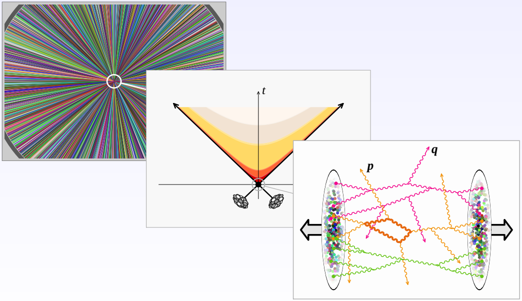
Heavy-ion collisions and QCD: the big picture

Quark Matter 2011, Annecy



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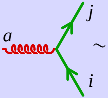
Outline

1. **QCD primer**
2. **Deconfinement**
3. **QCD in heavy ion collisions**
4. **Perturbative QCD**
5. **QCD at finite T, Lattice QCD**
6. **Color Glass Condensate**
7. **AdS/CFT**

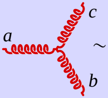
Quarks and gluons

Strong interactions : Quantum Chromo-Dynamics

- Matter : quarks ; Interaction carriers : gluons



$\sim \; g\,(t^a)_{ij}$



$\sim \; g\,(T^a)_{bc}$

- i, j : quark colors ; a, b, c : gluon colors
- $(t^a)_{ij}$: 3×3 SU(3) matrix ; $(T^a)_{bc}$: 8×8 SU(3) matrix

Lagrangian

$$\mathcal{L} = -\frac{1}{4}F^2 + \sum_f \overline{\psi}_f (i\not{D} - m_f)\psi_f$$

- Free parameters : quark masses m_f, scale $\Lambda_{\rm QCD}$

1 **QCD primer**
- Building blocks
- Asymptotic freedom
- Color confinement

2 **Deconfinement**

3 **QCD in heavy ion collisions**

4 **Perturbative QCD**

5 **QCD at finite T, Lattice QCD**

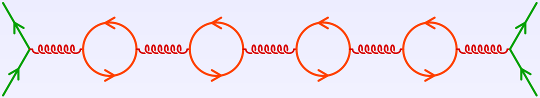
6 **Color Glass Condensate**

7 **AdS/CFT**

Asymptotic freedom

- Running coupling : $\alpha_s = g^2/4\pi$

$$\alpha_s(r) = \frac{2\pi N_c}{(11N_c - 2N_f)\log(1/r\Lambda_{QCD})}$$



- The effective charge seen at large distance is screened by fermionic fluctuations (as in QED)

Asymptotic freedom

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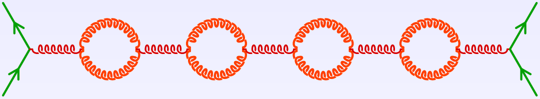
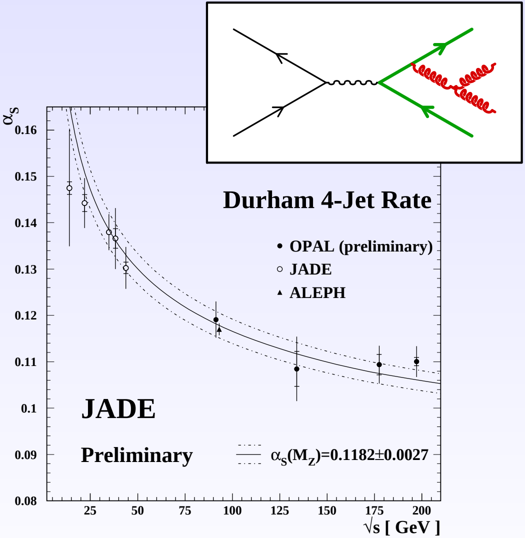


- The effective charge seen at large distance is screened by fermionic fluctuations (as in QED)
- But gluonic vacuum fluctuations produce an anti-screening (because of the non-abelian nature of their interactions)
- As long as $N_f < 11N_c/2 = 16.5$, the gluons win...

Asymptotic freedom



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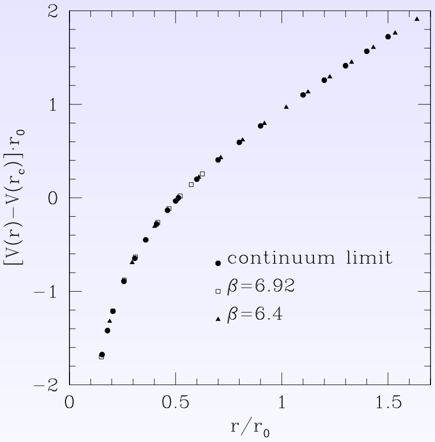
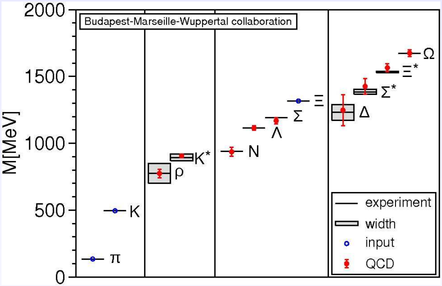
Color confinement



- The quark potential increases linearly with distance

Color confinement

- In nature, we do not see free quarks and gluons (the closest we have to actual quarks and gluons are jets)
- Instead, we see hadrons (quark-gluon bound states):



- The hadron spectrum is uniquely given by $\Lambda_{\rm QCD}, m_f$
- But this dependence is non-pertubative (it can now be obtained fairly accurately by lattice simulations)

1 QCD primer

2 Deconfinement

Debye screening

Deconfinement transition

Phase diagram

3 QCD in heavy ion collisions

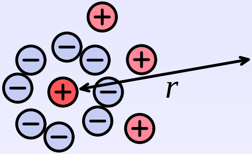
4 Perturbative QCD

5 QCD at finite T, Lattice QCD

6 Color Glass Condensate

7 AdS/CFT

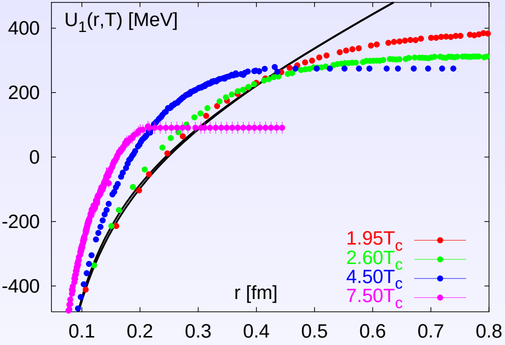
Debye screening



$$V(r) = \frac{\exp(-r/r_{\text{debye}})}{r}$$

- In a dense medium, color charges are screened by their neighbours
- The interaction potential decreases exponentially beyond the Debye radius r_{debye}
- Hadrons whose radius is larger than r_{debye} cannot bind

Debye screening



- In lattice calculations, one sees the $q\overline{q}$ potential flatten at long distance as T increases

1. QCD primer

2. **Deconfinement**
   - Debye screening
   - Deconfinement transition
   - Phase diagram

3. QCD in heavy ion collisions

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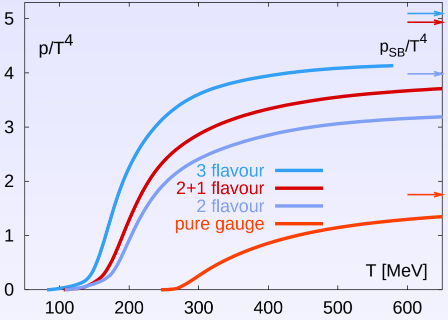
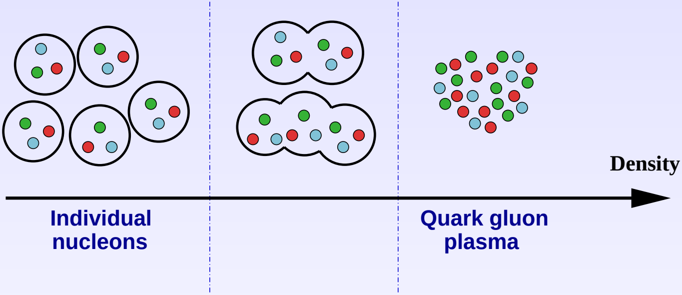
Deconfinement transition



- Rapid increase of the pressure :
  - at $T \sim 270$ MeV, with gluons only
  - at $T \sim 150$ to 180 MeV, with light quarks

▷ interpreted as the increase in the number of degrees of freedom due to the liberation of quarks and gluons

Deconfinement transition



- When the nucleon density increases, they merge, enabling quarks and gluons to hop freely from a nucleon to its neighbors
- This phenomenon extends to the whole volume when the phase transition ends
- Note: if the transition is first order, it goes through a mixed phase containing a mixture of nucleons and plasma

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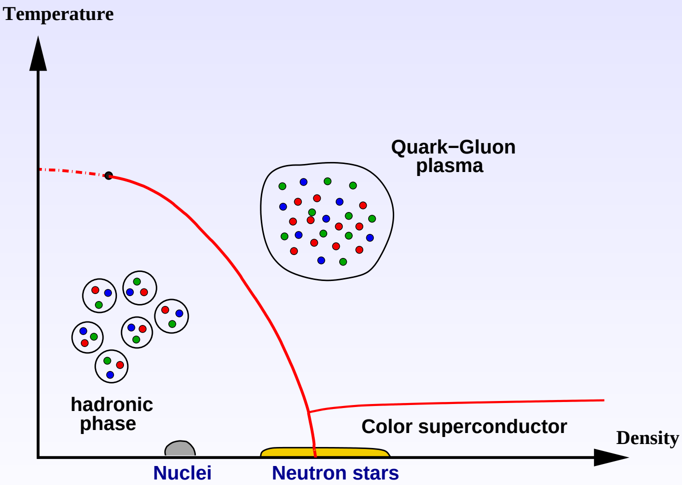
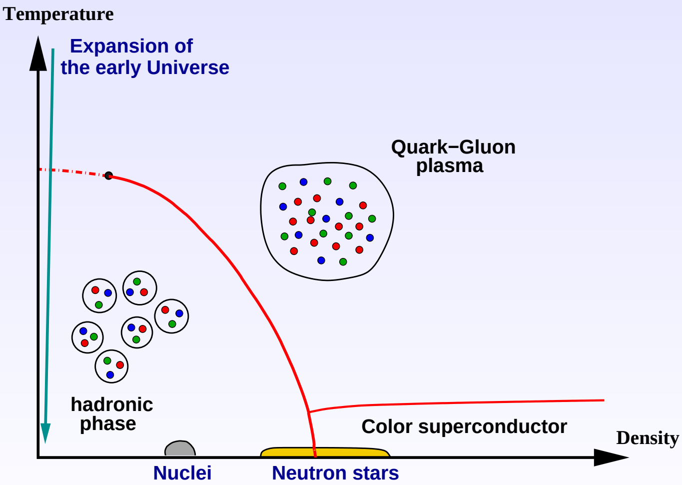
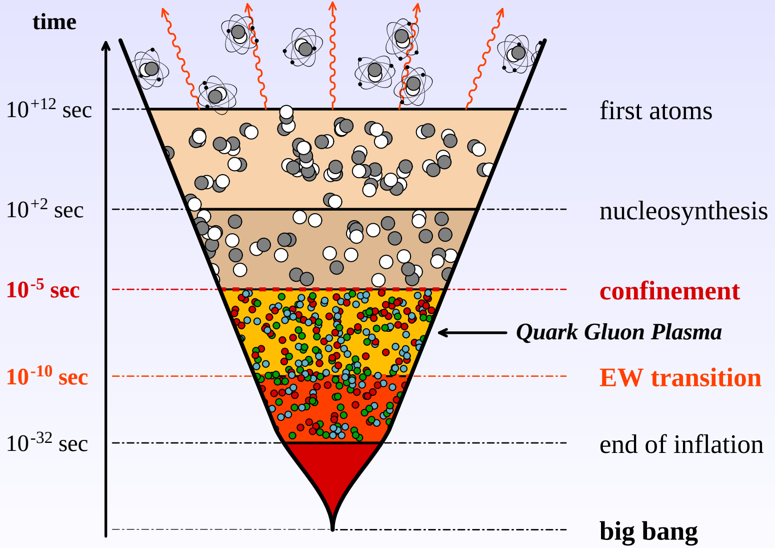
QCD phase diagram



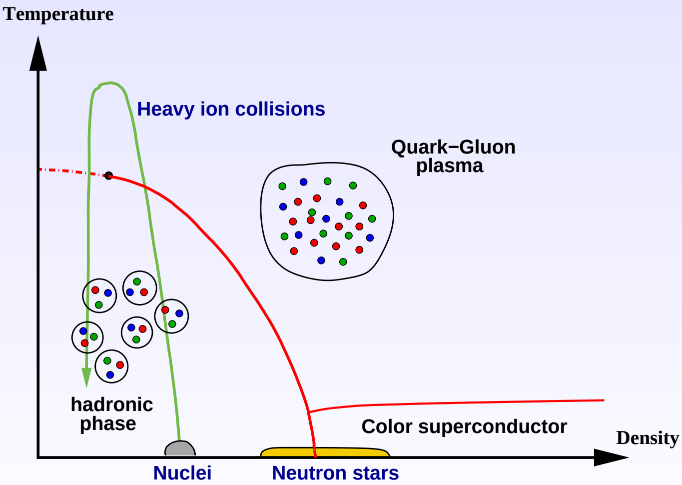
QGP in the early universe



QGP in the early universe



Heavy ion collisions



What would we like to learn?

- **i.** Establish the existence of a phase transition
- **ii.** Parameters of the transition: T_c, ϵ_c
- **iii.** Equation of state of nuclear matter
- **iv.** Transport properties of nuclear matter
- **v.** Do some hadrons survive in the QGP?
- **vi.** Dynamics of the collision, evolution at early time, formation of the QGP and thermalization

What we must get out of the way first...

- Unfortunately, heavy ion collisions also depend on a number of other trivial facts:

i. Lead nuclei are approximately spherical

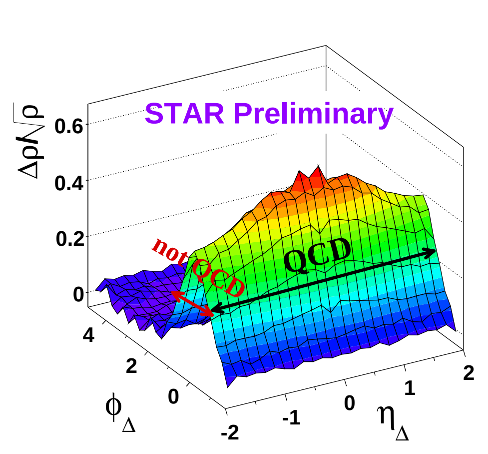
ii. Their diameter is about 12 fermis

iii. They contain $A \approx 200$ nucleons

iv. The positions of these nucleons fluctuate

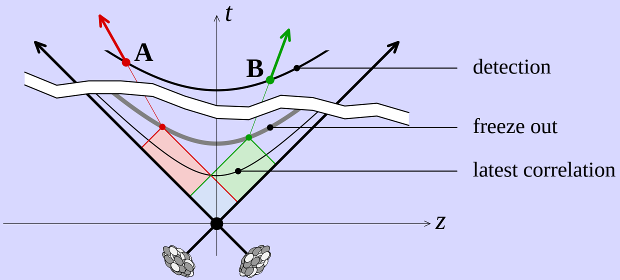
- These properties have all an incidence on observables
- None of them is interesting from the point of view of QCD
- We need ways to make observables independent of these trivial aspects of nuclear physics

Example: 2-hadron correlations (aka “the ridge”)



- Long range correlation in $\Delta\eta$ (rapidity)
- Narrow correlation in $\Delta\varphi$ (azimuthal angle)

Long range rapidity correlations



Long range rapidity correlations are created early

From causality, the latest time at which a correlation between two particles can be created is :

$$t_{\text{correlation}} \le t_{\text{freeze out}}\ e^{-\frac{1}{2}|y_A - y_B|}$$

With $t_{\text{freeze out}} = 10$ fm/c, $|y_A - y_B| = 6$: $\quad t_{\text{correlation}} \le 0.5$ fm/c

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Heavy Ion Collisions

Effective descriptions

Transport models

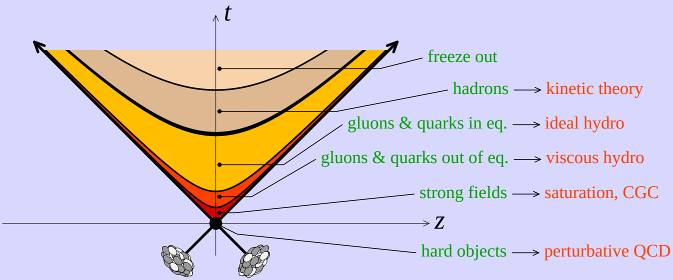
4 Perturbative QCD

5 QCD at finite T, Lattice QCD

6 Color Glass Condensate

7 AdS/CFT

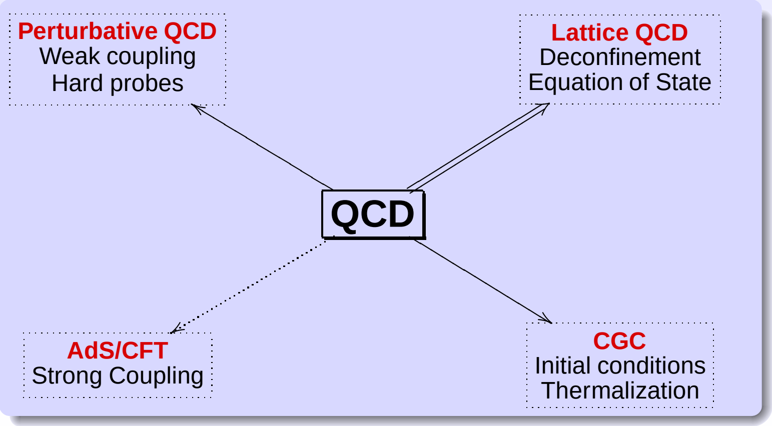
The multiple facets of QCD in HIC



- Except for the production of hard objects (jets, heavy quarks, direct photons) at the impact of the two nuclei, we have to deal with strong interactions in a non-perturbative regime
  NOTE: non-perturbative $\neq$ strongly coupled!!!
- One treats these situations with a range of effective descriptions (CGC, hydrodynamics, kinetic theory) that are more or less closely related to QCD, but always require some QCD input

The multiple facets of QCD in HIC

- The simple formulation of QCD is deceptive: Ab initio calculations are very difficult, and feasible only for a handful of questions
- In many instances, it is more efficient to use an effective theory in which inessential degrees of freedom have been integrated out



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The multiple facets of QCD in HIC

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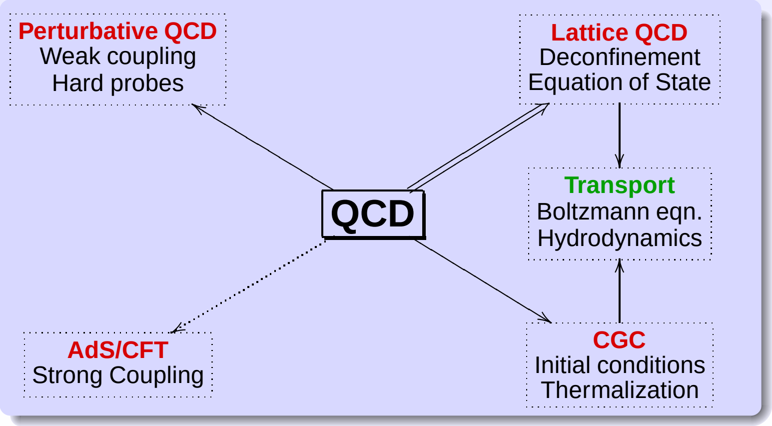

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Transport models

- In many cases, the description of the system can be done at a scale large enough for the microscopic details to become irrelevant:
  - Kinetic theory
  - Hydrodynamics
- To a large extent, the evolution of the system is driven by conservations laws (energy, momentum, baryon number...)
- The microscopic dynamics is relegated into a handful of quantities that enter in these mesoscopic descriptions

Kinetic theory

- The system is described by a particle distribution

$$f(t,\vec{\boldsymbol{x}},\vec{\boldsymbol{p}}) = \frac{dN}{d^3\vec{\boldsymbol{x}}d^3\vec{\boldsymbol{p}}}$$

  (in most cases, this distribution is spin and color averaged)

- The evolution of f is driven by the interactions between these particles

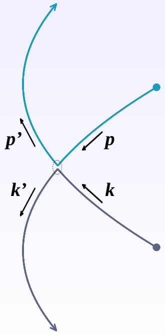
- The only QCD input is a set of cross-sections

Boltzmann equation

- The Boltzmann equation describes the evolution of a distribution of particles that undergo short range collisions

$$\left[\partial_t + \vec{\boldsymbol{v}}_{\boldsymbol{p}} \cdot \vec{\boldsymbol{\nabla}}_{\boldsymbol{x}}\right] f(t, \vec{\boldsymbol{x}}, \vec{\boldsymbol{p}}) = \underbrace{\mathcal{C}_{\boldsymbol{p}}[f]}_{\text{collisions}} \qquad \text{with } \vec{\boldsymbol{v}}_{\boldsymbol{p}} \equiv \frac{\vec{\boldsymbol{p}}}{E_{\boldsymbol{p}}}$$

- Elementary 2-body collision :



Boltzmann equation

- For $2 \to 2$ collisions, the collision term reads :

$$\mathcal{C}_{\boldsymbol{p}}[f] = \frac{1}{2E_{\boldsymbol{p}}} \int \frac{d^3\vec{\boldsymbol{p}}'}{(2\pi)^3 2E_{\boldsymbol{p}'}} \int \frac{d^3\vec{\boldsymbol{k}}}{(2\pi)^3 2E_{\boldsymbol{k}}} \int \frac{d^3\vec{\boldsymbol{k}}'}{(2\pi)^3 2E_{\boldsymbol{k}'}} \underbrace{(2\pi)^4\delta(p+k-p'-k')}_{E,\vec{\boldsymbol{p}}\ \text{conservation}}$$

$$\times \underbrace{\Big[f(\vec{\boldsymbol{p}}')f(\vec{\boldsymbol{k}}')(1+f(\vec{\boldsymbol{p}}))(1+f(\vec{\boldsymbol{k}})) - f(\vec{\boldsymbol{p}})f(\vec{\boldsymbol{k}})(1+f(\vec{\boldsymbol{k}}'))(1+f(\vec{\boldsymbol{p}}'))\Big]}_{\text{micro-reversibility, detailed balance}} \underbrace{\left|\mathcal{M}\right|^2}_{\text{QCD}}$$

▷ Most of the equation relies on conservation laws and general principles of statistical physics. Only the cross-section depends on QCD

Inputs

i. Cross-sections

ii. Initial condition $f(t_0, \vec{\boldsymbol{x}}, \vec{\boldsymbol{p}})$

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Hydrodynamics : limit of kinetic theory when $\ell_{\rm mfp} \longrightarrow 0$

Equations of hydrodynamics (conservation laws)

$$\partial_\mu T^{\mu\nu} = 0 \qquad , \qquad \partial_\mu J^\mu_B = 0$$

Assumptions and inputs

i. Near equilibrium form of $T^{\mu\nu}$:

$$T^{\mu\nu} = \underbrace{(p+\epsilon)\,v^\mu v^\nu - p\,g^{\mu\nu}}_{\text{ideal hydro}} \oplus \underbrace{(\eta,\zeta)\partial v}_{\text{viscous terms}} \oplus \cdots$$

ii. Equation of State: $p = f(\epsilon)$

iii. Transport coefficients: $\eta, \zeta, \cdots$

iv. Initial condition for ϵ and $\vec{v}$ at some t_0

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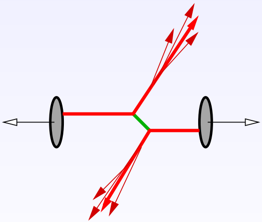
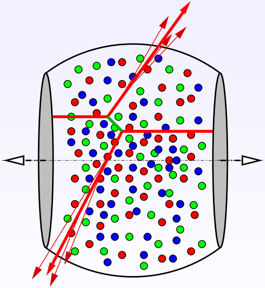
Jet quenching

- The basis of perturbative QCD is asymptotic freedom
- pQCD is the tool of choice for computing the production of hard objects (high $p_\perp$ jets, direct photons, heavy quarks)
- In heavy ion collisions, a new challenge for QCD is the study of the propagation of a hard object in a dense quark-gluon medium





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3. QCD in heavy ion collisions
4. Perturbative QCD
5. **QCD at finite T, Lattice QCD**
   - Partition function
   - Lattice QCD
6. Color Glass Condensate
7. AdS/CFT

Partition function

- Partition function :

$$Z \equiv \mathrm{Tr}\,(e^{-\beta H}) = \int [\mathcal{D}A^\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi]\; e^{-S_E[A^\mu,\overline{\psi},\psi]}$$

- S_E is the Euclidean action, with imaginary time in $[0, \beta = 1/T]$. The Matsubara formalism provides a way to do perturbative calculations at finite T

- Z knows everything about the QGP thermodynamics :

$$E = -\frac{\partial Z}{\partial \beta}$$

$$S = \beta E + \ln(Z)$$

$$F = E - TS = -\frac{1}{\beta}\ln(Z)$$

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Partition function

Lattice QCD

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Lattice QCD

- Lattice QCD : discretize space-time, and approximate the functional integration by a Monte-Carlo sampling
- “Sign problem” :
  - does not work for “real time” correlation functions
    ▷ limited to static properties of the QGP (thermodynamics)
  - does not work with a baryon chemical potential
- Light quarks with realistic masses are computationally expensive

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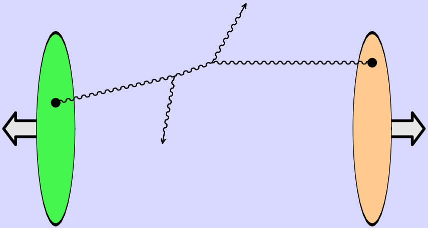
Longitudinal momentum fraction in AA collisions



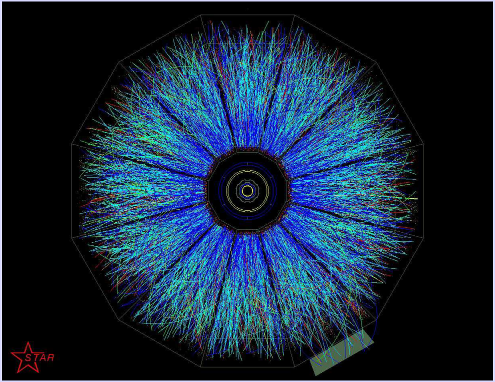
- The partons that are relevant for the process under consideration carry the longitudinal momentum fractions:

$$x_{1,2} = \frac{P_\perp}{\sqrt{s}} e^{\pm Y}$$

  - $P_\perp$: transverse momentum
  - Y : rapidity
  - $\sqrt{s}$: collision energy

Longitudinal momentum fraction in AA collisions

Nucleus-Nucleus collision

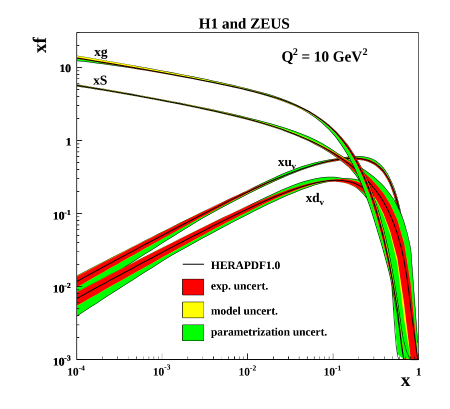


- 99% of the multiplicity below $p_\perp \sim 2$ GeV
- $x \sim 10^{-2}$ at RHIC ($\sqrt{s} = 200$ GeV)
- $x \sim 4.10^{-4}$ at the LHC ($\sqrt{s} = 5.5$ TeV)

  ▷ partons at small x are the most important

Growth of the gluon distribution at small x

Parton distributions at small x



- Gluons dominate at any $x \leq 10^{-1}$

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Why small-x gluons matter

Gluon saturation

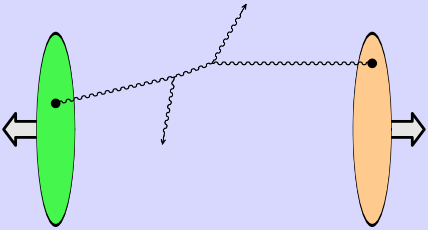
Saturation domain

7 AdS/CFT

Multiple scatterings and gluon recombination

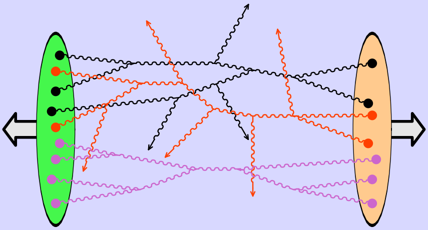
- Main difficulty: How to treat collisions involving a large number of partons?

Multiple scatterings and gluon recombination



- Dilute regime : one parton in each projectile interact
  - ▷ large Q^2, no small-x effects
  - ▷ usual PDFs + DGLAP evolution

Multiple scatterings and gluon recombination



- Dense regime : multiparton processes become crucial
  - ▷ gluon recombinations are important (saturation)
  - ▷ multi-parton distributions + JIMWLK evolution
  - ▷ new techniques are required (Color Glass Condensate):

$$\mathcal{L} = -\frac{1}{4}F^2 + J \cdot A$$

(gluons only, field A for $k^+ < \Lambda$, classical source J for $k^+ > \Lambda$)

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Criterion for gluon recombination

Gribov, Levin, Ryskin (1983)

Number of gluons per unit area :

$$\rho \sim \frac{xG_A(x, Q^2)}{\pi R_A^2}$$

Recombination cross-section :

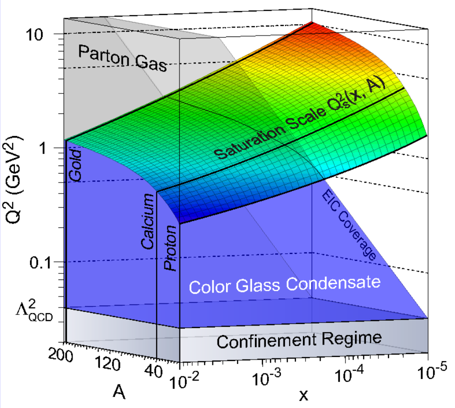
$$\sigma_{gg\to g} \sim \frac{\alpha_s}{Q^2}$$

Recombination happens if $\rho\sigma_{gg\to g} \gtrsim 1$, i.e. $Q^2 \lesssim Q_s^2$, with :

$$Q_s^2 \sim \frac{\alpha_s xG_A(x, Q_s^2)}{\pi R_A^2} \sim A^{1/3}\frac{1}{x^{0.3}}$$

Saturation domain

Saturation scale as a function of x and A



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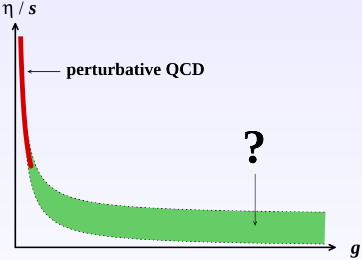
7 **AdS/CFT**
- Gauge-gravity duality
- Viscosity in N=4 SYM
- Limitations

Viscosity at weak coupling

- The shear viscosity has been calculated in QCD at weak coupling ($g \to 0$), and it is quite large :

$$\frac{\eta}{s} = \frac{5.12}{g^4 \ln\left(\frac{2.42}{g}\right)}$$



- However, η/s decreases quickly when the coupling increases $\triangleright$ Can we calculate it?

AdS/CFT duality at T=0

- Maximally super-symmetric $SU(N)$ Yang-Mills theories in the limit $g^2N \to +\infty$ are dual to classical super-gravity on an $\mathrm{AdS}_5 \times S_5$ manifold with metric

$$ds^2 = \frac{R^2}{z^2}(\underbrace{-dt^2 + d\vec{x}^2}_{\text{we live here... (at z=0)}} + dz^2) + R^2 d\Omega_5^2$$

- If an operator $\mathcal{O}$ of our world is coupled on the boundary to a field φ_0 that extends in the bulk, the duality states that :

$$e^{-S_{\rm cl}[\phi]} = \left\langle e^{\int_{\rm boundary} \mathcal{O}\varphi_0} \right\rangle$$

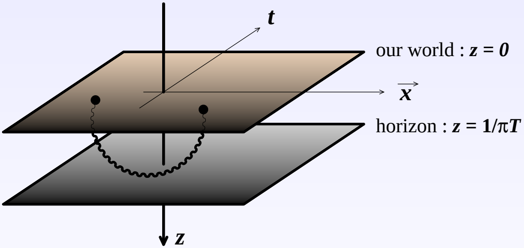
  - The right hand side is a generating functional for the correlators of operators $\mathcal{O}$ in the 4-dim gauge theory
  - The left hand side is calculable in the gravity dual (solve the classical EOM for ϕ with the boundary condition φ_0)

AdS/CFT duality at high T

- At finite temperature T :

$$-dt^2 + dz^2 \to -f(z)dt^2 + dz^2/f(z) \quad \text{with } f(z) = 1 - (\pi z T)^4$$

- $f(z) = 0$ at $z = 1/\pi T \quad \Rightarrow$ black hole horizon



- Ordinary particles in 4-dimensions are the end points of strings living in the bulk. Temperature effects occur when a string gets close to the BH horizon

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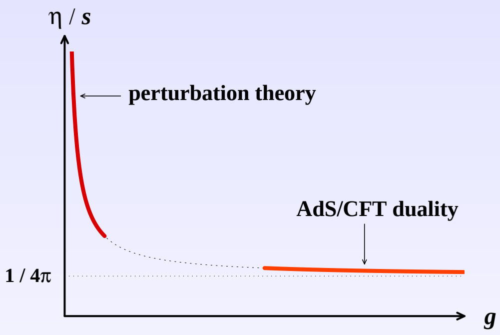
Viscosity in SUSY Yang-Mills



- In SYM at $g^2N \to \infty$, one gets $\eta/s = 1/4\pi$
- Conjecture : $1/4\pi$ is the lowest possible value for η/s
- Note: all the known substances have a viscosity to entropy ratio (much) larger than that

1 QCD primer

2 Deconfinement

3 QCD in heavy ion collisions

4 Perturbative QCD

5 QCD at finite T, Lattice QCD

6 Color Glass Condensate

7 AdS/CFT

Gauge-gravity duality

Viscosity in N=4 SYM

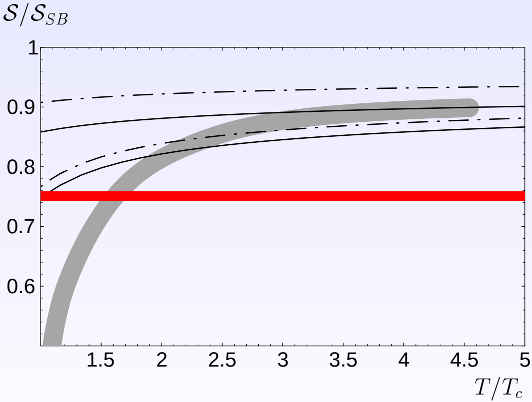
Limitations

SUSY YM $\neq$ QCD

- AdS/CFT only applies to maximally super-symmetric Yang-Mills theories. Such theories are scale invariant, have no running coupling, no chiral symmetry breaking, and no confinement
- Whether what we learn about these theories is accurate for QCD (that has broken scale invariance, running coupling, chiral symmetry breaking, confinement, and quite different matter fields...) is at best a wishful thinking
- Nevertheless an interesting playground in order to realize how wrong one's weak coupling prejudices may be...

Importance of scale violations

- Is the QGP at $T/T_c \sim 2-3$ really strongly coupled? For quantities such as the entropy, perturbative techniques (+resummations) lead to sensible results in this region



- At $T < 3T_c$, the coupling may indeed be strong, but scale violations make AdS/CFT unreliable

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Summary

- QCD in heavy ion collisions displays a very rich spectrum of phenomena
- Ab initio methods (lattice) are often impractical in QCD
- The consequence of this is the diversity of tools and techniques that have been developed to study various aspects of strong interactions in heavy ion collisions
- QCD also plays a role in providing inputs into a number of effective descriptions such as kinetic theory and hydrodynamics

Thank You!

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