Strongly coupled QGP or turbulent thermalization ?

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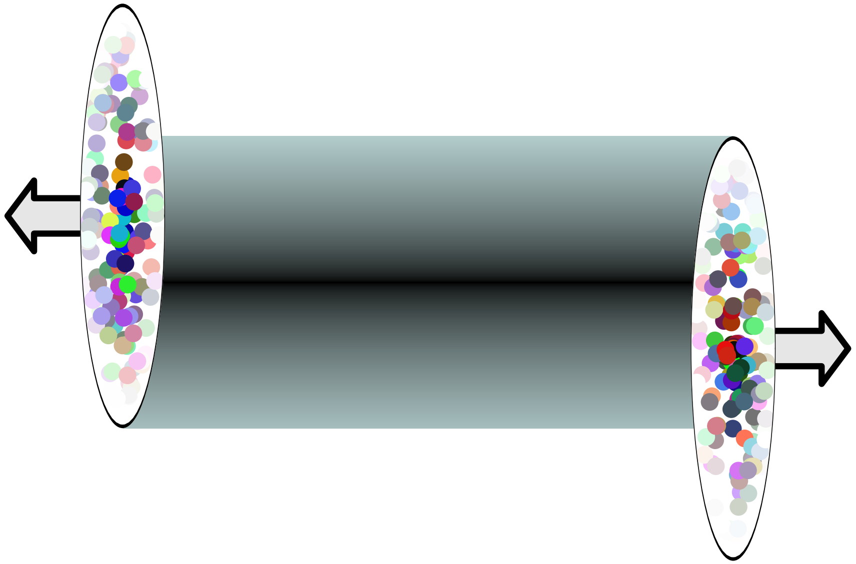
Outline

- What have we learned from RHIC?
- AdS/CFT duality and the strongly coupled QGP
- Ab initio perturbative approach and turbulent thermalization

What have we learned from RHIC?

Collective flow

- Consider a non-central collision :



![CERN]

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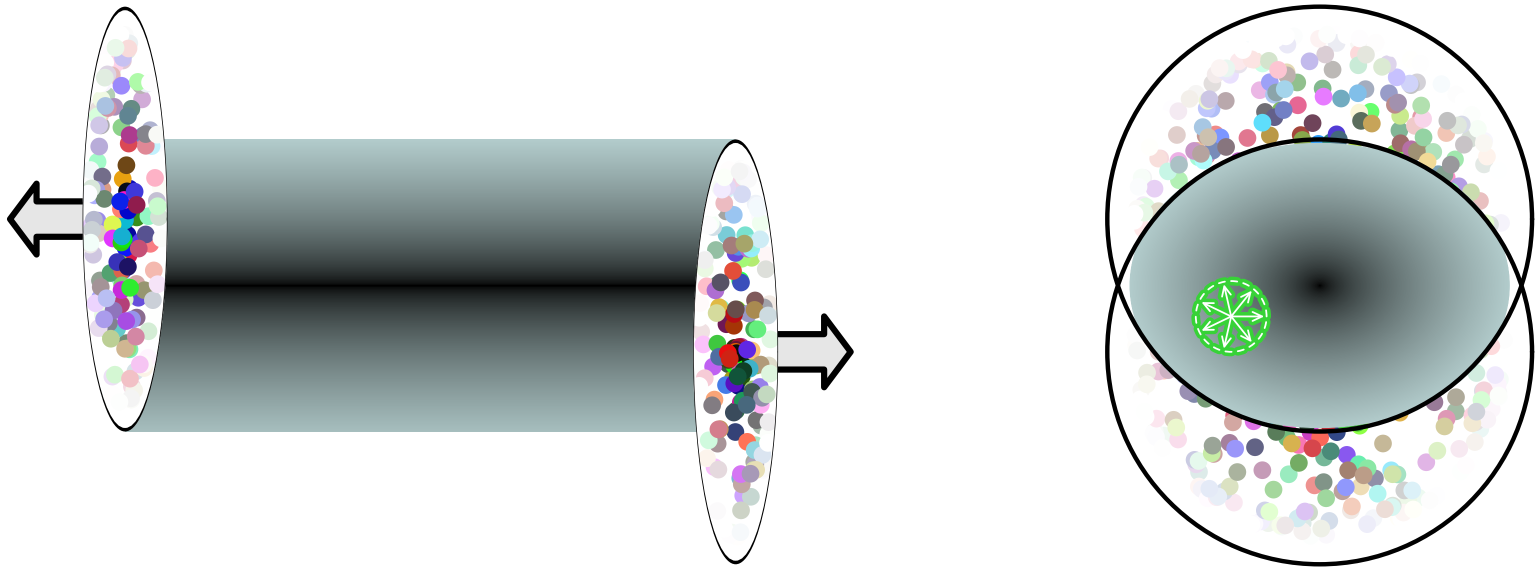

- Initially, the momentum distribution of particles is isotropic in the transverse plane, because their production comes from local partonic interactions

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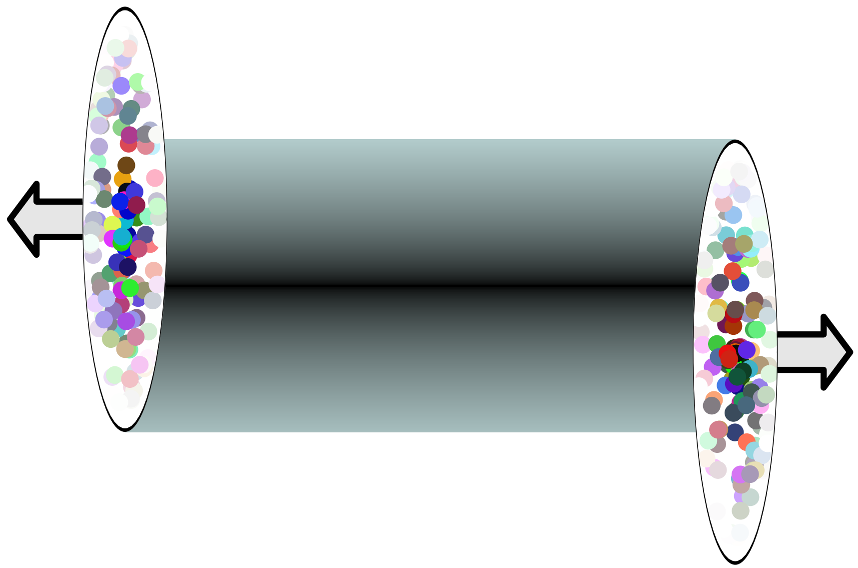

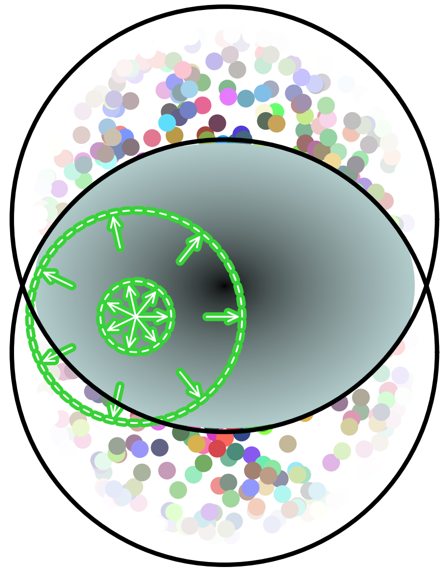

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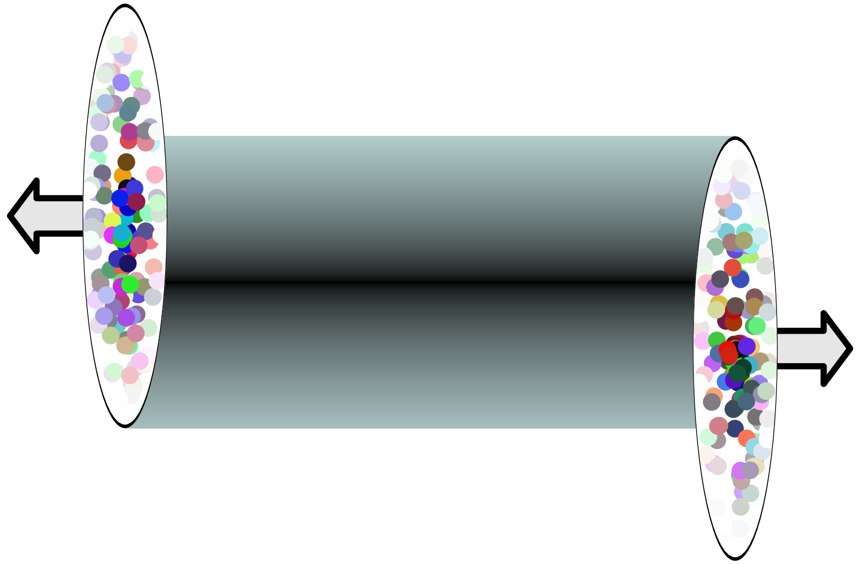
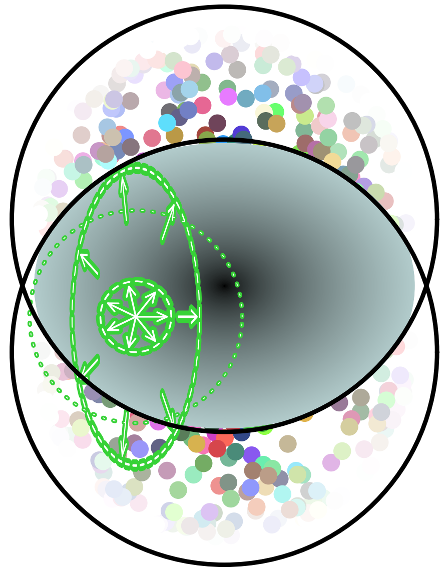




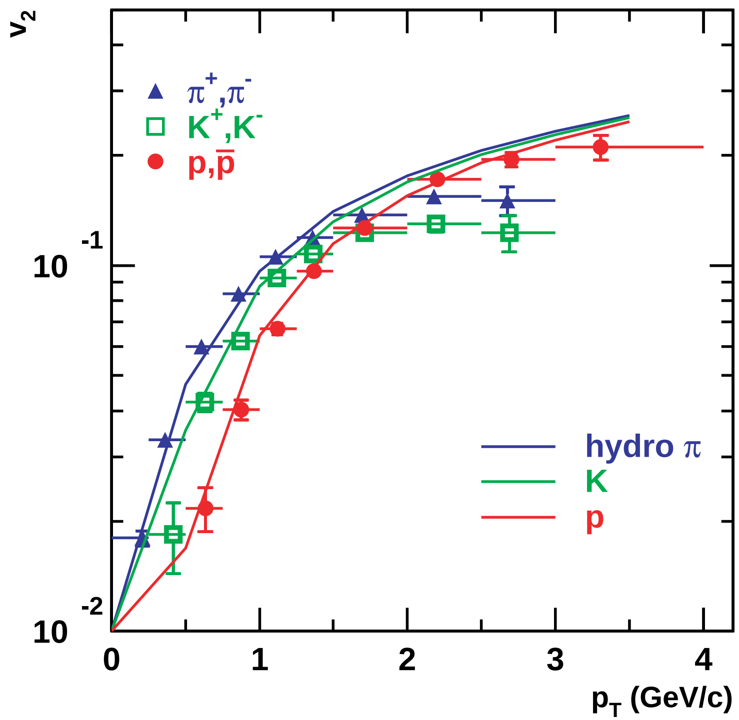
  - Initially, the momentum distribution of particles is isotropic in the transverse plane, because their production comes from local partonic interactions
  - If these particles were escaping freely, the distribution would remain isotropic at all times
  - If the system has a small mean free path, pressure gradients are anisotropic and induce an anisotropy of the distribution

Collective flow and ideal hydrodynamics

- Observable: 2^{nd} harmonic of the azimuthal distribution

$$dN/d\varphi \sim 1 + 2v_1 \cos(\varphi) + 2v_2 \cos(2\varphi) + \cdots$$

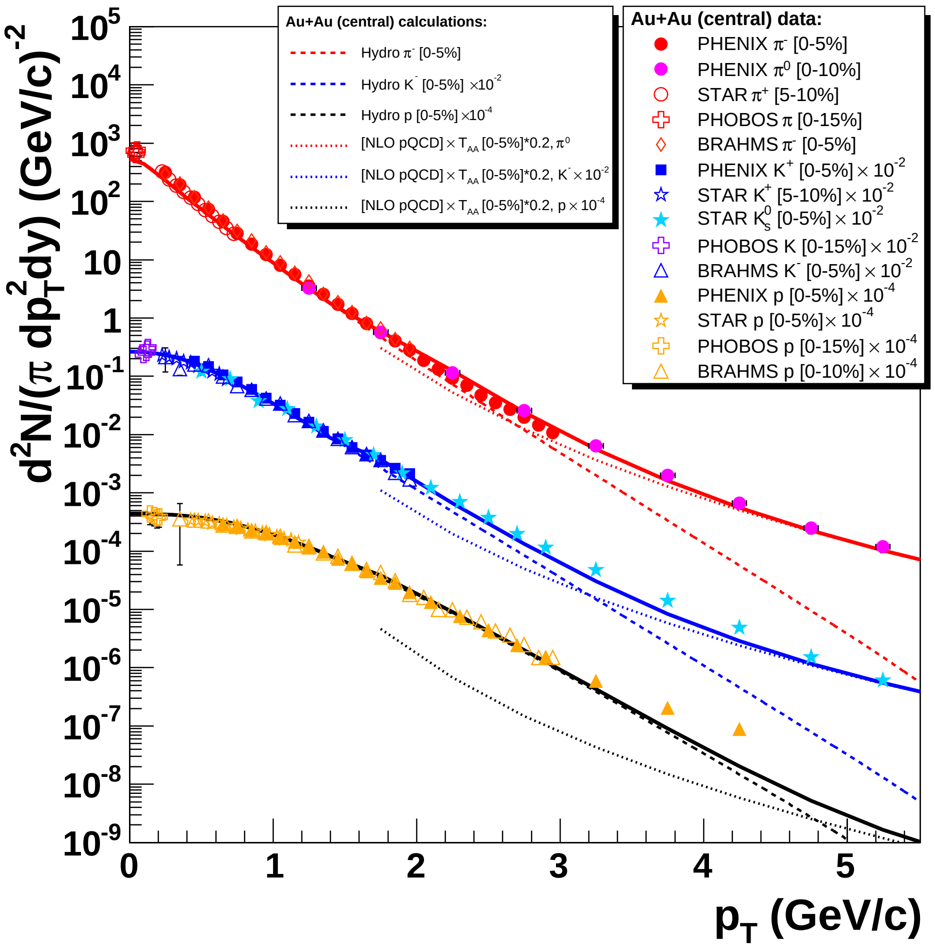
▷ v_2 measures the ellipticity of the momentum distribution



- Note : even heavy quarks seem to follow this flow

Another success of hydrodynamics

- Hydrodynamics reproduces the hadron spectra at low $p_\perp$



Is the QGP a perfect fluid?

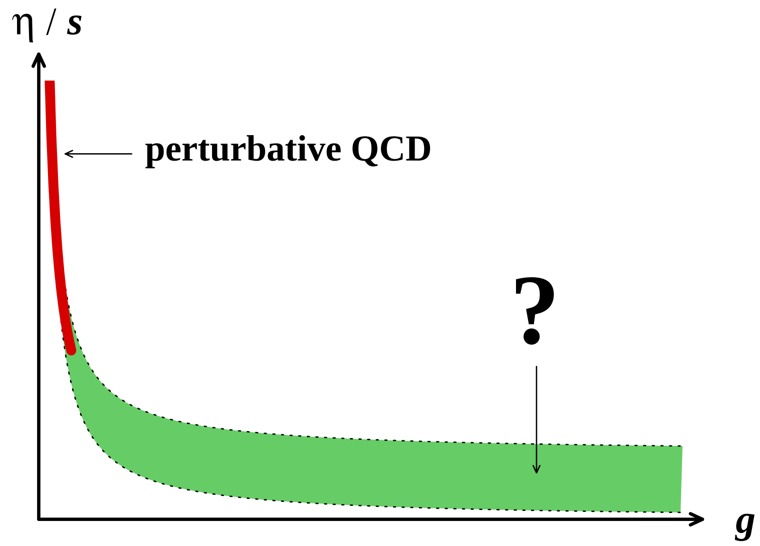
- Note: a perfect fluid is a fluid with a very small viscosity, that can be described with Euler equations (ideal hydrodynamics)
- The elliptic flow coefficient v_2 measured at RHIC is well reproduced by ideal hydrodynamics, that has no viscosity
  - In hydrodynamics, the relevant parameter is the dimensionless ratio η/s of the shear viscosity to the entropy density
  - It has been concluded from there that the QGP must have a very small ratio η/s
- In the weakly coupled QGP, η/s is all but small...

AdS/CFT duality and the sQGP

Weak coupling viscosity

- The shear viscosity has been calculated in QCD at weak coupling ($g \to 0$), and it is quite large :

$$\frac{\eta}{s} = \frac{5.12}{g^4 \ln\left(\frac{2.42}{g}\right)}$$



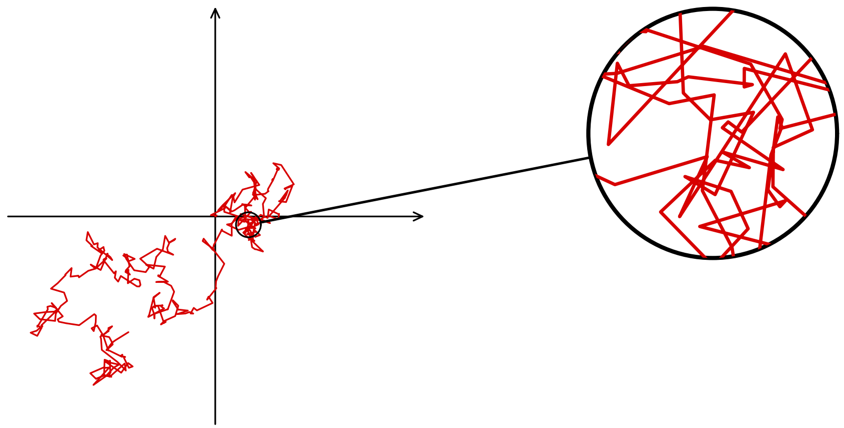
- However, η/s decreases quickly when the coupling increases $\triangleright$ one way to have a small viscosity is to have a large coupling. Problem : how to calculate it?

Uncertainty bound on η/s

- $\eta \sim \lambda\epsilon$ ($\lambda =$ mean free path, $\epsilon =$ energy density). Thus,

$$\frac{\eta}{s} \sim \lambda \underbrace{\frac{\epsilon}{s}}_{\text{energy per particle}}$$

- Heisenberg inequalities forbid the mean free path to be smaller than the De Broglie wavelength of the particles. Scatterings by an $\mathcal{O}(1)$ angle can occur only every $\lambda_{\rm Broglie}$ at most :



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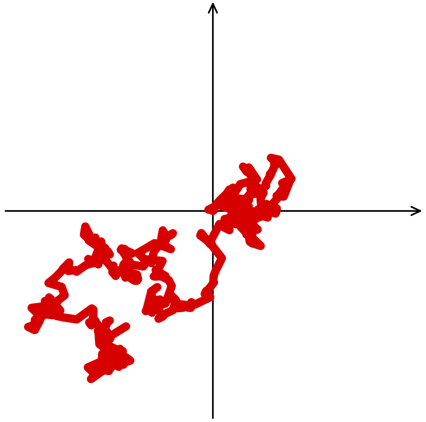

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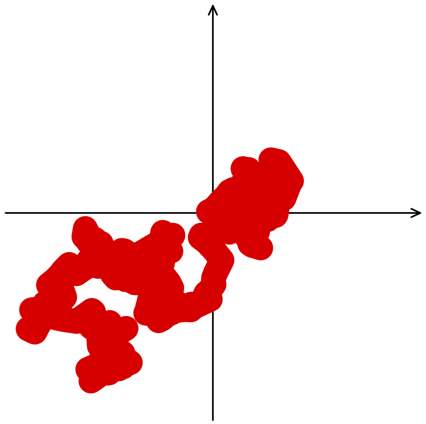


- Hence, $\dfrac{\eta}{s} \geq \mathcal{O}(1)$

AdS/CFT duality at T=0

- In QCD, we cannot compute the strong coupling limit
- Maximally super-symmetric $SU(N)$ Yang-Mills theories in the limit $g^2N \to +\infty$ are dual to classical super-gravity on an $\mathrm{AdS}_5 \times S_5$ manifold with metric

$$ds^2 = \frac{R^2}{z^2}\big(\underbrace{-dt^2 + d\vec{x}^2}_{\text{we live here... (at z=0)}} + dz^2\big) + R^2 d\Omega_5^2$$

- If an operator $\mathcal{O}$ of our world is coupled on the boundary to a field ϕ that lives in the bulk, the duality states that :

$$e^{-S_{\rm cl}[\phi]} = \left\langle e^{\int_{\rm boundary} \mathcal{O}\ \phi(z=0)} \right\rangle$$

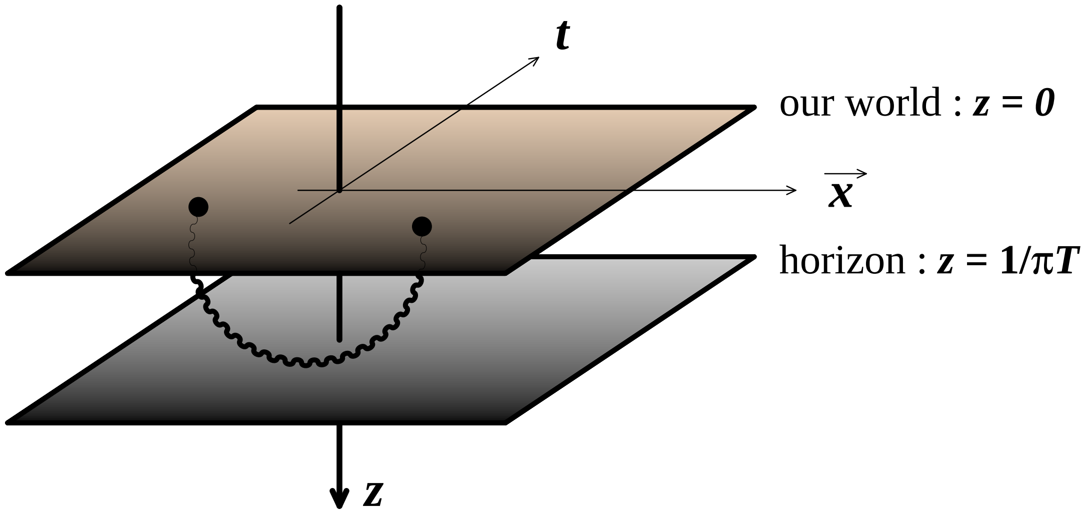
  - The right hand side is a generating functional for the correlators of operators $\mathcal{O}$ in the 4-dim super Yang-Mills theory
  - The left hand side is calculable in the gravity dual (solve the classical EOM for ϕ with the boundary condition $\phi(z=0)$)

AdS/CFT duality at high T

- At finite temperature T :

  $-dt^2 + dz^2 \to -f(z)dt^2 + dz^2/f(z)$ with $f(z) = 1 - (\pi z T)^4$

- $f(z) = 0$ at $z = 1/\pi T \quad \Rightarrow$ black hole horizon



- Ordinary particles in 4-dimensions are the end points of open strings living in the bulk

Viscosity in SUSY Yang-Mills

- The shear viscosity can be obtained from correlations of the energy-momentum tensor :

$$\eta \propto \int dt d^3\vec{\boldsymbol{x}} \; \left\langle T_{xy}(t,\vec{\boldsymbol{x}}) \; T_{xy}(0,\vec{\boldsymbol{0}}) \right\rangle$$

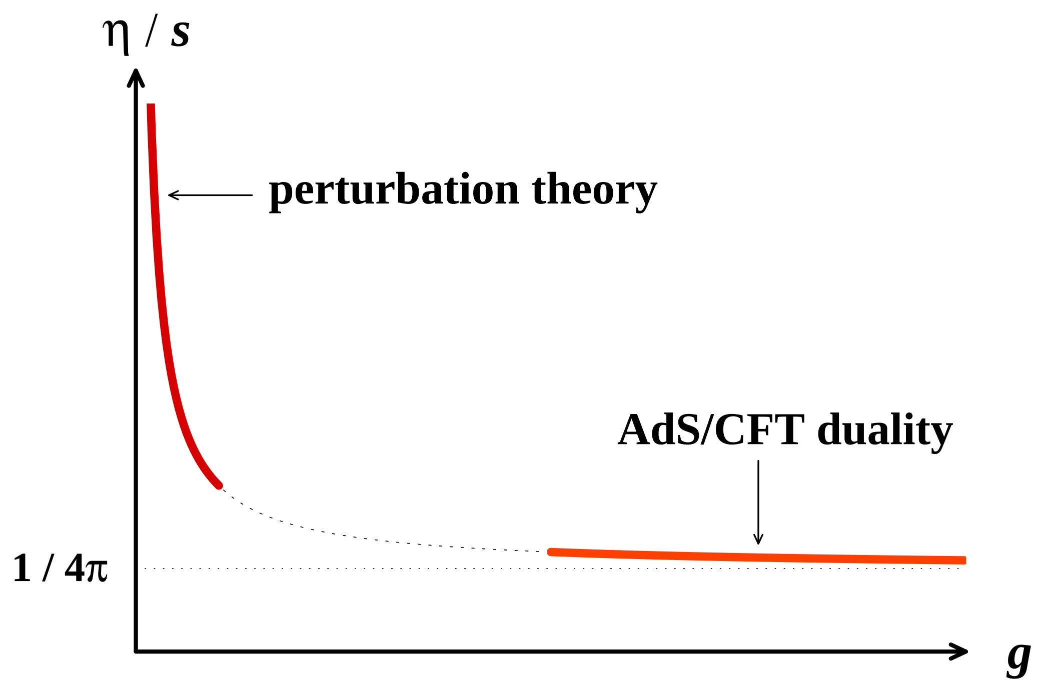
(linear response theory)

- In the dual theory, T_{xy} couples to metric perturbations, i.e. to the graviton. The above correlation function is also the absorption cross-section of a graviton (of zero frequency) by the black hole. Hence :

$$\eta \propto \sigma_{\rm abs}$$

- In the classical limit, $\sigma_{\rm abs}$ is the area of the horizon. Moreover, the area of a black-hole horizon is its entropy
- Combining everything, one obtains $\eta/s = 1/4\pi$

Viscosity in SUSY Yang-Mills



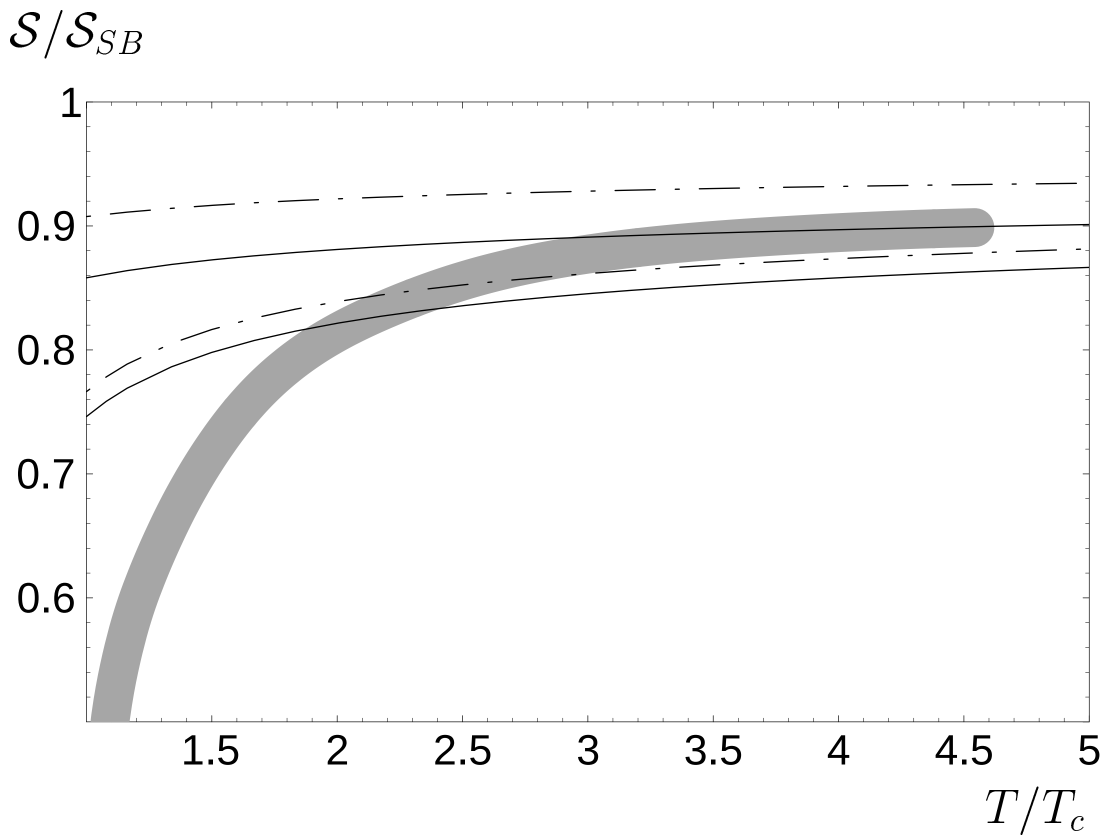
- Conjecture : $1/4\pi$ is the lowest possible value for η/s
- Note: all the known substances have a viscosity to entropy ratio (much) larger than the bound
  - ▷ led to the idea that the QGP may be the "most perfect fluid"

Caveats of AdS/CFT: SUSY YM $\neq$ QCD

- AdS/CFT only applies to maximally super-symmetric Yang-Mills theories. Such theories are scale invariant, have no running coupling, no chiral symmetry breaking, and no confinement
- Whether what we learn about these theories is accurate for QCD (that has broken scale invariance, running coupling, chiral symmetry breaking, confinement, and quite different matter fields...) is at best a wishful thinking
- Nevertheless an interesting playground in order to realize how wrong one's weak coupling prejudices may be...
- Note : in the strong coupling limit of any sensible field theory, η/s is probably close to the uncertainty principle limit

Caveats of AdS/CFT: is g really large?

- There are some dissenting views about whether the physics of the QGP at $T/T_{\rm crit} \sim 2-3$ is really strongly coupled. For quantities such as the entropy, perturbative techniques (+resummations) lead to accurate results in this region
  Blaizot, Iancu, Rebhan (1999-2000)



Ab initio perturbative approach

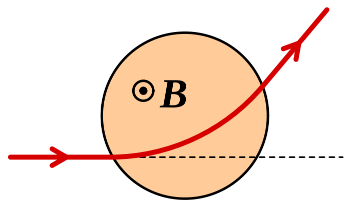
Small η/s in weak coupling ?

Asakawa, Bass, Muller (2006)

- Assume that $\alpha_s = \dfrac{g^2}{4\pi} \ll 1$
- Consider a domain of size Q_s^{-1}, in which the magnetic field is uniform and large, of order $B \sim Q_s^2/g$
- Let a particle of energy $E \sim Q_s$ go through this domain. The Lorenz force deflects its trajectory by an angle of order unity :

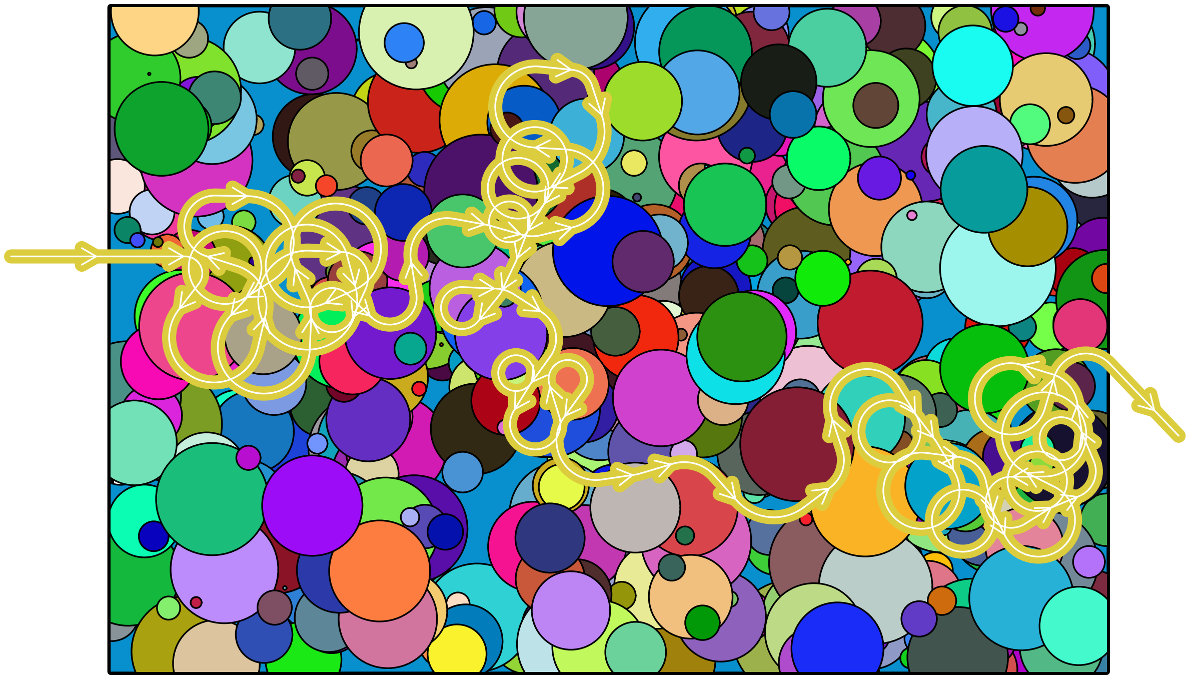
$$\frac{d\vec{\boldsymbol{p}}}{dt} = g\,\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}} \quad \Rightarrow \quad \dot{\theta} = \frac{gB}{E} \sim Q_s$$

time spent in the domain : $\delta\tau \sim Q_s^{-1}$



Small η/s in weak coupling ?

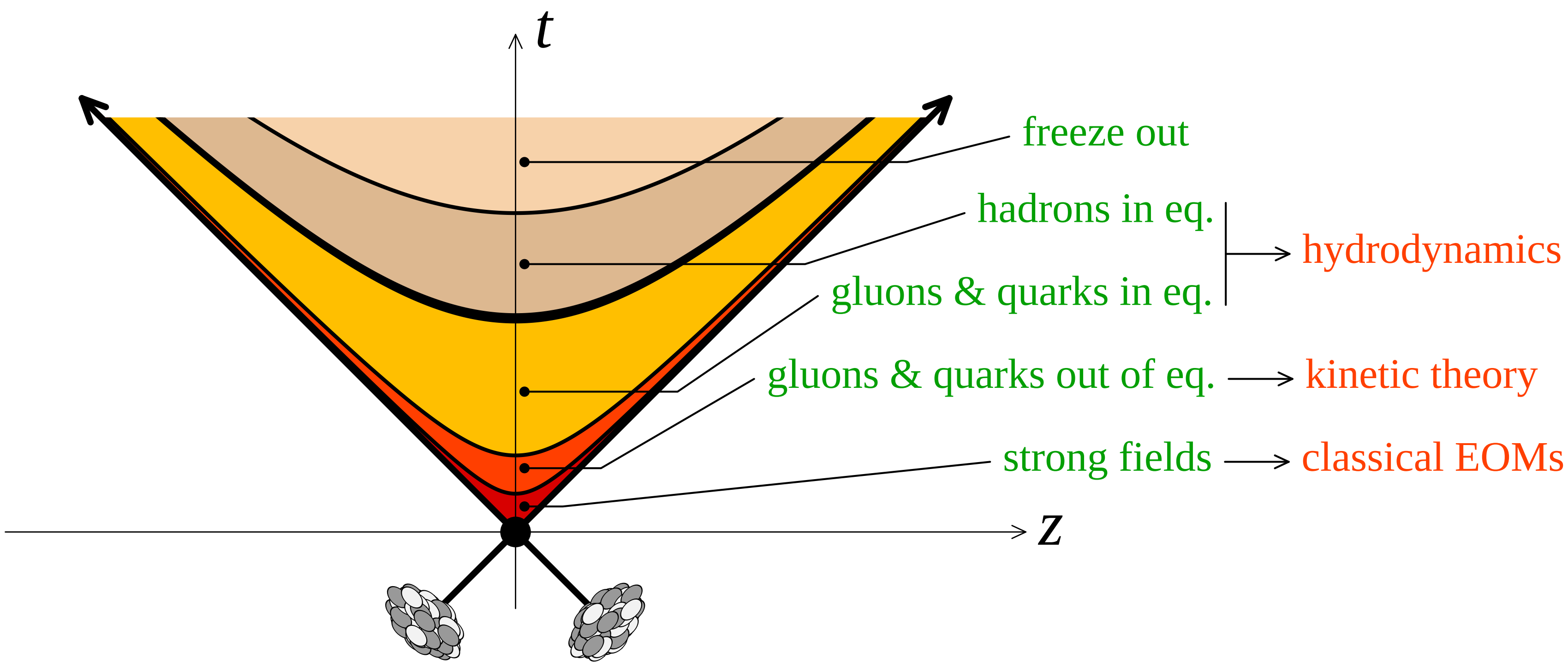
- Consider now a region filled with such domains, with random orientations for the magnetic field in each domain



▷ In such a medium, the mean free path of a particle of energy Q_s is of order Q_s^{-1}, i.e. as low as permitted by the uncertainty principle

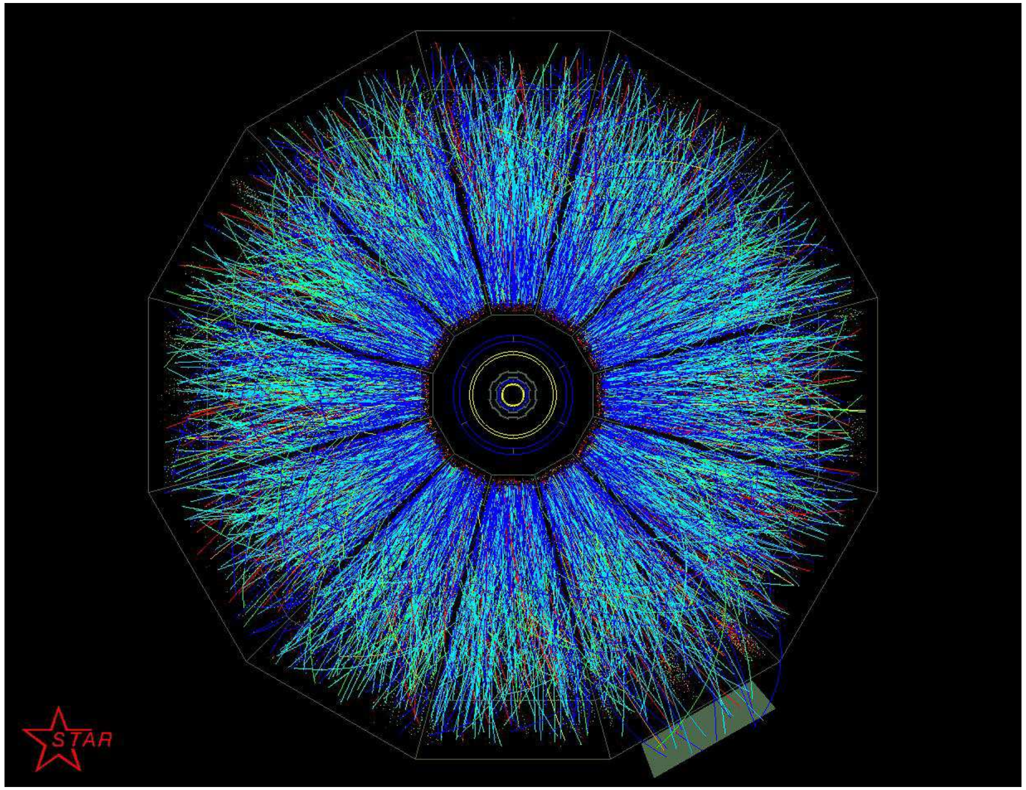
▷ η/s must be close to the lower bound

Ab initio perturbative approach



- Start from the beginning with perturbative QCD, and see whether large random magnetic fields are produced

Initial state: gluon saturation



- 99% of the multiplicity below $p_\perp \sim 2$ GeV
- the bulk of of particle production comes from (very) low x

  ▷ high gluon density (even more so in nuclei : $G_A / G_{\rm p} \approx A$)

Criterion for gluon recombination

Gribov, Levin, Ryskin (1983)

- Number of gluons per unit area:

$$\rho \sim \frac{xG_A(x, Q^2)}{\pi R_A^2}$$

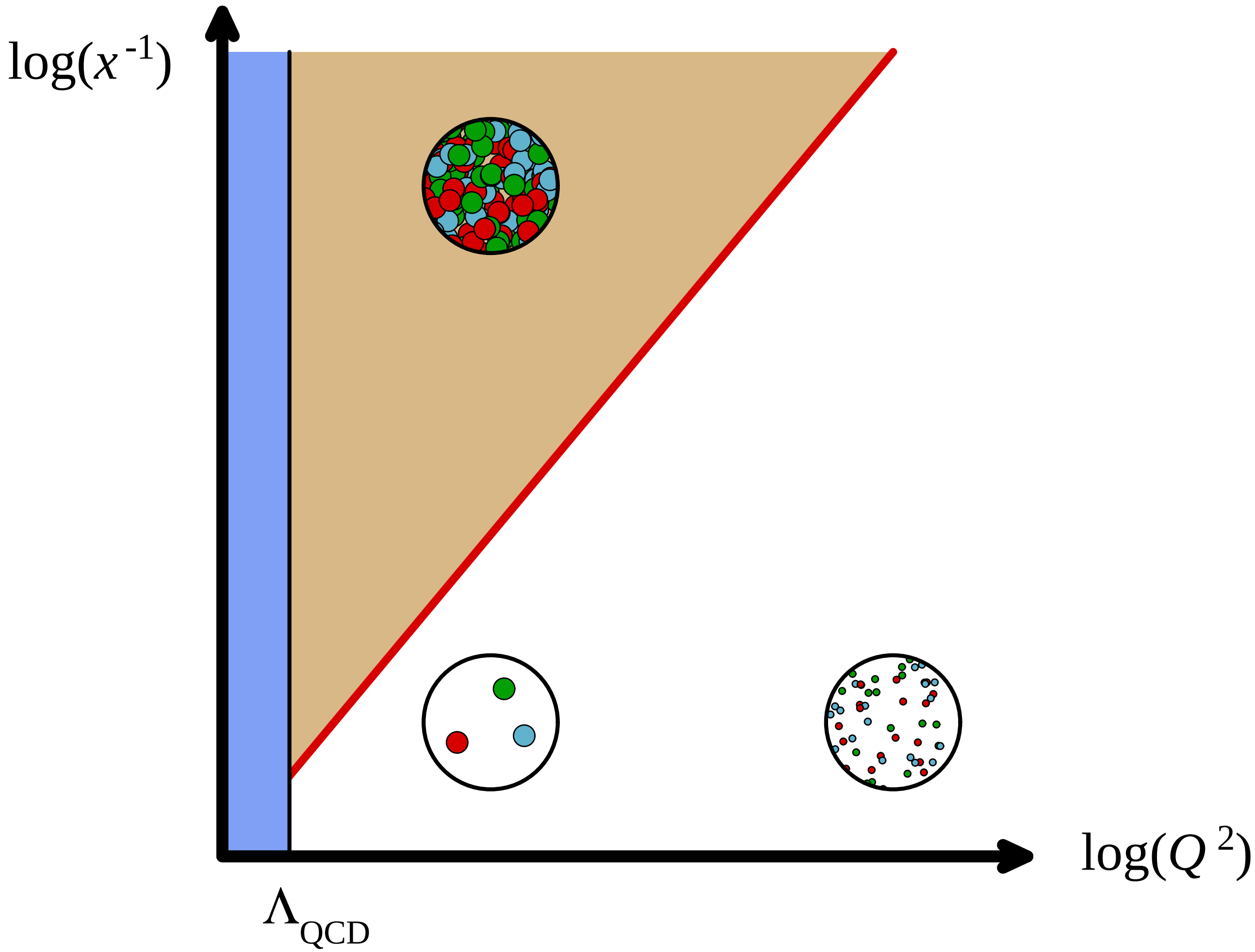
- Recombination cross-section:

$$\sigma_{gg\to g} \sim \frac{\alpha_s}{Q^2}$$

- Recombination happens if $\rho\sigma_{gg\to g} \gtrsim 1$, i.e. $Q^2 \lesssim Q_s^2$, with:

$$Q_s^2 \sim \frac{\alpha_s xG_A(x, Q_s^2)}{\pi R_A^2} \sim A^{1/3} \frac{1}{x^{0.3}}$$

Saturation domain



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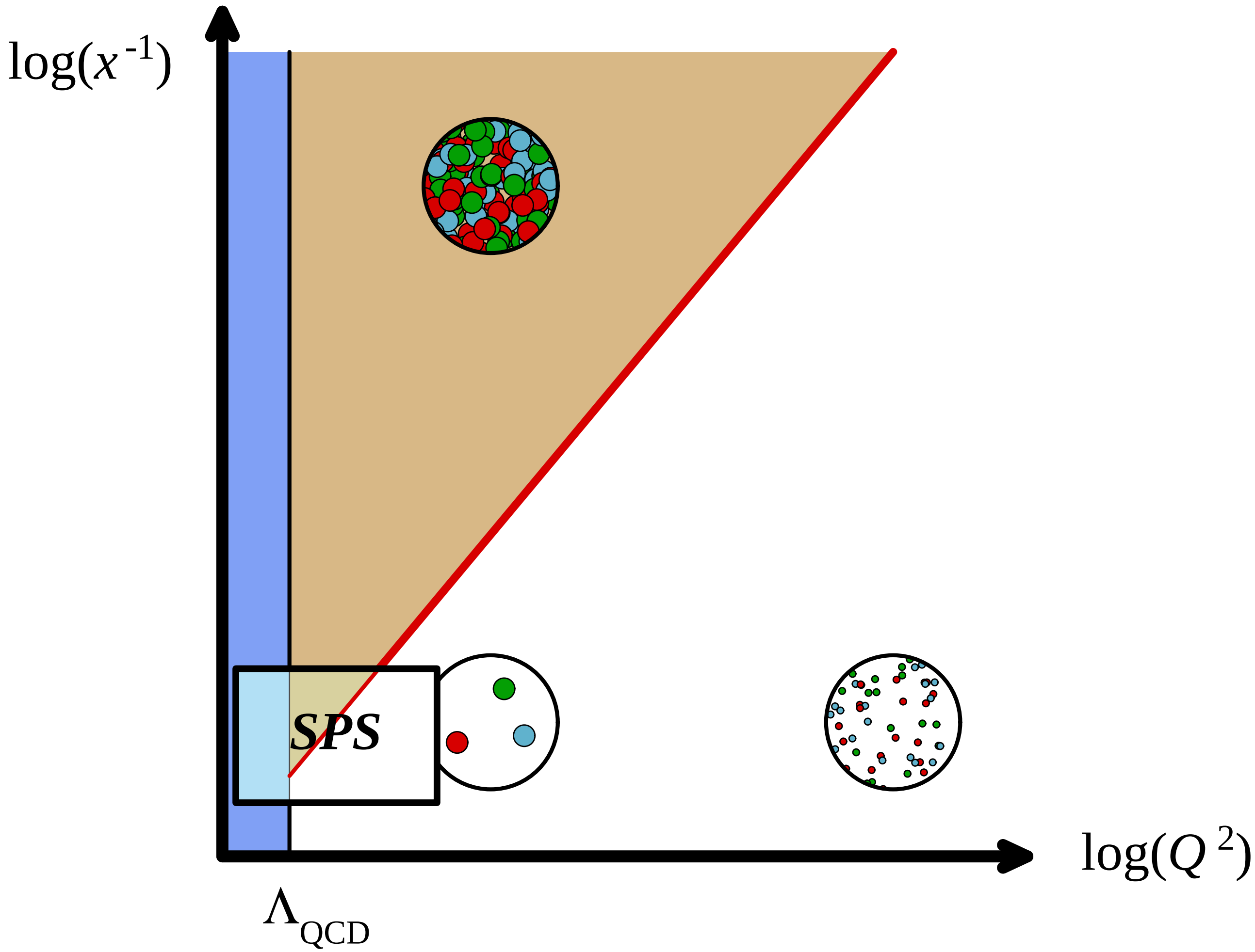

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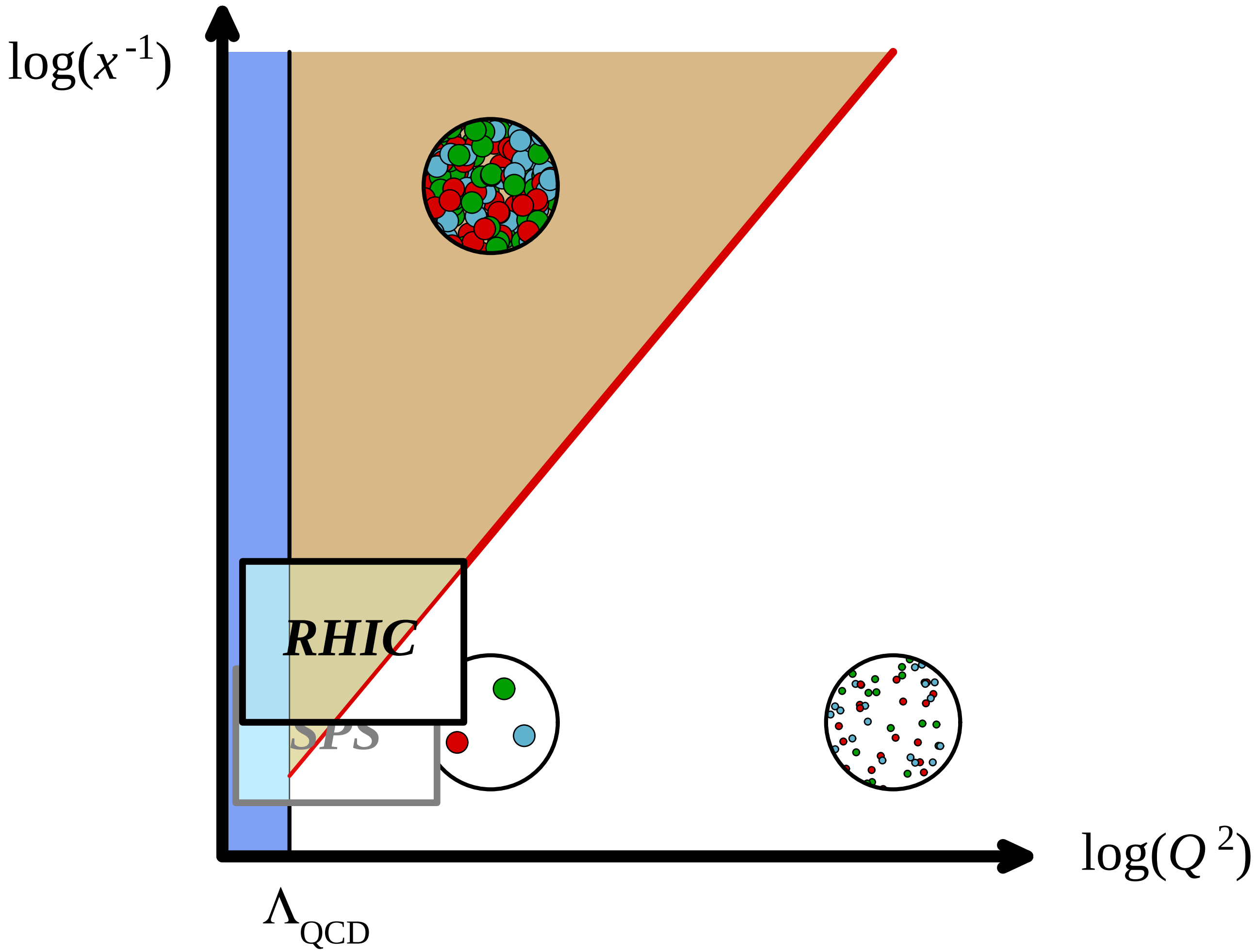

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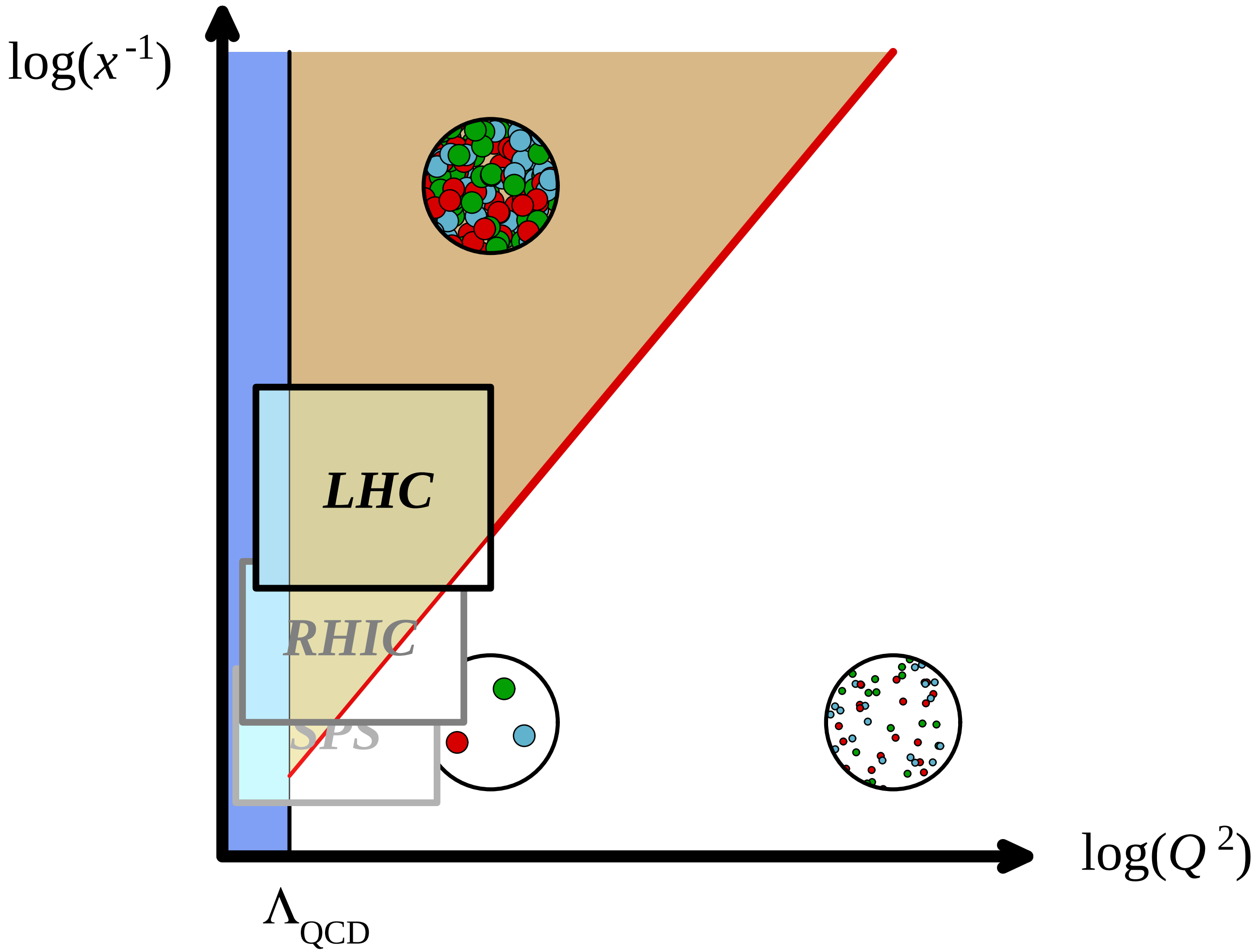

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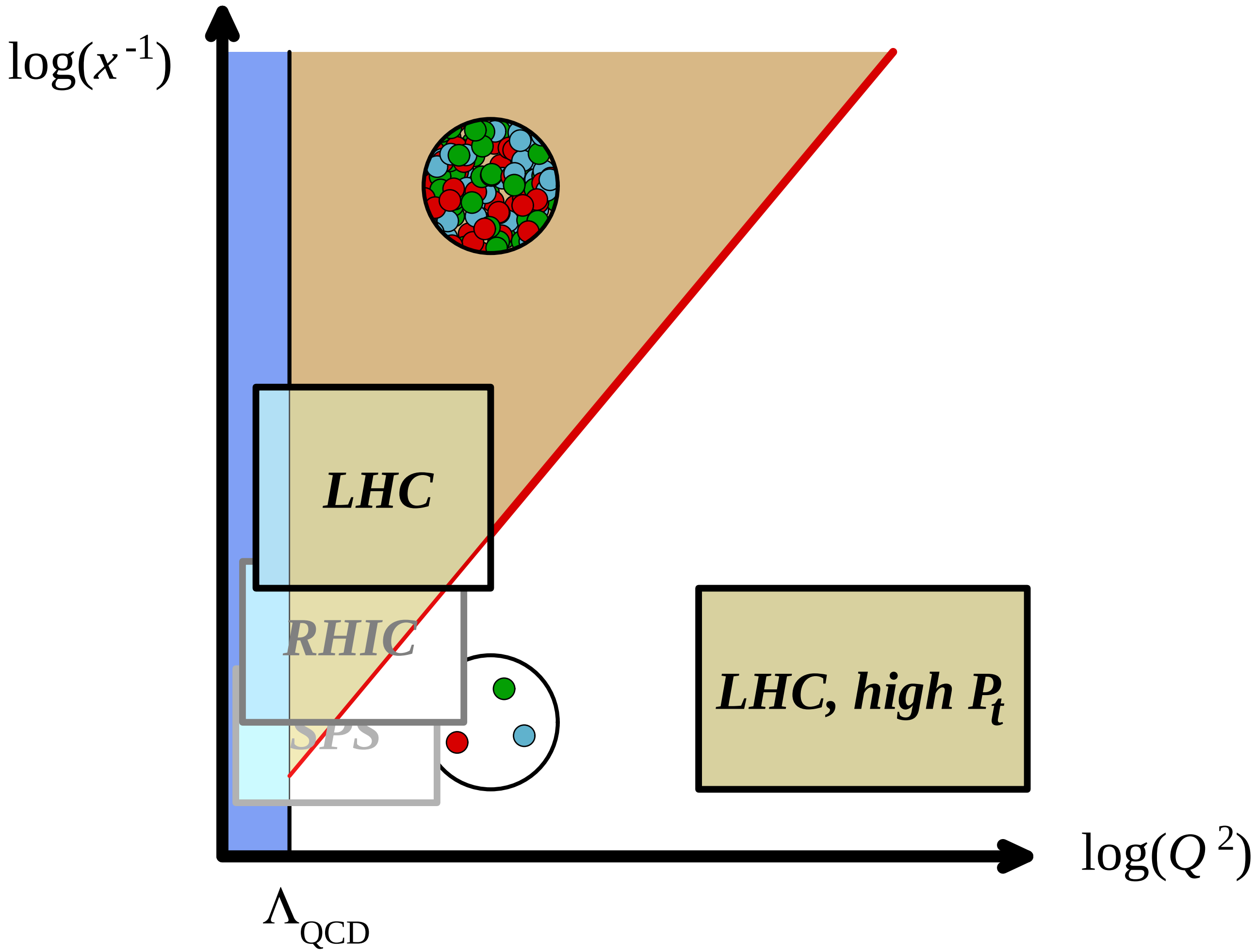
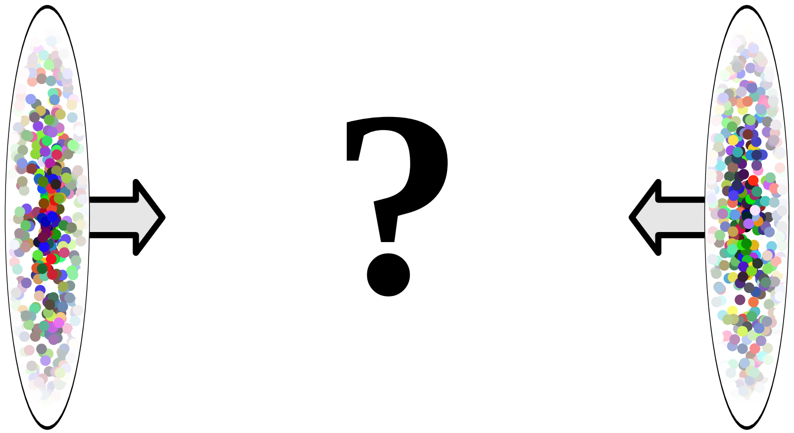


Initial particle production

- Main difficulty : the formalism for studying the collision of two densely occupied projectiles is quite involved



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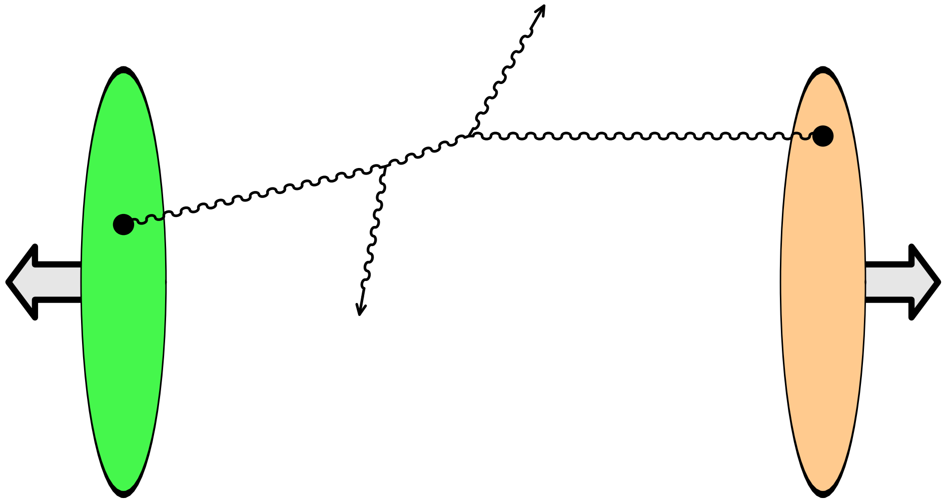
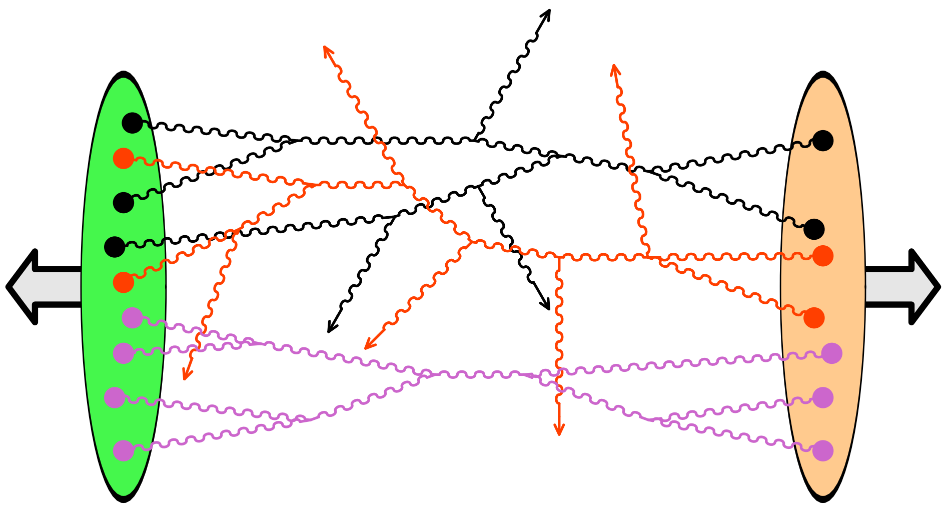


- Dilute regime : one parton in each projectile interact

Initial particle production



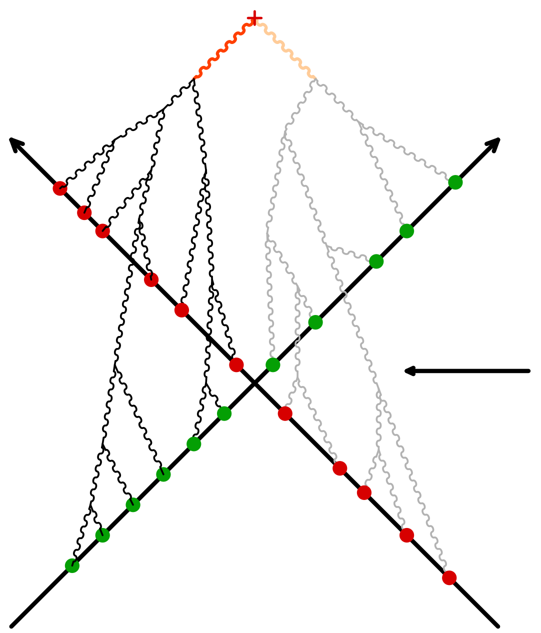
- Dilute regime : one parton in each projectile interact
- Dense regime : multiparton processes become crucial
  + pileup of many partonic scatterings in every AA collision

Initial particle production - LO

Krasnitz, Nara, Venugopalan (1999 – 2001), Lappi (2003)

$$\frac{d\overline{N}_{LO}}{d^3\vec{\boldsymbol{p}}} \propto \int_{x,y} e^{ip\cdot(x-y)} \quad \cdots \quad \mathcal{A}_\mu(x)\mathcal{A}_\nu(y)$$

- $\mathcal{A}^\mu(x) =$ classical solution of Yang-Mills equations with color sources ρ_1 and ρ_2 on the light-cone



only tree diagrams at LO

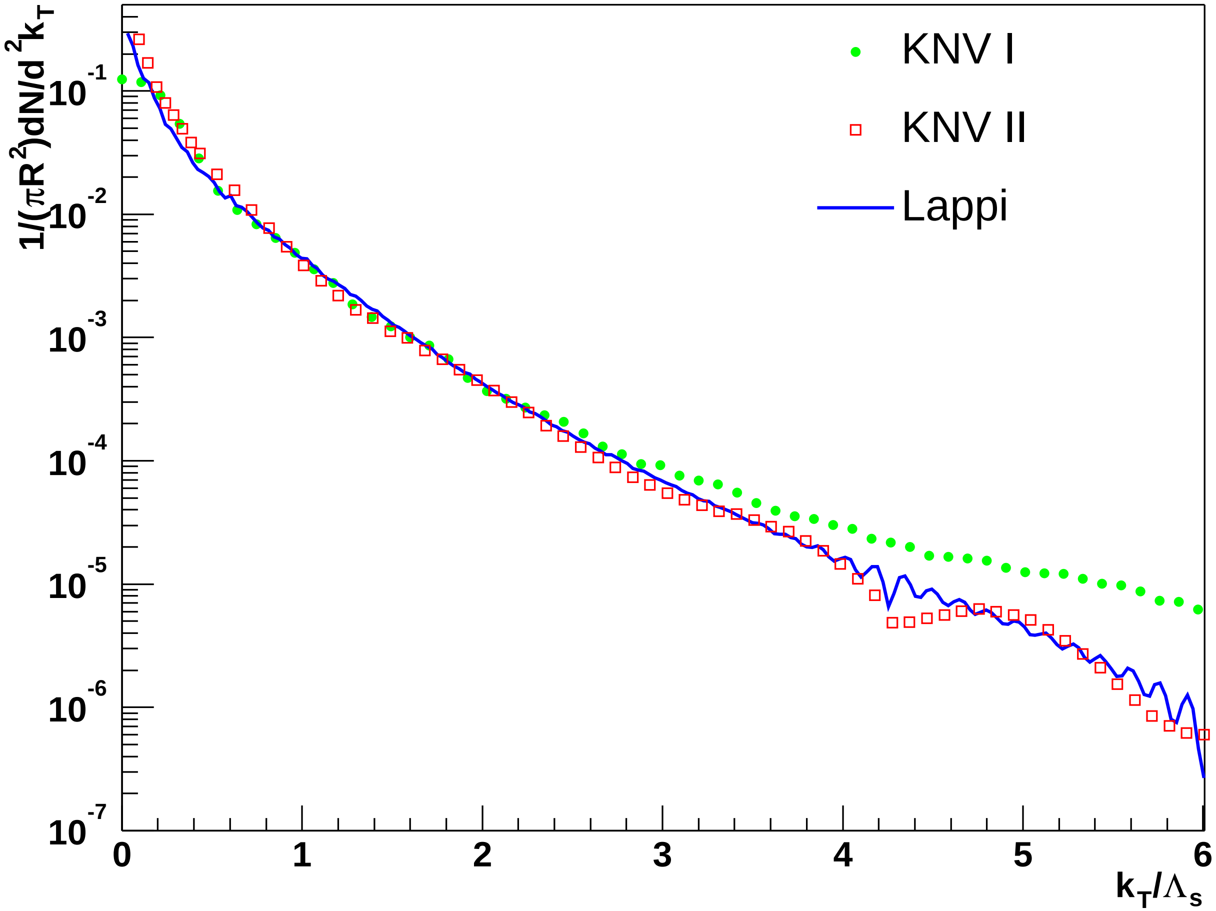
Initial particle production - LO

RHIC results on flow

AdS/CFT duality and the sQGP

Ab initio perturbative approach
- Small eta/s in weak coupling
- Gluon saturation
- Initial particle production
- Initial state factorization
- Glasma instability

Summary

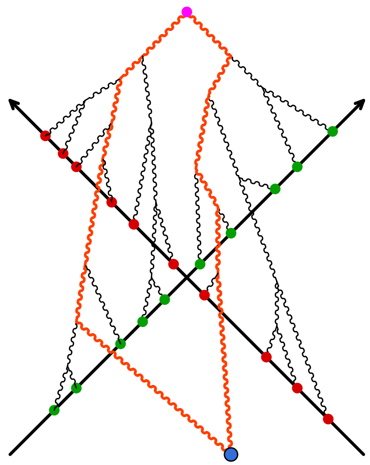


- Important softening at small $k_\perp$ compared to pQCD (saturation)
- Quark production has also been computed (FG, Kajantie, Lappi (2005))

Initial particle production - NLO

FG, Venugopalan (2006), FG, Lappi, Venugopalan (work in progress)

- Typical graph :



- Why is it important ?
  - Questions such as factorization can only be answered by looking at loop corrections
  - Instabilities in the classical solutions may inflate the effect of small perturbations

Initial particle production - NLO

- The sum of all the 1-loop contributions can be written in terms of classical fields, and small field fluctuations above the classical field
- The expressions can be rearranged in a way that clearly separates the initial and final state :

$$\delta\overline{N} = \underbrace{\left[\int_{\vec{u}\in\text{light cone}} \delta\mathcal{A}_{\rm in}(\vec{u})\ \boldsymbol{T}_{\vec{u}} + \int_{\vec{u},\vec{v}\in\text{light cone}} \frac{1}{2}\,\boldsymbol{\Sigma}(\vec{u},\vec{v})\ \boldsymbol{T}_{\vec{u}}\,\boldsymbol{T}_{\vec{v}} \right]}_{\text{operator that depends only on the initial state}} \underbrace{\overline{N}_{LO}[\mathcal{A}_{\rm in}]}_{\text{final state}}$$

- Notes :

$\mathcal{A}_{\rm in} =$ value of the classical field at $\tau = 0^+$

$\delta\mathcal{A}_{\rm in}$ and $\boldsymbol{\Sigma}$ depend only on the dynamics before the collision

$\boldsymbol{T}_{\vec{u}} \sim \dfrac{\delta}{\delta\mathcal{A}_{\rm in}(\vec{u})}$ (generator of shifts of $\mathcal{A}_{\rm in}(\vec{u})$)

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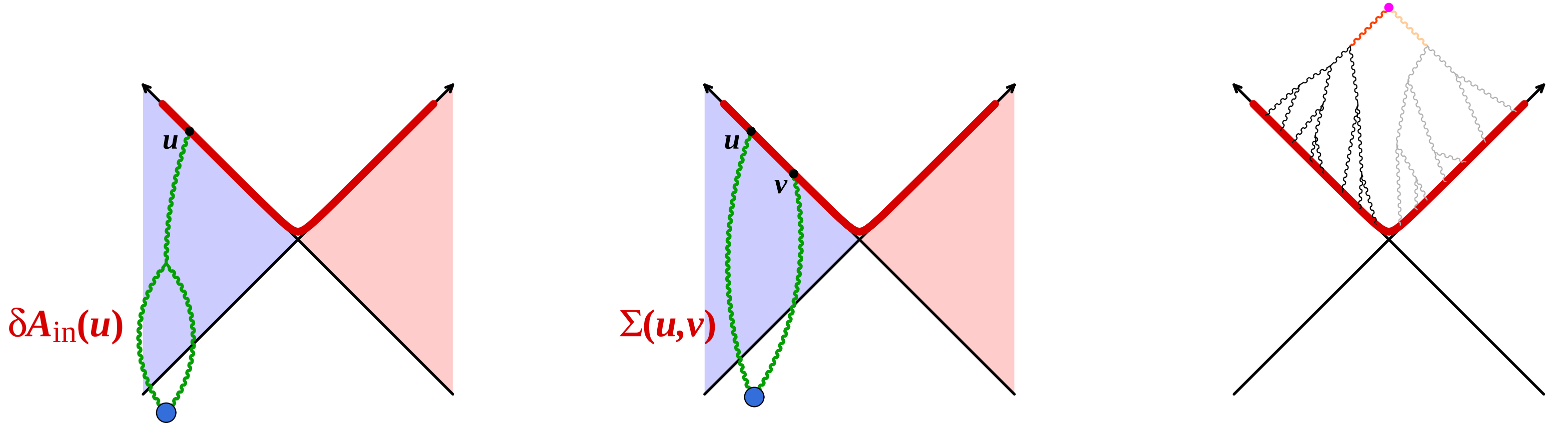


Initial state factorization

- The coefficients $\delta\mathcal{A}_{\rm in}$ and $\boldsymbol{\Sigma}$ in the initial state factor contain divergences, that manifest themselves as large logarithms $\log(1/x_{1,2})$

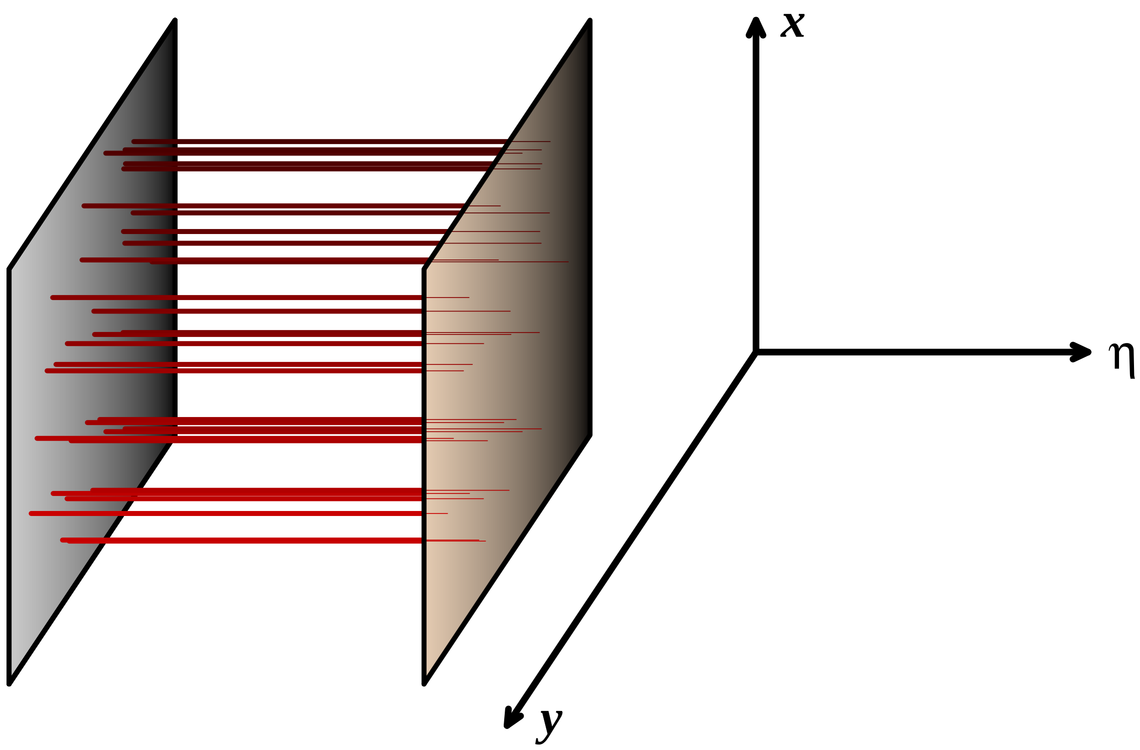
- These large logs invalidate the naive perturbative expansion, because $\alpha_s \log(1/x_{1,2})$ may be large even if α_s is small
  - ▷ All the terms in $\big[\alpha_s \log(\cdot)\big]^n$ should be collected and resummed

- It is expected that these large logs can be factorized in the distributions of color sources $W[\rho_{1,2}]$:

$$\frac{d\overline{N}}{dY\,d^2\vec{\boldsymbol{p}}_\perp} = \int \underbrace{[D\rho_1]\,[D\rho_2]\;W_{Y_{\rm beam}-Y}[\rho_1]\;W_{Y_{\rm beam}+Y}[\rho_2]}_{\text{projectiles source distributions}}\;\underbrace{\frac{d\overline{N}[\mathcal{A}_{\rm in}(\rho_1,\rho_2)]}{dY\,d^2\vec{\boldsymbol{p}}_\perp}}_{\text{“event-by-event” gluon spectrum}}$$

Glasma instability

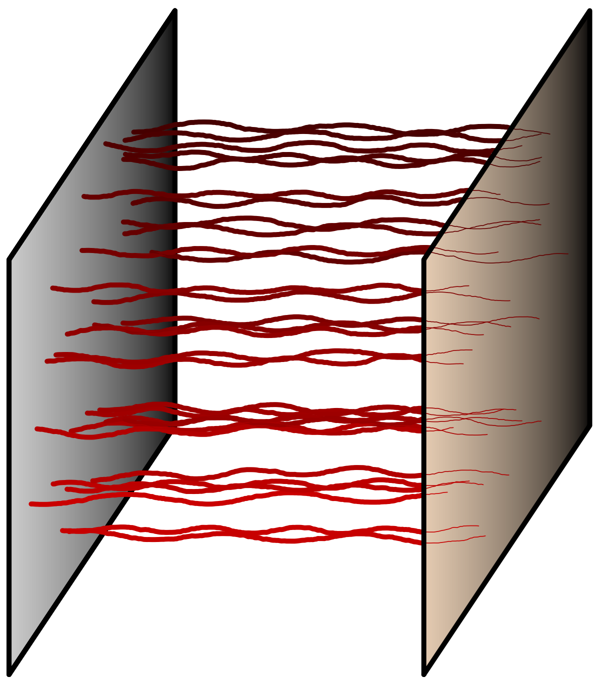
- Leading order magnetic fields at $\tau = 0^+$:



- At $\tau = 0^+$, the classical chromo-electric and chromo-magnetic fields are longitudinal (Lappi, McLerran (2006))
- They are also boost invariant (independent of η)

Glasma instability

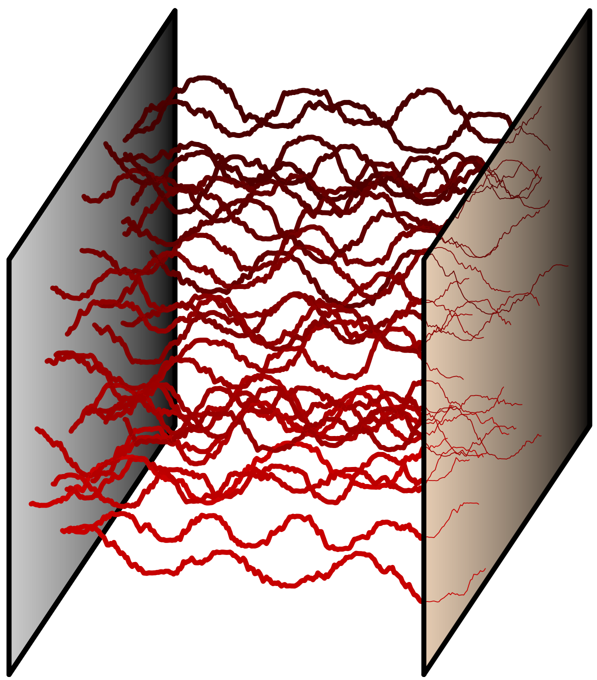
- Leading order + quantum fluctuations at $\tau = 0^+$:



- Loop corrections bring quantum fluctuations in this picture
- In the weak coupling regime, they are small corrections
- The spectrum of fluctuations is encoded in $\delta\mathcal{A}_{\rm in}$ and $\boldsymbol{\Sigma}$

Glasma instability

- Effect of the instability :



- η-dependent perturbations grow quickly in time, like $\exp\left(\sqrt{\mu\tau}\right)$
- Breakdown of the CGC approach at $\tau_{\rm max} \sim Q_s^{-1}\ln^2(1/\alpha_s)$?
- At $\tau \sim \tau_{\rm max}$, one gets patches where $\vec{B}$ is large and random

Glasma instability

- In order to push the CGC description beyond $\tau_{\max}$, one must resum all the corrections in $\left[\alpha_s e^{\sqrt{\mu\tau}}\right]^n$
- This resummation amounts to add fluctuations to the color fields at $\tau = 0$. The gluon spectrum becomes :

$$\frac{d\overline{N}}{dY\,d^2\vec{\boldsymbol{p}}_\perp} = \int [D\rho_1]\,[D\rho_2]\;W_{Y_{\rm beam}-Y}[\rho_1]\,W_{Y_{\rm beam}+Y}[\rho_2] \times \int \underbrace{[Da]\quad Z[a]}_{\text{fluctuation spectrum}}\;\frac{d\overline{N}[\mathcal{A}_{\rm in}(\rho_1,\rho_2)+a]}{dY\,d^2\vec{\boldsymbol{p}}_\perp}$$

- The spectrum of fluctuations $Z[a]$ has been calculated (Fukushima, FG, McLerran (2006)). Open questions :
  - Does the instability make the spectrum locally isotropic?
  - Does this system have a small η/s?

Summary

Summary

- The data seems to indicate that the matter formed at RHIC has a small η/s and thermalizes early
- The uncertainty principle gives a lower bound to η/s
- In the strong coupling limit of gauge theories, η/s is close to the minimal value

  For super-symmetric Yang-Mills theory, one can compute it explicitly in this limit by using the AdS/CFT correspondence
- One can also have a small η/s if the system has large random magnetic fields, even if the coupling is small

  In the (perturbative) CGC framework, the Glasma instability enhances quantum noise, which leads to such magnetic fields