Quark-Gluon Plasma and Heavy Ion Collisions

II – QCD at finite temperature

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General outline

Field theory at finite T
Length scales in the QGP
Collective phenomena
Thermal photon rate

- I : Introduction to Heavy Ion Collisions
- II : QCD at finite temperature
- III : Hydrodynamical behavior
- IV : Kinetic theory

Lecture II

Field theory at finite T
Length scales in the QGP
Collective phenomena
Thermal photon rate

- Field Theory at finite temperature
- Length scales in the QGP
- Collective phenomena
- Thermal photon rate

Field theory at finite T
- QFT at zero T
- Nonzero T
- Imaginary time formalism

Length scales in the QGP

Collective phenomena

Thermal photon rate

Field theory at finite T

Quantum field theory at T=0

Field theory at finite T
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Length scales in the QGP

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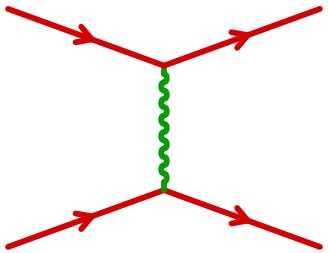
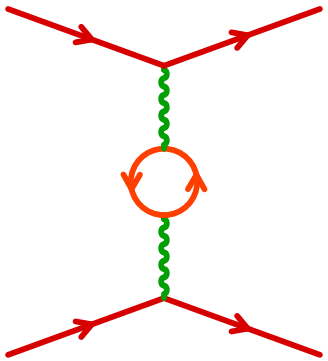
Thermal photon rate

- In order to study collision processes involving a small number of particles, one uses Quantum Field Theory at zero temperature
- It can be used to calculate scattering amplitudes, such as $\left\langle \vec{p}_1 \vec{p}_{2\mathrm{out}} \middle| \vec{k}_1 \vec{k}_{2\mathrm{in}} \right\rangle$



- Besides the incoming particles, the only other fields that can be involved in the scattering process are quantum fluctuations of the vacuum



Quantum field theory at T=0

Field theory at finite T
- QFT at zero T
- Nonzero T
- Imaginary time formalism

Length scales in the QGP

Collective phenomena

Thermal photon rate

- A QFT is specified by its Lagrangian, that describes the interactions among its elementary constituents

- When the interactions are weak, one can compute observables in perturbation theory, i.e. as a series in the coupling constants

- LSZ reduction formulas : scattering amplitudes are obtained from the Fourier transform of the time-ordered correlators

$$\left\langle \vec{p}_1 \vec{p}_{2\,\mathrm{out}} \middle| \vec{k}_1 \vec{k}_{2\,\mathrm{in}} \right\rangle = \int_{x_1,x_2,y_1,y_2} e^{i(k_1\cdot x_1 + k_2\cdot x_2 - p_1\cdot y_1 - p_2\cdot y_2)} \times \Box_{x_1}\Box_{x_2}\Box_{y_1}\Box_{y_2} \underbrace{\left\langle 0_{\mathrm{out}} \middle| \mathrm{T}\,\phi(x_1)\phi(x_2)\phi(y_1)\phi(y_2) \middle| 0_{\mathrm{in}} \right\rangle}_{\text{can be calculated perturbatively}}$$

Note : $\mathrm{T} =$ time ordering

Generalization to T>0

Field theory at finite T
- QFT at zero T
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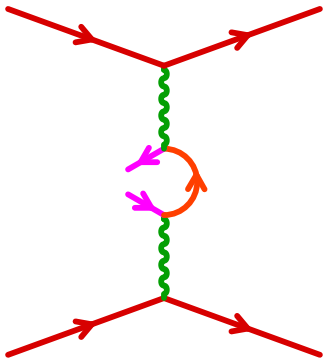
Length scales in the QGP

Collective phenomena

Thermal photon rate

- Contrary to $T = 0$, particles from the thermal environment can participate in reactions :



- This phenomenon gives their temperature dependence to correlators
- The time-ordered correlators are now defined as

$$G(x_1, \cdots, x_n) \equiv \frac{\mathrm{Tr}\left(e^{-\beta H}\, \mathrm{T}\, \phi(x_1) \cdots \phi(x_n)\right)}{\mathrm{Tr}\left(e^{-\beta H}\right)}$$

(with $\beta \equiv 1/T$)

T=0 limit

Field theory at finite T
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Thermal photon rate

- The thermal correlators can be rewritten in terms of eigenstates of the Hamiltonian :

$$G(x_1, \cdots, x_n) = \frac{1}{\mathrm{Tr}\,(e^{-\beta H})} \sum_{\text{states } n} e^{-\beta E_n} \left\langle n \middle| \mathrm{T}\, \phi(x_1) \cdots \phi(x_n) \middle| n \right\rangle$$

- When $T \to 0$ (i.e. $\beta \to +\infty$), only the vacuum state $\left|0\right\rangle$ survives since it has the lowest energy. Thus

$$\lim_{T \to 0} G(x_1, \cdots, x_n) = \left\langle 0 \middle| \mathrm{T}\, \phi(x_1) \cdots \phi(x_n) \middle| 0 \right\rangle$$

- Therefore, our definition of the thermal correlators is a natural extension of the definition used at zero temperature

Thermodynamical quantities

Field theory at finite T
- QFT at zero T
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Length scales in the QGP

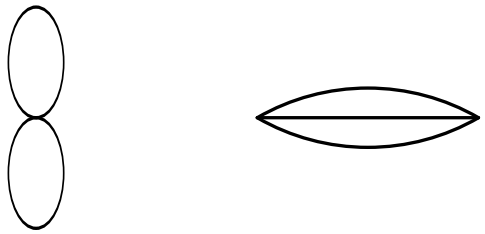
Collective phenomena

Thermal photon rate

- The sum of all the vacuum diagrams provides the partition function

$$Z = \mathrm{Tr}\left(e^{-\beta H}\right)$$

- Vacuum diagrams are diagrams without any external legs



- From Z, one can obtain other thermodynamical quantities :

$$E = -\frac{\partial Z}{\partial \beta}$$

$$S = \beta E + \ln(Z)$$

$$F = E - TS = -\frac{1}{\beta}\ln(Z)$$

Imaginary time frequencies

Field theory at finite T
- QFT at zero T
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Length scales in the QGP

Collective phenomena

Thermal photon rate

- The propagator – and more generally the integrand for any diagram – is β-periodic in the imaginary time $\tau = -ix^0$

- Therefore, one can go to Fourier space by decomposing the time dependence in Fourier series and by doing an ordinary Fourier transform in space :

$$G^0(\tau_x, \vec{\boldsymbol{x}}, \tau_y, \vec{\boldsymbol{y}}) = T \sum_{n=-\infty}^{+\infty} \int \frac{d^3\vec{\boldsymbol{p}}}{(2\pi)^3}\, e^{i\omega_n(\tau_x-\tau_y)} e^{-i\vec{\boldsymbol{p}}\cdot(\vec{\boldsymbol{x}}-\vec{\boldsymbol{y}})}\, G^0(\omega_n, \vec{\boldsymbol{p}})$$

with $\omega_n \equiv 2\pi n T$. Note : for fermions, $\omega_n = 2\pi(n+\frac{1}{2})T$

- Exercise : an explicit calculation gives :

$$G^0(\omega_n, \vec{\boldsymbol{p}}) = \frac{1}{\omega_n^2 + \vec{\boldsymbol{p}}^2 + m^2}$$

Imaginary time formalism

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Length scales in the QGP

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Thermal photon rate

- Feynman rules :
  - Propagators : $G^0(\omega_n, \vec{\boldsymbol{p}}) = 1/(\omega_n^2 + \vec{\boldsymbol{p}}^2 + m^2)$
  - Vertices : g + conservation of ω_n and $\vec{\boldsymbol{p}}$
  - Loops : $T \sum_n \int \frac{d^3\vec{\boldsymbol{p}}}{(2\pi)^3}$

- Examples (written here in the massless case) :

$$= \lambda T^2 \sum_{m,n} \int \frac{d^3\vec{\boldsymbol{p}}}{(2\pi)^3} \frac{d^3\vec{\boldsymbol{q}}}{(2\pi)^3} \frac{1}{(\omega_m^2 + \vec{\boldsymbol{p}}^2)(\omega_n^2 + \vec{\boldsymbol{q}}^2)}$$

$$= g^2 T^2 \sum_{m,n} \int \frac{d^3\vec{\boldsymbol{p}}}{(2\pi)^3} \frac{d^3\vec{\boldsymbol{q}}}{(2\pi)^3} \frac{1}{(\omega_m^2 + \vec{\boldsymbol{p}}^2)(\omega_n^2 + \vec{\boldsymbol{q}}^2)(\omega_{m+n}^2 + (\vec{\boldsymbol{p}} + \vec{\boldsymbol{q}})^2)}$$

Imaginary time formalism

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Length scales in the QGP

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Thermal photon rate

- The calculation of the discrete sums can be a bit tedious...
- Replace each propagator by

$$G^0(\omega_n, \vec{\boldsymbol{p}}) = \frac{1}{2E_{\boldsymbol{p}}} \int_0^\beta d\tau \; e^{-i\omega_n \tau} \left[(1+n_B(E_{\boldsymbol{p}})) \, e^{-E_{\boldsymbol{p}}\tau} + n_B(E_{\boldsymbol{p}}) \, e^{E_{\boldsymbol{p}}\tau} \right]$$

- One should combine this trick with the formula

$$\sum_n e^{i\omega_n \tau} = \beta \sum_n \delta(\tau - n\beta)$$

which turns all the time dependence into combinations of delta functions. Then, all the time integrations are trivial

Example

Field theory at finite T
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Length scales in the QGP

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Thermal photon rate

- Exercise. Tadpole in a $\lambda\phi^4$ theory :

$$
\begin{aligned}
&= \frac{\lambda T}{2}\sum_n \int \frac{d^3\vec{\boldsymbol{p}}}{(2\pi)^3}\frac{1}{\omega_n^2+\vec{\boldsymbol{p}}^2} \\
&= \frac{\lambda T}{2}\sum_n \int \frac{d^3\vec{\boldsymbol{p}}}{(2\pi)^3}\frac{1}{2E_{\boldsymbol{p}}}\int_0^\beta d\tau\; e^{-i\omega_n\tau}\left[(1+n_B(E_{\boldsymbol{p}}))e^{-E_{\boldsymbol{p}}\tau}+n_B(E_{\boldsymbol{p}})e^{E_{\boldsymbol{p}}\tau}\right] \\
&= \frac{\lambda}{2}\int \frac{d^3\vec{\boldsymbol{p}}}{(2\pi)^3 2E_{\boldsymbol{p}}}\int_0^\beta d\tau \sum_n \delta(\tau-n\beta)\left[(1+n_B(E_{\boldsymbol{p}}))e^{-E_{\boldsymbol{p}}\tau}+n_B(E_{\boldsymbol{p}})e^{E_{\boldsymbol{p}}\tau}\right] \\
&= \frac{\lambda}{2}\int \frac{d^3\vec{\boldsymbol{p}}}{(2\pi)^3 2E_{\boldsymbol{p}}}\left[1+2n_B(E_{\boldsymbol{p}})\right]
\end{aligned}
$$

(the remaining integral is “elementary”)

- Note : in the last formula, the 1 gives the usual ultraviolet divergence, and the n_B gives a finite contribution that vanishes if $T\to 0$ $\triangleright$ this term is a medium effect

Field theory at finite T

Length scales in the QGP
- Degrees of freedom
- Length scales

Collective phenomena

Thermal photon rate

Length scales in the QGP

Degrees of freedom

- Quarks : $2\ (\mathrm{spin}) \times 3\ (\mathrm{color}) = 6$ (per flavor)

$$\frac{dN_{\mathrm{q}}}{d^3\vec{\boldsymbol{x}}d^3\vec{\boldsymbol{k}}} = \frac{1}{e^{\omega/T}+1}$$ (Fermi-Dirac)

- Gluons : $2\ (\mathrm{spin}) \times 8\ (\mathrm{color}) = 16$

$$\frac{dN_{\mathrm{g}}}{d^3\vec{\boldsymbol{x}}d^3\vec{\boldsymbol{k}}} = \frac{1}{e^{\omega/T}-1}$$ (Bose-Einstein)

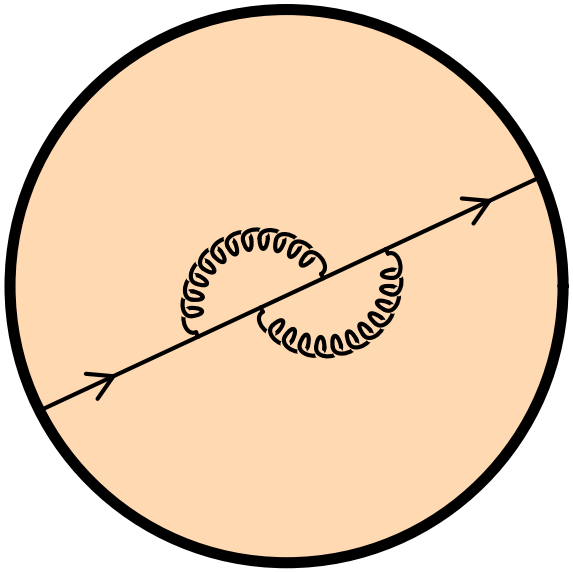
- Average energy per particle : $\langle\omega\rangle \sim T$
- Particle density : $\rho \sim T^3$
- Average distance between particles : $\ell \sim 1/T$

Length scales

- $1/T$: wavelength of particles in the plasma
- $1/gT$: typical distance for collective phenomena
  - Thermal masses of quasi-particles
  - Screening phenomena
  - Damping of waves
- $1/g^2T$: distance between two small angle scatterings
  - Color transport
  - Photon emission
- $1/g^4T$: distance between two large angle scatterings
  - Momentum, electric charge transport
    - ▷ characteristic scale of hydrodynamic modes
- In the weak coupling limit ($g \ll 1$), there is a clear hierarchy between these scales
- Distinct effective theories according to the characteristic scale of the problem under study

Vacuum fluctuations

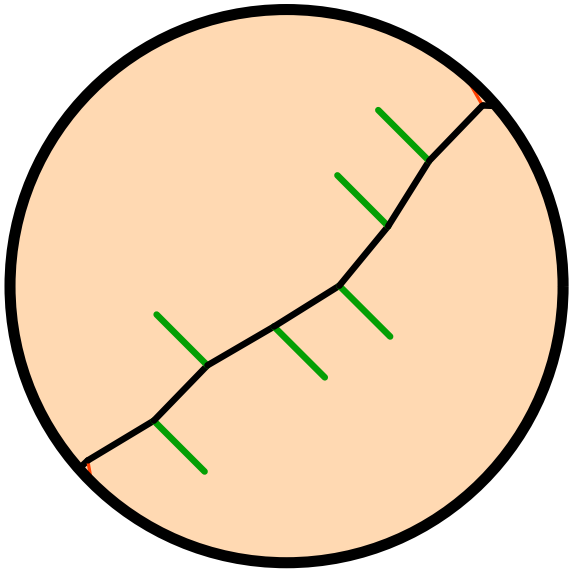
Field theory at finite T
Length scales in the QGP
● Degrees of freedom
● Length scales
Collective phenomena
Thermal photon rate



- At distances scales $\ell \lesssim 1/T$, medium effects are irrelevant
- At such scales the dynamics is simply described by the usual QCD in the vacuum

Thermal fluctuations

Field theory at finite T
Length scales in the QGP
● Degrees of freedom
● Length scales
Collective phenomena
Thermal photon rate



- Distance scales $1/T \lesssim \ell \lesssim 1/gT$ control the bulk thermodynamic properties. The system can be studied by QCD at finite temperature
- The leading thermal effects can be treated by an effective theory that encompasses the main collective effects, and that has the form of a collision-less Vlasov equation

Small angle scatterings

Field theory at finite T
Length scales in the QGP
● Degrees of freedom
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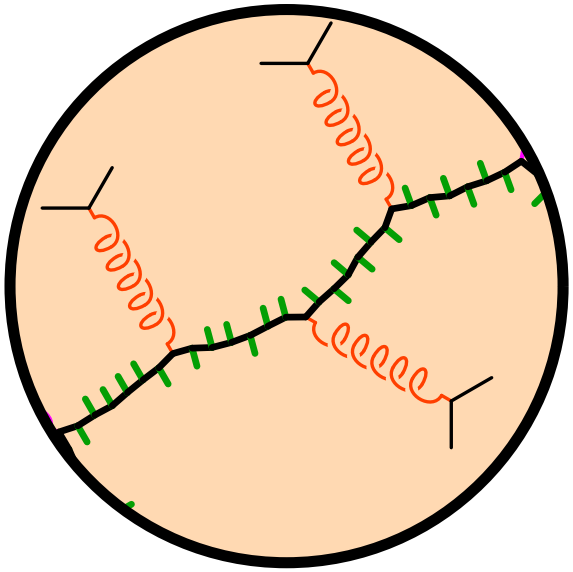
Thermal photon rate

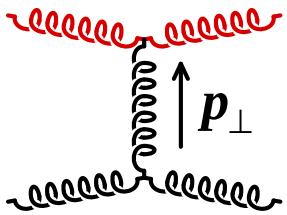


- When it is necessary to follow a plasma particle over distances $1/g^2T \lesssim \ell$, we must take into account soft (small angle) collisions with other particles of the plasma
- This can be done simply by adding a collision term to the previous Vlasov equation

Collision rate

- Collisional width :

$$\Gamma_{\rm coll} = \left| \quad \right|^2 \sim g^4\, T^3 \int_{m_{\rm debye}} \frac{d^2\vec{\boldsymbol{p}}_\perp}{p_\perp^4} \sim g^2 T$$



- $\lambda \equiv 1/\Gamma_{\rm coll}$ is the mean free path between two small angle scatterings ($\theta \sim g$)

- Note : the mean free path between two large angle scatterings ($\theta \sim 1$) is $\sim 1/g^4 T$

Large angle scatterings

Field theory at finite T
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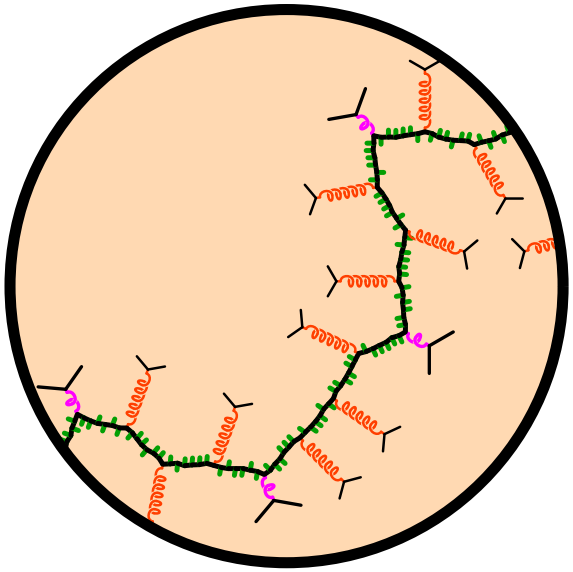

- Over distance scales $\ell \sim 1/g^4 T$, one must take into account the large angle collisions, that change significantly the direction of motion of the particle (this is necessary e.g. for calculating transport coefficients)
- The most efficient way to describe the system over these scales is via a Boltzmann equation for color/spin averaged particle distributions

Hydrodynamical regime

Field theory at finite T
Length scales in the QGP
● Degrees of freedom
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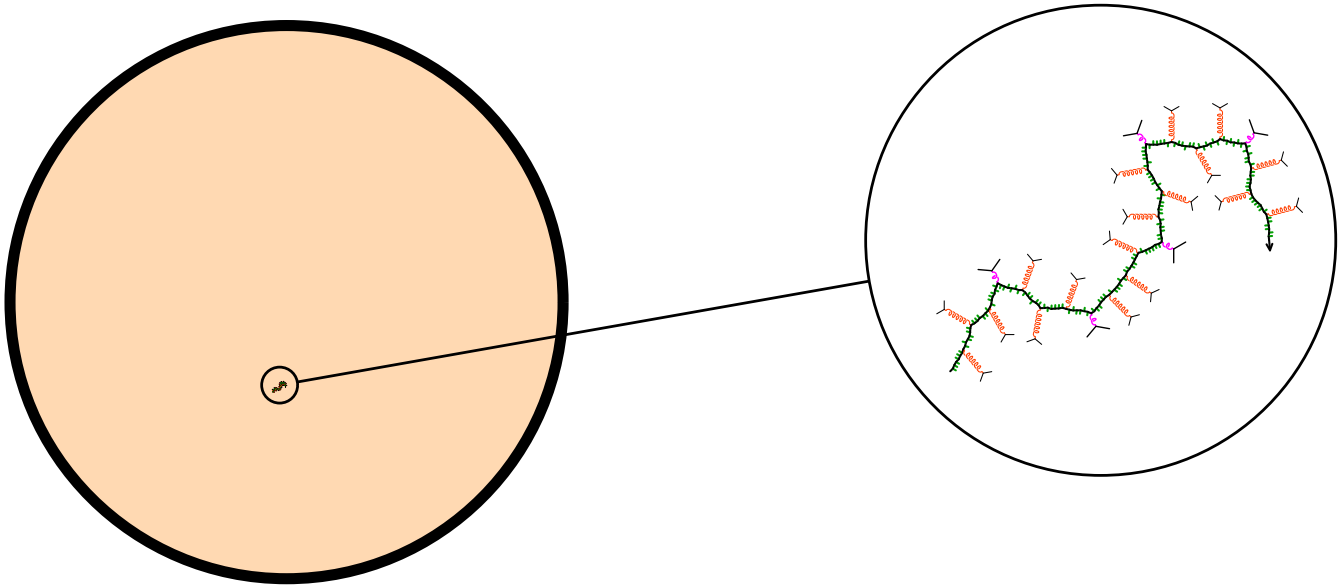
Thermal photon rate



- The hydrodynamical regime is reached for length scales that are much larger than the mean free path : $1/g^4T \ll \ell$
- In order to describe the system at such scales, one needs :
  - Hydrodynamical equations (Euler, Navier-Stokes)
  - Conservation equations for the various currents
  - Equation of state, viscosity

Field theory at finite T

Length scales in the QGP

Collective phenomena
- Dressed propagator
- Quasi-particles
- Debye screening

Thermal photon rate

Collective phenomena in the QGP

Collective phenomena

Field theory at finite T
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Thermal photon rate

- Phenomena involving many elementary constituents
- Long wavelength compared to the typical distance between constituents
- Small frequency or energy
- Major collective phenomena :
  - Quasi-particles
  - Debye screening
  - Landau damping
  - Collisional width

Dressed propagator

Field theory at finite T
Length scales in the QGP
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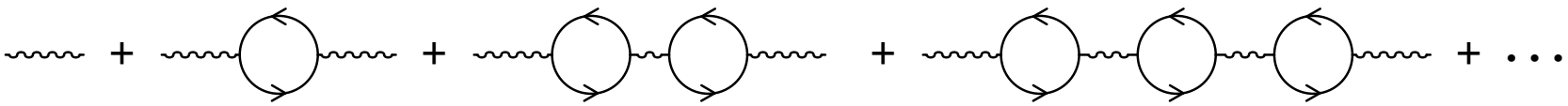
Thermal photon rate

- In order to see how the medium affects the propagation of excitations, one must compute the photon (gluon) polarization tensor $\Pi^{\mu\nu}(x,y) \equiv \left\langle J^\mu(x) J^\nu(y) \right\rangle$

- The photon (or gluon for QCD) self-energy can be resummed on the propagator. Diagrammatically, this amounts to summing :



- The properties of the medium can be read off the analytic properties of this resummed propagator (cuts, poles, ...)

Quasi-particles

Field theory at finite T
Length scales in the QGP
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Thermal photon rate

- Quasi-particles correspond to poles in the propagator. Their dispersion relation is the function $p_0 = \omega(\vec{\boldsymbol{p}})$ that defines the location of the pole

- The inverse of the imaginary part of p_0 is the lifetime of the quasi-particles (If $\mathrm{Im}(p_0) = 0$, they are stable). In order to be able to talk about quasi-particles, one must have $\mathrm{Im}(p_0) \ll \mathrm{Re}(p_0)$

Quasi-particles

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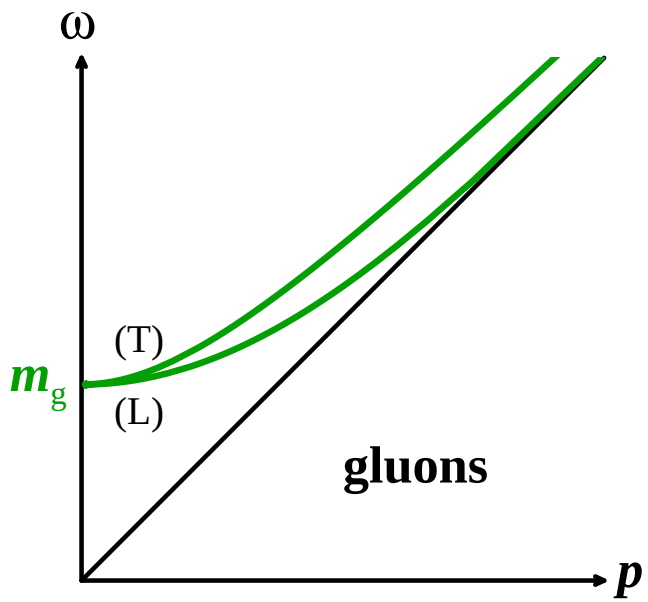
Thermal photon rate

- Dispersion curves of particles in the plasma :



- Thermal masses due to interactions with the other particles in the plasma :

$$m_q \sim m_g \sim gT$$

- At this order, the quasi-particles are stable

Debye screening

Field theory at finite T
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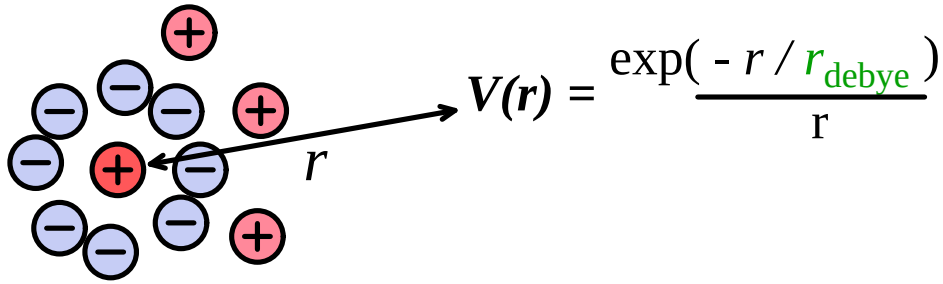
Thermal photon rate

- A test charge polarizes the particles of the plasma in its vicinity, in order to screen its charge :



- The Coulomb potential of the test charge decreases exponentially at large distance. The effective interaction range is :

$$\ell \sim 1/m_{\rm debye} \sim 1/gT$$

- Note : static magnetic fields are not screened by this mechanism (they are screened over length-scales $\ell_{\rm mag} \sim 1/g^2T$)

Debye screening

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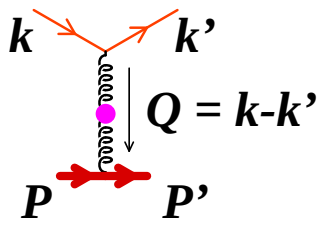
Thermal photon rate

- Place a quark of mass M at rest in the plasma, at $\vec{r} = 0$
- Scatter another quark off it. The scattering amplitude reads

$$\mathcal{M} = \left[g\bar{u}(\vec{k}')\gamma_\mu u(\vec{k})\right]\left[g\bar{u}(\vec{P}')\gamma_\nu u(\vec{P})\right] \sum_{\alpha=T,L} \frac{P_\alpha^{\mu\nu}(Q)}{Q^2 - \Pi_\alpha(Q)}$$



  - If $\vec{P} = 0$ (test charge at rest), only $\alpha = L$ contributes
  - From $(P+Q)^2 = M^2$, we get a $2\pi\delta(q_0)/2M$

- For the scattering off an external potential $\mathcal{A}^\mu$, the amplitude is $\mathcal{M} = \left[g\bar{u}(\vec{k}')\gamma_\mu u(\vec{k})\right] \mathcal{A}^\mu(Q)$
- Thus, the potential created by the test charge at rest is :

$$\mathcal{A}^\mu(Q) = g\frac{\bar{u}(\vec{P}')\gamma_\nu u(\vec{P})}{2M}\,\frac{2\pi\delta(q_0)P_L^{\mu\nu}(0,\vec{q})}{\vec{q}^2 + \Pi_L(0,\vec{q})} = \frac{2\pi g\delta^{\mu 0}\delta(q_0)}{\vec{q}^2 + \Pi_L(0,\vec{q})}$$

Debye screening

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Thermal photon rate

- By a Fourier transform, we obtain the Coulomb potential :

$$\mathcal{A}^0(t,\vec{\boldsymbol{r}}) = g \int \frac{d^3\vec{\boldsymbol{q}}}{(2\pi)^3} \frac{e^{i\vec{\boldsymbol{q}}\cdot\vec{\boldsymbol{r}}}}{\vec{\boldsymbol{q}}^2 + \Pi_L(0,\vec{\boldsymbol{q}})}$$

- If we are in the vacuum, $\Pi_L = 0$, and the Fourier transform gives the usual Coulomb law :

$$\mathcal{A}^0_{\text{vac}}(t,\vec{\boldsymbol{r}}) = g \int \frac{d^3\vec{\boldsymbol{q}}}{(2\pi)^3} \frac{e^{i\vec{\boldsymbol{q}}\cdot\vec{\boldsymbol{r}}}}{\vec{\boldsymbol{q}}^2} = \frac{g}{4\pi|\vec{\boldsymbol{r}}|}$$

- In a plasma, $\Pi_L(0,\vec{\boldsymbol{q}}) = \frac{g^2T^2}{3} \equiv m_D^2$. The Fourier transform can also be done exactly

$$\mathcal{A}^0(t,\vec{\boldsymbol{r}}) = g \int \frac{d^3\vec{\boldsymbol{q}}}{(2\pi)^3} \frac{e^{i\vec{\boldsymbol{q}}\cdot\vec{\boldsymbol{r}}}}{\vec{\boldsymbol{q}}^2 + m_D^2} = \frac{g}{4\pi|\vec{\boldsymbol{r}}|} e^{-m_D|\vec{\boldsymbol{r}}|}$$

▷ the potential is unmodified at $r \ll 1/m_D$, but exponentially suppressed at large distance

Field theory at finite T

Length scales in the QGP

Collective phenomena

Thermal photon rate
- How to compute a rate?
- 2-to-2 processes
- Bremsstrahlung
- LPM effect
- Thermal photon rates
- Initial temperature

Thermal photon rate

How to compute a rate?

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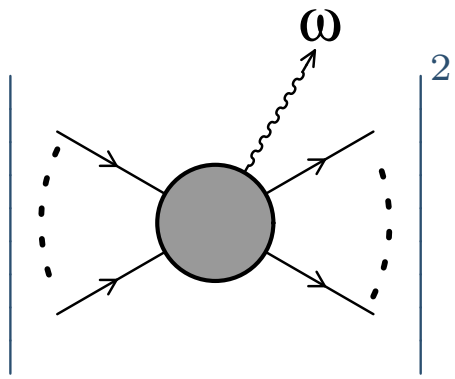
- Pedestrian approach:

$$\omega \frac{dN_\gamma}{dt dV d^3\vec{q}} \propto \int_{\substack{\text{unobserved}\\ \text{particles}}}$$



$$\times\, n(\omega_1)\cdots n(\omega_n)$$
$$\times\, (1 \pm n(\omega_1'))\cdots(1 \pm n(\omega_p'))$$

- From the photon polarization tensor:
  Weldon (1983) - Gale, Kapusta (1991)

$$\omega \frac{dN_\gamma}{dt dV d^3\vec{q}} \propto \frac{1}{e^{\omega/T}-1}\, \mathrm{Im}\,\Pi_{\mathrm{ret}}{}^{\mu}{}_{\mu}(\omega, \vec{q})$$

2-to-2 processes

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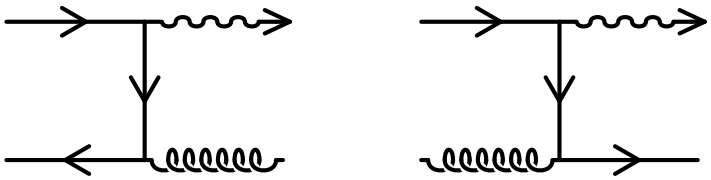
- $q\bar{q} \to g\gamma$ and $qg \to q\gamma$:



▷ For real photons ($Q^2 \equiv \omega^2 - \boldsymbol{q}^2 \to 0$), divergence if the quarks are massless:

$$\mathrm{Im}\,\Pi_{\mathrm{ret}}(\omega, \vec{\boldsymbol{q}}) \propto \alpha\alpha_s \ln(\omega T/Q^2)$$

- The thermal mass of the quarks cures the singularity:

$$\mathrm{Im}\,\Pi_{\mathrm{ret}}{}^{\mu}{}_{\mu}(\omega, \vec{\boldsymbol{q}}) = 4\pi\alpha\alpha_s T^2 \left[\ln\left(\frac{\omega T}{m_{\mathrm{q}}^2}\right) - 0.03\right]$$

Bremsstrahlung

Field theory at finite T
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Thermal photon rate
- How to compute a rate?
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- Thermal photon rates
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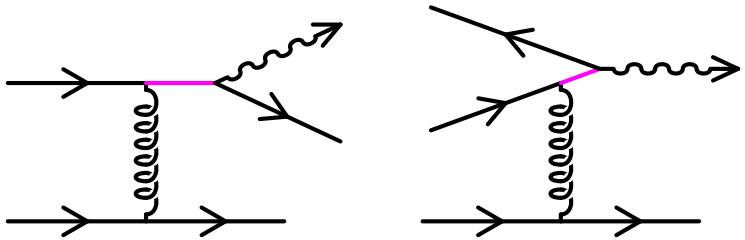
- Higher order, but enhanced by a collinear divergence:



▷ Singularity when a real photon is emitted collinearly to the quark: $\alpha_s^2 \frac{T^2}{m_{\rm q}^2} \sim \alpha_s$

- For 3 colors and 2 flavors, the bremsstrahlung contribution is:

$$\mathrm{Im}\,\Pi_{\rm ret}{}^{\mu}{}_{\mu}(\omega, \vec{q}) = \frac{32}{3\pi}\alpha\alpha_s \left[\pi^2 \frac{T^3}{\omega} + \omega T\right]$$

Landau Pomeranchuk Migdal effect

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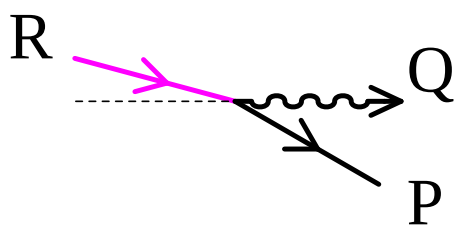
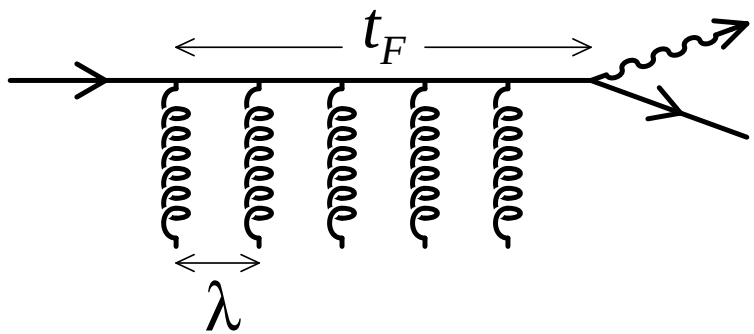
- Photon formation time



$$R \equiv P + Q$$
$$(r_0 = p_0 + \omega)$$
$$P^2 = m_{\rm q}^2$$

$$t_F^{-1} \sim r_0 - \sqrt{\vec{r}^2 + m_{\rm q}^2} \sim \frac{\omega}{2p_0 r_0}\left[\vec{p}_\perp^2 + m_{\rm q}^2\right]$$

- t_F should be compared to the mean free path of the quarks.
  LPM effect: multiple scatterings are important if $t_F \gtrsim \lambda$



Landau Pomeranchuk Migdal effect

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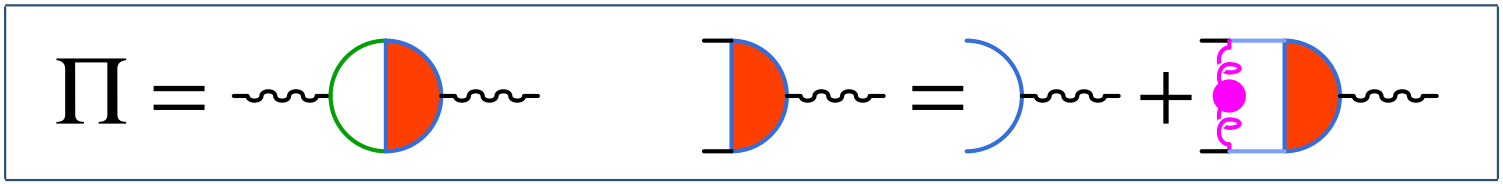
- Resummation of ladder diagrams:



▷ Dyson equation:

$$\mathrm{Im}\,\Pi_{\mathrm{ret}}{}^{\mu}_{\mu} = \alpha\ \mathrm{Im} \int dp_0\, d^2\vec{p}_\perp\, [\cdots]\ 2\vec{p}_\perp \cdot \vec{f}(\vec{p}_\perp)$$

▷ Bethe-Salpeter equation:

$$\frac{\vec{f}(\vec{p}_\perp)}{t_F} = 2\vec{p}_\perp + i\alpha_s T \int d^2\vec{l}_\perp\, \mathcal{C}(\vec{l}_\perp)[\vec{f}(\vec{p}_\perp) - \vec{f}(\vec{p}_\perp + \vec{l}_\perp)]$$

with

$$\mathcal{C}(\vec{l}_\perp) = \frac{1}{\vec{l}_\perp^{\,2}} - \frac{1}{\vec{l}_\perp^{\,2} + m^2_{\mathrm{debye}}}$$

Thermal photon rates

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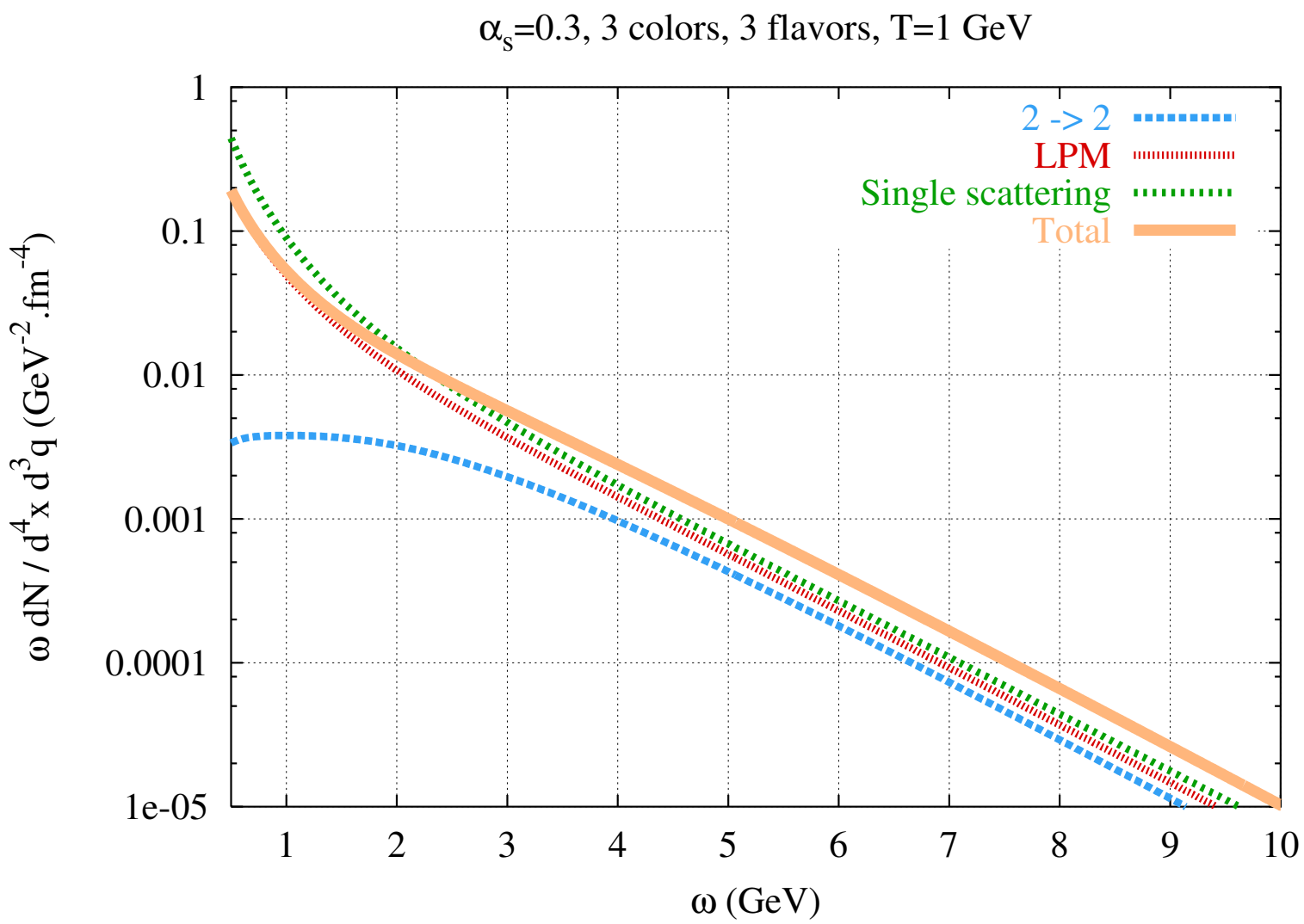
- Photon rate at $\mathcal{O}(\alpha\alpha_s)$



Initial temperature

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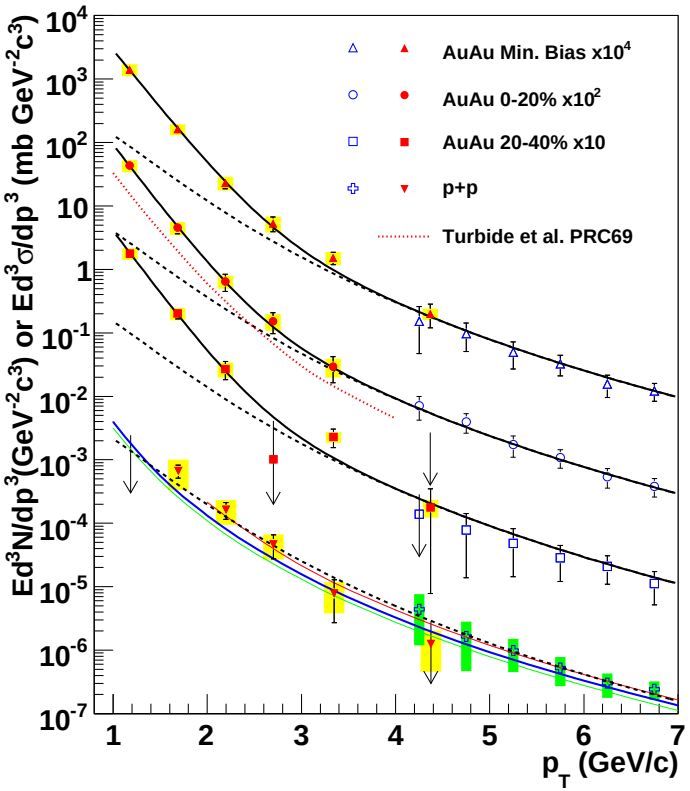
- RHIC direct photon spectrum:



- The photon excess (compared to the rescaled yield in proton-proton collisions) can be explained if the temperature is $T \approx 300 - 600$ MeV at a time $\tau \approx 0.6 - 0.15$ fm/c

Lecture III

Field theory at finite T
Length scales in the QGP
Collective phenomena
Thermal photon rate
Outline of lecture III

- Relativistic hydrodynamics
- Phenomenological results
- Viscous corrections

Lecture IV

Field theory at finite T
Length scales in the QGP
Collective phenomena
Thermal photon rate
Outline of lecture IV

- Collisionless transport
- Boltzmann equation
- Transport coefficients