# Gluon saturation from DIS to AA collisions

## I – Gluon saturation, Color Glass Condensate

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# General outline



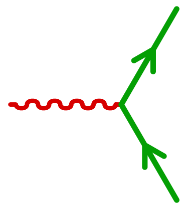
- Lecture I : Gluon saturation, Color Glass Condensate
- Lecture II : DIS and proton-nucleus collisions
- Lecture III : Saturation in nucleus-nucleus collisions

- Testing QCD - Factorization
- Parton model
- Parton saturation
- Phenomenology of saturation

# Testing QCD

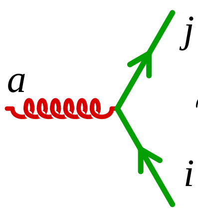
# Quarks and gluons

- Electromagnetic interaction : Quantum electrodynamics
  - Matter : electron , interaction carrier : photon
  - Interaction :

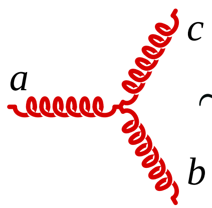


$\sim \quad e$ (electric charge of the electron)

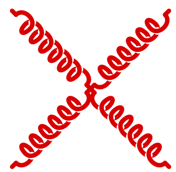
- Strong interaction : Quantum chromo-dynamics
  - Matter : quarks , interaction carriers : gluons
  - Interactions :




$\sim \quad g\,(t^a)_{ij}$




$\sim \quad g\,(T^a)_{bc}$



  - $i, j$ : colors of the quarks (3 possible values)
  - $a, b, c$ : colors of the gluons (8 possible values)
  - $(t^a)_{ij}$ : $3 \times 3$ matrix , $(T^a)_{bc}$ : $8 \times 8$ matrix

# QCD Lagrangian

- QCD Lagrangian :

$$\mathcal{L} = -\frac{1}{2}\mathrm{tr}\,(F_{\mu\nu}F^{\mu\nu}) + \overline{\psi}(i\not{D} - m)\psi$$

  - the gauge field $A^\mu$ belongs to $SU(3)$
  - $D^\mu \equiv \partial^\mu - igA^\mu$ is the covariant derivative
  - $F^{\mu\nu} \equiv i[D^\mu, D^\nu]/g = \partial^\mu A^\nu - \partial^\nu A^\mu - ig[A^\mu, A^\nu]$

- The Lagrangian is invariant under gauge transformations :

$$A^\mu(x) \to \Omega(x)A^\mu(x)\Omega^{-1}(x) + \frac{i}{g}\Omega(x)\partial^\mu\Omega^{-1}(x)$$

$$\psi(x) \to \Omega(x)\psi(x)$$

where $\Omega(x) \in SU(3)$

  - Note: the field strength is not invariant but transforms as :

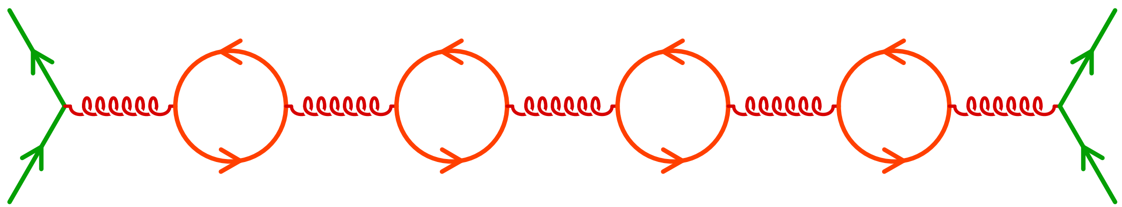
$$F^{\mu\nu}(x) \to \Omega(x)F^{\mu\nu}(x)\Omega^{-1}(x)$$

# Asymptotic freedom



- Running coupling : $\alpha_s = g^2/4\pi$

$$\alpha_s(r) = \frac{2\pi N_c}{(11N_c - 2N_f)\log(1/r\Lambda_{QCD})}$$



- The effective charge seen at large distance is screened by fermionic fluctuations (as in QED)

# Asymptotic freedom



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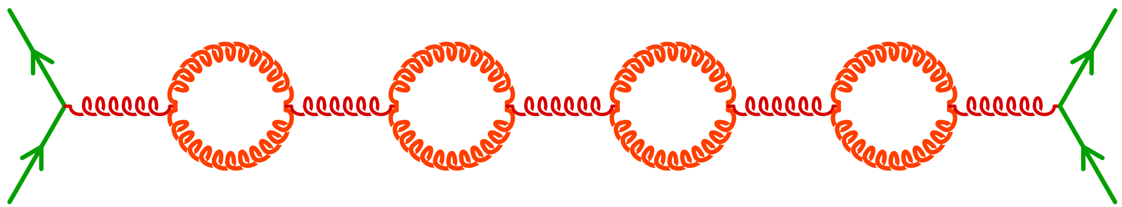
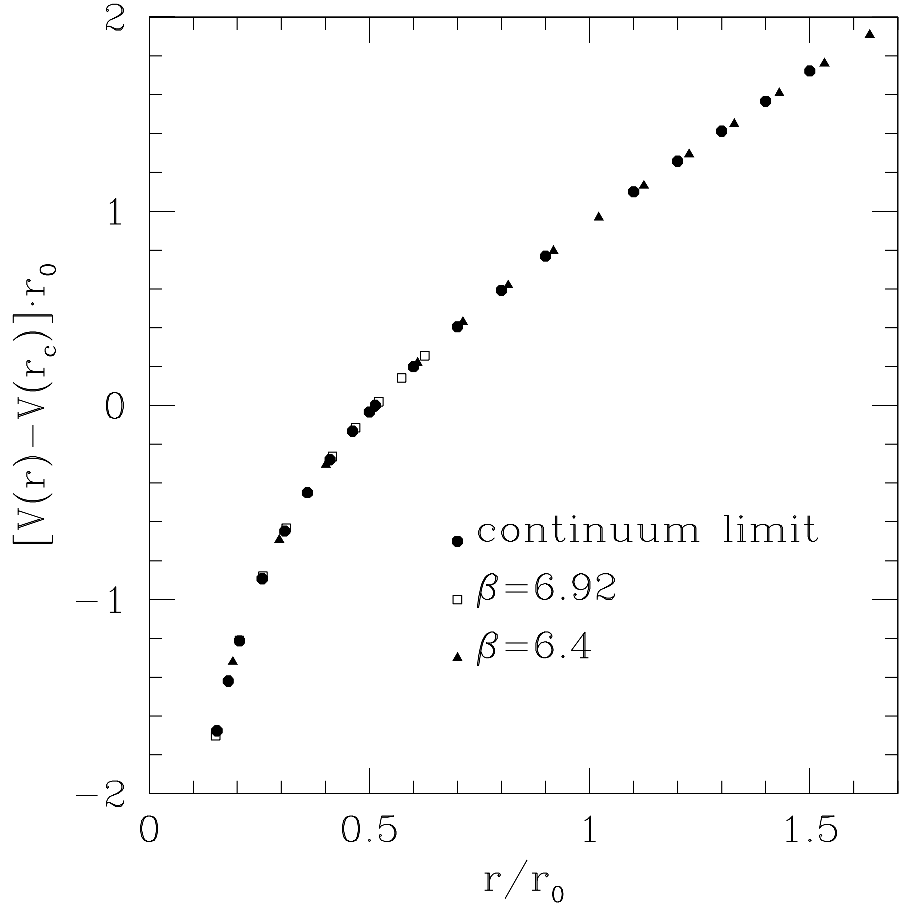


- The effective charge seen at large distance is screened by fermionic fluctuations (as in QED)
- But gluonic vacuum fluctuations produce an anti-screening (because of the non-abelian nature of their interactions)
- As long as $N_f < 11N_c/2 = 16.5$, the gluons win...

# Quark confinement




- The quark potential increases linearly with distance
- Color singlet hadrons : no free quarks and gluons in nature

# How to test QCD?



- QCD is the fundamental theory of strong interactions among quarks and gluons
- Experiments involve hadrons in their initial and final states, not quarks and gluons
- Hadrons cannot be described perturbatively in QCD
- Scattering amplitudes with time-like on-shell momenta cannot be computed on the lattice

▷ How can we compare theory and experiments?

▷ **Factorization :** separation of short distances (perturbative) and long distance (non perturbative)

# Factorization



- At a superficial level, factorization means that :

$$\mathcal{O}_{\text{hadrons}} = F \otimes \mathcal{O}_{\text{partons}}$$

  - $F =$ parton distribution
  - $\mathcal{O}_{\text{partons}} =$ observable at the partonic level (calculable in perturbation theory)

- For this to be useful, $F$ must be universal (i.e. independent of the observable $\mathcal{O}$)

- In order to test QCD experimentally, measure as many observables as possible, and try to find common $F$'s that fit all the data
Note : at this stage, by looking at only one observable, it is impossible to perform any meaningful test, since it is always possible to adjust $F$ so that it works

# Factorization

- Some loop corrections in $\mathcal{O}_{\rm partons}$ are enhanced by large logarithms, e.g.

$$\alpha_s \ln\left(\frac{M^2}{m_{_H}^2}\right) \quad , \qquad \alpha_s \ln\left(\frac{s}{M^2}\right) \sim \alpha_s \ln\left(\frac{1}{x}\right)$$

  Note : the log that occurs depends on the details of the kinematics
  - Bjorken limit: $s, M^2 \to +\infty$ with $s/M^2$ fixed
  - Regge limit: $s \to +\infty$, $M^2$ fixed

- These logs upset a naive application of perturbation theory when $\alpha_s \ln(\cdot) \sim 1$ $\triangleright$ they must be resummed

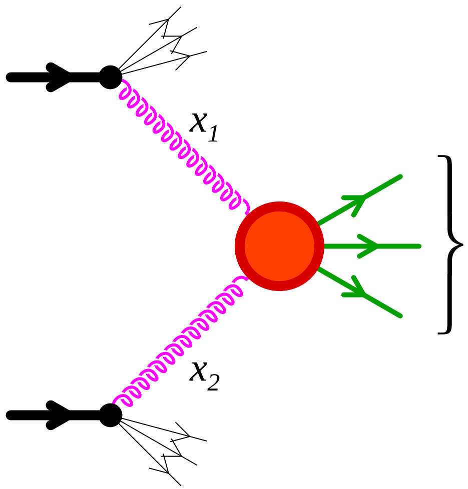
- This resummation can be performed analytically
  - the result of the resummation is universal
  - all the leading logs can be absorbed in $F$
    - $\triangleright$ the factorization formula remains true
    - $\triangleright$ this summation dictates how $F$ evolves with $M^2$ or $x$

# Factorization



- These logarithms tell us that the relevant parton distributions depend on the resolution scales (in time and in transverse momentum) associated to a given process

- Calculation of some process at LO :



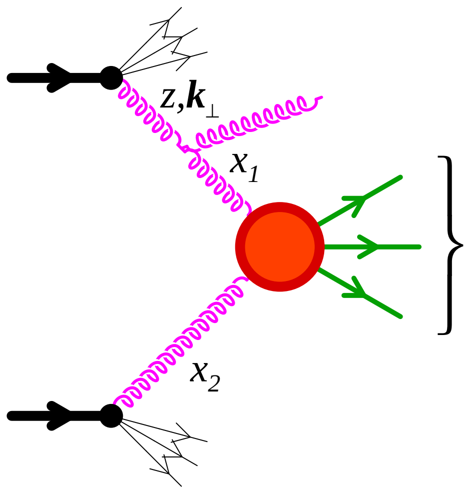

$$\Big\} \; (M_\perp \, , \, Y\,) \qquad \left\{ \begin{aligned} x_1 &= M_\perp \, e^{+Y} / \sqrt{s} \\ x_2 &= M_\perp \, e^{-Y} / \sqrt{s} \end{aligned} \right.$$

# Factorization



- These logarithms tell us that the relevant parton distributions depend on the resolution scales (in time and in transverse momentum) associated to a given process

- Radiation of an extra gluon :




$$(M_\perp\,, Y\,) \quad \Longrightarrow \quad \alpha_s \int_{x_1} \frac{dz}{z} \int^{M_\perp} \frac{d^2\vec{\boldsymbol{k}}_\perp}{\boldsymbol{k}_\perp^2}$$

- Practical consequence : pQCD predicts not only $\mathcal{O}_{\rm partons}$ but also the evolution $\partial_{_M} F$ (or $\partial_x F$)
  - ▷ the only required non-perturbative input is $F(x, M_0)$ or $F(x_0, M)$

# Collinear factorization

- Logs of $M_\perp \Longrightarrow$ DGLAP. Important when :
  - $M_\perp \gg \Lambda_{QCD}$, while $x_1, x_2$ are rather large

- Cross-sections read :

$$\frac{d\sigma}{dY\, d^2\vec{\boldsymbol{P}}_\perp} \propto F(x_1, M_\perp^2)\; F(x_2, M_\perp^2)\; |\mathcal{M}|^2$$

with $x_{1,2} = M_\perp \exp(\pm Y)/\sqrt{s}$

- Note : there are convolutions in $x_1$ and $x_2$ if some particles are integrated out in the final state

- The factorization of logarithms has been proven to all orders for sufficiently inclusive quantities (see Collins, Soper, Sterman, 1984–1985)

# Kt-factorization

Collins, Ellis (1991), Catani, Ciafaloni, Hautmann (1991)

- Logs of $1/x \Longrightarrow$ BFKL. Important when :
  - $M_\perp$ remains moderate, while $x_1$ or $x_2$ (or both) are small
- The BFKL equation is non-local in transverse momentum
  - ▷ it applies to non-integrated gluon distributions $\varphi(x, \vec{\boldsymbol{k}}_\perp)$

$$xG(x, Q^2) = \int^{Q^2} \frac{d^2\vec{\boldsymbol{k}}_\perp}{(2\pi)^2}\, \varphi(x, \vec{\boldsymbol{k}}_\perp)$$

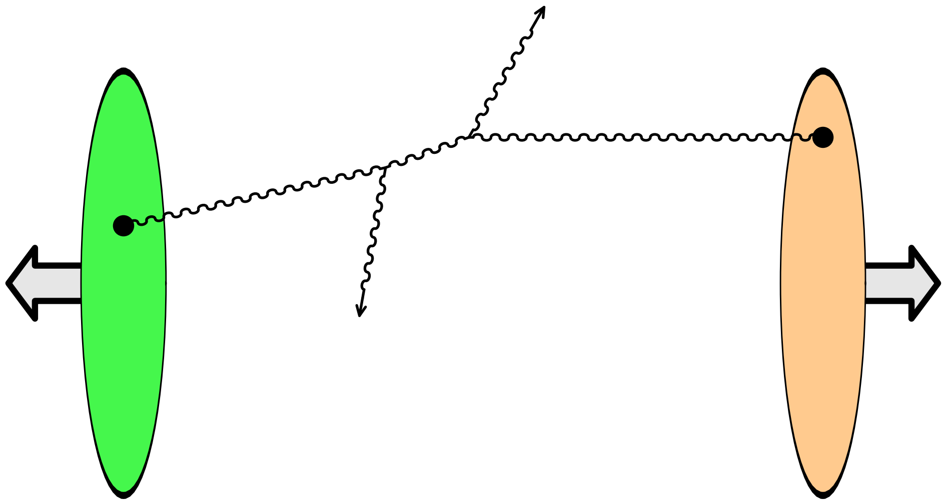
  - ▷ the matrix element is calculated for (off-shell) gluons with $\vec{\boldsymbol{k}}_\perp \neq \vec{0}$
- In this framework, cross-sections read :

$$\frac{d\sigma}{dY\,d^2\vec{\boldsymbol{P}}_\perp} \propto \int_{\vec{\boldsymbol{k}}_{1\perp}, \vec{\boldsymbol{k}}_{2\perp}} \delta(\vec{\boldsymbol{k}}_{1\perp} + \vec{\boldsymbol{k}}_{2\perp} - \vec{\boldsymbol{P}}_\perp)\, \varphi_1(x_1, k_{1\perp})\, \varphi_2(x_2, k_{2\perp})\, \frac{|\mathcal{M}|^2}{k_{1\perp}^2 k_{2\perp}^2}$$

$$(x_{1,2} = M_\perp\, e^{\pm Y} \,/\, \sqrt{s})$$

# Multi-parton interactions?

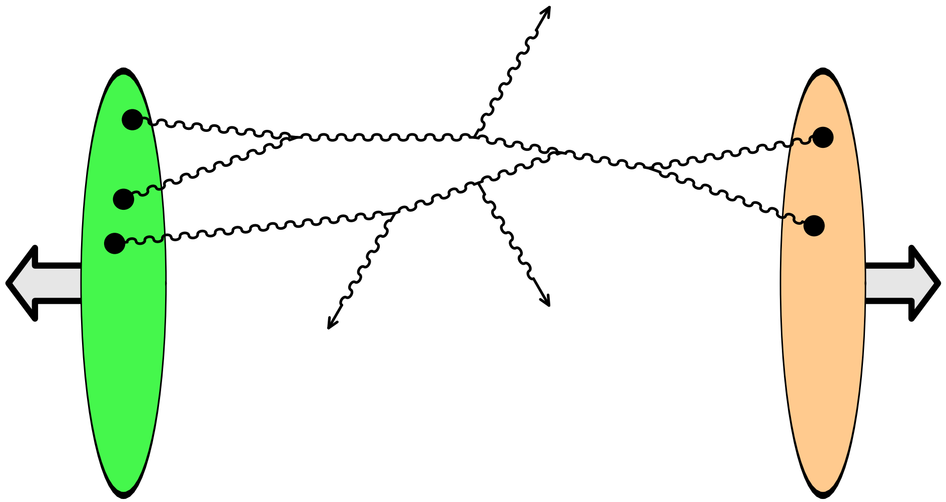




- Collinear or kt-factorization : only one parton in each projectile take part in the process of interest

# Multi-parton interactions?

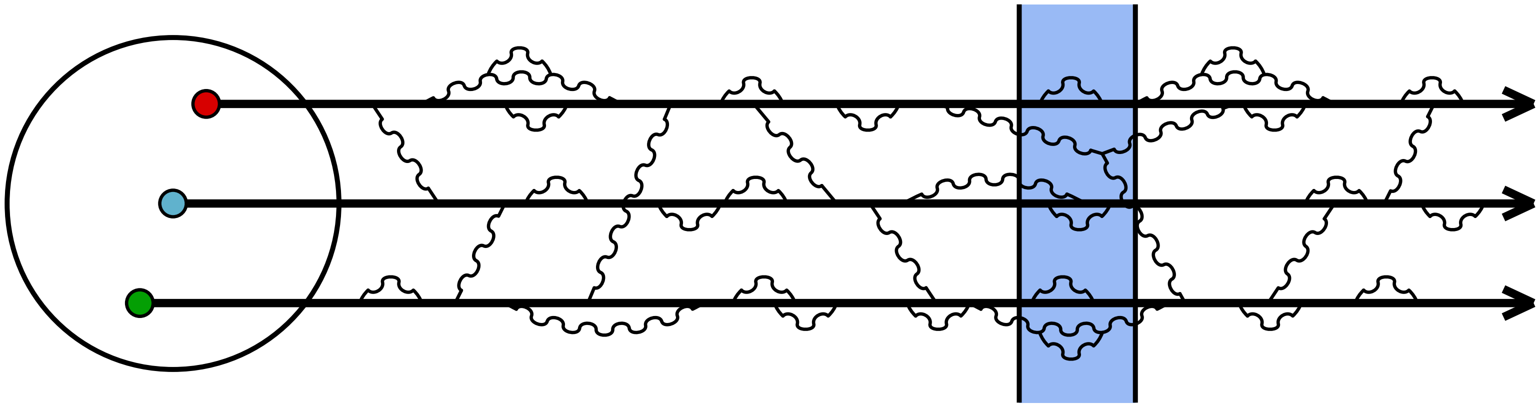




- Collinear or kt-factorization : only one parton in each projectile take part in the process of interest
- If multiparton interactions are important : the above forms of factorization cannot work anymore, because the only information they retain about the distribution of partons is their 2-point correlations (i.e. the number of partons)

# Parton model

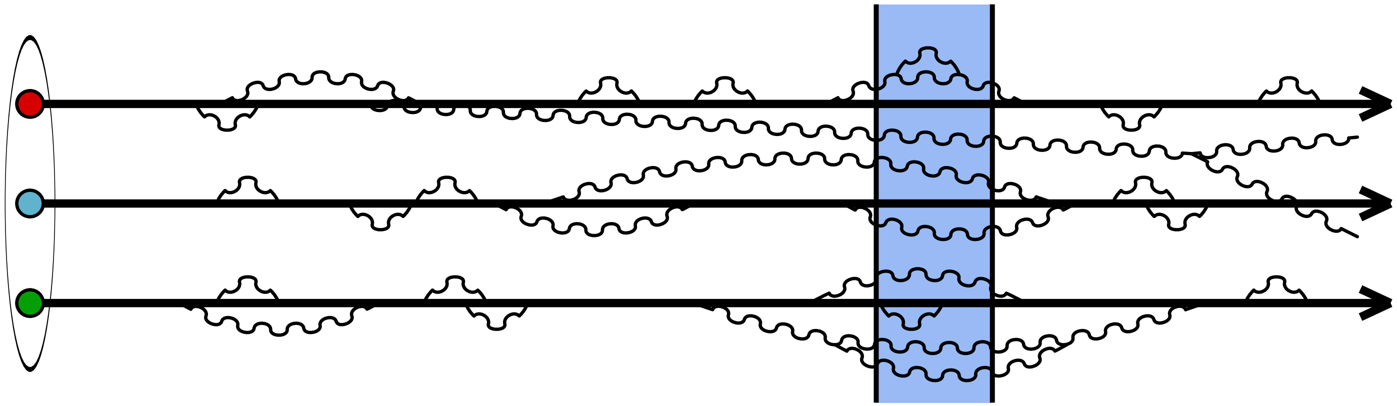
# Nucleon at low energy



- A nucleon at rest is a very complicated object...
- Contains fluctuations at all space-time scales smaller than its own size
- Only the fluctuations that are longer lived than the external probe participate in the interaction process
- The only role of short lived fluctuations is to renormalize the masses and couplings
- Interactions are very complicated if the constituents of the nucleon have a non trivial dynamics over time-scales comparable to those of the probe

# Nucleon at high energy





- Dilation of all internal time-scales for a high energy nucleon
- Interactions among constituents now take place over time-scales that are longer than the characteristic time-scale of the probe
  ▷ the constituents behave as if they were free
- Many fluctuations live long enough to be seen by the probe. The nucleon appears denser at high energy (the gluon distribution grows at small $x$)

# Parton model

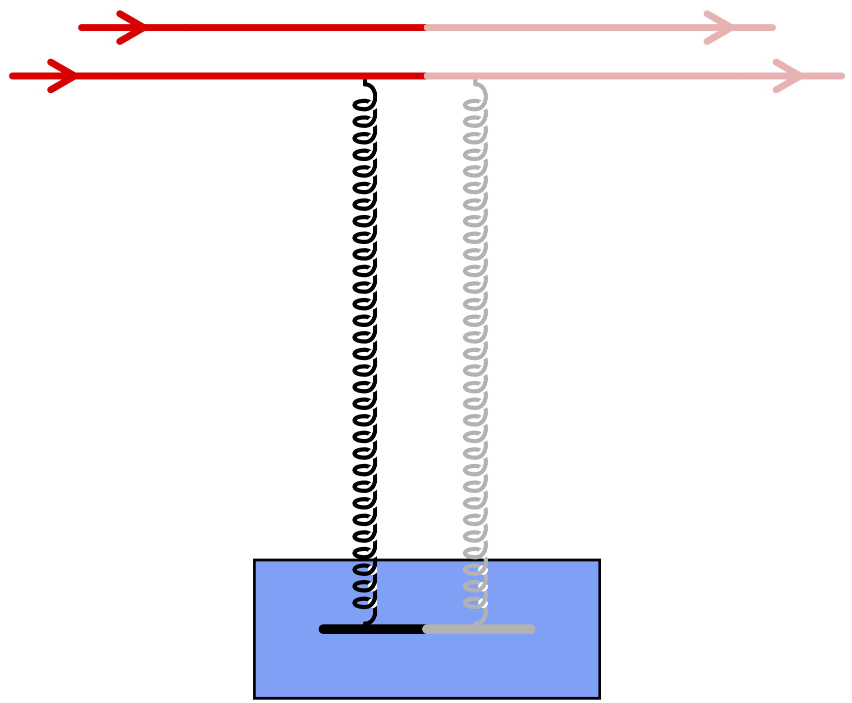


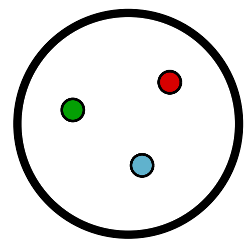
- At the time of the interaction, the nucleon can be seen as a collection of free constituents, called partons
- It can be described by non-perturbative parton distributions that depend on the momentum fraction $x$ of the partons and on some transverse resolution scale
- One can separate the perturbative hard scattering from the non-perturbative distribution functions, because the strong interactions that are responsible for these non-perturbative aspects occur on much larger timescales (factorization)
- All these properties are based only on kinematics and causality, and should remain true in the saturation regime
  - what we use as the “parton distribution” must contain information about multiparton configurations
  - the calculation of the “hard process”” will be more involved

# Gluon saturation

# Parton evolution at small x







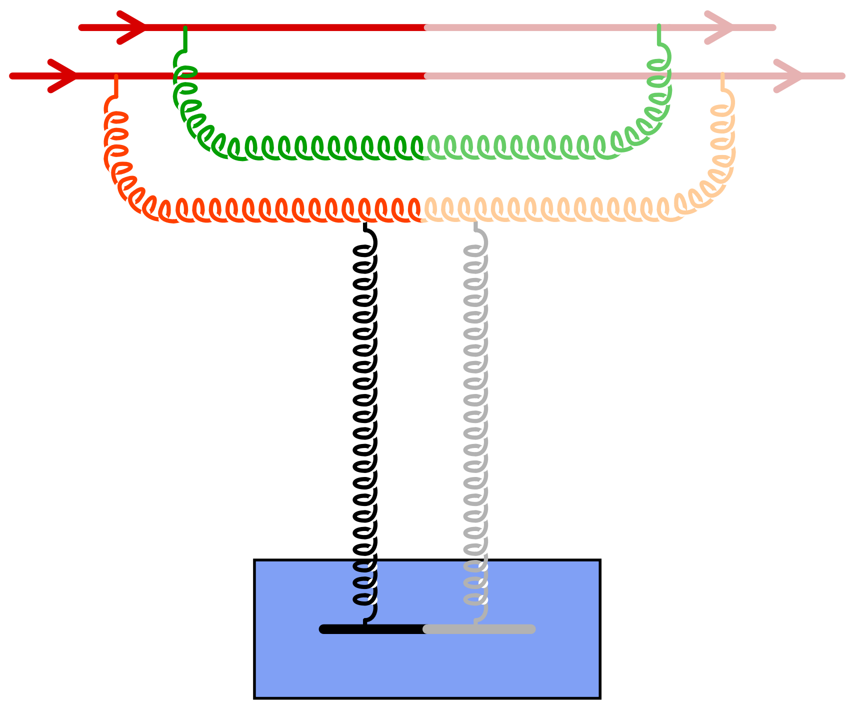
▷ assume that the projectile is big, e.g. a nucleus, and has many valence quarks (only two are represented)

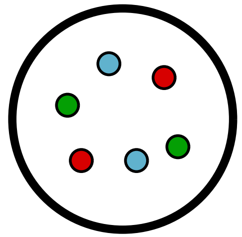
▷ on the contrary, consider a small probe, with few partons

▷ at low energy, only valence quarks are present in the hadron wave function

# Parton evolution







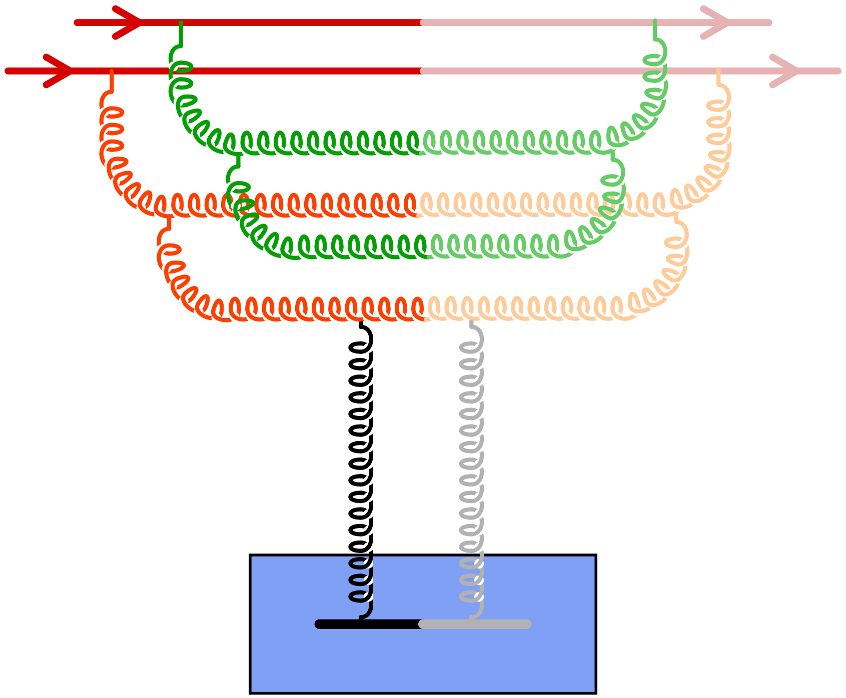
▷ when energy increases, new partons are emitted

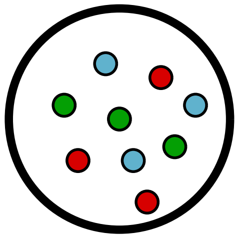
▷ the emission probability is $\alpha_s \int \frac{dx}{x} \sim \alpha_s \ln(\frac{1}{x})$, with $x$ the longitudinal momentum fraction of the gluon

▷ at small-$x$ (i.e. high energy), these logs need to be resummed

# Parton evolution

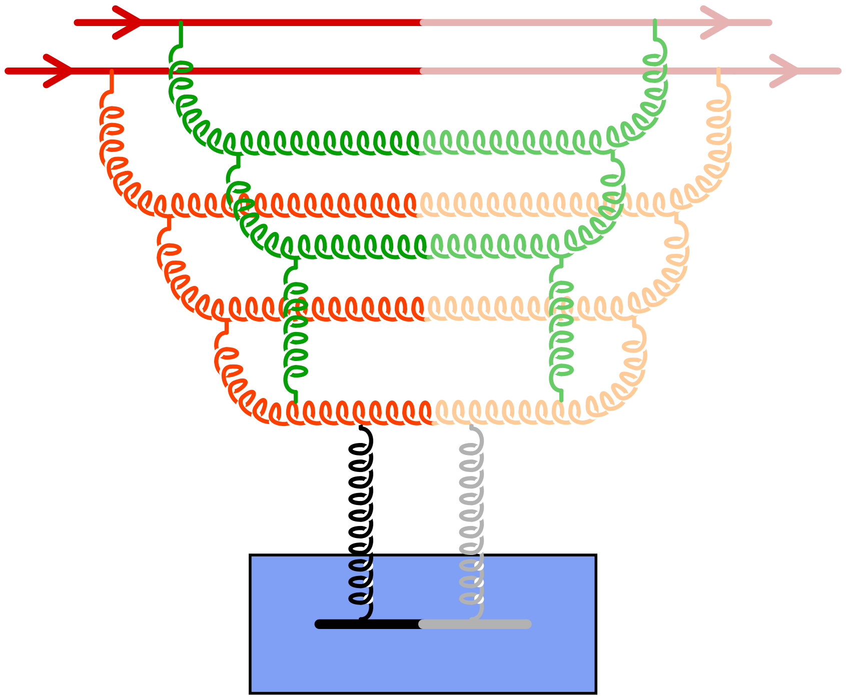


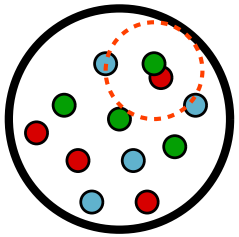




▷ as long as the density of constituents remains small, the evolution is linear: the number of partons produced at a given step is proportional to the number of partons at the previous step (BFKL)

# Parton evolution





- ▷ eventually, the partons start overlapping in phase-space
- ▷ parton recombination becomes favorable
- ▷ after this point, the evolution is non-linear:

the number of partons created at a given step depends non-linearly on the number of partons present previously

# Saturation criterion



Gribov, Levin, Ryskin (1983)

- Number of gluons per unit area:

$$\rho \sim \frac{xG_A(x, Q^2)}{\pi R_A^2}$$

- Recombination cross-section:

$$\sigma_{gg\to g} \sim \frac{\alpha_s}{Q^2}$$

- Recombination happens if $\rho\sigma_{gg\to g} \gtrsim 1$, i.e. $Q^2 \lesssim Q_s^2$, with:

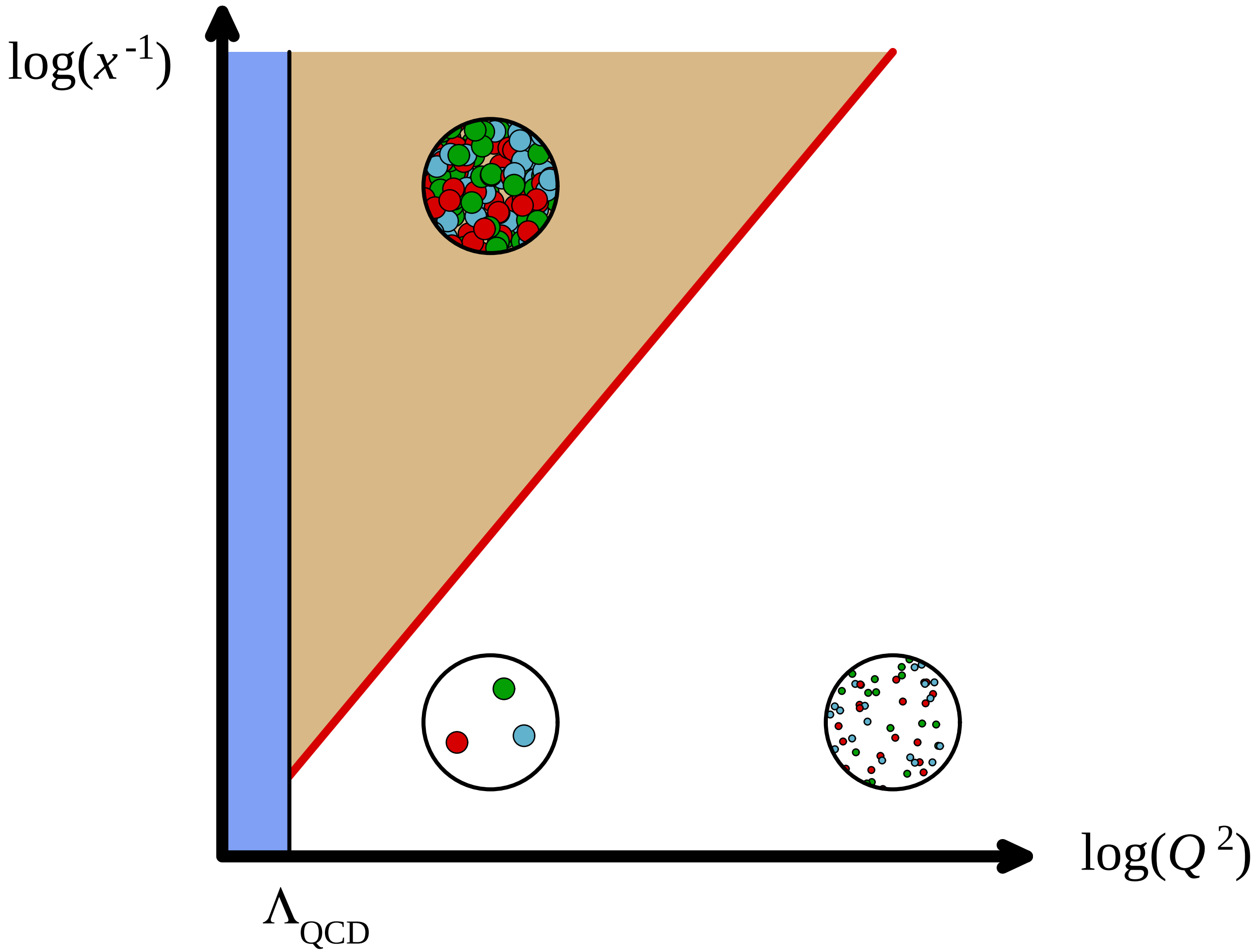
$$Q_s^2 \quad \sim \quad \frac{\alpha_s xG(x, Q_s^2)}{\pi R_A^2} \quad \sim \quad A^{1/3}\frac{1}{x^{0.3}}$$

- At saturation, the phase-space density is:

$$\frac{dN_g}{d^2\vec{\boldsymbol{x}}_\perp d^2\vec{\boldsymbol{p}}_\perp} \sim \frac{\rho}{Q^2} \sim \frac{1}{\alpha_s}$$

# Saturation domain

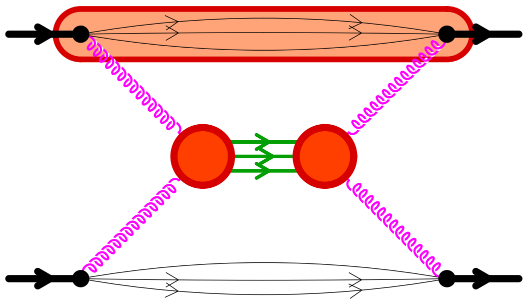




# Multiple scatterings



- Single scattering :



▷ 2-point function in the projectile ▷ gluon number

# Multiple scatterings



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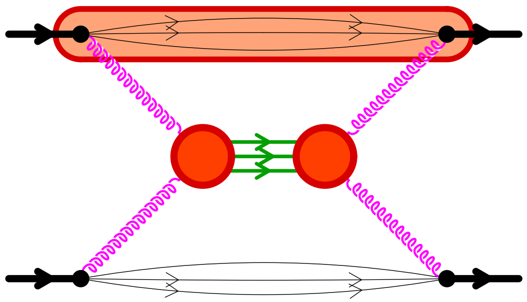
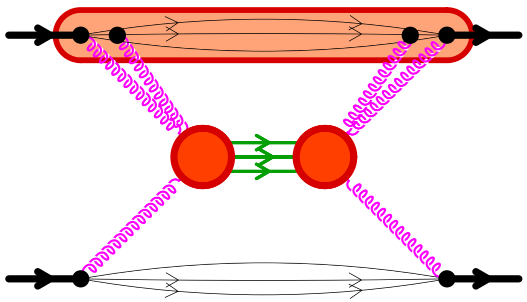


▷ 2-point function in the projectile ▷ gluon number

- Multiple scatterings :



▷ 4-point function in the projectile ▷ higher correlation
▷ multiple scatterings in the projectile

# Multiple scatterings

- Power counting : rescattering corrections are suppressed by inverse powers of the typical mass scale in the process :

$$\left[\frac{\mu^2}{M_\perp^2}\right]^n$$

- The parameter $\mu^2$ has a factor of $\alpha_s$, and a factor proportional to the gluon density $\triangleright$ rescatterings are important at high density
- Relative order of magnitude :

$$\frac{\text{2 scatterings}}{\text{1 scattering}} \sim \frac{Q_s^2}{M_\perp^2} \quad \text{with} \quad Q_s^2 \sim \alpha_s \frac{xG(x,Q_s^2)}{\pi R^2}$$

- When this ratio becomes $\sim 1$, all the rescattering corrections become important $\triangleright$ one must resum all $\left[Q_s/M_\perp\right]^n$
- These effects are not accounted for in DGLAP or BFKL

# Color Glass Condensate

- The fast partons (large $x$) are frozen by time dilation
  ▷ described as static color sources on the light-cone :

$$J_a^\mu = \delta^{\mu +}\delta(x^-)\rho_a(\vec{\boldsymbol{x}}_\perp) \qquad\qquad (x^- \equiv (t-z)/\sqrt{2})$$

- Slow partons (small $x$) are radiated by the fast ones. They have a large occupation number ▷ described by a classical color field $A^\mu$ that obeys Yang-Mills's equation:

$$[D_\nu, F^{\nu\mu}]_a = J_a^\mu$$

- The color sources $\rho_a$ are random, and described by a distribution functional $W_Y[\rho]$, with $Y$ the rapidity that separates "soft" and "hard"

# Color Glass Condensate



- Evolution equation (JIMWLK, 2001) :

$$\frac{\partial W_Y[\rho]}{\partial Y} = \mathcal{H}[\rho] \;\; W_Y[\rho]$$

$$\mathcal{H}[\rho] = \int_{\vec{\boldsymbol{x}}_\perp} \sigma(\vec{\boldsymbol{x}}_\perp) \frac{\delta}{\delta\rho(\vec{\boldsymbol{x}}_\perp)} + \frac{1}{2} \int_{\vec{\boldsymbol{x}}_\perp, \vec{\boldsymbol{y}}_\perp} \chi(\vec{\boldsymbol{x}}_\perp, \vec{\boldsymbol{y}}_\perp) \frac{\delta^2}{\delta\rho(\vec{\boldsymbol{x}}_\perp)\delta\rho(\vec{\boldsymbol{y}}_\perp)}$$

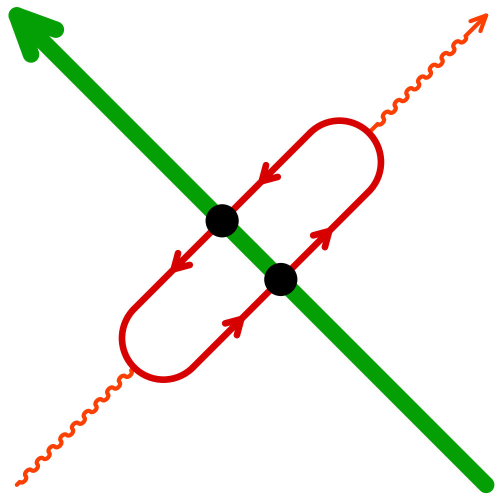
- $\sigma$ and $\chi$ are non-linear functions of $\rho$
- When the source density $\rho$ is small, one can expand $\sigma$ and $\chi$
  - $\triangleright$ JIMWLK simplifies into BFKL provided one defines

$$\varphi(x, k_\perp) \sim g^2 \int_{\vec{\boldsymbol{r}}_\perp} \frac{e^{-i\vec{\boldsymbol{k}}_\perp \cdot \vec{\boldsymbol{r}}_\perp}}{k_\perp^2} \int [D\rho] \; W_{Y=\ln(\frac{1}{x})}[\rho] \;\; \rho_a(0)\rho_a(\vec{\boldsymbol{r}}_\perp)$$

- To recover $k_T$-factorization, $|\mathcal{M}|^2$ must be calculated at order $\rho_1^2 \, \rho_2^2$

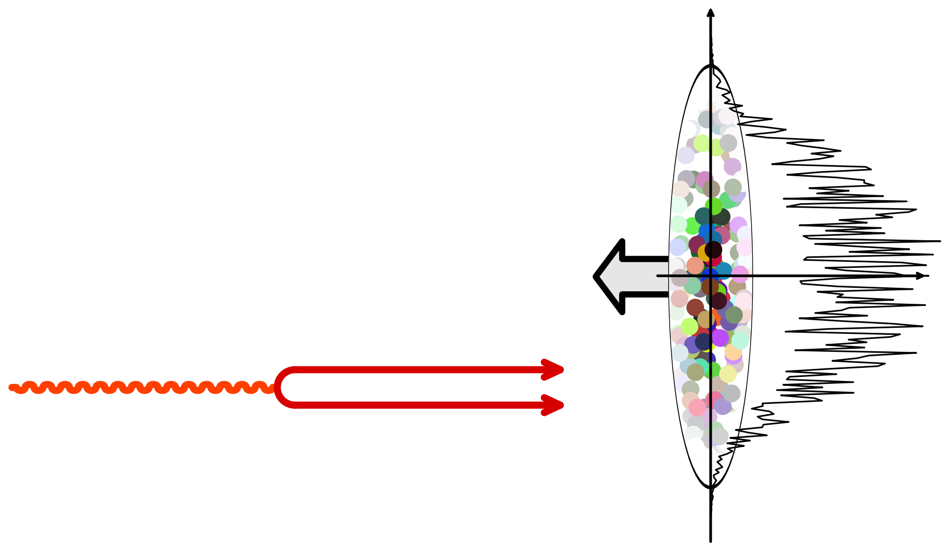
# Deep Inelastic Scattering

- Reactions involving a hadron or nucleus and an "elementary" projectile are fairly straightforward to study
- The archetype is the forward DIS amplitude :



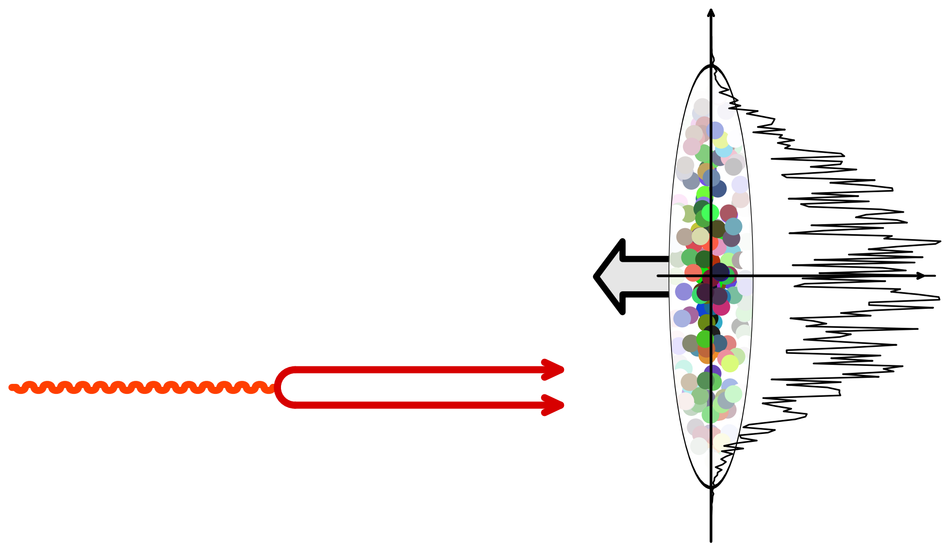
$$\langle \boldsymbol{T}(\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp)\rangle = \int [D\rho]\; W_Y[\rho]\; \left[1-\frac{1}{N_c}\mathrm{tr}(U(\vec{\boldsymbol{x}}_\perp)U^\dagger(\vec{\boldsymbol{y}}_\perp))\right]$$

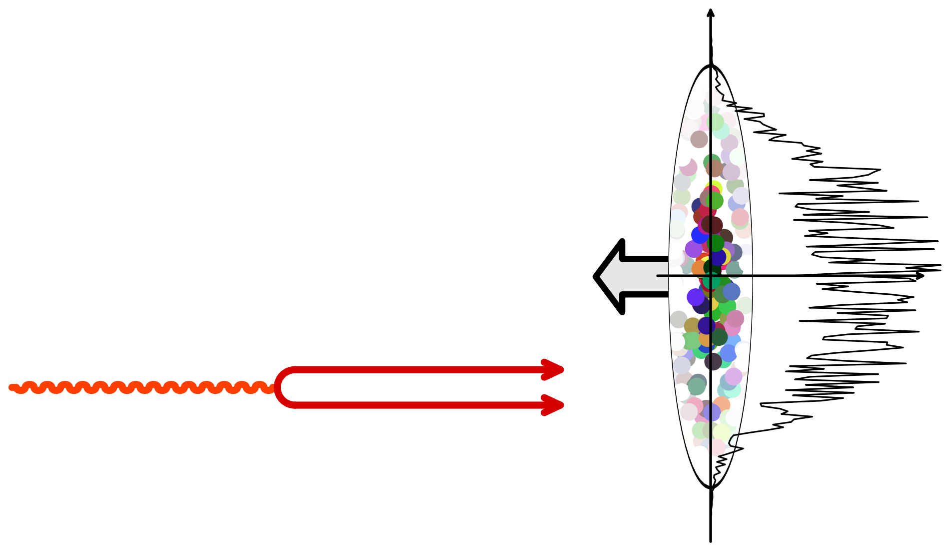
$\triangleright$ this formula resums all the $[\alpha_s \ln(1/x)]^m [Q_s/p_\perp]^n$ for the inclusive DIS cross-section



# Deep Inelastic Scattering







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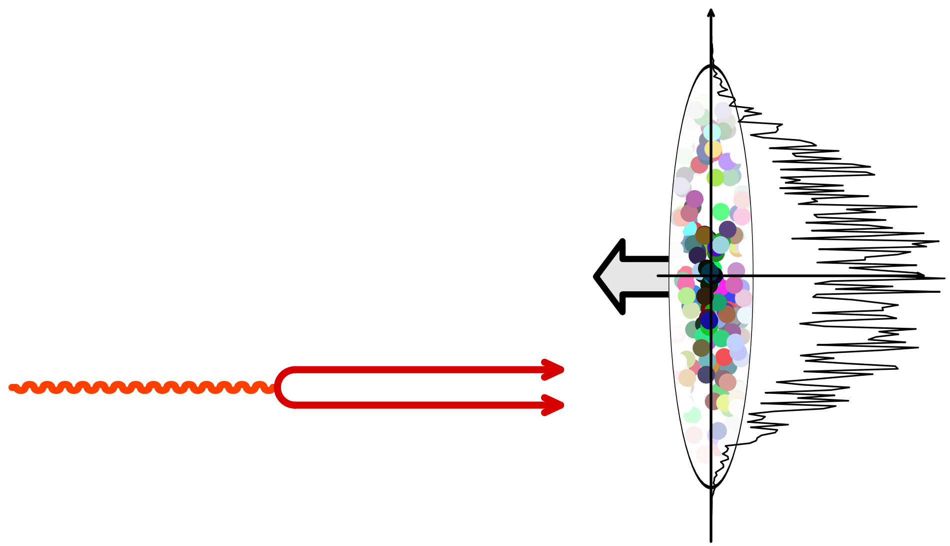

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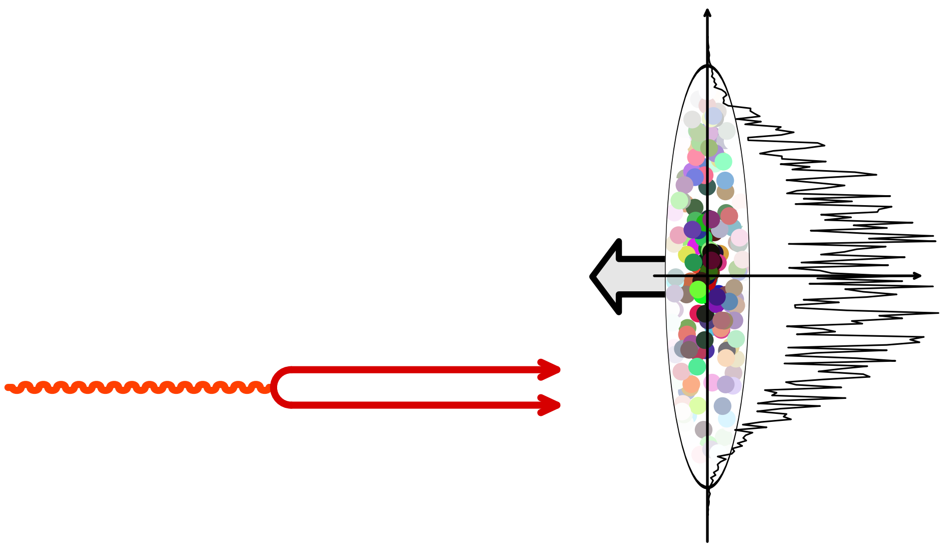
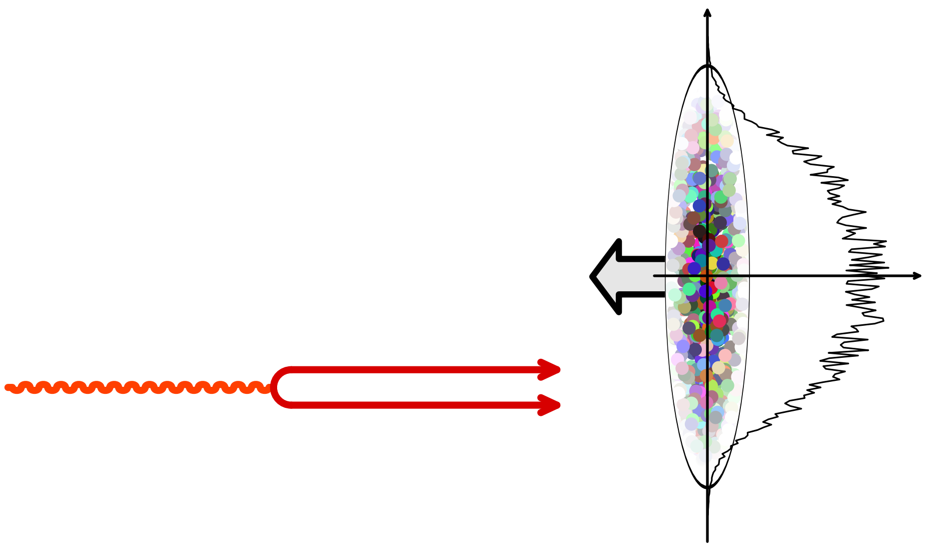
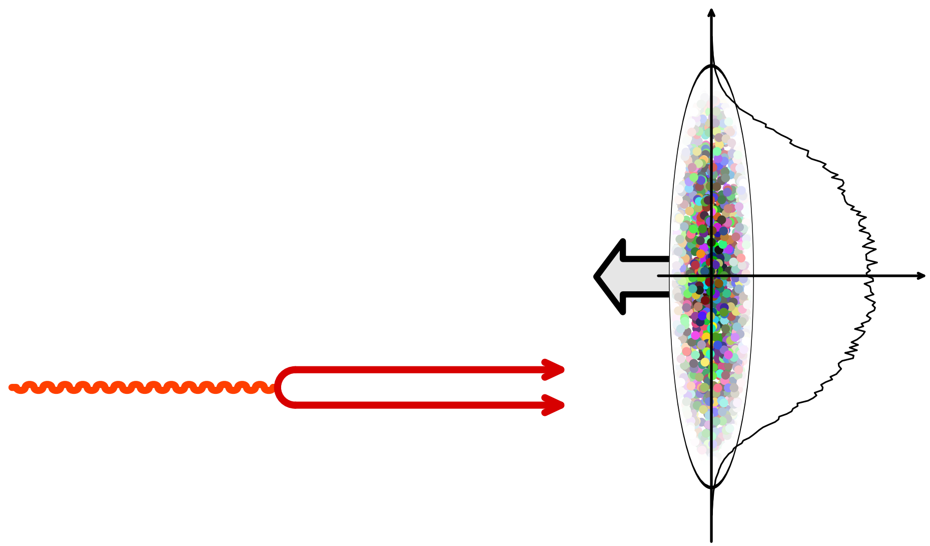




**10 configurations**



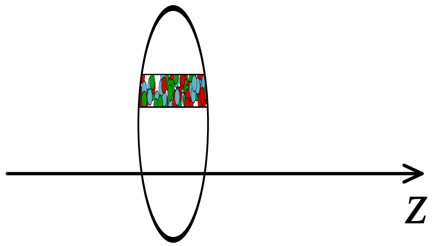
**100 configurations**

# Phenomenology of saturation

# Initial condition - MV model



- The JIMWLK equation must be completed by an initial condition, given at some moderate $x_0$
- As with DGLAP, the problem of finding the initial condition is non-perturbative
- The McLerran-Venugopalan model is often used as an initial condition at moderate $x_0$ for a large nucleus :




- partons distributed randomly
- many partons in a small area
- no correlations at different $\vec{\boldsymbol{x}}_\perp$

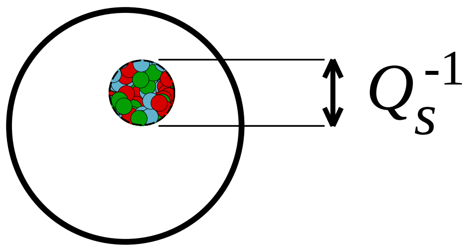
- The MV model assumes that the density of color charges $\rho(\vec{\boldsymbol{x}}_\perp)$ has a Gaussian distribution :

$$W_{x_0}[\rho] = \exp\left[-\int d^2\vec{\boldsymbol{x}}_\perp \frac{\rho_a(\vec{\boldsymbol{x}}_\perp)\rho_a(\vec{\boldsymbol{x}}_\perp)}{2\mu^2(\vec{\boldsymbol{x}}_\perp)}\right]$$

# Color correlation length



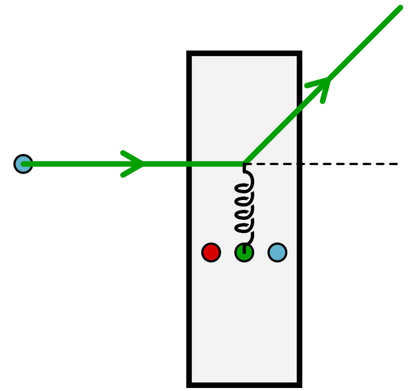
- In a nucleon at low energy, the typical correlation length among color charges is of the order of the nucleon size, i.e. $\Lambda_{QCD}^{-1} \sim 1\ \mathrm{fm}$. This is because the typical color screening distance is $\Lambda_{QCD}^{-1}$. At low energy, color screening is due to confinement, and thus non-perturbative
- At high energy (small $x$), partons are much more densely packed, and it can be shown that color neutralization occurs in fact over distances of the order of $Q_s^{-1} \ll \Lambda_{QCD}^{-1}$




- This implies that all hadrons, and nuclei, behave in the same way at high energy. In this sense, the small $x$ regime described by the CGC is universal

# Multiple scatterings

- Single scattering dominates at high $p_\perp$ :

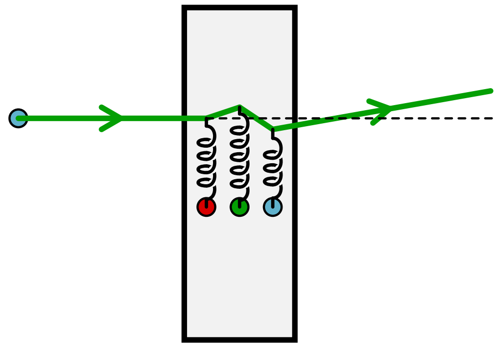


  - Differential cross-sections between a parton and a nucleus at high $p_\perp$ should scale like the atomic number $A$ (volume scaling)

# Multiple scatterings



- Multiple scatterings at low $p_\perp$ :

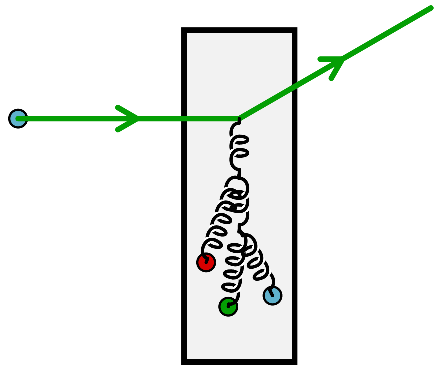


- One of the scatterings “produces” the final state, while the others merely change its momentum
- Each extra scattering corresponds to a correction $\alpha_s A^{1/3} \Lambda^2 / p_\perp^2$
  $\triangleright$ important correction at low $p_\perp$, despite the $\alpha_s$ suppression
- When this effect is extremal, differential cross-sections at low $p_\perp$ scale like $A^{2/3}$ (area scaling)
- Multiple scatterings only affect the momentum distribution of the final states, not the yield $\triangleright$ the suppression at low $p_\perp$ is compensated by an increase at higher $p_\perp$ (Cronin effect)

# Shadowing



- Interactions among the partons in the nuclear target (shadowing) :



  - Modification of the single scattering contribution due to the non-linear interactions of partons inside the target
  - At low $x$, this effect induces a suppression of the differential cross-section : $d\sigma_{pA}/d^2\vec{\boldsymbol{p}}_\perp \sim A^\alpha$ with $\alpha < 1$

# Lecture II : DIS and pA collisions



- Eikonal scattering
- Energy dependence
- Geometrical scaling
- Fits of DIS data
- Proton-Nucleus collisions

# Lecture III : Nucleus-nucleus collisions



- Introduction to nucleus-nucleus collisions
- Power counting and bookeeping
- Inclusive gluon spectrum
- Loop corrections, factorization, unstable modes