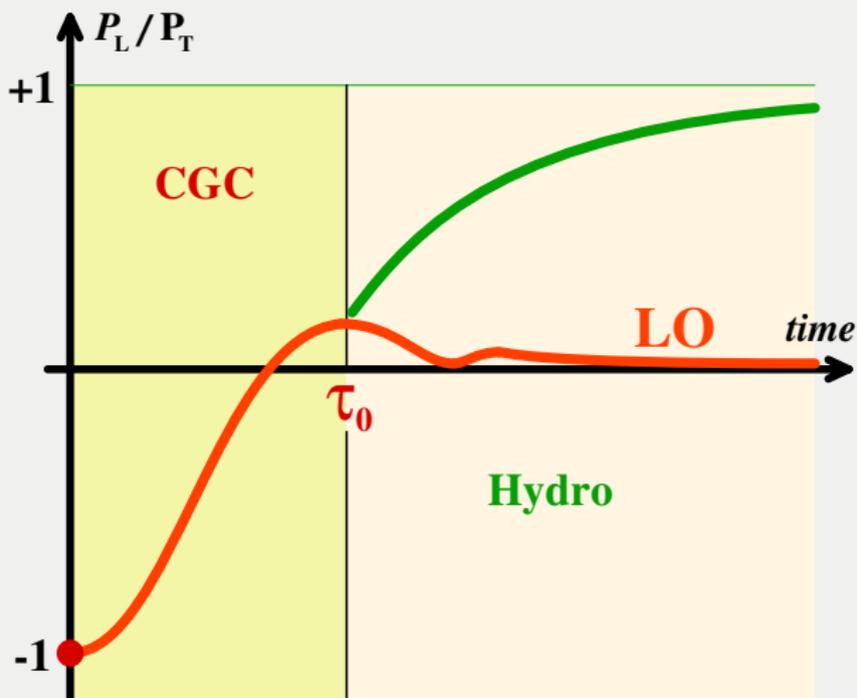


$$\mathbf{E} \parallel \mathbf{B} \text{ at } \tau = 0^+ :$$
$$P_T = \epsilon, P_L = -\epsilon$$

$P_L$  rises to positive values, but never becomes comparable to  $P_T$



# LO : UNSATISFACTORY MATCHING TO HYDRODYNAMICS



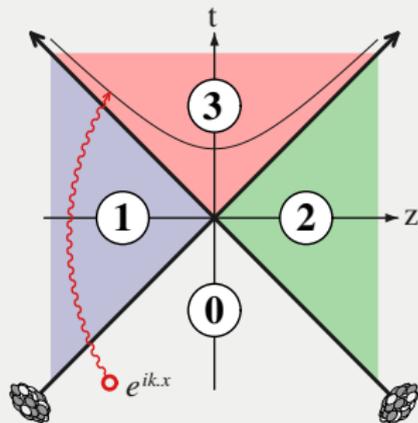
# CGC AT NEXT-TO-LEADING ORDER

- LO : deterministic classical field  $A_{LO}^\mu \sim Q_s/g$
- NLO : Gaussian quantum fluctuations  $a^\mu$ . At  $\tau = 0^+$  :
  - $a^\mu \sim Q_s \ll Q_s/g \Rightarrow$  Coherent state
  - Variance  $\sigma(\mathbf{u}, \mathbf{v})$  of the fluctuations known analytically

$$\sigma(\mathbf{u}, \mathbf{v}) = \frac{1}{2} \int_{\text{modes } \mathbf{k}} a_{\mathbf{k}}(\mathbf{u}) a_{\mathbf{k}}^*(\mathbf{v})$$

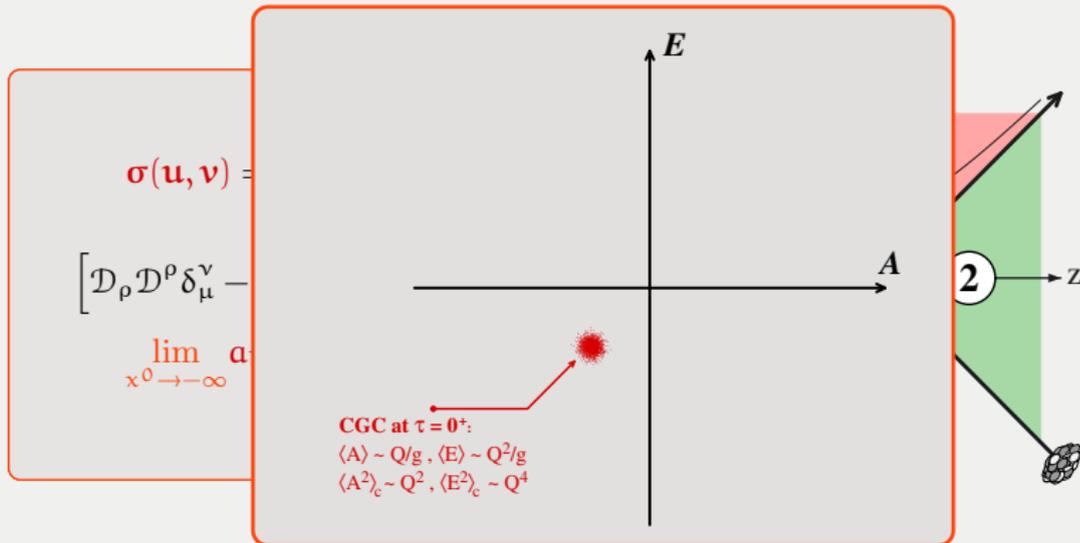
$$\left[ \mathcal{D}_\rho \mathcal{D}^\rho \delta_\mu^\nu - \mathcal{D}_\mu \mathcal{D}^\nu + ig \mathcal{F}_\mu{}^\nu \right] a_{\mathbf{k}}^\mu = 0$$

$$\lim_{x^0 \rightarrow -\infty} a_{\mathbf{k}}(x) = e^{i\mathbf{k} \cdot \mathbf{x}}$$



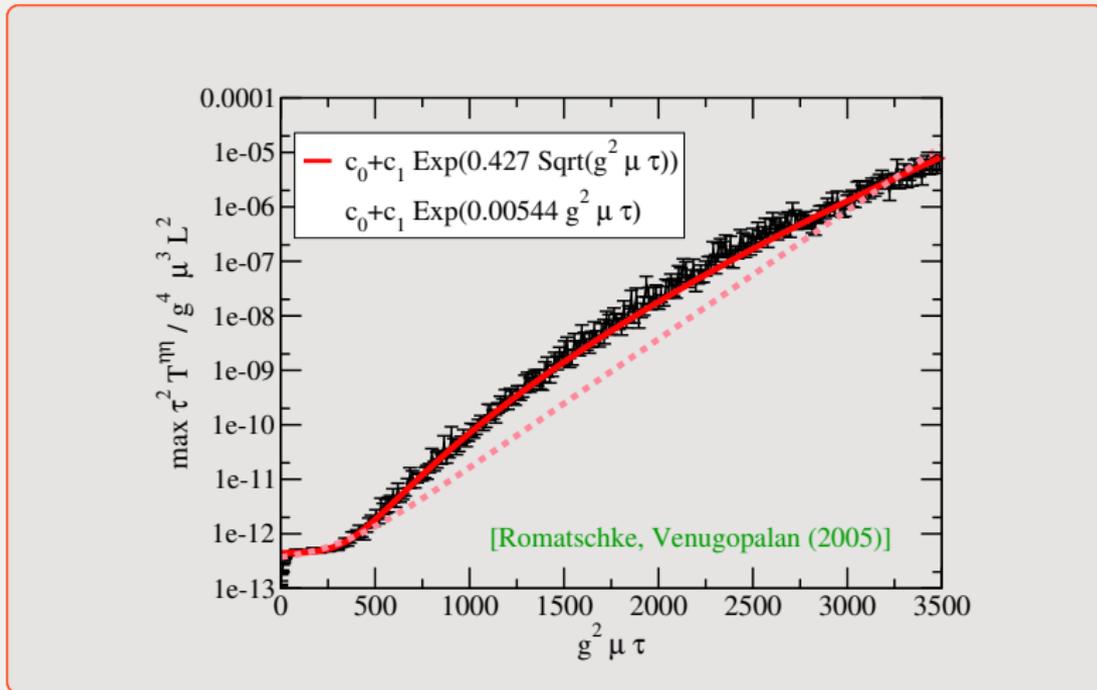
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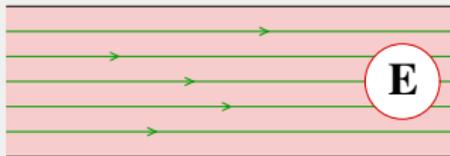


# **Instabilities and Resummation**

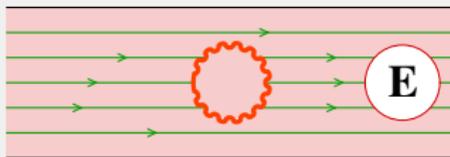
# INSTABILITY OF CLASSICAL SOLUTIONS



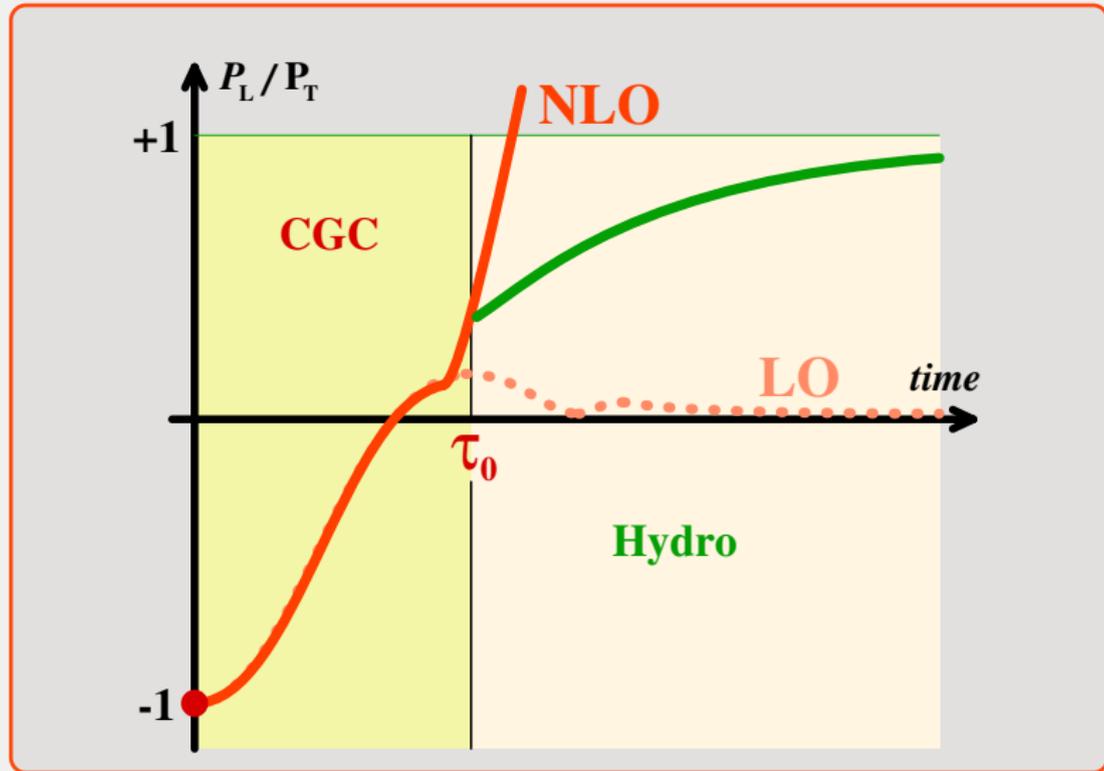
- LO = longitudinal chromo-E and chromo-B fields



- NLO = gluon loop embedded in this field



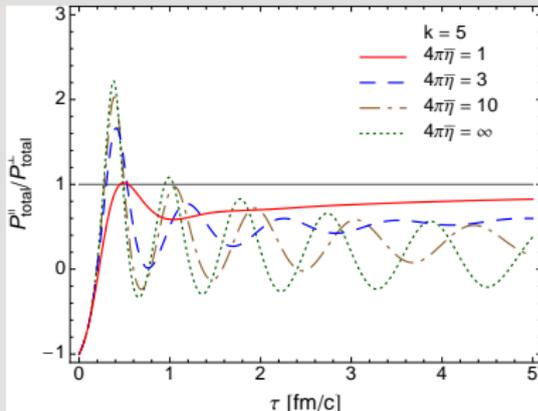
- instability  $\sim$  imaginary part of the loop  $\sim$  gluon pair production
- BUT : at NLO, no feedback of the produced gluons on the LO field!



## Color flux tube model : [Ryblewski, Florkowski (2013)]

$$\underbrace{(p^\mu \partial_\mu + g F^{\mu\nu} p_\nu)}_{\text{Lorentz force}} \partial_p^\mu G = \underbrace{\frac{dN}{d\Gamma}}_{\text{Schwinger}} + \underbrace{C_p[G]}_{\text{collisions}}$$

$$\partial_\mu F^{\mu\nu} = j^\nu \quad (\text{feedback})$$



- Field converted into particles by instability
- Nearly constant  $P_L/P_T$
- Ratio depends on  $\bar{\eta} \equiv \eta/s$

[FG, Lappi, Venugopalan (2007–2008)]

- Observables at NLO can be obtained from the LO by “fiddling” with the initial condition of the classical field :

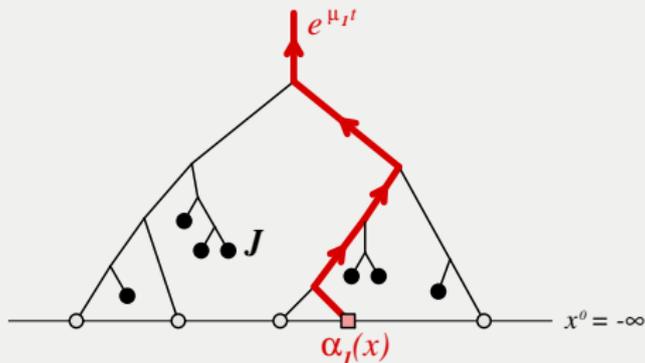
$$\mathcal{O}_{\text{NLO}} = \frac{\hbar}{2} \int_{\mathbf{u}, \mathbf{v}} \sigma(\mathbf{u}, \mathbf{v}) \frac{\partial}{\partial \mathcal{A}_{\text{init}}(\mathbf{u})} \frac{\partial}{\partial \mathcal{A}_{\text{init}}(\mathbf{v})} \mathcal{O}_{\text{LO}}$$

- NLO : the time evolution remains classical;  
 $\hbar$  only enters in the initial condition
- NNLO :  $\hbar$  starts appearing in the time evolution itself

# IMPROVED POWER COUNTING

- For an unstable mode:

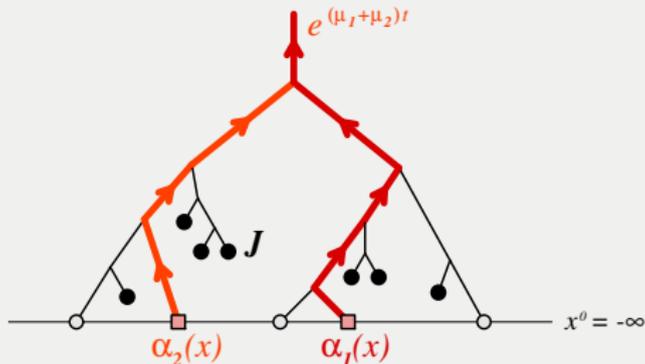
$$\alpha_k(x) \underset{x^0 \rightarrow +\infty}{\sim} e^{\mu_k x^0} \quad (\mu_k = \text{Lyapunov exponent})$$



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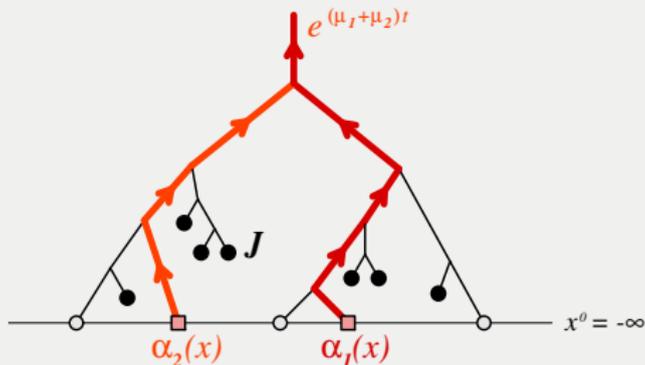
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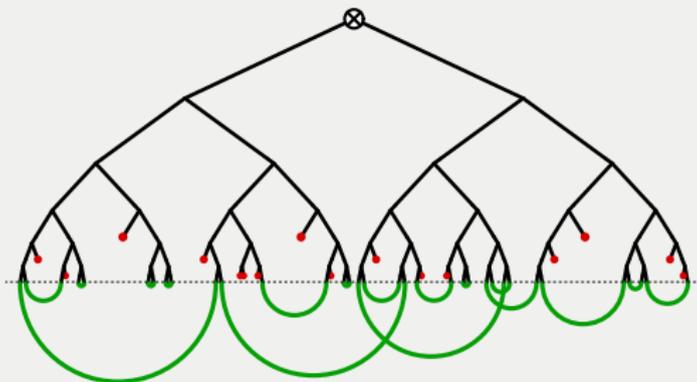
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- 1 loop :  $g^2 \hbar e^{2\mu_k t}$
- $n$  loops :  $(g^2 \hbar e^{2\mu_k t})^n$

# RESUMMATION OF THE LEADING TERMS



$$\mathcal{O}_{\text{resummed}} \equiv \exp \left[ \frac{\hbar}{2} \int_{\mathbf{u}, \mathbf{v}} \boldsymbol{\sigma}(\mathbf{u}, \mathbf{v}) \frac{\partial}{\partial \mathcal{A}_{\text{init}}(\mathbf{u})} \frac{\partial}{\partial \mathcal{A}_{\text{init}}(\mathbf{v})} \right] \mathcal{O}_{\text{LO}}$$

$$\mathcal{O}_{\text{resummed}} = \mathcal{O}_{\text{LO}} + \mathcal{O}_{\text{NLO}} + \text{subset of all higher orders}$$

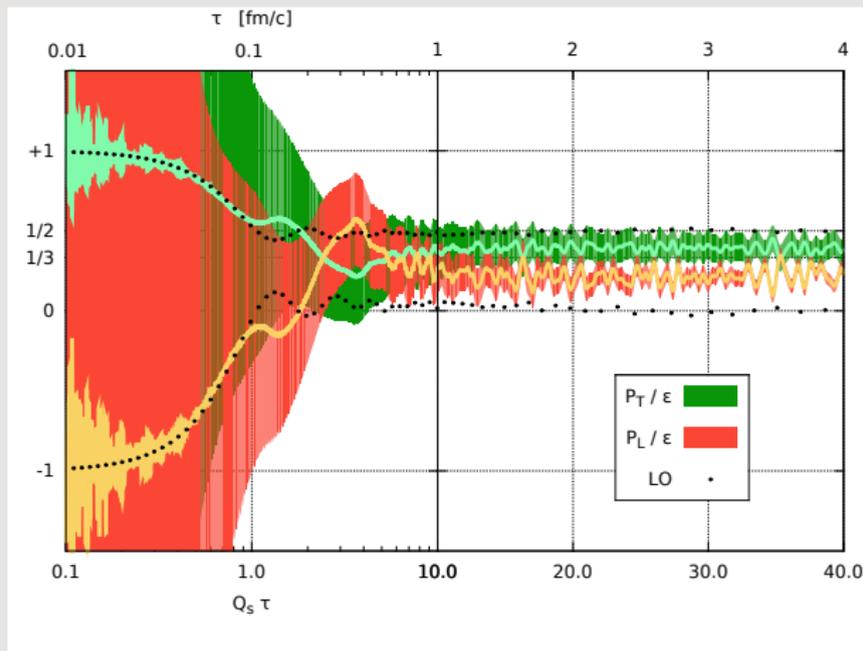
## RESUMMATION : CLASSICAL STATISTICAL APPROXIMATION

$$\begin{aligned} & \exp \left[ \frac{\hbar}{2} \int_{\mathbf{u}, \mathbf{v}} \underbrace{\boldsymbol{\sigma}(\mathbf{u}, \mathbf{v}) \frac{\partial}{\partial \mathcal{A}_{\text{init}}(\mathbf{u})} \frac{\partial}{\partial \mathcal{A}_{\text{init}}(\mathbf{v})}}_{\sim \text{Laplacian}} \right] \mathcal{O}_{\text{LO}}(\mathcal{A}_{\text{init}}) \\ &= \int [\mathbf{D}\mathbf{a}(\mathbf{u})] \exp \left[ -\frac{1}{2\hbar} \int_{\mathbf{u}, \mathbf{v}} \mathbf{a}(\mathbf{u}) \boldsymbol{\sigma}^{-1}(\mathbf{u}, \mathbf{v}) \mathbf{a}(\mathbf{v}) \right] \mathcal{O}_{\text{LO}}(\mathcal{A}_{\text{init}} + \mathbf{a}) \end{aligned}$$

- In this resummation, the observable is obtained as an average over classical fields with fluctuating initial conditions
- The variance of the fluctuations ( $\hbar \sigma$ ) is prescribed by the NLO

# Evolution at small coupling : $g = 0.5$ ( $\lambda \equiv g^2 N_c = 0.5$ )

[Epelbaum, FG (2013)]



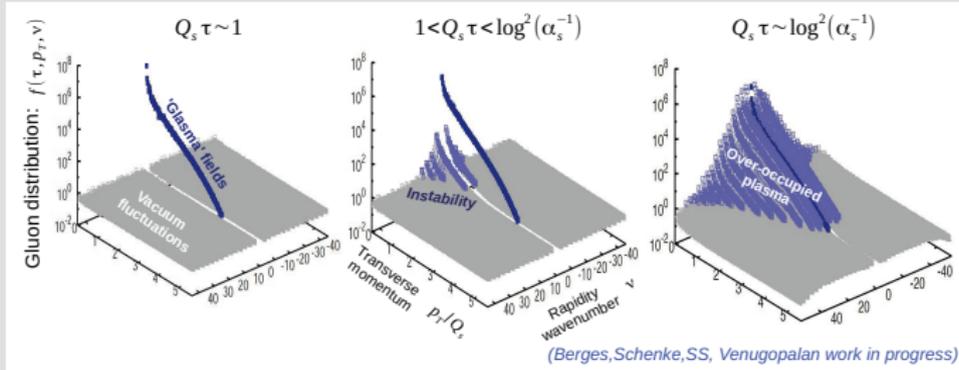
# IS IT POSSIBLE TO START FROM A DECOHERED STATE ?

## CGC at $\tau = 0^+$ : coherent initial state

$$A = \mathcal{A}_{\text{LO}} + \int_{\mathbf{p}} c_{\mathbf{p}} a_{\mathbf{p}} \quad \langle c_{\mathbf{p}} c_{\mathbf{p}'} \rangle \sim \delta_{\mathbf{p}\mathbf{p}'} \frac{1}{2}$$

$$\langle \tilde{A} \tilde{A}^* \rangle_{\tau=0^+} = \underbrace{\tilde{\mathcal{A}}_{\text{LO}} \tilde{\mathcal{A}}_{\text{LO}}^*}_{\sim \delta(\nu) f(\mathbf{p}_{\perp})} + \frac{1}{2} \underbrace{\sum_{\mathbf{p}} \tilde{a}_{\mathbf{p}} \tilde{a}_{\mathbf{p}}^*}_{\frac{1}{2} \text{ for all modes}}$$

# IS IT POSSIBLE TO START FROM A DECOHERED STATE ?



- At  $\tau \gtrsim Q_s^{-1}$ , large occupation in a broad range of  $\nu, k_\perp$

# IS IT POSSIBLE TO START FROM A DECOHERED STATE ?

## Incoherent distribution of particles :

$$A = \int_{\mathbf{p}} c_{\mathbf{p}} a_{\mathbf{p}} \quad \langle c_{\mathbf{p}} c_{\mathbf{p}'} \rangle \sim \delta_{\mathbf{p}\mathbf{p}'} \left[ \frac{1}{2} + f_0(\mathbf{p}) \right]$$

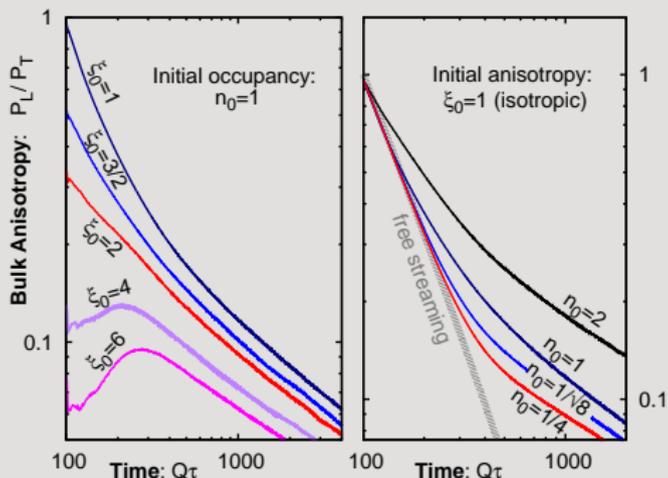
$\frac{1}{2}$   $\iff$  zero point fluctuations

$f_0(\mathbf{p})$   $\iff$  initial particle distribution ( $\sim g^{-2}$ )

If  $f_0(\mathbf{p}) \gg 1$ , approximate  $\frac{1}{2} + f_0 \rightarrow f_0$  ?

# IS IT POSSIBLE TO START FROM A DECOHERED STATE ?

[Berges, Boguslavski, Schlichting, Venugopalan (2013)]



- No dependence on the coupling (can be scaled out)
- $P_T/P_L \sim \tau^{-2/3}$  (until  $Q\tau \approx \alpha_s^{-3/2}$ )

# Classical Statistical Approximation

$$\langle \mathcal{O} \rangle = \int [D\phi_+ D\phi_-] \mathcal{O}[\phi] e^{i(S[\phi_+] - S[\phi_-])}$$

- $\phi_+$  = amplitude     $\phi_-$  = conjugate amplitude
- $\phi_+ - \phi_-$  = quantum interference

- Introduce :  $\phi_1 \equiv \phi_+ - \phi_-$ ,  $\phi_2 \equiv \frac{1}{2}(\phi_+ + \phi_-)$

$$\underbrace{S[\phi_+] - S[\phi_-]}_{\text{odd in } \phi_1} = \phi_1 \cdot \frac{\delta S[\phi_2]}{\delta \phi_2} + \text{terms cubic in } \phi_1$$

- Strong field regime :  $\phi_{\pm}$  large, but  $\phi_+ - \phi_-$  small  
→ Neglect the terms cubic in  $\phi_1$   
 $D\phi_1$  → classical Euler-Lagrange equation for  $\phi_2$
- Remaining fluctuations : initial condition for  $\phi_2$

## Schwinger-Keldysh perturbation theory

$$G_{++}(p) = \frac{i}{p^2 + i\epsilon} + 2\pi f_0(p)\delta(p^2) \quad G_{--}(p) = [G_{++}^*(p)]^*$$

$$G_{-+}(p) = 2\pi(\theta(p^0) + f_0(p))\delta(p^2) \quad G_{+-}(p) = G_{-+}(-p)$$

$$\Gamma_{++++} = -ig^2 \quad \Gamma_{----} = +ig^2$$

After rotation  $\phi_{\pm} \rightarrow \phi_{1,2}$  :

$$G_{21}(p) = \frac{i}{p^2 + ip^0 \epsilon} \quad G_{12}(p) = \frac{i}{p^2 - ip^0 \epsilon}$$
$$G_{22}(p) = 2\pi \left( \frac{1}{2} + f_0(p) \right) \delta(p^2) \quad G_{11}(p) = 0$$

$$\Gamma_{1222} = -ig^2 \quad \Gamma_{1112} = -\frac{i}{4}g^2$$

- **Weak CSA** : drop  $\Gamma_{1112}$
- **Strong CSA** : drop  $\Gamma_{1112}$  AND the  $1/2$  in  $\frac{1}{2} + f_0(p)$

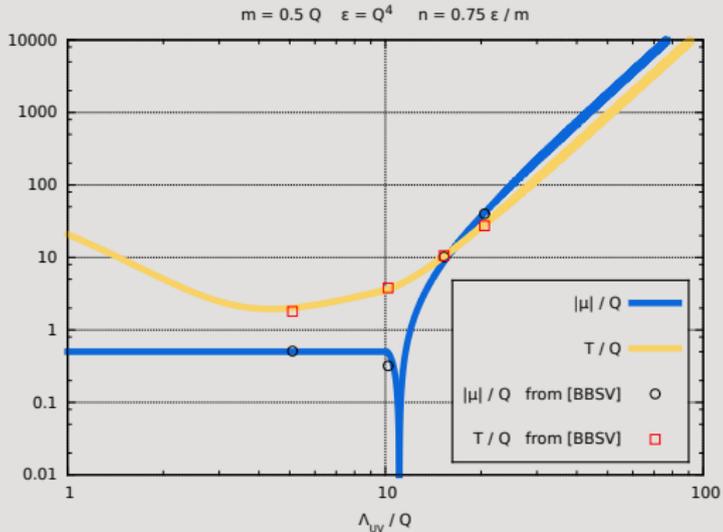
- CSA  $\neq$  underlying theory at 2-loops and beyond
- Vacuum fluctuations make the **Weak CSA non-renormalizable**  
Example of problematic graph :

$$\text{Im} \frac{1}{2} \frac{2}{2} \frac{1}{2} \frac{2}{2} = -\frac{g^4}{1024\pi^3} \left( \Lambda_{\text{UV}}^2 - \frac{2}{3}p^2 \right)$$


$\implies$  divergence in an operator not present in the Lagrangian

- Strong CSA has no such problem of UV sensitivity

# ULTRAVIOLET SENSITIVITY



- Weak cutoff dependence if  $\Lambda_{UV} \sim (3 - 6) \times (\text{physical scales})$

# Classical approximations in Kinetic Theory

Dyson-Schwinger  
equations

→

Boltzmann  
equation :  $p^\mu \partial_\mu f = C_p[f]$

- Collision term in the  $\phi_{1,2}$  basis:

$$C_p[f] = \frac{i}{2} \left[ \Sigma_{11}(p) + \left(\frac{1}{2} + f(p)\right) \left( \Sigma_{21}(p) - \Sigma_{12}(p) \right) \right]$$



$$\begin{aligned} \implies C_p[f] = & \frac{g^4}{4E_p} \int_{\mathbf{k}} \int_{\mathbf{p}'} \int_{\mathbf{k}'} (2\pi)^4 \delta(P + K - P' - K') \\ & \times \left[ f(\mathbf{p}') f(\mathbf{k}') (1 + f(\mathbf{p})) (1 + f(\mathbf{k})) \right. \\ & \left. - f(\mathbf{p}) f(\mathbf{k}) (1 + f(\mathbf{p}')) (1 + f(\mathbf{k}')) \right] \end{aligned}$$

**Weak CSA** collision term :

$$C_{\mathbf{p}}[f] = \frac{g^4}{4E_{\mathbf{p}}} \int_{\mathbf{k}} \int_{\mathbf{p}'} \int_{\mathbf{k}'} (2\pi)^4 \delta(\mathbf{P} + \mathbf{K} - \mathbf{P}' - \mathbf{K}') \\ \times \left[ \left(\frac{1}{2} + f(\mathbf{p}')\right) \left(\frac{1}{2} + f(\mathbf{k}')\right) \left(\frac{1}{2} + f(\mathbf{p}) + \frac{1}{2} + f(\mathbf{k})\right) \right. \\ \left. - \left(\frac{1}{2} + f(\mathbf{p})\right) \left(\frac{1}{2} + f(\mathbf{k})\right) \left(\frac{1}{2} + f(\mathbf{p}') + \frac{1}{2} + f(\mathbf{k}')\right) \right]$$

(Terms in  $f^3$  and  $f^2$  correct, but spurious  $f^1$  terms)

**Strong CSA** : drop also all the  $\frac{1}{2}$  (Terms in  $f^3$  correct)

$$\times \left[ f(\mathbf{p}') f(\mathbf{k}') (1 + f(\mathbf{p})) (1 + f(\mathbf{k})) \right. \\ \left. - f(\mathbf{p}) f(\mathbf{k}) (1 + f(\mathbf{p}')) (1 + f(\mathbf{k}')) \right]$$

## SHOULD WE KEEP OR DROP THE VACUUM $1/2$ ?

- The  $1/2$ 's are responsible for UV problems, but...
- They ensure that the collision term is correct at orders  $f^3$  and  $f^2$
- They are important in certain kinematic situations

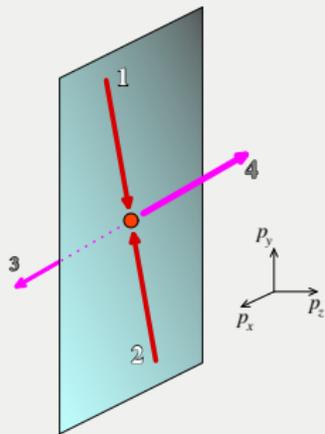
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  - They are important in certain kinematic situations
- 
- No  $f^2$  terms without the 1/2's :

$$\partial_t f_4 \sim g^4 \int_{123} \cdots [f_1 f_2 (f_3 + f_4) - f_3 f_4 (f_1 + f_2)] \\ + \cdots [\cancel{f_1 f_2} - \cancel{f_3 f_4}]$$

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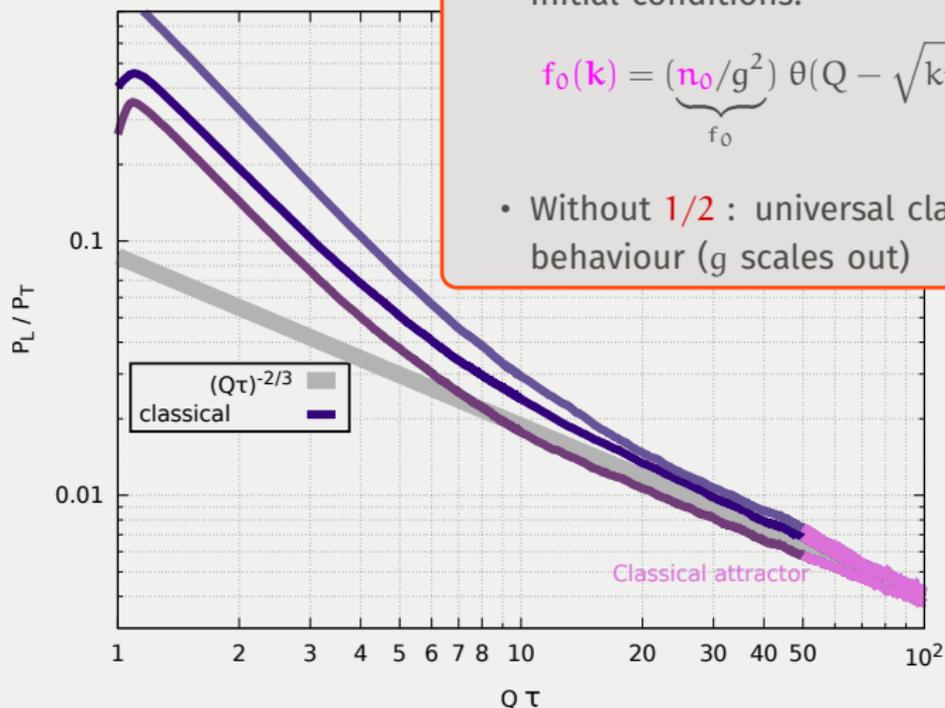
- No  $f^2$  terms without the  $1/2$ 's :

$$\partial_t f_4 \sim g^4 \int_{123} \dots \left[ \cancel{f_1 f_2 (f_3 + f_4)} - \cancel{f_3 f_4 (f_1 + f_2)} \right] + \dots \left[ f_1 f_2 - f_3 f_4 \right]$$

- When the distribution is very anisotropic, trying to produce the particle 4 at large angle results in  $f_3 \approx f_4 \approx 0 \Rightarrow$  nothing left
- Cubic terms  $\Leftrightarrow$  stimulated emission : ineffective to produce particles in empty regions of phase-space

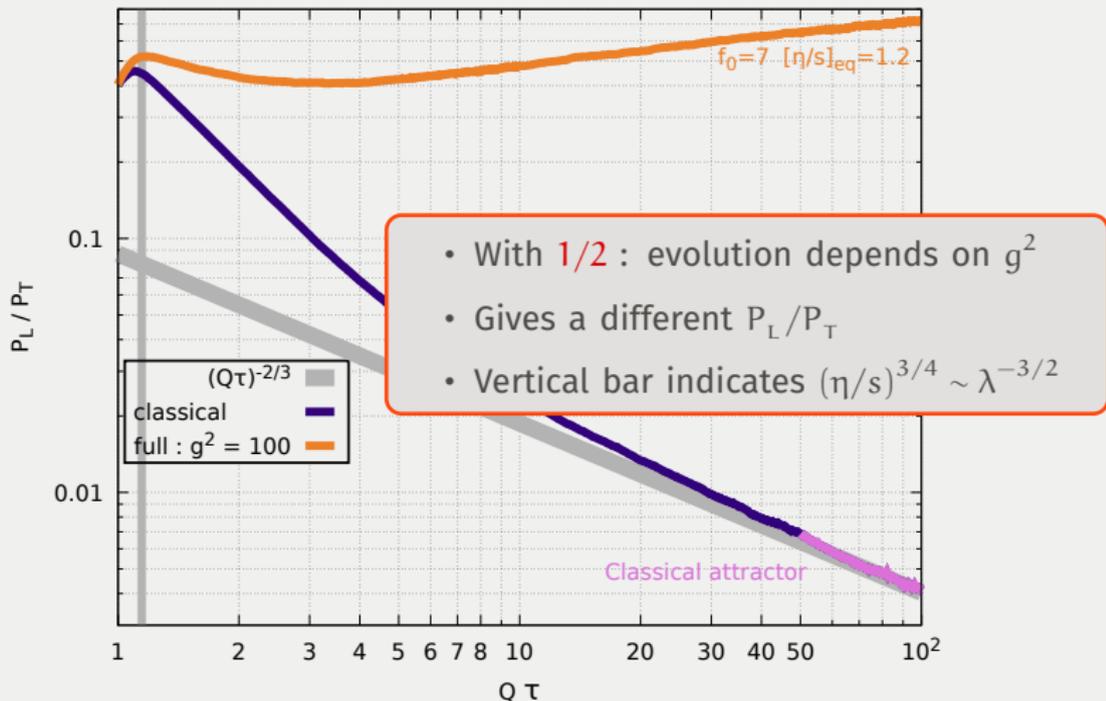
# ISOTROPIZATION IN A LONGITUDINALLY EXPANDING SYSTEM

[Epelbaum, FG, Jeon, Moore, Wu (2015)]



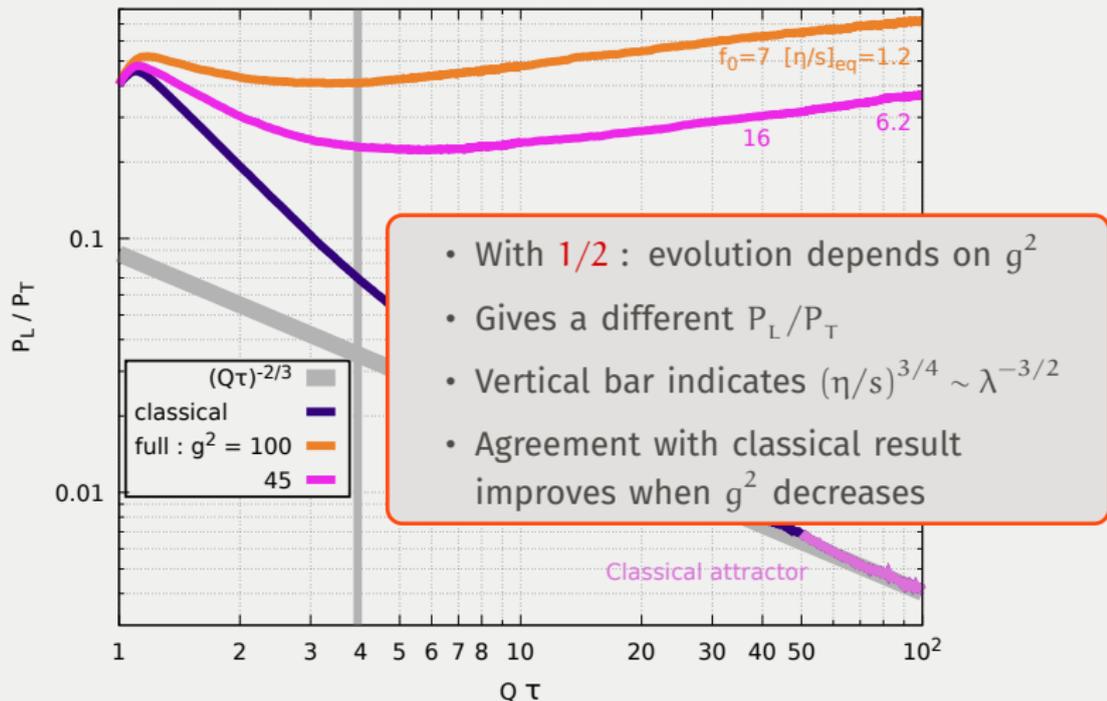
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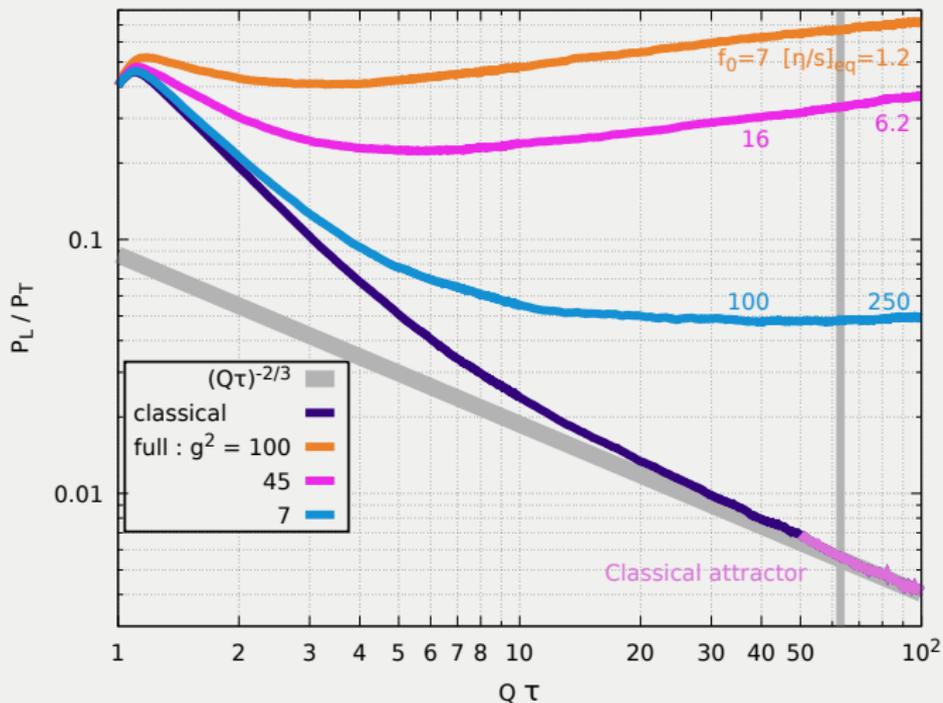
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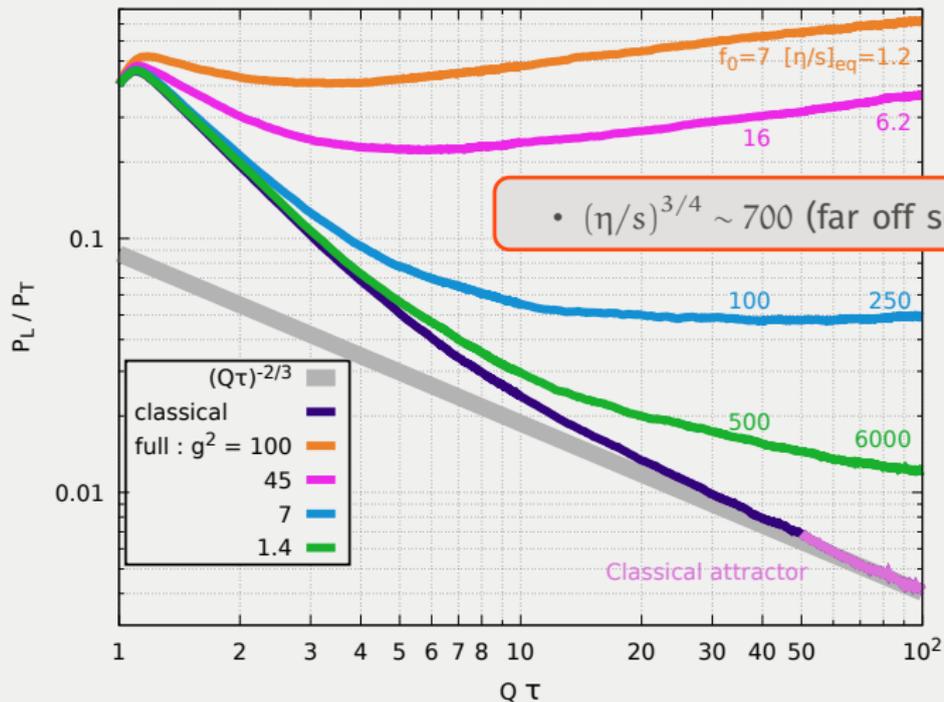
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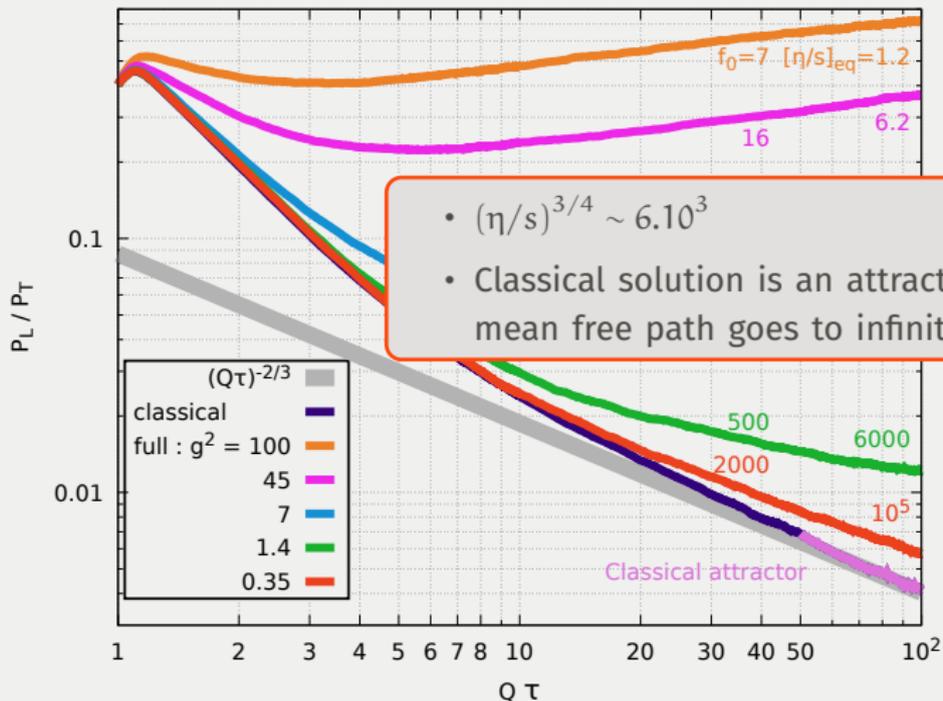
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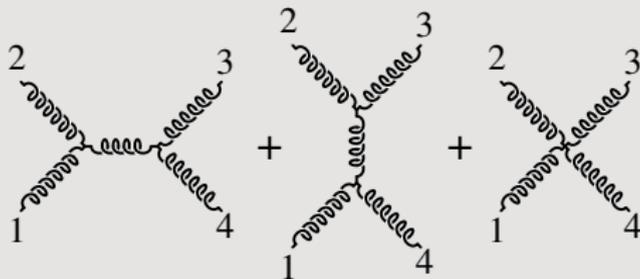
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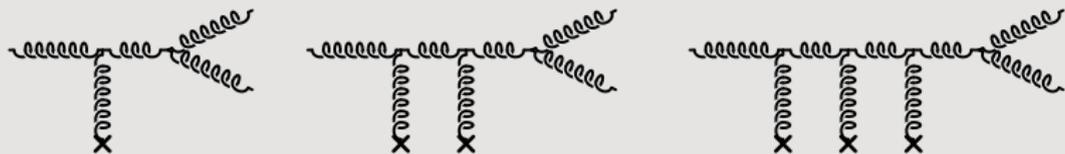


# **State of the art Kinetic approach**

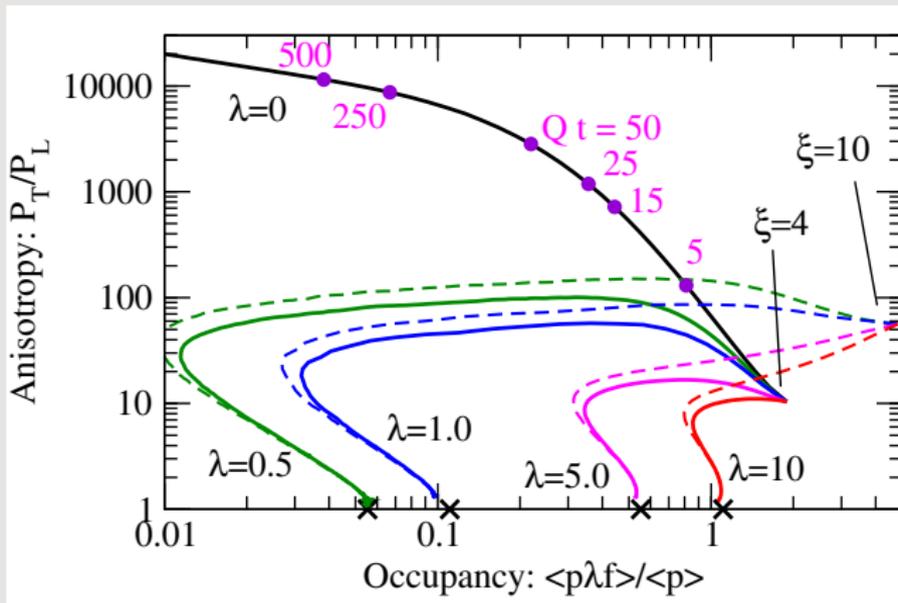
$2 \rightarrow 2$



$1 \rightarrow 2, 2 \rightarrow 1$  + LPM resummation

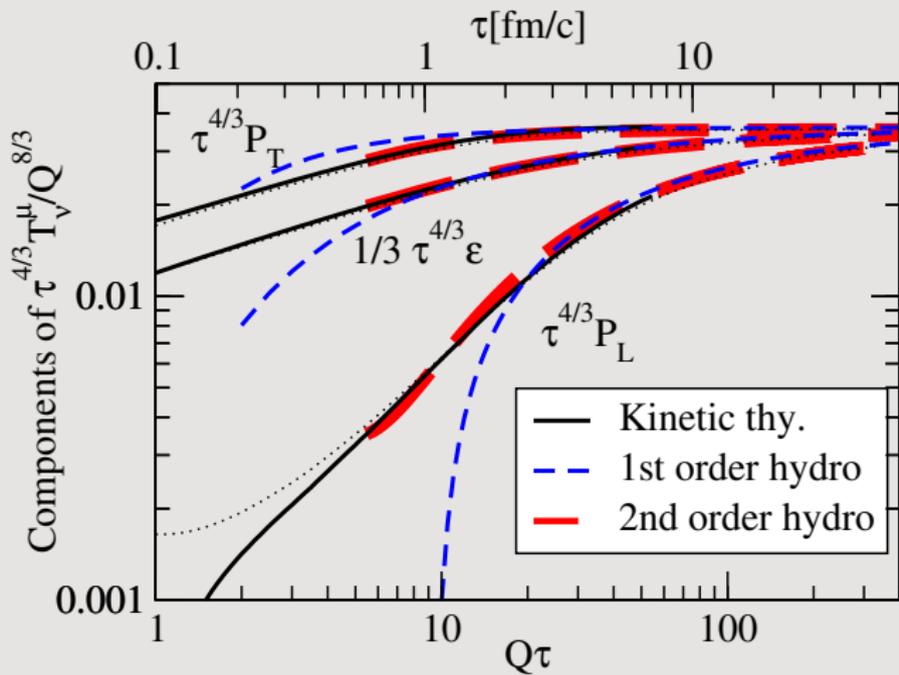


# KINETIC THEORY RESULTS FOR GLUONS [Kirkela, Zhu (2015)]



For  $\lambda = 0.5$ , the **Strong CSA** breaks down at  $Q\tau \approx 2$ , while simple estimates suggested that it would be valid up to  $Q\tau \approx \alpha_s^{-3/2} \approx 350$

## Consistent with hydrodynamics before full isotropization



# Two-Particle Irreducible framework

# IS SOMETHING MISSING ?

	Weak CSA	Strong CSA	Kinetic th.
UV finite	X	✓	✓
$f^2$ terms	✓	X	✓

# IS SOMETHING MISSING ?

	Weak CSA	Strong CSA	Kinetic th.
UV finite	X	✓	✓
$f^2$ terms	✓	X	✓
Screening	✓	✓	X

# IS SOMETHING MISSING ?

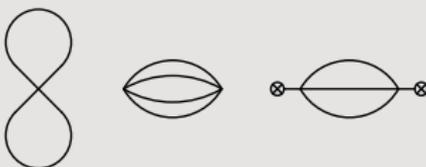
	Weak CSA	Strong CSA	Kinetic th.	2-PI
UV finite	X	✓	✓	✓
$f^2$ terms	✓	X	✓	✓
Screening	✓	✓	X	✓

## Quantum effective action

$$\Gamma[\varphi, G] = S[\varphi] - \frac{i}{2} \text{tr} \log G + \frac{i}{2} \text{tr} \left( (G_0^{-1} - G^{-1}) G \right) + \Phi[\varphi, G]$$

$\Phi[\varphi, G] =$  sum of vacuum 2PI graphs

## Truncation at order $g^4$

$$\Phi[\varphi, G] =$$


## Equations of motion

$$\frac{\delta\Gamma}{\delta\varphi_x} = \frac{\delta\Gamma}{\delta G_{xy}} = 0$$

$$\sqrt{-g_x} \left\{ \nabla_\mu \nabla^\mu \varphi_x + V'(\varphi_x) + V'''(\varphi_x) G_{xx} \right\} = \frac{\delta\Phi}{\delta\varphi_x}$$

$$\begin{aligned} \left( \nabla_\mu^x \nabla_x^\mu + V''(\varphi_x) \right) G_{xy} &= -i \frac{1}{\sqrt{-g_x}} \delta(x-y) \\ &\quad - \int d^4z \sqrt{-g_z} \underbrace{-2 \frac{1}{\sqrt{-g_x}} \frac{\delta\Phi}{\delta G_{xz}} \frac{1}{\sqrt{-g_z}}}_{\Sigma_{xz}} G_{zy} \end{aligned}$$

# COMPUTATIONAL COST PER TIME STEP

## Kinetic theory (deterministic algorithm)

*3d-isotropic* :  $N^3$

*Long. expansion + azimuthal symm.* :  $N^3 N_z^3$

## 2PI with $N_{\text{mem}}$ -deep time memory integral

*3d-isotropic* :  $N \log(N) \times N_{\text{mem}}^2$

*Long. expansion + azimuthal symm.* :  $NN_z \log(NN_z) \times N_{\text{mem}}^2$

[So far, one implementation : **Hatta, Nishiyama (2013)**]

# Summary

- LO : no pressure isotropization, NLO : instabilities
- Beyond NLO : **Classical statistical approximation**
  - **Weak CSA** :  
non-renormalizable, sensitive to UV cutoff
  - **Strong CSA** :  
underestimates large angle scatterings  
breaks rapidly unless  $\eta/s$  very large
- **Kinetic theory** : avoids all these difficulties, but does not cope well with screening effects in the soft region
- **Two-PI for longitudinally expanding systems** :  
Important to properly treat screening effects



## Energy-momentum tensor

$$\begin{aligned} T^{\mu\nu} = & \nabla^\mu \varphi \nabla^\nu \varphi - g^{\mu\nu} \mathcal{L} + \left[ \nabla_x^\mu \nabla_y^\nu G_{xy} \right]_{x=y} \\ & + \frac{1}{2} g^{\mu\nu} \left\{ V''(\varphi_x) G_{xx} - \left[ \nabla_\alpha^x \nabla_y^\alpha G_{xy} \right]_{x=y} \right\} \\ & - g^{\mu\nu} \frac{\delta \Phi}{\delta \sqrt{-g}} \end{aligned}$$

# ISOTROPIZATION IN A FIXED BOX

