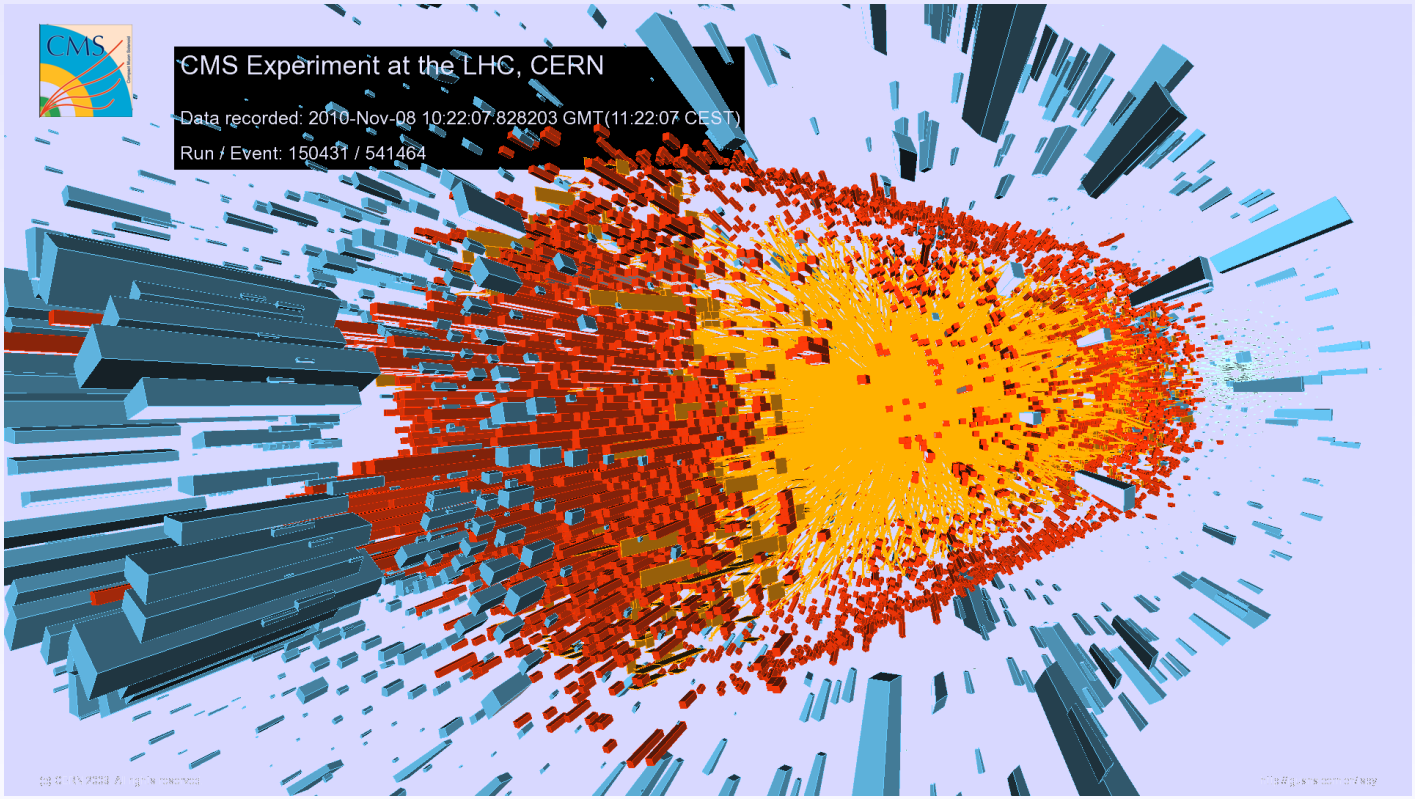
The Early Stages of Heavy Ion Collisions

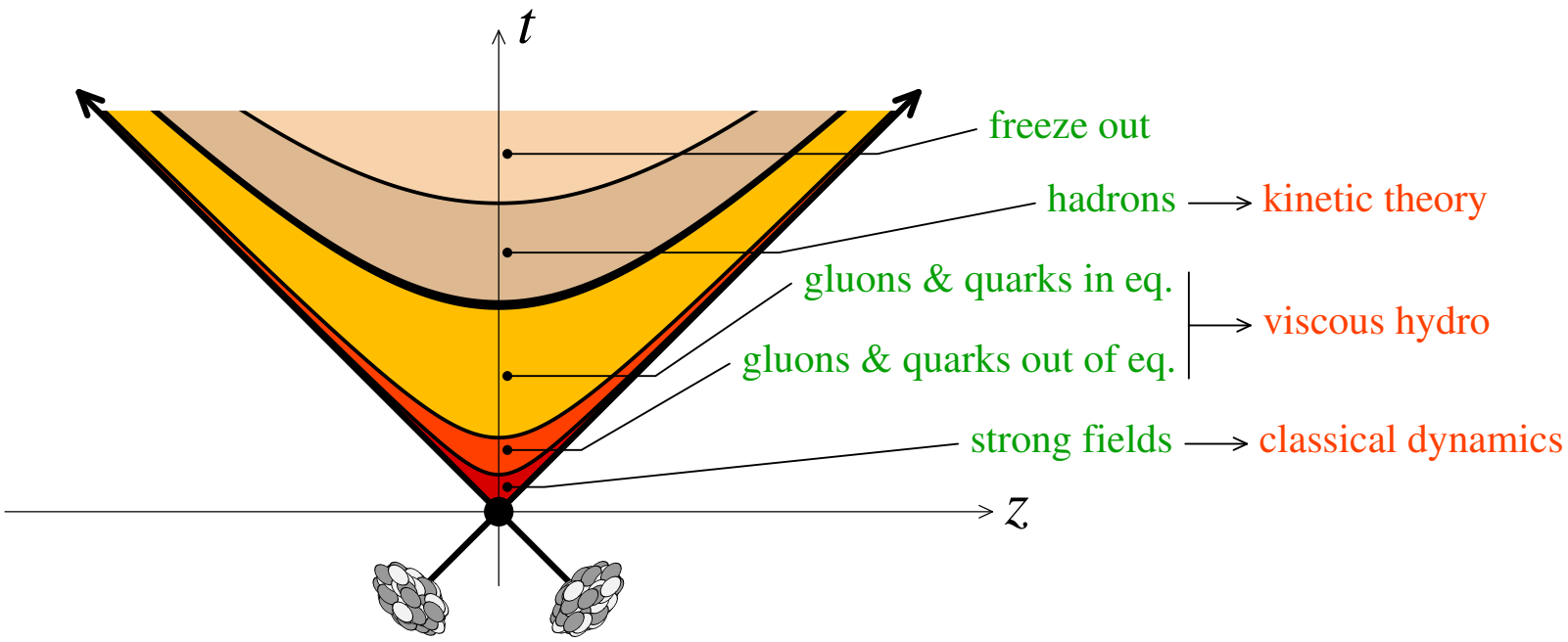
YITP, October 2015

François Gelis
IPhT, Saclay

Heavy ion collision at the LHC



Stages of a nucleus-nucleus collision



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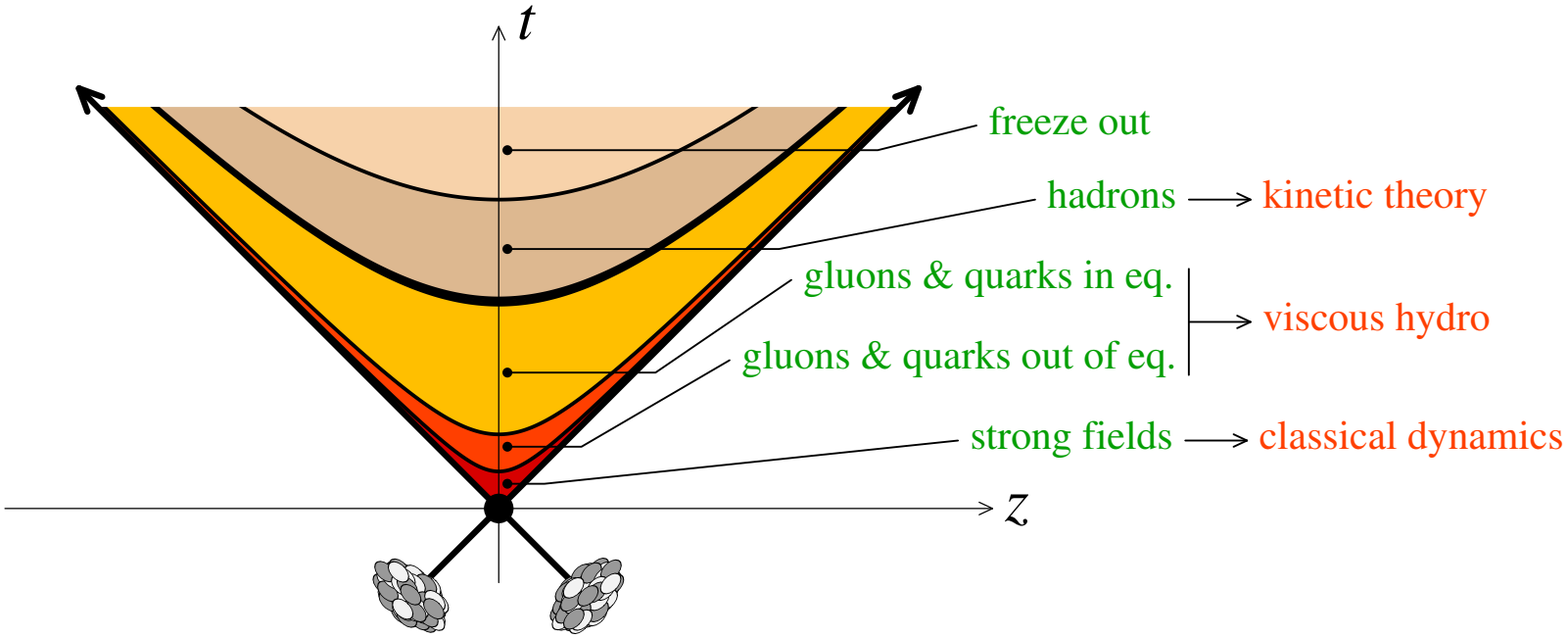

- Well described as a nearly ideal fluid expanding into vacuum according to relativistic hydrodynamics

What is hydrodynamics?

- Hydrodynamics is a macroscopic description based on energy-momentum conservation :

$$\partial_\mu T^{\mu\nu} = 0$$

True in any quantum field theory

Not closed : 4 equations, 10 independent components in $T^{\mu\nu}$

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- Additional assumption : at macroscopic scales, $T^{\mu\nu}$ is expressible in terms of ϵ (energy density), P (pressure) and u^μ (fluid velocity field)
- For a frictionless fluid : $T^{\mu\nu}_{\text{ideal}} = (\epsilon + P)\, u^\mu u^\nu - P\, g^{\mu\nu}$

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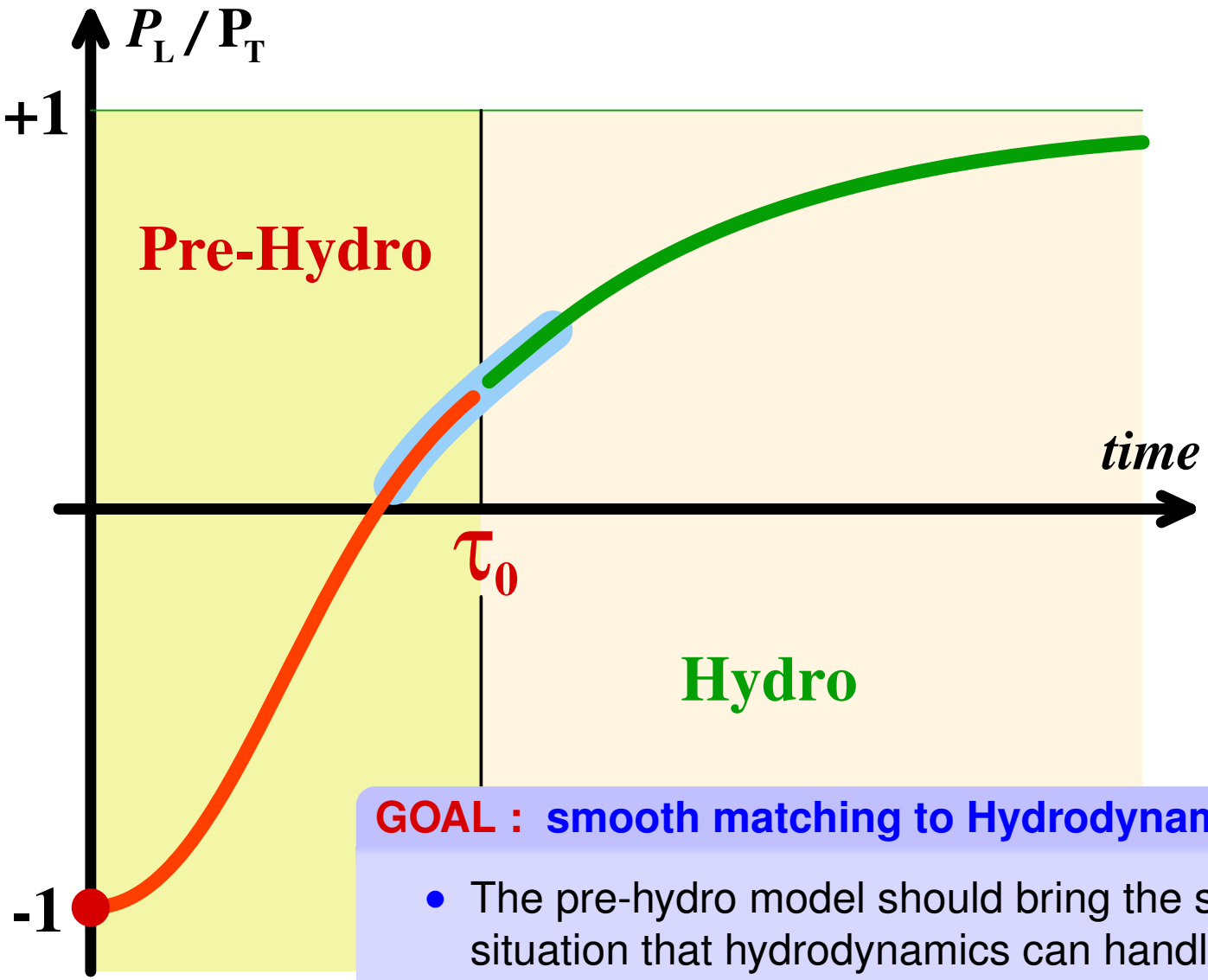
- Microscopic inputs : $\epsilon = f(P)$ (EoS), $\eta, \zeta, \cdots$ (transport coeff.)

Conditions for hydrodynamics

- The difference between P_L and P_T should not be too large (for the expansion to make sense)

- The ratio η/s should be very small (fits require $\eta/s \sim 0.1$) (for an efficient transfer from spatial to momentum anisotropy)

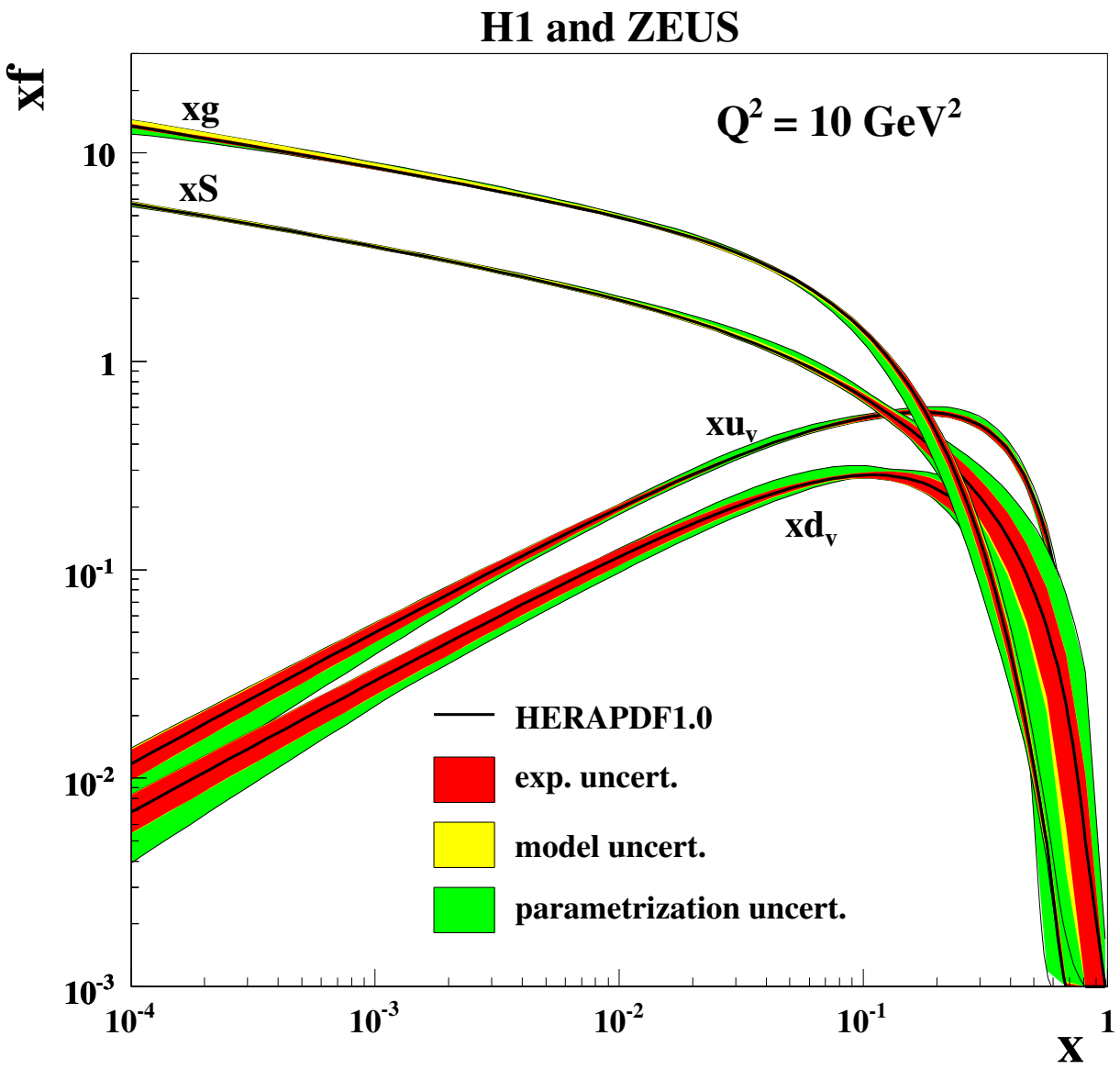


GOAL : smooth matching to Hydrodynamics

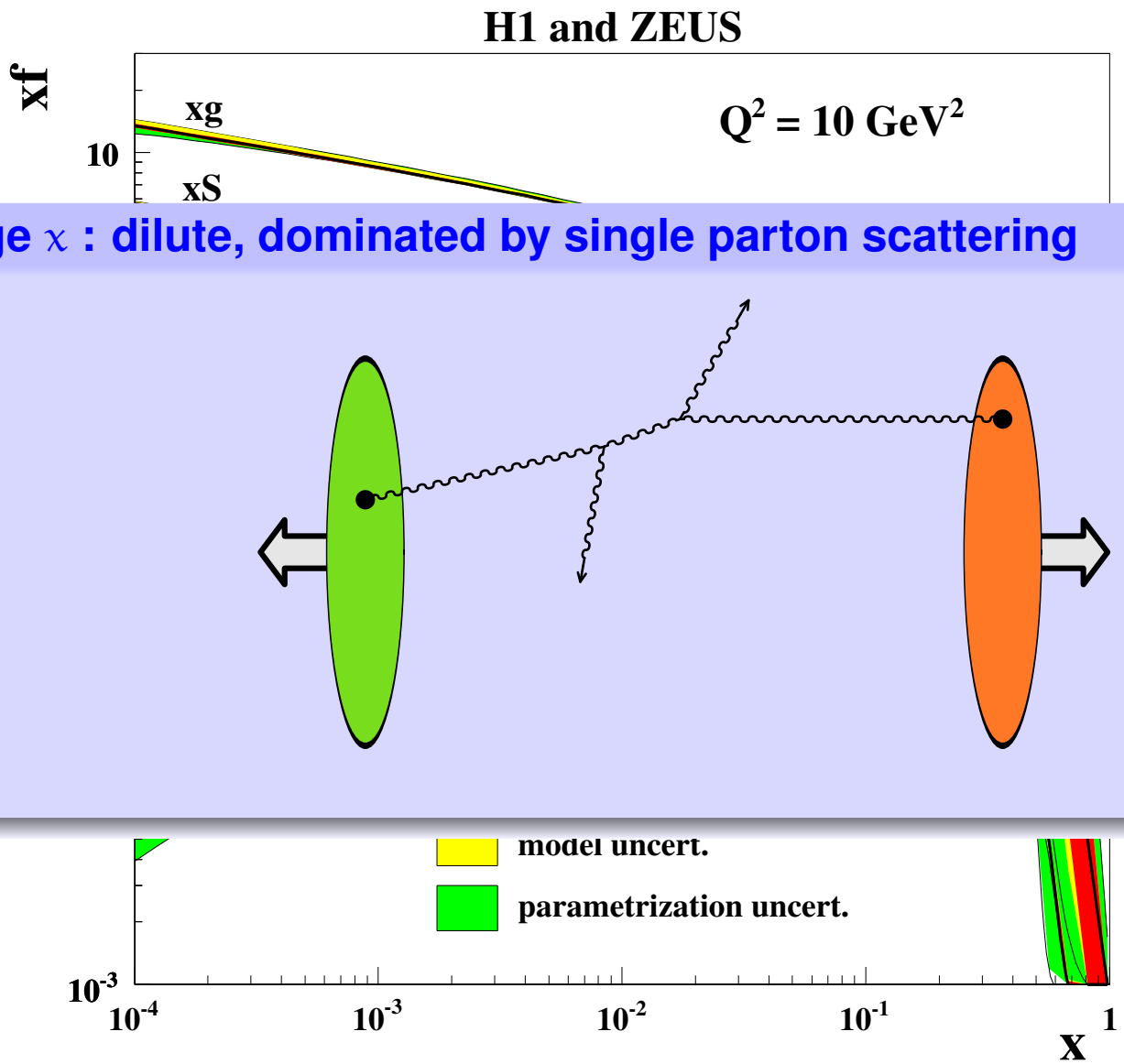
- The pre-hydro model should bring the system to a situation that hydrodynamics can handle
- Pre-hydro and hydro should agree over some range of time $\Rightarrow$ no τ_0 dependence

Color Glass Condensate in Heavy Ion Collisions

Parton distributions in a nucleon

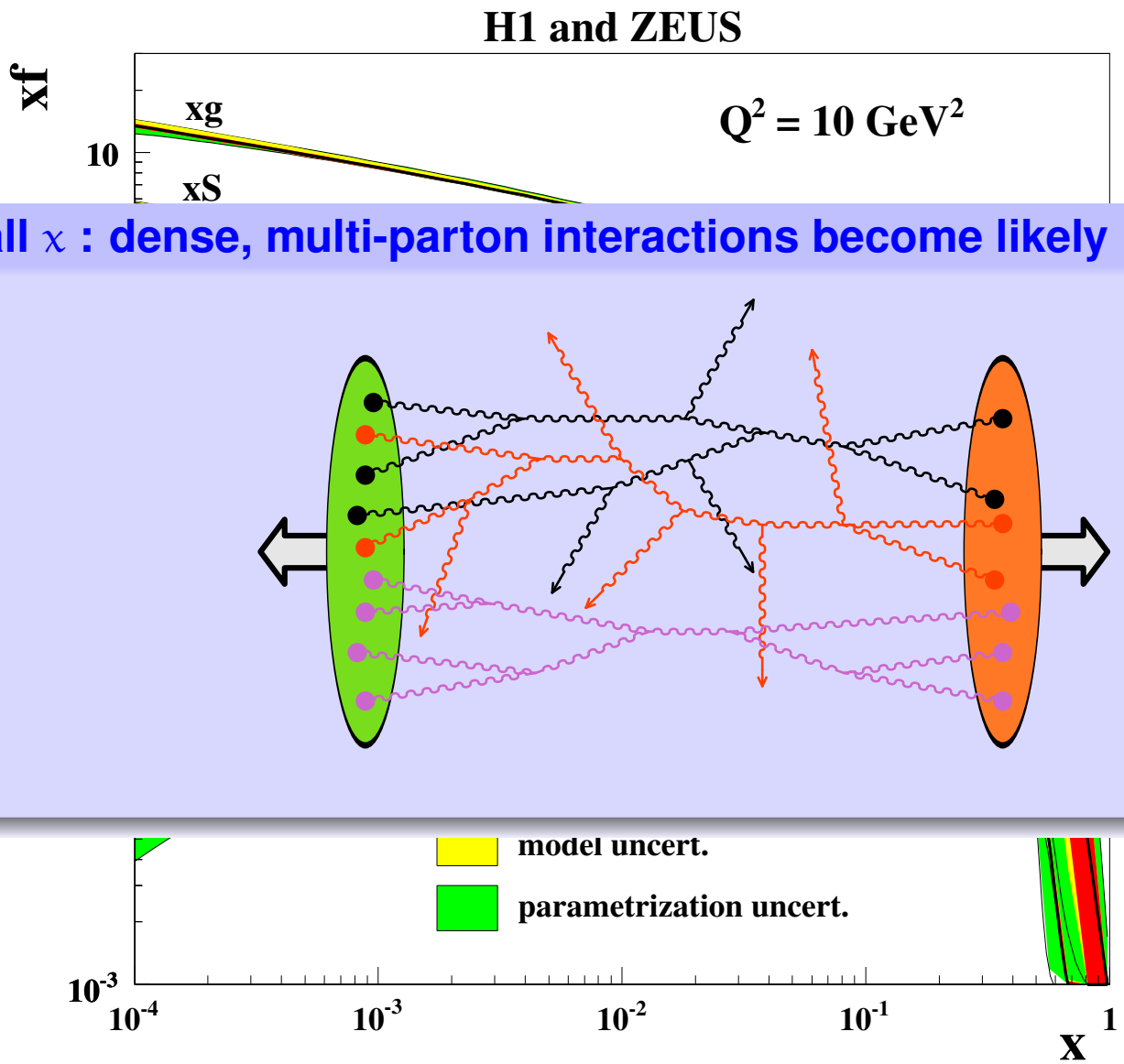


Parton distributions in a nucleon



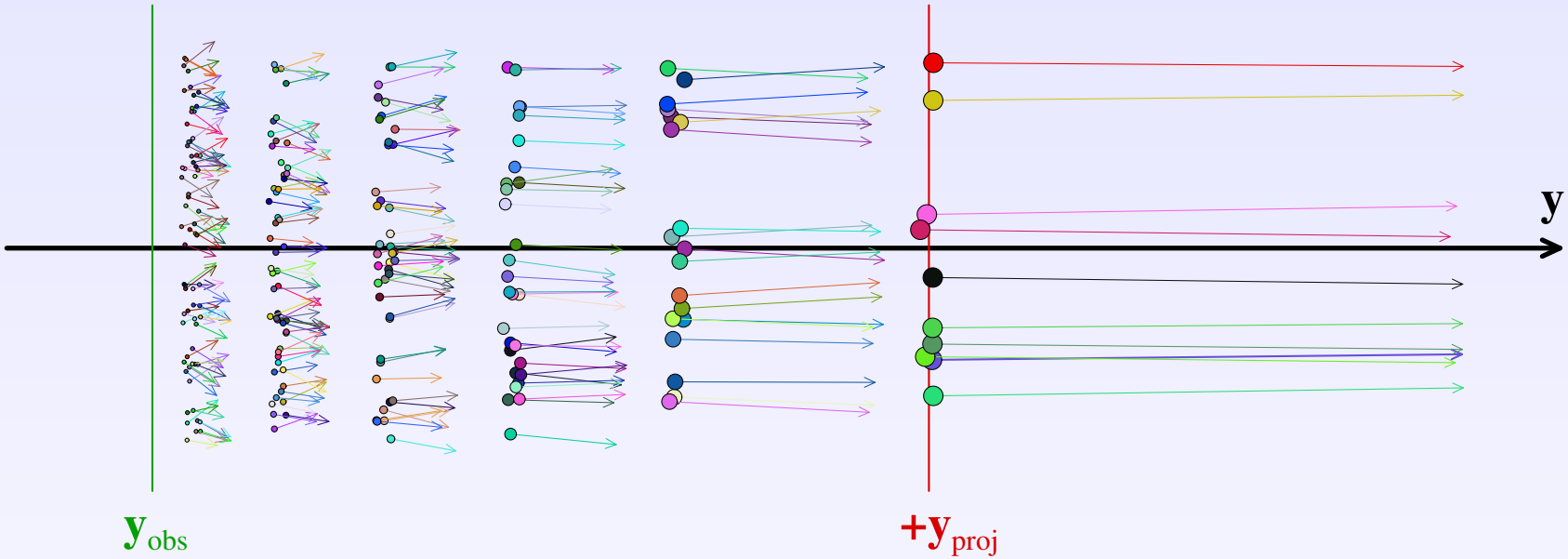
Large x : dilute, dominated by single parton scattering

Parton distributions in a nucleon



Small χ : dense, multi-parton interactions become likely

Degrees of freedom [McLerran, Venugopalan (1994)]



- $p_\perp^2 \sim Q_s^2 \sim \Lambda_{QCD}\, e^{\lambda(y_{proj}-y)}$, $p_z \sim Q_s\, e^{y-y_{obs}}$
- Fast partons : frozen dynamics, negligible $p_\perp$ $\Rightarrow$ classical sources
- Slow partons : evolve with time $\Rightarrow$ gauge fields

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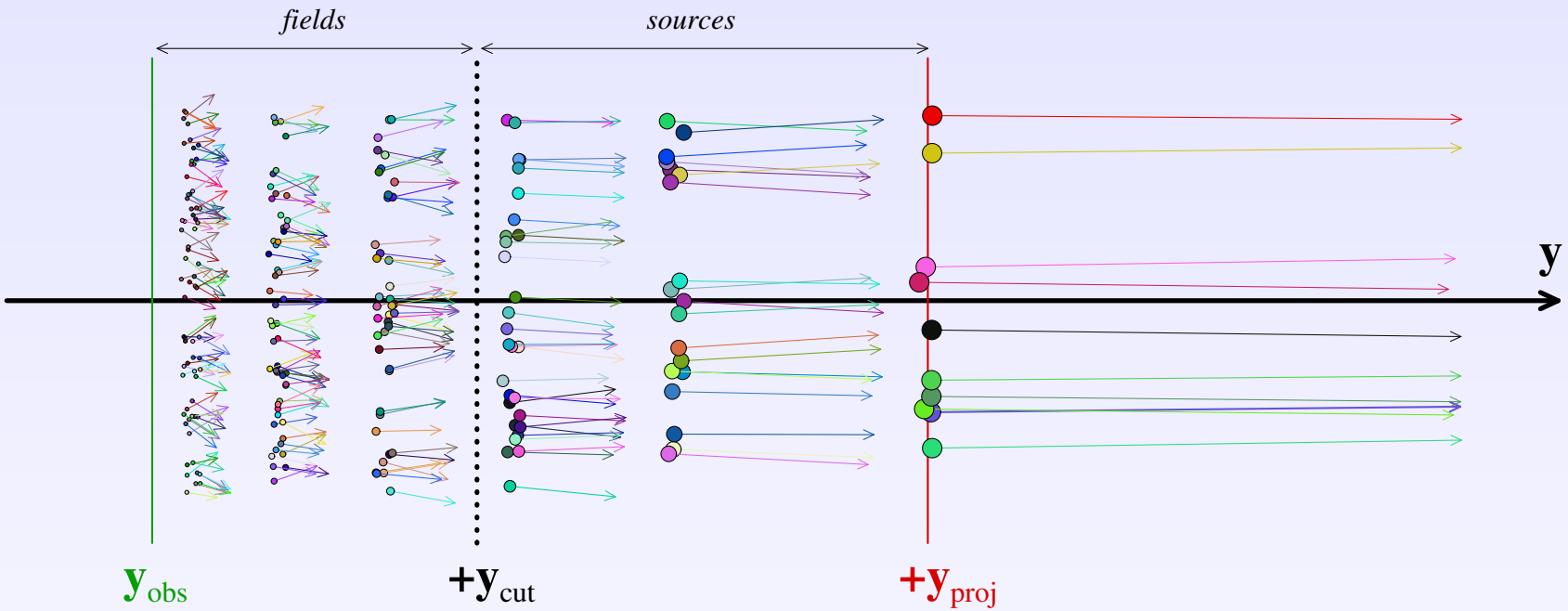

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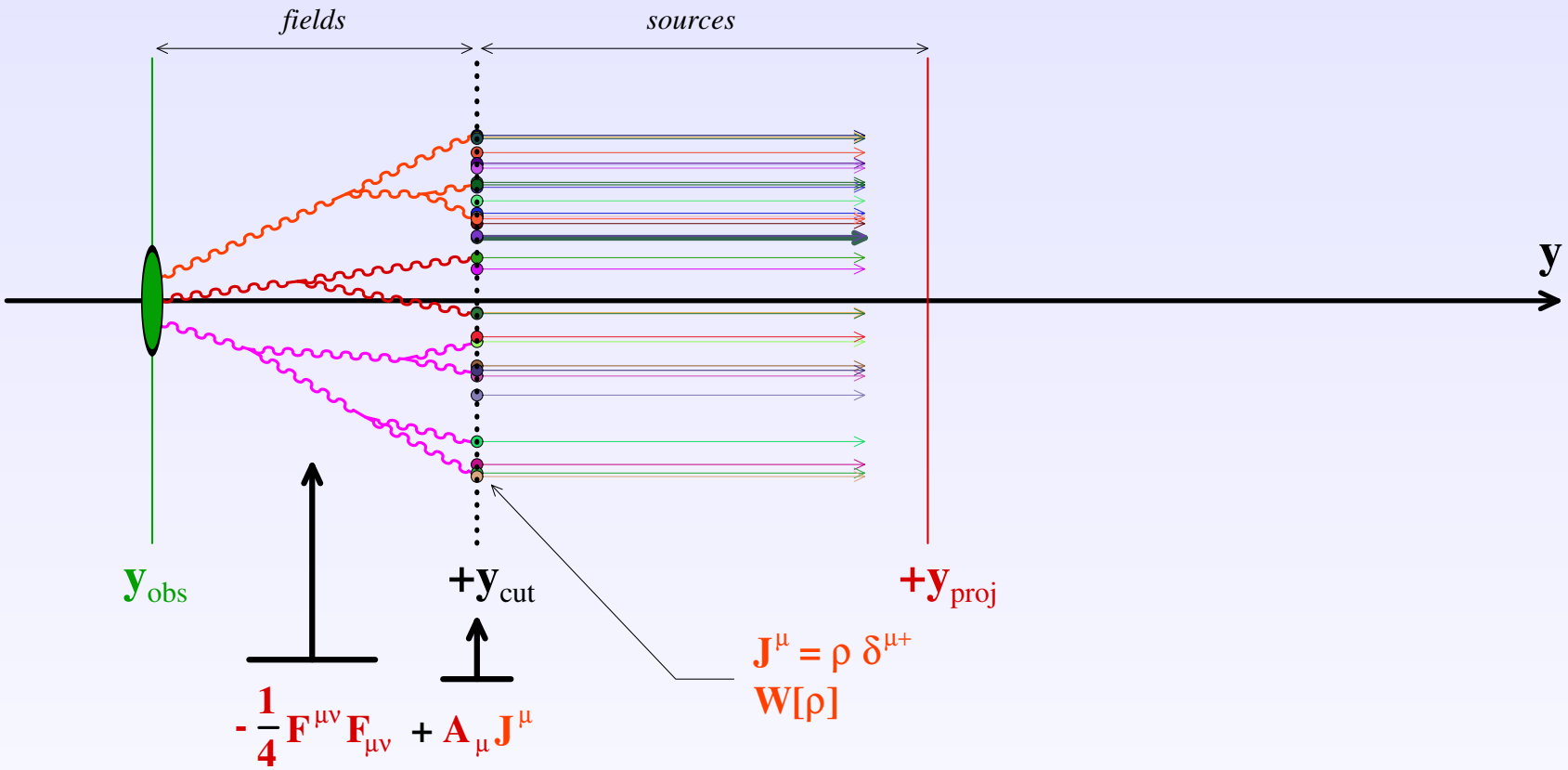

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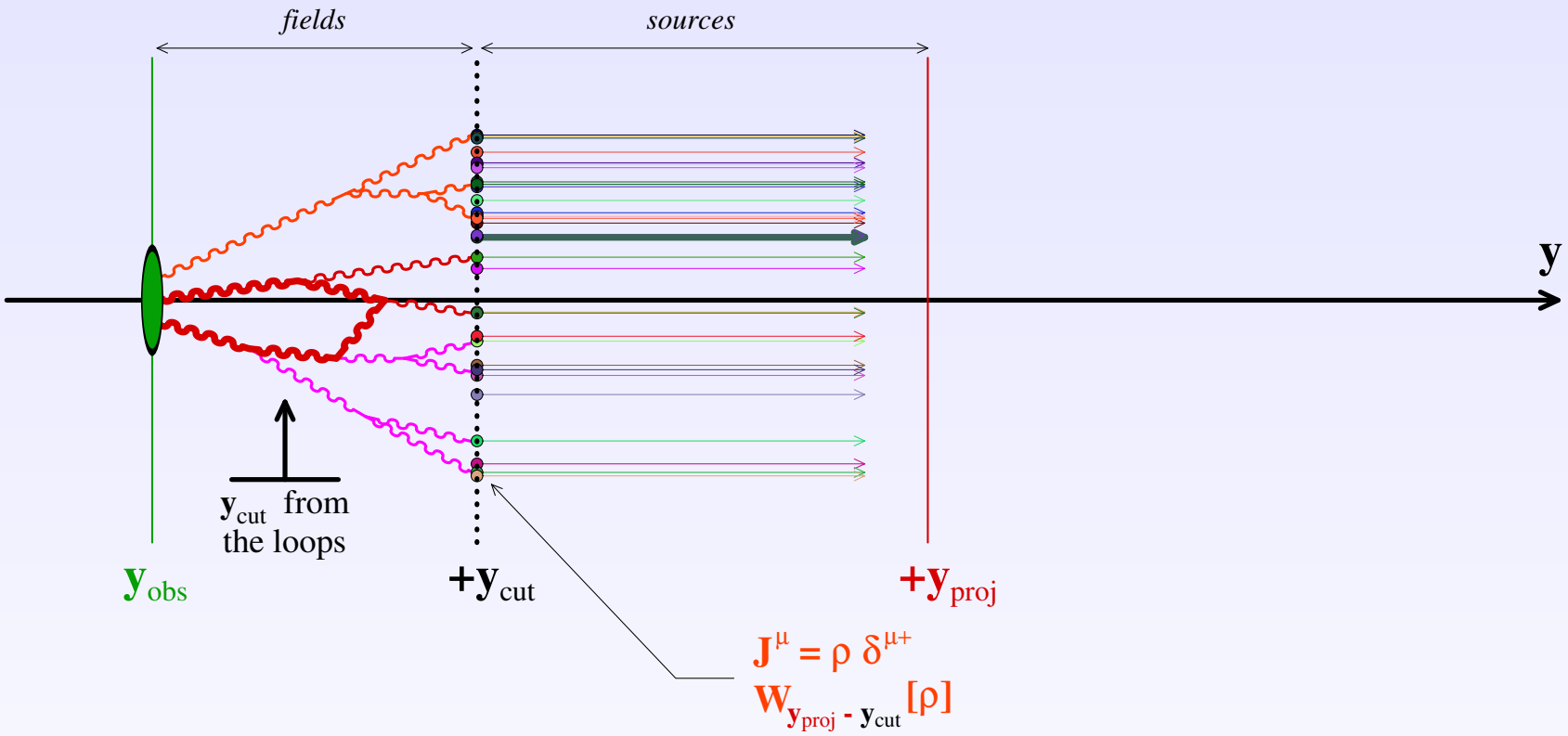
Cancellation of the cutoff dependence



- The cutoff y_{cut} is arbitrary and should not affect the result
- The probability density $W[\rho]$ changes with the cutoff
- Loop corrections cancel the cutoff dependence from $W[\rho]$

B-JIMWLK evolution equation

[Jalilian-Marian, Kovner, Leonidov, Weigert (1998)]
[Balitsky (1996)] [Iancu, Leonidov, McLerran (2001)]

B-JIMWLK equation at Leading Log

$$\frac{\partial W_Y[\rho]}{\partial Y} = \underbrace{\frac{1}{2}\int_{\vec{x}_\perp,\vec{y}_\perp} \frac{\delta}{\delta\rho_a(\vec{x}_\perp)}\chi_{ab}(\vec{x}_\perp,\vec{y}_\perp)\frac{\delta}{\delta\rho_b(\vec{y}_\perp)}}_{\mathcal{H}\quad\text{(JIMWLK Hamiltonian)}} W_Y[\rho]$$

- Mean field approx. (BK equation) : **[Kovchegov (1999)]**
- Langevin form of B-JIMWLK : **[Blaizot, Iancu, Weigert (2003)]**
- First numerical solution : **[Rummukainen, Weigert (2004)]**

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B-JIMWL

Recent developments :

Running coupling correction

[Lappi, Mäntysaari (2012)]

B-JIMWLK equation at Next to Leading Log

[Kovner, Lublinsky, Mulian (2013)]

[Caron-Huot (2013)][Balitsky, Chirilli (2013)]

- Me
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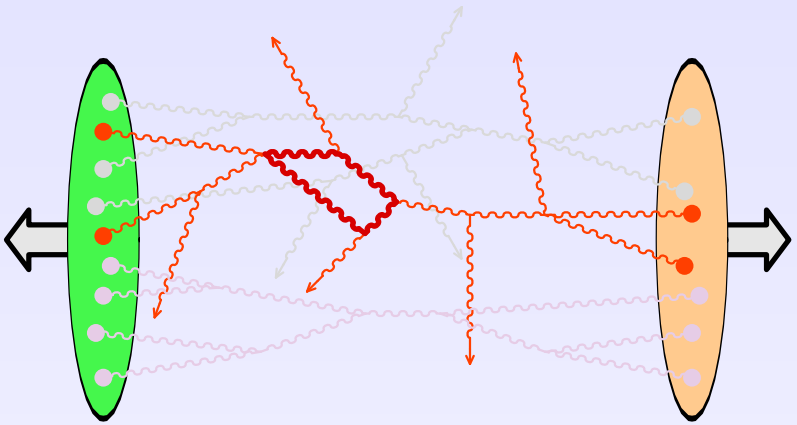
Power counting in the saturated regime

$$\mathcal{S} = \underbrace{-\frac{1}{4}\int F_{\mu\nu}F^{\mu\nu}}_{\text{slow gluons}} + \int \underbrace{(J_1^\mu + J_2^\mu)}_{\text{fast partons}} A_\mu$$



In the saturated regime: $J^\mu \sim g^{-1}$

$$g^{-2}\ g^{\#\text{ of external gluons}}\ g^{2\times(\#\text{ of loops})}$$

- No dependence on the number of sources J^μ
  - ▷ infinite number of graphs at each order in g^2

Example : expansion of $T^{\mu\nu}$ in powers of g^2

$$T^{\mu\nu} \sim \frac{1}{g^2}\left[c_0 + c_1\, g^2 + c_2\, g^4 + \cdots\right]$$

Inclusive observables at Leading Order

[FG, Venugopalan (2006)]

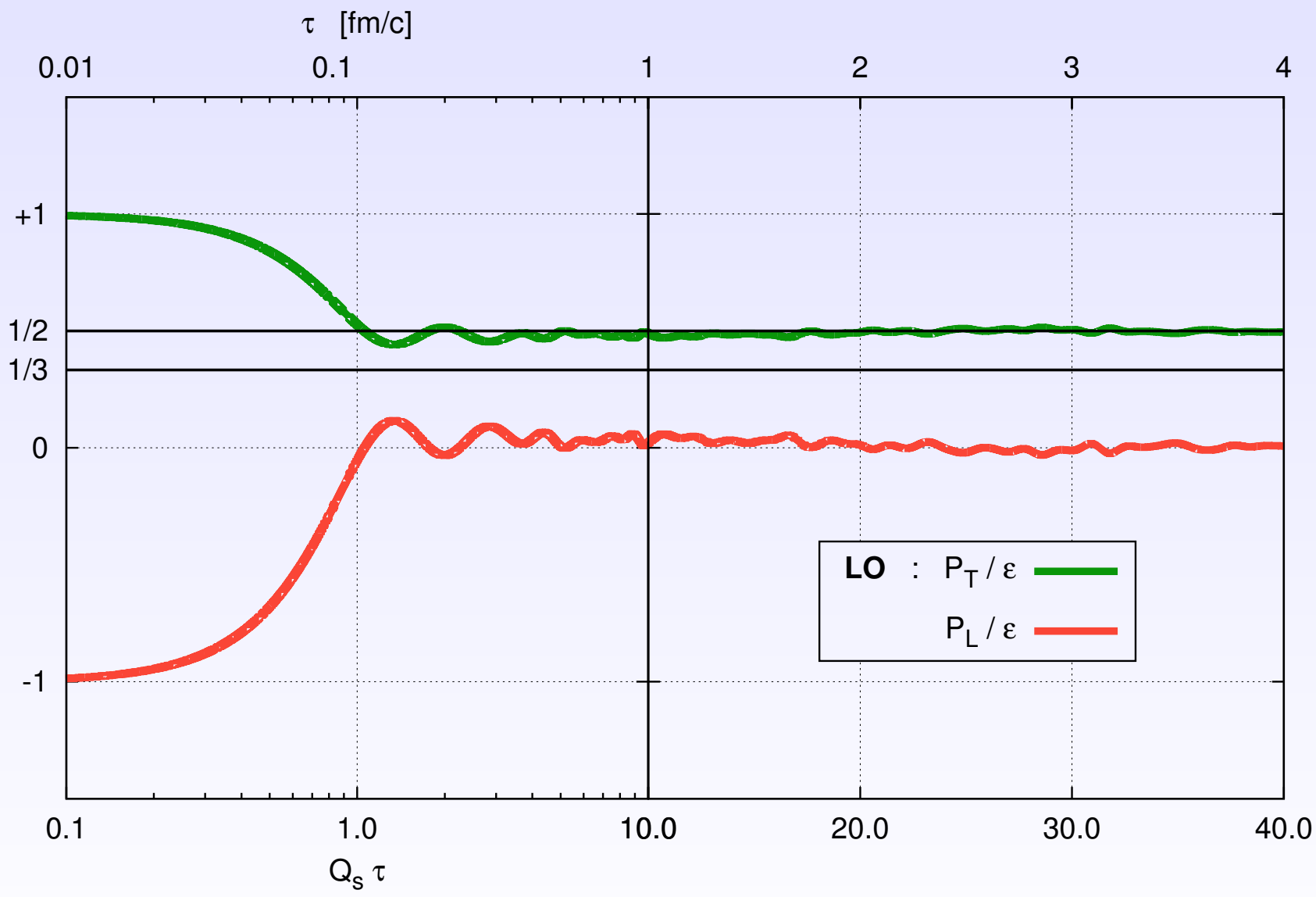
- The Leading Order is the sum of all the tree diagrams
  Expressible in terms of classical solutions of Yang-Mills equations :
  $$\mathcal{D}_\mu \mathcal{F}^{\mu\nu} = J_1^\nu + J_2^\nu$$

- Boundary conditions : $\lim_{x^0 \to -\infty} \mathcal{A}^\mu(x) = 0$
  (WARNING : this is not true for exclusive observables!)

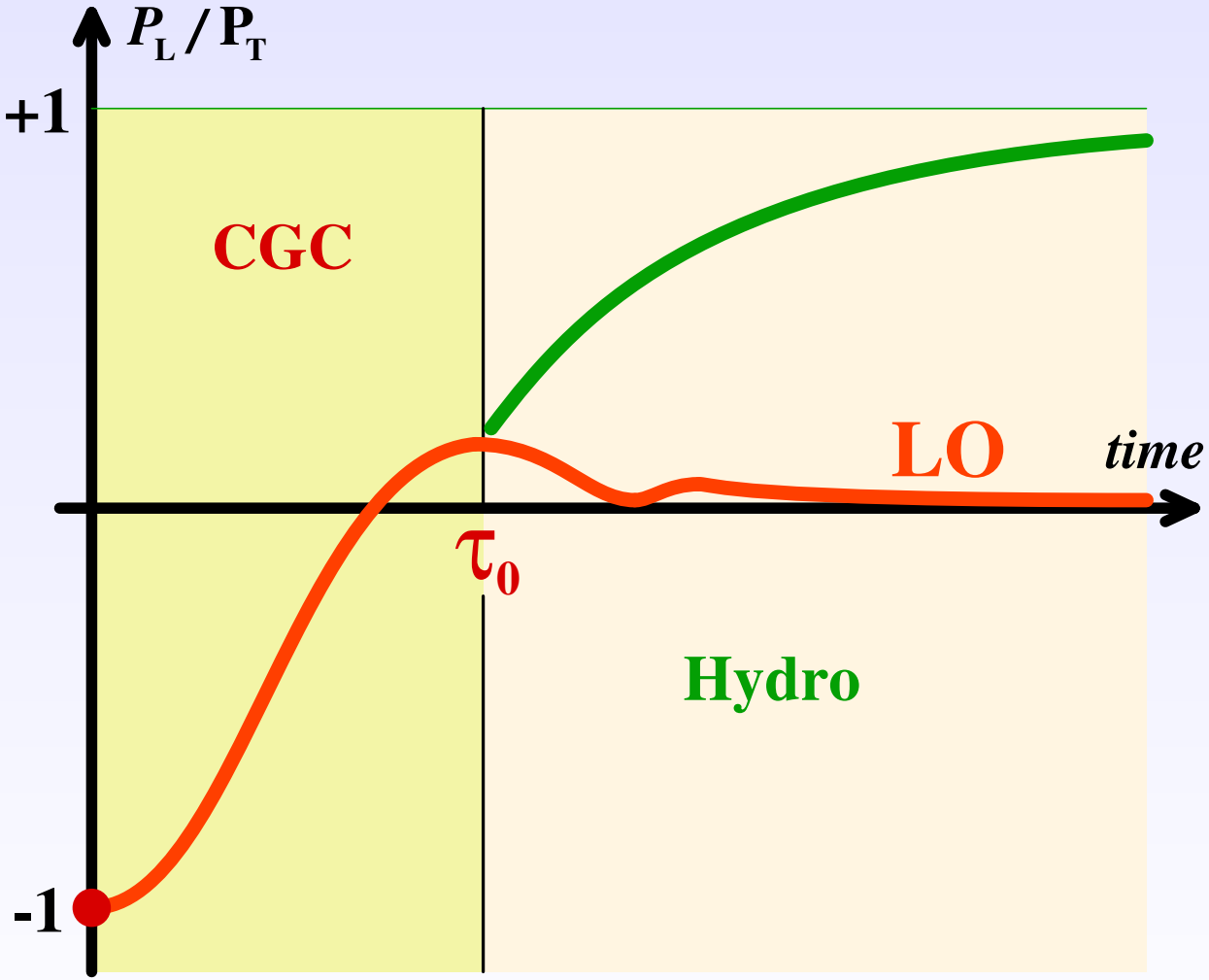
Components of the energy-momentum tensor at LO :

$$T^{00}_{\rm LO} = \frac{1}{2}\big[\underbrace{\mathbf{E}^2 + \mathbf{B}^2}_{\text{class. fields}}\big] \qquad T^{0i}_{\rm LO} = \big[\mathbf{E} \times \mathbf{B}\big]^i$$

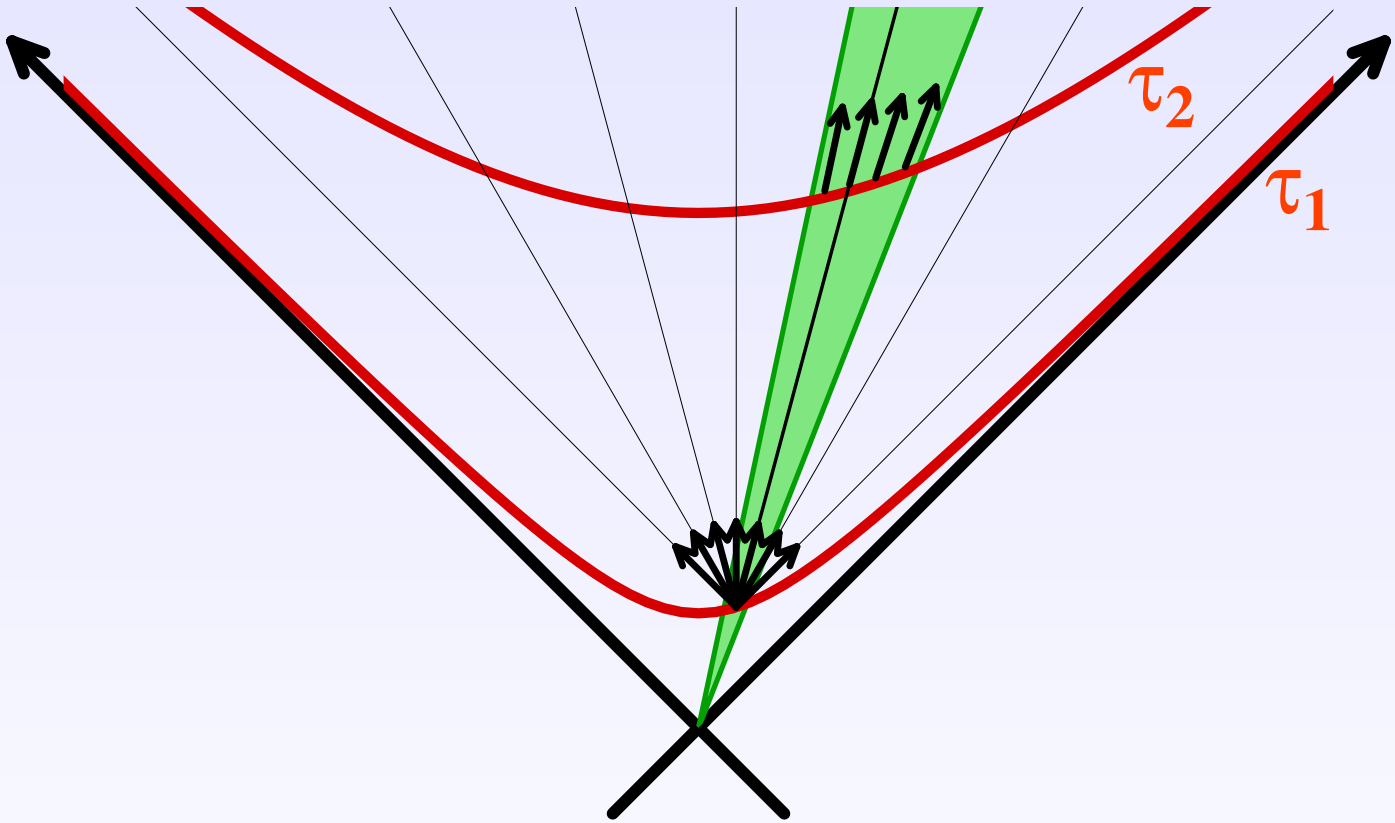
$$T^{ij}_{\rm LO} = \frac{\delta^{ij}}{2}\big[\mathbf{E}^2 + \mathbf{B}^2\big] - \big[\mathbf{E}^i\mathbf{E}^j + \mathbf{B}^i\mathbf{B}^j\big]$$


τ [fm/c]
0.01
0.1
1
2
3
4
+1
1/2
1/3
0
-1
LO : P_T / ε
P_L / ε
0.1
1.0
10.0
20.0
30.0
40.0
Q_s τ

CGC at LO : unsatisfactory matching to hydrodynamics



Competition between Expansion and Isotropization

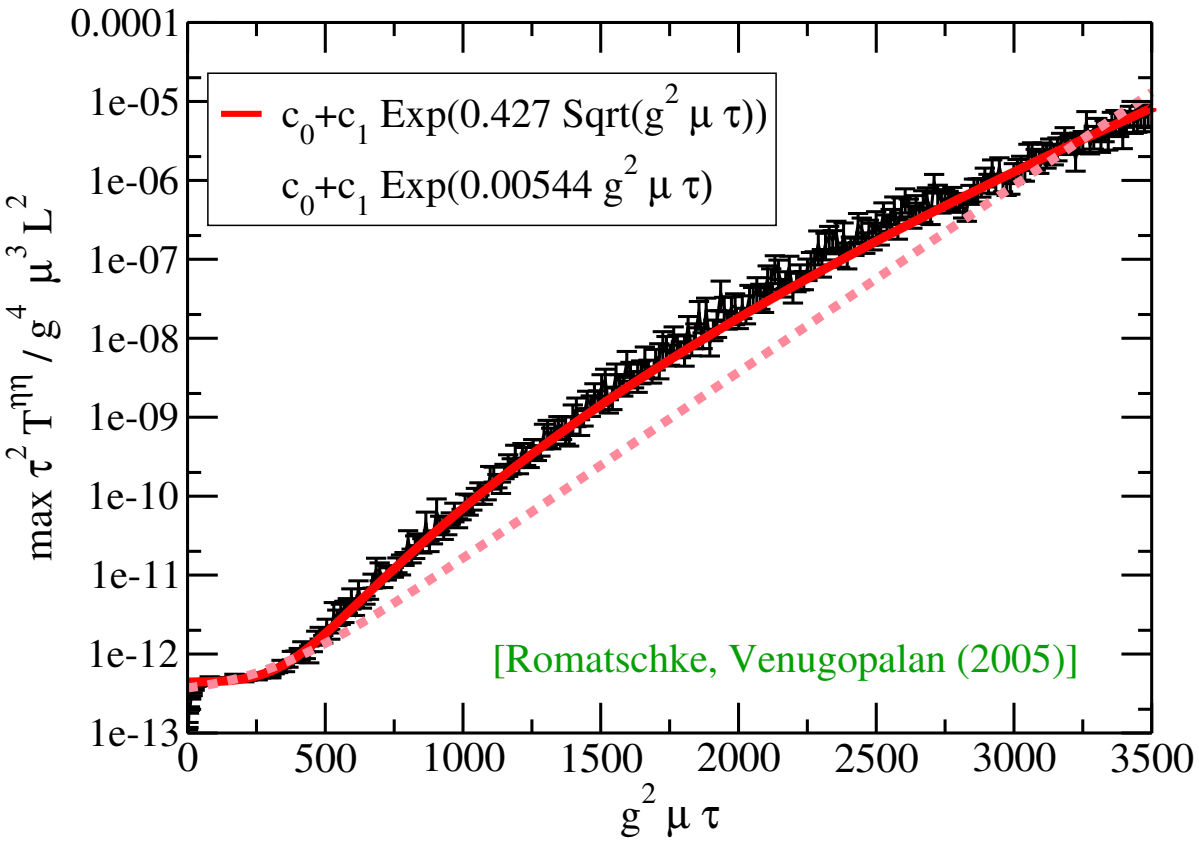


- CGC at LO is very close to free streaming

Does it get better at Next-to-Leading Order?

CGC at NLO : instabilities

[Mrowczynski (1988), Romatschke, Strickland (2003), Arnold, Lenaghan, Moore (2003), Rebhan, Romatschke, Strickland (2005), Arnold, Lenaghan, Moore, Yaffe (2005), Romatschke, Rebhan (2006), Bodeker, Rummukainen (2007), Fujii, Itakura (2008),...,**Attems, Rebhan, Strickland (2012)**, **Fukushima (2013)**]



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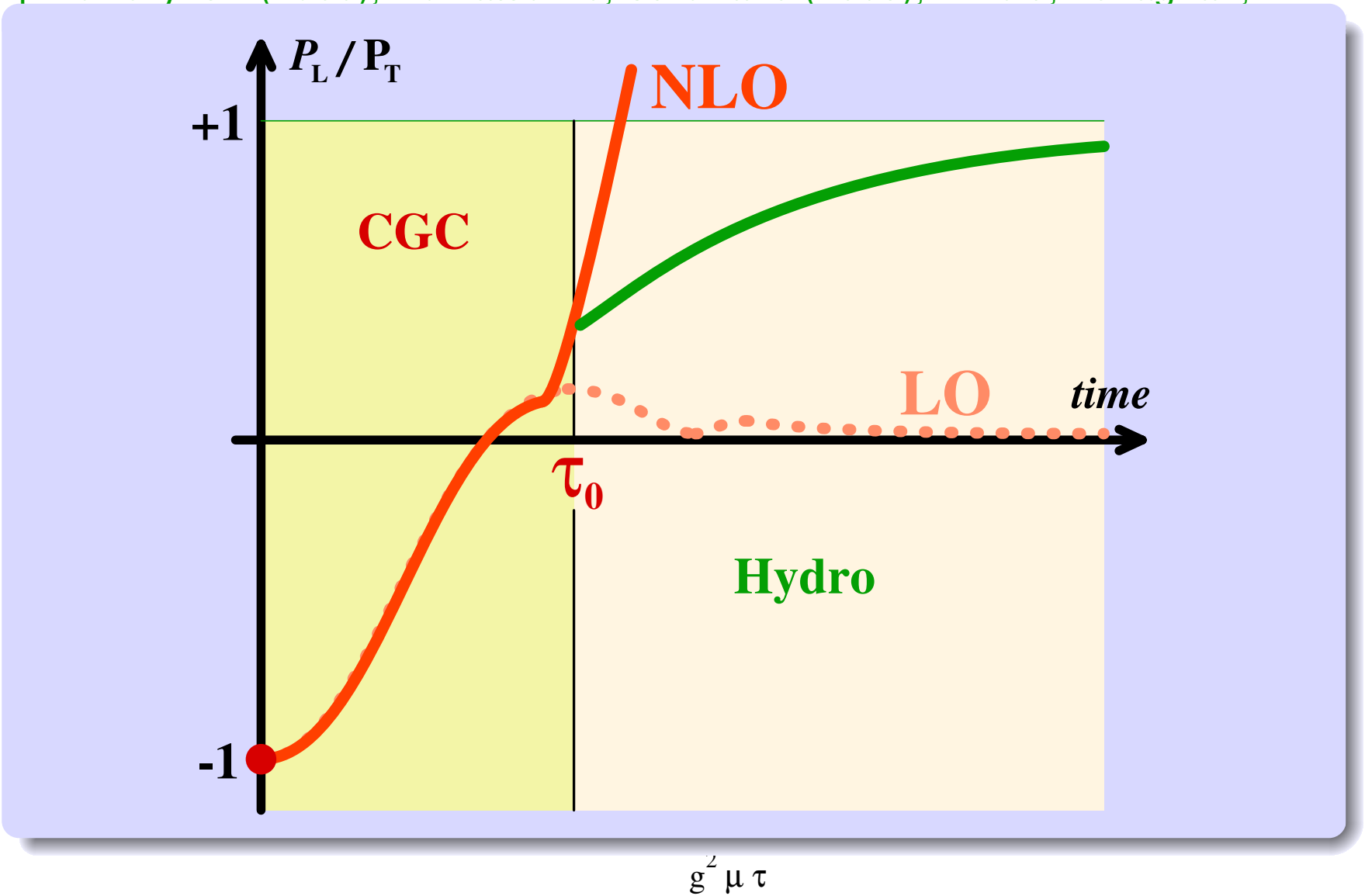
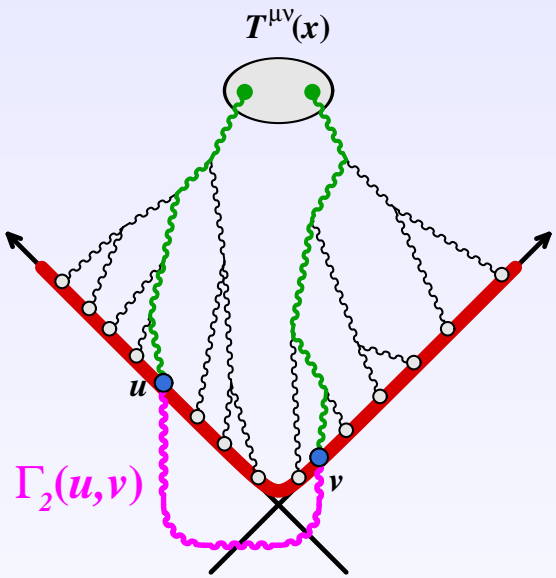


Beyond NLO :

Classical Statistical Approximation

Improved power counting and resummation

Loop $\sim g^2$, Small perturbation $\sim e^{\sqrt{\mu\tau}}$



- 1 loop : $(ge^{\sqrt{\mu\tau}})^2$

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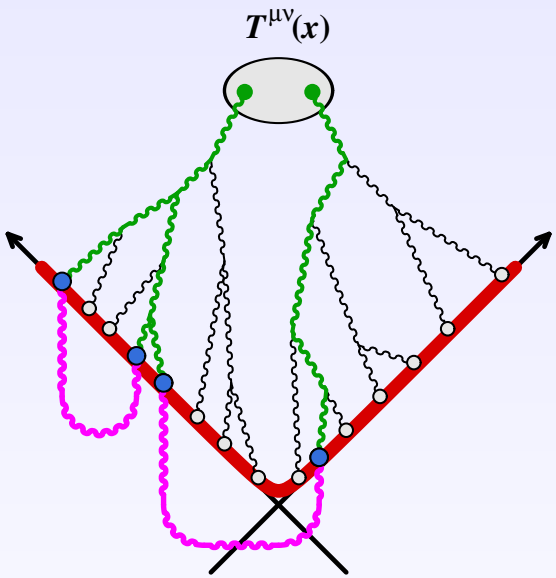

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- 2 disconnected loops : $(ge^{\sqrt{\mu\tau}})^4$

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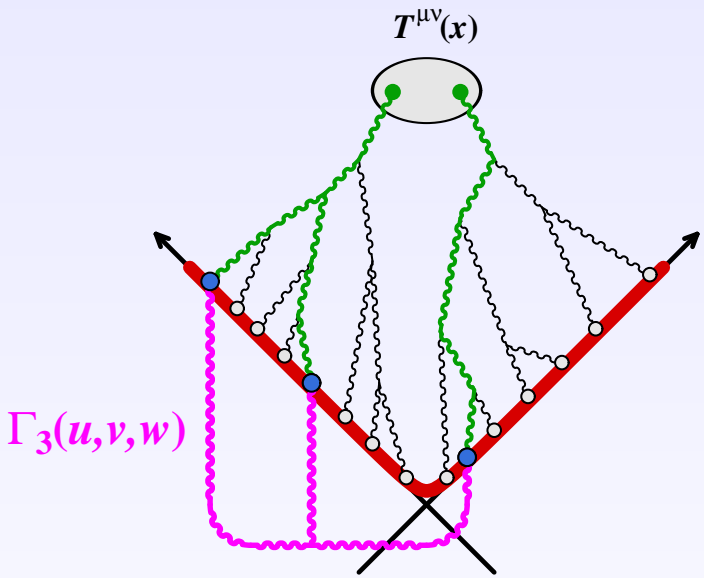


- 1 loop : $(ge^{\sqrt{\mu\tau}})^2$
- 2 disconnected loops : $(ge^{\sqrt{\mu\tau}})^4$
- 2 entangled loops : $g(ge^{\sqrt{\mu\tau}})^3$ ▷ subleading

Leading terms

- All disconnected loops to all orders
  ▷ exponentiation of the 1-loop result

Classical Statistical Approximation (CSA)

$$
\begin{aligned}
T^{\mu\nu}_{\text{resummed}} &= \underbrace{T^{\mu\nu}_{\text{LO}} + T^{\mu\nu}_{\text{NLO}}}_{\text{in full}} + \underbrace{T^{\mu\nu}_{\text{NNLO}} + \cdots}_{\text{partially}} \\
&= \int [\mathrm{D}a] \, \exp\left[-\frac{1}{2} \int_{u,v} a(u) \Gamma_2^{-1}(u,v) a(v) \right] T^{\mu\nu}_{\text{LO}}[\mathcal{A}_{\text{init}} + a]
\end{aligned}
$$

- The exponentiation of the 1-loop result collects all the terms with the worst time behavior
- Equivalent to Gaussian fluctuations of the initial field + classical time evolution
- Note: This is the Wigner distribution of a coherent state

CSA in Quantum Mechanics

- Von Neumann equation for the density operator :

$$\frac{\partial \widehat{\rho}_\tau}{\partial \tau} = i\hbar \left[\widehat{H}, \widehat{\rho}_\tau\right]$$

- Wigner transform :

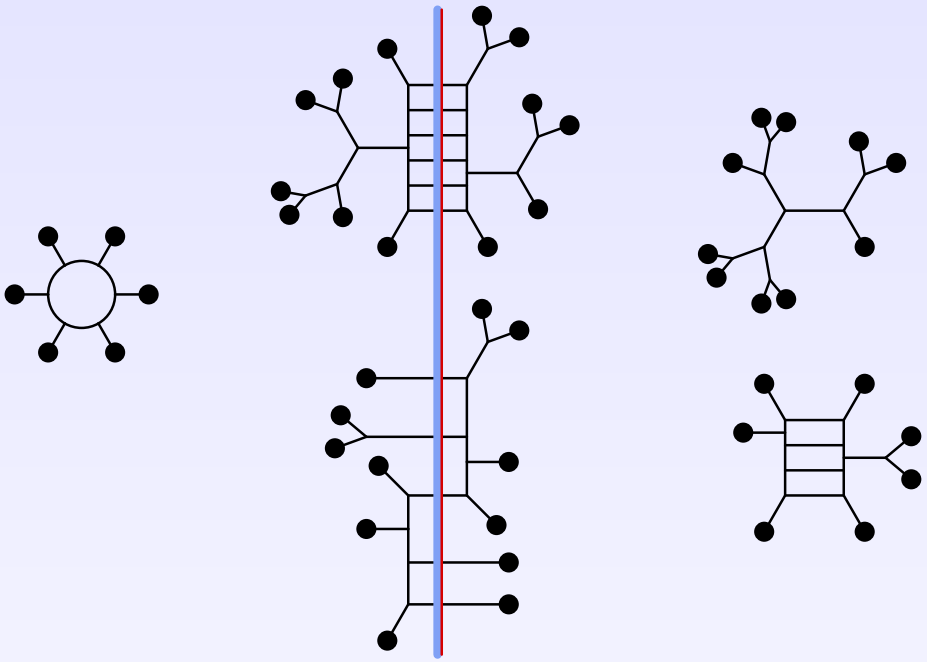
$$\begin{aligned} W_\tau(\mathbf{x},\mathbf{p}) &\equiv \int d\mathbf{s}\; e^{i\mathbf{p}\cdot\mathbf{s}} \left\langle \mathbf{x} + \frac{\mathbf{s}}{2} \middle| \widehat{\rho}_\tau \middle| \mathbf{x} - \frac{\mathbf{s}}{2} \right\rangle \\ \mathcal{H}(\mathbf{x},\mathbf{p}) &\equiv \int d\mathbf{s}\; e^{i\mathbf{p}\cdot\mathbf{s}} \left\langle \mathbf{x} + \frac{\mathbf{s}}{2} \middle| \widehat{H} \middle| \mathbf{x} - \frac{\mathbf{s}}{2} \right\rangle \quad \text{(classical Hamiltonian)} \end{aligned}$$

- Moyal equation (equivalent to Von Neumann) :

$$\begin{aligned} \frac{\partial W_\tau}{\partial \tau} &= \mathcal{H}(\mathbf{x},\mathbf{p})\, \frac{2}{i\hbar}\, \sin\left(\frac{i\hbar}{2} \left(\overleftarrow{\partial}_{\mathbf{p}} \overrightarrow{\partial}_{\mathbf{x}} - \overleftarrow{\partial}_{\mathbf{x}} \overrightarrow{\partial}_{\mathbf{p}} \right) \right) W_\tau(\mathbf{x},\mathbf{p}) \\ &= \underbrace{\{\mathcal{H}, W_\tau\}}_{\text{Poisson bracket}} + \mathcal{O}(\hbar^2) \end{aligned}$$

- Classical time evolution $\Longleftrightarrow$ $\mathcal{O}(\hbar^2)$ error
- $\mathcal{O}(\hbar^1)$ corrections come from the initial condition

CSA in Quantum Field Theory (I)



Amplitude : ϕ_+ G_{++} G_{+-} (Amplitude)* : ϕ_- G_{--}

- From the Schwinger-Keldysh formalism, define new fields as:

$$\phi_1 \equiv \frac{1}{2}(\phi_+ + \phi_-) \qquad \phi_2 \equiv \phi_+ - \phi_-$$

- Vertices : $\phi_1^3\phi_2$ $\phi_1\phi_2^3$ (odd terms in ϕ_2 only)
- ϕ_2 encodes quantum interferences

CSA in Quantum Field Theory (II)

CSA : drop the vertex $\phi_1\phi_2^3$

- No $\phi_1\phi_2^3$ vertex $\Longrightarrow$ classical time evolution
- Differences with the original QFT start appearing at 2 loops
- Equivalent to classical runs averaged over the initial conditions

Ensemble of initial classical fields

- This approximation does not specify the initial fields: controlled by the observable under consideration (e.g. $\langle 0_{\rm in}|T^{\mu\nu}|0_{\rm in}\rangle$)
- Initial 2-point correlations encoded in G_{11}. Generically:

$$G_{11}(p) \sim \Big(f_0(\mathbf{p}) + \frac{1}{2}\Big)\,\delta(p^2)$$

quasiparticles $\hookleftarrow$ $\hookrightarrow$ vacuum zero point fluctuations

- The $1/2$ generates the 1-loop quantum corrections
- Dropping the $1/2$: nothing quantum left

$\mathrm{CSA}_{\mathrm{vac}}$: vacuum fluctuations perturbed by CGC background

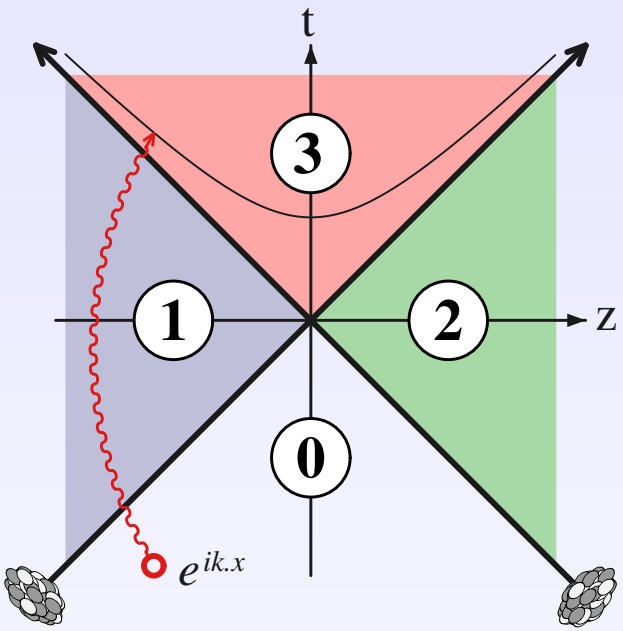
[Epelbaum, FG (2013)]

CGC at $\tau \ll Q_s^{-1}$ (1-loop accurate)

$$\langle \mathcal{A}^\mu \rangle = \mathcal{A}^\mu_{\mathrm{LO}} \qquad \text{Var.} = \frac{1}{2} \int_{\text{modes } \mathbf{k}} a_{\mathbf{k}}(u) a^*_{\mathbf{k}}(v)$$

$$\left[\mathcal{D}_\rho \mathcal{D}^\rho \delta^\nu_\mu - \mathcal{D}_\mu \mathcal{D}^\nu + ig\, \mathcal{F}_\mu{}^\nu \right] a^\mu_{\mathbf{k}} = 0$$

$$\lim_{x^0 \to -\infty} a_{\mathbf{k}}(x) = e^{i\mathbf{k}\cdot\mathbf{x}}$$



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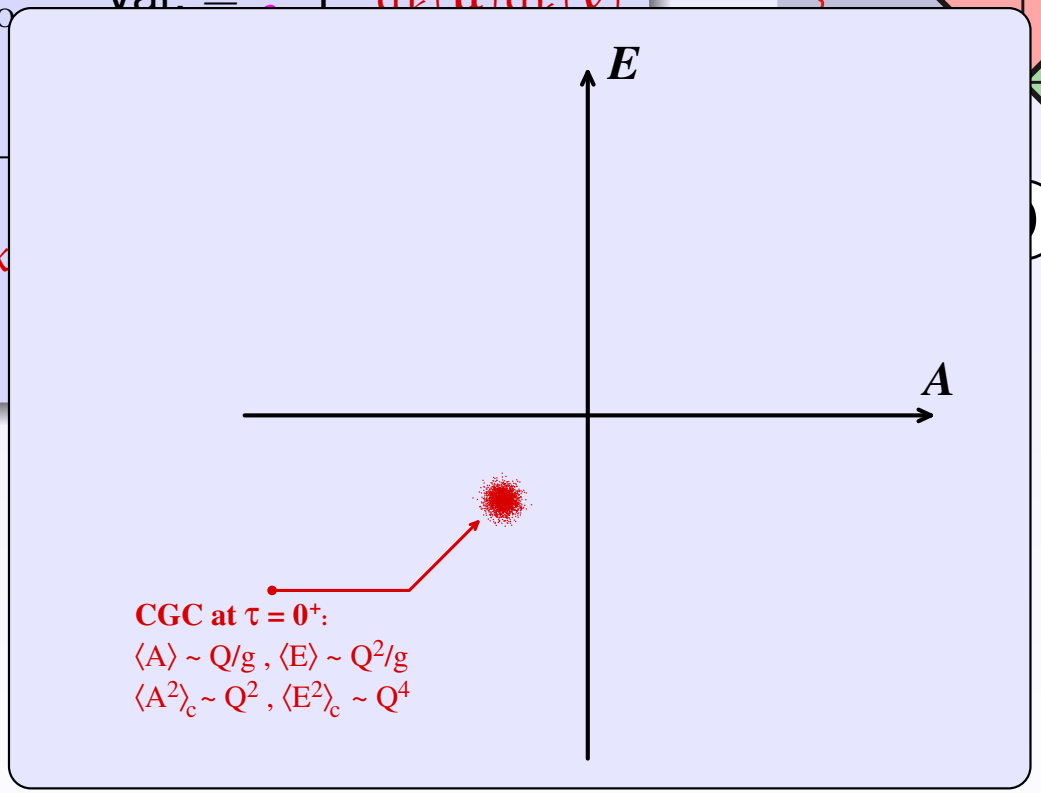
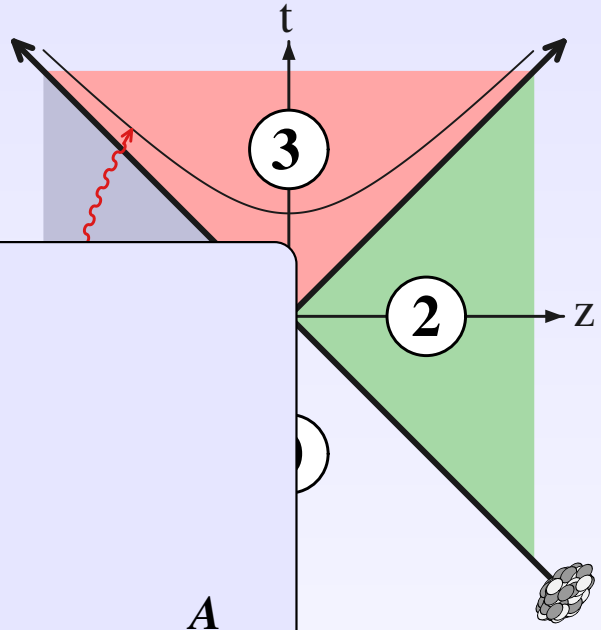
$$\Big[\mathcal{D}_\rho \mathcal{D}^\rho \delta^\nu_\mu - \ldots$$

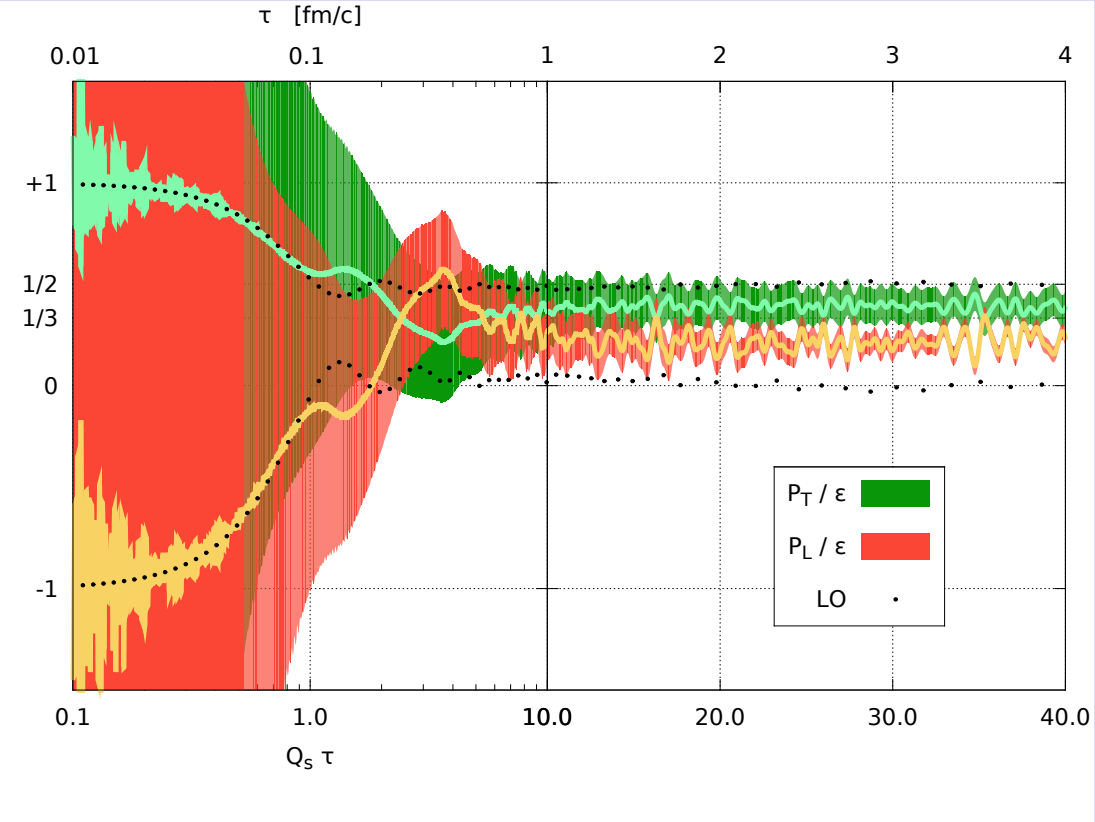
$$\lim_{x^0 \to -\infty} a_k(x \ldots$$





$\mathrm{CSA}_{\mathrm{vac}}$: vacuum fluctuations perturbed by CGC background

$g = 0.5$



- Nearly constant P_L/P_T for $Q_s\tau \gtrsim 2$

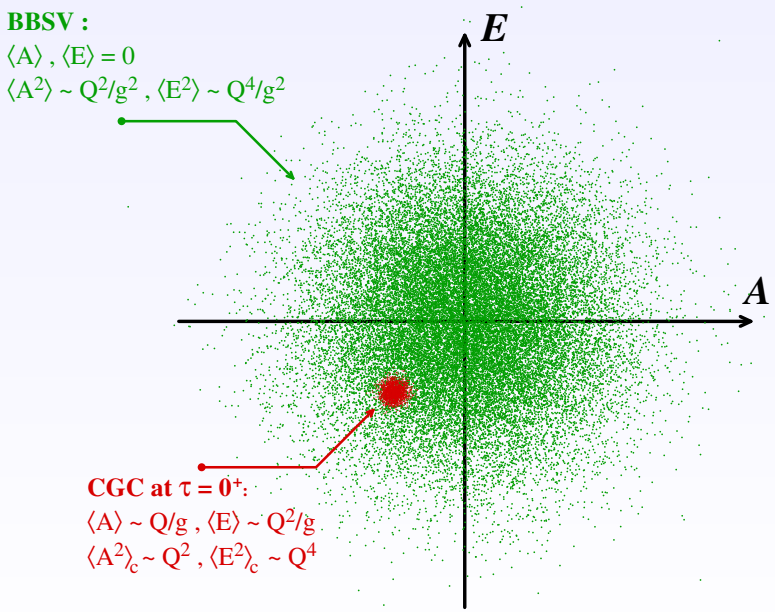
$\mathrm{CSA}_{\mathrm{part}}$: particles only, no quantum fluctuations

[Berges, Boguslavski, Schlichting, Venugopalan (2013)]

Dense gas of free gluons

$$\langle \mathcal{A}^\mu \rangle = 0 \qquad \text{Var.} = \int_{\text{modes } \mathbf{k}} f_0(\mathbf{k})\, a_{\mathbf{k}}(\mathbf{u}) a^*_{\mathbf{k}}(\mathbf{v}) \qquad a_{\mathbf{k}}(x) \equiv e^{i\mathbf{k}\cdot x}$$

$$f_0(\mathbf{k}) \sim (n_0/g^2) \times \theta(Q - \sqrt{k_\perp^2 + \xi_0\, k_z^2})$$



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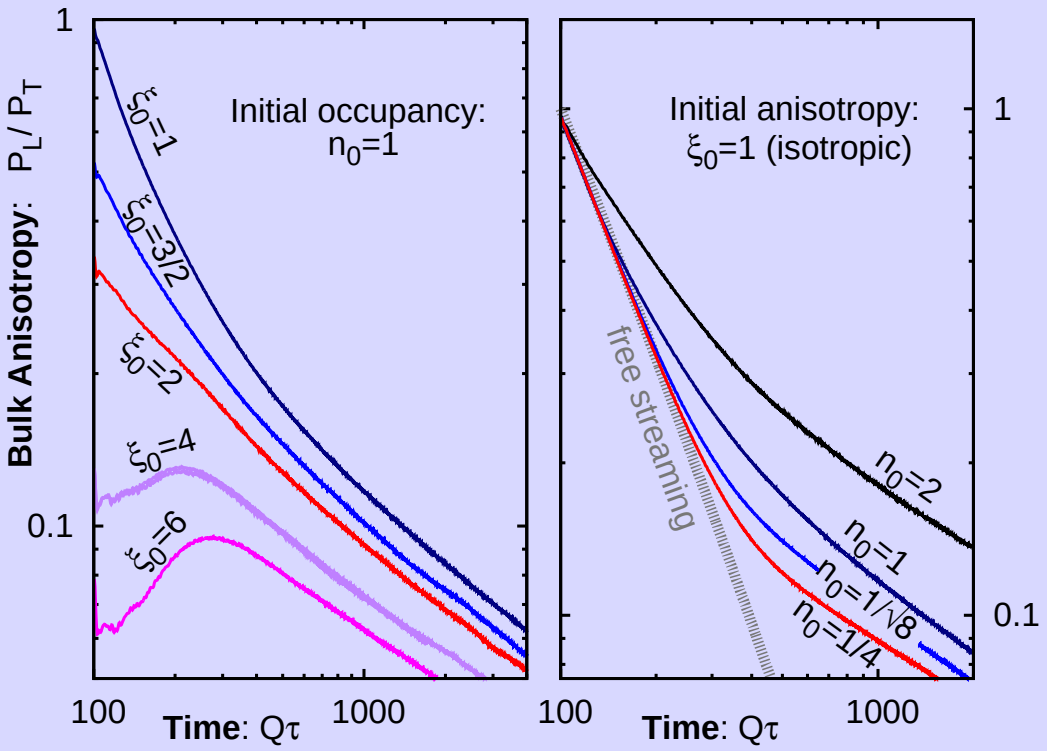
Same result for all g's



- Self-similar evolution with scaling laws consistent with the small angle scattering analysis of **Baier, Mueller, Schiff, Son (2002)** :

$$f(t, p_\perp, p_z) \sim \tau^{-2/3}\, f_s(\tau^0 p_\perp, \tau^{1/3} p_z) \qquad \frac{P_L}{P_T} \sim \tau^{-2/3}$$

Lessons from Kinetic Theory

CSA in Kinetic Theory

- Boltzmann equation for the elastic process $12 \to 34$:

$$\partial_t f_1 = \frac{1}{\omega_1} \int_{2,3,4} \delta^{(4)}(P_1+P_2-P_3-P_4)\ |\mathcal{M}_{12,34}|^2 \ \Big[f_3 f_4 (1+f_1)(1+f_2) - f_1 f_2 (1+f_3)(1+f_4)\Big]$$

- Kinetic analogue of CSA_{part} : keep only the cubic terms in f

$$f_3 f_4 (1+f_1)(1+f_2) - f_1 f_2 (1+f_3)(1+f_4) \quad \to \quad f_3 f_4 (f_1+f_2) - f_1 f_2 (f_3+f_4)$$

- Kinetic analogue of CSA_{vac} : from the cubic terms, do $f \to f + \frac{1}{2}$

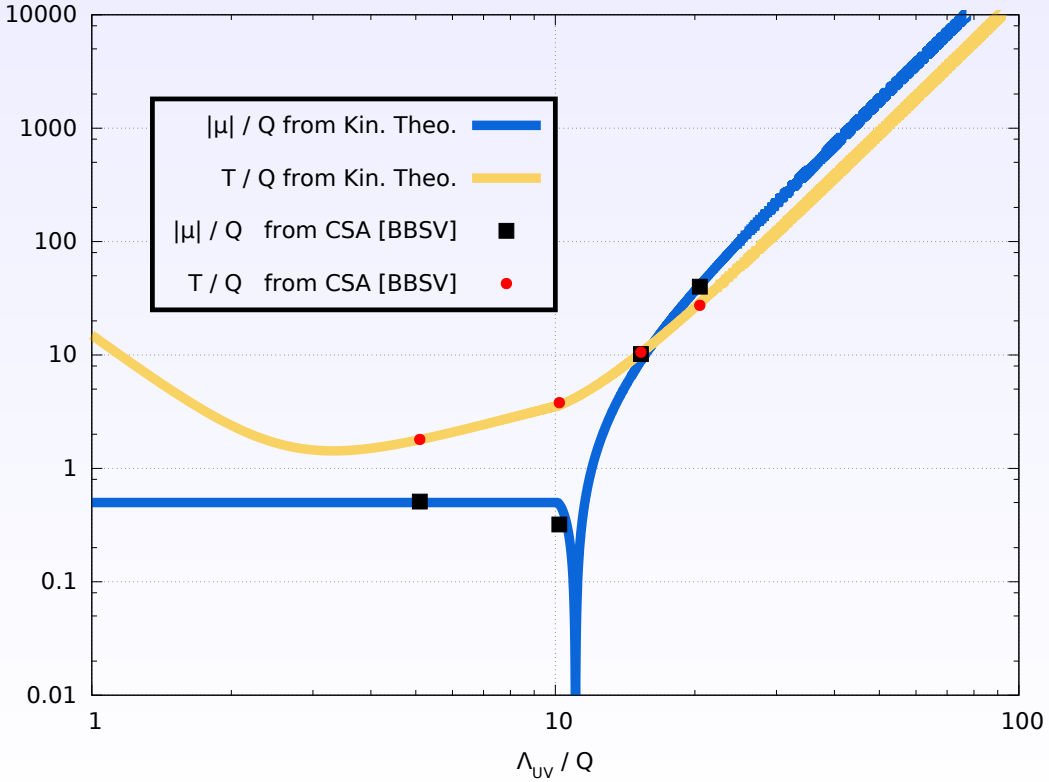
$$\begin{aligned} f_3 f_4 (1+f_1)(1+f_2) - f_1 f_2 (1+f_3)(1+f_4) \quad \to \quad & (\tfrac{1}{2}+f_3)(\tfrac{1}{2}+f_4)(1+f_1+f_2) \\ & -(\tfrac{1}{2}+f_1)(\tfrac{1}{2}+f_2)(1+f_3+f_4) \end{aligned}$$

- The Boltzmann equation can be used to assess the effect of these two approximations by comparing their solutions with that of the unapproximated equation

UV cutoff dependence of CSA_{vac}

[Berges, Boguslavski, Schlichting, Venugopalan (2013)]
[Epelbaum, FG, Tanji, Wu (2014)]

- CSA_{vac} is a nonrenormalizable approximation of the original QFT
- Dependence on the UV cutoff. Can also be seen in kinetic theory
- At late times, $f(\mathbf{p}) = \dfrac{T}{\omega_{\mathbf{p}} - \mu} - \dfrac{1}{2}$, but T and μ depend on Λ_{UV}



UV cutoff dependence of $\mathrm{CSA}_{\mathrm{vac}}$

[Berges, Boguslavski, Schlichting, Venugopalan (2013)]
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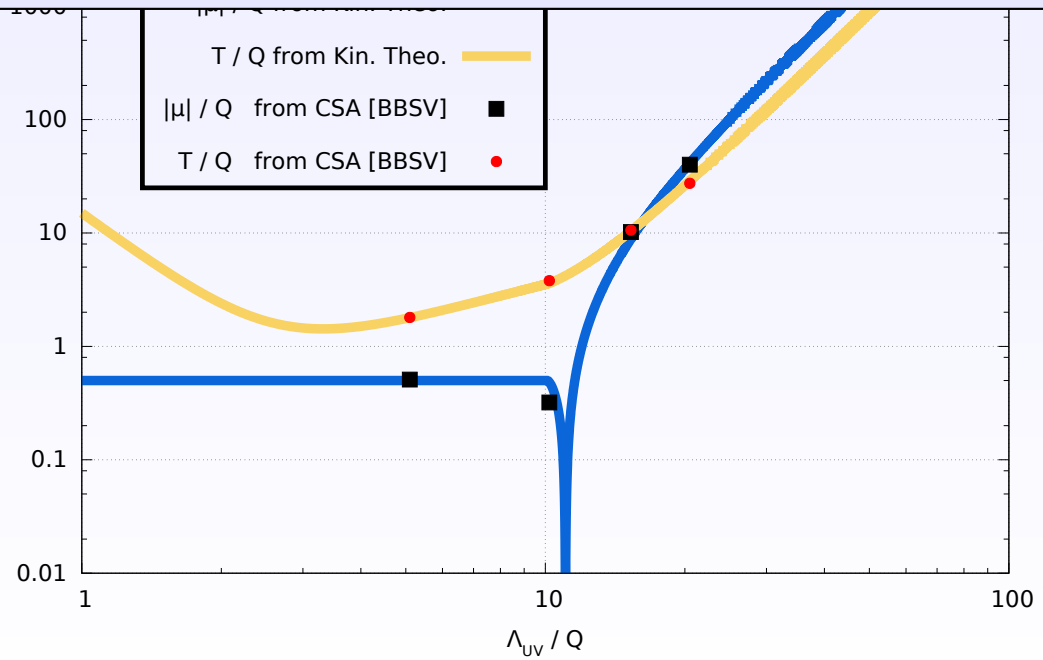
- $\mathrm{CSA}_{\mathrm{vac}}$ is a nonrenormalizable approximation of the original QFT
- Dependence on the UV cutoff. Can also be seen in kinetic theory
- Strong cutoff dependence if $\Lambda_{\mathrm{uv}} \gg$ physical scales
- Mild sensitivity if $\Lambda_{\mathrm{uv}} \sim [3-6] \times$ (physical scales)



$\mathrm{CSA}_{\mathrm{part}}$: misses large angle scatterings in anisotropic systems

[Epelbaum, FG, Jeon, Moore, Wu (2015)]

- Kinetic version of $\mathrm{CSA}_{\mathrm{part}}$:

$$\partial_t f_4 \sim g^4 \int_{123} \cdots \left[f_1 f_2 (f_3 + f_4) - f_3 f_4 (f_1 + f_2) \right] + \cdots \left[\cancel{f_1 f_2 - f_3 f_4} \right]$$

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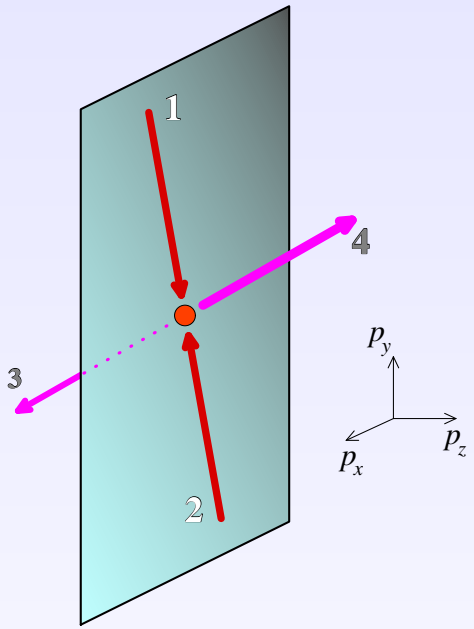

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- If the distribution becomes very anisotropic, trying to produce the particle 4 at large angle results in $f_3 \approx f_4 \approx 0 \;\Rightarrow$ nothing left
- The same argument applies also to any inelastic $n \to n'$ scattering

The CSA without vacuum fluctuations underestimates large angle scatterings when the distribution is anisotropic, and may lead to wrong conclusions regarding isotropization

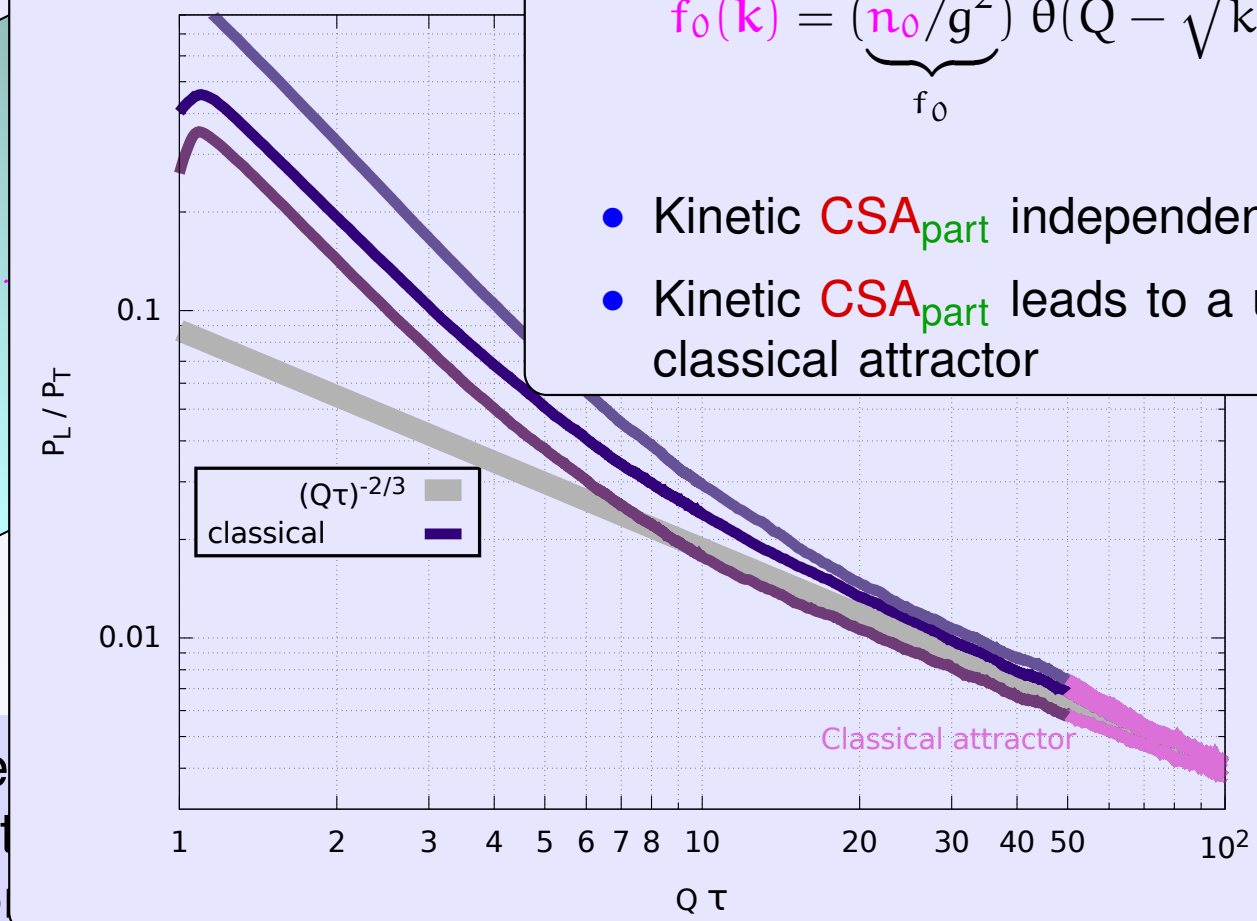
$\mathrm{CSA}_{\mathrm{part}}$: misses large angle scatterings in anisotropic systems

[Epelbaum, FG, Jeon, Moore, Wu (2015)]

- Initial conditions:

$$f_0(\mathbf{k}) = \underbrace{(n_0/g^2)}_{f_0}\,\theta(Q - \sqrt{k_\perp^2 + \xi_0\, k_z^2})$$

- Kinetic $\mathrm{CSA}_{\mathrm{part}}$ independent of g^2
- Kinetic $\mathrm{CSA}_{\mathrm{part}}$ leads to a universal classical attractor



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CSA_{part} : misses large angle scatterings in anisotropic systems

[Epelbaum, FG, Jeon, Moore, Wu (2015)]

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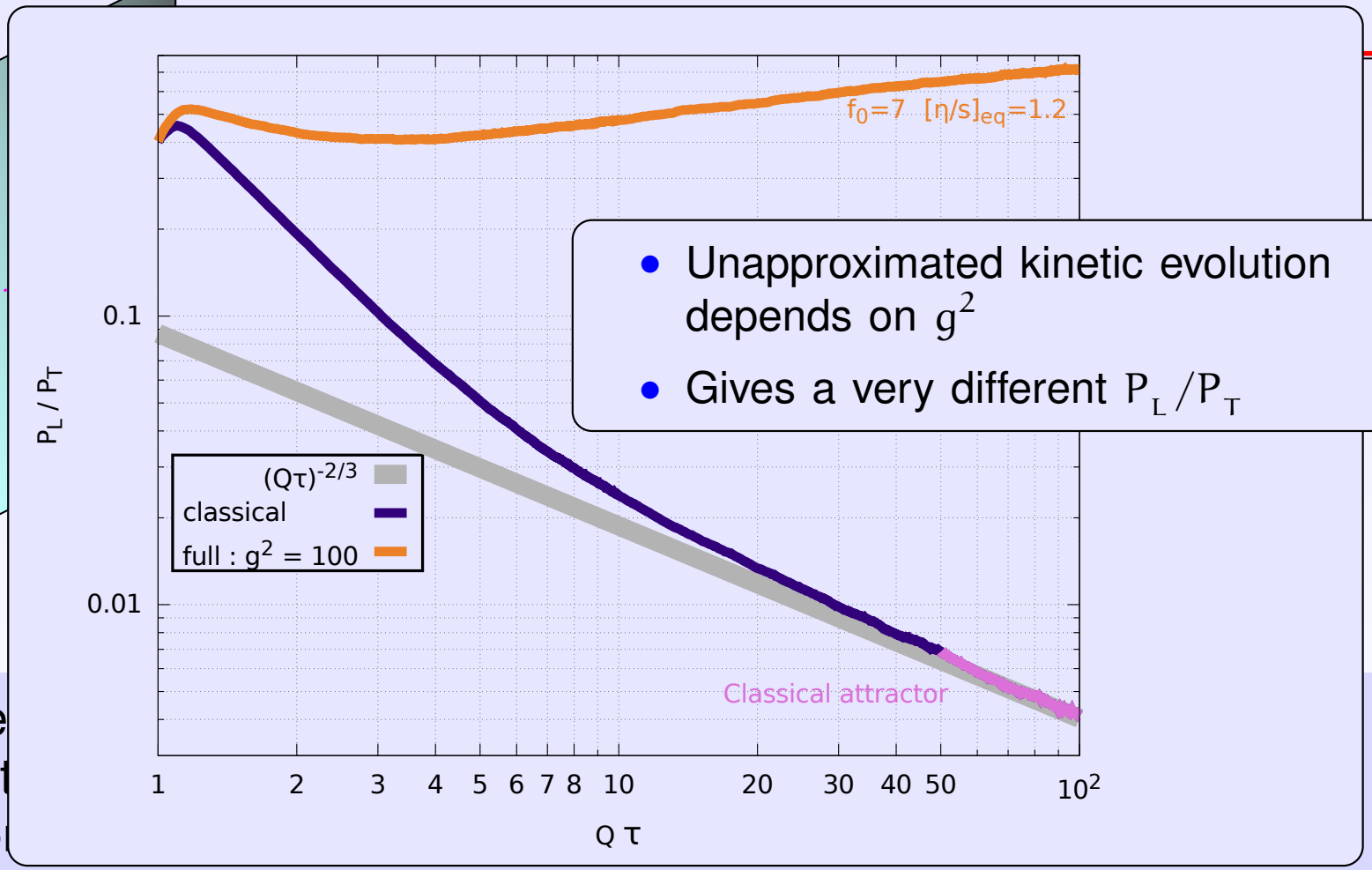

- Unapproximated kinetic evolution depends on g^2
- Gives a very different P_L/P_T

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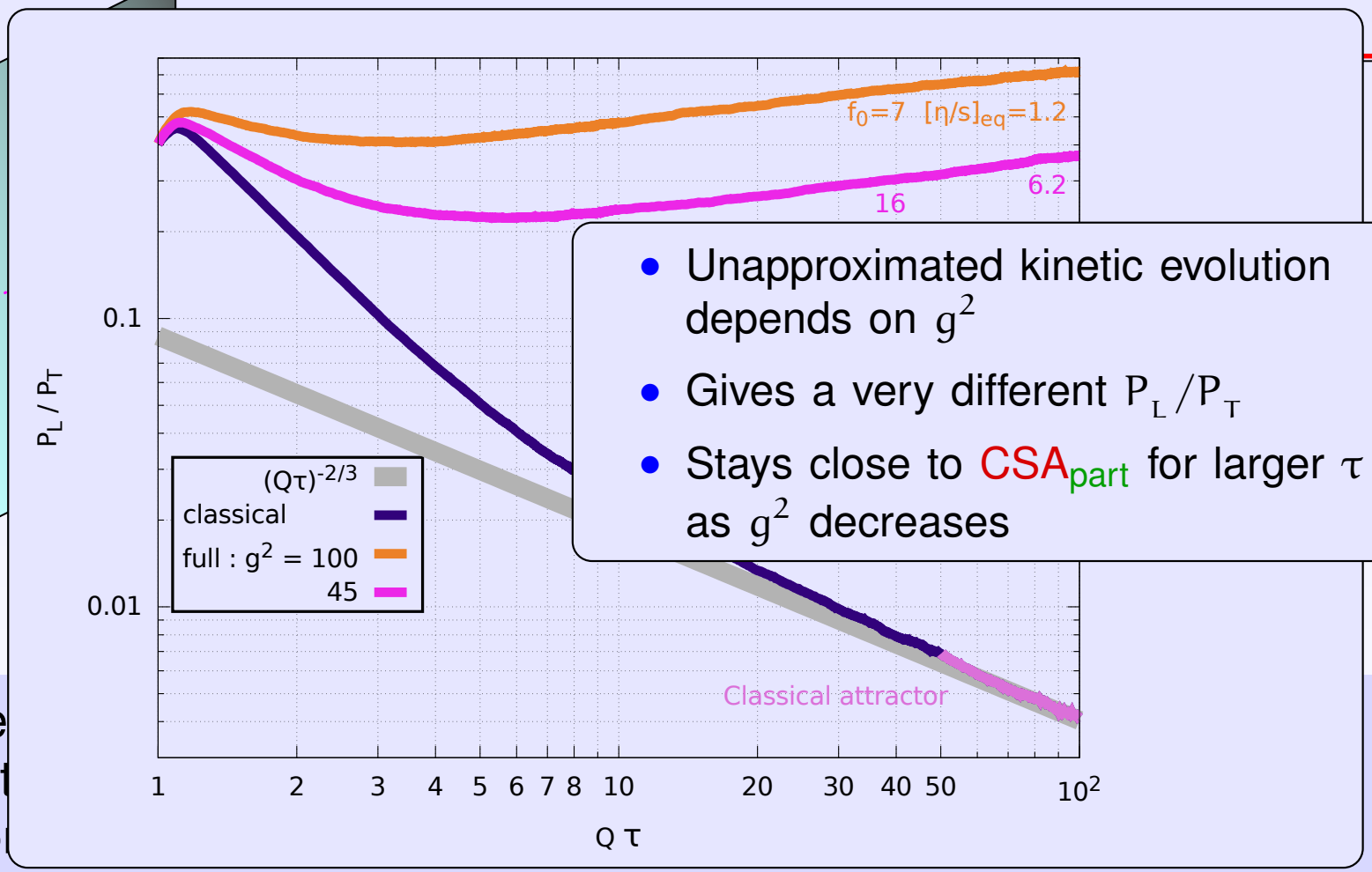

- Unapproximated kinetic evolution depends on g^2
- Gives a very different P_L/P_T
- Stays close to CSA_{part} for larger τ as g^2 decreases

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CSA_{part} : misses large angle scatterings in anisotropic systems

[Epelbaum, FG, Jeon, Moore, Wu (2015)]

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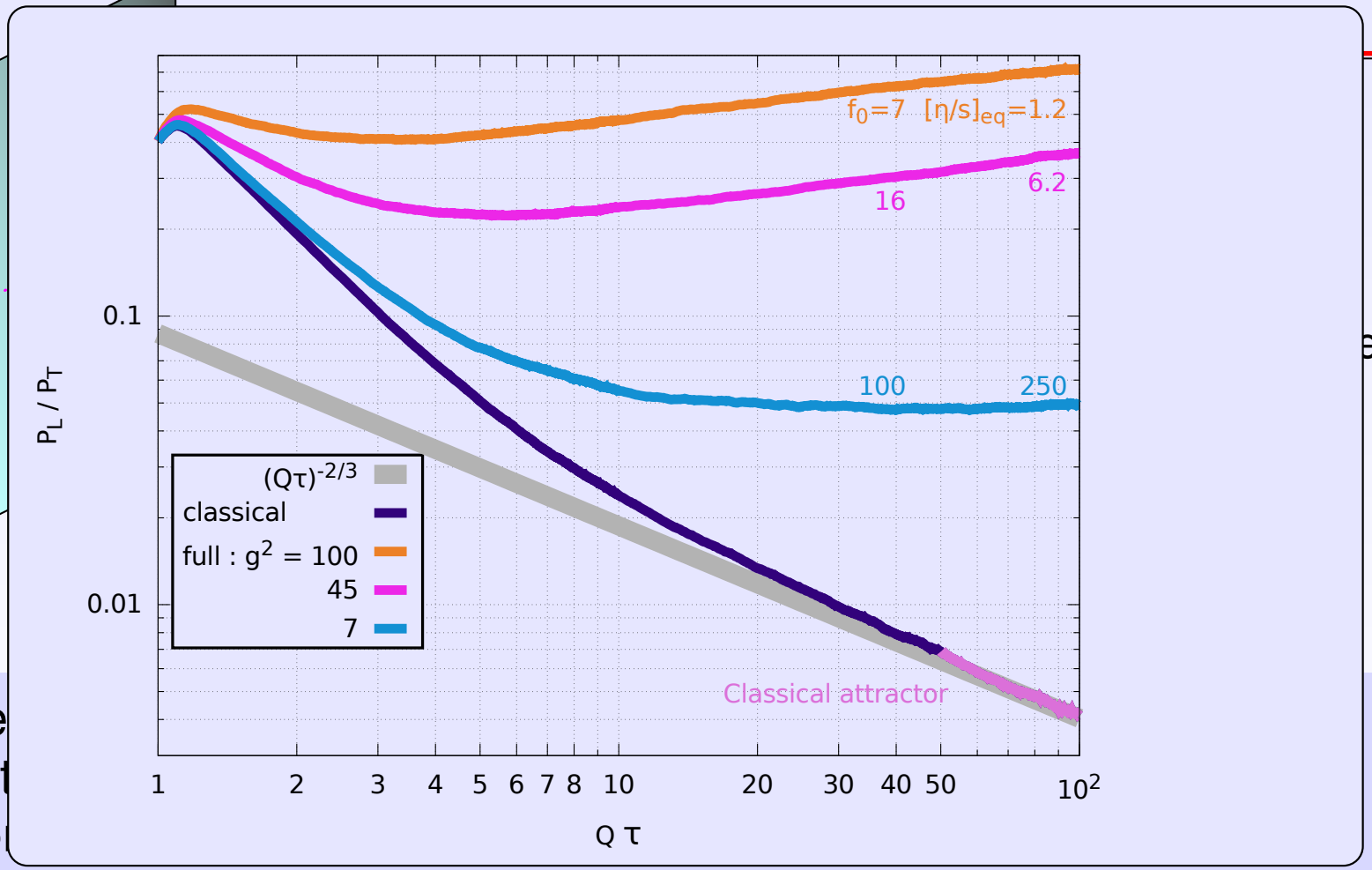

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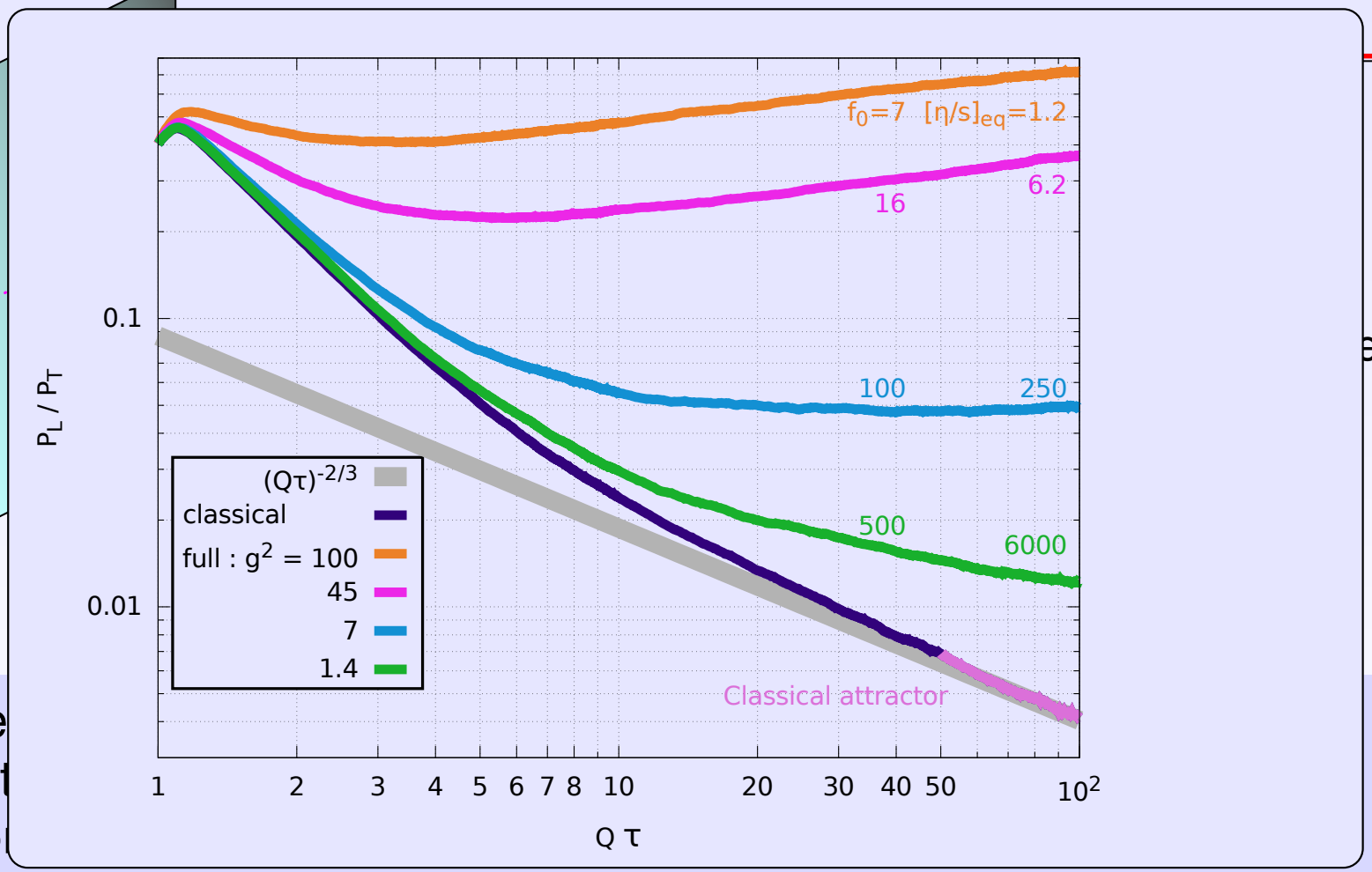

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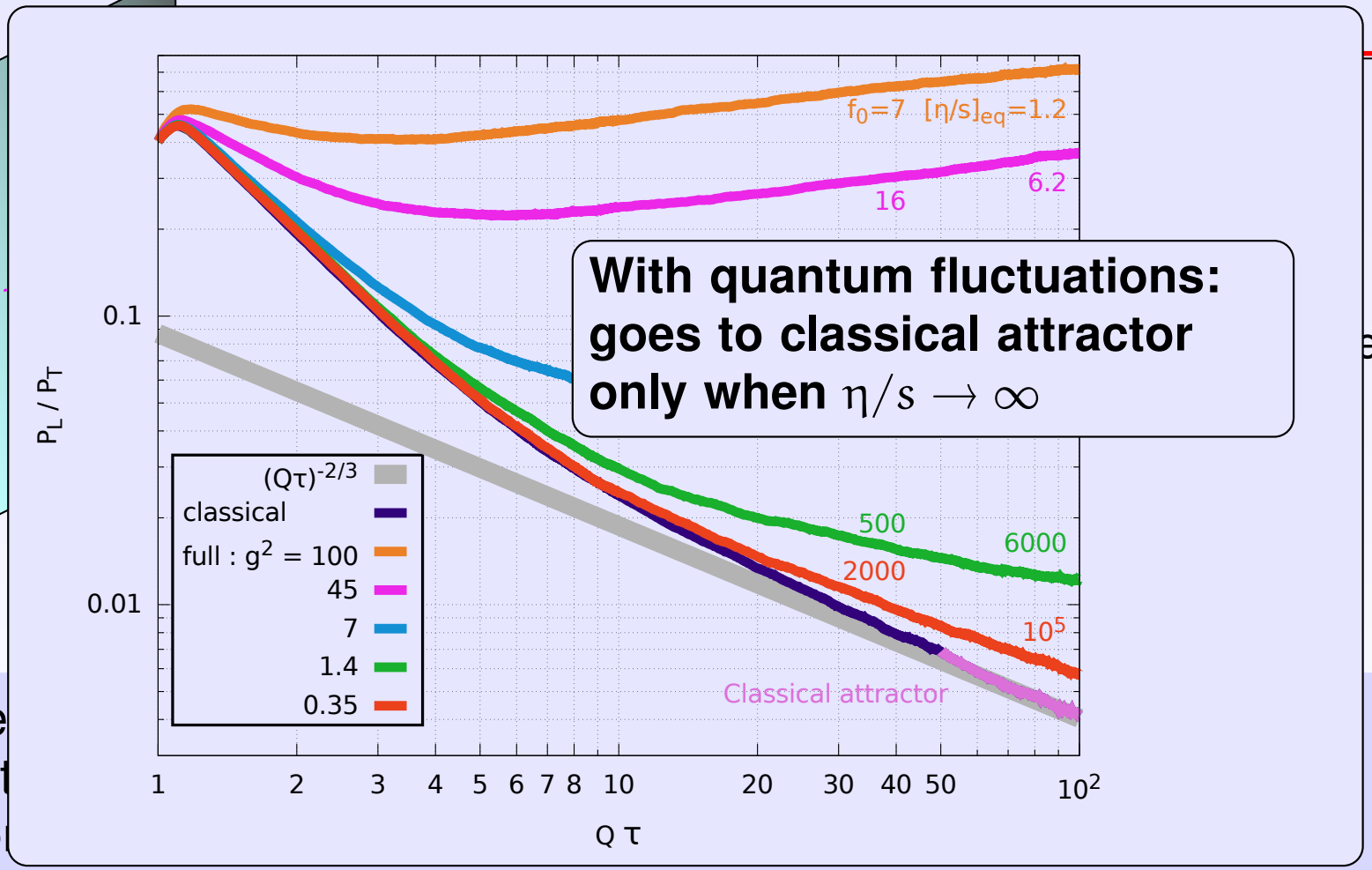

With quantum fluctuations: goes to classical attractor only when $\eta/s \to \infty$

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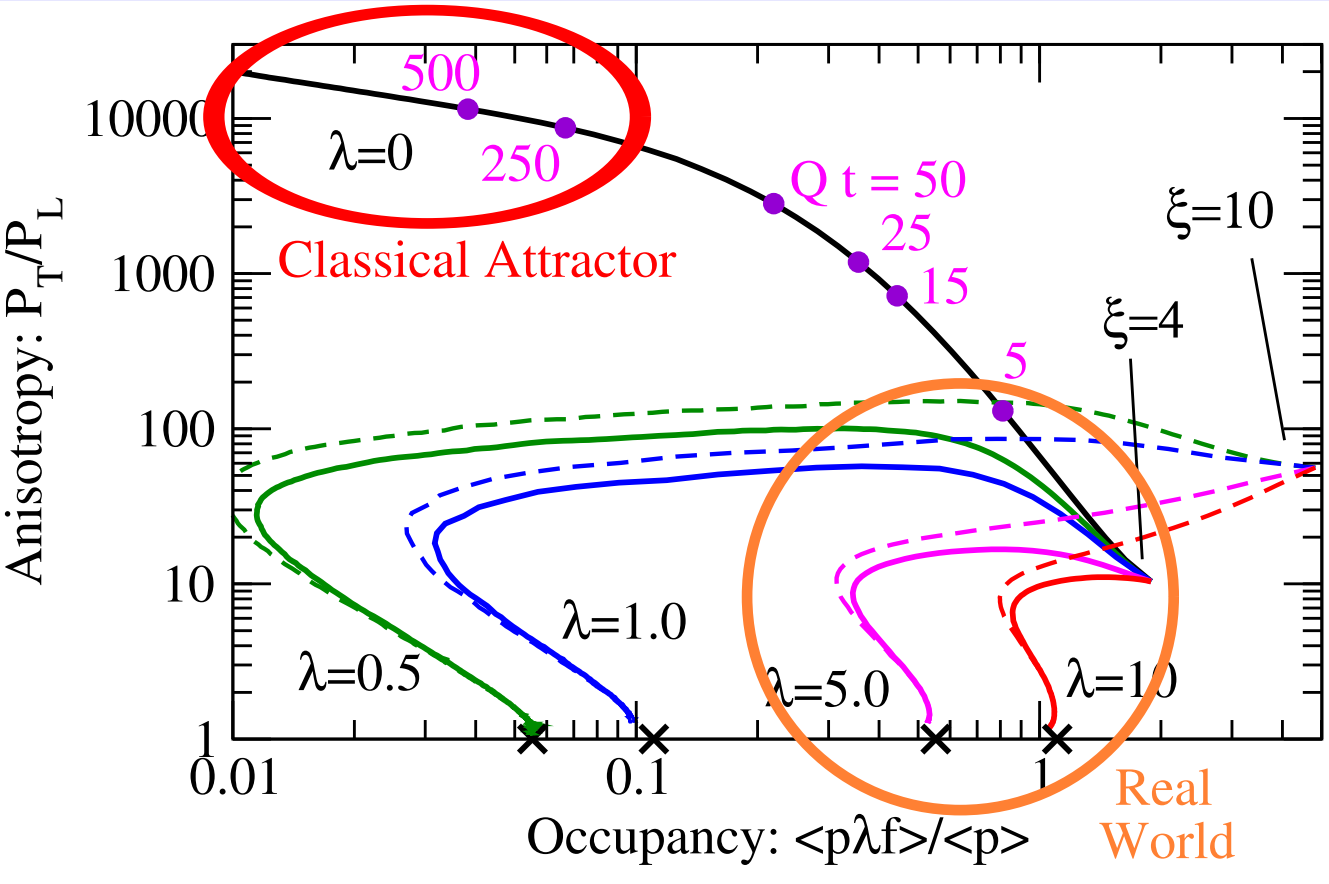
Kinetic theory results for gluons [Kurkela, Zhu (2015)]



- At $g^2 N_c = 1/2$, the CSA$_{\text{part}}$ approximation breaks down before $Q\tau = 5$, while naive estimates suggest that it is valid up to $Q\tau \approx \alpha_s^{-3/2} \approx 350$

Summary

Summary

- LO : no pressure isotropization, NLO : instabilities
- Resummation beyond NLO : Classical statistical approximation
- Two implementations... and two different results :
  - $\mathrm{CSA}_{\mathrm{vac}}$: vacuum-like CGC initial conditions
    roughly constant P_L/P_T,
    non-renormalizable approximation, very sensitive to UV cutoff
  - $\mathrm{CSA}_{\mathrm{part}}$: particle-like initial conditions
    universal classical attractor, P_L/P_T decreases forever,
    underestimates large angle scatterings,
    breaks long before reaching the attractor even for quite large η/s
- In the present situation, classical field simulations need to be corroborated and validated by other approaches
- Highly needed : ways to overcome the problems of the classical statistical approximation (Kinetic theory, 2-PI,...)