QCD at small $x$
and Nucleus-Nucleus collisions

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Outline

QCD at small $x$
- Nucleons at high energy
- Parton evolution and saturation
- Color Glass Condensate
- What is the present evidence?
- The present frontiers of the CGC

Initial conditions for nucleus-nucleus collisions
- Issues in particle production
- Factorization of leading logarithms
- Effect of unstable modes

Related talks:
- M. Strickland (next talk)
- S. Mrowczynski, T. Hirano (Nov. 18th), H. Fujii (Nov. 19th)
QCD at small x
Nucleon at rest

- Very complicated non-perturbative object, that contains fluctuations at all space-time scales smaller than its own size.

- Only the fluctuations that are longer lived than the external probe participate in the interaction process.

- Interactions are very complicated if the constituents of the nucleon have a non trivial dynamics over time-scales comparable to those of the probe.
- **Dilation** of all internal time-scales of the nucleon

- The constituents behave as if they were free over time-scales comparable to the interaction time

- Many fluctuations live long enough to be seen by the probe. The nucleon appears **denser at high energy**. Pre-existing fluctuations act as static sources of new partons

- In a nucleus, soft gluons (long wavelength) belonging to different nucleons overlap in the longitudinal direction
  
  ▶ coherent effects ▶ saturation
Parton distributions in a proton

- Nucleons at high energy
- Parton saturation
- Color Glass Condensate
- Experimental hints
- Present Frontiers

Summary
Parton evolution

▷ assume that the projectile is big, e.g. a nucleus, and has many valence quarks (only two are represented)
▷ on the contrary, consider a small probe, with few partons
▷ at low energy, only valence quarks are present in the hadron wave function
Parton evolution

- when energy increases, new partons are emitted

- the emission probability is \( \alpha_s \int \frac{dx}{x} \sim \alpha_s \ln(\frac{1}{x}) \), with \( x \) the longitudinal momentum fraction of the gluon

- at small-\( x \) (i.e. high energy), these logs need to be resummed
Parton evolution

\[ \text{as long as the density of constituents remains small, the evolution is } \textbf{linear}: \text{ the number of partons produced at a given step is proportional to the number of partons at the previous step (BFKL)} \]
eventually, the partons start overlapping in phase-space

parton recombination becomes favorable

after this point, the evolution is non-linear: the number of partons created at a given step depends non-linearly on the number of partons present previously
Saturation criterion

Gribov, Levin, Ryskin (1983)

- Number of gluons per unit area:

\[ \rho \sim \frac{xG_A(x, Q^2)}{\pi R_A^2} \]

- Recombination cross-section:

\[ \sigma_{gg \rightarrow g} \sim \frac{\alpha_s}{Q^2} \]

- Recombination happens if \( \rho \sigma_{gg \rightarrow g} \gtrsim 1 \), i.e. \( Q^2 \lesssim Q_s^2 \), with:

\[ Q_s^2 \sim \frac{\alpha_s xG_A(x, Q_s^2)}{\pi R_A^2} \sim A^{1/3} \frac{1}{x^{0.3}} \]
Saturation domain

\[ \log(x^{-1}) \]

\[ \Lambda_{QCD} \]

\[ \log(Q^2) \]
Saturation domain

\[ \log(x^{-1}) \]

\[ \log(Q^2) \]

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Saturation

SPS

Y = 0

François Gelis – 2006

Saturation domain

\[ \log(x^{-1}) \]

\[ \Lambda_{QCD} \]

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- Saturation domain
- \( \log(Q^2) \)
- \( \log(x^{-1}) \)
- \( \Lambda_{QCD} \)
Saturation domain

$log(x^{-1})$

$log(Q^2)$

$\Lambda_{\text{QCD}}$

$\log(Q^2)$

$log(x^{-1})$

$\Lambda_{\text{QCD}}$

$\text{LHC}$

$Y = 0$

$\text{RHIC}$

$Y = 0$

$Y = 0$

$Y = 0$

$\text{LHC}$

$\text{Large Y}$
Saturation domain

\[ \log(Q^2) \]

\[ \Lambda_{QCD} \]

\[ \log(x^{-1}) \]
Degrees of freedom and their interplay


- Small-$x$ modes have a large occupation number
  - they are described by a classical color field $A^\mu$ that obeys Yang-Mills’s equation:
    \[ [D_\nu, F^{\nu \mu}]_a = J^\mu_a \]

- The source term $J^\mu_a$ comes from the faster partons. The large-$x$ modes, slowed down by time dilation, are described as frozen color sources $\rho_a$. Hence:
  \[ J^\mu_a = \delta^{-}(x) \rho_a(\vec{x}_\perp) \]

- The color sources $\rho_a$ are random, and described by a distribution $W_Y[\rho]$, with $Y \equiv \ln(1/x_0)$, $x_0$ being the frontier between “small-$x$” and “large-$x$”. JIMWLK equation:
  \[ \frac{\partial W_Y[\rho]}{\partial Y} = \mathcal{H}[\rho] \ W_Y[\rho] \]
Hadronic collisions

- In order to study the collisions of two hadrons at leading order, the color current must have two terms:

\[
J^\mu \equiv \delta^{\mu+} \delta(x^-) \rho_1(\vec{x}_\perp) + \delta^{\mu-} \delta(x^+) \rho_2(\vec{x}_\perp)
\]

- Compute the observable \( O \) of interest in the color field created by a configuration \((\rho_1, \rho_2)\) of the sources. Note: the sources are of order \( 1/\sqrt{\alpha_s} \) \( \triangleright \) very non-linear problem

- Average over the sources \( \rho_1, \rho_2 \)

\[
\langle O \rangle_Y = \int [D\rho_1] [D\rho_2] W_{Y_{\text{beam}}-Y}[\rho_1] W_{Y_{\text{beam}}+Y}[\rho_2] O[\rho_1, \rho_2]
\]
Low $x$ ($x < 10^{-2}$) data displayed as a function of $\tau = x^{0.3} Q^2$

Stasto, Golec-Biernat, Kwiecinski (2000)
Iancu, Itakura, McLerran (2002)
Limiting fragmentation

- Inclusive hadron spectrum at RHIC, shifted by the beam rapidity ($\sqrt{s} = 19.6, 64, 130, 200$ GeV)

(data from PHOBOS, STAR and BRAHMS):

\[ MV, \lambda_0=0.0, \lambda_s=0.46 \]


- Limiting fragmentation is natural in the framework of gluon saturation. It follows from:
  - Approximate Bjorken scaling in the nucleus at large $x$
  - Unitarization of scattering amplitudes in the nucleus at small $x$
High pt suppression at large $Y$

- Results of the BRAHMS experiment at RHIC for deuteron-gold collisions:

$$R_{dAu} \equiv \frac{1}{N_{coll}} \frac{dN}{dp_{\perp} d\eta} |_{dAu} \frac{dN}{dp_{\perp} d\eta} |_{pp}$$

Albacete, Armesto, Kovner, Salgado, Wiedemann ('03), Kharzeev, Levin, McLerran ('03), Iancu, Itakura, Triantafyllopoulos ('04)

- At small rapidity, suppression at low $p_{\perp}$ and enhancement at high $p_{\perp}$ (multiple scatterings – Cronin effect)
- At large rapidity, suppression at all $p_{\perp}$'s (shadowing)
Multiplicity at RHIC

- Predictions from different approaches vs. data:

[Graph showing multiplicity predictions from various models compared to data, labeled "Eskola, QM 2001".]
Multiplicity at RHIC

- $N_{\text{part}}$ scaling and energy dependence:
  
  Kharzeev, Levin, Nardi (2001)

See also: Armesto, Salgado, Wiedemann (2004)
The present Frontiers of the CGC

Two aspects of QCD at high energy are under active study, but have not yet been applied to heavy ion collisions:

- **Beyond mean field, fluctuations of $Q_s$ and pomeron loops**:
  - Evolution equations with a stochastic term:
    - Hatta, Iancu, Marquet, Soyez, Triantafyllopoulos (2006)
    - Marquet, Soyez, Xiao (2006)
  - Toy models in 1+1 dimensions:
    - Blaizot, Iancu, Triantafyllopoulos (2006)
  - Applications to diffractive reactions:
    + many more...

- **Towards NLO evolution equations**:
  - Albacete, Armesto, Milhano (2006)
Initial conditions for nucleus-nucleus collisions ("Glasma")
What do we mean by Initial Conditions?

- calculate the initial production of semi-hard particles
- prepare the stage for kinetic theory or hydrodynamics
Typical e+e- or pp collision
Why is pQCD predictive there?

- More precisely, why is pQCD predictive despite the fact that hadrons are non-perturbative bound states?

- **Factorization**:
  - (Collinear) divergences in loop corrections can be absorbed into the (DGLAP) evolution of parton distributions and fragmentation functions

- **Universality** : parton distributions are process independent
Can we set up an equally systematic framework for semi-hard particle production in nucleus-nucleus collisions?
Gluon multiplicity at LO


\[
\frac{dN_{LO}}{d^3 \vec{p}} \propto \int_{x,y} e^{i \vec{p} \cdot (x-y)} \cdots \mathcal{A}_\mu(x) \mathcal{A}_\nu(y)
\]

\[\mathcal{A}_\mu(x) = \text{retarded solution of Yang-Mills equations}\]
Gluon multiplicity at LO


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\]

\[ A^\mu(x) = \text{retarded solution of Yang-Mills equations} \]
\[ \uparrow \text{can be cast into an initial value problem on the light-cone} \]
Gluon multiplicity at LO

- Important softening at small $k_\perp$ compared to pQCD (saturation)
- Quark production has also been computed (FG, Kajantie, Lappi (2005))
The color field at $\tau = 0$ does not depend on the rapidity $\eta$.

- It remains independent of $\eta$ at all times (invariance under boosts in the $z$ direction).
- Numerical resolution performed in $2 + 1$ dimensions.
Systematics of particle production

- Dilute regime: one source in each projectile interact
Systematics of particle production

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- **Dense regime**: non linearities are important
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Systematics of particle production

- **Dilute regime**: one source in each projectile interact
- **Dense regime**: non linearities are important
- Many gluons can be produced from the same diagram
- There can be many simultaneous disconnected diagrams
- Some of them may not produce anything (**vacuum diagrams**)
- All these diagrams can have loops (not at LO though)
In the saturated regime, the sources are of order $1/\sqrt{\alpha_s}$, and the order of each disconnected diagram is given by:

$$\alpha_s^{-1} \left( \frac{1}{\alpha_s} \right)^{\frac{1}{2} (# \text{ produced gluons})} \alpha_s^{# \text{ loops}}$$

Total order = product of the orders of the subdiagrams

▷ summing all the contributions to the spectrum at a given order requires powerful bookkeeping tools (FG, Venugopalan (2006))
The 1-loop correction to $\overline{N}$ can be written as a perturbation of the initial value problem encountered at LO:
The 1-loop correction to \( \overline{N} \) can be written as a perturbation of the initial value problem encountered at LO:

\[
\delta \overline{N} = \left[ \int_{\vec{x} \in \text{light cone}} \delta A(\vec{x}) \ T_{\vec{x}} \right] \overline{N}_{LO}
\]

- \( \overline{N}_{LO} \) is a functional of the initial fields \( A_{\text{in}}(\vec{x}) \) on the light-cone.
- \( T_{\vec{x}} \) is the generator of shifts of the initial condition at the point \( \vec{x} \) on the light-cone, i.e.: \( T_{\vec{x}} \sim \delta / \delta A_{\text{in}}(\vec{x}) \).
1-loop correction to $N$

The 1-loop correction to $N$ can be written as a perturbation of the initial value problem encountered at LO:

$$
\delta N = \left[ \int_{\vec{x} \in \text{light cone}} \delta A(\vec{x}) \ T_{\vec{x}} + \int_{\vec{x}, \vec{y} \in \text{light cone}} \frac{1}{2} \Sigma(\vec{x}, \vec{y}) \ T_{\vec{x}} \ T_{\vec{y}} \right] N_{LO}
$$

- $N_{LO}$ is a functional of the initial fields $A_{in}(\vec{x})$ on the light-cone
- $T_{\vec{x}}$ is the generator of shifts of the initial condition at the point $\vec{x}$ on the light-cone, i.e. $T_{\vec{x}} \sim \delta / \delta A_{in}(\vec{x})$
- $\delta A(\vec{x})$ and $\Sigma(\vec{x}, \vec{y})$ are in principle calculable analytically
Divergences

- If taken at face value, this 1-loop correction is plagued by several divergences:

  - The two coefficients $\delta A(\vec{x})$ and $\Sigma(\vec{x}, \vec{y})$ are infinite, because of an unbounded integration over a rapidity variable.

  - At late times, $T_{\vec{x}} A(\tau, \vec{y})$ diverges exponentially,

    $$T_{\vec{x}} A(\tau, \vec{y}) \sim e^{\sqrt{\mu \tau}}$$

    because of an instability of the classical solution of Yang-Mills equations under rapidity dependent perturbations (Romatschke, Venugopalan (2005))
Initial state factorization

Anatomy of the full calculation:

\[ W_{Y_{beam} - Y_0} [\rho_1] \]

\[ W_{Y_{beam} + Y_0'} [\rho_2] \]

\[ N[ A_{in} (\rho_1, \rho_2) ] + \delta N \]

By putting arbitrary frontiers \( Y_0, Y_0' \) between the “observable” and the “source distributions”, the divergent coefficients become finite.

For the final result to be independent of \( Y_0, Y_0' \), one needs:

\[
\left[ \delta N \right]_{\text{divergent coefficients}} = \left( (Y_0 - Y) \mathcal{H}^\dagger[\rho_1] + (Y - Y_0') \mathcal{H}^\dagger[\rho_2] \right) N_{LO}
\]

where \( \mathcal{H}[\rho] \) is the Hamiltonian that governs the rapidity dependence of the source distribution \( W_Y[\rho] \):

\[
\partial_Y W_Y[\rho] = \mathcal{H}[\rho] W_Y[\rho]
\]

FG, Lappi, Venugopalan (work in progress)
Unstable modes

Romatschke, Venugopalan (2005)

- Rapidity dependent perturbations to the classical fields grow like $\exp\left(\#\sqrt{\tau}\right)$ until the non-linearities become important:

\[
\text{max} \frac{T^4}{g^2 \mu \tau} / \frac{L^2}{g^2 \mu \tau} = c_0 + c_1 \exp(0.427 \sqrt{g^2 \mu \tau})
\]

\[
\text{max} \frac{T^4}{g^2 \mu \tau} / \frac{L^2}{g^2 \mu \tau} = c_0 + c_1 \exp(0.00544 g^2 \mu \tau)
\]
Unstable modes

One can sum the contribution of the unstable modes by:

$$\left[ \delta N \right]_{\text{unstable modes}} = \int [Da] \mathcal{D}_{\text{fluct}} [a] \bar{N}_{\text{LO}} [A_{\text{in}} (\rho_1, \rho_2) + a]$$
Unstable modes

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Unstable modes

- One can sum the contribution of the unstable modes by:

$$\left[ \delta N \right]_{\text{unstable modes}} = \int \left[ D a \right] D_{\text{fluct}} [a] \, N_{LO} [A_{\text{in}}(\rho_1, \rho_2) + a]$$

- The distribution of fluctuations has been calculated recently
  Fukushima, FG, McLerran (2006)

- Still open issue: can these instabilities fight efficiently against the expansion of the system?
Summary
Summary

- Gluon recombination is important at small $x$, and affects initial particle production in high-energy AA collisions.

- Thanks to the large density of color sources, calculating the initial particle spectrum can be done via semi-classical techniques.

- The resummation of the divergences at 1-loop tells us to:
  - average over the initial sources with the weight $W_Y[\rho]$
  - average over fluctuations with a distribution $D_{\text{fluct}}[a]$

  ▶ Provides a self-consistent framework based on the $\{\text{JIMWLK} + \text{classical field approximation}\}$ combination.

  ▶ Somewhat analogous to factorization in conventional pQCD:

  $W_Y[\rho] \leftrightarrow \text{parton distribution}$
  $D_{\text{fluct}}[a] \leftrightarrow \text{fragmentation function}$
Extra bits

- Limiting frag.
- dA collisions I
- dA collisions II
- Local anisotropy
- Unstable modes
Extrapolation to LHC energy

\[ \frac{dN}{d\eta} \approx 1000 - 1400 \]
dA collisions at RHIC

Kharzeev, Kovchegov, Tuchin (2005)
**dA collisions at RHIC**


![Graph showing dN/dy distribution for dAu collisions at RHIC](image-url)
Local anisotropy

- After some time, the gluons have a longitudinal velocity tied to their space-time rapidity by $v_z = \tanh(\eta)$:
Local anisotropy

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\[ v_z = \tanh(\eta) \]

—at late times: if particles fly freely, only one longitudinal velocity can exist at a given \( \eta \): \( v_z = \tanh(\eta) \)
Unstable modes

- The coefficient \( \delta A(\vec{x}) \) is boost invariant, and does not trigger the instability. When summed to all orders, the contribution of the unstable modes exponentiates:

\[
\begin{bmatrix} \delta N \end{bmatrix}_{\text{unstable modes}} = e^{\frac{1}{2} \int_{\vec{x}, \vec{y}} \Sigma(\vec{x}, \vec{y}) T_{\vec{a}} T_{\vec{b}} N_{\text{LO}} [A_{\text{in}}(\rho_1, \rho_2)]}
\]

- By rewriting the Gaussian in \( T_{\vec{x}} \) as a Fourier transform:

\[
\begin{bmatrix} \delta N \end{bmatrix}_{\text{unstable modes}} = \int [Da] \left( e^{\frac{1}{2} \int_{\vec{x}, \vec{y}} \Sigma(\vec{x}, \vec{y}) a(\vec{a}) a(\vec{b})} e^{i \int_{\vec{x}} a(\vec{a}) T_{\vec{a}} N_{\text{LO}} [A_{\text{in}}(\rho_1, \rho_2)]} \right) D_{\text{fluct}}[a]
\]

\[
= \int [Da] D_{\text{fluct}}[a] \bar{N}_{\text{LO}} [A_{\text{in}}(\rho_1, \rho_2) + a]
\]

▷ summing the instabilities simply requires to add Gaussian fluctuations to the initial condition for the classical field