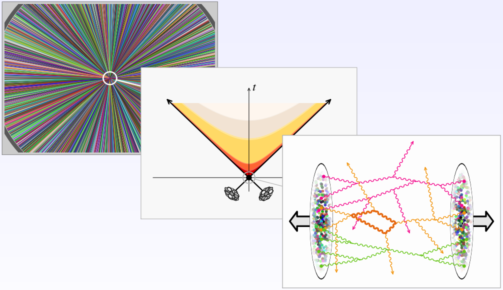
The initial stages of high energy Heavy Ion Collisions

Institut de Physique Théorique, Saclay, December 2011



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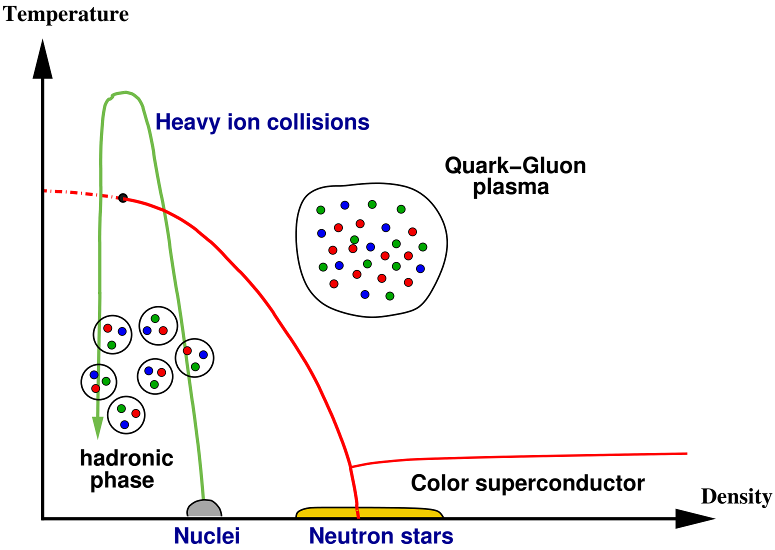
Outline

1. **Quantum field theory with strong sources**
2. **Initial state factorization**
3. **Phenomenology**
4. **Final state evolution**

Introduction

QCD phase diagram – Heavy ion collisions



QCD phase diagram – Heavy ion collisions

What would we like to learn?

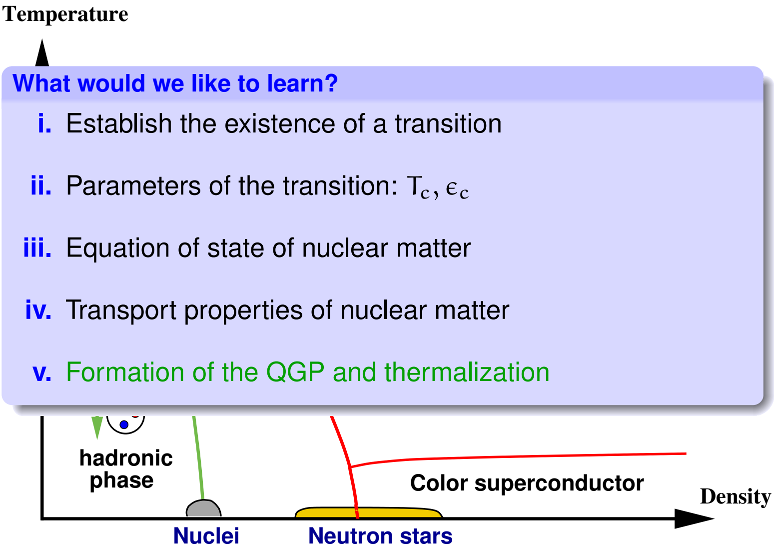
i. Establish the existence of a transition

ii. Parameters of the transition: T_c, ϵ_c

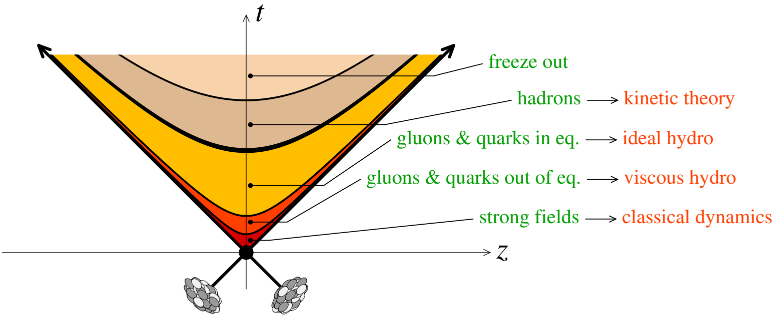
iii. Equation of state of nuclear matter

iv. Transport properties of nuclear matter

v. Formation of the QGP and thermalization



Stages of a collision



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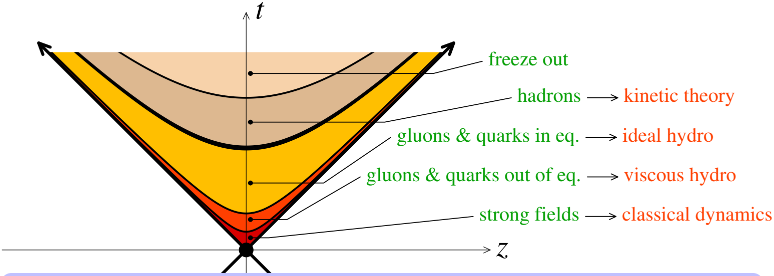
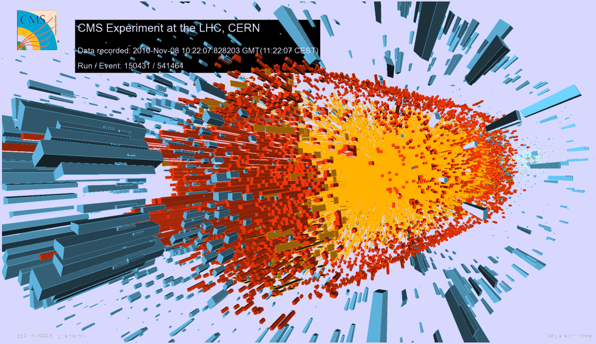


This talk : evolution up to times ~ 1 fm/c

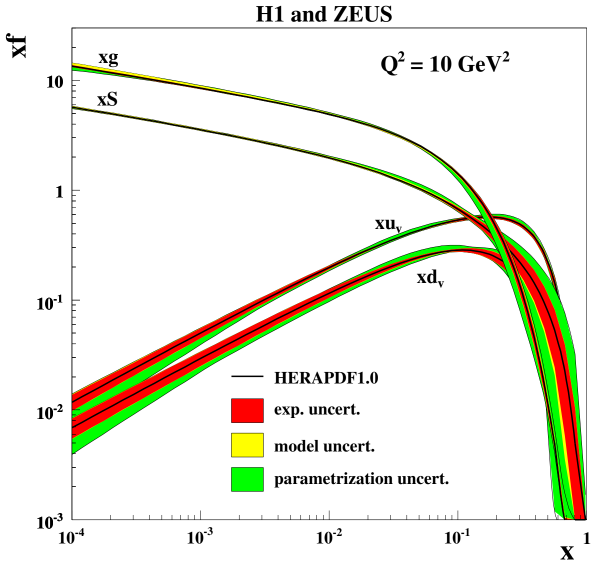
- **i.** Partonic content of high energy nuclei
- **ii.** Gluon production in the collision
- **iii.** Evolution shortly after the collision, Thermalization

Kinematics



- Low typical final state transverse momentum $p_\perp \lesssim 1$ GeV
- Incoming partons have low momentum fractions $x \sim p_\perp / E$
  - $x \sim 10^{-2}$ at RHIC ($E = 200$ GeV)
  - $x \sim 4.10^{-4}$ at the LHC ($E = 2.76 - 5.5$ TeV)

Nucleon parton distributions



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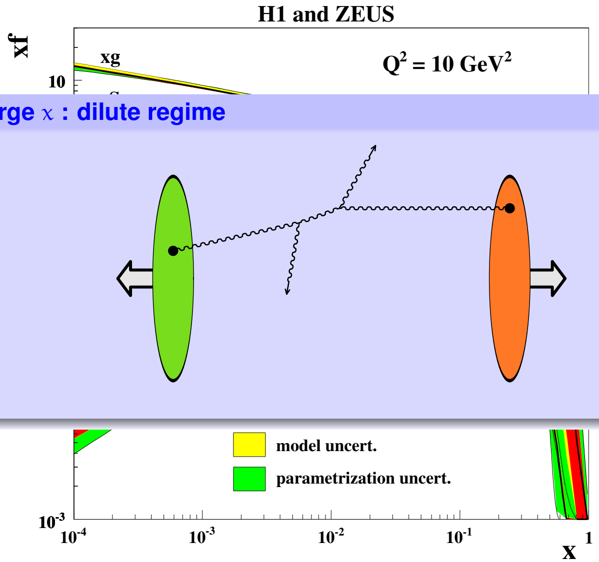
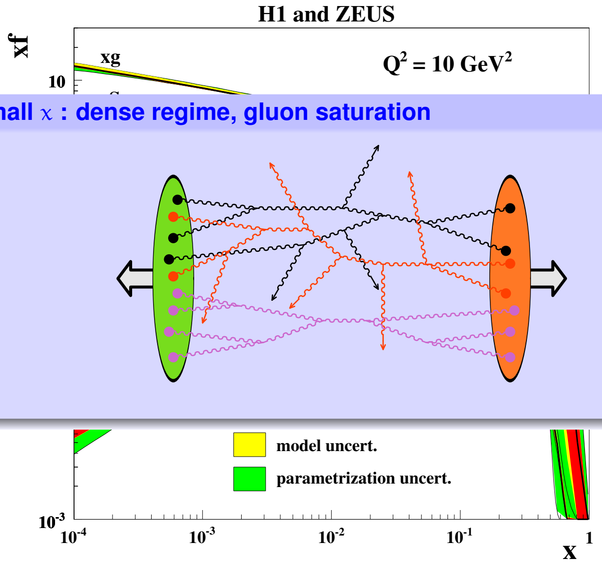


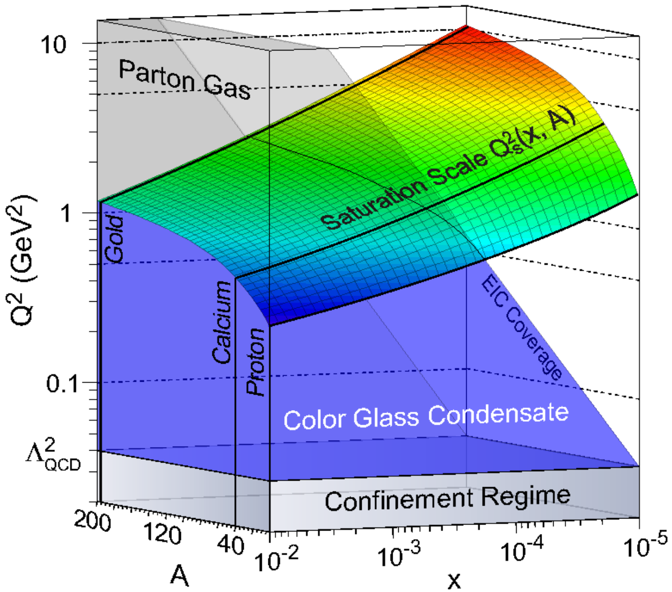
Large x : dilute regime

Nucleon parton distributions



Small x : dense regime, gluon saturation

Saturation domain



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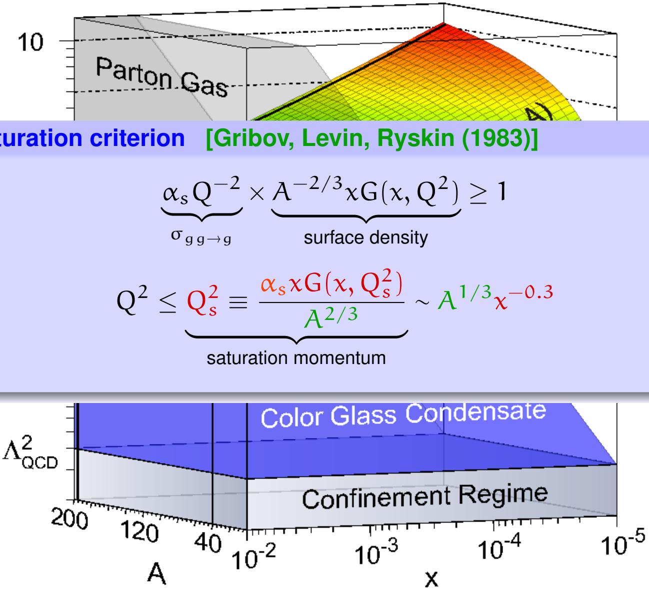

Saturation criterion **[Gribov, Levin, Ryskin (1983)]**

$$\underbrace{\alpha_s Q^{-2}}_{\sigma_{gg\to g}} \times \underbrace{A^{-2/3} xG(x,Q^2)}_{\text{surface density}} \geq 1$$

$$Q^2 \leq \underbrace{Q_s^2 \equiv \frac{\alpha_s xG(x,Q_s^2)}{A^{2/3}}}_{\text{saturation momentum}} \sim A^{1/3} x^{-0.3}$$

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Color Glass Condensate = effective theory of small x gluons

[McLerran, Venugopalan (1994), Jalilian-Marian, Kovner, Leonidov, Weigert (1997), Iancu, Leonidov, McLerran (2001)]

- The fast partons ($k^+ > \Lambda^+$) are frozen by time dilation
  ▷ described as static color sources on the light-cone :

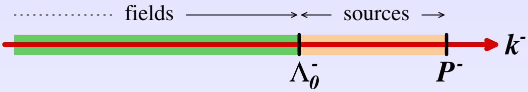
$$J^\mu = \delta^{\mu +} \rho(x^-, \vec{x}_\perp) \qquad (0 < x^- < 1/\Lambda^+)$$

- The color sources ρ are random, and described by a probability distribution $W_{\Lambda^+}[\rho]$

- Slow partons ($k^+ < \Lambda^+$) may evolve during the collision
  ▷ treated as standard gauge fields
  ▷ eikonal coupling to the current J^μ : $J_\mu A^\mu$

$$\mathcal{S} = \underbrace{-\frac{1}{4}\int F_{\mu\nu}F^{\mu\nu}}_{\mathcal{S}_{YM}} + \int \underbrace{J^\mu A_\mu}_{\text{fast partons}}$$

Renormalization group evolution, JIMWLK equation



- The cutoff between the sources and the fields is not physical, and should not enter in observables
- Loop corrections contain logs of the cutoff
- These logs can be cancelled by letting the distribution of the sources depend on the cutoff

$$\Lambda \frac{\partial W[\rho]}{\partial \Lambda} = \mathcal{H}\left(\rho, \frac{\delta}{\delta \rho}\right) W[\rho] \qquad \text{(JIMWLK equation)}$$

- So far, proven in situations involving only one nucleus

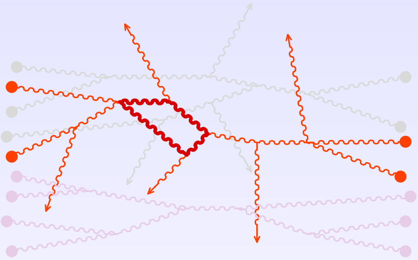
  What about nucleus-nucleus collisions?

  Do the logs mix the sources of the two nuclei?

Quantum Field Theory with strong sources

[FG, Venugopalan (2006)]

Power counting



In the saturated regime: $J \sim g^{-1}$

$$g^{-2}\ g^{\#\text{ of external legs}}\ g^{2\times(\#\text{ of loops})}$$

- No dependence on the number of sources J
  - ▷ infinite number of graphs at each order

Inclusive observables

- Inclusive observables do not veto any final state
  Example: moments of the transition probabilities :

$$\frac{dN_1}{d^3\vec{p}} \sim \sum_{n=0}^{\infty}(n+1)\int \frac{1}{(n+1)!}\Big[\underbrace{d\Phi_1\cdots d\Phi_n}_{n \text{ part. phase-space}}\Big]\,\Big|\big\langle \mathbf{p}\mathbf{p}_1\cdots\mathbf{p}_{n\,\mathrm{out}}\big|0_{\mathrm{in}}\big\rangle\Big|^2$$

  (single inclusive particle distribution)

- Completeness of the out-states ▷ expectation value of some out-operator in the in-vacuum state :

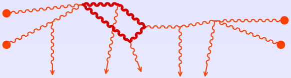
$$\frac{dN_1}{d^3\vec{p}} \sim \big\langle 0_{\mathrm{in}}\big| a^{\dagger}_{\mathrm{out}}(\mathbf{p})\,a_{\mathrm{out}}(\mathbf{p})\big|0_{\mathrm{in}}\big\rangle$$

$$\uparrow$$

$$\overbrace{\sum_n \frac{1}{n!}\int\Big[d\Phi_1\cdots d\Phi_n\Big]\,\big|\mathbf{p}_1\cdots\mathbf{p}_{n\,\mathrm{out}}\big\rangle\big\langle\mathbf{p}_1\cdots\mathbf{p}_{n\,\mathrm{out}}\big| = 1}$$

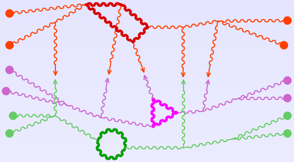
- Disconnected vacuum-vacuum graphs cancel in $\big\langle 0_{\mathrm{in}}\big|\cdots\big|0_{\mathrm{in}}\big\rangle$ expectation values

Bookkeeping



- Start with transition amplitudes : sources $\longrightarrow$ particles

Bookkeeping



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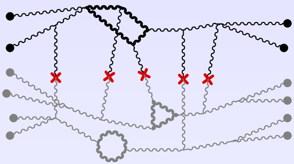

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- See them as cuts through vacuum diagrams
  Cut propagator $\sim \delta(p^2)$

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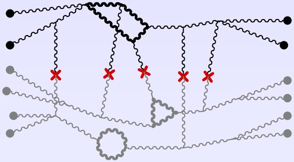

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Weight each cut by $z(p)$ $\longrightarrow$ generating functional

$$F[z] \equiv \sum_n \frac{1}{n!} \int \left[d\Phi_1 \cdots d\Phi_n\right] z(\mathbf{p}_1) \cdots z(\mathbf{p}_n) \left|\left\langle \mathbf{p}_1 \cdots \mathbf{p}_{n\,\mathrm{out}} \middle| 0_{\mathrm{in}} \right\rangle\right|^2$$

Generating functional

- Observables are given by derivatives of $F[z]$, e.g.

$$\frac{dN_1}{d^3\vec{p}} = \frac{\delta F[z]}{\delta z(\mathbf{p})}\Big|_{z=1}$$

(inclusive observables are derivatives at the point $z = 1$)

- Unitarity implies $F[1] = 1$
- Exclusive observables involve derivatives at $z = 0$

Reduction formula

$$\frac{\delta \log F[z]}{\delta z(\mathbf{p})} = \int d^4x d^4y \; e^{ip\cdot(x-y)} \; \square_x \square_y \left[A_+(x)A_-(y) + G_{+-}(x,y)\right]$$

$A_\pm$ and G_{+-} are connected Schwinger-Keldysh 1- and 2-point functions, with cut propagators weighted by $z(\mathbf{p})$

Leading Order

- Structure of the expansion in g^2 :

$$A_\pm = \frac{1}{g}\Big[\underbrace{a_0}_{\text{tree}} + \underbrace{a_1 g^2}_{\text{1-loop}} + \cdots\Big] \qquad G_{+-} = \underbrace{b_0}_{\text{tree}} + \underbrace{b_1 g^2}_{\text{1-loop}} + \cdots$$

- LO : we need only $A_+(x)$ and $A_-(y)$, at tree level
- These functions obey the classical equation of motion :

$$\square\mathcal{A} + V'(\mathcal{A}) = J$$

- Boundary conditions : retarded, with $\mathcal{A} \to 0$ at $x_0 = -\infty$

Inclusive spectra at LO

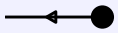
$$\left.\frac{dN_1}{d^3\vec{p}}\right|_{\text{LO}} \sim \int d^4x d^4y \; e^{ip\cdot(x-y)} \; \square_x \square_y \; \mathcal{A}(x)\,\mathcal{A}(y)$$

$$\left.\frac{dN_n}{d^3\vec{p}_1\cdots d^3\vec{p}_n}\right|_{\text{LO}} = \left.\frac{dN_1}{d^3\vec{p}_1}\right|_{\text{LO}} \cdots \left.\frac{dN_1}{d^3\vec{p}_n}\right|_{\text{LO}}$$

Retarded classical fields

$$\square\mathcal{A} + V'(\mathcal{A}) = J \qquad , \qquad \lim_{x_0\to-\infty} \mathcal{A}(x) = 0$$

- Perturbative expansion :



- Built with retarded propagators

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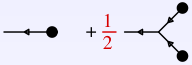

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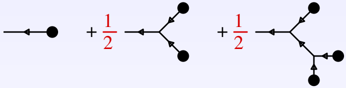

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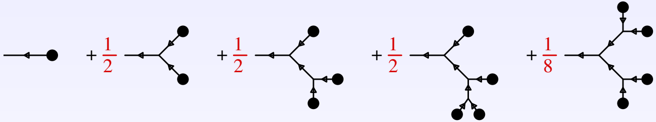


- Built with retarded propagators

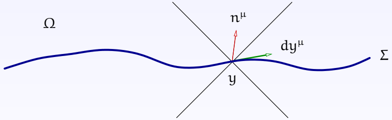
- Classical fields resum the full series of tree diagrams

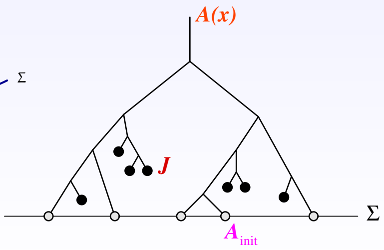
Cauchy problem for classical fields

- In some situations, one needs to express the classical field in terms of the source J and its value on a surface Σ

Green's formula

$$\mathcal{A}(x) = i \int_{y\in\Omega} G_R^0(x,y)\,\Big[J(y) - V'(\mathcal{A}(y))\Big] + i \int_{y\in\Sigma} G_R^0(x,y)\,(n\cdot \overleftrightarrow{\partial}_y)\,\mathcal{A}_{\rm init}(y)$$





Small perturbations of a classical field

Wave propagating over a classical background

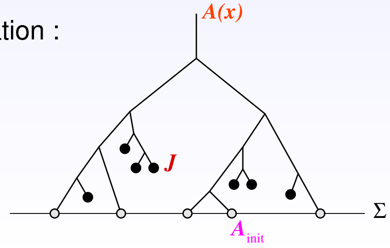
$$\Big[\square_x + V''(\mathcal{A}(x))\Big]\, a(x) = 0 \quad , \quad a(x) = \alpha(x) \;\text{ on }\; \Sigma$$

Formal solution

$$[\alpha\,\mathbb{T}]_y \equiv \alpha(y)\frac{\delta}{\delta\mathcal{A}_{\rm init}(y)} + (n\cdot\partial\alpha(y))\frac{\delta}{\delta(n\cdot\partial\mathcal{A}_{\rm init}(y))}$$

$$a(x) \equiv \int\limits_{y\in\Sigma} [\alpha\,\mathbb{T}]_y \;\; \mathcal{A}(x)$$

- Diagrammatic interpretation :



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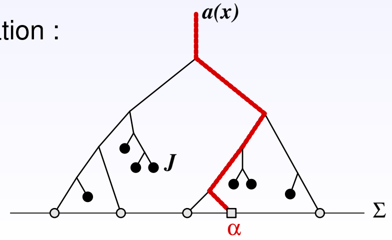


Next to Leading Order

- What do we need at NLO?

$$A_\pm = \frac{1}{g}\left[a_0 + \underline{\underline{a_1 g^2}} + \cdots\right] \qquad G_{+-} = \underline{\underline{b_0}} + b_1 g^2 + \cdots$$

- These two subleading quantities can be expressed in terms of perturbations to the retarded classical field

- For instance, at tree level:

$$G_{+-}(x,y) = \int \frac{d^3k}{(2\pi)^3 2k}\; a_k(x)\, a_k^*(y)$$

$$\left[\square_x + V''(\mathcal{A}(x))\right] a_k(x) = 0 \qquad , \qquad \lim_{x_0 \to -\infty} a_k(x) = e^{ik\cdot x}$$

Next to Leading Order

Master relation between LO and NLO

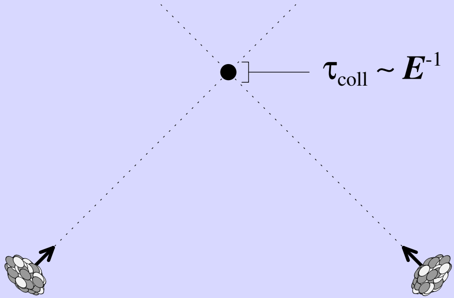
$$\left.\frac{dN_1}{d^3\vec{p}}\right|_{NLO} = \left[\frac{1}{2}\int_{u,v\in\Sigma}\int_k \left[a_k\,\mathbb{T}\right]_u\left[a_k^*\,\mathbb{T}\right]_v + \int_{u\in\Sigma}\left[\alpha\,\mathbb{T}\right]_u\right]\left.\frac{dN_1}{d^3\vec{p}}\right|_{LO}$$

- Valid for all inclusive multi-gluon spectra, and for the energy-momentum tensor
- Valid for any Cauchy surface Σ
- Not specific to scalar theories
- In the CGC, upper cutoff on the loop momentum : $k^\pm < \Lambda$, to avoid double counting with the sources $J_{1,2}$
  ▷ large logarithms of the cutoff

Factorization

[FG, Lappi, Venugopalan (2008)]

Handwaving argument for factorization



- The duration of the collision is very short: $\tau_{coll} \sim E^{-1}$

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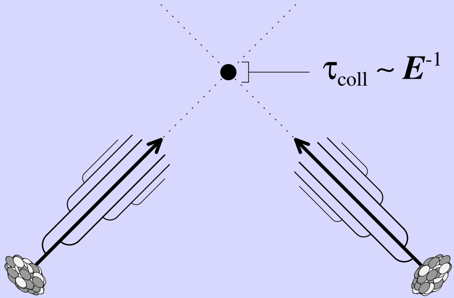

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- The logarithms we want to resum are due to the radiation of soft gluons, which takes a long time
  ▷ it must happen (long) before the collision

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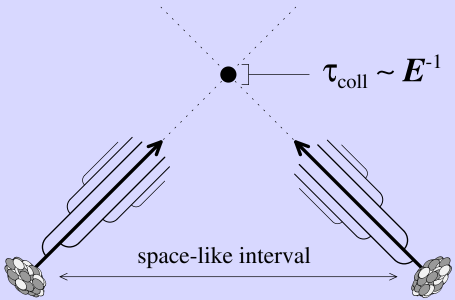

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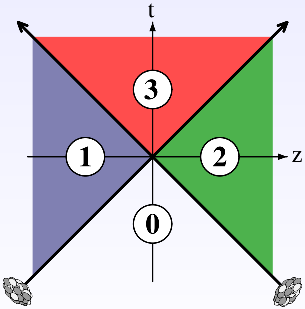
- The logarithms we want to resum are due to the radiation of soft gluons, which takes a long time
  ▷ it must happen (long) before the collision
- The projectiles are not in causal contact before the impact
  ▷ the logarithms are intrinsic properties of the projectiles, independent of the measured observable

Space-time evolution of the classical field

- Sources located on the light-cone:

$$J^\mu = \delta^{\mu+} \underbrace{\rho_1(x^-, \boldsymbol{x}_\perp)}_{\sim\delta(x^-)} + \delta^{\mu-} \underbrace{\rho_2(x^+, \boldsymbol{x}_\perp)}_{\sim\delta(x^+)}$$



- Region 0 : $\mathcal{A}^\mu = 0$
- Regions 1,2 : $\mathcal{A}^\mu$ depends only on ρ_1 or ρ_2 (known analytically)
- Region 3 : $\mathcal{A}^\mu$ = radiated field after the collision, only known numerically

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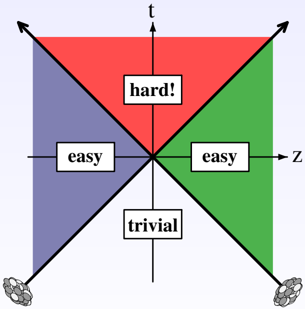

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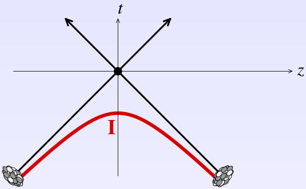
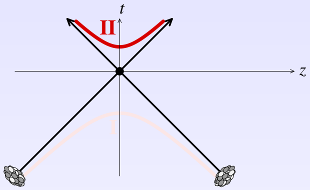
Choice of the Cauchy surface Σ



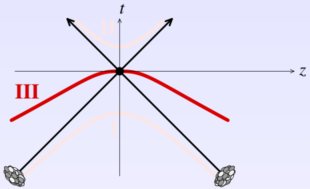
- **I** : too trivial to be useful

Choice of the Cauchy surface Σ



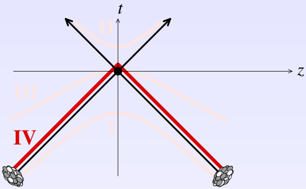
- **I** : too trivial to be useful
- **II** : too hard to be usable

Choice of the Cauchy surface Σ



- **I** : too trivial to be useful
- **II** : too hard to be usable
- **III** : tractable

Choice of the Cauchy surface Σ



- **I** : too trivial to be useful
- **II** : too hard to be usable
- **III** : tractable
- **IV** : simplest choice

Strategy

- Compute the fluctuations $a_{\mathbf{k}}$ and α on Σ
- Extract the logarithms in the integration over $\mathbf{k}$
- Check that they are universal

Initial state logarithms

Central result

$$\frac{1}{2}\int_{u,v\in\Sigma}\int_{k}\big[a_k\,\mathbb{T}\big]_u\big[a_k^*\,\mathbb{T}\big]_v+\int_{u\in\Sigma}\big[\alpha\,\mathbb{T}\big]_u=$$

$$=\log\left(\Lambda^+\right)\,\mathcal{H}_1+\log\left(\Lambda^-\right)\,\mathcal{H}_2+\text{terms w/o logs}$$

$\mathcal{H}_{1,2}=$ JIMWLK Hamiltonians of the two nuclei

- No mixing between the logs of Λ^+ and Λ^-
- Since the LO$\leftrightarrow$NLO relationship is the same for all inclusive observables, these logs have a universal structure

Factorization of the logarithms

- By integrating over $\rho_{1,2}$'s, one can absorb the logarithms into universal distributions $W_{1,2}[\rho_{1,2}]$
- $\mathcal{H}$ is a self-adjoint operator :

$$\int [D\rho]\; W\,(\mathcal{H}\,\mathcal{O}) = \int [D\rho]\; (\mathcal{H}\,W)\;\mathcal{O}$$

Single inclusive gluon spectrum at Leading Log accuracy

$$\left\langle \frac{dN_1}{d^3\vec{p}} \right\rangle_{\text{Leading Log}} = \int \left[D\rho_1\; D\rho_2\right]\; W_1\left[\rho_1\right]\; W_2\left[\rho_2\right]\; \underbrace{\left.\frac{dN_1[\rho_{1,2}]}{d^3\vec{p}}\right|_{\text{LO}}}_{\text{fixed }\rho_{1,2}}$$

- Logs absorbed into the evolution of $W_{1,2}$ with the scales

$$\Lambda\frac{\partial W}{\partial \Lambda} = \mathcal{H}\,W \qquad \text{(JIMWLK equation)}$$

Multi-gluon correlations at Leading Log

- The previous factorization can be extended to multi-particle inclusive spectra :

$$\left\langle \frac{dN_n}{d^3\vec{\mathbf{p}}_1 \cdots d^3\vec{\mathbf{p}}_n} \right\rangle_{\text{Leading Log}} =$$

$$= \int [D\rho_1\, D\rho_2]\; W_1[\rho_1]\; W_2[\rho_2]\; \frac{dN_1[\rho_{1,2}]}{d^3\vec{\mathbf{p}}_1} \cdots \frac{dN_1[\rho_{1,2}]}{d^3\vec{\mathbf{p}}_n}\bigg|_{\text{LO}}$$

- At Leading Log accuracy, all the rapidity correlations come from the evolution of the distributions $W[\rho_{1,2}]$

  ▷ they are a property of the pre-collision initial state

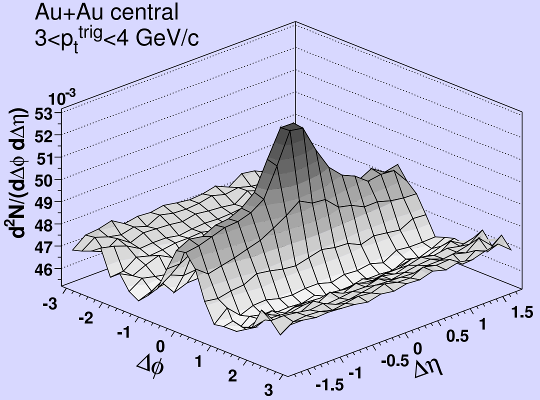
- Predicts long range ($\Delta y \sim \alpha_s^{-1}$) correlations in rapidity

Phenomenology

[Dumitru, FG, McLerran, Venugopalan (2008)]
[Dusling, FG, Lappi, Venugopalan (2010)]

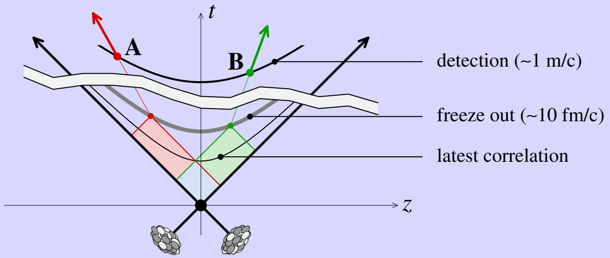
2-particle correlations in AA collisions

[STAR Collaboration, RHIC]



- Long range rapidity correlation
- Narrow correlation in azimuthal angle
- Narrow jet-like correlation near $\Delta y = \Delta\varphi = 0$

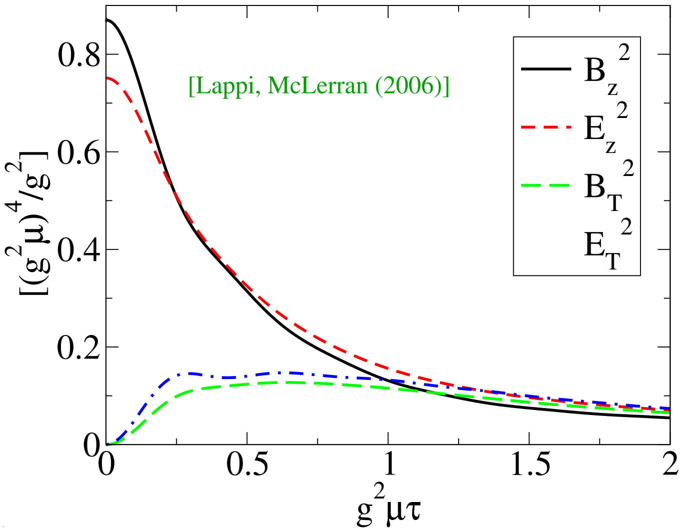
Probing early times with rapidity correlations



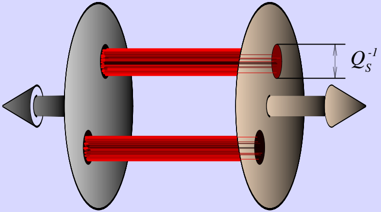
- By causality, long range rapidity correlations are sensitive to the dynamics of the system at early times :

$$\tau_{\text{correlation}} \leq \tau_{\text{freeze out}} \; e^{-|\Delta y|/2}$$

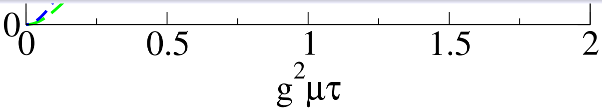
Color field at early time



Color field at early time



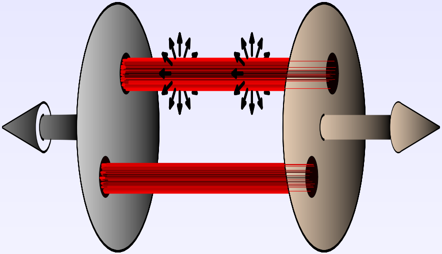
- The field lines form tubes of transverse size $\sim Q_s^{-1}$
- Rapidity correlation length : $\Delta\eta \sim \alpha_s^{-1}$



François Gelis

2-hadron correlations from color flux tubes

- η-independent fields lead to long range correlations :



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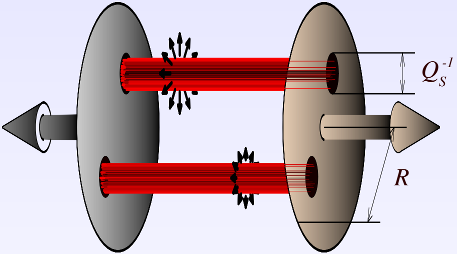

- Particles emitted by different flux tubes are not correlated
  ▷ $(RQ_s)^{-2}$ sets the strength of the correlation

2-hadron correlations from color flux tubes

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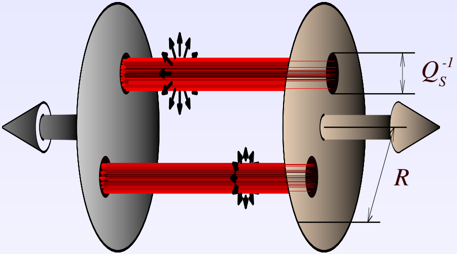

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2-hadron correlations from color flux tubes

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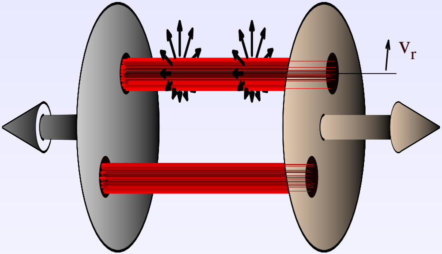
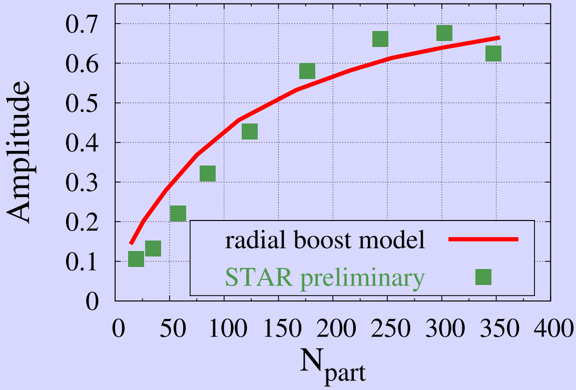


- Particles emitted by different flux tubes are not correlated
  $\triangleright$ $(RQ_s)^{-2}$ sets the strength of the correlation
- At early times, the correlation is flat in $\Delta\varphi$

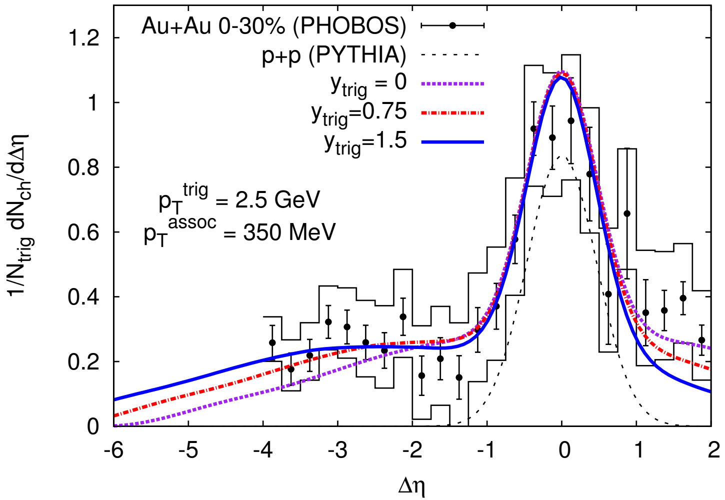
  The collimation in $\Delta\varphi$ is produced later by radial flow

Centrality dependence



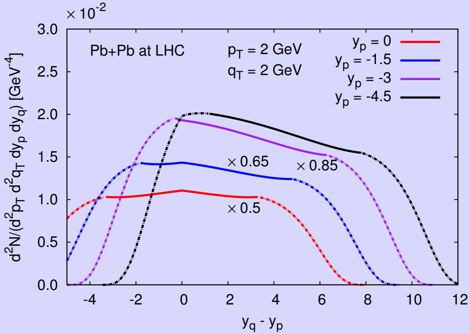
- Main effect : increase of the radial flow velocity with the centrality of the collision

Rapidity dependence



Rapidity dependence

Prediction at LHC energy



-6 -5 -4 -3 -2 -1 0 1 2

Δη

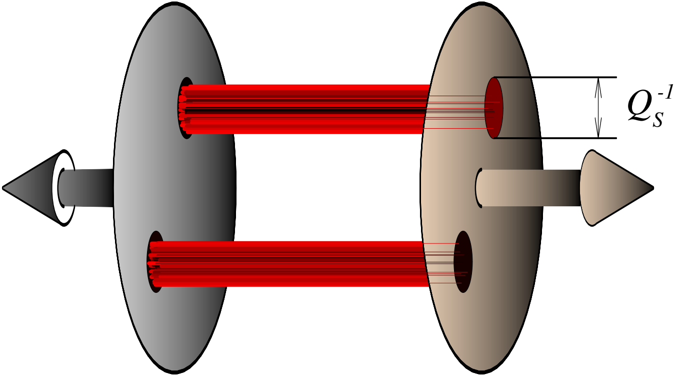
Final state evolution

[Dusling, Epelbaum, FG, Venugopalan (2010)]

[Dusling, FG, Venugopalan (2011)]

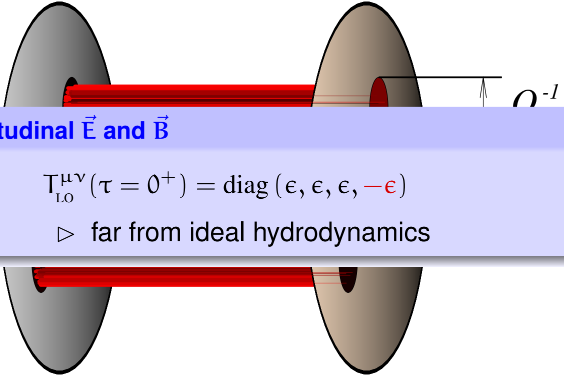
[Epelbaum, FG (2011)]

Energy momentum tensor at LO



François Gelis

Energy momentum tensor at LO



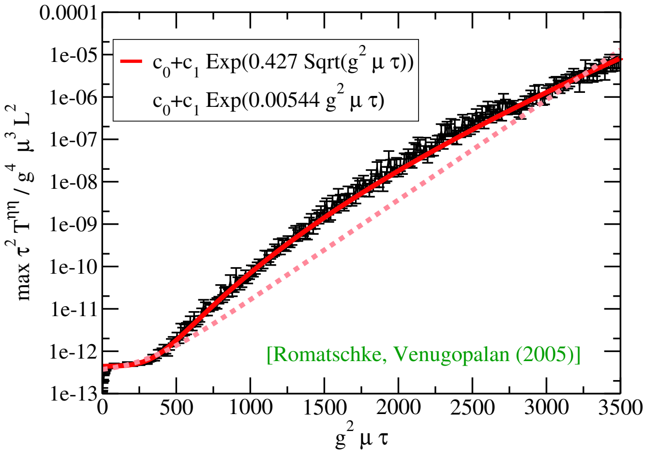
$T^{\mu\nu}$ for longitudinal $\vec{E}$ and $\vec{B}$

$$T^{\mu\nu}_{LO}(\tau = 0^+) = \mathrm{diag}\,(\epsilon, \epsilon, \epsilon, -\epsilon)$$

▷ far from ideal hydrodynamics

Weibel instabilities for small perturbations

[Mrowczynski (1988), Romatschke, Strickland (2003), Arnold, Lenaghan, Moore (2003), Rebhan, Romatschke, Strickland (2005), Arnold, Lenaghan, Moore, Yaffe (2005), Romatschke, Rebhan (2006),...]



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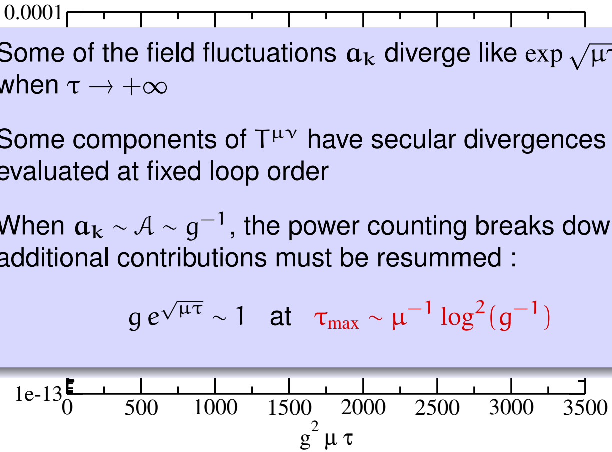
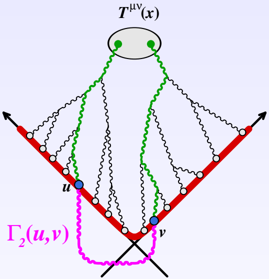


- Some of the field fluctuations a_k diverge like $\exp\sqrt{\mu\tau}$ when $\tau \to +\infty$
- Some components of $T^{\mu\nu}$ have secular divergences when evaluated at fixed loop order
- When $a_k \sim \mathcal{A} \sim g^{-1}$, the power counting breaks down and additional contributions must be resummed :

$$g\, e^{\sqrt{\mu\tau}} \sim 1 \quad \text{at} \quad \tau_{max} \sim \mu^{-1} \log^2(g^{-1})$$

Improved power counting

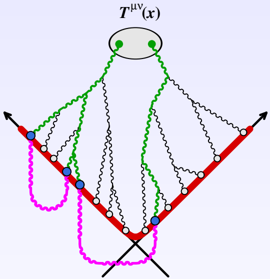
$$\text{Loop} \sim g^2 \qquad , \qquad \mathbb{T}_{\mathbf{u}} \sim e^{\sqrt{\mu\tau}}$$



- 1 loop : $(g e^{\sqrt{\mu\tau}})^2$

Improved power counting

Loop $\sim g^2$, $\mathbb{T}_{\mathbf{u}} \sim e^{\sqrt{\mu\tau}}$

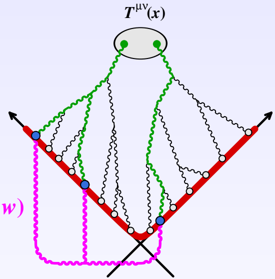


- 1 loop : $(g e^{\sqrt{\mu\tau}})^2$
- 2 disconnected loops : $(g e^{\sqrt{\mu\tau}})^4$

Improved power counting

François Gelis

Loop $\sim g^2$, $\mathbb{T}_u \sim e^{\sqrt{\mu\tau}}$



- 1 loop : $(ge^{\sqrt{\mu\tau}})^2$
- 2 disconnected loops : $(ge^{\sqrt{\mu\tau}})^4$
- 2 nested loops : $g(ge^{\sqrt{\mu\tau}})^3$ ▷ subleading

Leading terms at τ_{max}

- All disjoint loops to all orders
  ▷ exponentiation of the 1-loop result

Resummation of the leading secular terms

$$T^{\mu\nu}_{\text{resummed}} = \exp\left[\frac{1}{2}\int_{u,v\in\Sigma}\underbrace{\int_{k}[a_k\mathbb{T}]_u[a^*_k\mathbb{T}]_v}_{\mathcal{G}(u,v)} + \int_{u\in\Sigma}[\alpha\mathbb{T}]_u\right] T^{\mu\nu}_{\text{LO}}[\mathcal{A}_{\text{init}}]$$

Resummation of the leading secular terms

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$$= \int[D\chi]\,\exp\left[-\frac{1}{2}\int_{u,v\in\Sigma}\chi(u)\mathcal{G}^{-1}(u,v)\chi(v)\right] T^{\mu\nu}_{\text{LO}}[\mathcal{A}_{\text{init}}+\chi+\alpha]$$

- The evolution remains classical, but we must average over a Gaussian ensemble of initial conditions
- Note : the constant shift α can be absorbed into a redefinition of $\mathcal{A}_{\text{init}}$

More on this resummation

- The Gaussian fluctuations around the classical field $\mathcal{A}_{\rm init}$ promote it to a coherent quantum state
(they add $1/2$ particle to every mode)

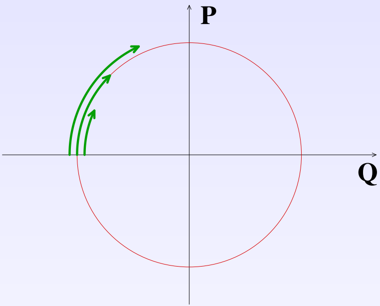
- Dual formulation of QM in the classical phase-space :

Density	$\widehat{\rho}$		$W(Q,P)$
Evolution	$\partial_t\widehat{\rho}+i[\widehat{H},\widehat{\rho}]=0$	$\xrightarrow[\text{trans.}]{\text{Wigner}}$	$\partial_t W+\{\{W,H\}\}=0$
Initial condition	$\big\|\mathcal{A}_{\rm init}\big\rangle\big\langle\mathcal{A}_{\rm init}\big\|$		$\exp-\frac{1}{2}\int\chi\mathcal{G}^{-1}\chi$

Approximations :
  - Moyal bracket $\{\{\cdot,\cdot\}\}$ replaced by classical Poisson bracket
  - Non-gaussianities of the initial distribution are ignored

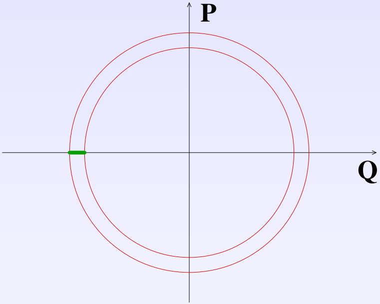
- Independent (and anterior..) uses of this scheme :
  - Cosmology [Polarski, Starobinsky (1995), Son (1996), Khlebnikov, Tkachev (1996)]
  - Cold atoms [Davis, Morgan, Burnett (2002), Norrie, Ballagh, Gardiner (2004)]

Anharmonicity and decoherence



- The oscillation frequency depends on the initial condition

Anharmonicity and decoherence



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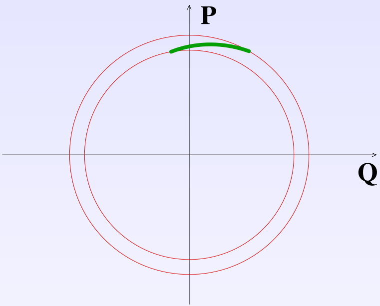
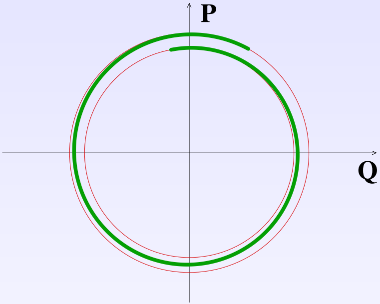
Anharmonicity and decoherence



- The oscillation frequency depends on the initial condition
- An ensemble of initial configurations spreads in time

Anharmonicity and decoherence



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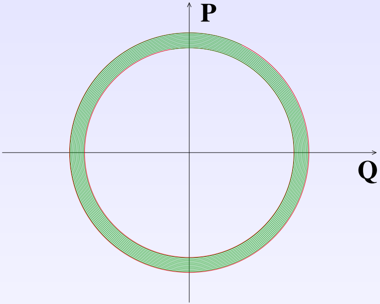


- The oscillation frequency depends on the initial condition
- An ensemble of initial configurations spreads in time
- At large times, the ensemble fills densely all the region allowed by energy conservation

Similar problem in a simpler toy model

François Gelis

ϕ^4 field theory coupled to a source

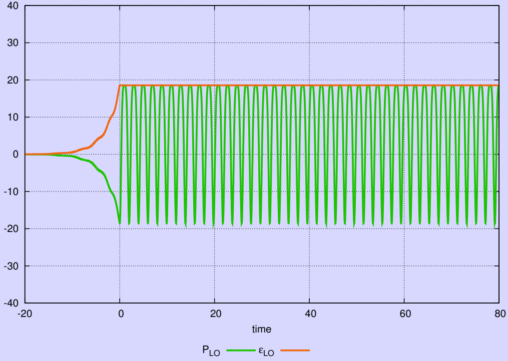
$$\mathcal{L} = \frac{1}{2}(\partial_\alpha \phi)^2 - \frac{g^2}{4!}\phi^4 + J\phi$$

$$J \quad \propto \quad \theta(-x^0)\,\frac{Q^3}{g}$$

- In 3+1-dim, g is dimensionless, and the only scale in the problem is Q, provided by the external source
- The source is active only at $x^0 < 0$, and is turned off adiabatically when $x^0 \to -\infty$
- This theory has unstable modes (parametric resonance)

Secular divergences in fixed order calculations

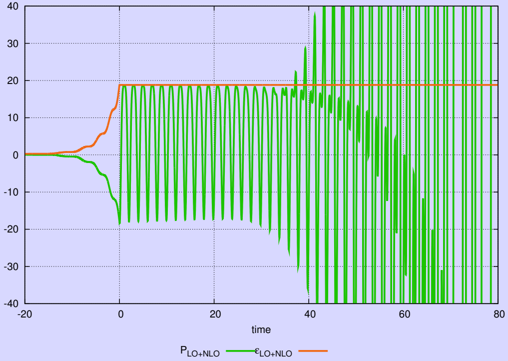
Tree



- Oscillating pressure at LO : no equation of state

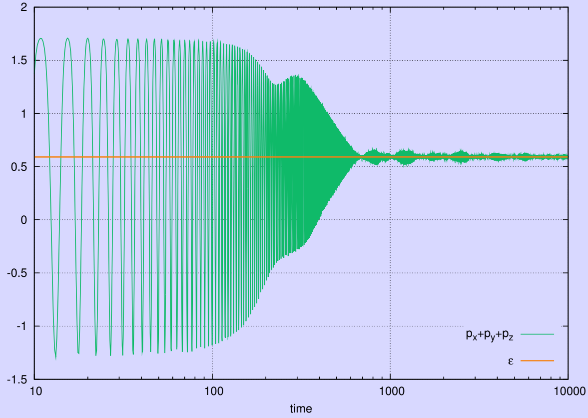
Secular divergences in fixed order calculations

Tree + 1-loop



- Oscillating pressure at LO : no equation of state
- Small NLO correction to the energy density (protected by energy conservation)
- Secular divergence in the NLO correction to the pressure

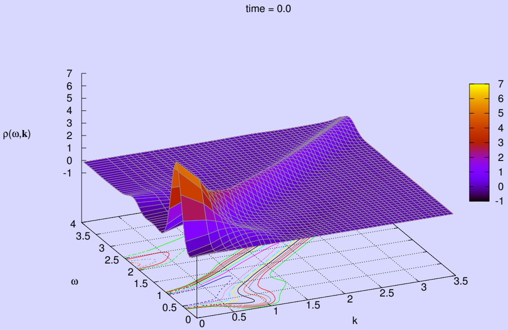
Resummed energy momentum tensor



- No secular divergence in the resummed pressure
- The pressure relaxes to the equilibrium equation of state

Spectral density

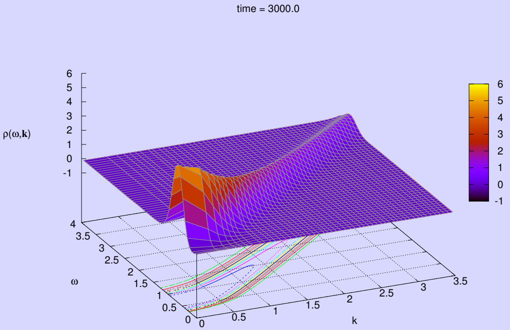
$\tau = 0$



- Complicated spectral density at early times

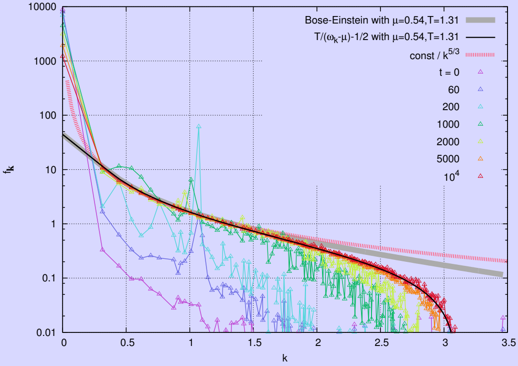
Spectral density

$\tau = 3000$



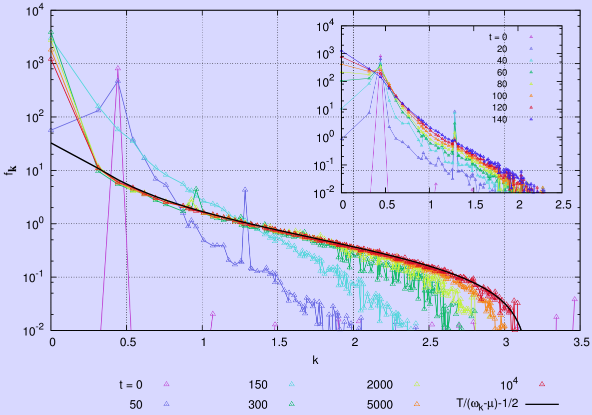
- Complicated spectral density at early times
- Single quasiparticle peak at late times

Time evolution of the occupation number



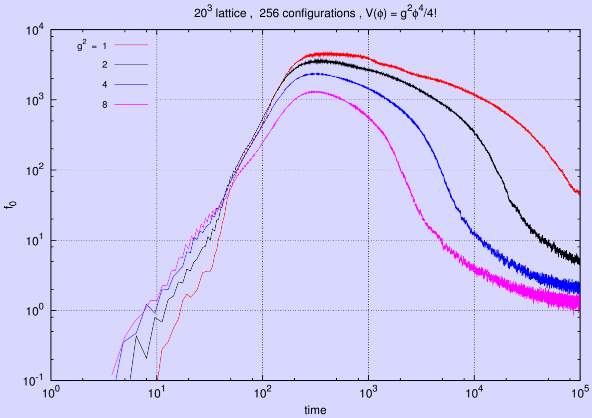
- Resonant peak at early times
- Turbulent Kolmogorov spectrum in the intermediate k-range?
- Late times : classical equilibrium with a chemical potential
- $\mu \approx m$ + excess at $k = 0$: Bose-Einstein condensation?

Bose-Einstein condensation



- Start with the same energy density, but an empty zero mode
- Very quickly, the zero mode becomes highly occupied
- Same distribution as before at late times

Evolution of the condensate



- Formation time almost independent of the coupling
- Condensate lifetime much longer than its formation time
- Smaller amplitude and faster decay at large coupling

Summary and Outlook

Summary

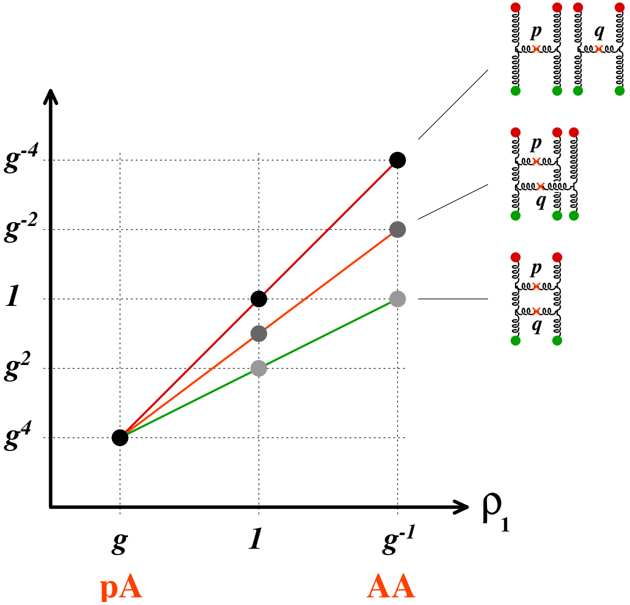
- Factorization of high energy logarithms in AA collisions
  - limited to inclusive observables
  - leads to the rapidity dependence of correlations
  - links nucleus-nucleus collisions to other reactions (pA, DIS)
- Resummation of secular terms in the final state evolution
  - stabilizes the NLO calculation
  - leads to the equilibrium equation of state
  - full thermalization on much longer time-scales
  - Bose-Einstein condensation for overoccupied initial states

Outlook

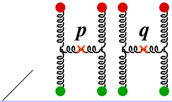
- factorization for dense-dilute collisions?
- can it be extended to exclusive observables?
- thermalization in QCD, w/ longitudinal expansion?
- if a BEC is formed, phenomenological implications?
- links to quantum chaos?

Extra

Dense-dilute collisions



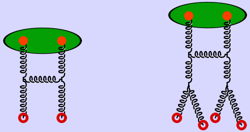
Dense-dilute collisions



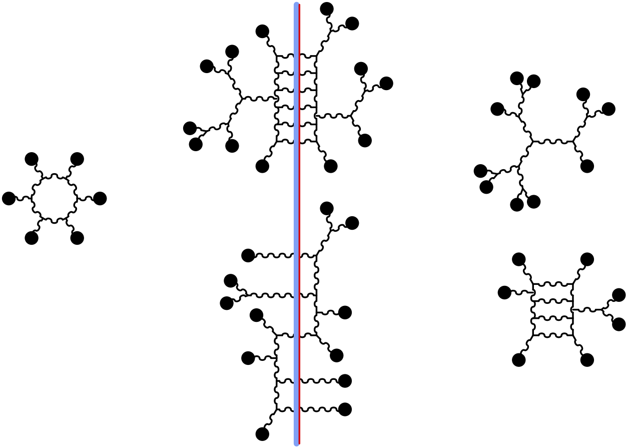
Expected complications

- More diagrams to consider even at Leading Order
- More terms in the evolution Hamiltonian if $\rho \sim g$:

$$g^2 \rho^2 \left(\frac{\partial}{\partial\rho}\right)^2 \sim g^4 \rho^2 \left(\frac{\partial}{\partial\rho}\right)^4$$



Exclusive processes

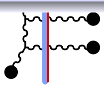


Exclusive processes

Example : differential probability to produce 1 particle at LO

$$\left.\frac{dP_1}{d^3\vec{p}}\right|_{LO} = F[0] \times \int d^4x d^4y \; e^{ip\cdot(x-y)} \, \square_x \square_y \, \mathcal{A}_+(x)\mathcal{A}_-(y)\Big|_{z=0}$$

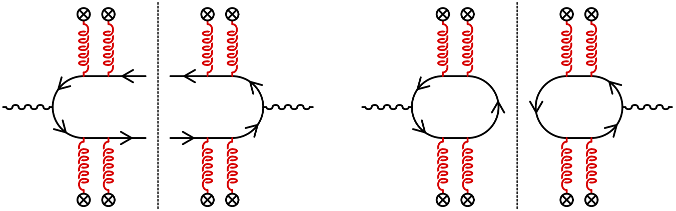
- The vacuum-vacuum graphs do not cancel in exclusive quantities : $F[0] \neq 1$ (in fact, $F[0] = \exp(-c/g^2) \ll 1$)
- $\mathcal{A}_+$ and $\mathcal{A}_-$ are classical solutions of the Yang-Mills equations, but with non-retarded boundary conditions



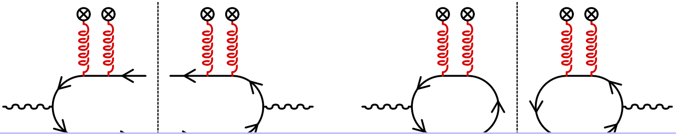
Thermalization in Yang-Mills theory

- Recent analytical work : Kurkela, Moore (2011)
- Going from scalars to gauge fields :
  - More fields per site (3 Lorentz components $\times$ 8 colors)
  - More complicated spectrum of initial conditions
  - Expansion : UV overflow on a fixed grid in η

BEC and dilepton production



BEC and dilepton production



Two topologies for virtual photons at LO

Connected	$\omega \sim M_{inv} \sim Q_s$	$k_\perp \sim Q_s$
Disconnected	$\omega \sim M_{inv} \sim Q_s$	$k_\perp \ll Q_s$

▷ excess of dileptons with $k_\perp \ll M_{inv}$

Links to Quantum Chaos

- Quantum Chaos : how does the chaos at the classical level manifests itself in quantum mechanics?
- Berry's conjecture [M.V. Berry (1977)]

  High lying eigenstates of such systems have nearly random wavefunctions. The corresponding Wigner distribution is almost uniform on the energy surface
- Srednicki's eigenstate thermalization hypothesis [M. Srednicki (1994)]

  For sufficiently inclusive measurements, these high lying eigenstates look thermal. If the system starts in a coherent state, decoherence is the main mechanism to thermalization

Backup

Boundary conditions

$$\mathcal{A}_+(x) = \int_y G^0_{++}(x,y)\Big[J(y) - V'(\mathcal{A}_+(y))\Big] - G^0_{+-}(x,y)\Big[J(y) - V'(\mathcal{A}_-(y))\Big]$$

$$\mathcal{A}_-(x) = \int_y G^0_{-+}(x,y)\Big[J(y) - V'(\mathcal{A}_+(y))\Big] - G^0_{--}(x,y)\Big[J(y) - V'(\mathcal{A}_-(y))\Big]$$

$$\widetilde{G}^0_{+-}(p) = 2\pi\, z(\mathbf{p})\, \theta(-p_0)\delta(p^2)$$

- $\mathcal{A}_+$ and $\mathcal{A}_-$ are solutions of the classical EoM
- Decompose the fields in Fourier modes :

$$\mathcal{A}_\epsilon(x) \equiv \int \frac{d^3\vec{\mathbf{p}}}{(2\pi)^3 2E_{\mathbf{p}}} \Big[a^{(+)}_\epsilon(x_0,\vec{\mathbf{p}})\, e^{-ip\cdot x} + a^{(-)}_\epsilon(x_0,\vec{\mathbf{p}})\, e^{+ip\cdot x}\Big]$$

Boundary conditions

$$x^0 = -\infty \;:\quad a^{(+)}_+(\vec{\mathbf{p}}) = a^{(-)}_-(\vec{\mathbf{p}}) = 0$$

$$x^0 = +\infty \;:\quad a^{(+)}_-(\vec{\mathbf{p}}) = z(\mathbf{p})\, a^{(+)}_+(\vec{\mathbf{p}})\,,\quad a^{(-)}_+(\vec{\mathbf{p}}) = z(\mathbf{p})\, a^{(-)}_-(\vec{\mathbf{p}})$$

Moyal bracket

- Wigner transform of the commutator
- Explicit expression :

$$\{\{A,B\}\} = \frac{2}{\hbar} A(Q,P)\ \sin\left(\frac{\hbar}{2}(\overleftarrow{\nabla}_Q\overrightarrow{\nabla}_P - \overleftarrow{\nabla}_P\overrightarrow{\nabla}_Q)\right)\ B(Q,P)$$

- Quantum deformation of the Poisson bracket :

$$\{\{A,B\}\} = \{A,B\} + \mathcal{O}(\hbar^2)$$