Gluon saturation from DIS to AA collisions

II – Proton-nucleus collisions

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General outline

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Solution of YM equations

Gluon production

Heavy quark production

- Lecture I : Gluon saturation in DIS
- Lecture II : Proton-nucleus collisions
- Lecture III : AA collisions : gluon production
- Lecture IV : AA collisions : glasma instabilities

Lecture II : Proton-nucleus collisions

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- Solution of Yang-Mills equations
- Gluon production
- Heavy quark production

Introduction

Solution of YM equations

Gluon production

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Introduction

Probing saturation in ideal conditions

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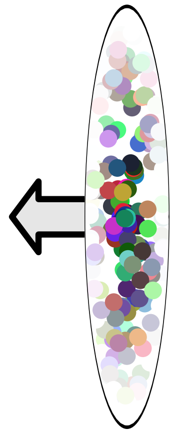
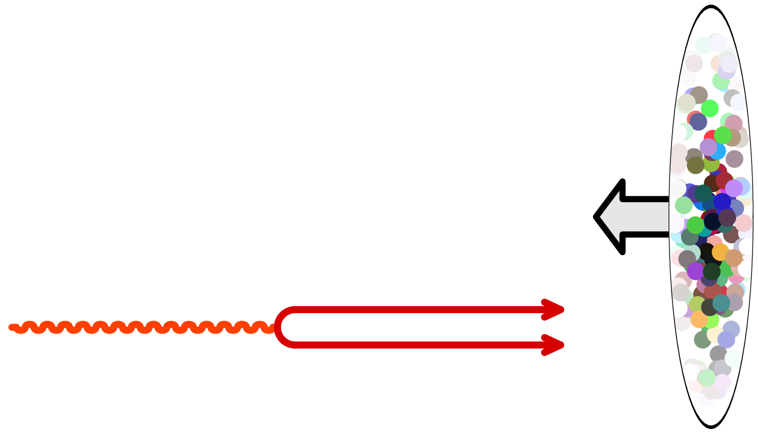


Probing saturation in ideal conditions



- Ideally, one would like to collide the saturated nucleon/nucleus with a well known simple probe that does not involve QCD at all ▷ Deep Inelastic Scattering

Probing saturation in ideal conditions

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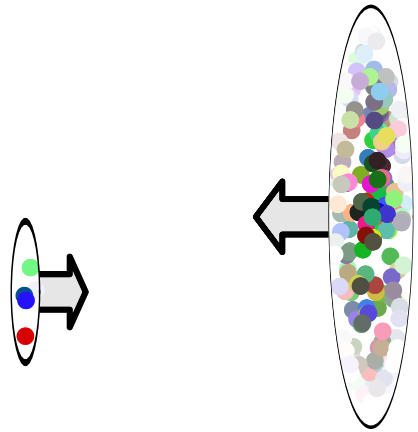

- Ideally, one would like to collide the saturated nucleon/nucleus with a well known simple probe that does not involve QCD at all ▷ Deep Inelastic Scattering
- The next best thing is to probe a saturated hadron with another hadron which is not saturated
  - preferably, the probe should be a nucleon – not a nucleus – whose relevant parton content is not at small x

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- Order 1
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Solution of YM equations

YM equations in covariant gauge

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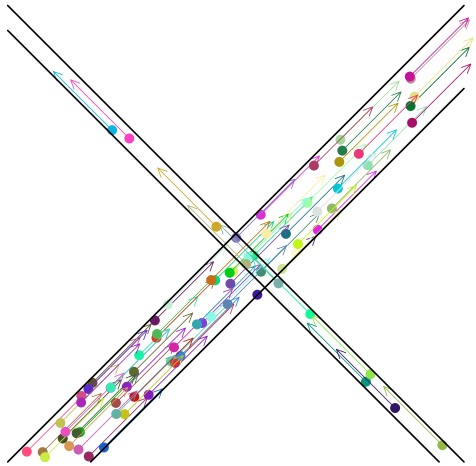
Blaizot, FG, Venugopalan (2004)

- We must solve the Yang-Mills equations with the current :

$$J^\mu(x) \equiv \delta^{\mu +}\,\delta(x^-)\rho_{_A}(\vec{\boldsymbol{x}}_\perp) + \delta^{\mu -}\,\delta(x^+)\rho_{_{\rm p}}(\vec{\boldsymbol{x}}_\perp)$$



- Initial condition : the gauge field vanishes at $x^0 \to -\infty$
- The proton source density $\rho_{_{\rm p}}$ is much smaller than the nuclear one. We will only keep the first order in $\rho_{_{\rm p}}$

Order 0 in the proton source

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- In covariant gauge, $\partial_\mu A^\mu = 0$, the YM equations can be rewritten as :

$$\square A^\nu = J^\nu + ig[A_\mu, F^{\mu\nu} + \partial^\mu A^\nu]$$

- One must also enforce current conservation :

$$[D_\mu, J^\mu] = 0$$

- Reminder : solution at order ρ_{p}^0 (nucleus alone)

$$A_{_0}^+ = -\delta(x^-)\,\frac{1}{\boldsymbol{\partial}_\perp^2}\,\rho_{_A}(\boldsymbol{x}_\perp) \quad , \qquad A_{_0}^- = A_{_0}^i = 0$$

(covariant current conservation is trivial at this order)

- Note : the color field of the proton alone is :

$$A_{\mathrm{p}}^\mu = -\,\delta^{\mu -}\,\delta(x^+)\,\frac{1}{\boldsymbol{\partial}_\perp^2}\,\rho_{\mathrm{p}}(\vec{\boldsymbol{x}}_\perp)$$

Order 1 in the proton source

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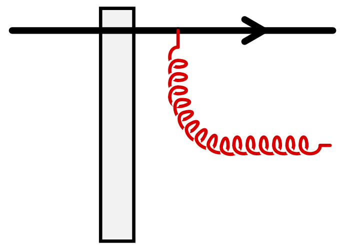
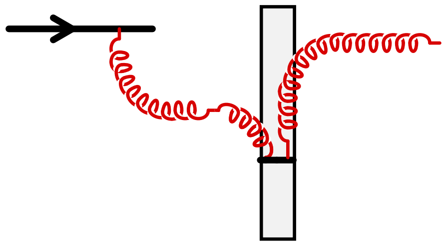
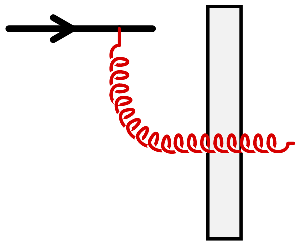
- Color precession of the proton current J^- :



- Color precession of the nuclear current J^+ :



- Multiple scatterings of a gluon in the nuclear field :



Order 1 in the proton source

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- Solution at order $\rho_{\rm p}^1$ in momentum space :

$$A_1^\mu(k) = A_{\rm p}^\mu(k) + \frac{ig}{k^2}\int \frac{d^2\vec{\boldsymbol{k}}_{1\perp}}{(2\pi)^2}\Big\{C_U^\mu\Big[U(\vec{\boldsymbol{k}}_{2\perp}) - (2\pi)^2\delta(\vec{\boldsymbol{k}}_{2\perp})\Big] + C_V^\mu\Big[V(\vec{\boldsymbol{k}}_{2\perp}) - (2\pi)^2\delta(\vec{\boldsymbol{k}}_{2\perp})\Big]\Big\}\frac{\rho_{\rm p}(\vec{\boldsymbol{k}}_{1\perp})}{\boldsymbol{k}_{1\perp}^2}$$

$$C_U^- \equiv -\frac{\boldsymbol{k}_{1\perp}^2}{k^+}\,,\; C_U^+ \equiv \frac{\boldsymbol{k}_{2\perp}^2 - \boldsymbol{k}_\perp^2}{k^-}\,,\; C_U^i \equiv -2\boldsymbol{k}_1^i$$

$$C_V^- \equiv 2k^-\,,\; C_V^+ \equiv -2k^+ + 2\frac{\boldsymbol{k}_\perp^2}{k^-}\,,\; C_V^i \equiv 2\boldsymbol{k}^i$$

$$U(\vec{\boldsymbol{k}}_{2\perp}) \equiv \int_{\vec{\boldsymbol{x}}_\perp} e^{-i\vec{\boldsymbol{k}}_{2\perp}\cdot\vec{\boldsymbol{x}}_\perp}\;\mathrm{T}\exp ig\int dz^-\; A_0^+(z^-,\vec{\boldsymbol{x}}_\perp)$$

$$V(\vec{\boldsymbol{k}}_{2\perp}) \equiv \int_{\vec{\boldsymbol{x}}_\perp} e^{-i\vec{\boldsymbol{k}}_{2\perp}\cdot\vec{\boldsymbol{x}}_\perp}\;\mathrm{T}\exp i\frac{g}{2}\int dz^-\; A_0^+(z^-,\vec{\boldsymbol{x}}_\perp)$$

Note the weird factor $1/2$ in the exponential in V...

Order 1 in the proton source

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- Properties of C_U^μ and C_V^μ :

For any k : $k \cdot C_U = k \cdot C_V = 0$

For k on-shell : $C_U \cdot C_V = C_V^2 = 0$

$$C_U^2 = -4\frac{\boldsymbol{k}_{1\perp}^2 \boldsymbol{k}_{2\perp}^2}{\boldsymbol{k}_\perp^2} \quad \text{(Lipatov's vertex)}$$

- Properties of U and V :

Define U and V with bounds in the integration over z^-, e.g.

$$U(y^-, x^-|\vec{\boldsymbol{x}}_\perp) \equiv \mathrm{T}\,\exp ig \int_{x^-}^{y^-} dz^-\; A_0^+(z^-, \vec{\boldsymbol{x}}_\perp)$$

Exercise : prove that we have :

$$U(y^-, x^-) - V(y^-, x^-) = \frac{ig}{2}\int_{x^-}^{y^-} dz^- U(y^-, z^-) A_0^+(z^-) V(z^-, x^-)$$

Solution in other gauges

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- Schwinger gauge : $x^+ A^- + x^- A^+ = 0$

  Dumitru, McLerran (2002)

- Light-cone gauge of the proton : $A^- = 0$

  FG, Mehtar-Tani (2006)

  The advantage of this gauge is that the proton does not affect the sources of the nucleus. The nuclear field can be treated as a background that one calculates once for all

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Gluon production

Amplitude

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Heavy quark production

- The gluon production amplitude is given by :

$$\mathcal{M}_{\rm g}^{(\lambda)}(\boldsymbol{k}) = k^2\ A_1^\mu(k)\ \epsilon_\mu^{(\lambda)}(\boldsymbol{k})$$

- Sum over the polarizations :

$$\sum_\lambda \epsilon_\mu^{(\lambda)}(\boldsymbol{q})\epsilon_\nu^{(\lambda)*}(\boldsymbol{q}) = -g_{\mu\nu}$$

(using this formula includes non physical polarizations as well, but they do not contribute thanks to the transversality of the color field)

- When we square the amplitude, we only get a correlator $\big\langle UU^\dagger\big\rangle$, thanks to the properties of C_U^μ and C_V^μ (in particular, V does not contribute)

Gluon yield

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- After squaring the amplitude, one gets :

$$\frac{dN_g}{d^2\vec{\boldsymbol{k}}_\perp dy} \sim \frac{\alpha_s}{\boldsymbol{k}_\perp^2} \int \frac{d^2\boldsymbol{p}_\perp}{(2\pi)^2} \; \phi_p(\boldsymbol{k}_\perp - \boldsymbol{p}_\perp) \; \frac{d\phi_A(\boldsymbol{p}_\perp|\boldsymbol{b})}{d^2\boldsymbol{X}_\perp}$$

  Note : this formula is compatible with $k_\perp$-factorization (see Kovchegov, Tuchin (2002) for a proof that this formula is valid at leading log)

- ϕ_p is the non integrated gluon distribution of the proton

- $d\phi_A/d^2\boldsymbol{X}_\perp$ is the non integrated gluon distribution of the nucleus, at the impact parameter $\boldsymbol{b}$:

$$\frac{d\phi_A(\vec{\boldsymbol{p}}_\perp|\boldsymbol{b})}{d^2\vec{\boldsymbol{X}}_\perp} = \frac{\boldsymbol{p}_\perp^2}{4\alpha_s N_c} \int d^2\vec{\boldsymbol{r}}_\perp \; e^{i\vec{\boldsymbol{p}}_\perp\cdot\vec{\boldsymbol{r}}_\perp} \; \mathrm{tr}\left\langle U(\boldsymbol{b}+\frac{\vec{\boldsymbol{r}}_\perp}{2})U^\dagger(\boldsymbol{b}-\frac{\vec{\boldsymbol{r}}_\perp}{2})\right\rangle$$

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- Limit of collinear factorization in the proton :

  If one assumes that the proton non integrated gluon distribution is much narrower than the nuclear one, we can assume

$$\left|\boldsymbol{k}_\perp - \boldsymbol{p}_\perp\right| \ll \left|\boldsymbol{p}_\perp\right|$$

  and thus

$$\boldsymbol{p}_\perp \approx \boldsymbol{k}_\perp$$

  Therefore,

$$\frac{dN_{\rm g}}{d^2\vec{\boldsymbol{k}}_\perp dy} \sim \frac{\alpha_s}{\boldsymbol{k}_\perp^2} \frac{d\phi_A(\boldsymbol{k}_\perp|\boldsymbol{b})}{d^2\boldsymbol{X}_\perp} \underbrace{\int^{|\boldsymbol{k}_\perp|} \frac{d^2\boldsymbol{q}_\perp}{(2\pi)^2} \;\phi_{\rm p}(\boldsymbol{q}_\perp)}_{x_1 G_{\rm p}(x_1, \boldsymbol{k}_\perp^2)}$$

McLerran-Venugopalan model

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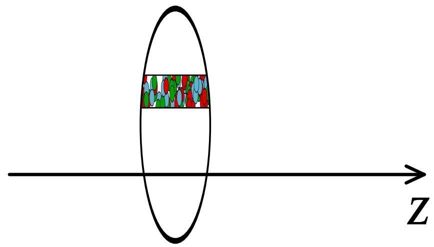
- The JIMWLK equation must be completed by an initial condition, given at some moderate x_0
- The McLerran-Venugopalan model is often used as an initial condition at moderate x_0 for a large nucleus :



  - partons distributed randomly
  - many partons in a small tube
  - no correlations at different $\vec{\boldsymbol{x}}_\perp$

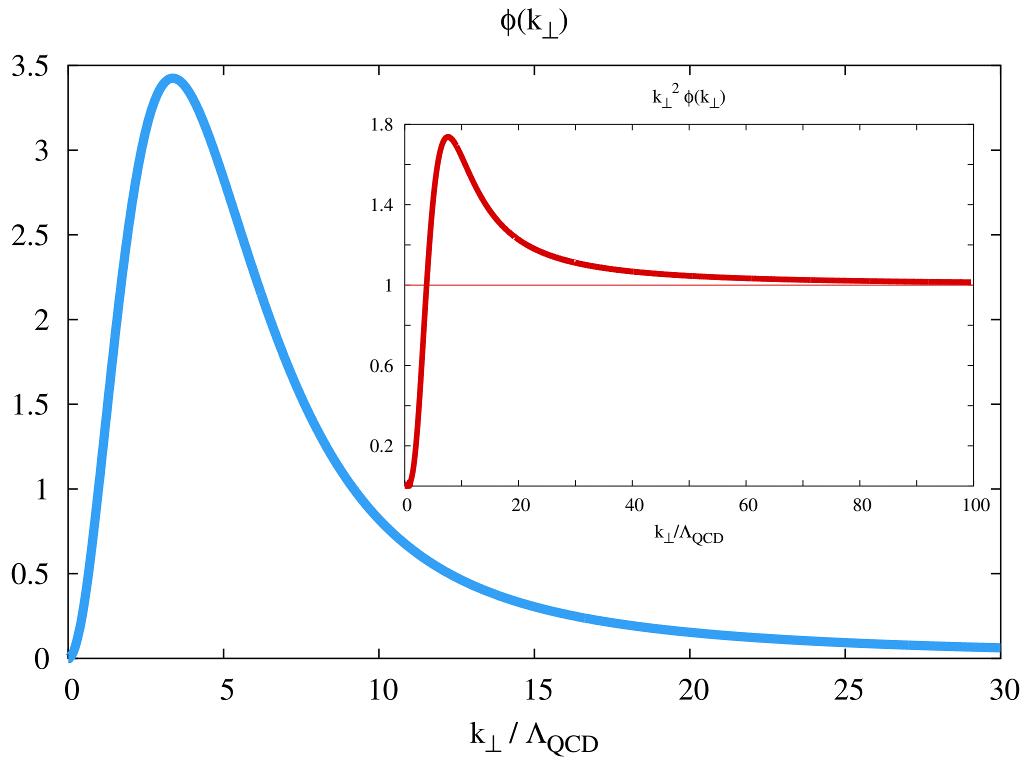
- The MV model assumes that the density of color charges $\rho(\vec{\boldsymbol{x}}_\perp)$ has a Gaussian distribution :

$$W_Y[\rho] = \exp\left[-\int d^2\vec{\boldsymbol{x}}_\perp \frac{\rho_a(\vec{\boldsymbol{x}}_\perp)\rho_a(\vec{\boldsymbol{x}}_\perp)}{2\mu^2(Y)}\right]$$

McLerran-Venugopalan model

- φ_A in the McLerran-Venugopalan model:

$$W_Y[\rho_A] = \exp\left[-\int_{\vec{x}_\perp} \frac{\rho_{A,a}(\vec{x}_\perp)\rho_{A,a}(\vec{x}_\perp)}{2\mu_A^2(Y)}\right]$$



McLerran-Venugopalan model

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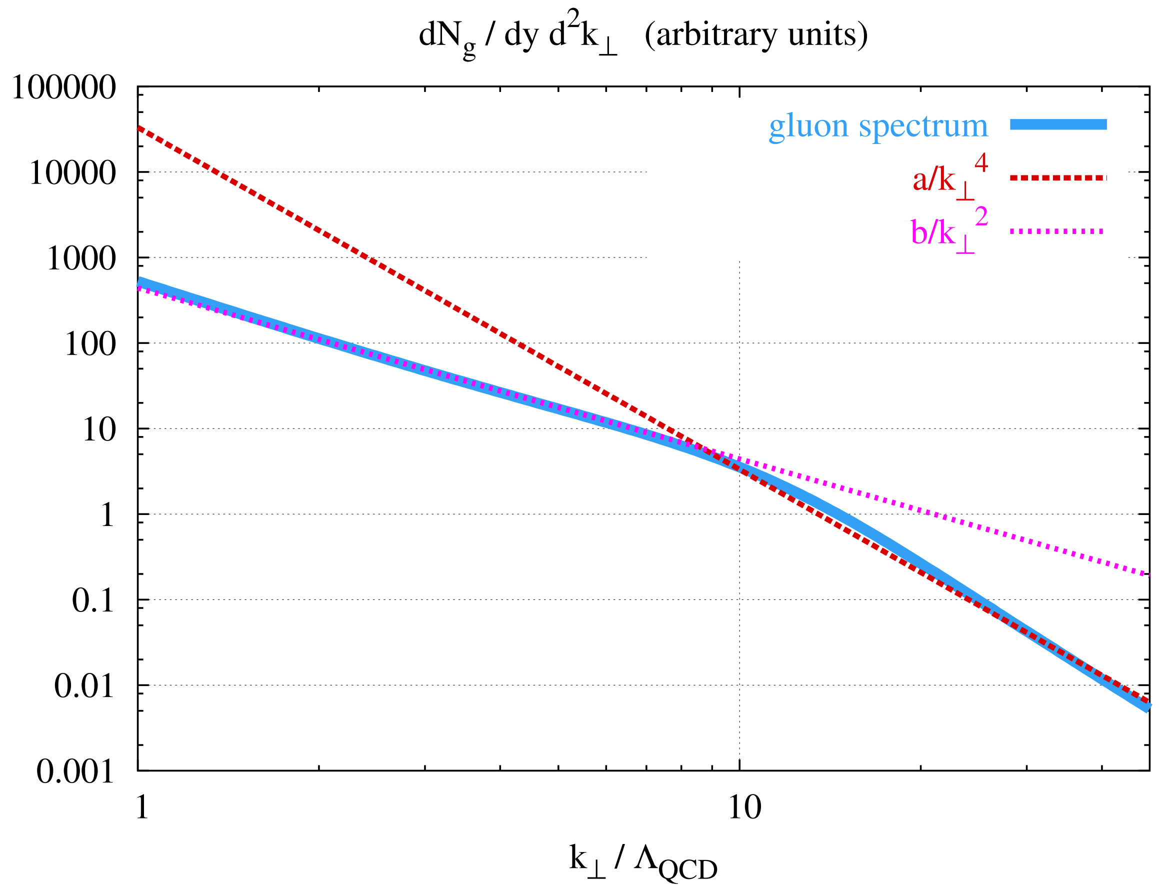
- Gluon spectrum in the MV model:



McLerran-Venugopalan model

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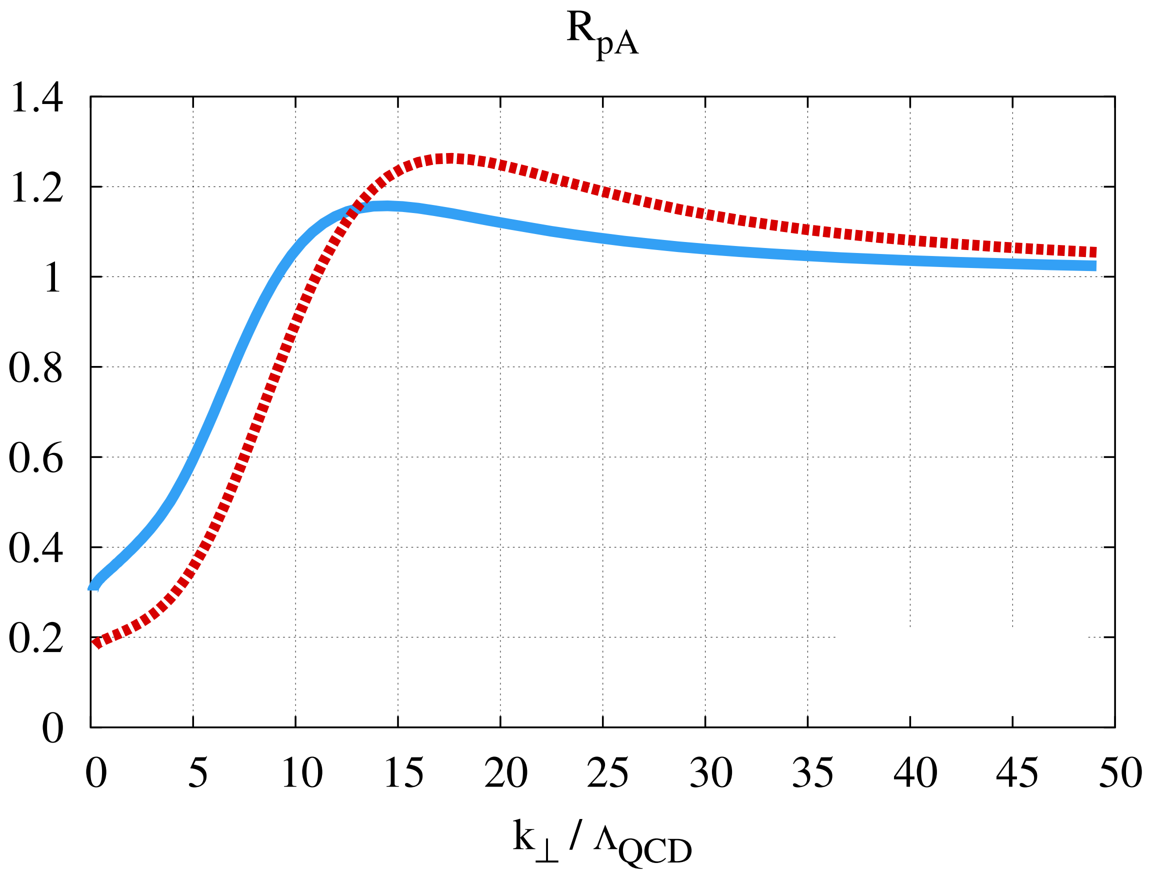
- The ratio R_{pA} in the MV model:



(stronger effect if Q_s is larger)

High pt suppression at large Y

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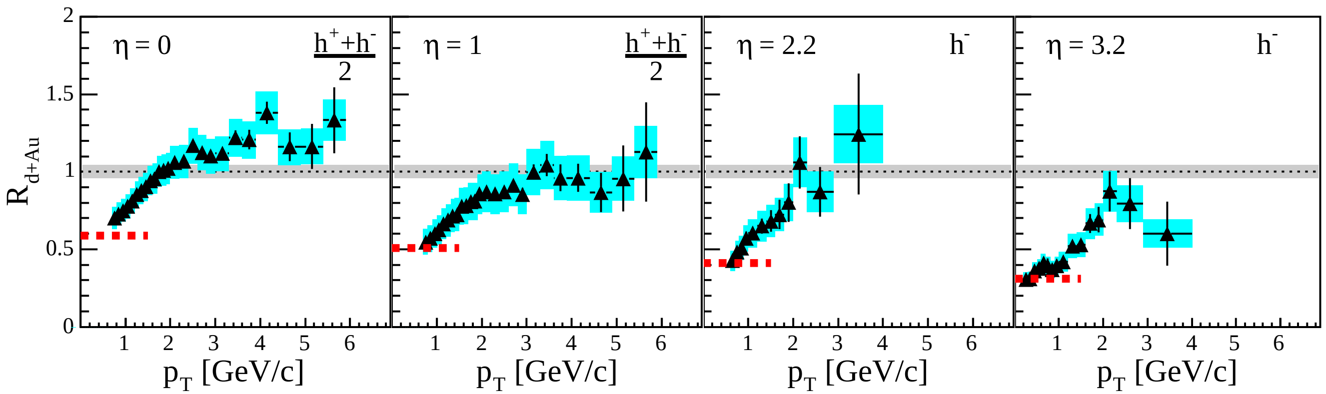
- Results of the BRAHMS experiment at RHIC for deuteron-gold collisions :

$$R_{\rm dAu} \equiv \frac{1}{N_{\rm coll}} \frac{\left.\frac{dN}{dp_\perp d\eta}\right|_{\rm dAu}}{\left.\frac{dN}{dp_\perp d\eta}\right|_{\rm pp}}$$



  - At small rapidity, suppression at low $p_\perp$ and enhancement at high $p_\perp$ (multiple scatterings – Cronin effect)
  - At large rapidity, suppression at all $p_\perp$'s (shadowing)

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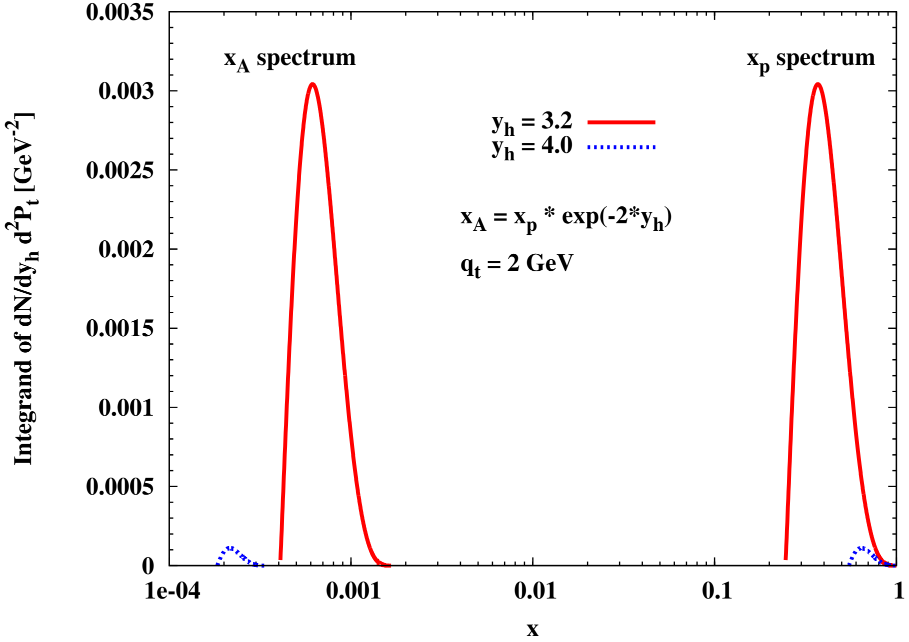
Kinematics

- Relevant values of $x_{1,2}$:



- Note : the MV model has some Cronin effect, but cannot lead to a suppression at forward rapidity
- Evolution to small-x (BK, JIMWLK) leads to a suppression

RdA at RHIC from the BK equation

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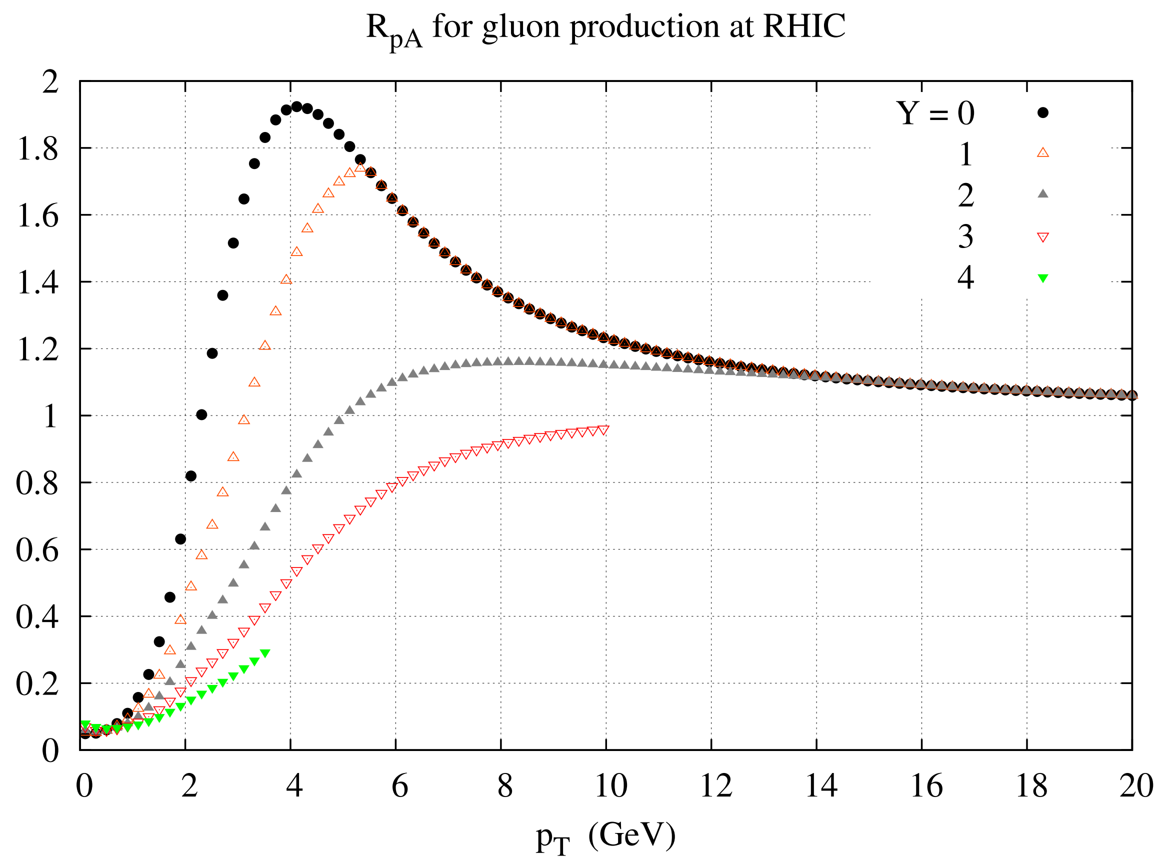

Albacete, Armesto, Kovner, Salgado, Wiedemann (2004)

RpA at LHC from the BK equation

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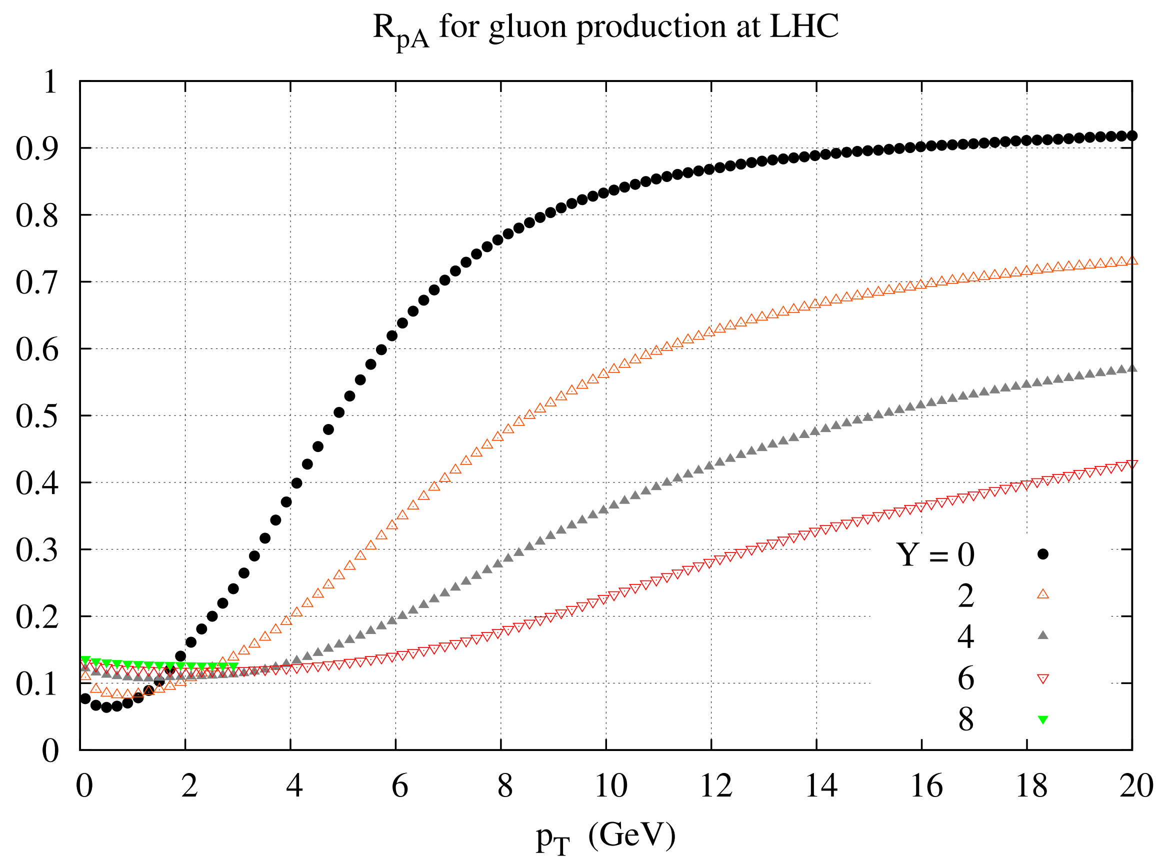

dA collisions at RHIC

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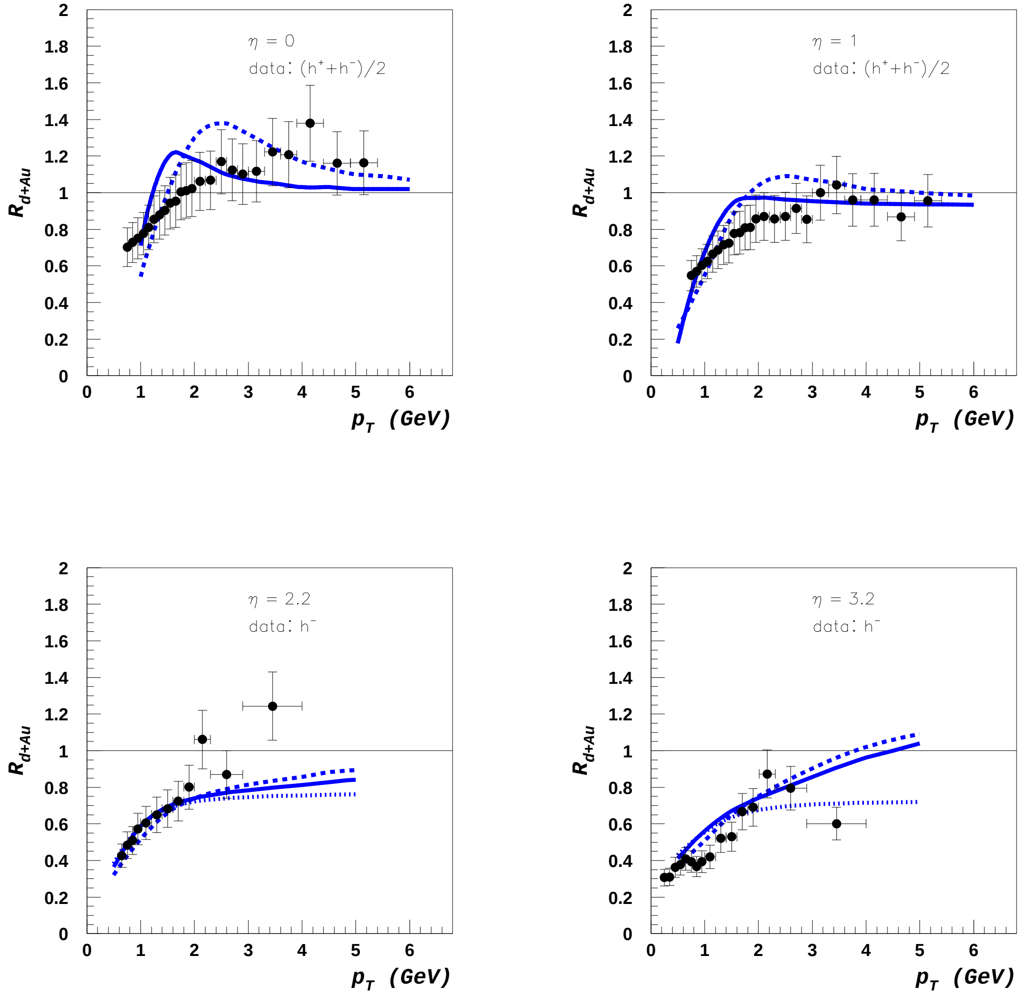
- Kharzeev, Kovchegov, Tuchin (2005)



dA collisions at RHIC

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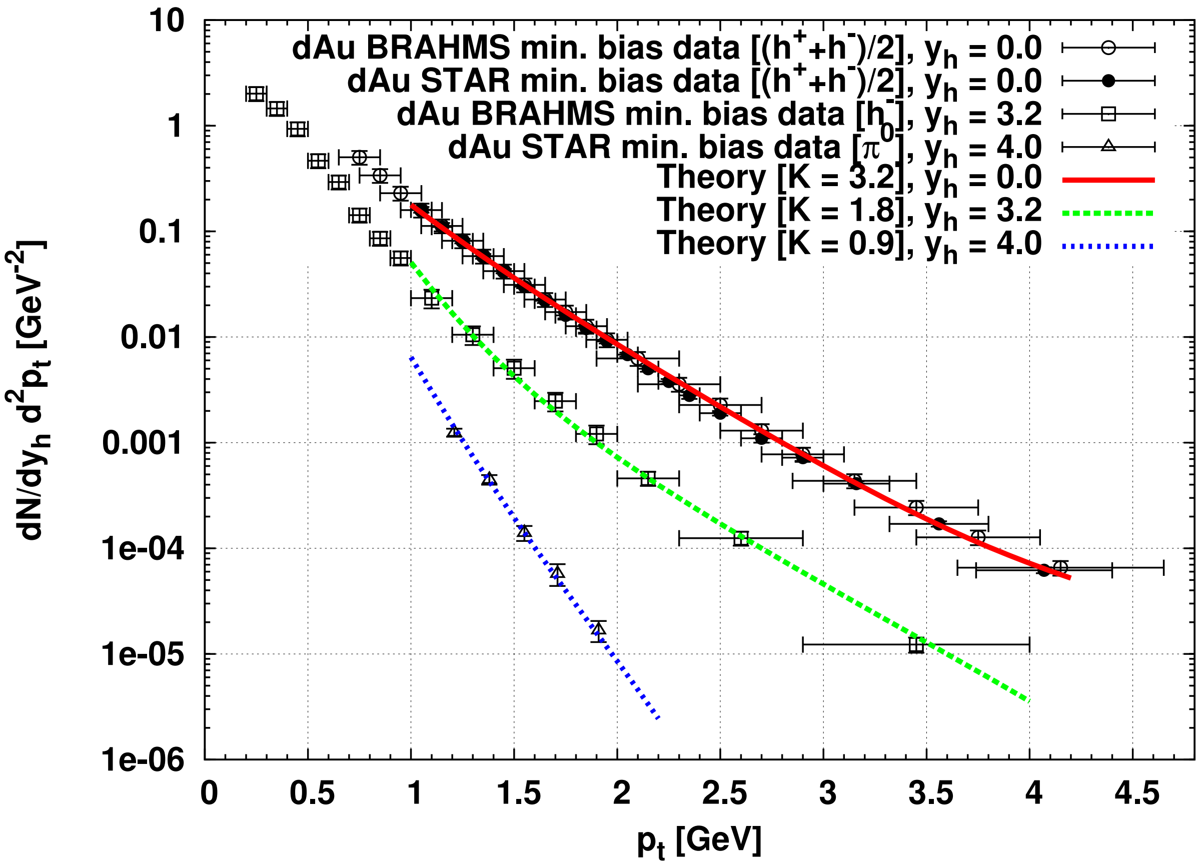
- Dumitru, Hayashigaki, Jalilian-Marian (2005 – 2006)



Note : the model predicts only the slope of the spectrum; its normalization is adjusted by a Y-dependent K-factor

Limiting fragmentation (RHIC)

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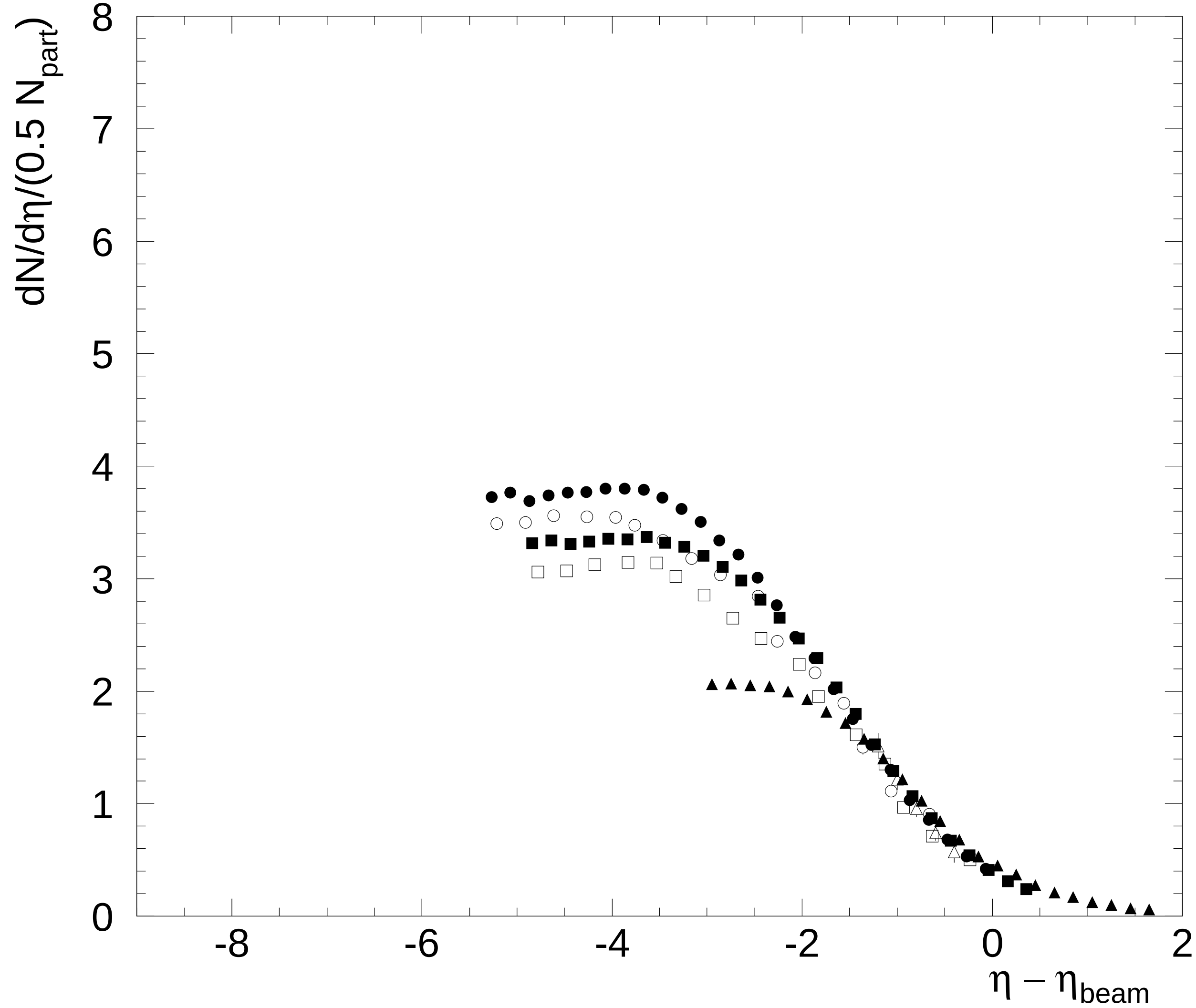

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Qualitative explanation

- The ratio of the two saturation scales is :
  $Q_s^2(x_2)/Q_s^2(x_1) \sim \exp(2\lambda Y) \sim 20$ with $\lambda \approx 0.3$ and $Y = 5$

  ▷ neglect the transverse momentum in the projectile at large x_1 compared to that in the projectile at small x_2

  ▷ use collinear factorization for projectile 1

- The spectrum reads :

$$\frac{dN_{\rm g}}{d^2\vec{\boldsymbol{p}}_\perp dY} \sim x_1 f(x_1, \boldsymbol{p}_\perp^2) \underline{\int d^2\vec{\boldsymbol{r}}_\perp\; e^{i\vec{\boldsymbol{p}}_\perp\cdot\vec{\boldsymbol{r}}_\perp} \left<{\rm tr}\left(U(0)U^\dagger(\vec{\boldsymbol{r}}_\perp)\right)\right>_{x_2}}$$

  Note : the underlined factor becomes independent of x_2 when integrated over $\vec{\boldsymbol{p}}_\perp$ because of the unitarity of the Wilson lines

- At large x_1, $x_1\, f(x_1, \boldsymbol{p}_\perp^2)$ is almost independent of $\boldsymbol{p}_\perp^2$ (Bjorken scaling), and the integration over $\vec{\boldsymbol{p}}_\perp$ leads to :

$$\frac{dN}{dY} \propto x_1\, f(x_1) \quad \Rightarrow \quad \text{depends only on } x_1 \sim \exp(Y - Y_{\rm beam})$$

Distribution of recoils

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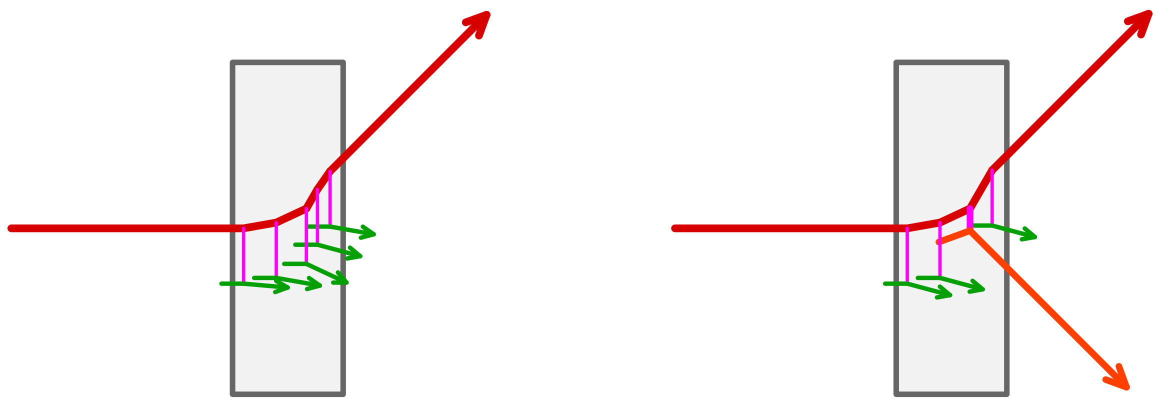
Heavy quark production

FG, Borghini (2006)

- Since in this description a pA collision amounts to multiple scatterings of a parton from the proton on those of the nucleus, an interesting issue is the distribution of the recoils when the incoming parton is scattered at a high $\boldsymbol{p}_\perp$



- If the recoil momentum is shared evenly between a large number of partons, the final state will look like a monojet
- If a single parton takes most of the recoil, then the final state will look like a standard di-jet event

Number of recoils above Ktmin

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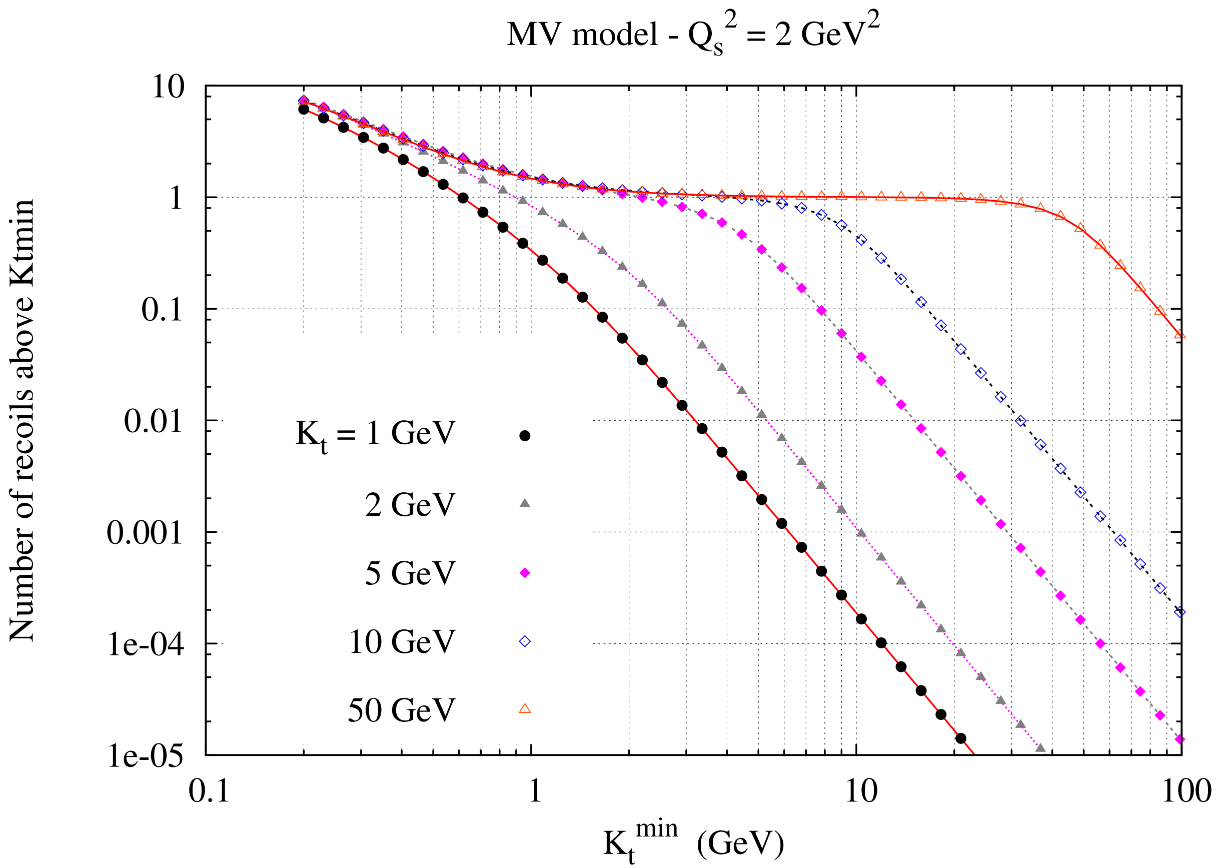

- When $Q_s \lesssim k_\perp^{\rm min} \lesssim k_\perp$, there is only one recoil

  ▷ the momentum of the scattered parton is absorbed by a single source ▷ pair of jets rather than a monojet

Levy random walks

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- Interpretation : the scattering of the incoming parton can be seen a a random walk in $\boldsymbol{p}_\perp$ space, with a probability $\mathcal{P}(k_\perp)$ to gain $\vec{\boldsymbol{k}}_\perp$ at each step of the random walk
- A crucial property of $\mathcal{P}(k_\perp)$ is whether its second moment,

$$\sigma \equiv \int d^2\vec{\boldsymbol{k}}_\perp \; k_\perp^2 \; \mathcal{P}(k_\perp) \; , \quad \text{is finite or not}$$

Levy random walks

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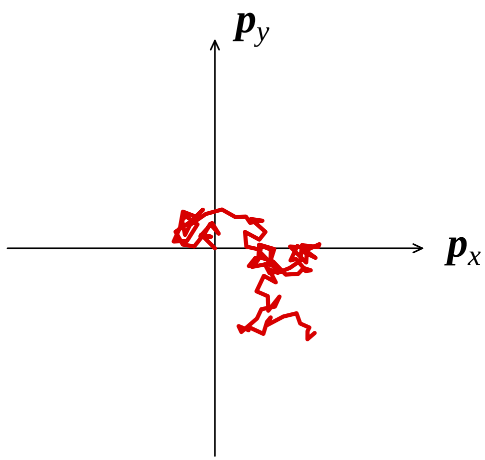
$$\sigma \equiv \int d^2\vec{\boldsymbol{k}}_\perp\; k_\perp^2\; \mathcal{P}(k_\perp)\;, \qquad \text{is finite or not}$$

- If σ is finite, the random walk takes an exponentially large number of steps to get far from the origin :



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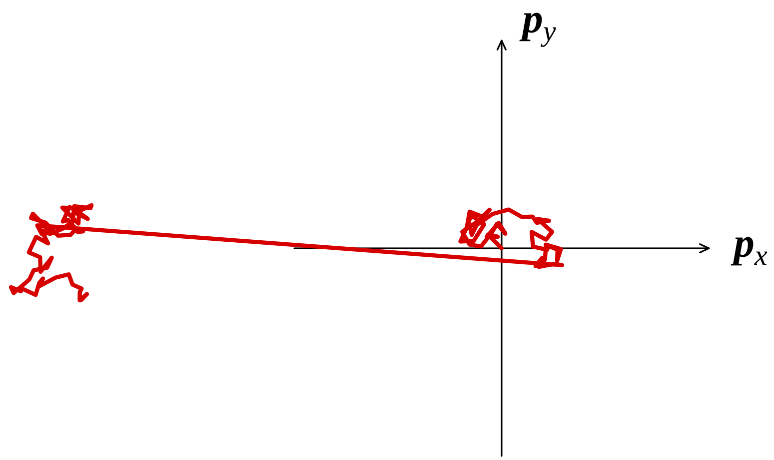
$$\sigma \equiv \int d^2\vec{\boldsymbol{k}}_\perp \; k_\perp^2 \; \mathcal{P}(k_\perp) \; , \qquad \text{is finite or not}$$

- If σ is infinite (true for the MV model), the random walk can go far from the origin in one big step and a few small ones :



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- Amplitude
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- Kt factorization
- Cronin effect
- Pair production

Heavy quark production

CERN

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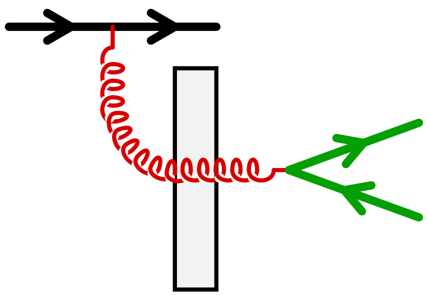
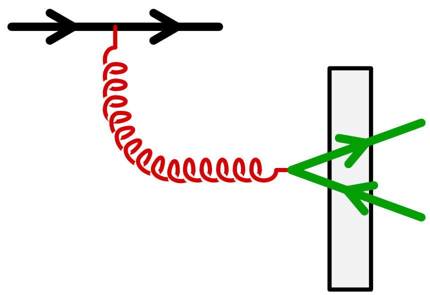
- Pair production

Blaizot, FG, Venugopalan (2004), Tuchin (2004)

- We expect that the pair is produced either before or after the collision with the nucleus. The production of the pair inside the nucleus should be suppressed by $s^{-1/2}$

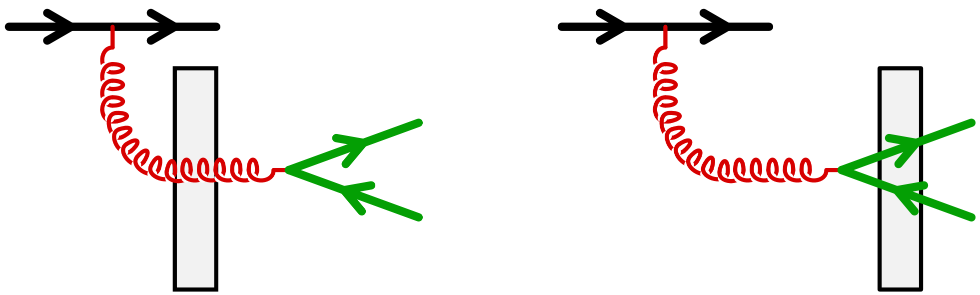




Heavy quark production

Blaizot, FG, Venugopalan (2004), Tuchin (2004)

- We expect that the pair is produced either before or after the collision with the nucleus. The production of the pair inside the nucleus should be suppressed by $s^{-1/2}$



- The manifestation of this property is somewhat obfuscated at the amplitude level :
  - True for the amplitude if the classical field A^μ inside the nucleus remains bounded when $s \to \infty$
  - This is not the case in covariant gauge...
  - One must split the field into a singular part (proportional to $\delta(x^-)$) and a regular part (that has no $\delta(x^-)$)

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Quark production amplitude

- Regular contributions to the amplitude:

$$\mathcal{M}^{\rm reg}_{Q\overline{Q}} = g^2 \int\limits_{\vec{\boldsymbol{k}}_{1\perp},\vec{\boldsymbol{k}}_\perp} \frac{\rho_{{\rm p},a}(\vec{\boldsymbol{k}}_{1\perp})}{k_{1\perp}^2} \int\limits_{\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp} e^{i\vec{\boldsymbol{k}}_\perp\cdot\vec{\boldsymbol{x}}_\perp} e^{i(\vec{\boldsymbol{p}}_\perp+\vec{\boldsymbol{q}}_\perp-\vec{\boldsymbol{k}}_\perp-\vec{\boldsymbol{k}}_{1\perp})\cdot\vec{\boldsymbol{y}}_\perp}$$

$$\times\ \overline{u}(\vec{\boldsymbol{q}}) \Bigg\{ \frac{\gamma^-(\not{q}-\not{k}+m)\gamma^+(\not{q}-\not{k}-\not{k}_1+m)\gamma^-[\widetilde{U}(\vec{\boldsymbol{x}}_\perp)t^a\widetilde{U}^\dagger(\vec{\boldsymbol{y}}_\perp)]}{2p^-[(\vec{\boldsymbol{q}}_\perp-\vec{\boldsymbol{k}}_\perp)^2+m^2]+2q^-[(\vec{\boldsymbol{q}}_\perp-\vec{\boldsymbol{k}}_\perp-\vec{\boldsymbol{k}}_{1\perp})^2+m^2]}$$

$$+t^b\Big[\frac{\not{C}_U(p+q,\vec{\boldsymbol{k}}_{1\perp})}{(p+q)^2}U_{ba}(\vec{\boldsymbol{x}}_\perp) - \frac{\gamma^-}{p^-+q^-}V_{ba}(\vec{\boldsymbol{x}}_\perp)\Big]\Bigg\} v(\vec{\boldsymbol{p}})$$

- Notes:
  - $\widetilde{U}$ is a Wilson line in the fundamental representation
  - the Wilson line V is still there !

Quark production amplitude

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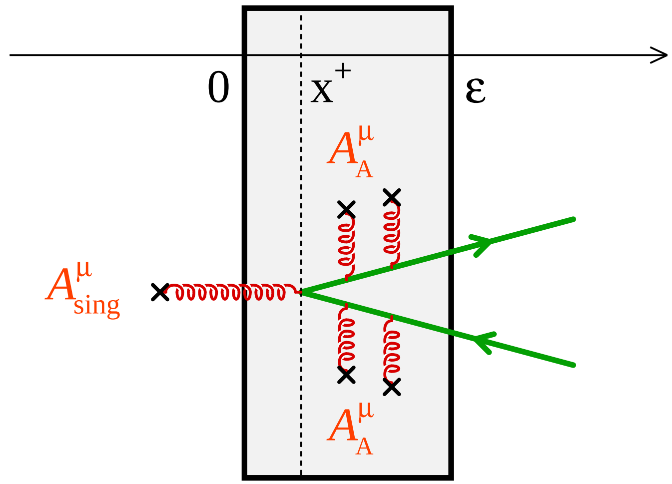
- Singular diagram :



- Corresponding term in the amplitude:

$$\mathcal{M}^{\rm sing}_{Q\overline{Q}} = g^2 \int_{\vec{\boldsymbol{k}}_{1\perp}} \frac{\rho_{{\rm p},a}(\vec{\boldsymbol{k}}_{1\perp})}{k_{1\perp}^2} \int_{\vec{\boldsymbol{x}}_\perp} e^{i(\vec{\boldsymbol{p}}_\perp + \vec{\boldsymbol{q}}_\perp - \vec{\boldsymbol{k}}_{1\perp})\cdot\vec{\boldsymbol{x}}_\perp}$$
$$\times \frac{\overline{u}(\vec{\boldsymbol{q}})\gamma^- t^b v(\vec{\boldsymbol{p}})}{p^- + q^-} \left[V_{ba}(\vec{\boldsymbol{x}}_\perp) - U_{ba}(\vec{\boldsymbol{x}}_\perp)\right]$$

Quark production amplitude

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- Total amplitude :

$$\mathcal{M}_F = g^2 \int_{\vec{\boldsymbol{k}}_{1\perp},\vec{\boldsymbol{k}}_\perp} \frac{\rho_{\mathrm{p},a}(\vec{\boldsymbol{k}}_{1\perp})}{k_{1\perp}^2} \int_{\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp} e^{i\vec{\boldsymbol{k}}_\perp\cdot\vec{\boldsymbol{x}}_\perp} e^{i(\vec{\boldsymbol{p}}_\perp+\vec{\boldsymbol{q}}_\perp-\vec{\boldsymbol{k}}_\perp-\vec{\boldsymbol{k}}_{1\perp})\cdot\vec{\boldsymbol{y}}_\perp}$$

$$\times\overline{u}(\vec{\boldsymbol{q}})\Big\{[\widetilde{U}(\vec{\boldsymbol{x}}_\perp)t^a\widetilde{U}^\dagger(\vec{\boldsymbol{y}}_\perp)]T_{q\bar{q}}(\vec{\boldsymbol{k}}_\perp) + [t^bU_{ba}(\vec{\boldsymbol{x}}_\perp)]\not{L}\Big\}v(\vec{\boldsymbol{p}})$$

with

$$T_{q\bar{q}}(\vec{\boldsymbol{k}}_\perp) \equiv \frac{\gamma^-(\not{q}-\not{k}+m)\gamma^+(\not{q}-\not{k}-\not{k}_1+m)\gamma^-}{2p^-[(\vec{\boldsymbol{q}}_\perp-\vec{\boldsymbol{k}}_\perp)^2+m^2] + 2q^-[(\vec{\boldsymbol{q}}_\perp-\vec{\boldsymbol{k}}_\perp-\vec{\boldsymbol{k}}_{1\perp})^2+m^2]}$$

- Notes:
  - The V's cancel between regular and singular contributions
  - The terms with the adjoint Wilson line U combine to be proportional to Lipatov's vertex L^μ
  - Our original expectations are now fulfilled...

Pair cross-section

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- Pair production cross-section:

$$\frac{d\sigma_{Q\overline{Q}}}{d^2\vec{\boldsymbol{p}}_\perp d^2\vec{\boldsymbol{q}}_\perp dy_p dy_q} = \frac{\alpha_s^2 N_c}{8\pi^4 d_A} \int_{\vec{\boldsymbol{k}}_{1\perp},\vec{\boldsymbol{k}}_{2\perp}} \frac{\delta(\vec{\boldsymbol{p}}_\perp + \vec{\boldsymbol{q}}_\perp - \vec{\boldsymbol{k}}_{1\perp} - \vec{\boldsymbol{k}}_{2\perp})}{\boldsymbol{k}_{1\perp}^2 \boldsymbol{k}_{2\perp}^2}$$

$$\times\Big\{ \int_{\vec{\boldsymbol{k}}_\perp,\vec{\boldsymbol{k}}'_\perp} \mathrm{tr}\Big[(\not{q}+m)T_{q\bar{q}}(\vec{\boldsymbol{k}}_\perp)(\not{p}-m)T^*_{q\bar{q}}(\vec{\boldsymbol{k}}'_\perp)\Big]\phi_A^{q\bar{q},q\bar{q}}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp,\vec{\boldsymbol{k}}'_\perp)$$

$$+\int_{\vec{\boldsymbol{k}}_\perp} \mathrm{tr}\Big[(\not{q}+m)T_{q\bar{q}}(\vec{\boldsymbol{k}}_\perp)(\not{p}-m)\not{L}^* + \mathrm{h.c.}\Big]\phi_A^{q\bar{q},g}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp)$$

$$+\mathrm{tr}\Big[(\not{q}+m)\not{L}(\not{p}-m)\not{L}^*\Big]\phi_A^{g,g}(\vec{\boldsymbol{k}}_{2\perp})\Big\}\phi_{\mathrm{p}}(\vec{\boldsymbol{k}}_{1\perp})$$

▷ compatible with $k_\perp$-factorization on the proton side, but not for the nucleus: one needs three different “distributions” in order to describe the nucleus

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- Nuclear “gluon distributions”:

$$\phi_A^{g,g}(\vec{\boldsymbol{k}}_{2\perp}) = \frac{k_{2\perp}^2}{4\alpha_s N_c} \int_{\vec{\boldsymbol{x}}_\perp, \vec{\boldsymbol{y}}_\perp} e^{i\vec{\boldsymbol{k}}_{2\perp}\cdot(\vec{\boldsymbol{x}}_\perp - \vec{\boldsymbol{y}}_\perp)} \; \mathrm{tr}\left\langle U(\vec{\boldsymbol{x}}_\perp)U^\dagger(\vec{\boldsymbol{y}}_\perp)\right\rangle$$

$$\phi_A^{q\bar{q},g}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp) = \frac{k_{2\perp}^2}{2\alpha_s N_c} \int_{\vec{\boldsymbol{x}}_\perp, \vec{\boldsymbol{y}}_\perp, \vec{\boldsymbol{z}}_\perp} e^{i\left[\vec{\boldsymbol{k}}_\perp\cdot\vec{\boldsymbol{x}}_\perp + (\vec{\boldsymbol{k}}_{2\perp} - \vec{\boldsymbol{k}}_\perp)\cdot\vec{\boldsymbol{y}}_\perp - \vec{\boldsymbol{k}}_{2\perp}\cdot\vec{\boldsymbol{z}}_\perp\right]} \times \mathrm{tr}\left\langle \widetilde{U}(\vec{\boldsymbol{x}}_\perp)t^a\widetilde{U}^\dagger(\vec{\boldsymbol{y}}_\perp)t^b U_{ba}(\vec{\boldsymbol{z}}_\perp)\right\rangle$$

$$\phi_A^{q\bar{q},q\bar{q}}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp, \vec{\boldsymbol{k}}'_\perp) = \frac{k_{2\perp}^2}{2\alpha_s N_c} \int_{\vec{\boldsymbol{x}}_\perp, \vec{\boldsymbol{y}}_\perp, \vec{\boldsymbol{x}}'_\perp, \vec{\boldsymbol{y}}'_\perp} e^{i\left[\vec{\boldsymbol{k}}_\perp\cdot\vec{\boldsymbol{x}}_\perp - \vec{\boldsymbol{k}}'_\perp\cdot\vec{\boldsymbol{x}}'_\perp + (\vec{\boldsymbol{k}}_{2\perp} - \vec{\boldsymbol{k}}_\perp)\cdot\vec{\boldsymbol{y}}_\perp - (\vec{\boldsymbol{k}}_{2\perp} - \vec{\boldsymbol{k}}'_\perp)\cdot\vec{\boldsymbol{y}}'_\perp\right]} \times \mathrm{tr}\left\langle \widetilde{U}(\vec{\boldsymbol{x}}_\perp)t^a\widetilde{U}^\dagger(\vec{\boldsymbol{y}}_\perp)\widetilde{U}(\vec{\boldsymbol{y}}'_\perp)t^a\widetilde{U}(\vec{\boldsymbol{x}}'_\perp)\right\rangle$$

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Pair cross-section

Sum rules and $k_\perp$-factorization:

- The 2-, 3- and 4-point functions are related by:

$$\int_{\vec{\boldsymbol{k}}_\perp,\vec{\boldsymbol{k}}'_\perp} \phi_A^{q\bar{q},q\bar{q}}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp,\vec{\boldsymbol{k}}'_\perp) = \int_{\vec{\boldsymbol{k}}_\perp} \phi_A^{q\bar{q},g}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp) = \phi_A^{g,g}(\vec{\boldsymbol{k}}_{2\perp})$$

- $k_\perp$-factorization would be valid if one could neglect the $\vec{\boldsymbol{k}}_\perp$ dependence in $T_{q\bar{q}}(\vec{\boldsymbol{k}}_\perp)$
- this happens if the $Q\bar{Q}$ pair has a small transverse size (compared to the typical scale in the nucleus, i.e. Q_s^{-1})
Note: physically, this means that the $Q\overline{Q}$ pair propagates through the nucleus as if it were a gluon
- $k_\perp$-factorization should be recovered in the following limits:
$m \to \infty$, $m(Q\overline{Q}) \to \infty$, $p_\perp(Q) \to \infty$

Quark cross-section

- Single quark production cross-section:

$$\frac{d\sigma_q}{d^2\vec{\boldsymbol{q}}_\perp dy_q} = \frac{\alpha_s^2 N}{8\pi^4 d_A}\int\frac{dp^+}{p^+}\int_{\vec{\boldsymbol{k}}_{1\perp},\vec{\boldsymbol{k}}_{2\perp}}\frac{1}{\boldsymbol{k}_{1\perp}^2\boldsymbol{k}_{2\perp}^2}$$
$$\times\Big\{\mathrm{tr}\Big[(\not{q}+m)T_{q\bar{q}}(\vec{\boldsymbol{k}}_{2\perp})(\not{p}-m)T^*_{q\bar{q}}(\vec{\boldsymbol{k}}_{2\perp})\Big]\frac{C_F}{N}\phi_A^{q,q}(\vec{\boldsymbol{k}}_{2\perp})$$
$$+\int_{\vec{\boldsymbol{k}}_\perp}\mathrm{tr}\Big[(\not{q}+m)T_{q\bar{q}}(\vec{\boldsymbol{k}}_\perp)(\not{p}-m)\not{L}^* + \mathrm{h.c.}\Big]\phi_A^{q\bar{q},g}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp)$$
$$+\mathrm{tr}\Big[(\not{q}+m)\not{L}(\not{p}-m)\not{L}^*\Big]\phi_A^{g,g}(\vec{\boldsymbol{k}}_{2\perp})\Big\}\phi_p(\vec{\boldsymbol{k}}_{1\perp})$$

  - $\phi_A^{q,q}$ is the analogue of $\phi_A^{g,g}$ for the fundamental representation
  - $k_\perp$-factorization still broken for the nucleus
  - contains only 2-point and 3-point correlators

Kt factorization

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Fujii, FG, Venugopalan (2005,2006)

- $\boldsymbol{k}_\perp$-factorization holds if the 3-point and 2-point functions are related by:

$$\phi_A^{q\bar{q},g}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp) = (2\pi)^2 \frac{1}{2}\Big[\delta(\vec{\boldsymbol{k}}_\perp) + \delta(\vec{\boldsymbol{k}}_\perp - \vec{\boldsymbol{k}}_{2\perp})\Big] \phi_A^{g,g}(\vec{\boldsymbol{k}}_{2\perp})$$

- This relation means that the $Q\overline{Q}$ pair interacts with the nucleus in such a way that all the momentum exchanged goes to the quark or to the antiquark
- The ratio $\phi_A^{q\bar{q},g}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp)/\phi_A^{g,g}(\vec{\boldsymbol{k}}_{2\perp})$ should be close to the sum of two delta functions for $k_\perp$-factorization to be a good approximation

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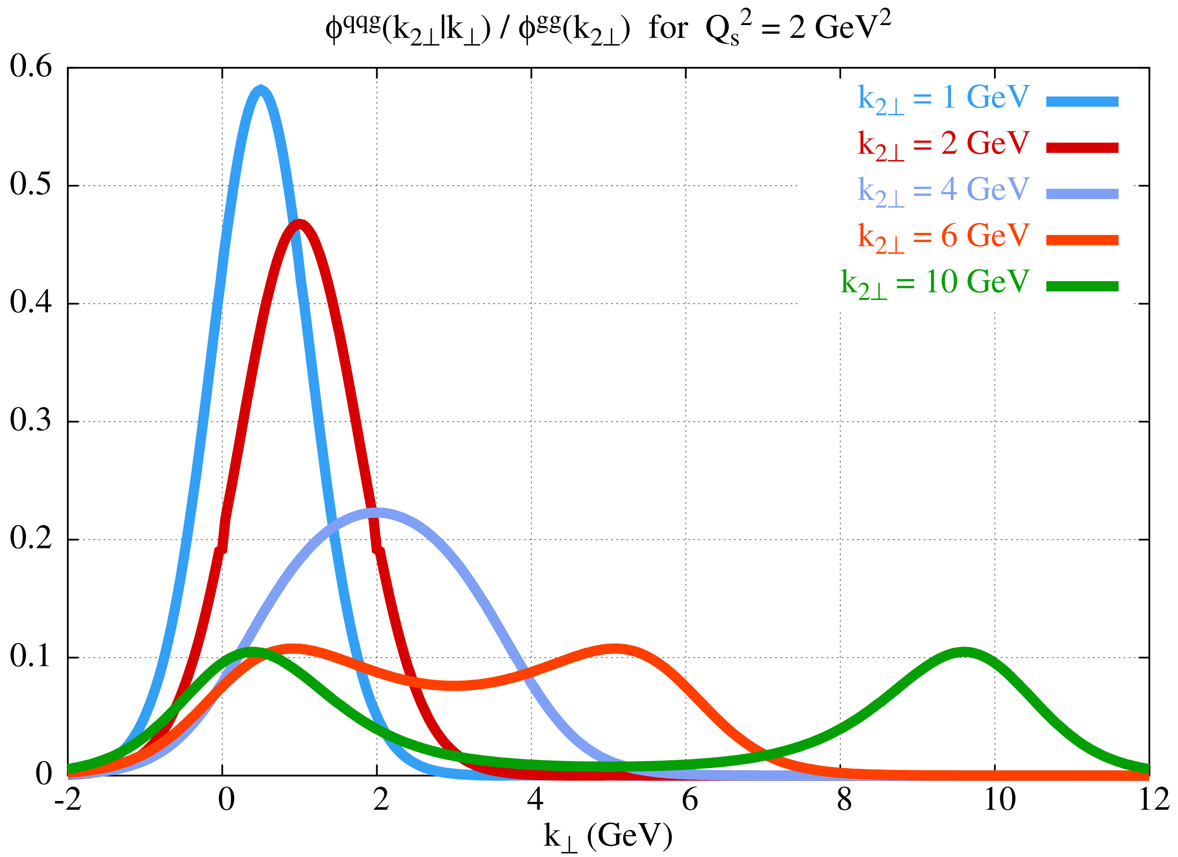
- 3-point function in the MV model:



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- For a large enough $k_{2\perp}$, there are two peaks localized at $\vec{\boldsymbol{k}}_\perp = \vec{0}$ and $\vec{\boldsymbol{k}}_\perp = \vec{\boldsymbol{k}}_{2\perp}$ respectively
- The width of the peaks is of the order of the saturation momentum Q_s
- The area under each peak is $1/2$ (when they are well separated...)
- When $k_{2\perp} \lesssim Q_s$, the two peaks merge into a single peak centered at $\vec{\boldsymbol{k}}_\perp = \vec{\boldsymbol{k}}_{2\perp}/2$
- $k_\perp$-factorization should be a good approximation if all the scales characterizing the final state are much larger than Q_s:
  - the typical $k_{2\perp}$ is large compared to Q_s
  - the width of the peaks can be neglected

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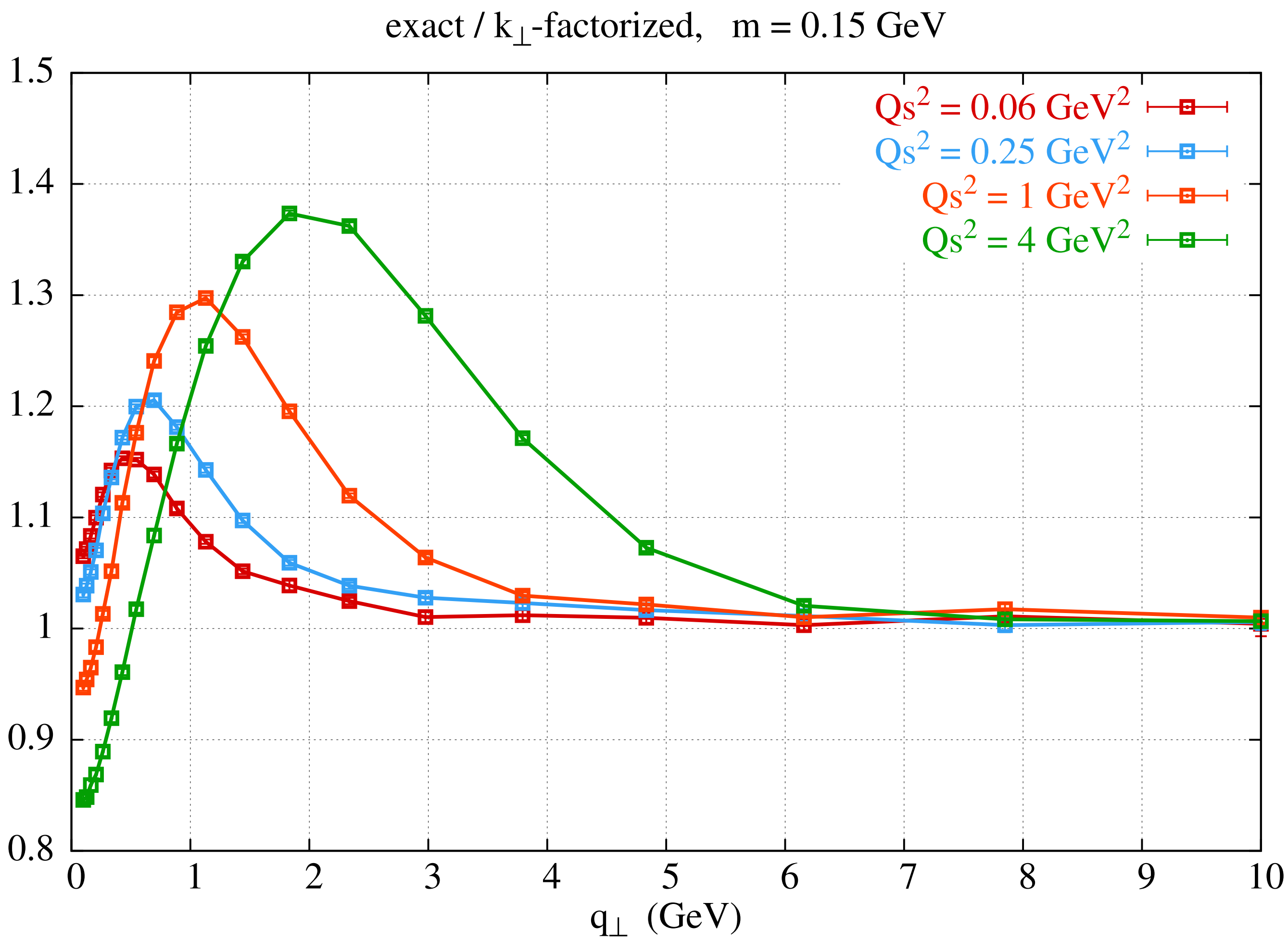
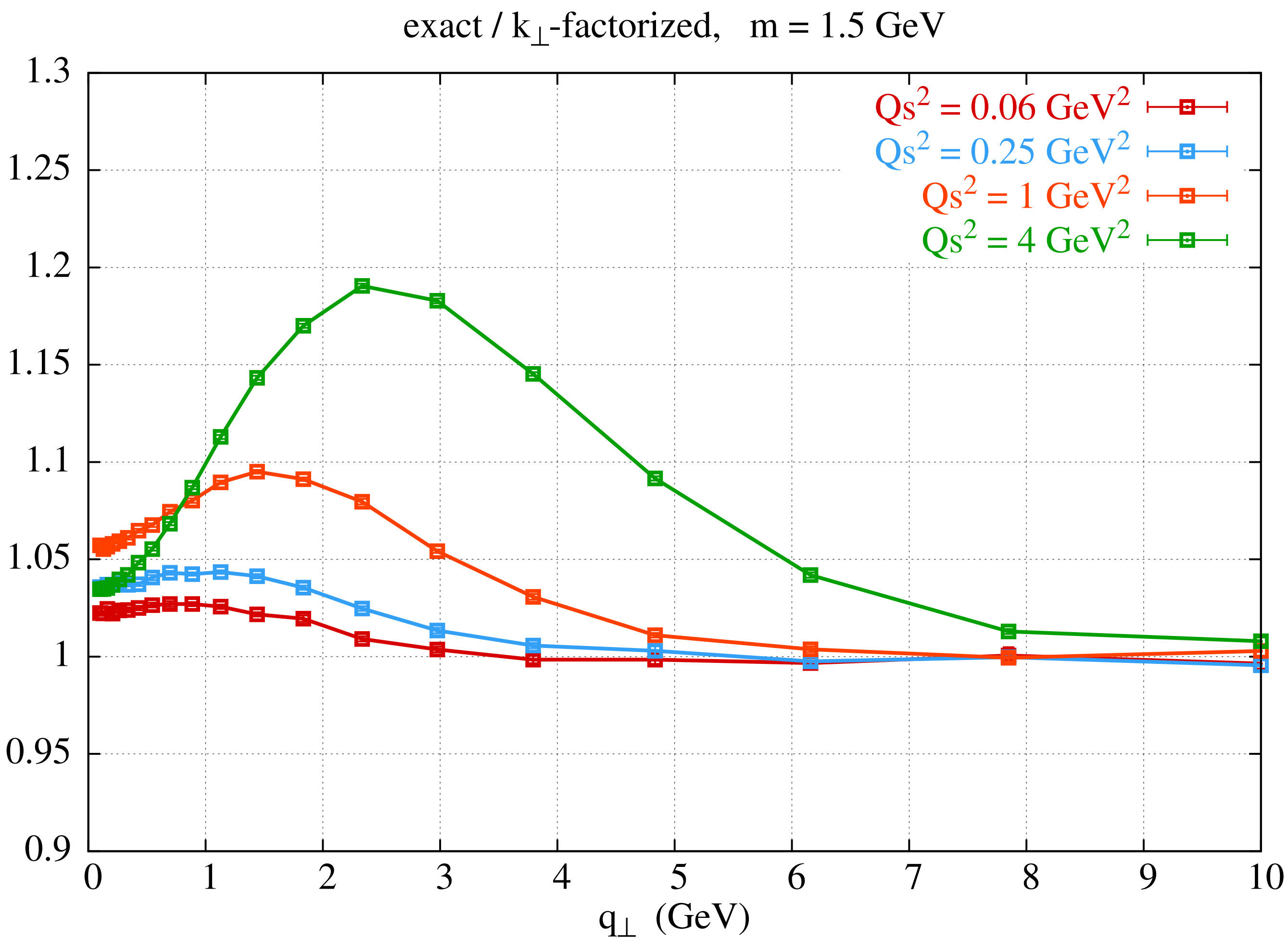
- Single s-quark cross-section :



Kt factorization

- Single c-quark cross-section :



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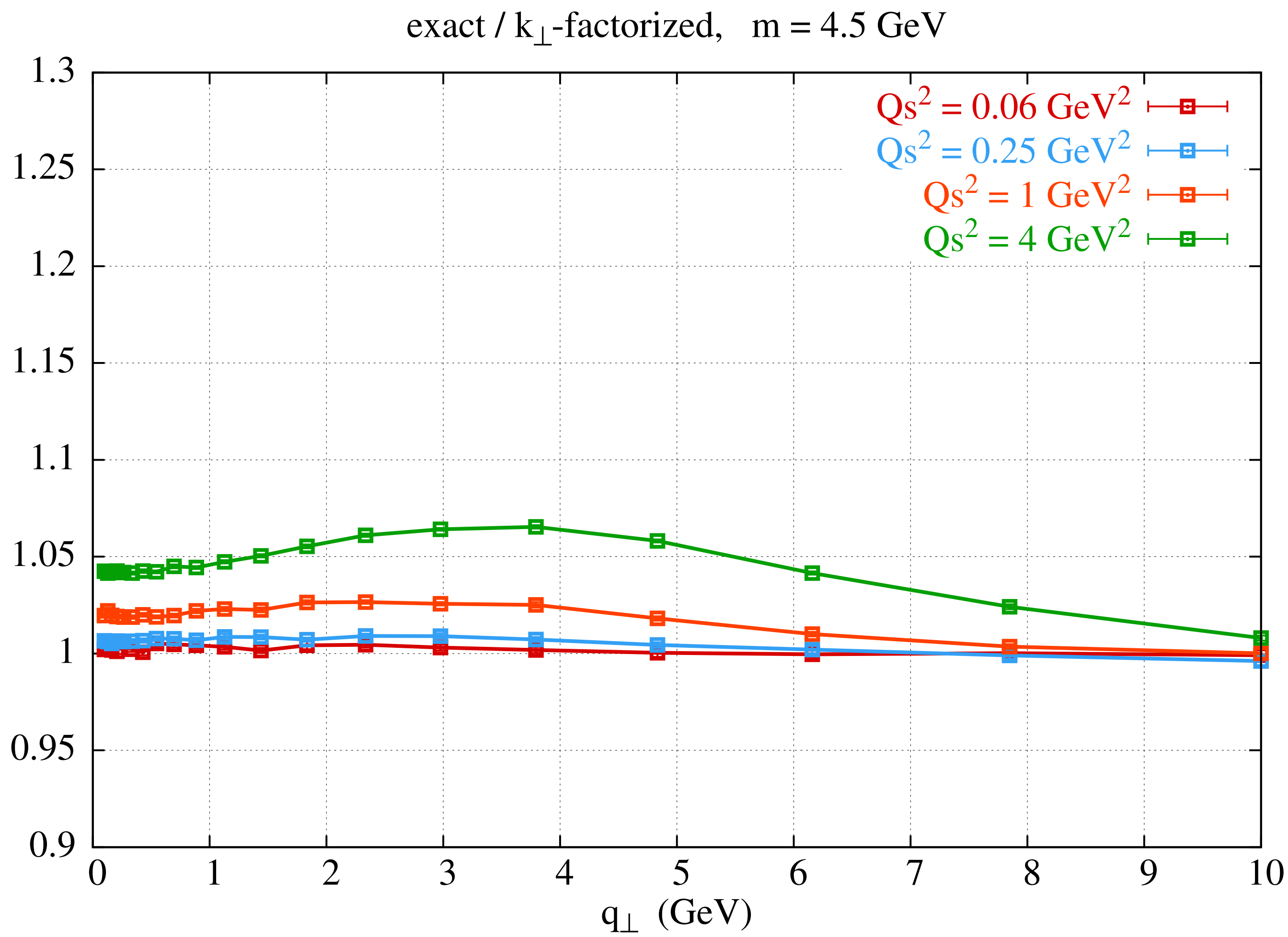
- Single b-quark cross-section :



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General trends for the breaking of $k_\perp$-factorization :

- The magnitude of the breaking increases as m decreases
- The magnitude of the breaking increases with Q_s
- The effect is maximum for $q_\perp \sim Q_s$
- As expected, $k_\perp$-factorization is recovered at large $q_\perp$
- If $Q_s \lesssim m, q_\perp$, the $k_\perp$-factorization breaking terms enhance the cross-section: having more scatterings pushes a few more pairs above the kinematical threshold
- If $Q_s \gg m, q_\perp$, the effect is a reduction of the cross-section: with a large Q_s it becomes less likely to produce a quark with a small transverse mass

$\triangleright$ These corrections tend to enhance the Cronin peak that one would obtain by using the $k_\perp$-factorized formula for quark production

Cronin effect for quark production

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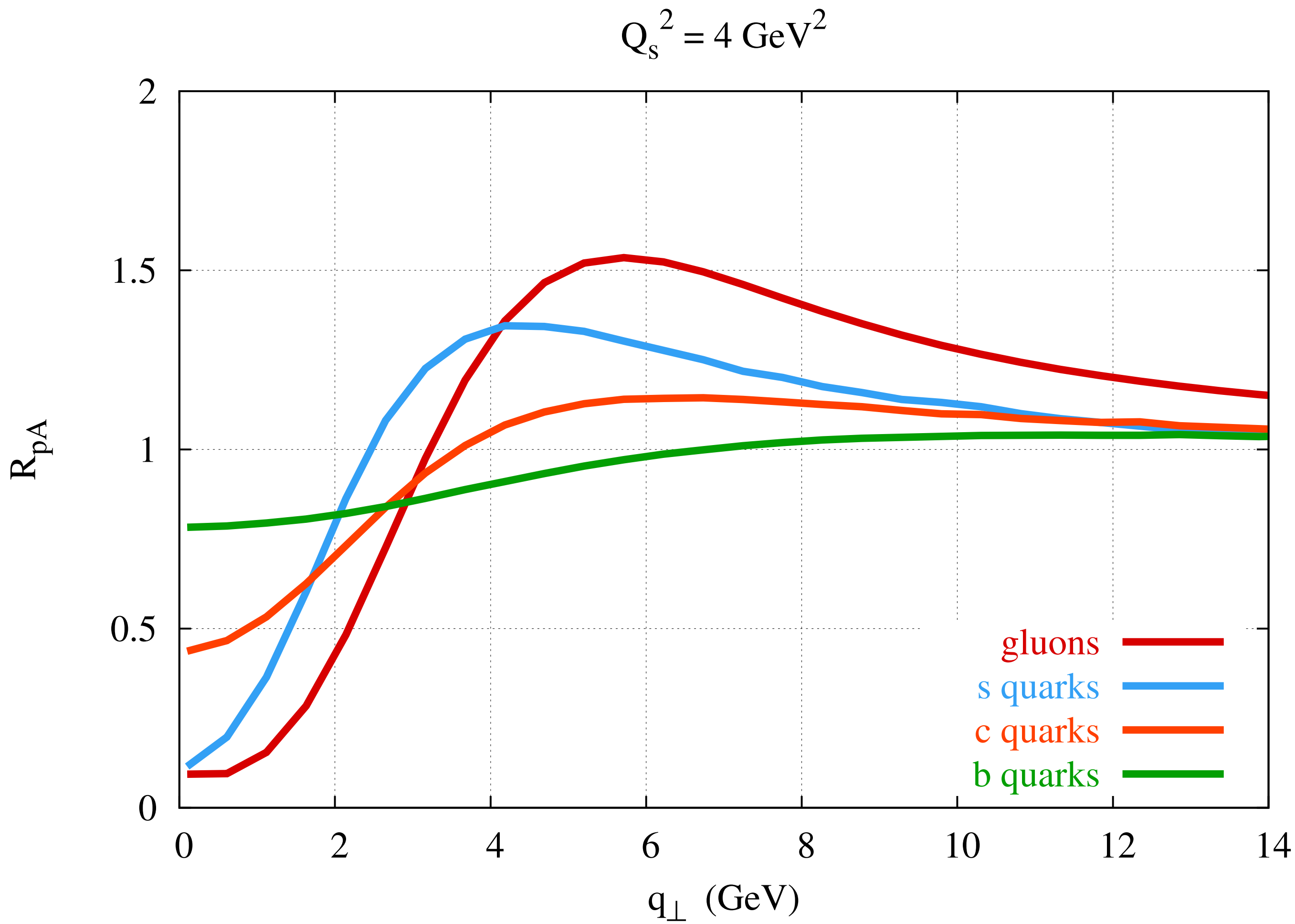
- MV model $(Q_s^2 = 4\ \text{GeV}^2)$



Cronin effect for pair production

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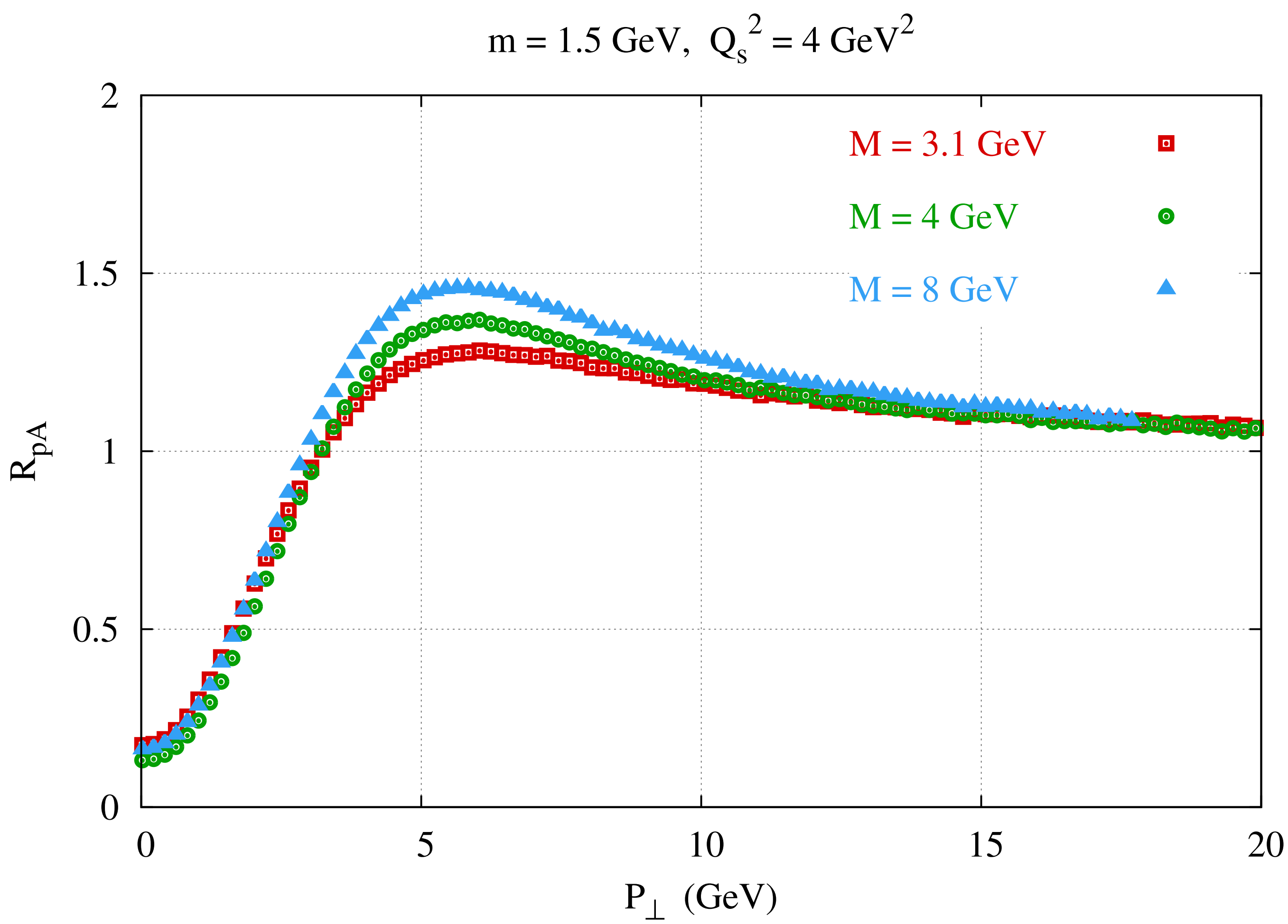
- MV model $(m = 1.5 \text{ GeV}, Q_s^2 = 4 \text{ GeV}^2)$



Invariant mass spectrum

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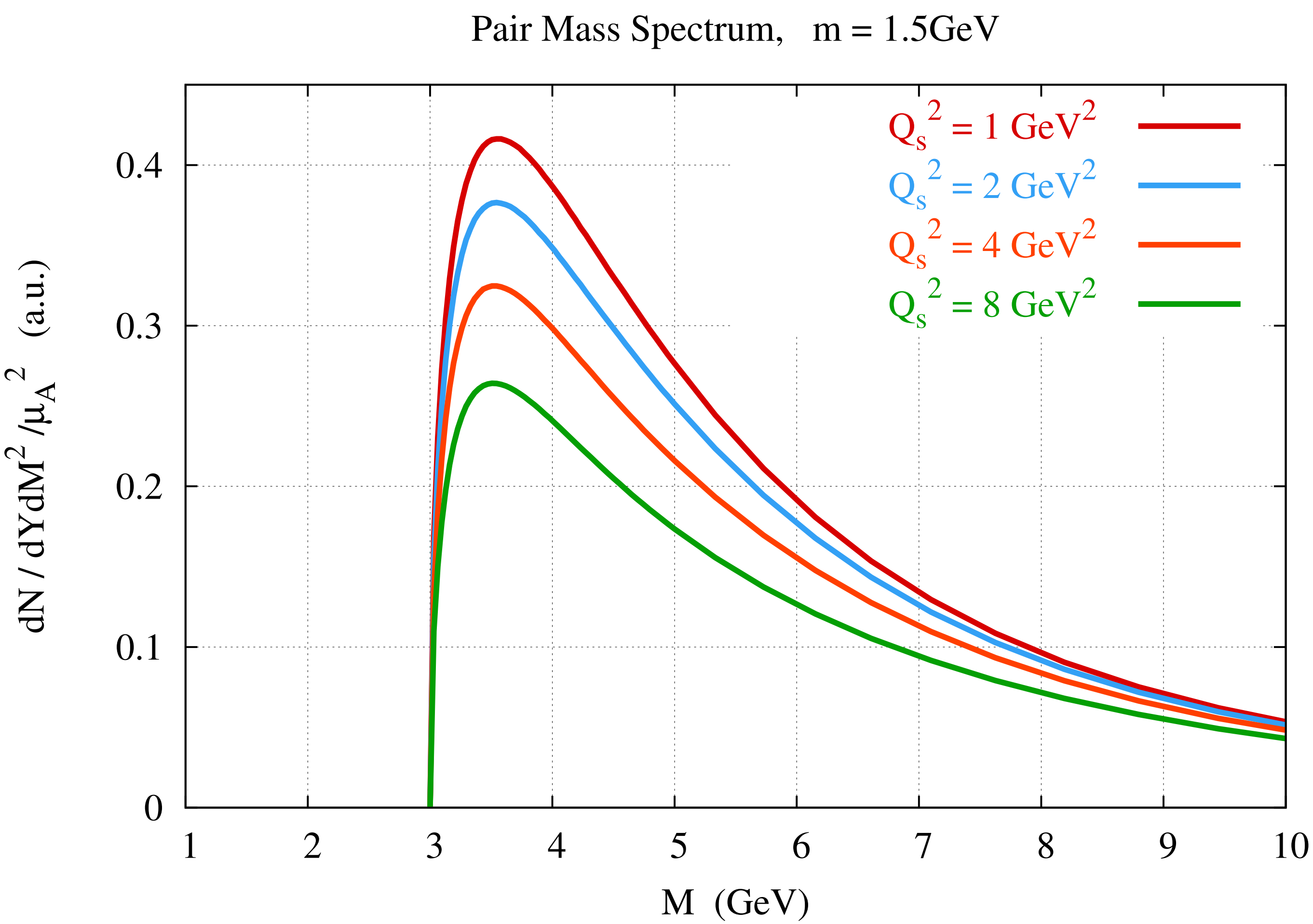
- MV model $(m = 1.5 \text{ GeV})$



JPsi in the Color Evaporation Model

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$$\frac{dN_{J/\psi}}{dY\,d^2\vec{\boldsymbol{P}}_\perp} = F_{J/\psi}\int_{4m_Q^2}^{4m_D^2} dM^2\;\frac{dN_{Q\overline{Q}}}{dM^2dY\,d^2\vec{\boldsymbol{P}}_\perp}$$