# QCD at small $x$
# and Nucleus-Nucleus collisions

**François Gelis**

**CEA / DSM / SPhT**

# Outline



- QCD at small $x$

  Nucleons at high energy

  Parton evolution and saturation

  Color Glass Condensate

  What is the present evidence?

  The present frontiers of the CGC

- Initial conditions for nucleus-nucleus collisions

  Issues in particle production

  Factorization of leading logarithms

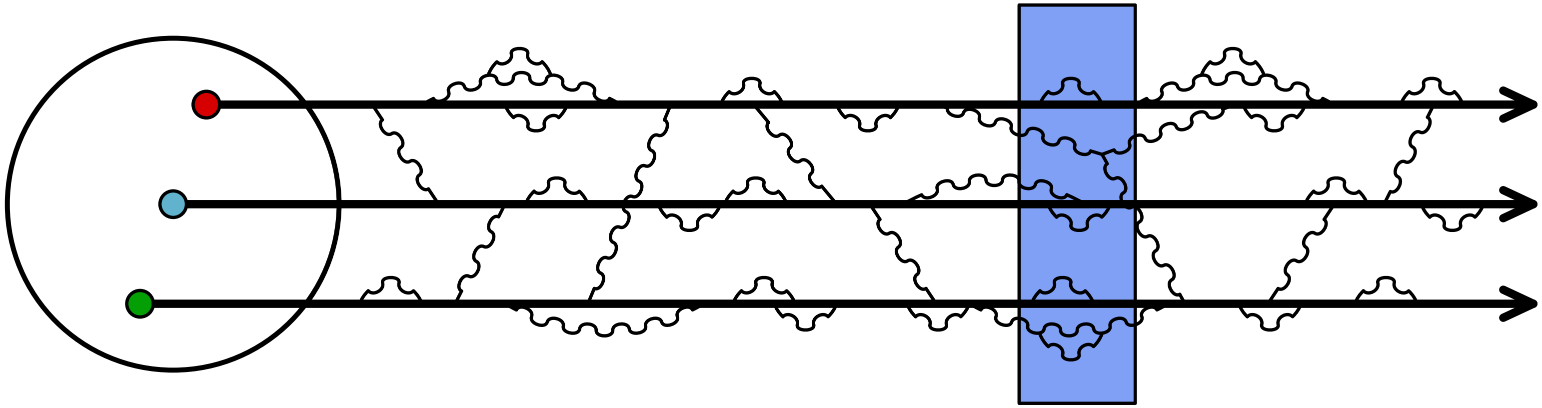
  Effect of unstable modes

  Related talks :

  - R. Venugopalan, N. Borghini, Z. Kang, J. Albacete, N. Armesto, L. Molnar, T. Larsen, J. Lee, H. Yang, D. d'Enterria (Nov. 15th)
  - M. Strickland (next talk)
  - S. Mrowczynski, T. Hirano (Nov. 18th), H. Fujii (Nov. 19th)

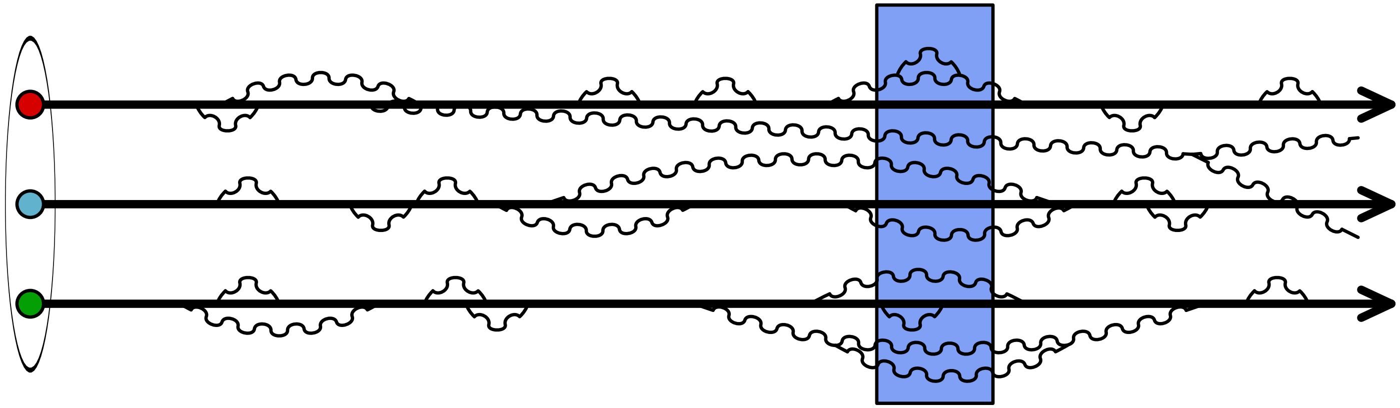
# QCD at small x

# Nucleon at rest



- Very complicated non-perturbative object, that contains fluctuations at all space-time scales smaller than its own size
- Only the fluctuations that are longer lived than the external probe participate in the interaction process
- Interactions are very complicated if the constituents of the nucleon have a non trivial dynamics over time-scales comparable to those of the probe

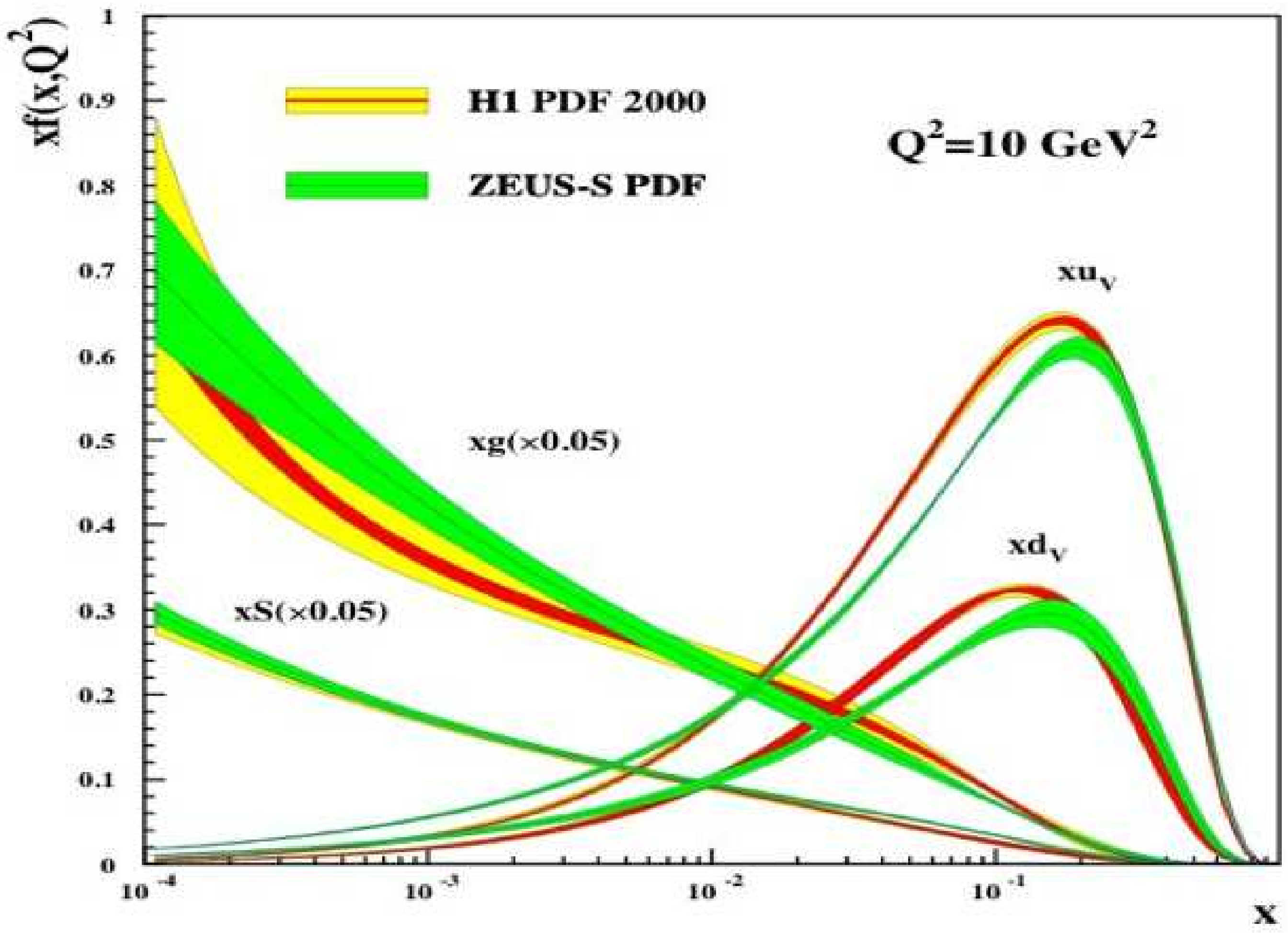
# Nucleon at high energy



- Dilation of all internal time-scales of the nucleon
- The constituents behave as if they were free over time-scales comparable to the interaction time
- Many fluctuations live long enough to be seen by the probe. The nucleon appears denser at high energy. Pre-existing fluctuations act as static sources of new partons
- In a nucleus, soft gluons (long wavelength) belonging to different nucleons overlap in the longitudinal direction
  ▷ coherent effects ▷ saturation

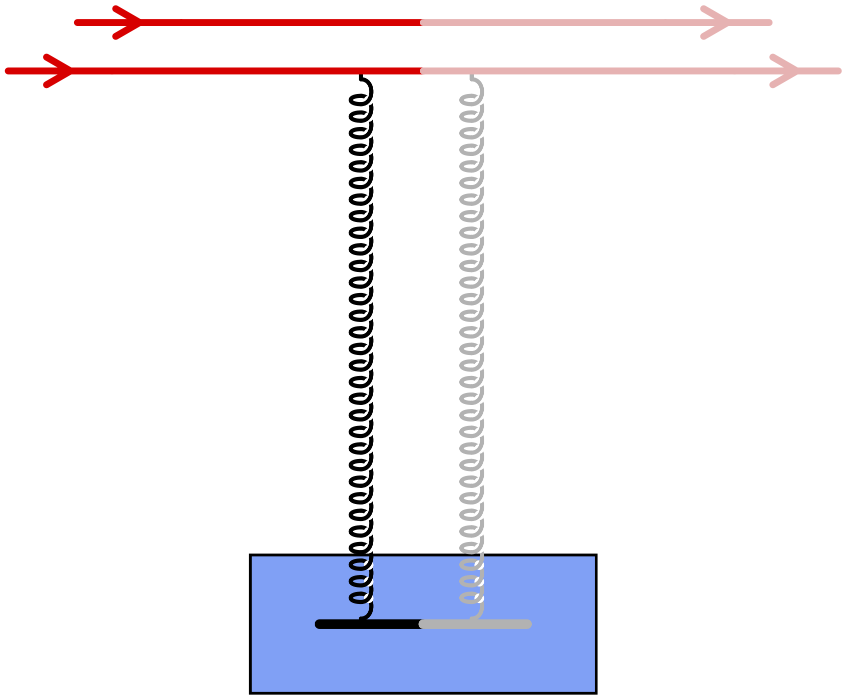
# Parton distributions in a proton

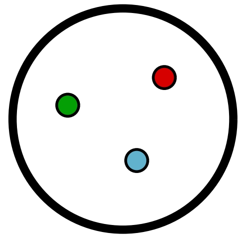




# Parton evolution







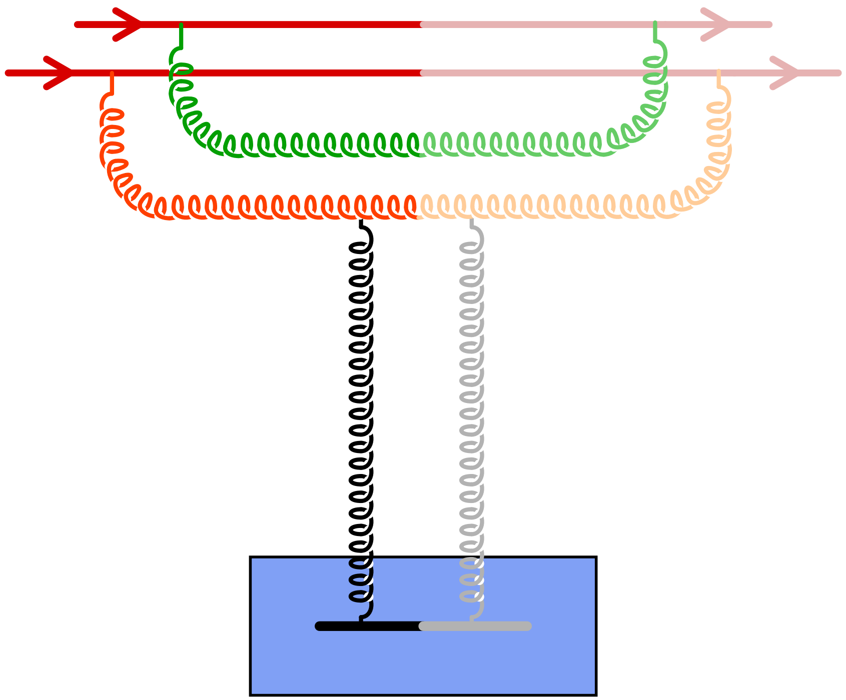
▷ assume that the projectile is big, e.g. a nucleus, and has many valence quarks (only two are represented)

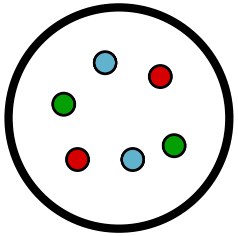
▷ on the contrary, consider a small probe, with few partons

▷ at low energy, only valence quarks are present in the hadron wave function

# Parton evolution







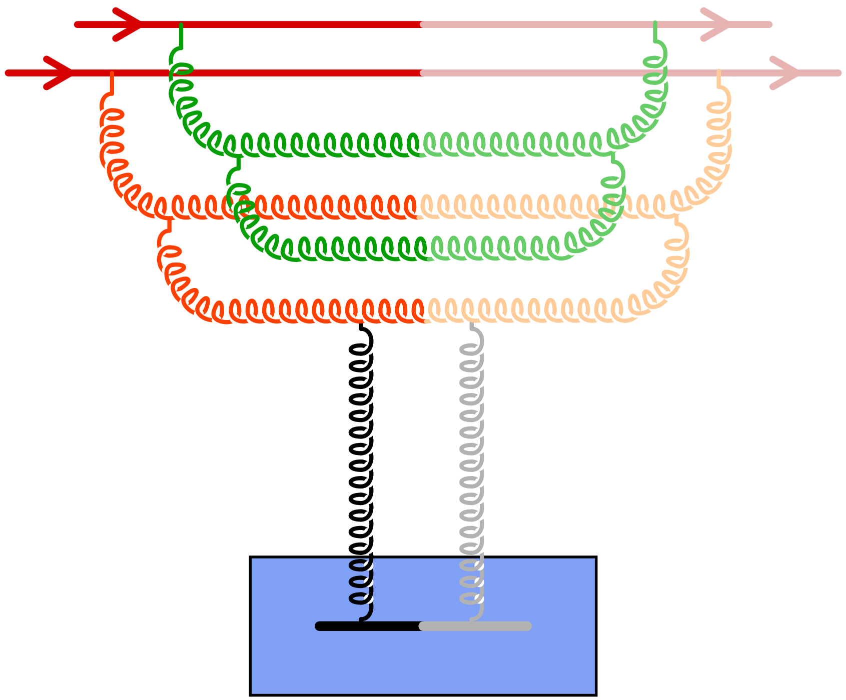
▷ when energy increases, new partons are emitted

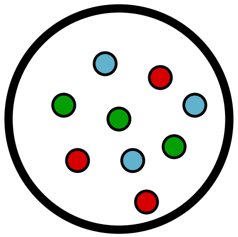
▷ the emission probability is $\alpha_s \int \frac{dx}{x} \sim \alpha_s \ln(\frac{1}{x})$, with $x$ the longitudinal momentum fraction of the gluon

▷ at small-$x$ (i.e. high energy), these logs need to be resummed

# Parton evolution

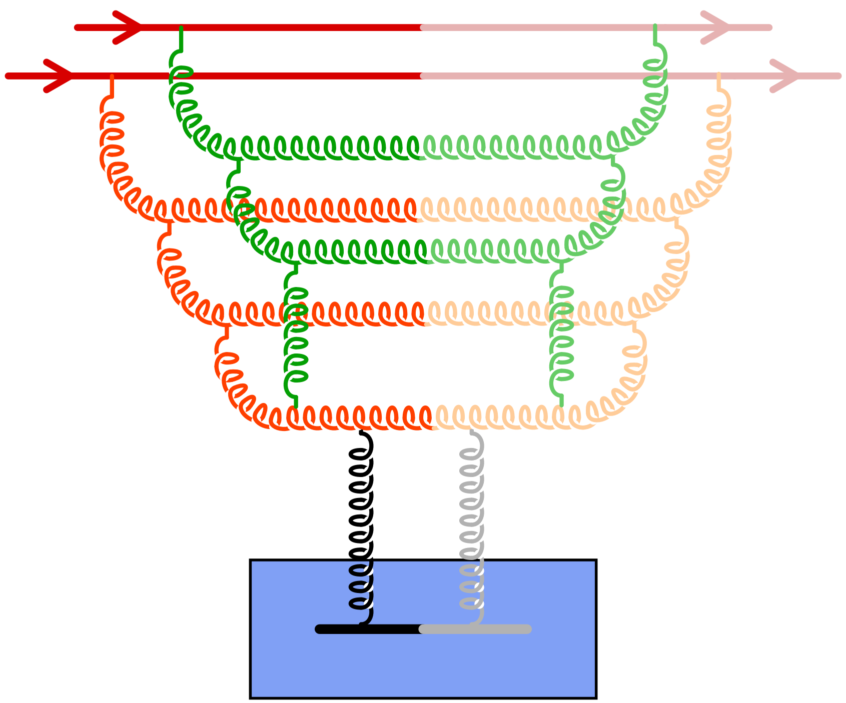


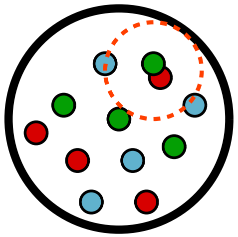




▷ as long as the density of constituents remains small, the evolution is linear: the number of partons produced at a given step is proportional to the number of partons at the previous step (BFKL)

# Parton evolution





▷ eventually, the partons start overlapping in phase-space

▷ parton recombination becomes favorable

▷ after this point, the evolution is non-linear:
the number of partons created at a given step depends non-linearly on the number of partons present previously

# Saturation criterion



Gribov, Levin, Ryskin (1983)

- Number of gluons per unit area:

$$\rho \sim \frac{xG_A(x,Q^2)}{\pi R_A^2}$$

- Recombination cross-section:

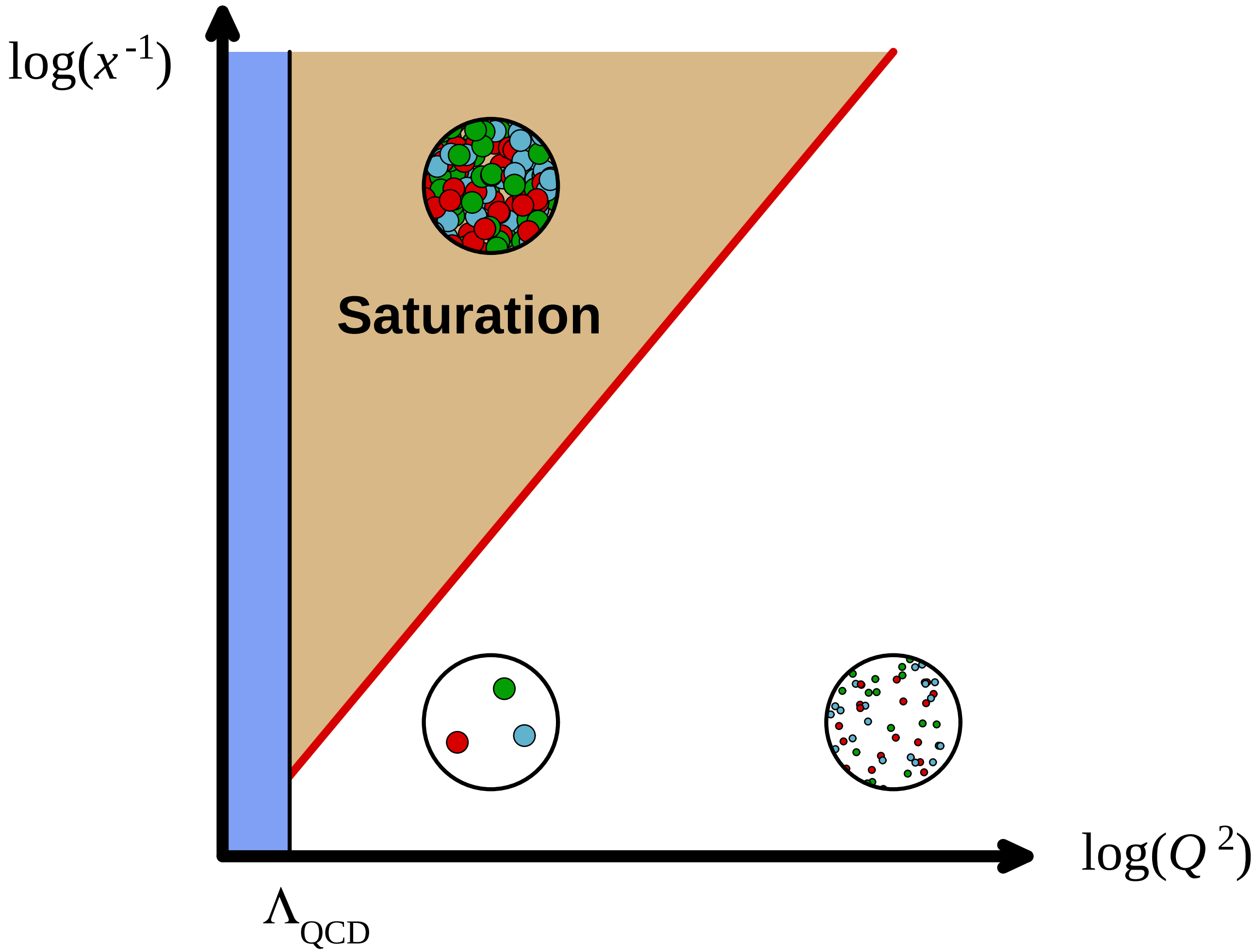
$$\sigma_{gg\to g} \sim \frac{\alpha_s}{Q^2}$$

- Recombination happens if $\rho\sigma_{gg\to g} \gtrsim 1$, i.e. $Q^2 \lesssim Q_s^2$, with:

$$Q_s^2 \quad \sim \quad \frac{\alpha_s xG_A(x,Q_s^2)}{\pi R_A^2} \quad \sim \quad A^{1/3}\frac{1}{x^{0.3}}$$

# Saturation domain





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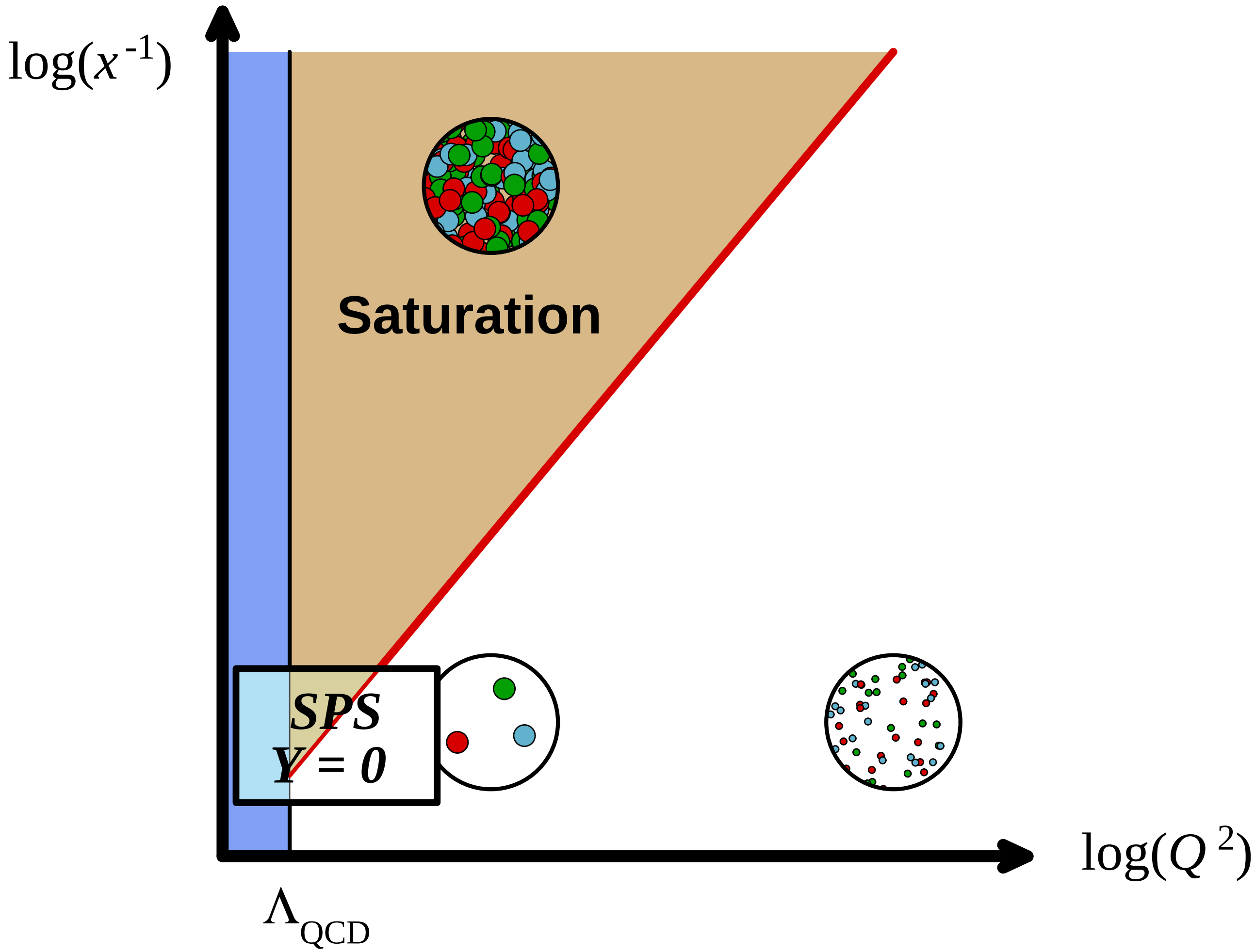

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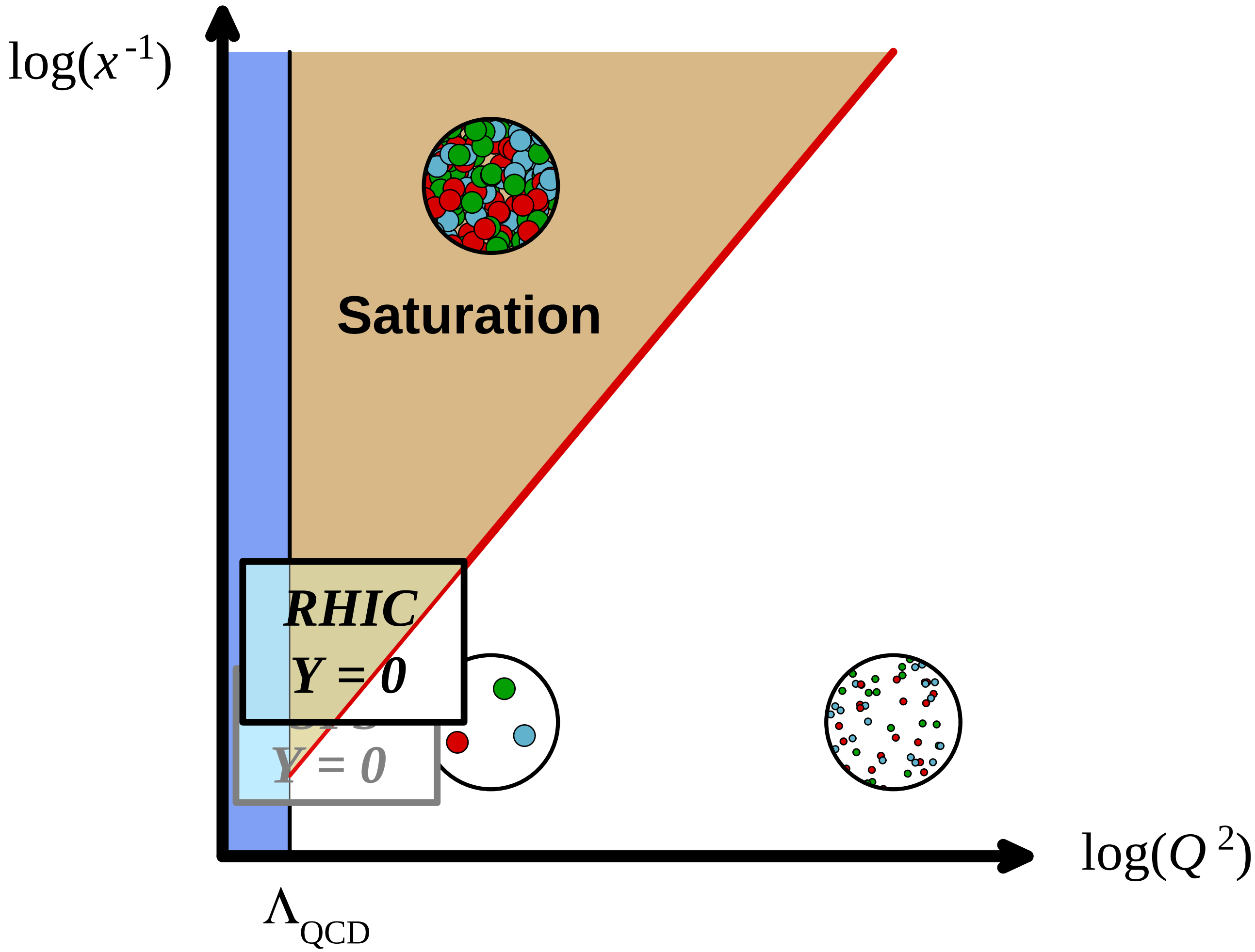

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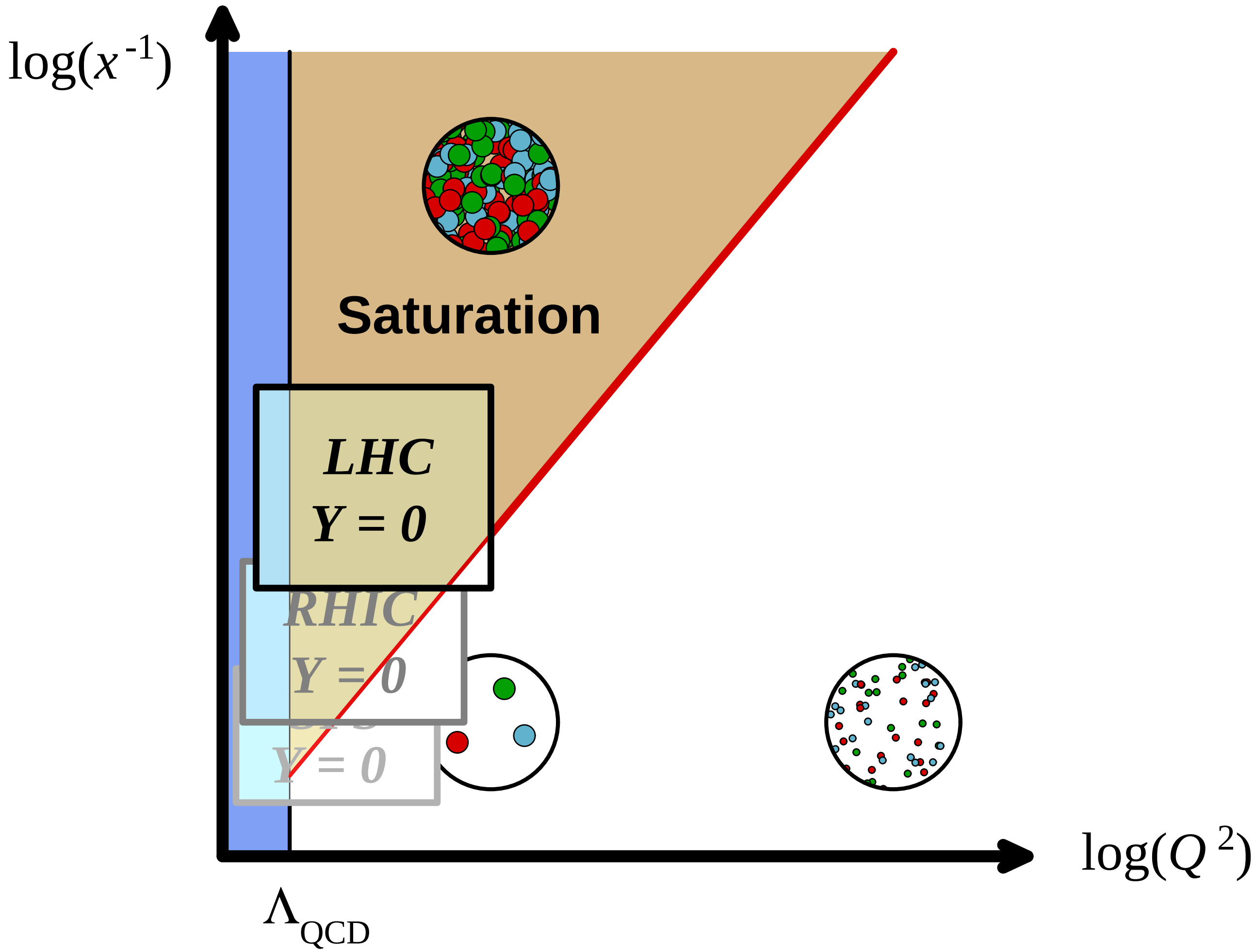

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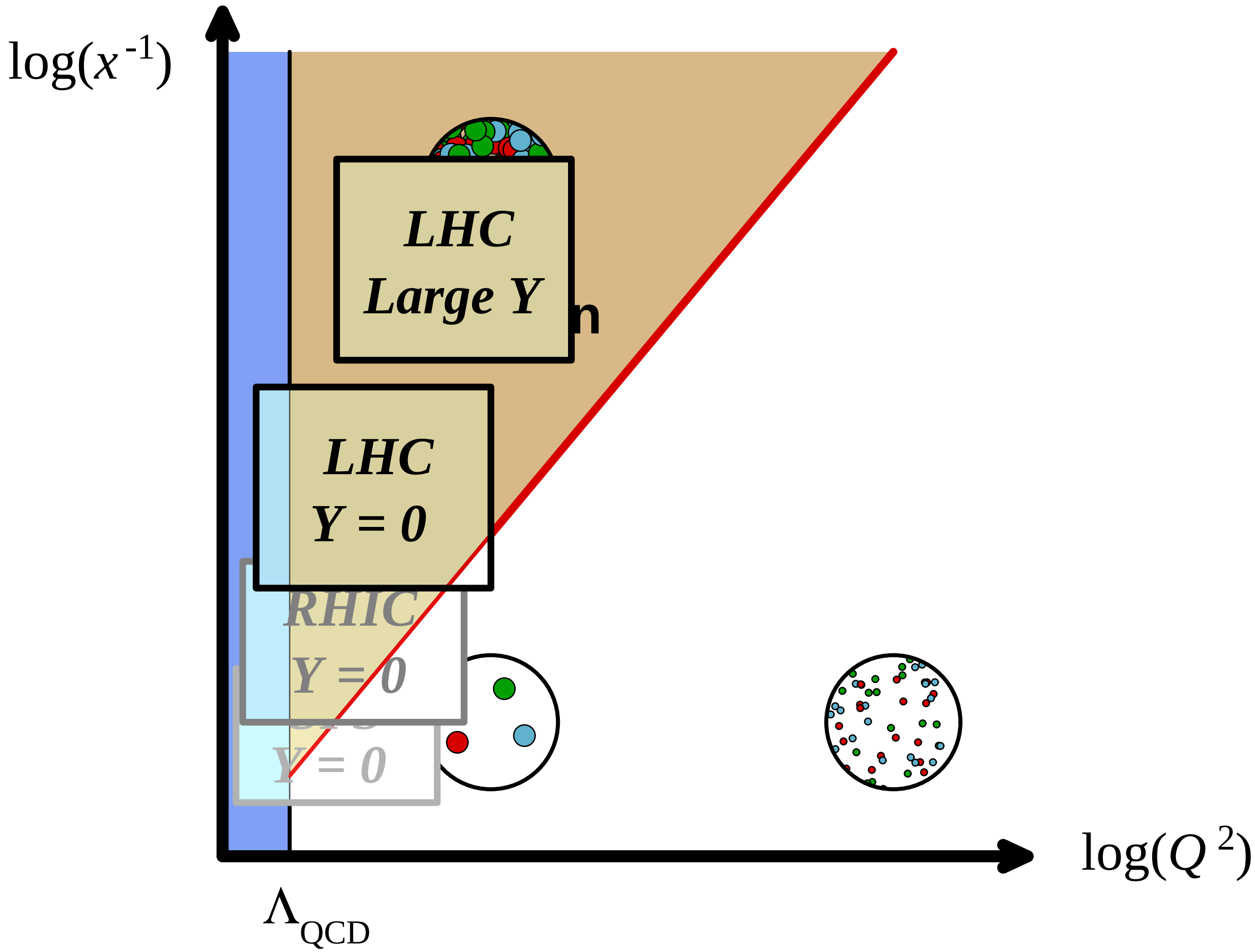

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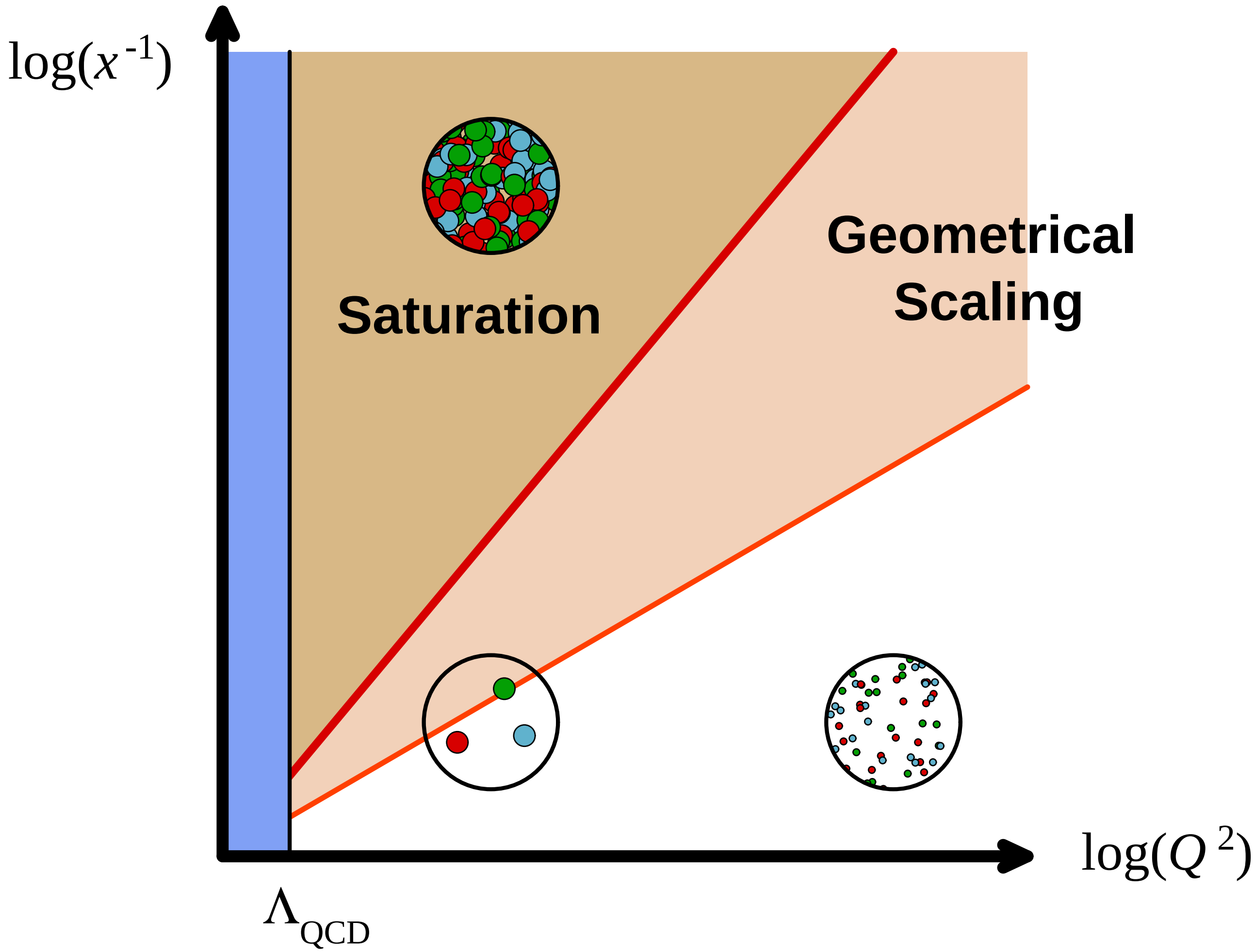


# Degrees of freedom and their interplay



McLerran, Venugopalan (1994), Iancu, Leonidov, McLerran (2001)

- Small-$x$ modes have a large occupation number
  ▷ they are described by a classical color field $A^\mu$ that obeys Yang-Mills's equation:

$$\left[D_\nu, F^{\nu\mu}\right]_a = J_a^\mu$$

- The source term $J_a^\mu$ comes from the faster partons. The large-$x$ modes, slowed down by time dilation, are described as frozen color sources $\rho_a$. Hence :

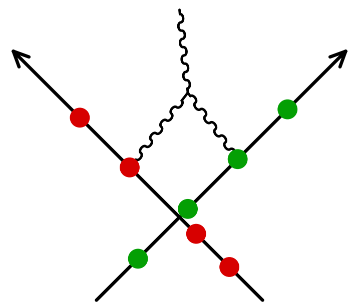
$$J_a^\mu = \delta^{\mu +}\delta(x^-)\rho_a(\vec{\boldsymbol{x}}_\perp)$$

- The color sources $\rho_a$ are random, and described by a distribution $W_{_Y}[\rho]$, with $Y \equiv \ln(1/x_0)$, $x_0$ being the frontier between "small-$x$" and "large-$x$". JIMWLK equation :

$$\frac{\partial W_{_Y}[\rho]}{\partial Y} = \mathcal{H}[\rho]\;\; W_{_Y}[\rho]$$

# Hadronic collisions

- In order to study the collisions of two hadrons at leading order, the color current must have two terms :



$$J^\mu \equiv \delta^{\mu +}\delta(x^-)\,\rho_1(\vec{\boldsymbol{x}}_\perp) + \delta^{\mu -}\delta(x^+)\,\rho_2(\vec{\boldsymbol{x}}_\perp)$$

- Compute the observable $\mathcal{O}$ of interest in the color field created by a configuration $(\rho_1, \rho_2)$ of the sources. Note : the sources are of order $1/\sqrt{\alpha_s}$ ▷ very non-linear problem

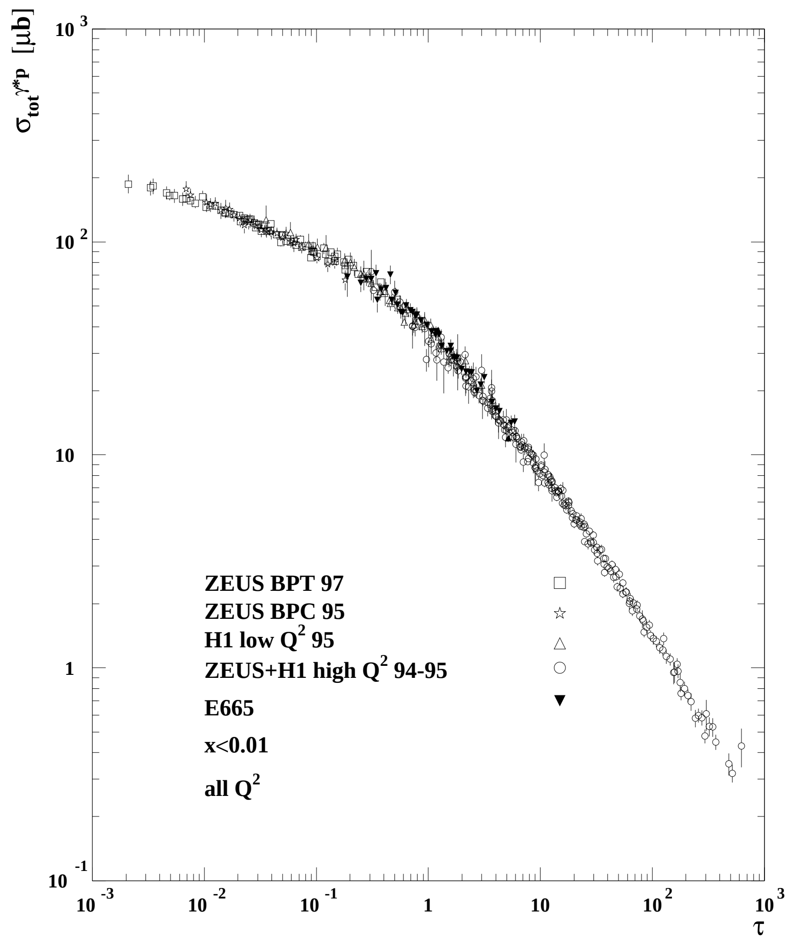
- Average over the sources $\rho_1$, $\rho_2$

$$\left\langle \mathcal{O} \right\rangle_Y = \int [D\rho_1]\,[D\rho_2]\,W_{Y_{\rm beam}-Y}[\rho_1]\,W_{Y_{\rm beam}+Y}[\rho_2]\,\mathcal{O}[\rho_1,\rho_2]$$

# Geometrical scaling in F2



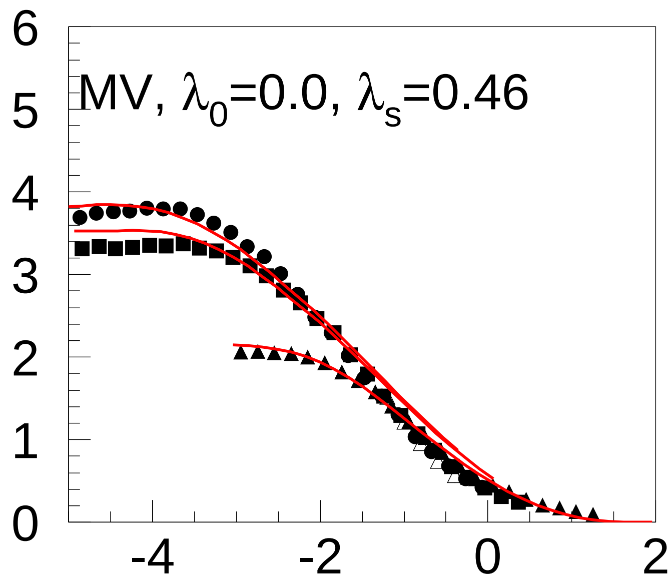
- Low $x$ ($x < 10^{-2}$) data displayed as a function of $\tau = x^{0.3}Q^2$
  Stasto, Golec-Biernat, Kwiecinski (2000)
  Iancu, Itakura, McLerran (2002)



# Limiting fragmentation



- Inclusive hadron spectrum at RHIC, shifted by the beam rapidity ($\sqrt{s} = 19.6, \ 64, \ 130, \ 200$ GeV) (data from PHOBOS, STAR and BRAHMS) :



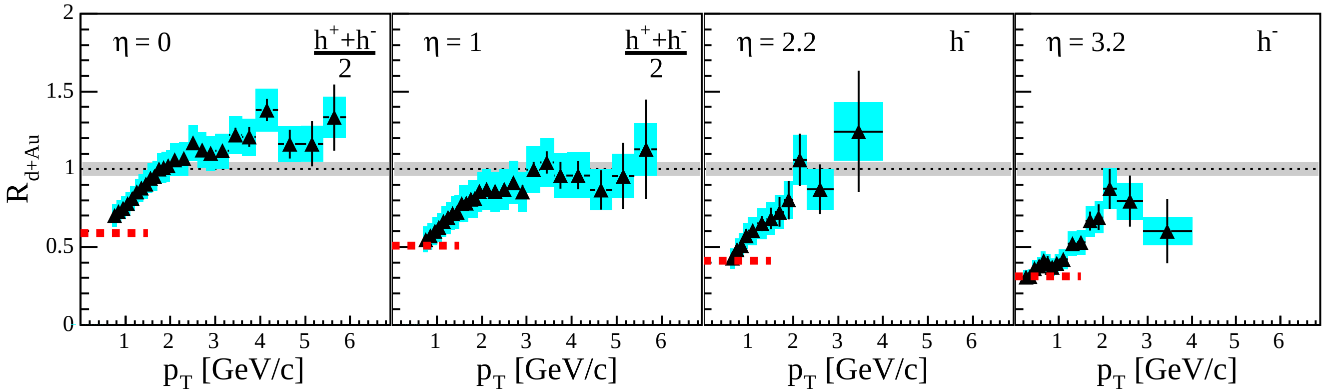

Jalilian-Marian (2002), FG, Stasto, Venugopalan (2006)

- Limiting fragmentation is natural in the framework of gluon saturation. It follows from :
  - Approximate Bjorken scaling in the nucleus at large $x$
  - Unitarization of scattering amplitudes in the nucleus at small $x$

# High pt suppression at large Y

- Results of the BRAHMS experiment at RHIC for deuteron-gold collisions :

$$R_{\rm dAu} \equiv \frac{1}{N_{\rm coll}} \frac{\left.\frac{dN}{dp_\perp d\eta}\right|_{\rm dAu}}{\left.\frac{dN}{dp_\perp d\eta}\right|_{\rm pp}}$$

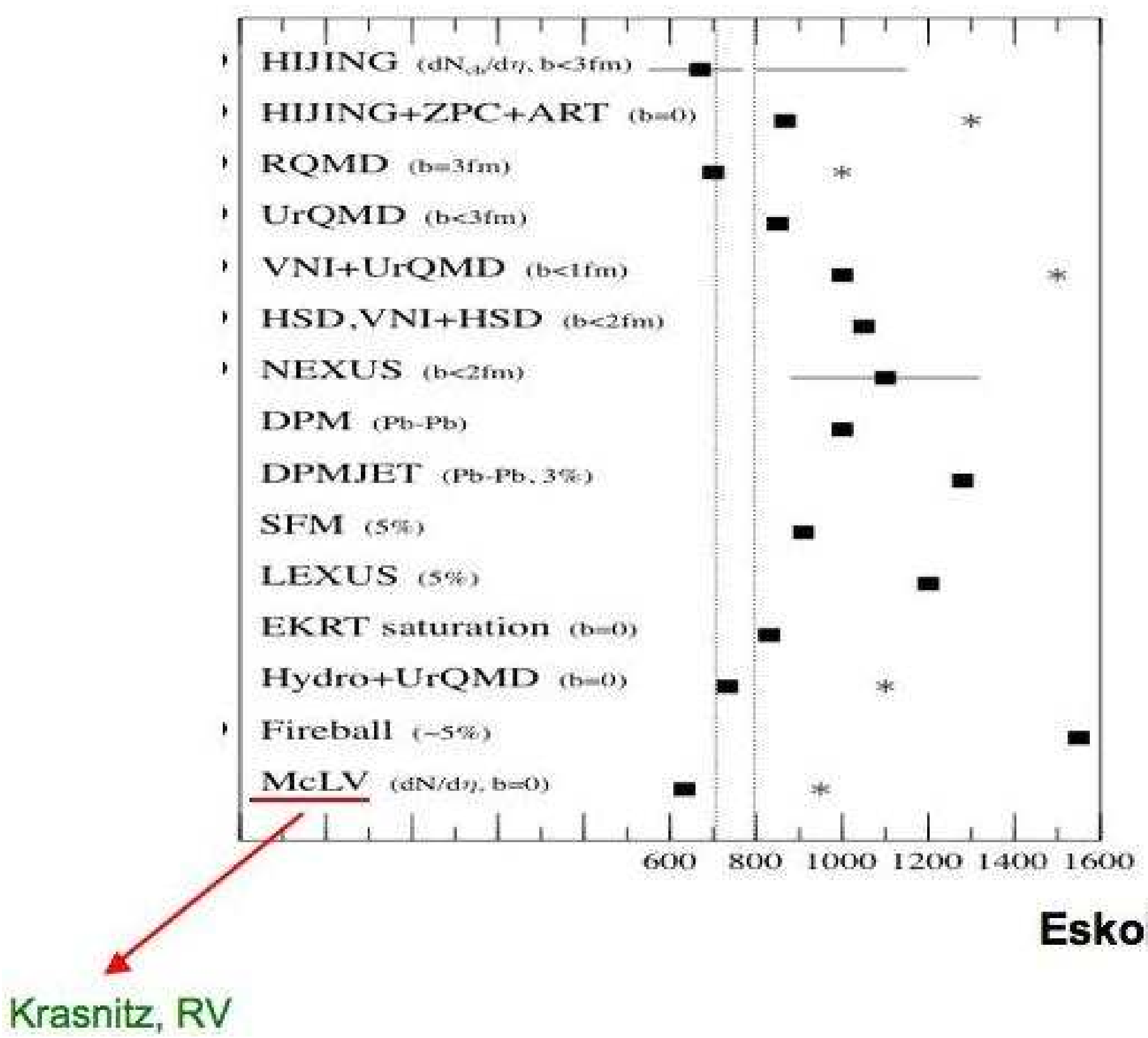



Albacete, Armesto, Kovner, Salgado, Wiedemann ('03), Kharzeev, Levin, McLerran ('03), Iancu, Itakura, Triantafyllopoulos ('04)

- At small rapidity, suppression at low $p_\perp$ and enhancement at high $p_\perp$ (multiple scatterings – Cronin effect)
- At large rapidity, suppression at all $p_\perp$'s (shadowing)

# Multiplicity at RHIC

- Predictions from different approaches vs. data :

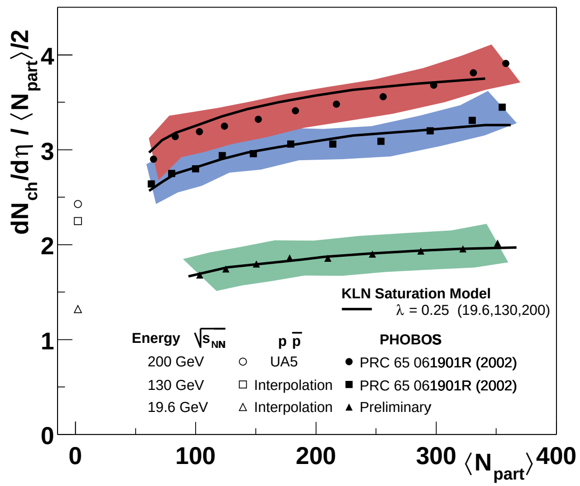


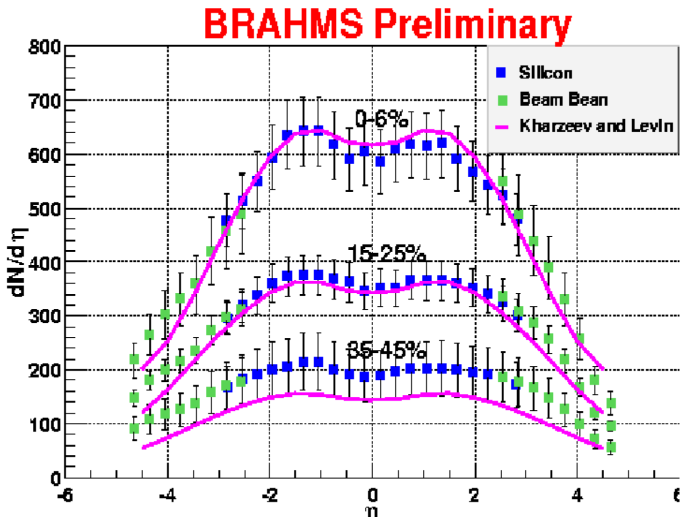

Eskola, QM 2001

# Multiplicity at RHIC

- $N_{\text{part}}$ scaling and energy dependence :

Kharzeev, Levin, Nardi (2001)




See also : Armesto, Salgado, Wiedemann (2004)

# The present Frontiers of the CGC



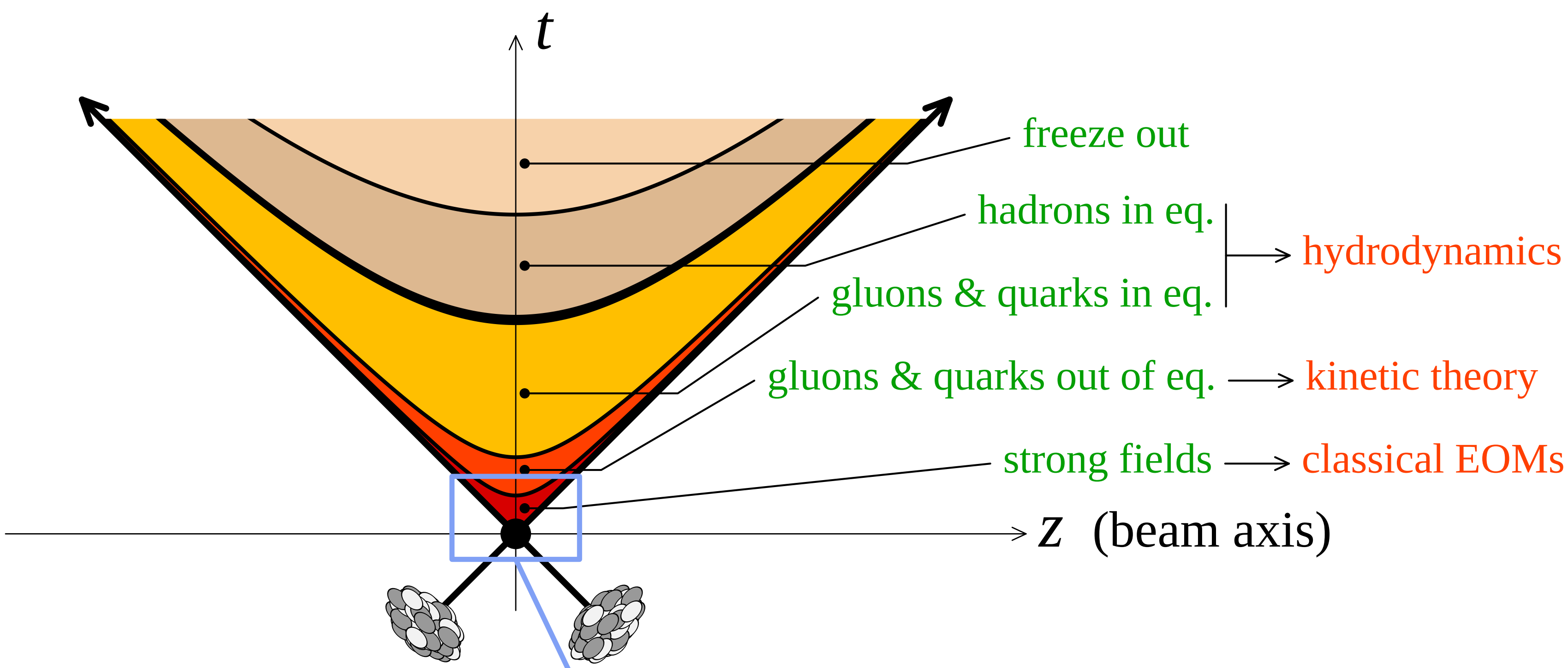
- Two aspects of QCD at high energy are under active study, but have not yet been applied to heavy ion collisions :
  - Beyond mean field, fluctuations of $Q_s$ and pomeron loops :
    - Evolution equations with a stochastic term :
      Hatta, Iancu, Marquet, Soyez, Triantafyllopoulos (2006)
      Marquet, Soyez, Xiao (2006)
    - Toy models in 1+1 dimensions :
      Shoshi, Xiao (2006), Kozlov, Levin, Khachtryan, Miller (2006)
      Blaizot, Iancu, Triantafyllopoulos (2006)
    - Applications to diffractive reactions :
      Iancu, Marquet, Soyez (2006), Shoshi, Xiao (2006)

    + many more...
  - Towards NLO evolution equations :
    Gardi, Kuokkanen, Rummukainen, Weigert (2006)
    Kovchegov, Weigert (2006), Balitsky (2006)
    Albacete, Armesto, Milhano (2006)

# Initial conditions for nucleus-nucleus collisions (“Glasma”)

# What do we mean by Initial Conditions?

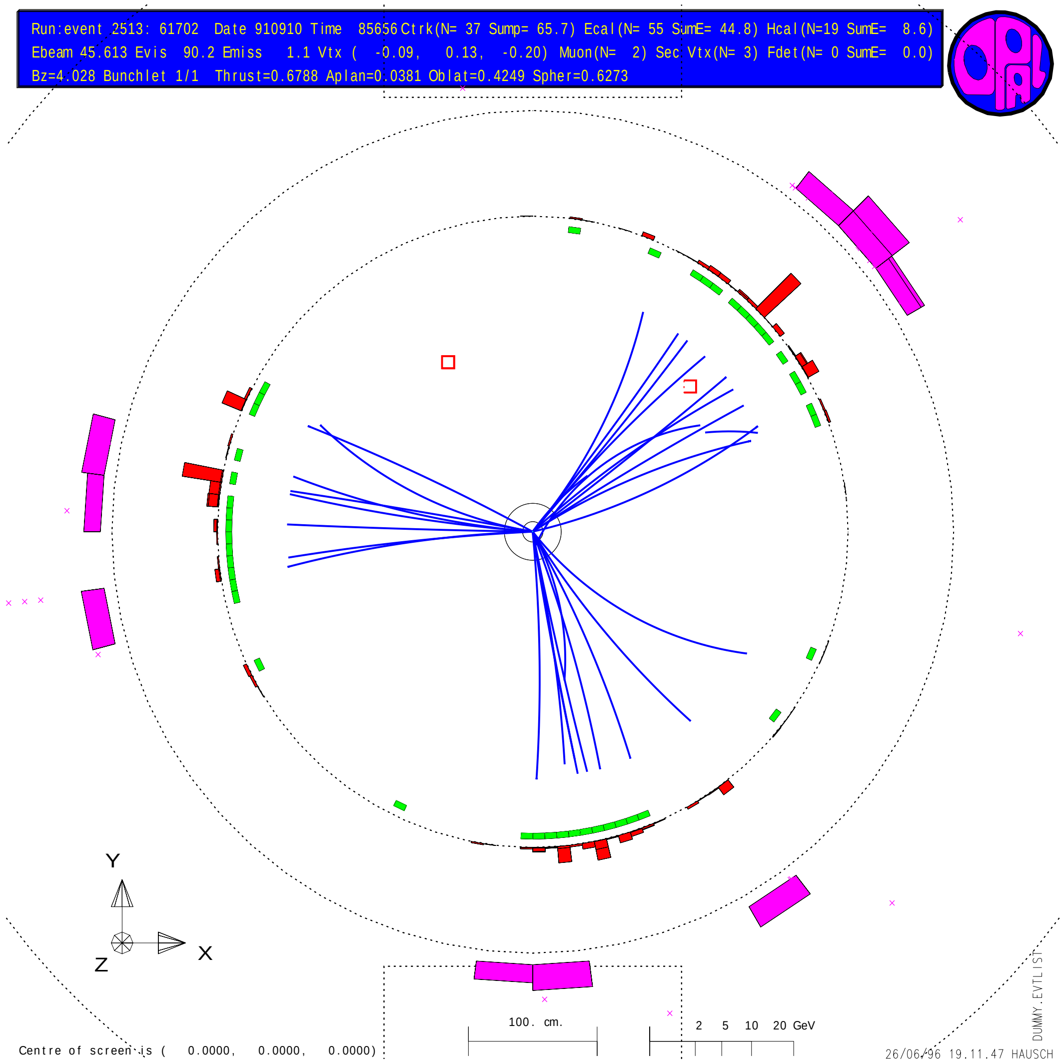





- calculate the initial production of semi-hard particles
- prepare the stage for kinetic theory or hydrodynamics

# Typical e+e- or pp collision



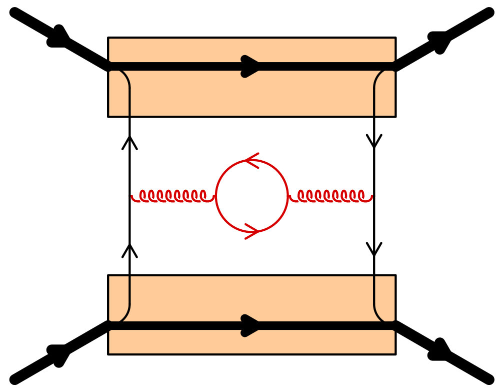


# Why is pQCD predictive there ?



- More precisely, why is pQCD predictive despite the fact that hadrons are non-perturbative bound states?

- Factorization :

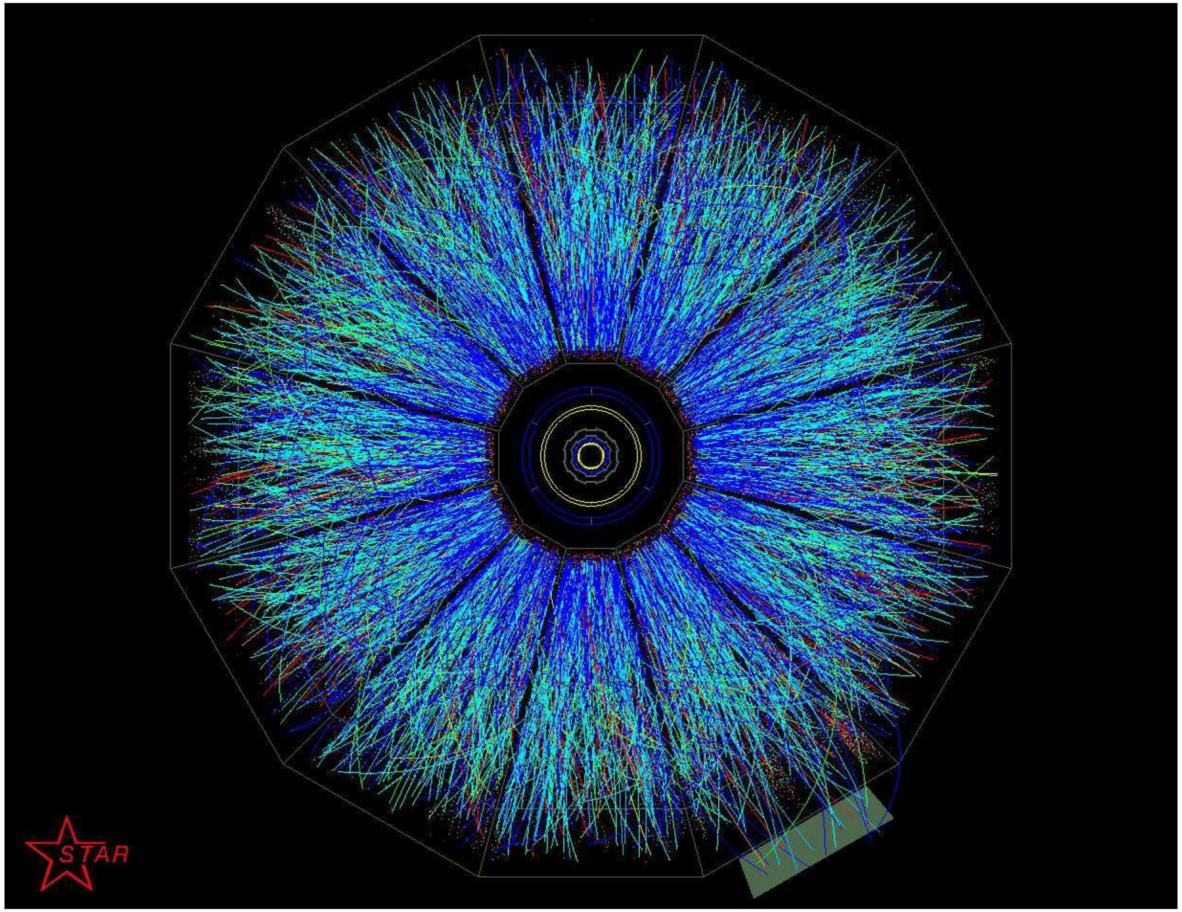


▷ (Collinear) divergences in loop corrections can be absorbed into the (DGLAP) evolution of parton distributions and fragmentation functions

- Universality : parton distributions are process independent

# Typical nucleus-nucleus collision





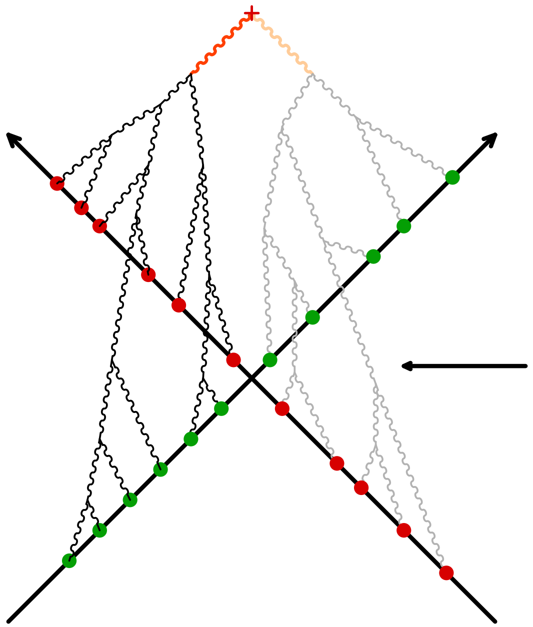

- Can we set up an equally systematic framework for semi-hard particle production in nucleus-nucleus collisions?

# Gluon multiplicity at LO

Krasnitz, Nara, Venugopalan (1999 – 2001), Lappi (2003)

$$\frac{d\overline{N}_{LO}}{d^3\vec{\boldsymbol{p}}} \propto \int_{x,y} e^{ip\cdot(x-y)} \cdots \mathcal{A}_\mu(x)\mathcal{A}_\nu(y)$$

- $\mathcal{A}^\mu(x) =$ retarded solution of Yang-Mills equations



only tree diagrams at LO

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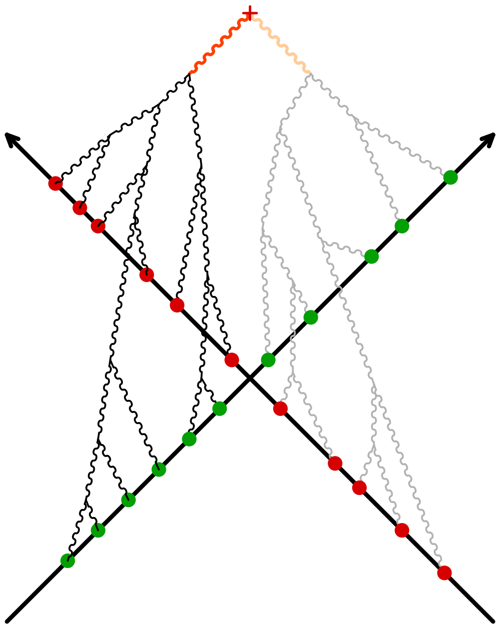
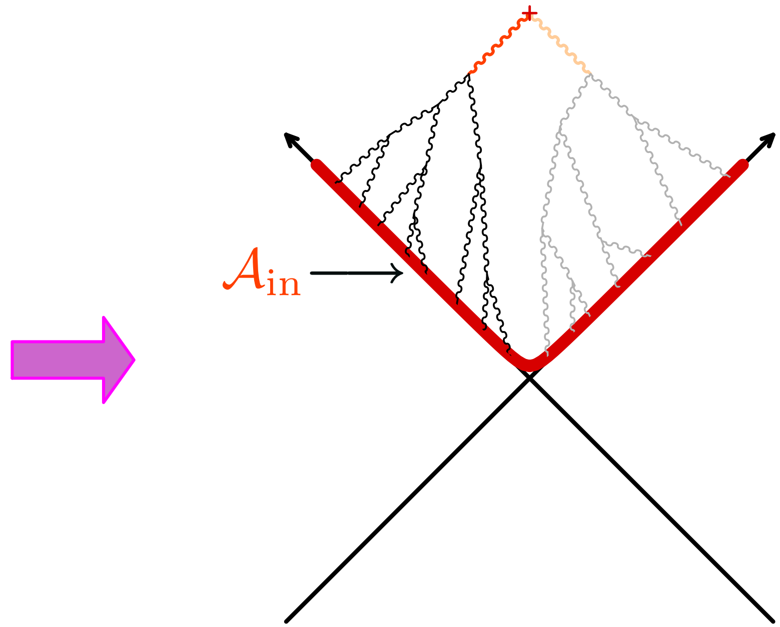
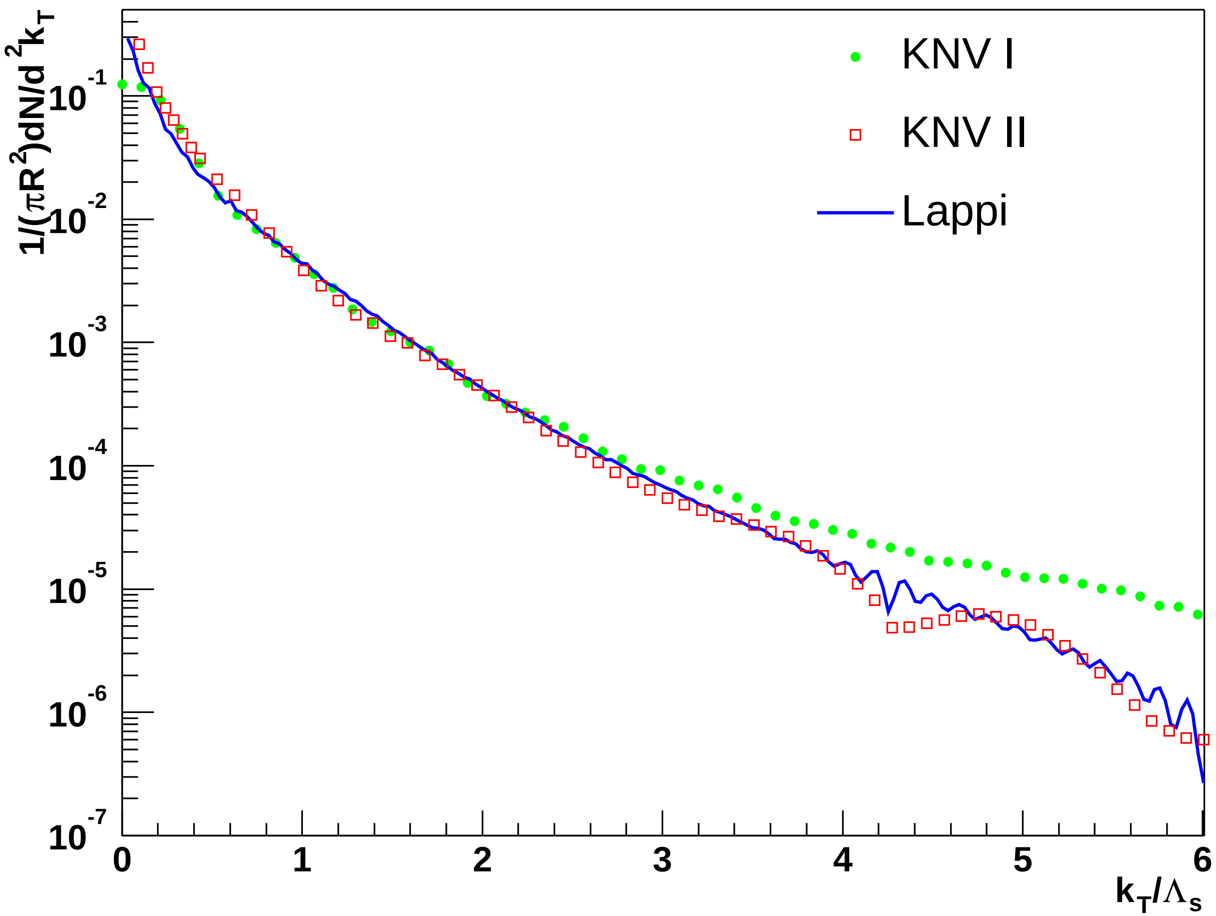
- $\mathcal{A}^\mu(x)$ = retarded solution of Yang-Mills equations
  - ▷ can be cast into an initial value problem on the light-cone





# Gluon multiplicity at LO

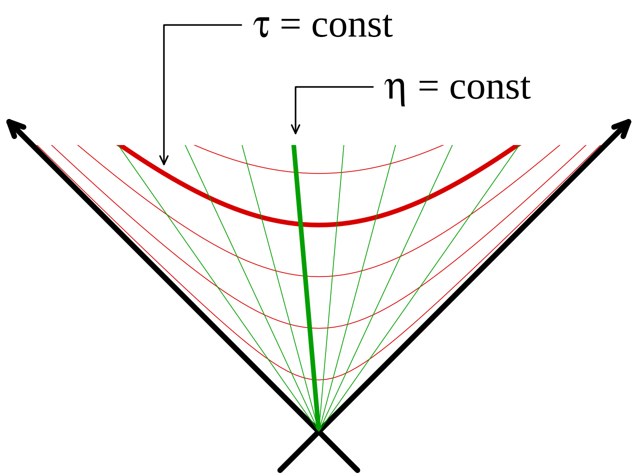





- Important softening at small $k_\perp$ compared to pQCD (saturation)
- Quark production has also been computed (FG, Kajantie, Lappi (2005))

# Initial conditions and boost invariance





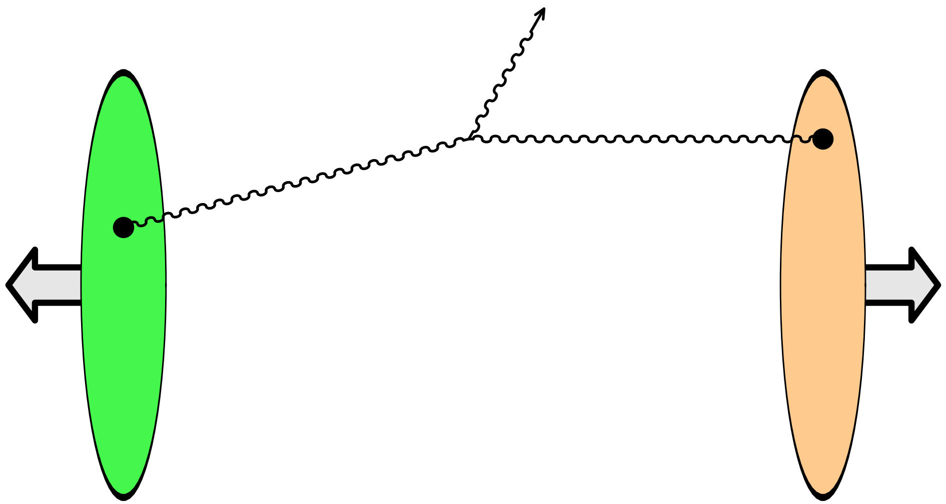

- The color field at $\tau = 0$ does not depend on the rapidity $\eta$

  ▷ it remains independent of $\eta$ at all times (invariance under boosts in the $z$ direction)

  ▷ numerical resolution performed in $2 + 1$ dimensions

# Systematics of particle production

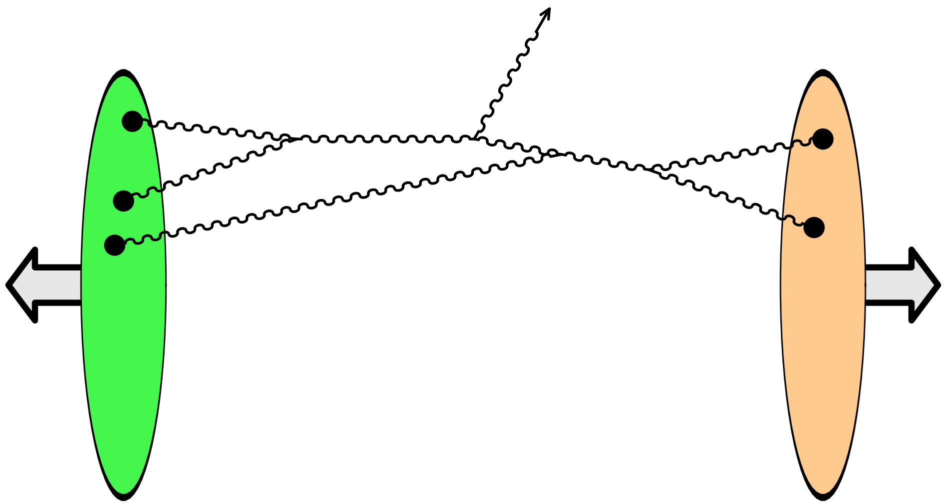




- Dilute regime : one source in each projectile interact

# Systematics of particle production





- Dilute regime : one source in each projectile interact
- Dense regime : non linearities are important

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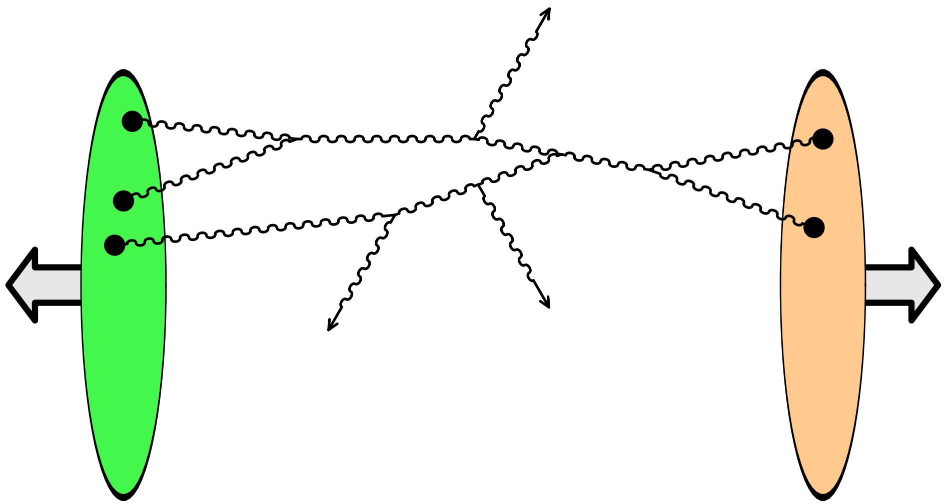

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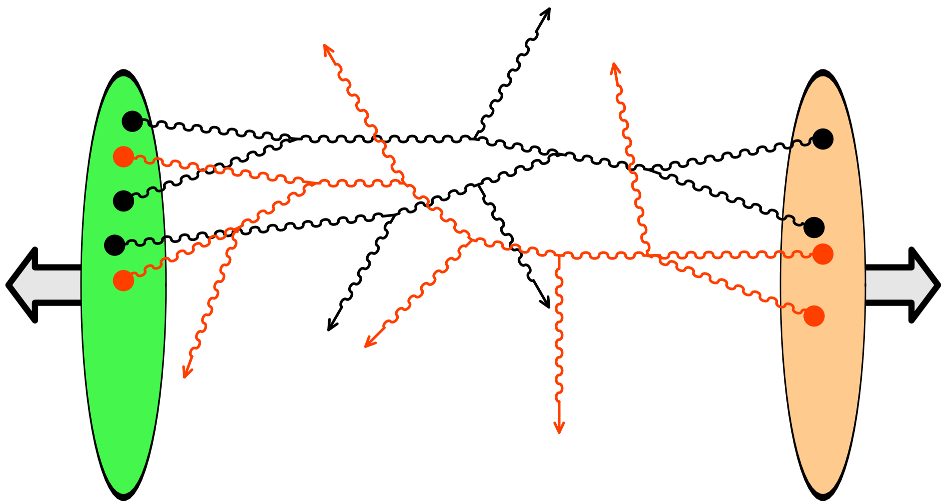

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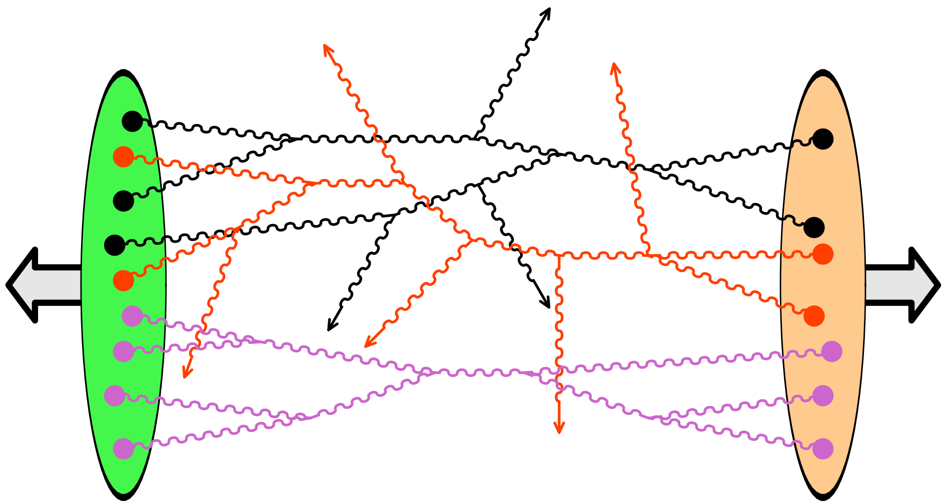

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- Some of them may not produce anything (vacuum diagrams)

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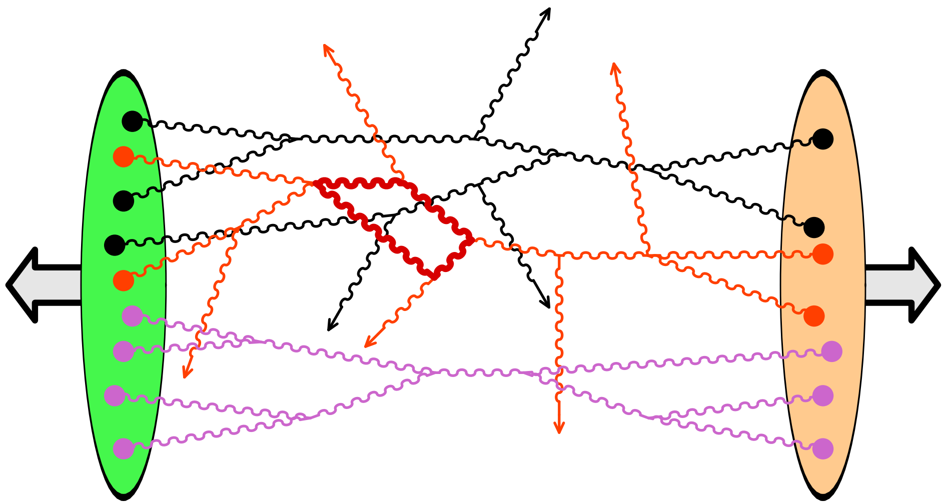
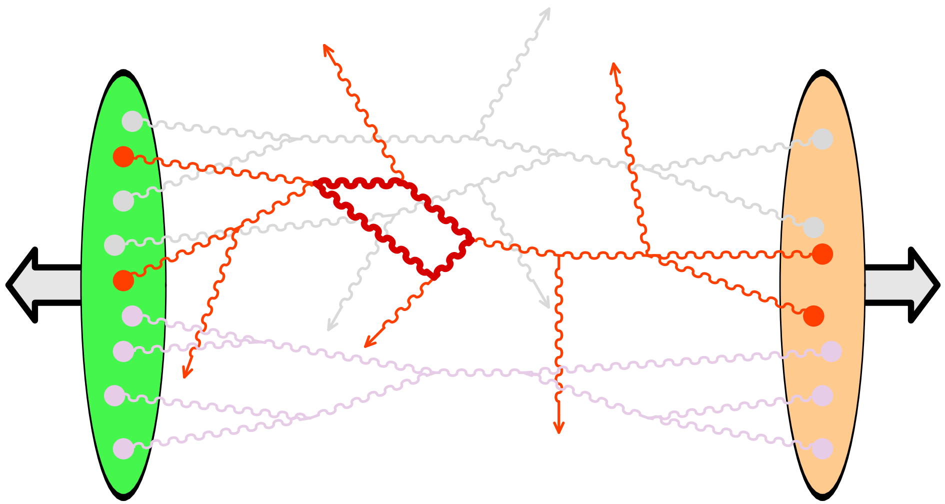


- Dilute regime : one source in each projectile interact
- Dense regime : non linearities are important
- Many gluons can be produced from the same diagram
- There can be many simultaneous disconnected diagrams
- Some of them may not produce anything (vacuum diagrams)
- All these diagrams can have loops (not at LO though)

# Power counting





- In the saturated regime, the sources are of order $1/\sqrt{\alpha_s}$, and the order of each disconnected diagram is given by :

$$\alpha_s^{-1} \quad \alpha_s^{\frac{1}{2}(\#\ \text{produced gluons})} \quad \alpha_s^{\#\ \text{loops}}$$

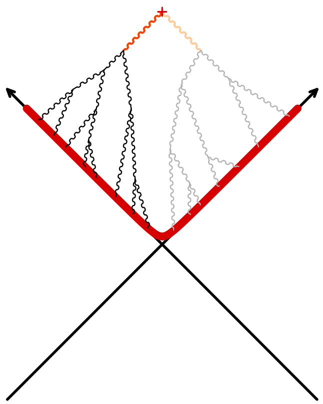
- Total order = product of the orders of the subdiagrams

▷ summing all the contributions to the spectrum at a given order requires powerful bookkeeping tools (FG, Venugopalan (2006))

# 1-loop correction to N



- The 1-loop correction to $\overline{N}$ can be written as a perturbation of the initial value problem encountered at LO :



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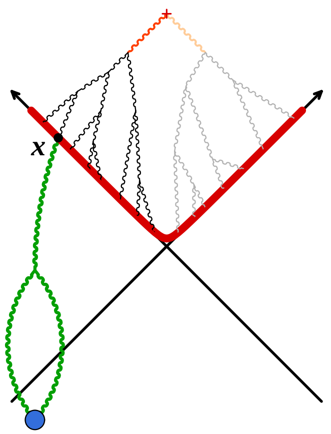


$$\delta\overline{N} = \left[ \int_{\vec{x}\in \text{light cone}} \delta\mathcal{A}(\vec{x})\ \boldsymbol{T}_{\vec{x}} \right] \overline{N}_{LO}$$

- $\overline{N}_{LO}$ is a functional of the initial fields $\mathcal{A}_{\rm in}(\vec{x})$ on the light-cone
- $\boldsymbol{T}_{\vec{x}}$ is the generator of shifts of the initial condition at the point $\vec{x}$ on the light-cone, i.e. : $\boldsymbol{T}_{\vec{x}} \sim \delta/\delta\mathcal{A}_{\rm in}(\vec{x})$

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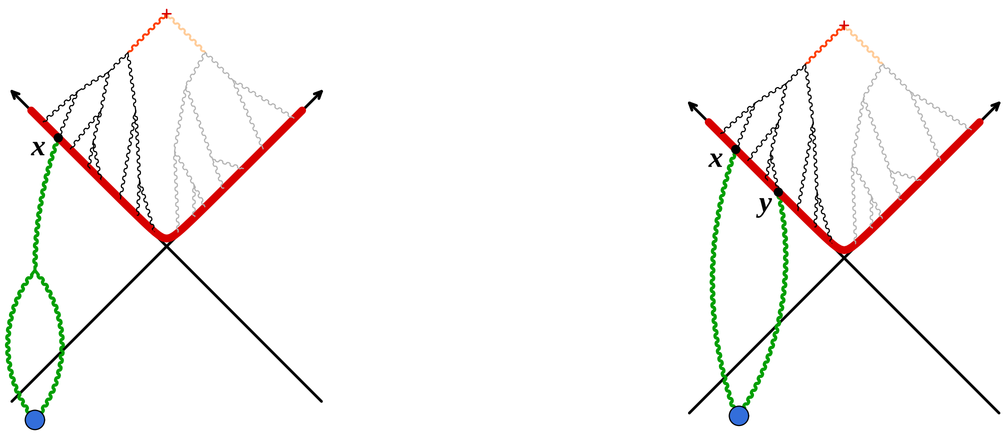



$$\delta\overline{N} = \left[ \int_{\vec{x}\in \text{light cone}} \delta\mathcal{A}(\vec{x})\; \boldsymbol{T}_{\vec{x}} + \int_{\vec{x},\vec{y}\in \text{light cone}} \frac{1}{2}\,\boldsymbol{\Sigma}(\vec{x},\vec{y})\; \boldsymbol{T}_{\vec{x}}\,\boldsymbol{T}_{\vec{y}} \right] \overline{N}_{LO}$$

- $\overline{N}_{LO}$ is a functional of the initial fields $\mathcal{A}_{\rm in}(\vec{x})$ on the light-cone
- $\boldsymbol{T}_{\vec{x}}$ is the generator of shifts of the initial condition at the point $\vec{x}$ on the light-cone, i.e. : $\boldsymbol{T}_{\vec{x}} \sim \delta/\delta\mathcal{A}_{\rm in}(\vec{x})$
- $\delta\mathcal{A}(\vec{x})$ and $\boldsymbol{\Sigma}(\vec{x},\vec{y})$ are in principle calculable analytically

# Divergences

- If taken at face value, this 1-loop correction is plagued by several divergences :
  - The two coefficients $\delta\mathcal{A}(\vec{\boldsymbol{x}})$ and $\boldsymbol{\Sigma}(\vec{\boldsymbol{x}},\vec{\boldsymbol{y}})$ are infinite, because of an unbounded integration over a rapidity variable
  - At late times, $\boldsymbol{T}_{\vec{\boldsymbol{x}}}\mathcal{A}(\tau,\vec{\boldsymbol{y}})$ diverges exponentially,

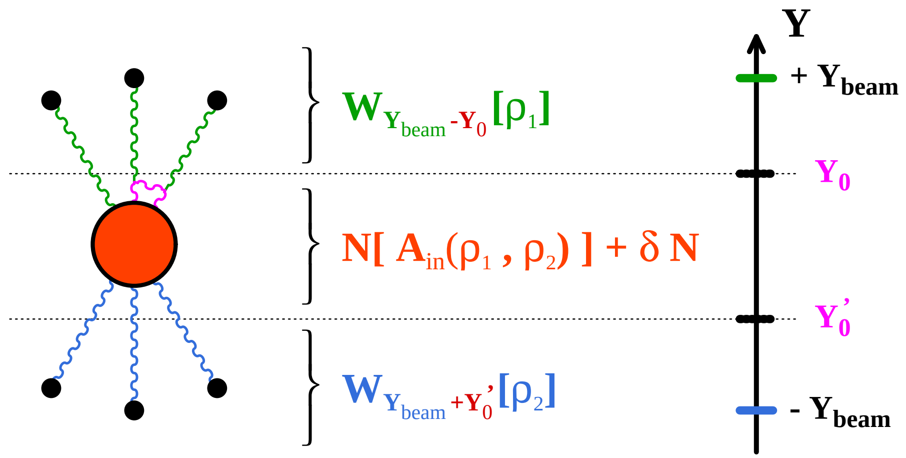
$$\boldsymbol{T}_{\vec{\boldsymbol{x}}}\mathcal{A}(\tau,\vec{\boldsymbol{y}}) \underset{\tau\to+\infty}{\sim} e^{\sqrt{\mu\tau}}$$

  because of an instability of the classical solution of Yang-Mills equations under rapidity dependent perturbations (Romatschke, Venugopalan (2005))

# Initial state factorization



- Anatomy of the full calculation :




- By putting arbitrary frontiers $Y_0$, $Y_0'$ between the "observable" and the "source distributions", the divergent coefficients become finite
- For the final result to be independent of $Y_0$, $Y_0'$, one needs :

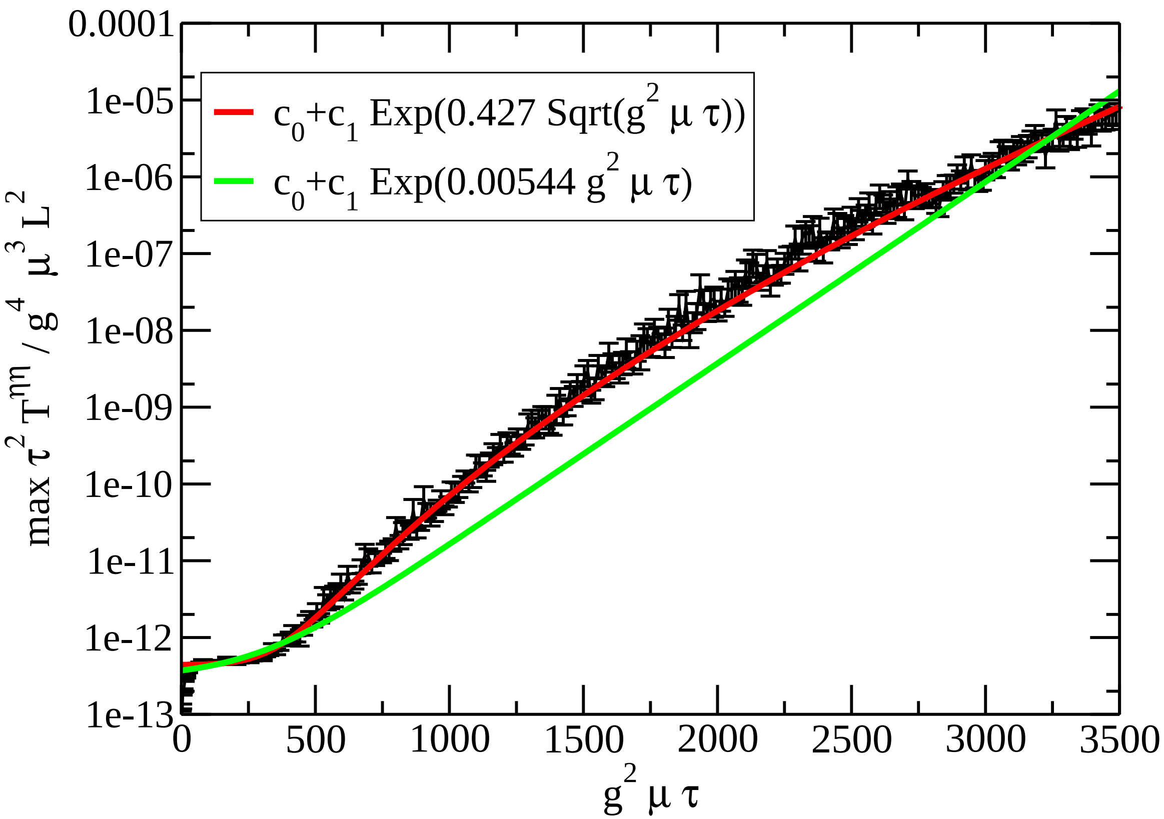
$$\left[\delta\overline{N}\right]_{\substack{\text{divergent}\\\text{coefficients}}} = \left[(Y_0 - Y)\,\mathcal{H}^\dagger[\rho_1] + (Y - Y_0')\,\mathcal{H}^\dagger[\rho_2]\right]\ \overline{N}_{LO}$$

where $\mathcal{H}[\rho]$ is the Hamiltonian that governs the rapidity dependence of the source distribution $W_Y[\rho]$ : $\quad \partial_Y W_Y[\rho] = \mathcal{H}[\rho]\ W_Y[\rho]$
FG, Lappi, Venugopalan (work in progress)

# Unstable modes

Romatschke, Venugopalan (2005)

- Rapidity dependent perturbations to the classical fields grow like $\exp(\#\sqrt{\tau})$ until the non-linearities become important :



# Unstable modes



- One can sum the contribution of the unstable modes by :

$$\Big[\delta\overline{N}\Big]_{\substack{\text{unstable}\\ \text{modes}}} = \int [Da]\; \mathcal{D}_{\text{fluct}}[a]\; \overline{N}_{LO}[\mathcal{A}_{\text{in}}(\rho_1,\rho_2)+a]$$

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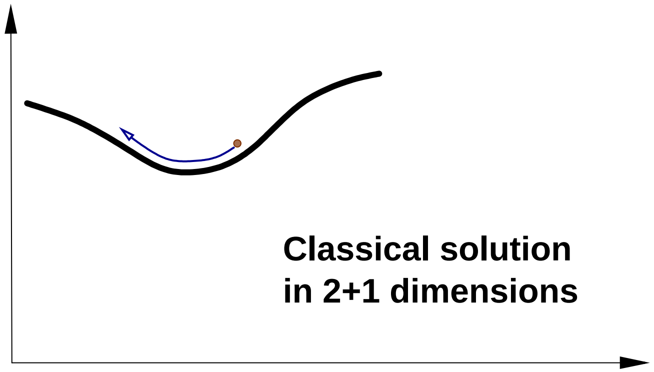

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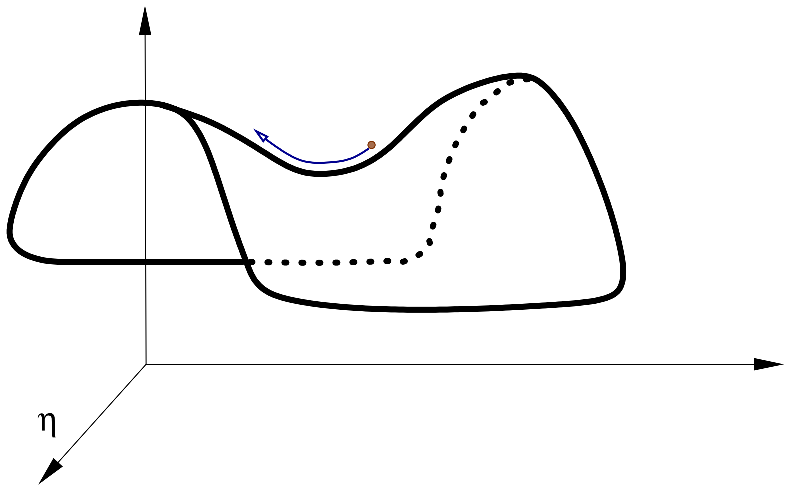

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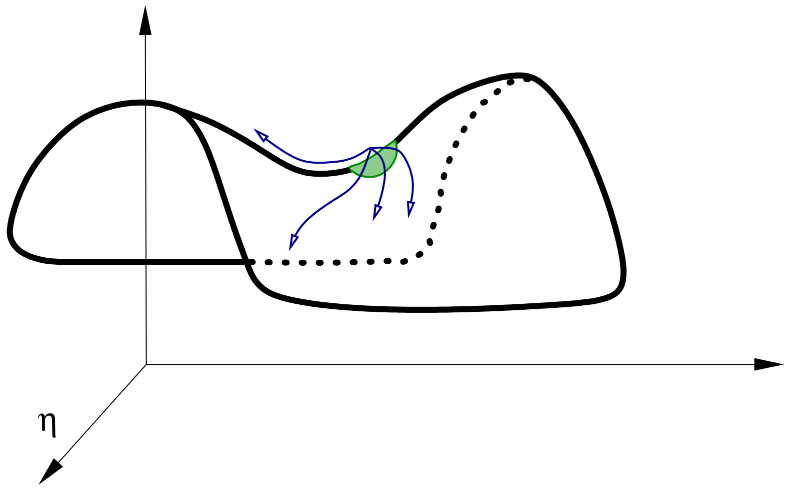



- The distribution of fluctuations has been calculated recently Fukushima, FG, McLerran (2006)
- Still open issue : can these instabilities fight efficiently against the expansion of the system ?

# Summary

# Summary

- Gluon recombination is important at small $x$, and affects initial particle production in high-energy AA collisions
- Thanks to the large density of color sources, calculating the initial particle spectrum can be done via semi-classical techniques
- The resummation of the divergences at 1-loop tells us to :
  - average over the initial sources with the weight $W_Y[\rho]$
  - average over fluctuations with a distribution $\mathcal{D}_{\rm fluct}[a]$

▷ Provides a self-consistent framework based on the {JIMWLK + classical field approximation} combination

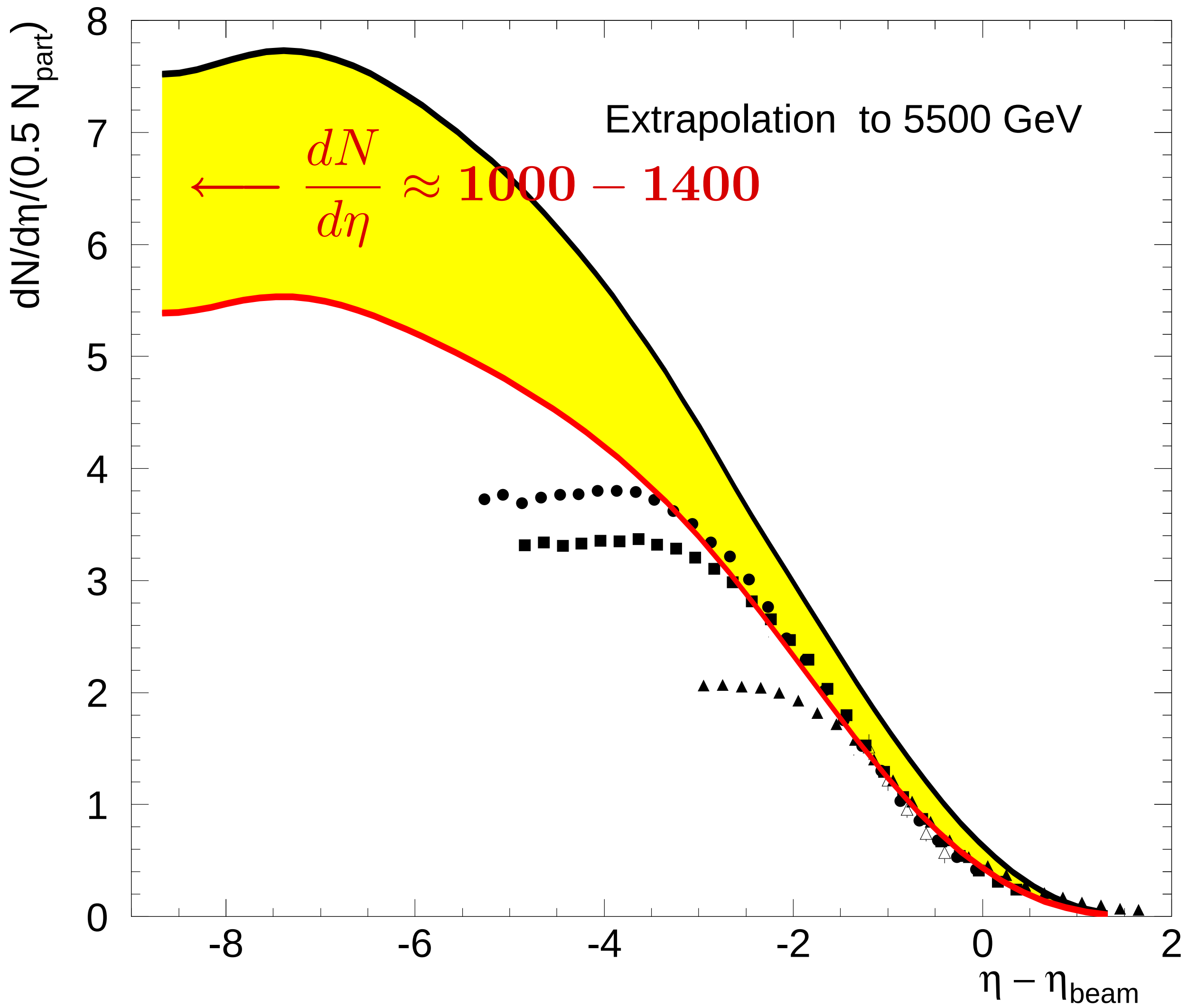
▷ Somewhat analogous to factorization in conventional pQCD :

$$W_Y[\rho] \quad \longleftrightarrow \quad \text{parton distribution}$$
$$\mathcal{D}_{\rm fluct}[a] \quad \longleftrightarrow \quad \text{fragmentation function}$$

# Extra bits

# Extrapolation to LHC energy

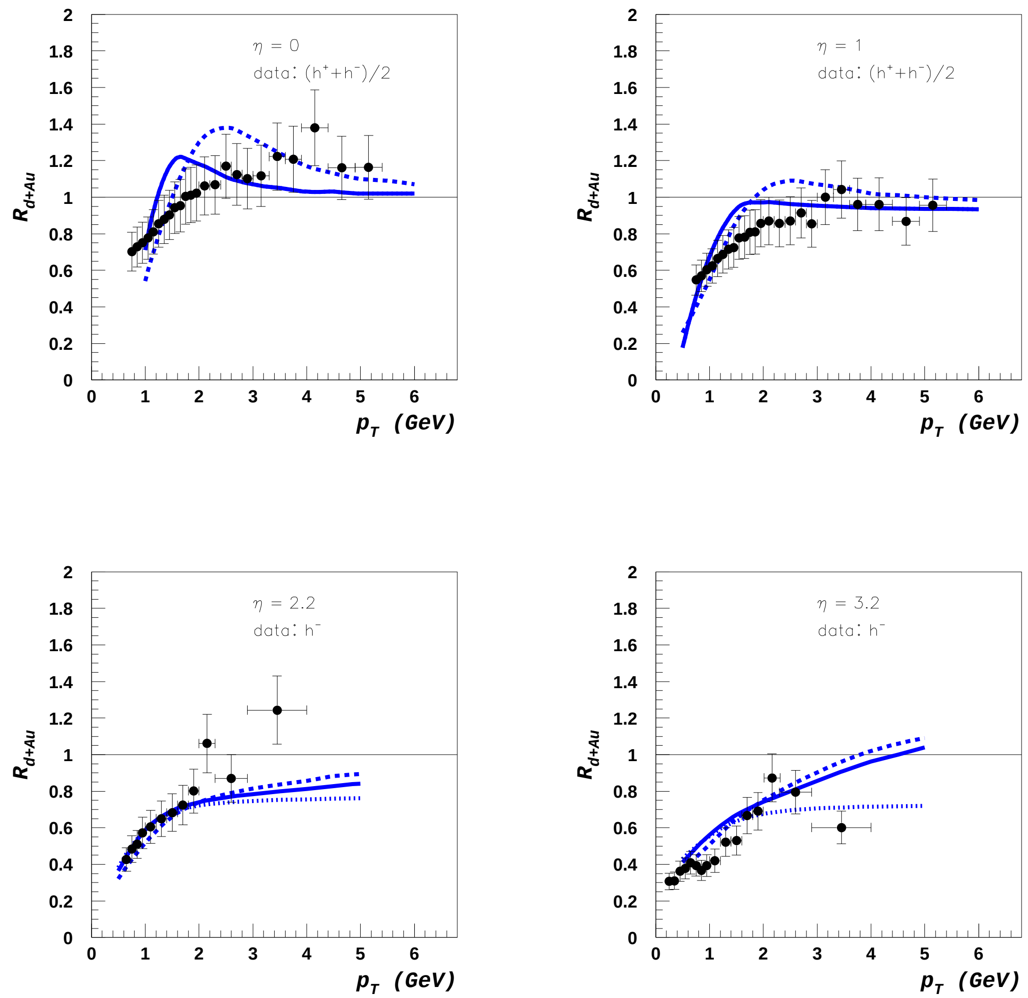




# dA collisions at RHIC



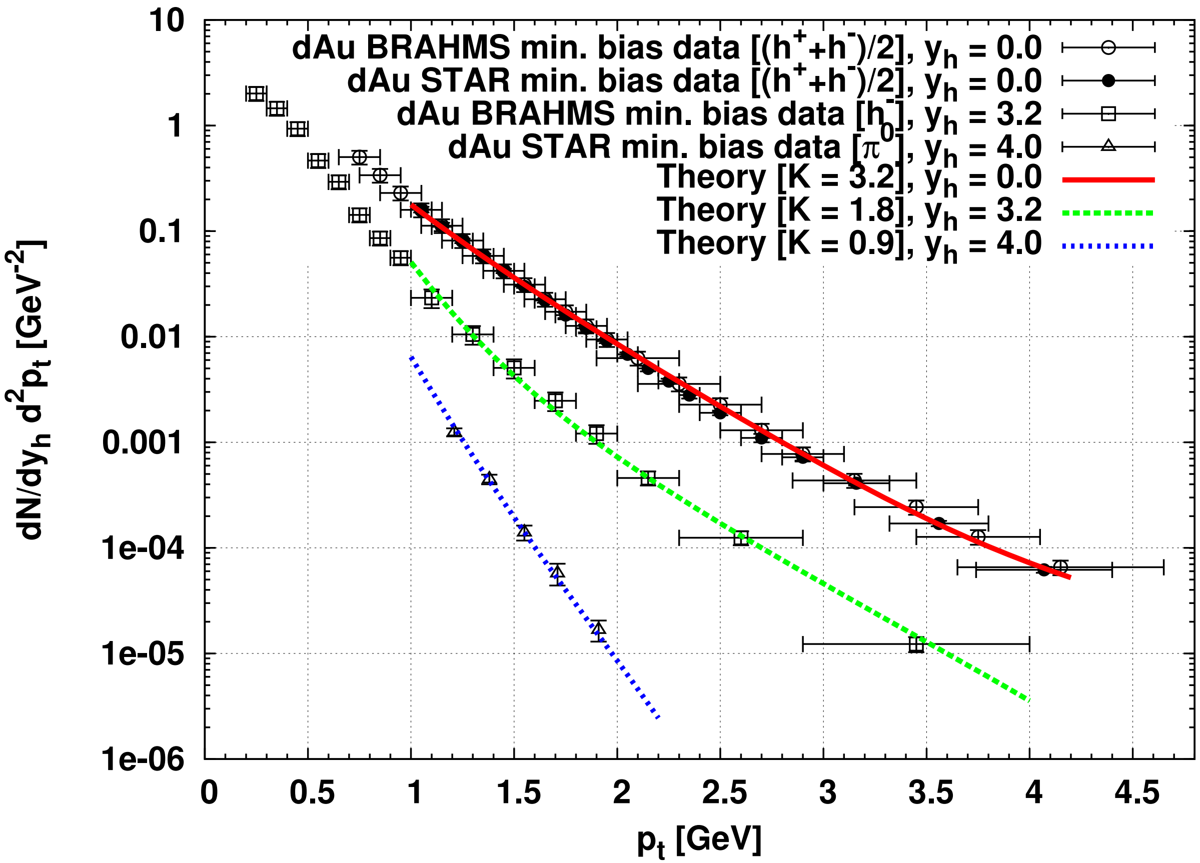
- Kharzeev, Kovchegov, Tuchin (2005)



# dA collisions at RHIC



- Dumitru, Hayashigaki, Jalilian-Marian (2005 – 2006)



# Local anisotropy



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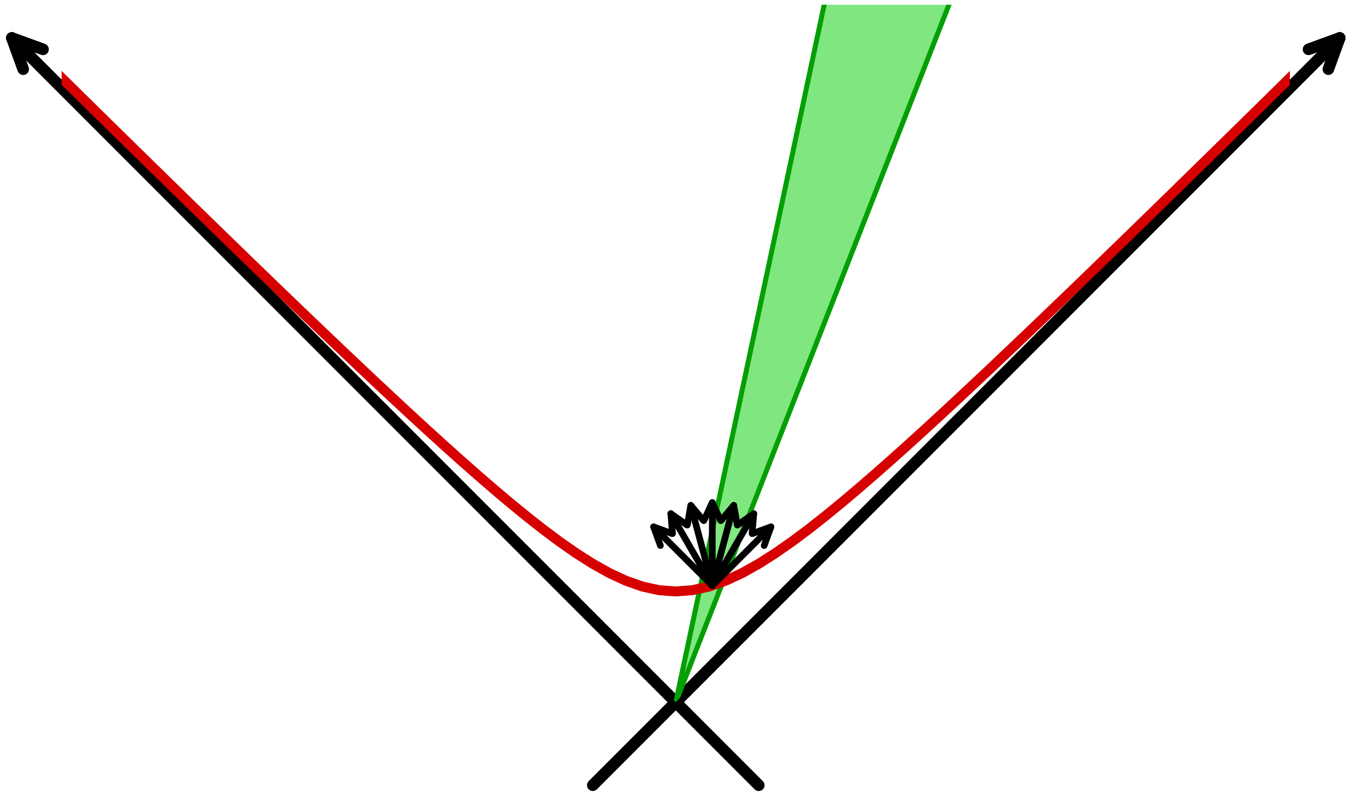

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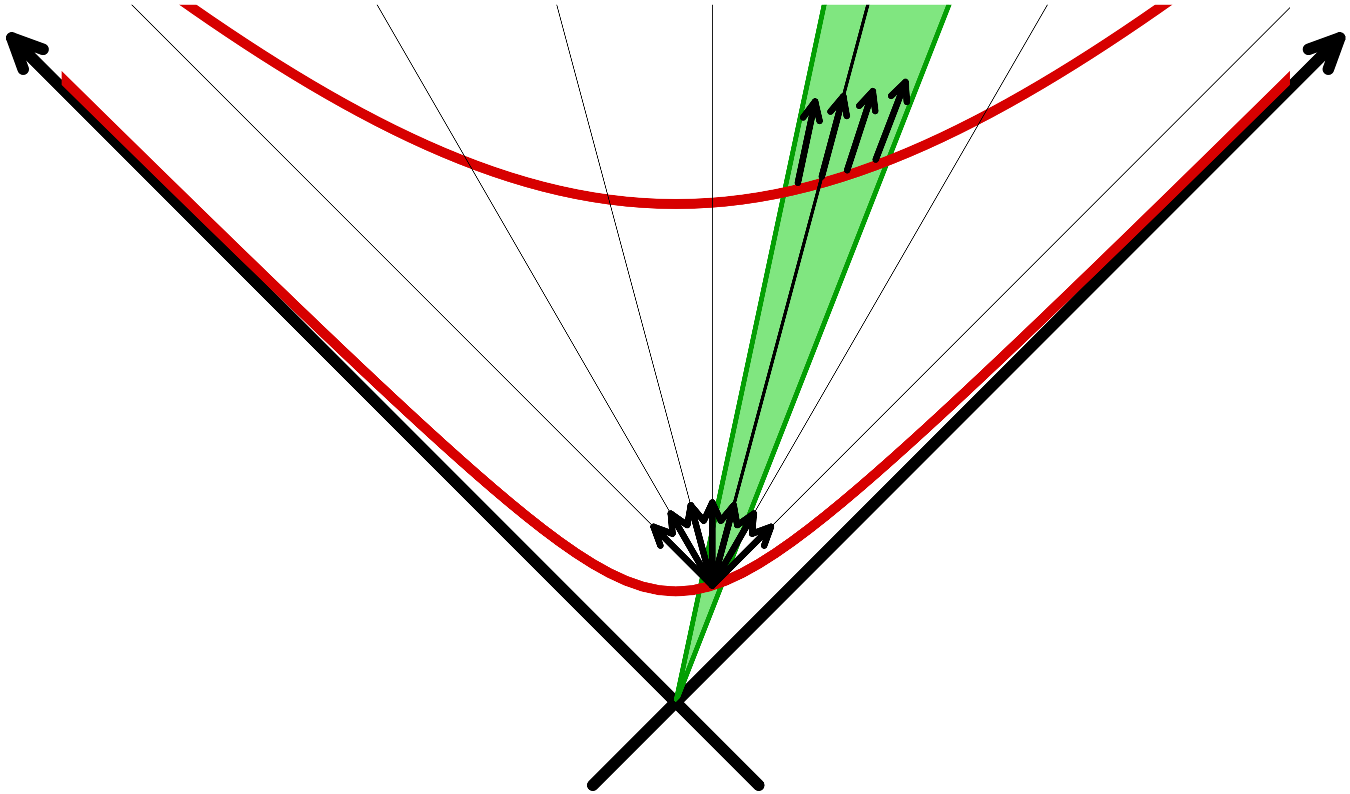

▷ at late times : if particles fly freely, only one longitudinal velocity can exist at a given $\eta$ : $v_z = \tanh(\eta)$

# Unstable modes



- The coefficient $\delta\mathcal{A}(\vec{x})$ is boost invariant, and does not trigger the instability. When summed to all orders, the contribution of the unstable modes exponentiates :

$$\left[\delta\overline{N}\right]_{\substack{\text{unstable}\\ \text{modes}}} = e^{\frac{1}{2}\int_{\vec{x},\vec{y}} \Sigma(\vec{x},\vec{y})\, \boldsymbol{T}_{\vec{x}}\boldsymbol{T}_{\vec{y}}}\ \overline{N}_{LO}[\mathcal{A}_{\rm in}(\rho_1,\rho_2)]$$

- By rewriting the Gaussian in $\boldsymbol{T}_{\vec{x}}$ as a Fourier transform :

$$\left[\delta\overline{N}\right]_{\substack{\text{unstable}\\ \text{modes}}} = \int [Da]\ \underbrace{e^{\frac{1}{2}\int_{\vec{x},\vec{y}} \frac{a(\vec{x})a(\vec{y})}{\Sigma(\vec{x},\vec{y})}}}_{\mathcal{D}_{\rm fluct}[a]}\ e^{i\int_{\vec{x}} a(\vec{x})\, \boldsymbol{T}_{\vec{x}}}\ \overline{N}_{LO}[\mathcal{A}_{\rm in}(\rho_1,\rho_2)]$$

$$= \int [Da]\ \mathcal{D}_{\rm fluct}[a]\ \overline{N}_{LO}[\mathcal{A}_{\rm in}(\rho_1,\rho_2)+a]$$

▷ summing the instabilities simply requires to add Gaussian fluctuations to the initial condition for the classical field