QCD at finite Temperature

II – Collective phenomena in the QGP

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General outline

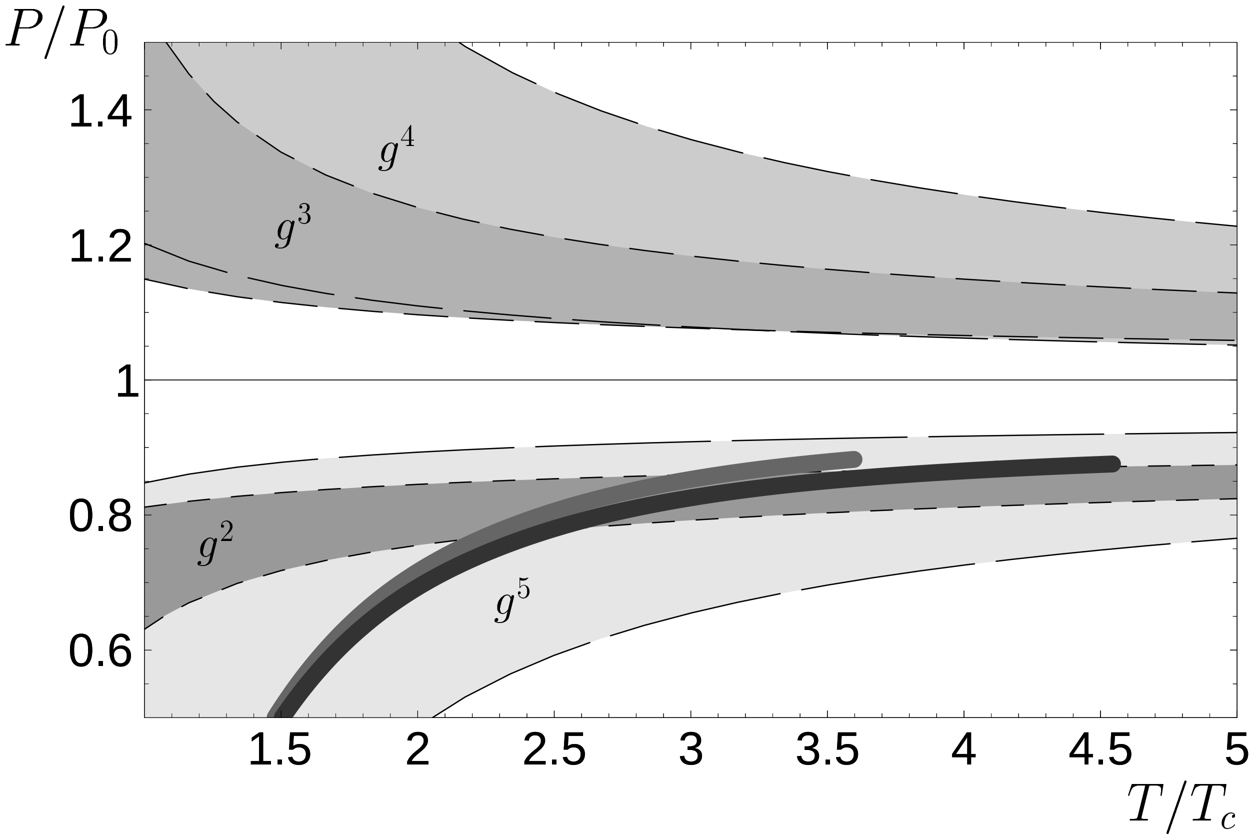
- Lecture I : Quantum field theory at finite T
- Lecture II : Collective phenomena in the QGP
- Lecture III : Out of equilibrium systems

Lecture II : Collective effects in the QGP

- Length scales in the QGP
- Long distance effective theories
- Collective phenomena in the QGP
- Anisotropic plasmas

Convergence ?

- Example: perturbative calculation of the QGP pressure :



  - Does not converge at all...
  - The bare quanta of the naive perturbative expansion are quite different from the actual (dressed) quanta in the QGP

Length scales in the QGP

Degrees of freedom

- Quarks : $2\,(\text{spin}) \quad \times \quad 3\,(\text{color}) \quad = \quad 6$ (per flavor)

$$\frac{dN_{\rm q}}{d^3\vec{\boldsymbol{x}}\,d^3\vec{\boldsymbol{k}}} = \frac{1}{e^{\omega/T}+1}$$ (Fermi-Dirac)

- Gluons : $2\,(\text{spin}) \quad \times \quad 8\,(\text{color}) \quad = \quad 16$

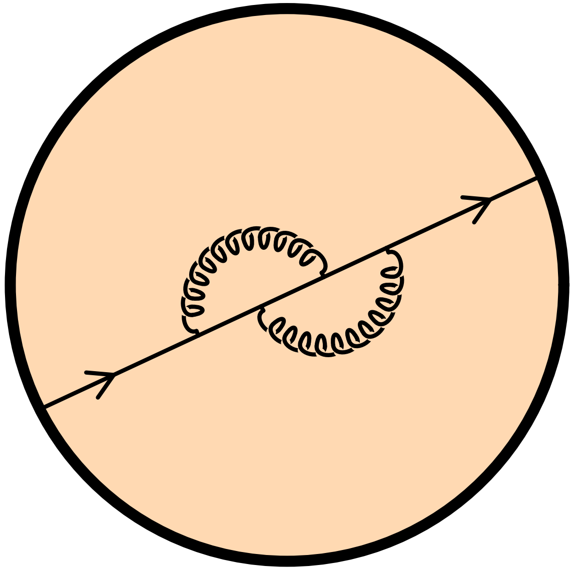
$$\frac{dN_{\rm g}}{d^3\vec{\boldsymbol{x}}\,d^3\vec{\boldsymbol{k}}} = \frac{1}{e^{\omega/T}-1}$$ (Bose-Einstein)

- Average energy per particle : $\langle E\rangle \sim T$
- Particle density : $\rho \sim T^3$
- Average distance between particles : $\ell \sim 1/T$

Length scales

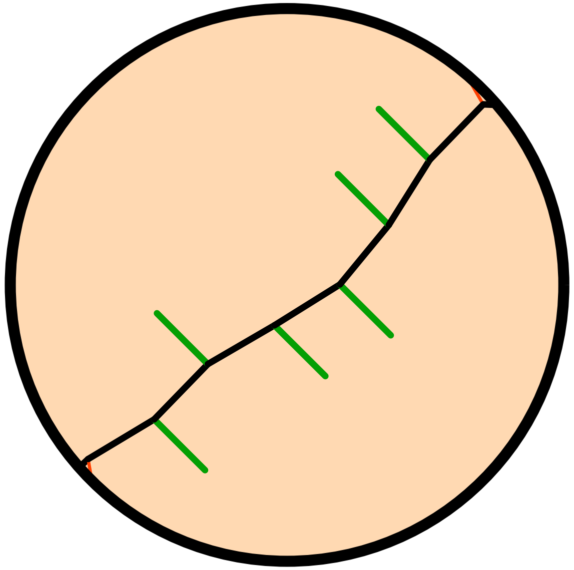
- $1/T$: wavelength of particles in the plasma
- $1/gT$: typical distance for collective phenomena
  - Thermal masses of quasi-particles
  - Screening phenomena
  - Damping of plasma waves
- $1/g^2T$: distance between two small angle scatterings
  - Color transport
  - Photon emission
- $1/g^4T$: distance between two large angle scatterings
  - Momentum, electric charge transport
    - ▷ characteristic scale of hydrodynamic modes
- In the weak coupling limit ($g \ll 1$), there is a clear hierarchy between these scales
- Distinct effective theories according to the characteristic scale of the problem under study

Vacuum fluctuations



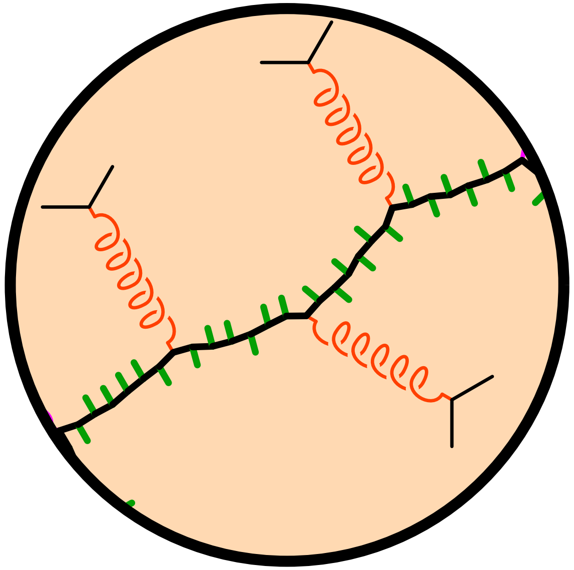
- At distances scales $\ell \lesssim 1/T$, medium effects are irrelevant
- At such scales the dynamics is simply described by QCD in the vacuum

Thermal fluctuations



- Distance scales $1/T \lesssim \ell \lesssim 1/gT$ control the bulk thermodynamic properties. The system can be studied by QCD at finite temperature
- The leading thermal effects can be treated by an effective theory that encompasses the main collective effects, and that has the form of a collision-less Vlasov equation

Small angle scatterings

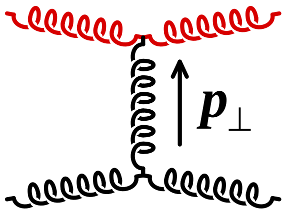


- When it is necessary to follow a plasma particle over distances $1/g^2T \lesssim \ell$, we must take into account soft (small angle) collisions with other particles of the plasma
- This can be done simply by adding a collision term to the previous Vlasov equation

Collision rate

- Collisional width (up to logs) :

$$\Gamma_{\rm coll} = \left| \right.$$

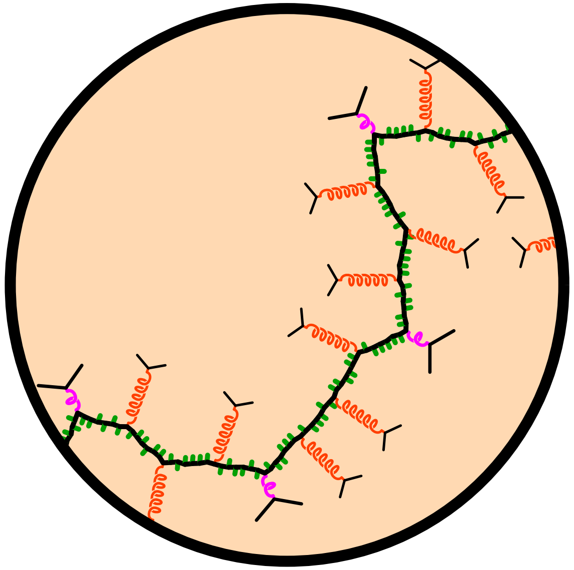


$$\left. \right|^2 \sim g^4\, T^3 \int_{m_{\rm debye}} \frac{d^2\vec{\boldsymbol{p}}_\perp}{p_\perp^4} \sim g^2 T$$

- $\lambda \equiv 1/\Gamma_{\rm coll}$ is the mean free path between two small angle scatterings ($\theta \sim g$)

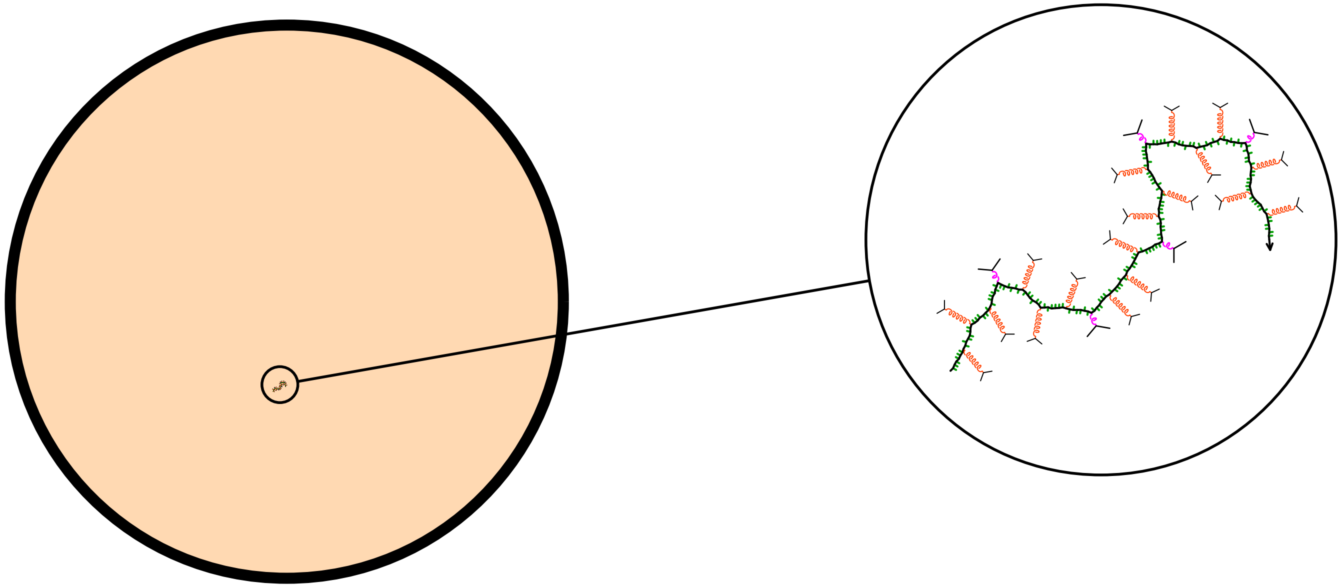
- Note : the mean free path between two large angle scatterings ($\theta \sim 1$) is $\sim 1/g^4 T$

Large angle scatterings



- Over distance scales $\ell \sim 1/g^4T$, one must take into account the large angle collisions, that change significantly the direction of motion of the particle (this is necessary e.g. for calculating transport coefficients)
- The most efficient way to describe the system at these scales is via a Boltzmann equation for color/spin averaged particle distributions

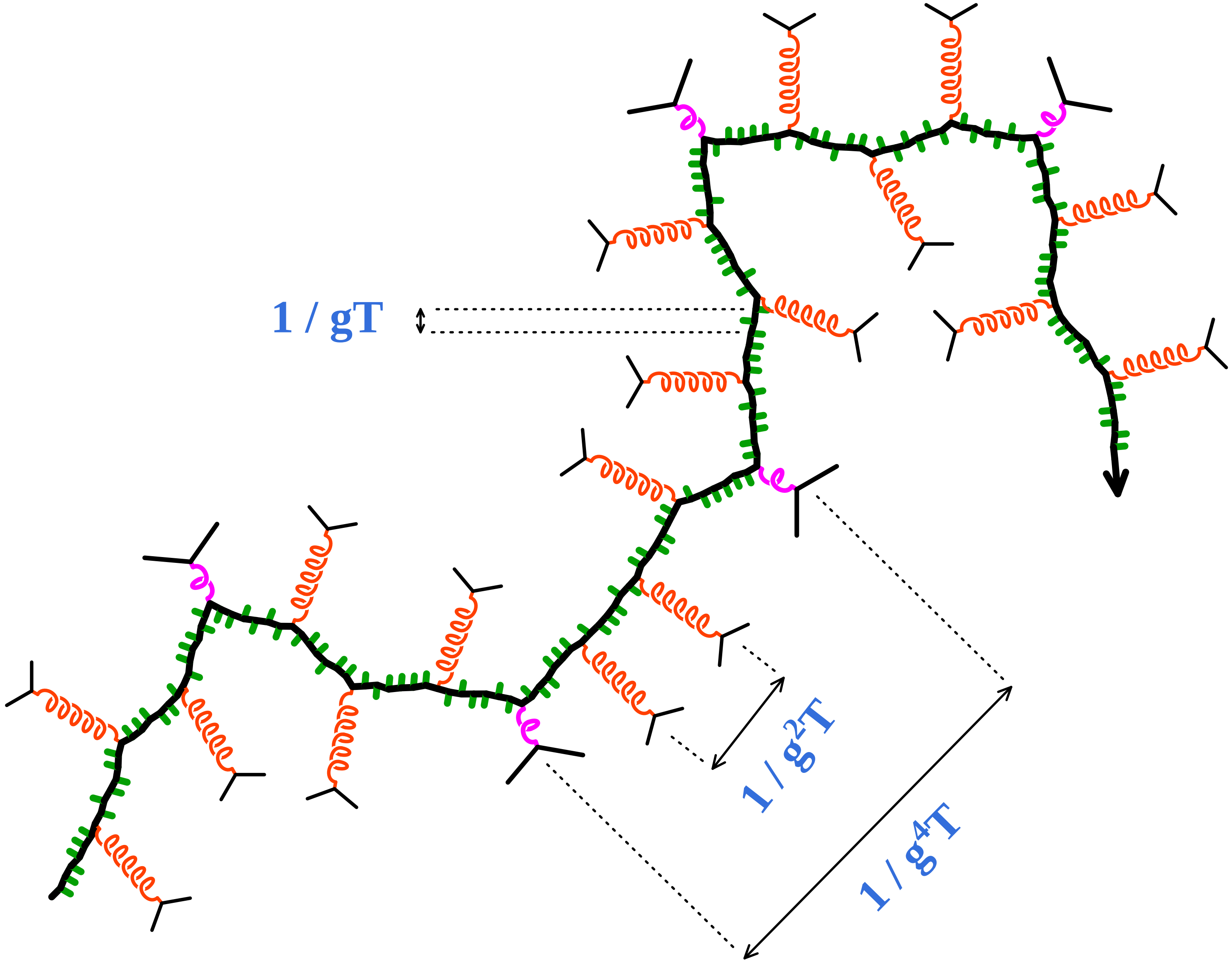
Hydrodynamical regime



- The hydrodynamical regime is reached for length scales that are much larger than the mean free path : $1/g^4T \ll \ell$
- In order to describe the system at such scales, one needs :
  - Hydrodynamical equations (Euler, Navier-Stokes)
  - Conservation equations for the various currents
  - Equation of state, viscosity

Long distance effective theories

Reminder : Length scales



Length scales

- A mode is perturbative if its kinetic energy is much larger than its potential energy
  - Kinetic energy : $\big\langle K\big\rangle \sim \big\langle(\partial A)^2\big\rangle \sim k^2\big\langle A^2\big\rangle$
  - Potential energy : $\big\langle U\big\rangle \sim g^2\big\langle A^4\big\rangle \sim g^2\big\langle A^2\big\rangle^2$
  - ▷ Thus, a mode k is perturbative if $g^2\big\langle A^2\big\rangle \ll k^2$
- When discussing the order of magnitude of $\big\langle A^2\big\rangle$, it is useful to distinguish the contribution of the various momentum scales by defining

$$\big\langle A^2\big\rangle_{\kappa^*} \sim \int^{\kappa^*} \frac{d^3\vec{\boldsymbol{k}}}{E_{\boldsymbol{k}}}\, n_B(E_{\boldsymbol{k}})$$

Length scales

- Hard modes : $k \sim T$, $\big\langle A^2 \big\rangle_{T} \sim T^2$. Thus, $\langle K \rangle \gg \langle U \rangle$
- Soft modes : $k \sim gT$, $k^2 \sim g^2 \big\langle A^2 \big\rangle_{T}$

  But the contribution of soft modes to $\langle A^2 \rangle$ is $\big\langle A^2 \big\rangle_{gT} \sim gT^2$, and $k^2 \gg g^2 \big\langle A^2 \big\rangle_{gT}$

  The soft modes interact strongly with the hard modes, but weakly among themselves ▷ they can be described perturbatively after the hard modes have been resummed
- Ultrasoft modes : $k \sim g^2 T$, $\big\langle A^2 \big\rangle_{g^2 T} \sim g^2 T^2$, $k^2 \sim g^2 \big\langle A^2 \big\rangle_{g^2 T}$

  The dynamics of the ultrasoft modes is completely non-perturbative, because their self-interactions are as large as their kinetic energy

Hard thermal loops

Braaten, Pisarski (1990), Frenkel, Taylor (1990)

- Obtained from the bare perturbative expansion by the resummation of Hard Thermal Loops (HTL) :

$$\Delta\mathcal{L}_{HTL}(\text{gluons}) = \frac{m_{\rm g}^2}{2}\int\frac{d\Omega_{\hat{\boldsymbol{v}}}}{4\pi}\; F_{\mu\alpha}\frac{v^\alpha v^\beta}{(v\cdot D)^2}F_\beta{}^\mu\;, \qquad v^\mu = (1,\hat{\boldsymbol{v}})$$

- Can be formulated as a (local) collisionless transport theory for classical particles (Blaizot, Iancu (1993-1995)) :

$$(1) \qquad [D_\mu, F^{\mu\nu}] = m_{\rm g}^2\int\frac{d\Omega_{\hat{\boldsymbol{v}}}}{4\pi}v^\nu W(x,\hat{\boldsymbol{v}})$$

$$(2) \qquad [v\cdot D, W(x,\hat{\boldsymbol{v}})] = \hat{\boldsymbol{v}}\cdot\boldsymbol{E}(x)$$

  - $W(x,\hat{\boldsymbol{v}})$ is the density of hard particles $(\omega\sim T)$ at the location x, with a velocity in the direction $\hat{\boldsymbol{v}}$
  - (1) : Yang-Mills equation for the soft field modes $(\omega\sim gT)$
  - (2) : Vlasov equation for the hard particles

Effective theory at the scale g2T ln(1/g)

Bödeker (1999)

- Soft collisions have a mean free path of $\lambda \sim (g^2 T \ln(1/g))^{-1}$
  - ▷ still perturbative after resummation of the gT modes
- By integrating out the modes $\sim gT$, one obtains a Boltzmann-Langevin equation for the long wavelength variations of the density $W(x, \hat{\boldsymbol{v}})$:

$$[v \cdot D, W(x, \hat{\boldsymbol{v}})] = \hat{\boldsymbol{v}} \cdot \boldsymbol{E}(x) + \xi(x, \hat{\boldsymbol{v}}) + g^2 N T \ln\left(\frac{gT}{\Lambda}\right) \int \frac{d\Omega_{\hat{\boldsymbol{v}}'}}{4\pi} I(\hat{\boldsymbol{v}}, \hat{\boldsymbol{v}}') W(x, \hat{\boldsymbol{v}}')$$

- ◆ $\xi(x, \hat{\boldsymbol{v}})$ is a Gaussian noise, of correlation :

$$\langle \xi(x_1, \hat{\boldsymbol{v}}_1), \xi(x_2, \hat{\boldsymbol{v}}_2) \rangle = -2 \frac{g^2 N T^2}{m_{\rm g}^2} \ln\left(\frac{gT}{\Lambda}\right) I(\hat{\boldsymbol{v}}_1, \hat{\boldsymbol{v}}_2) \delta(x_1 - x_2)$$

- ◆ $I(\hat{\boldsymbol{v}}, \hat{\boldsymbol{v}}')$ is a collision term due to the interactions with field modes of momentum $\sim gT$ (small angle collisions)

Dimensional reduction

- By summing the Matsubara modes whose frequency is non-zero (fermions, bosons for $n \neq 0$), one gets a 3-dimensional Yang-Mills theory coupled to an adjoint Higgs :

$$\mathcal{L}_E = \frac{1}{4} F_{ij}^2 + \mathrm{tr}[D_i, A_0]^2 + m_E^2 \mathrm{tr} A_0{}^2 + \frac{\lambda_E}{2} (\mathrm{tr} A_0{}^2)^2 + \cdots$$

  - A_0 is the gluon zero mode
  - m_E, λ_E are determined by matching to the underlying theory (i.e. QCD)

- By integrating out the massive A_0, one gets a 3-dimensional pure Yang-Mills theory :

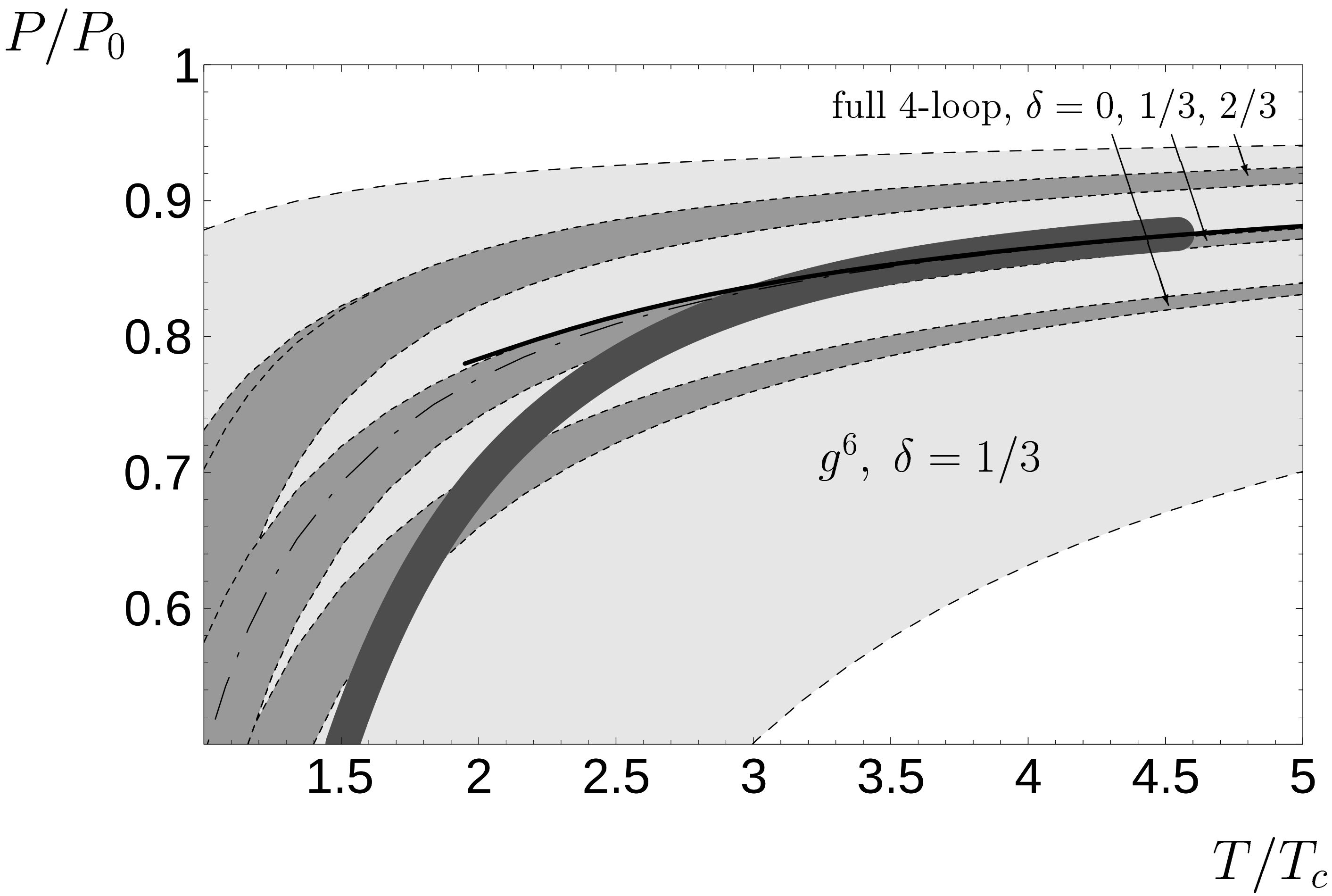
$$\mathcal{L}_M = \frac{1}{4} F_{ij}^2 + \cdots$$

  - its coupling g_M is determined order by order from $\mathcal{L}_E$
  - this Yang-Mills theory is non-perturbative, and must be simulated on a lattice (this is much simpler than simulations of 4-dim QCD)

Dimensional reduction

Kajantie, Laine, Rummukainen, Schröder (2002)

- Calculation of the QGP pressure to $g^6 \ln(1/g)$ (4 loops) :



Collective phenomena in the QGP

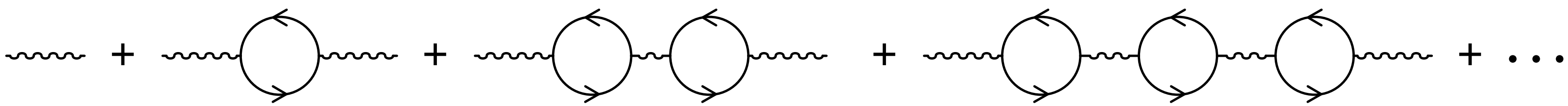
Collective phenomena

- Phenomena involving many elementary constituents
- Long wavelength compared to the typical distance between constituents
- Small frequency or energy
- Major collective phenomena :
  - Quasi-particles
  - Debye screening
  - Landau damping
  - Collisional width

Dressed propagator

- In order to assess how the medium affects the propagation of excitations, one must compute the photon (gluon) polarization tensor $\Pi^{\mu\nu}(x,y) \equiv \big\langle J^\mu(x) J^\nu(y) \big\rangle$

- The photon (or gluon for QCD) self-energy can be resummed on the propagator. Diagrammatically, this amounts to summing :



- The properties of the medium can be read off the analytic properties of this resummed propagator (cuts, poles, ...)

Dressed propagator

- Reminder : the photon polarization tensor $\Pi^{\mu\nu}$ is transverse. At $T = 0$, this implies :

$$\Pi^{\mu\nu}(P) = \left(g^{\mu\nu} - \frac{P^\mu P^\nu}{P^2}\right)\,\Pi(P^2)$$

  - this is due to gauge invariance and Lorentz invariance
  - Exercise : this property ensures that the photon remains massless at all orders of perturbation theory

- This formula is not valid at $T > 0$, because there is a preferred frame (in which the plasma velocity is zero)

  $\triangleright$ the tensorial decomposition of $\Pi^{\mu\nu}$ is more complicated, and the photon acquires an effective mass

Dressed propagator

- At finite T, the tensorial decomposition of $\Pi^{\mu\nu}$ is :

$$\Pi^{\mu\nu}(P) = P_T^{\mu\nu}(P)\,\Pi_T(P) + P_L^{\mu\nu}(P)\,\Pi_L(P)$$

with the following projectors (in the plasma rest frame)

$$P_T^{ij}(P) = g^{ij} + \frac{p^i p^j}{\vec{\boldsymbol{p}}^2}\,, \quad P_T^{0i}(P) = 0\,, \quad P_T^{00}(P) = 0$$

$$P_L^{ij}(P) = -\frac{(p^0)^2 p^i p^j}{\vec{\boldsymbol{p}}^2 P^2}\,, \quad P_L^{0i}(P) = -\frac{p^0 p^i}{P^2}\,, \quad P_L^{00}(P) = -\frac{\vec{\boldsymbol{p}}^2}{P^2}$$

- Therefore, we have

$$\Pi^{\mu}{}_{\mu}(P) = 2\Pi_T(P) + \Pi_L(P) \quad , \quad \Pi^{00}(P) = -\frac{\vec{\boldsymbol{p}}^2}{P^2}\,\Pi_L(P)$$

- This leads to the following propagator :

$$D^{\mu\nu}(P) = P_T^{\mu\nu}(P)\,\frac{1}{P^2 - \Pi_T(P)} + P_L^{\mu\nu}(P)\,\frac{1}{P^2 - \Pi_L(P)}$$

Dressed propagator - Exercise

- Check the following properties of the tensors $P_{T,L}^{\mu\nu}$:

$$P_{T\ \mu}^{\mu} = 2$$

$$P_{L\ \mu}^{\mu} = 1$$

$$P_{T\ \alpha}^{\mu}\, P_{T}^{\alpha\nu} = P_{T}^{\mu\nu}$$

$$P_{L\ \alpha}^{\mu}\, P_{L}^{\alpha\nu} = P_{L}^{\mu\nu}$$

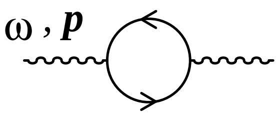
$$P_{T\ \alpha}^{\mu}\, P_{L}^{\alpha\nu} = 0$$

Dressed propagator

- The calculation of $\Pi^{\mu}{}_{\mu}$ and Π^{00} can be done for a discrete Matsubara frequency ω_p, and one performs the analytic continuation $i\omega_p \to p_0$ afterwards

- Because one is after the long distance properties of the plasma, one also makes the approximation $|\vec{\boldsymbol{p}}| \ll |\vec{\boldsymbol{k}}|$ (Hard Thermal Loops : Braaten, Pisarski - 1990)

- For instance, the fermionic contribution to the spatial part Π^{ij} of the polarization tensor reads :



$$= -\frac{g^2 N_{\rm f} T}{2} \int \frac{d^3\vec{\boldsymbol{k}}}{(2\pi)^3}\, \widehat{\boldsymbol{v}}^i_{\boldsymbol{k}} \frac{\partial n_F(\vec{\boldsymbol{k}})}{\partial \boldsymbol{k}^l} \left[\delta^{jl} - \frac{\widehat{\boldsymbol{v}}^j_{\boldsymbol{k}} \widehat{\boldsymbol{v}}^l_{\boldsymbol{k}}}{\omega - \widehat{\boldsymbol{v}}_{\boldsymbol{k}} \cdot \vec{\boldsymbol{p}} + i\epsilon}\right]$$

$(\widehat{\boldsymbol{v}}_{\boldsymbol{k}} \equiv \vec{\boldsymbol{k}}/|\vec{\boldsymbol{k}}|)$

  - Note : with the gluon loop, the only change is $N_{\rm f} \to N_{\rm f} + 2N_{\rm c}$

Quasi-particles

- The functions $\Pi_{T,L}(P)$ read :

$$\Pi_T(P) = \frac{e^2 T^2}{6}\left[\frac{p_0^2}{p^2} + \frac{p_0}{2p}\left(1 - \frac{p_0^2}{p^2}\right)\ln\left(\frac{p_0+p}{p_0-p}\right)\right]$$

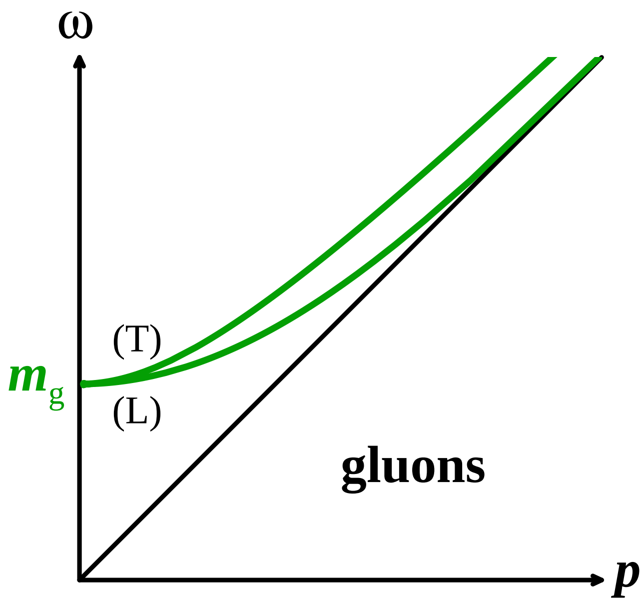
$$\Pi_L(P) = \frac{e^2 T^2}{3}\left[1 - \frac{p_0^2}{p^2}\right]\left[1 - \frac{p_0}{2p}\ln\left(\frac{p_0+p}{p_0-p}\right)\right]$$

- Quasi-particles correspond to poles in the propagator. Their dispersion relation is the function $p_0 = \omega(\vec{\boldsymbol{p}})$ that defines the location of the pole

- The inverse of the imaginary part of p_0 is the lifetime of the quasi-particles (If $\mathrm{Im}(p_0) = 0$, they are stable). In order to be able to talk about quasi-particles, one must have $\mathrm{Im}(p_0) \ll \mathrm{Re}(p_0)$

Quasi-particles

- Dispersion curves of particles in the plasma :



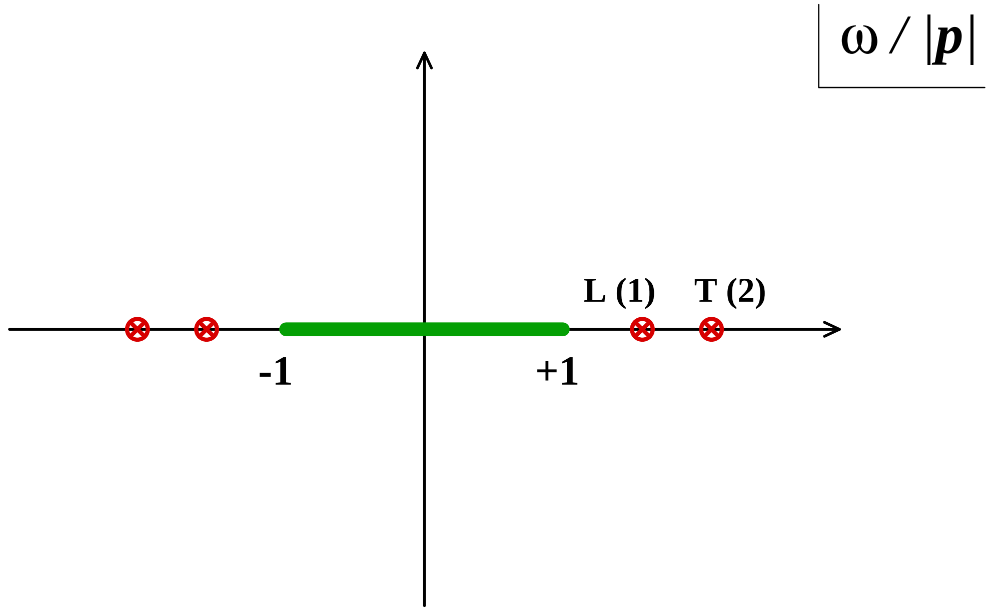
- Thermal masses due to interactions with the other particles in the plasma :

$$m_{\rm q} \sim m_{\rm g} \sim gT$$

- At this order, the quasi-particles are stable

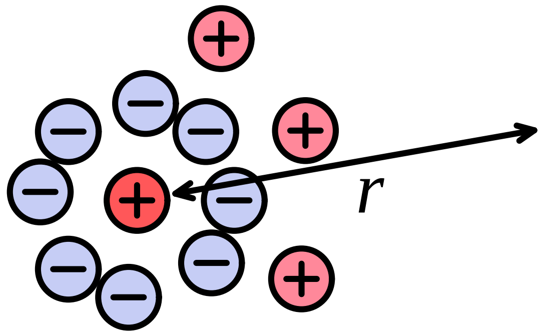
Singularities

- In the complex plane of $\omega/|\vec{p}|$, the dressed propagator has poles (quasi-particles) and a cut (Landau damping) :



Debye screening

- A test charge polarizes the particles of the plasma in its vicinity, in order to screen its charge :



$$V(r) = \frac{\exp(-m_{\text{debye}}\, r)}{r}$$

- The Coulomb potential of the test charge decreases exponentially at large distance. The effective interaction range is :

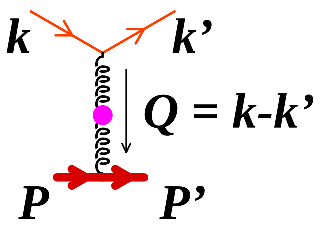
$$\ell \sim 1/m_{\text{debye}} \sim 1/gT$$

- Note : static magnetic fields are not screened by this mechanism (they are screened over length-scales $\ell_{\text{mag}} \sim 1/g^2T$)

Debye screening

- Place a quark of mass M at rest in the plasma, at $\vec{r} = 0$
- Scatter another quark off it. The scattering amplitude reads

$$\mathcal{M} = \left[g\overline{u}(\vec{k}')\gamma_\mu u(\vec{k})\right]\left[g\overline{u}(\vec{P}')\gamma_\nu u(\vec{P})\right] \sum_{\alpha=T,L} \frac{P_\alpha^{\mu\nu}(Q)}{Q^2 - \Pi_\alpha(Q)}$$



  - If $\vec{P} = 0$ (test charge at rest), only $\alpha = L$ contributes
  - From $(P+Q)^2 = M^2$, we get a $2\pi\delta(q_0)/2M$

- For the scattering off an external potential A^μ, the amplitude is $\mathcal{M} = \left[g\overline{u}(\vec{k}')\gamma_\mu u(\vec{k})\right] A^\mu(Q)$
- Thus, the potential created by the test charge at rest is :

$$A^\mu(Q) = g\frac{\overline{u}(\vec{P}')\gamma_\nu u(\vec{P})}{2M}\;\frac{2\pi\delta(q_0)P_L^{\mu\nu}(0,\vec{q})}{\vec{q}^2 + \Pi_L(0,\vec{q})} = \frac{2\pi g\delta^{\mu 0}\delta(q_0)}{\vec{q}^2 + \Pi_L(0,\vec{q})}$$

Debye screening

- By a Fourier transform, we obtain the Coulomb potential :

$$A^0(t,\vec{\boldsymbol{r}}) = g \int \frac{d^3\vec{\boldsymbol{q}}}{(2\pi)^3} \, \frac{e^{i\vec{\boldsymbol{q}}\cdot\vec{\boldsymbol{r}}}}{\vec{\boldsymbol{q}}^2 + \Pi_L(0,\vec{\boldsymbol{q}})}$$

- If we are in the vacuum, $\Pi_L = 0$, and the Fourier transform gives the usual Coulomb law :

$$A^0_{\rm vac}(t,\vec{\boldsymbol{r}}) = g \int \frac{d^3\vec{\boldsymbol{q}}}{(2\pi)^3} \, \frac{e^{i\vec{\boldsymbol{q}}\cdot\vec{\boldsymbol{r}}}}{\vec{\boldsymbol{q}}^2} = \frac{g}{4\pi|\vec{\boldsymbol{r}}|}$$

- In a plasma, $\Pi_L(0,\vec{\boldsymbol{q}}) = \frac{g^2T^2}{3} \equiv m_D^2$. The Fourier transform can also be done exactly

$$A^0(t,\vec{\boldsymbol{r}}) = g \int \frac{d^3\vec{\boldsymbol{q}}}{(2\pi)^3} \, \frac{e^{i\vec{\boldsymbol{q}}\cdot\vec{\boldsymbol{r}}}}{\vec{\boldsymbol{q}}^2 + m_D^2} = \frac{g}{4\pi|\vec{\boldsymbol{r}}|} \, e^{-m_D|\vec{\boldsymbol{r}}|}$$

▷ the potential is unmodified at $r \ll 1/m_D$, but exponentially suppressed at large distance

Debye screening

- It is easy to see here why the naive perturbation theory works pretty badly

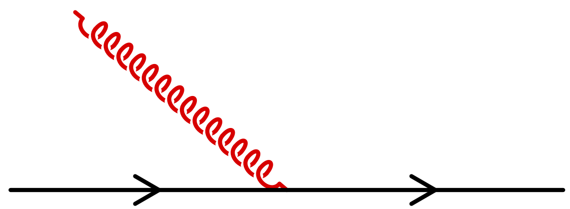
- Suppose we want to calculate the Coulomb potential of a test charge in the QGP in perturbation theory. The term of order g^{2n+1} would be :

$$A^0_{2n+1}(t, \vec{\boldsymbol{r}}) = (-1)^n \, g \, m_D^{2n} \int \frac{d^3\vec{\boldsymbol{q}}}{(2\pi)^3} \, \frac{e^{i\vec{\boldsymbol{q}}\cdot\vec{\boldsymbol{r}}}}{\boldsymbol{q}^{2n+2}}$$

$\triangleright$ all these corrections are very divergent in the infrared. No truncation in the series over n gives the correct long distance behavior of the potential

Landau damping

- The self-energies $\Pi_{L,T}(p_0, \vec{\boldsymbol{p}})$ have an imaginary part when $|p_0| \le |\vec{\boldsymbol{p}}|$. This implies that the propagation of space-like modes is attenuated
- A wave propagating through the plasma is damped because its quanta may be absorbed by particles of the plasma :



- The characteristic frequency of this damping is :

$$\omega_c \sim gT$$

Anisotropic plasmas

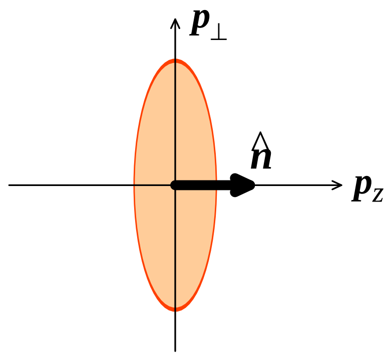
Medium effects (anisotropic)

- Most of the previous analysis can be carried through in the case of a plasma with an anisotropic distribution of particles. In particular, the formula for the polarization tensor in terms of $f(\vec{\boldsymbol{k}})$ remains valid :

$$\Pi^{ij}(\omega,\vec{\boldsymbol{p}}) = -g^2 T \int \frac{d^3\vec{\boldsymbol{k}}}{(2\pi)^3}\, \widehat{\boldsymbol{v}}^i_{\boldsymbol{k}}\, \frac{\partial f(\vec{\boldsymbol{k}})}{\partial \boldsymbol{k}^l} \left[\delta^{jl} - \frac{\widehat{\boldsymbol{v}}^j_{\boldsymbol{k}}\widehat{\boldsymbol{v}}^l_{\boldsymbol{k}}}{\omega - \widehat{\boldsymbol{v}}_{\boldsymbol{k}}\cdot\vec{\boldsymbol{p}} + i\epsilon}\right]$$

- Model for an anisotropic distribution : start from a generic isotropic distribution $f(\boldsymbol{k}^2)$ and squeeze it :

$$f(\boldsymbol{p}^2) \quad \longrightarrow \quad f(\boldsymbol{p}^2 + \xi(\widehat{\boldsymbol{n}}\cdot\vec{\boldsymbol{p}})^2)$$



Medium effects (anisotropic)

- Within this model, it is easy to factorize the integration over the argument of n (i.e. $\boldsymbol{p}^2 + \xi(\widehat{\boldsymbol{n}} \cdot \vec{\boldsymbol{p}})^2$) :

$$\Pi^{ij}(\omega, \vec{\boldsymbol{p}}) = m_D^2 \int \frac{d^2\widehat{\boldsymbol{v}}_{\boldsymbol{k}}}{4\pi} \, \widehat{\boldsymbol{v}}_{\boldsymbol{k}}^i \, \frac{\widehat{\boldsymbol{v}}_{\boldsymbol{k}}^l + \xi(\widehat{\boldsymbol{v}}_{\boldsymbol{k}} \cdot \widehat{\boldsymbol{n}})\widehat{\boldsymbol{n}}^l}{(1 + \xi(\widehat{\boldsymbol{v}}_{\boldsymbol{k}} \cdot \widehat{\boldsymbol{n}})^2)^2} \left[\delta^{jl} - \frac{\widehat{\boldsymbol{v}}_{\boldsymbol{k}}^j \widehat{\boldsymbol{v}}_{\boldsymbol{k}}^l}{\omega - \widehat{\boldsymbol{v}}_{\boldsymbol{k}} \cdot \vec{\boldsymbol{p}} + i\epsilon}\right]$$

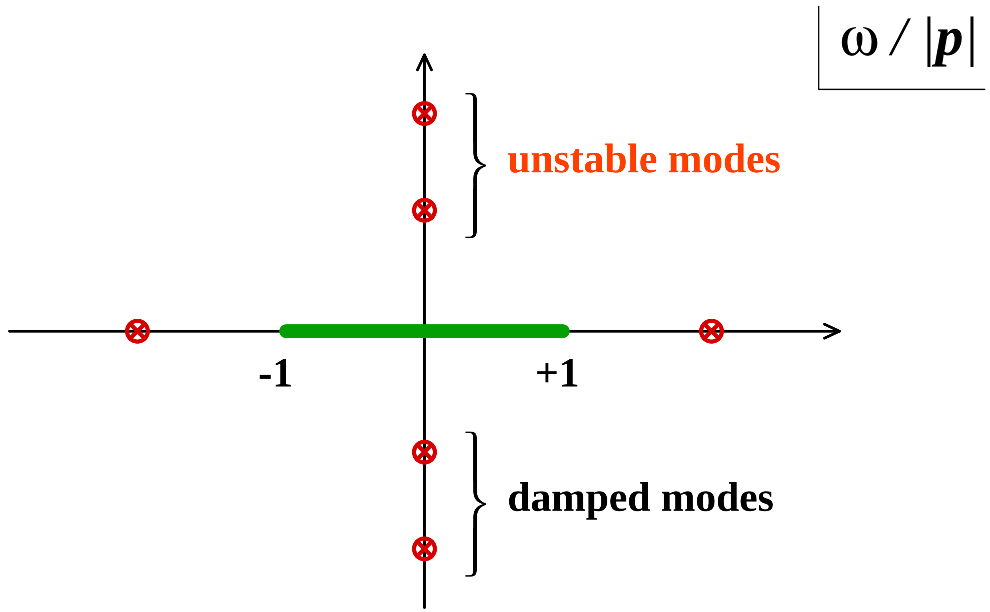
with

$$m_D^2 \equiv -\frac{g^2}{2\pi^2} \int_0^\infty dk \; k^2 \; \frac{df(k^2)}{dk^2}$$

- m_D sets the magnitude of all the medium effects on the gauge bosons
- Only the remaining integral over the unit vector $\widehat{\boldsymbol{v}}_{\boldsymbol{k}}$ is affected by the anisotropy $(\xi \neq 0)$
- The tensorial decomposition of $\Pi^{\mu\nu}$ is more complicated than in the isotropic case, because the vector $\widehat{\boldsymbol{n}}^\mu$ can be used in the construction of the basis

Singularities (anisotropic)

- In the anisotropic case, some poles of the dressed propagator have moved away from the real axis :



- Some poles have migrated to the upper half plane, and lead to instabilities
- These imaginary poles exist no matter how small the squeezing parameter ξ is (but their imaginary part goes to zero when $\xi \to 0$)

Instability

- The poles in the upper half plane lead to the indefinite growth of some small fluctuations. Indeed, the forward propagation of a small fluctuation $a(x)$ in the medium is given by :

$$a(x) = \int d^3\vec{\boldsymbol{y}}\; G_R(x,y) \left[\overleftarrow{\partial}_y^0 - \overrightarrow{\partial}_y^0 \right] a_{\rm in}(y_0, \vec{\boldsymbol{y}})$$

  where $a_{\rm in}(y)$ is the initial condition for the fluctuation, and G_R the retarded propagator in the medium

- Consider a toy model for a propagator with such a pole :

$$G_R(k) \equiv \frac{1}{(k_0 - i\Gamma) - |\vec{\boldsymbol{k}}|^2} \qquad \text{with } \Gamma > 0$$

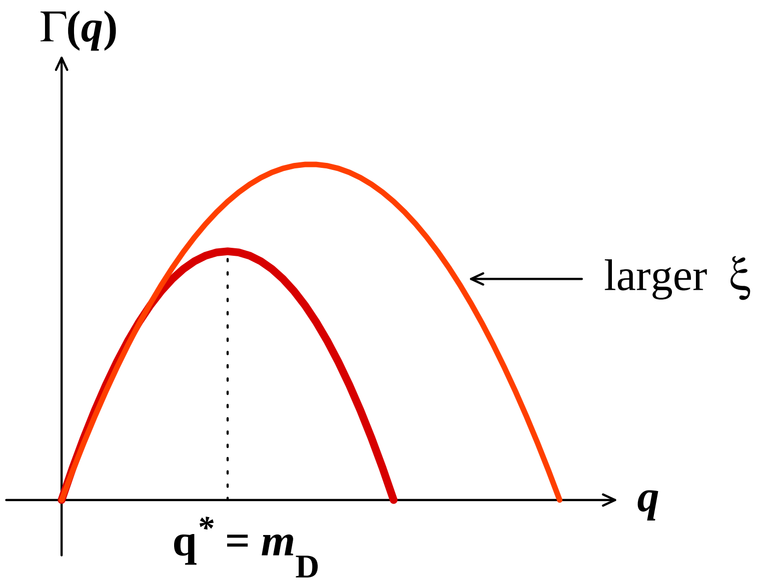
  and a plane wave as the initial condition $a_{\rm in}(y) \equiv \exp(-iq\cdot y)$

- One finds :

$$a(x) = e^{-i(q_0 x_0 - \vec{\boldsymbol{q}}\cdot\vec{\boldsymbol{x}})} \underbrace{e^{\Gamma(x_0 - y_0)}}_{\text{unbounded growth of the fluctuation}}$$

Growth rate spectrum

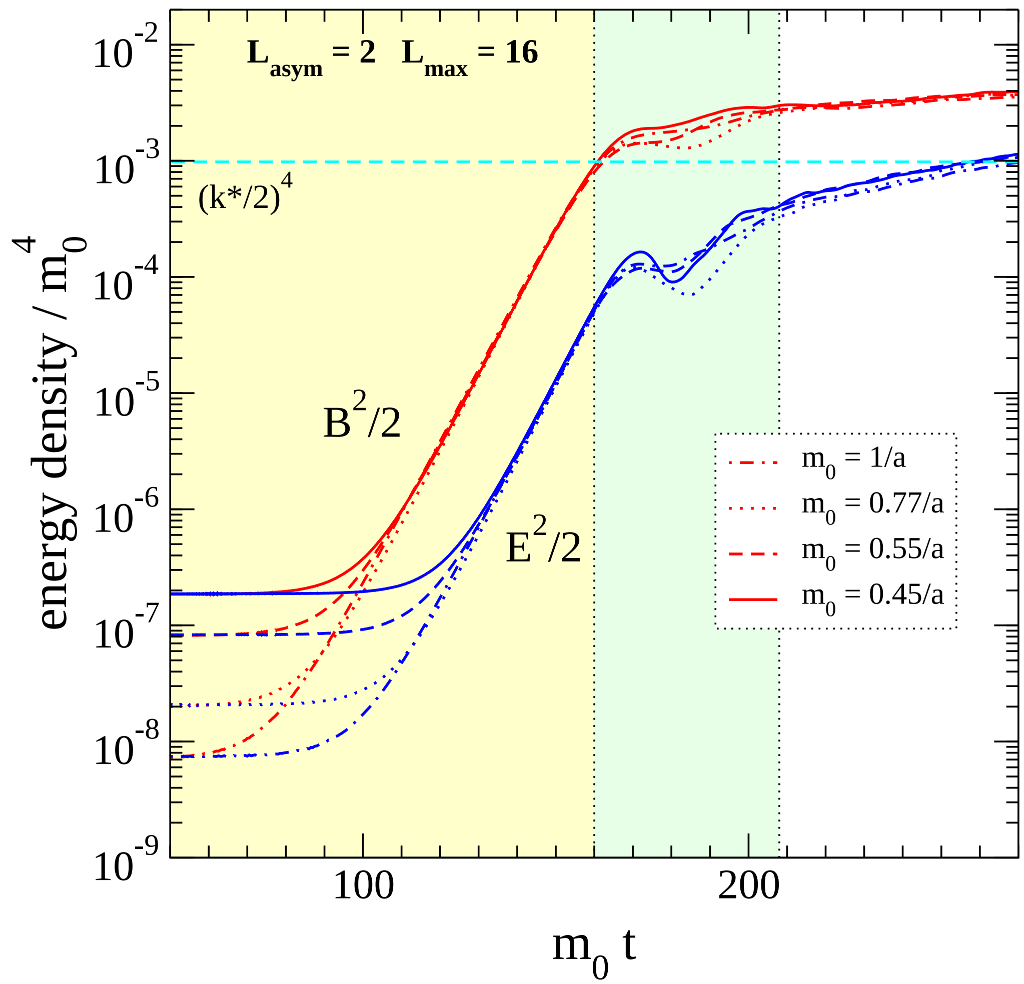
- In QED and QCD, the growth rate Γ depends on the momentum q of the plane wave fluctuation :



- At moderate anisotropies, the most unstable mode is of the same order as the Debye mass
- At very large anisotropies, all modes from soft to hard are unstable

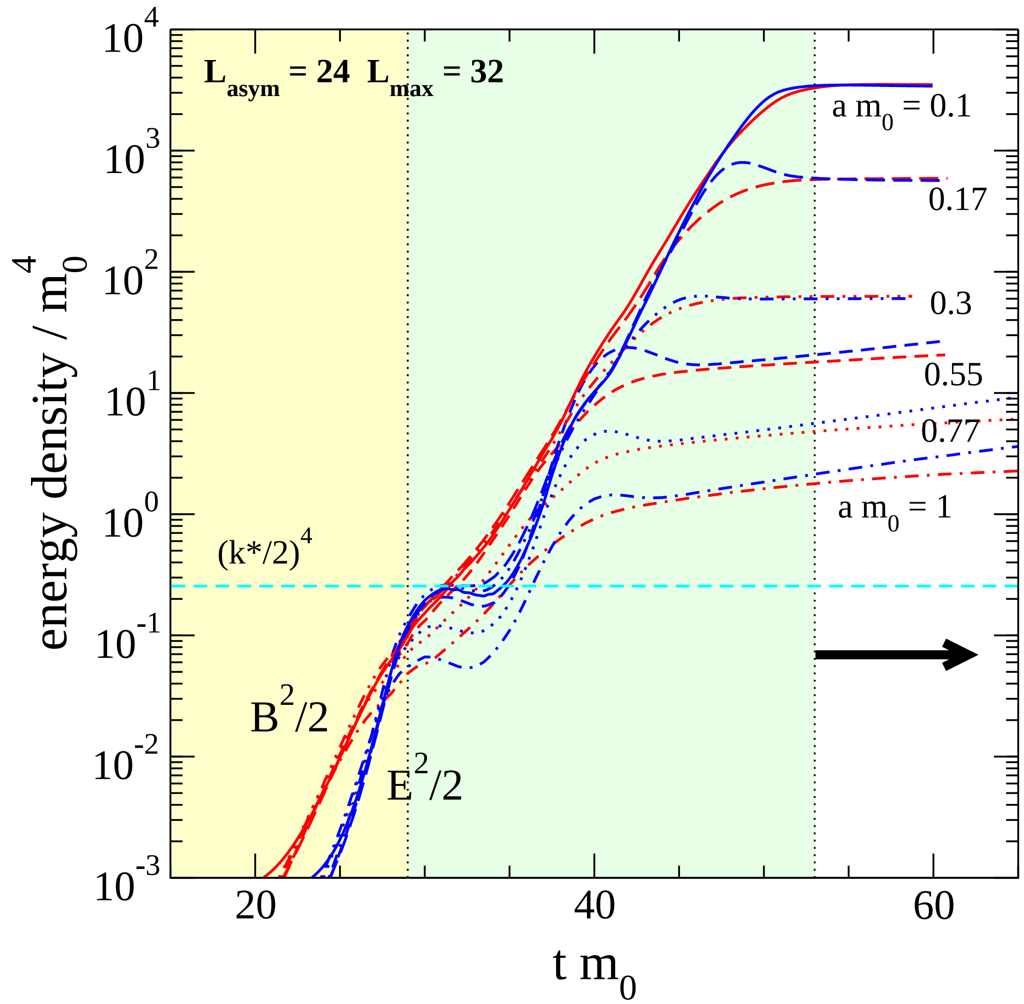
Numerical results

- Small anisotropy. From : Rummukainen (Trento, Jan. 2007)



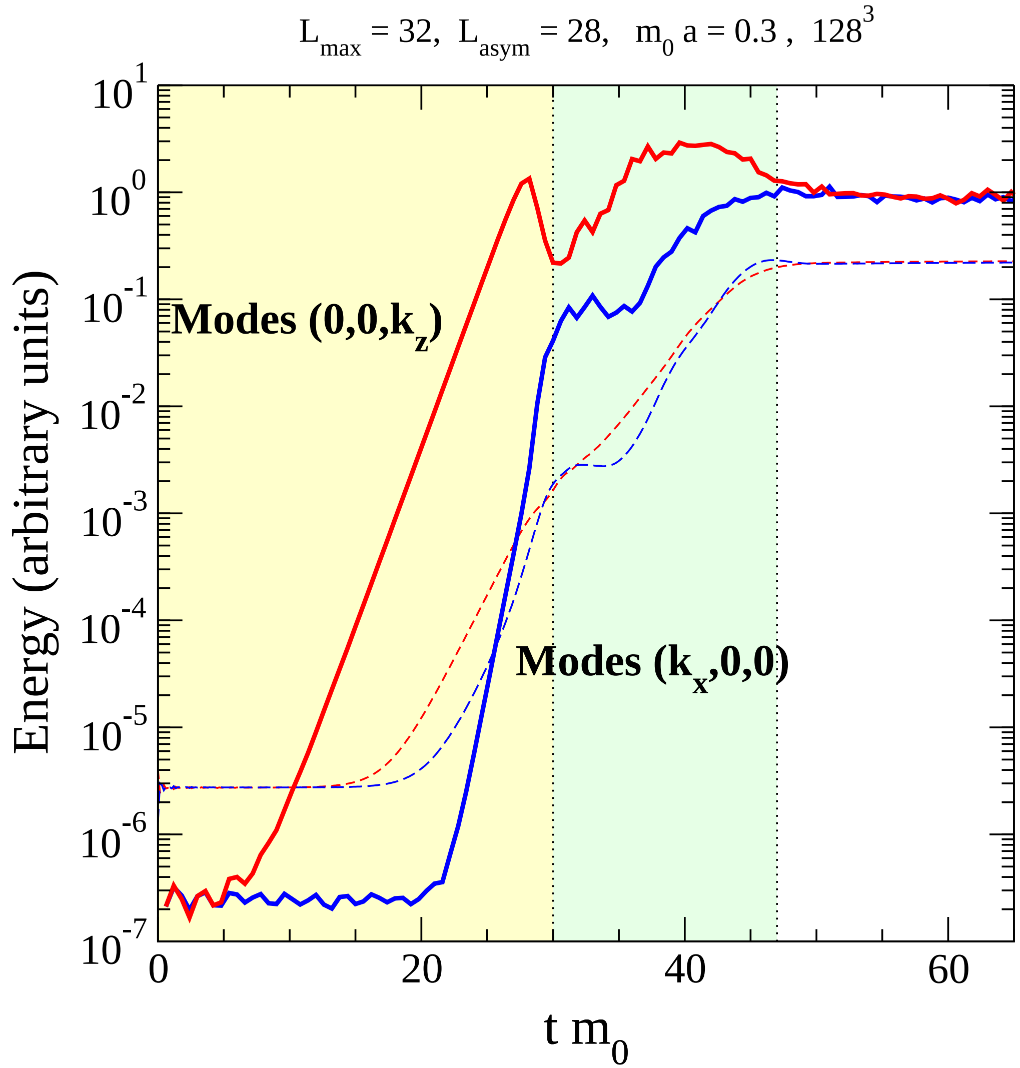
Numerical results

- Large anisotropy. From : Rummukainen (Trento, Jan. 2007)



Numerical results

- Isotropization. From : Rummukainen (Trento, Jan. 2007)



Lecture III : Out of equilibrium systems

- Schwinger-Keldysh formalism, Long time pathologies
- From fields to kinetic theory
- Collisionless kinetic equations
- Boltzmann equation
- Transport coefficients