Photon and dilepton emission in a quark-gluon plasma

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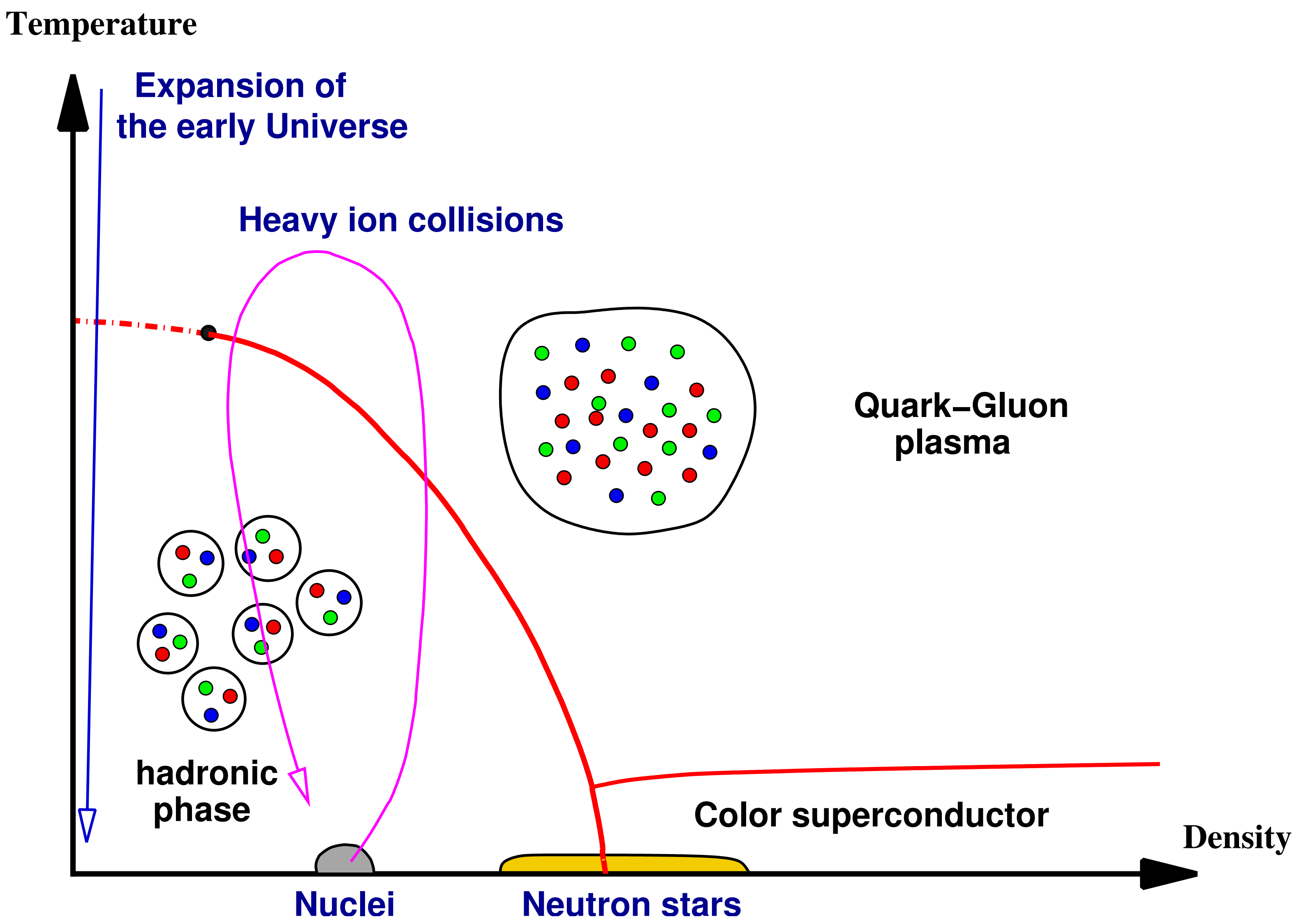
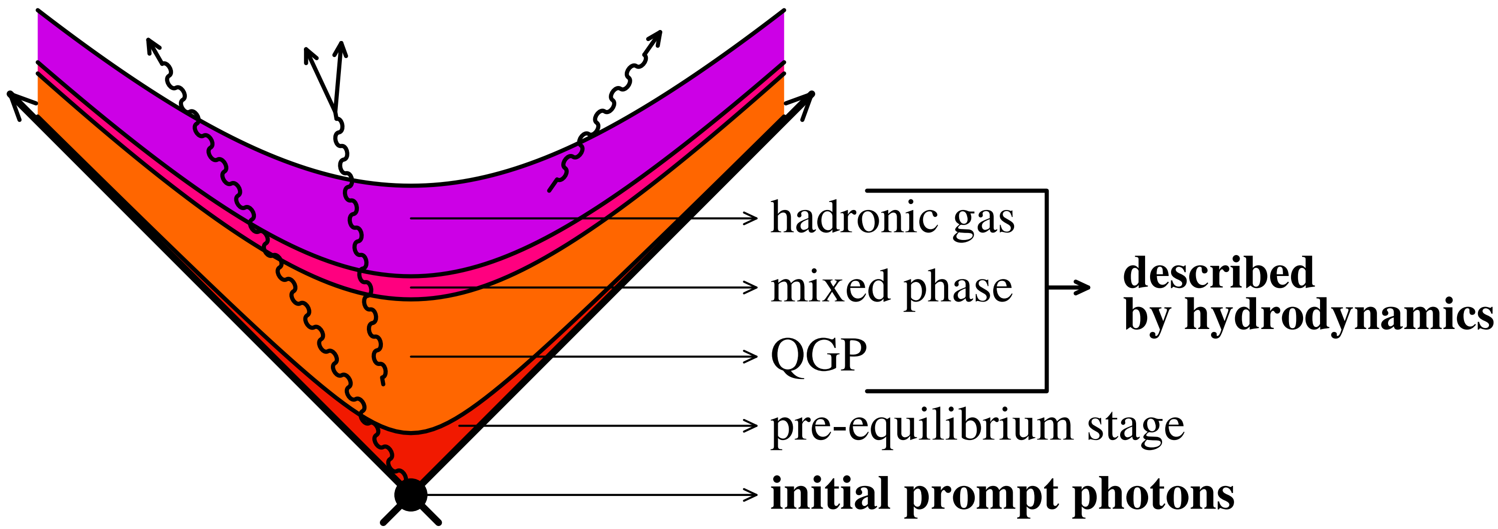
Outline

- Introduction
- How to compute a thermal rate ?
- Old calculations
  - 1-loop calculation
  - 2-loop calculation, Sum rules and exact result
- Landau-Pomeranchuk-Migdal effect
  - Resummation of multiple scatterings
  - Fast numerical resolution
- Dilepton production
- Non chemical equilibrium
- More on the Debye mass

QCD phase diagram



Sketch of a heavy ion collision



▷ Separate description of prompt photons and thermal photons

▷ Usual assumption: very few quarks in the pre-equilibrium phase + short duration → almost no photon production

▷ One needs:

- ▷ prompt photon cross-section
- ▷ rates from QGP
- ▷ rates from hadronic gas

What can we learn ?

- Photon mean free path $\gg$ system size:
  $\triangleright$ photons escape easily
  $\triangleright$ no final state interactions
- Thermal photon rates are very sensitive to the temperature:

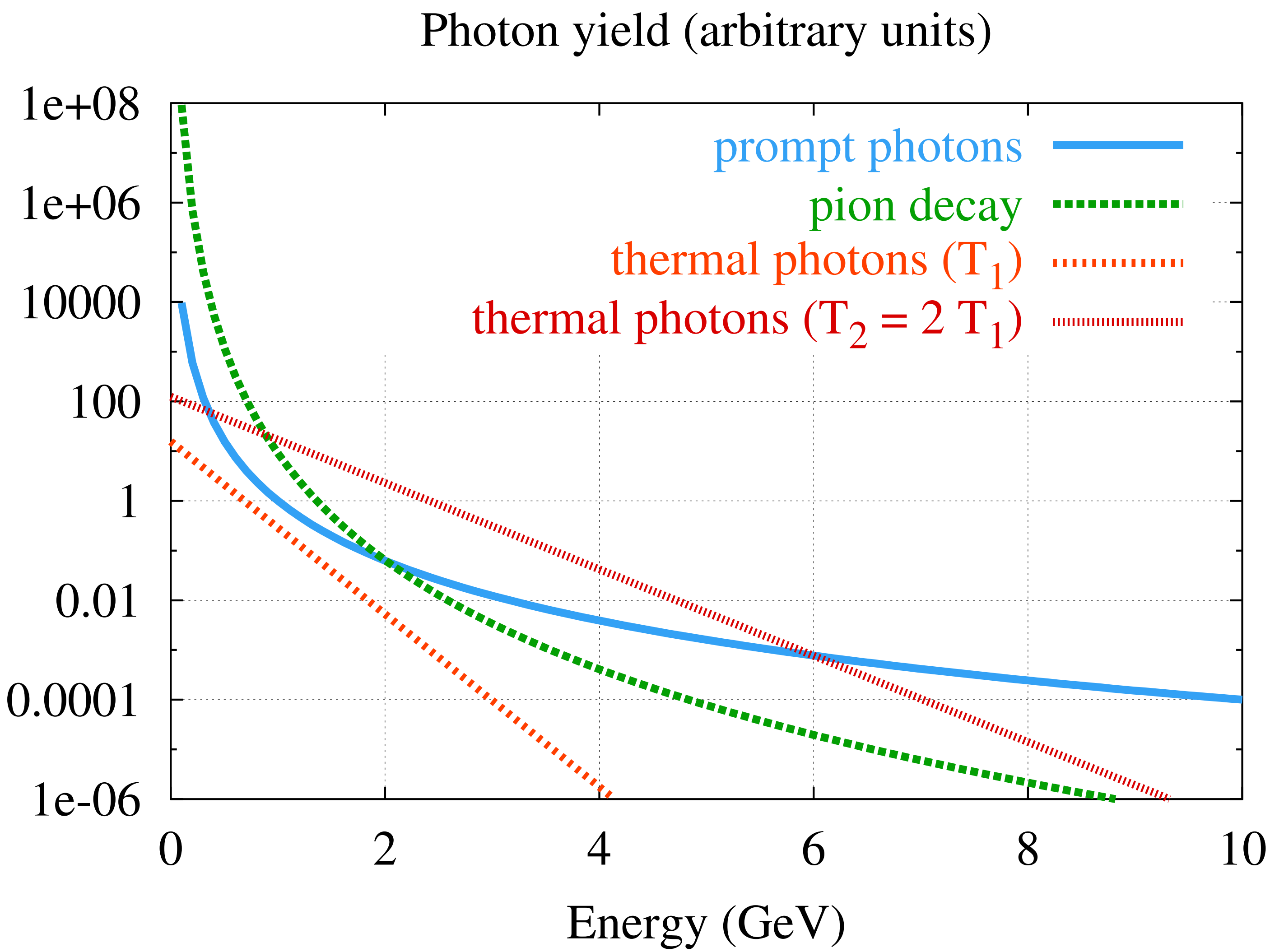
$$\frac{dN_\gamma}{dtdV} \sim T^4$$

  $\triangleright$ T drops with time $\Rightarrow$ sensitive to the early stages of the collision

- Drawback: π^0 decay is a huge source of photons
  $\triangleright$ difficult measurement

Photon spectrum

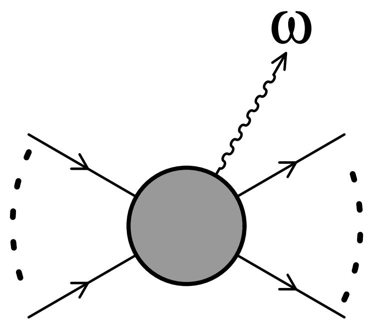
- Comparison of the various contributions



How to compute a rate ?

- Pedestrian approach:

$$\omega \frac{dN_\gamma}{dt dV d^3\vec{\boldsymbol{q}}} \propto \int_{\binom{\text{unobserved}}{\text{particles}}} \left| \quad \cdots \quad \right|^2$$



$$\times\, n(\omega_1)\cdots n(\omega_n)$$

$$\times\, (1 \pm n(\omega'_1))\cdots(1 \pm n(\omega'_p))$$

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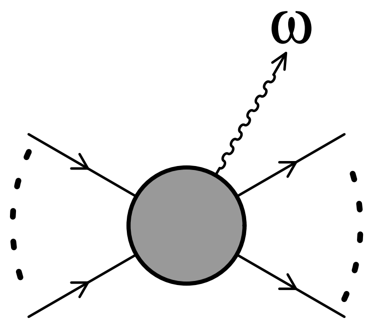


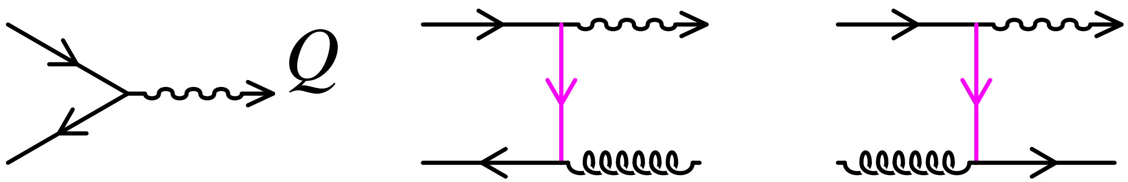
$$\times\, n(\omega_1)\cdots n(\omega_n)$$
$$\times\, (1 \pm n(\omega'_1))\cdots(1 \pm n(\omega'_p))$$

- Using Thermal Field Theory:
  Weldon (1983) - Gale, Kapusta (1991)

$$\omega \frac{dN_\gamma}{dt dV d^3\vec{\boldsymbol{q}}} \propto \frac{1}{e^{\omega/T} - 1}\ \mathrm{Im}\, {\Pi_{\rm ret}}^{\mu}{}_{\mu}(\omega, \vec{\boldsymbol{q}})$$

Early calculations

McLerran, Toimela (1985) - Baier, Pire, Schiff (1988)
Altherr, Aurenche, Becherrawy (1989)



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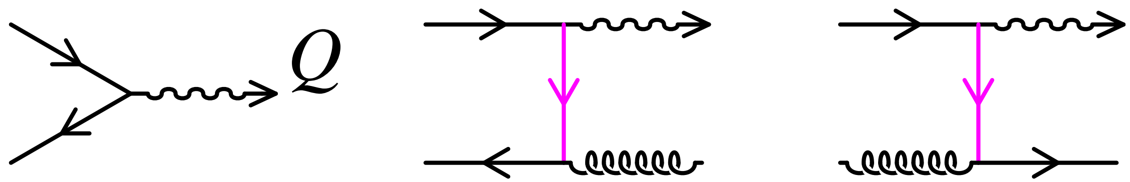


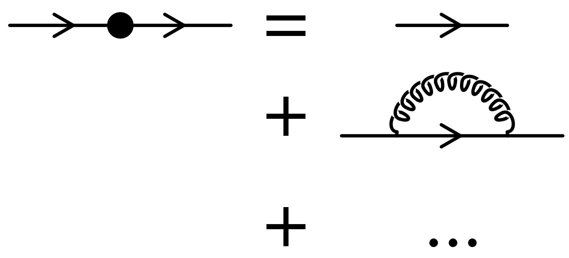
▷ For real photons ($Q^2 = 0$), infrared divergence when the exchanged quark is massless:

$$\mathrm{Im}\,\Pi_{\mathrm{ret}}(\omega, \vec{\boldsymbol{q}}) \propto \alpha\alpha_s \ln(\omega T/Q^2)$$

Hard Thermal Loops (1/2)

Braaten, Pisarski (1990) - Frenkel, Taylor (1990)

- In-medium quarks



Medium effects give a mass to quasi-particles:
$m_q^2 = \frac{4\pi}{3}\alpha_s T^2$

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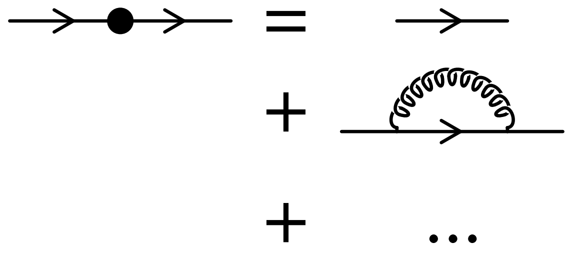

Medium effects give a mass to quasi-particles:
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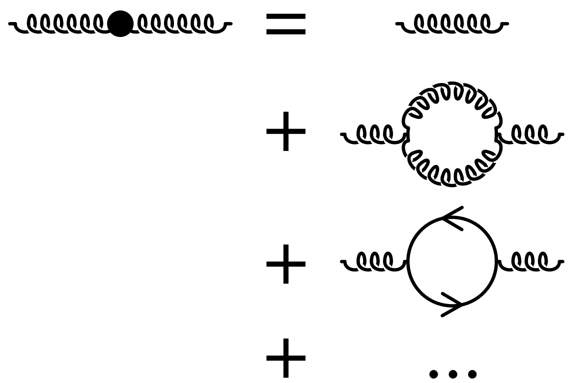
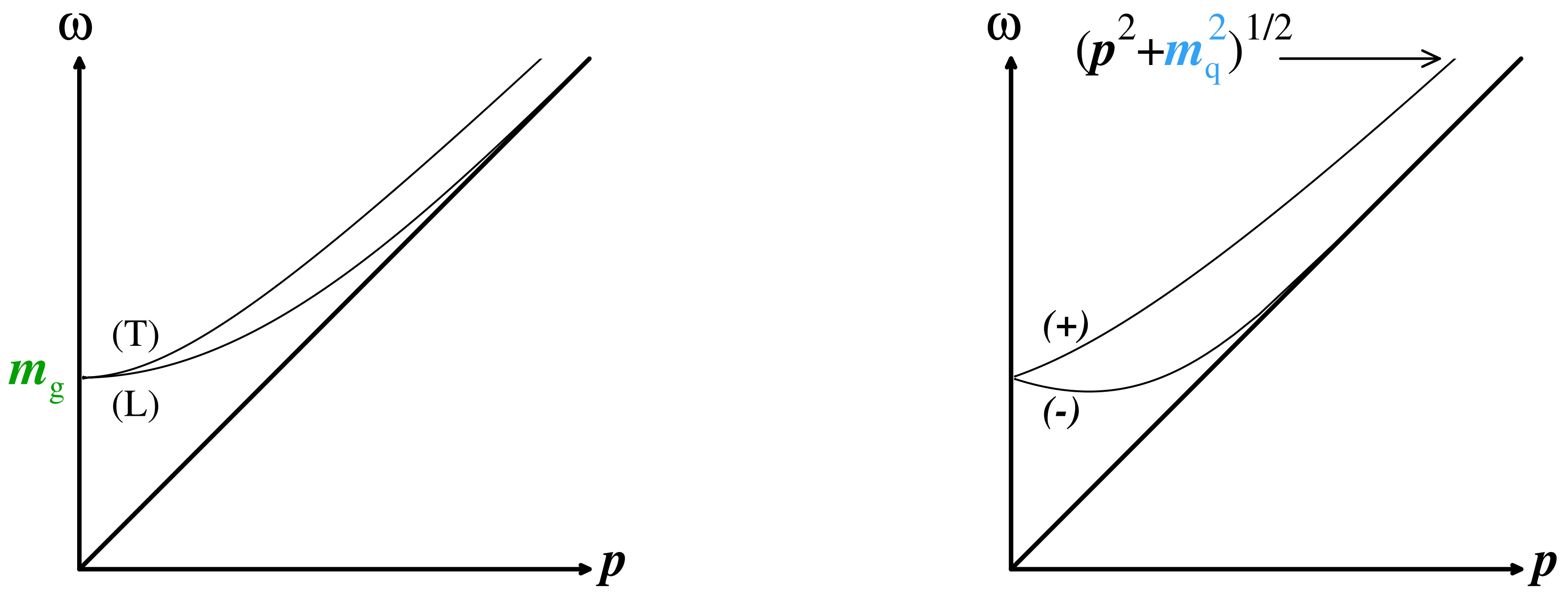
- In-medium gluons



$m_{\rm g}^2 = \frac{4\pi}{3}\alpha_s(1+\frac{N_f}{6})T^2$
(N_f = # of quark flavors)

Hard Thermal Loops (2/2)

- In-medium dispersion relations



Hard Thermal Loops (2/2)

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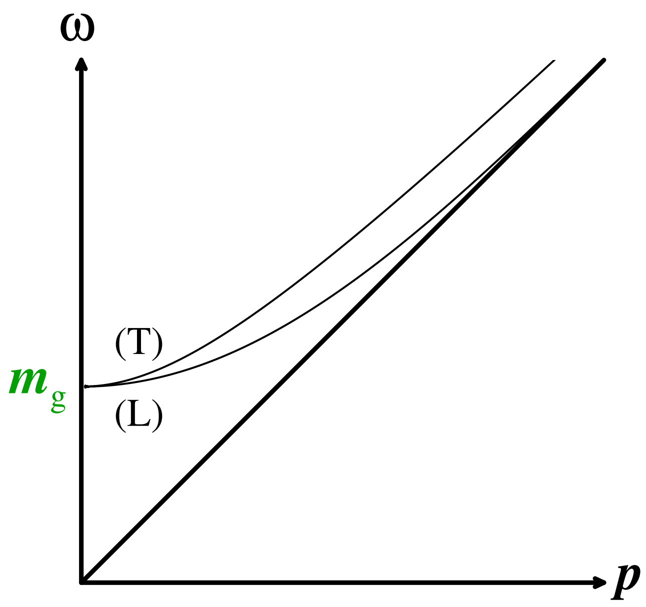

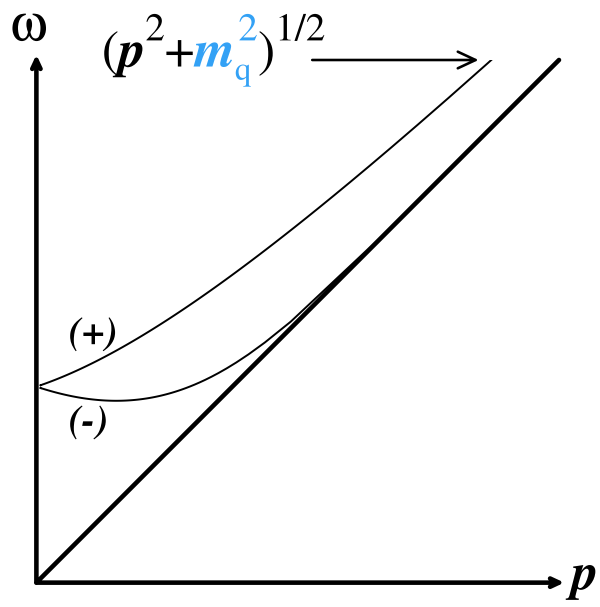

- Debye screening:

$$\lim_{|\vec{\boldsymbol{p}}|\to 0}\Pi_L(0,\vec{\boldsymbol{p}}) = 3m_g^2\ ,\ \ \lim_{|\vec{\boldsymbol{p}}|\to 0}\Pi_T(0,\vec{\boldsymbol{p}}) = 0$$

Hard Thermal Loops (2/2)

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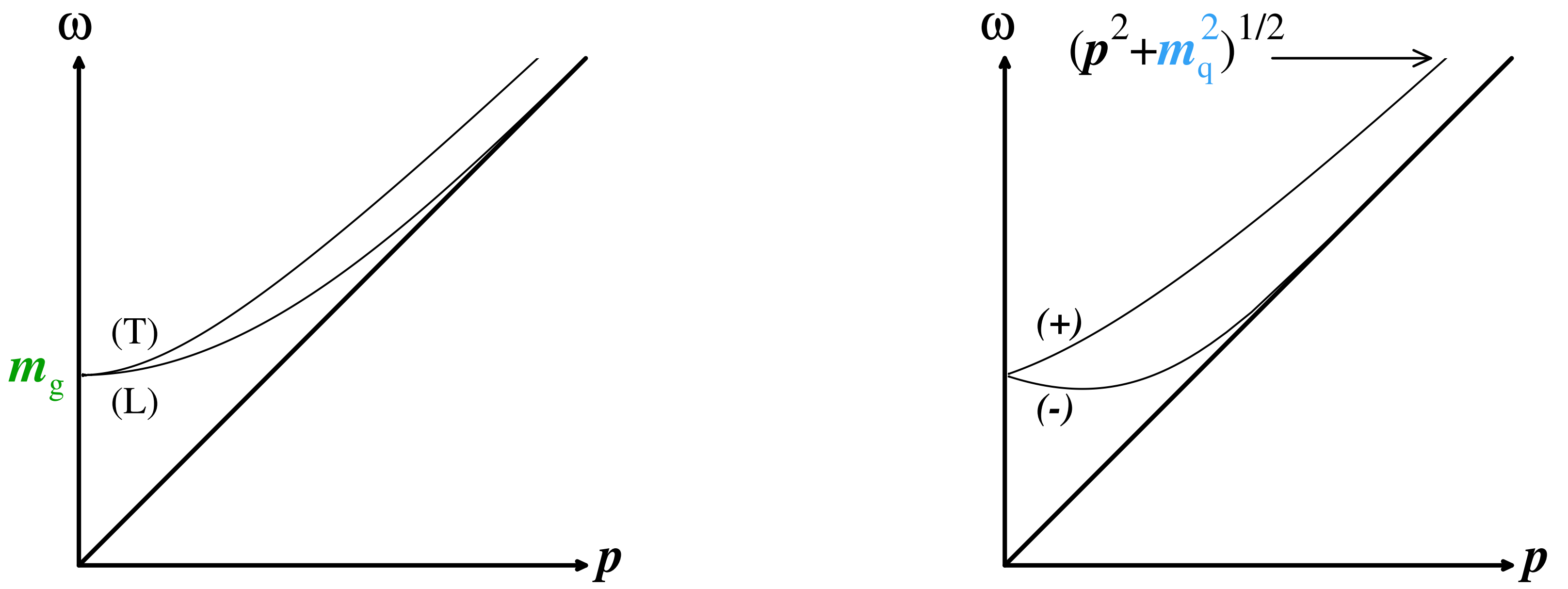
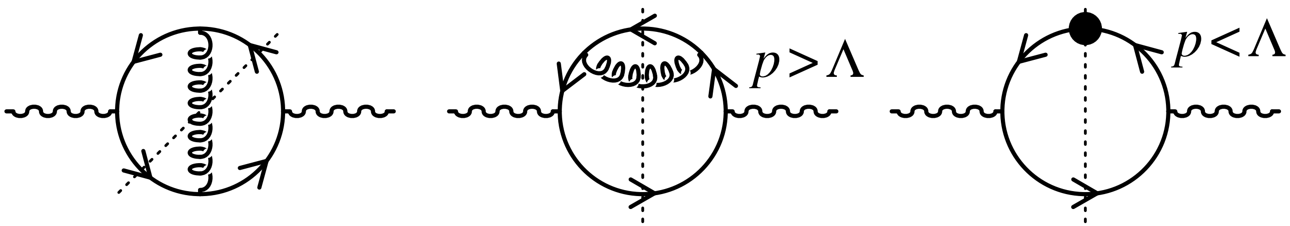


- Debye screening:
  $\lim_{|\vec{p}|\to 0} \Pi_L(0,\vec{p}) = 3m_g^2$, $\lim_{|\vec{p}|\to 0} \Pi_T(0,\vec{p}) = 0$
- Landau damping: $\mathrm{Im}\,\Pi(\omega,\vec{p}) \neq 0 \quad \text{if } |\omega| < |\vec{p}|$

1-loop resummed result

- Diagrams



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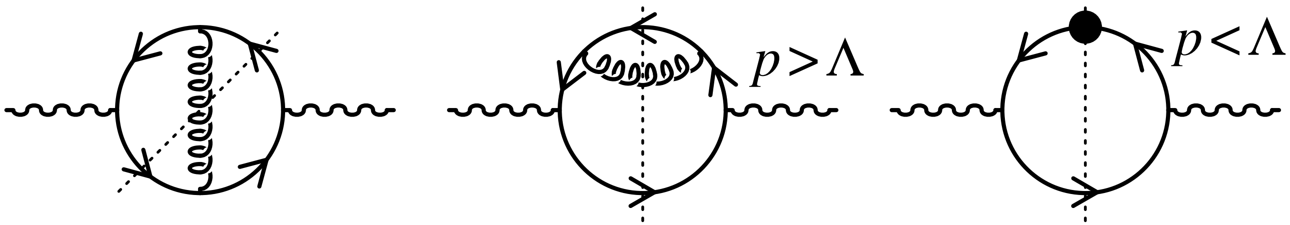


- m_q screens the infrared divergence

Kapusta, Lichard, Seibert (1991)

Baier, Nakkagawa, Niegawa, Redlich (1992)

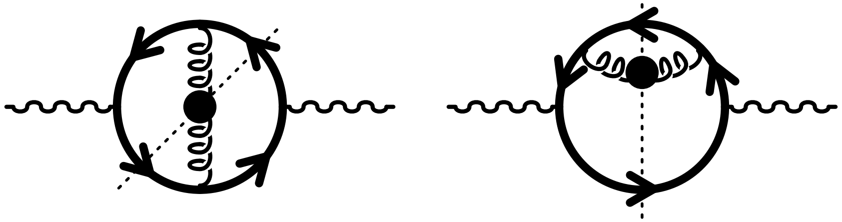
$$\mathrm{Im}\,\Pi_{\mathrm{ret}}{}^{\mu}{}_{\mu}(\omega, \vec{\boldsymbol{q}}) =$$

$$= 4\pi \frac{5\alpha\alpha_s}{9} T^2 \left[\ln\left(\frac{\omega T}{m_q^2}\right) - \frac{1}{2} - \gamma_E + \frac{7}{3}\ln(2) + \frac{\zeta'(2)}{\zeta(2)} \right]$$

(for 3 flavors: $5/9 \to 6/9$)

2-loop resummed result (1/5)

- Diagrams (the gluon is space-like)



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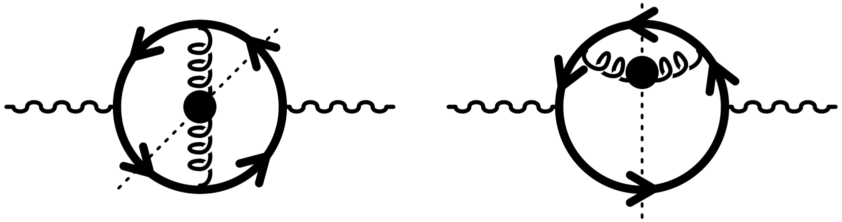
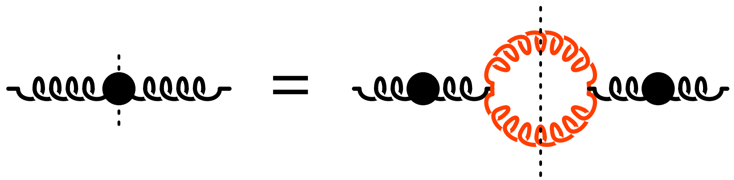


- Space-like gluon:



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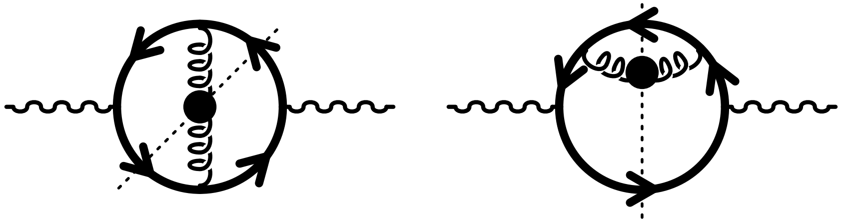
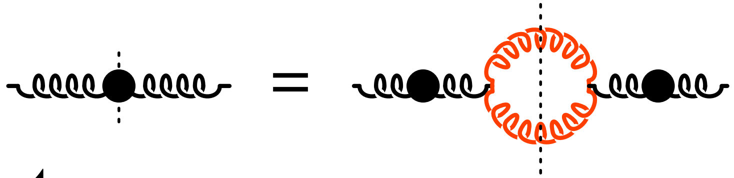
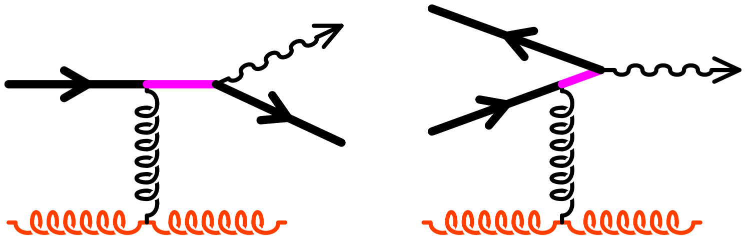


- Space-like gluon: 
- Collinear enhancement

  Aurenche, FG, Kobes, Petitgirard (1996)

  Aurenche, FG, Kobes, Zaraket (1998)



singularity when a real photon is emitted forward: $\alpha_s^2 \frac{T^2}{m_q^2} \sim \alpha_s$

2-loop resummed result (2/5)

- Result:

$$\mathrm{Im}\,\Pi_{\mathrm{ret}}{}^{\mu}{}_{\mu}(\omega,\vec{\boldsymbol{q}}) =$$

$$= \frac{64\alpha\alpha_s}{3\pi^2}\Big(\sum_f e_f^2\Big)(J_{_T}+J_{_L}+K_{_T}+K_{_L})\left[\pi^2\frac{T^3}{\omega}+\omega T\right]$$

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- The $J_{T,L}$, $K_{T,L}$ are numerical constants that depend only on $m_{\mathrm{q}}^2/m_{\mathrm{g}}^2$

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- The $J_{_{T,L}}$, $K_{_{T,L}}$ are numerical constants that depend only on $m_{\mathrm{q}}^2/m_{\mathrm{g}}^2$
- Term in T^3/ω: mainly from $qS \longrightarrow q\gamma S$ (S = scattering center from the medium)
- Term in ωT: mainly from $q\bar{q}S \longrightarrow \gamma S$

2-loop resummed result (3/5)

$$J_{T,L} \equiv \int_{-\infty}^{0} dt\, \mathcal{C}_{T,L}(t) \sqrt{\frac{-t}{4m_q^2 - t}} \tanh^{-1} \sqrt{\frac{-t}{4m_q^2 - t}}$$

$$K_{T,L} \equiv 2m_q^2 \int_{-\infty}^{0} \frac{dt}{t} \mathcal{C}_{T,L}(t) \left[\sqrt{\frac{-t}{4m_q^2 - t}} \tanh^{-1} \sqrt{\frac{-t}{4m_q^2 - t}} + \frac{t}{4m_q^2} \right]$$

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$$\mathcal{C}_{T,L}(t) \equiv \int_0^1 \frac{dx}{x} \frac{\left|\mathrm{Im}\,\Pi_{T,L}(x)\right|}{(t - \mathrm{Re}\,\Pi_{T,L}(x))^2 + (\mathrm{Im}\,\Pi_{T,L}(x))^2}$$

$$\Pi_T(x) = 3m_g^2 \left[\frac{x^2}{2} + \frac{x(1-x^2)}{4} \ln\left(\frac{x+1}{x-1}\right)\right]$$

$$\Pi_L(x) = 3m_g^2 (1-x^2) \left[1 - \frac{x}{2} \ln\left(\frac{x+1}{x-1}\right)\right]$$

Sum rule (1/4)

Aurenche, FG, Zaraket (2002)

- If:

$$\begin{aligned} \mathrm{Im}\,\Pi(0) &= 0\;, \\ \mathrm{Im}\,\Pi(x) &= 0 \quad \text{if } x \geq 1\;, \\ \mathrm{Re}\,\Pi(x) &\geq 0 \quad \text{if } x \geq 1\;, \end{aligned}$$

then

$$\mathcal{C}(t) \equiv \int_0^1 \frac{dx}{x} \frac{\mathrm{Im}\,\Pi(x)}{(t-\mathrm{Re}\,\Pi(x))^2 + (\mathrm{Im}\,\Pi(x))^2} = \\ = \frac{\pi}{2}\left[\frac{1}{t-\mathrm{Re}\,\Pi(0)} - \frac{1}{t-\mathrm{Re}\,\Pi(\infty)}\right]$$

Sum rule (2/4)

- Spectral representation of a propagator:

$$\frac{1}{p_0^2 - \boldsymbol{p}^2 - \Sigma(p_0, \boldsymbol{p})} = \int_0^\infty \frac{d\omega}{\pi} \omega \frac{\rho(\omega, \boldsymbol{p})}{p_0^2 - \omega^2 + i\epsilon}$$

with

$$\rho(\omega, \boldsymbol{p}) = \frac{-2\,\mathrm{Im}\,\Sigma(\omega, \boldsymbol{p})}{(\omega^2 - \boldsymbol{p}^2 - \mathrm{Re}\,\Sigma(\omega, \boldsymbol{p}))^2 + (\mathrm{Im}\,\Sigma(\omega, \boldsymbol{p}))^2}$$

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- Taking the real part gives:

$$\int_0^\infty \frac{d\omega}{\pi} \omega \frac{\rho(\omega, \boldsymbol{p})}{p_0^2 - \omega^2} = \frac{p_0^2 - \boldsymbol{p}^2 - \mathrm{Re}\,\Sigma(p_0, \boldsymbol{p})}{(p_0^2 - \boldsymbol{p}^2 - \mathrm{Re}\,\Sigma(p_0, \boldsymbol{p}))^2 + (\mathrm{Im}\,\Sigma(p_0, \boldsymbol{p}))^2}$$

Sum rule (3/4)

- $p_0 = 0$: if $\mathrm{Im}\,\Sigma(0, \boldsymbol{p}) = 0$,

$$\int_0^\infty \frac{d\omega}{\omega} \rho(\omega, \boldsymbol{p}) = \frac{\pi}{\boldsymbol{p}^2 + \mathrm{Re}\,\Sigma(0, \boldsymbol{p})}$$

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- $p_0 \to \infty$: if $\mathrm{Im}\,\Sigma(p_0, \boldsymbol{p}) = 0$ for $p_0 > |\boldsymbol{p}|$,

$$\int_0^\infty d\omega\, \omega \rho(\omega, \boldsymbol{p}) = \lim_{p_0 \to \infty} \frac{\pi p_0^2}{p_0^2 - \mathrm{Re}\,\Sigma(p_0, \boldsymbol{p})}$$

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$$\int_0^\infty d\omega\,\omega\rho(\omega,\boldsymbol{p}) = \lim_{p_0\to\infty} \frac{\pi p_0^2}{p_0^2 - \mathrm{Re}\,\Sigma(p_0,\boldsymbol{p})}$$

- Contribution of the time-like region:

$$\int_{|\boldsymbol{p}|}^\infty \frac{d\omega}{\omega} f(\omega)\rho(\omega,\boldsymbol{p}) = \pi \sum_{\text{poles }\omega_i} Z(\omega_i)\frac{f(\omega_i)}{\omega_i^2}$$

Sum rule (4/4)

- Then:

$$\mathcal{C}(t) = -\frac{1}{2t}\int_0^{|\boldsymbol{p}|}\frac{d\omega}{\omega}(\omega^2 - \boldsymbol{p}^2)\rho(\omega, \boldsymbol{p})$$

with $\Sigma(p_0, \boldsymbol{p}) \equiv t^{-1}(p_0^2 - \boldsymbol{p}^2)\Pi(p_0/|\boldsymbol{p}|)$

Sum rule (4/4)

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- The propagator of self-energy Σ has no pole with $\omega > |\boldsymbol{p}|$ (here we need $\mathrm{Re}\,\Sigma(\omega,\boldsymbol{p}) \geq 0$ if $\omega > |\boldsymbol{p}|$)

Sum rule (4/4)

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- The propagator of self-energy Σ has no pole with $\omega > |\boldsymbol{p}|$ (here we need $\mathrm{Re}\,\Sigma(\omega,\boldsymbol{p}) \ge 0$ if $\omega > |\boldsymbol{p}|$)
- Finally:

$$\mathcal{C}(t) = \frac{\pi}{2}\left[\frac{1}{t - \mathrm{Re}\,\Pi(0)} - \frac{1}{t - \mathrm{Re}\,\Pi(\infty)}\right]$$

- All the conditions are satisfied by $\Pi_{T,L}(x)$

2-loop resummed result (4/5)

- The sum rule implies:

$$\mathcal{C}_T(t) = \frac{\pi}{2}\left[\frac{1}{t-m_g^2} - \frac{1}{t}\right] \; , \; \mathcal{C}_L(t) = \frac{\pi}{2}\left[\frac{1}{t-3m_g^2} - \frac{1}{t-m_g^2}\right]$$

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- $$J_T + J_L = \pi F(4m_{\rm q}^2/3m_{\rm g}^2)$$
  $$K_T + K_L = \pi\left[\frac{1}{2} + \frac{1}{4}\ln\left(\frac{m_{\rm q}^2}{3m_{\rm g}^2}\right) - \frac{2m_{\rm q}^2}{3m_{\rm g}^2}F(4m_{\rm q}^2/3m_{\rm g}^2)\right]$$
  with $F(x) \equiv \int_0^1 du\, \frac{\tanh^{-1}(u)}{1+(x-1)u^2}$

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- $$J_{T} + J_{L} = \pi F(4m_{\rm q}^2/3m_{\rm g}^2)$$
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  with $F(x) \equiv \int_0^1 du\, \frac{\tanh^{-1}(u)}{1+(x-1)u^2}$

- For 3 colors and 2 flavors ($m_{\rm q}^2/m_{\rm g}^2 = 3/4$):

$$\mathrm{Im}\,\Pi_{\rm ret}{}^{\mu}{}_{\mu}(\omega,\vec{\boldsymbol{q}}) = \frac{32}{3\pi}\frac{5\alpha\alpha_s}{9}\left[\pi^2\frac{T^3}{\omega} + \omega T\right]$$

2-loop resummed result (5/5)

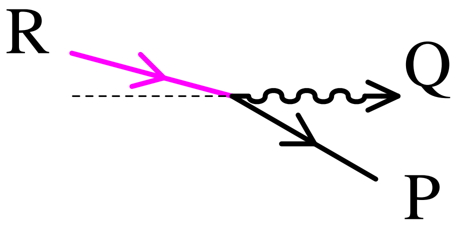
- For 3 colors and 3 flavors ($m_{\rm q}^2/m_{\rm g}^2 = 2/3$):

$$\mathrm{Im}\,\Pi_{\rm ret}{}^{\mu}{}_{\mu}(\omega,\vec{\boldsymbol{q}}) = \frac{32}{3\pi}\left[1+\frac{5\pi^2}{36}+\ln\left(\frac{\sqrt{2}}{3}\right)-\frac{55}{12}\ln^2(2)\right.$$
$$\left.+\frac{10}{3}\ln(2)\ln(3)-\frac{5}{3}\mathrm{Li}_2\left(\frac{3}{4}\right)-\frac{5}{3}\mathrm{Li}_2\left(-\frac{1}{2}\right)\right]$$
$$\times\frac{6\alpha\alpha_s}{9}\left[\pi^2\frac{T^3}{\omega}+\omega T\right]$$

with $\mathrm{Li}_2(z)\equiv\sum_{n=1}^{+\infty} z^n/n^2$.

LPM effect (1/6)

- Photon formation time

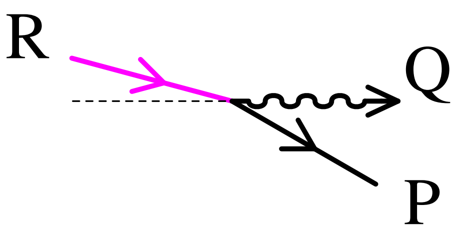


$$R \equiv P + Q$$
$$(r_0 = p_0 + \omega)$$
$$P^2 = m_{\rm q}^2$$

$$t_{_F}^{-1} \sim \delta E = r_0 - \sqrt{\vec{\boldsymbol{r}}^2 + m_{\rm q}^2}$$

LPM effect (1/6)

- Photon formation time



$$R \equiv P + Q$$
$$(r_0 = p_0 + \omega)$$
$$P^2 = m_{\rm q}^2$$

$$t_{_F}^{-1} \sim \delta E = r_0 - \sqrt{\vec{\boldsymbol{r}}^2 + m_{\rm q}^2}$$

$$\boxed{t_{_F}^{-1} = \frac{\omega}{2p_0 r_0}\left[\vec{\boldsymbol{p}}_\perp^2 + m_{\rm q}^2\right]}$$

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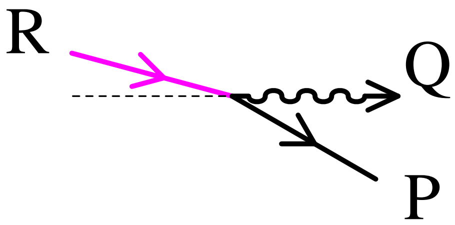

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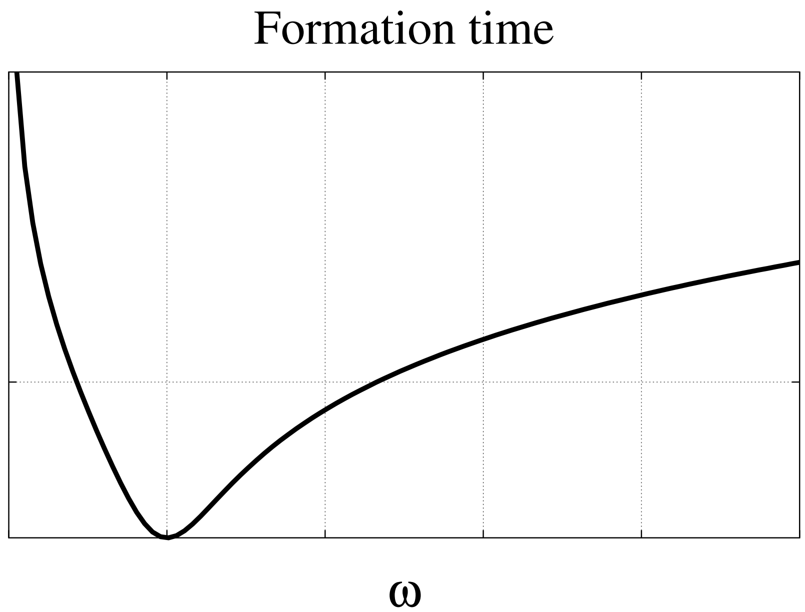

LPM effect (2/6)

- Other scales
  - ▷ Mean free path: $\lambda \sim (\alpha_s T \ln(1/\alpha_s))^{-1}$
  - ▷ Electric screening: $\ell_{\rm elec} \sim (\sqrt{\alpha_s} T)^{-1}$
  - ▷ Magnetic screening: $\ell_{\rm mag} \sim (\alpha_s T)^{-1}$

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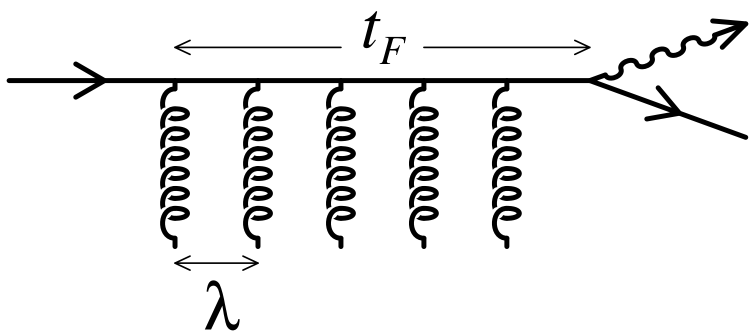
- LPM effect: multiple scatterings are important if

$$\boxed{t_F \gtrsim \lambda}$$



LPM effect (3/6)

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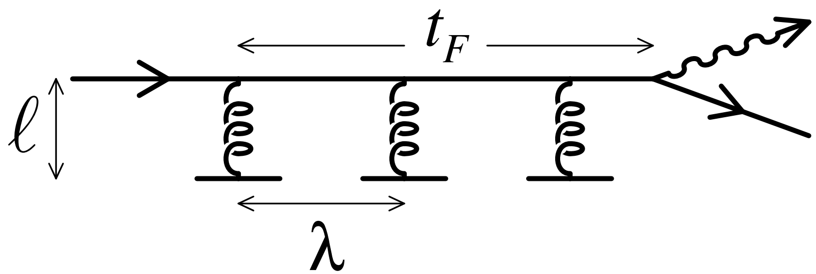

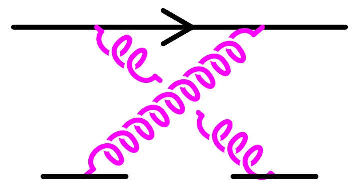

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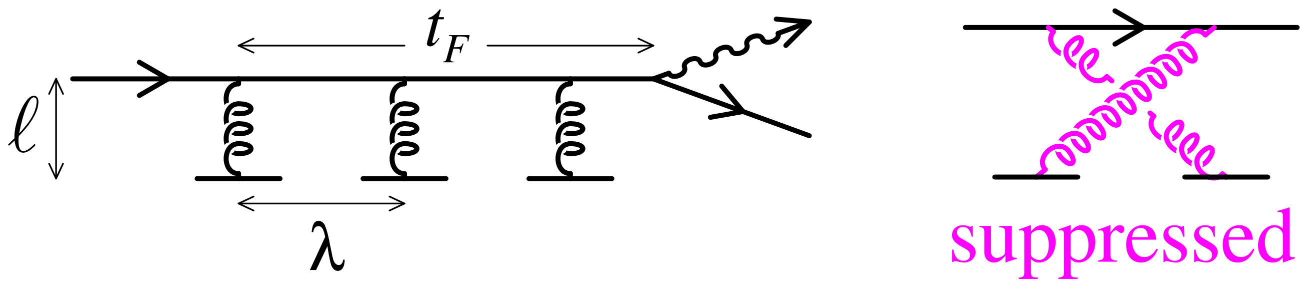
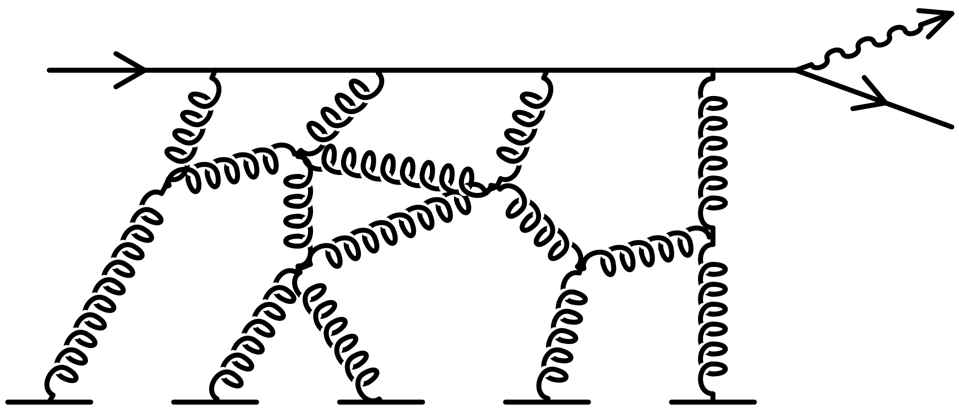


  ▷ *Long ranged interactions:* $\ell \gtrsim \lambda$



LPM effect (4/6)

Arnold, Moore, Yaffe (2001,2002)

- A cancellation between vertex and self-energy topologies prevents long ranged interactions
  ▷ Only ladder topologies contribute to $\Pi_{\rm ret}$

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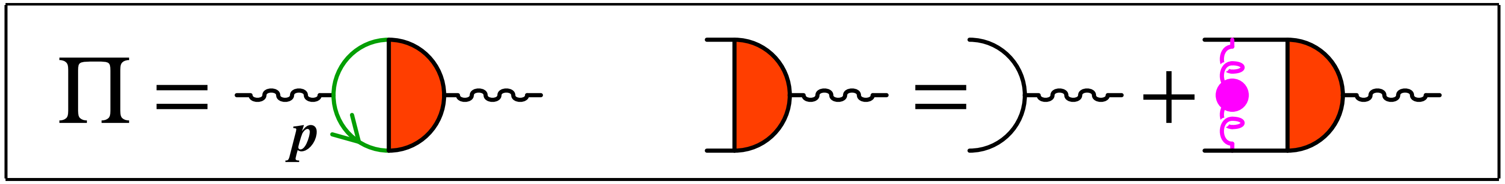
- A cancellation between vertex and self-energy topologies prevents long ranged interactions
  ▷ Only ladder topologies contribute to $\Pi_{\rm ret}$
  ▷ Interpretation: long ranged interactions correspond to very small momentum transfers:

$$l_\perp \sim \ell^{-1} \; ,$$

  which are inefficient to induce the emission of a photon.

LPM effect (5/6)

- Resummation of ladder diagrams:



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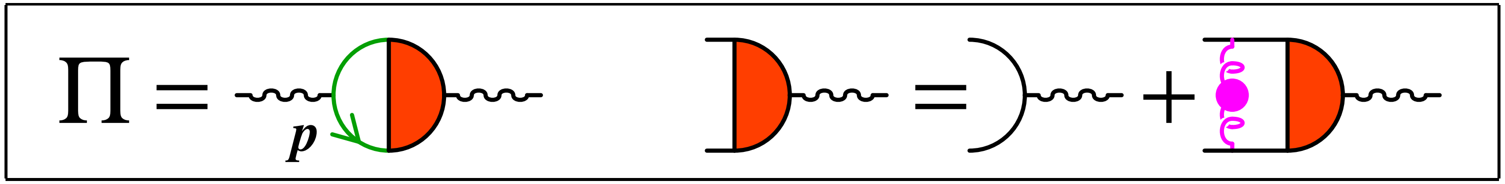

▷ Dyson equation:

$$\mathrm{Im}\,\Pi_{\mathrm{ret}\,\mu}^{\;\mu} = \alpha\;\mathrm{Im}\int dp_0\, d^2\vec{\boldsymbol{p}}_\perp\,[\cdots]\; 2\vec{\boldsymbol{p}}_\perp\cdot\vec{\boldsymbol{f}}(\vec{\boldsymbol{p}}_\perp)$$

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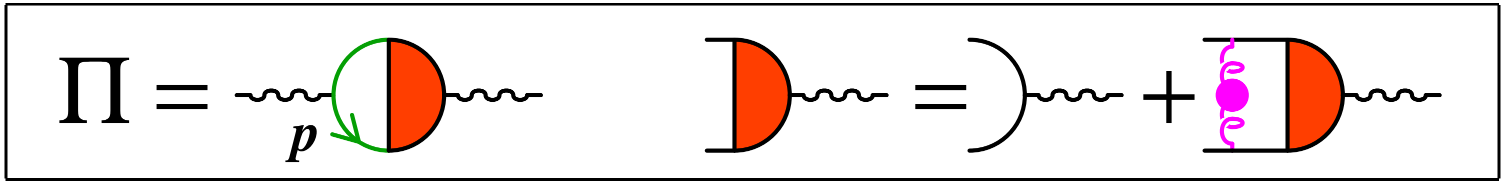

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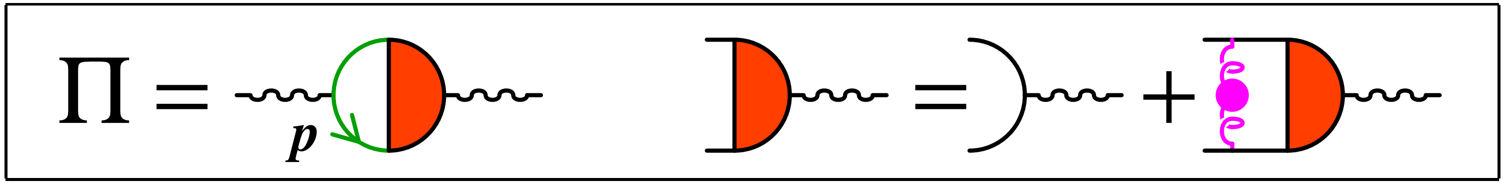

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- **Note:** the term in $\mathcal{C}(-\boldsymbol{l}_\perp^2)\vec{\boldsymbol{f}}(\vec{\boldsymbol{p}}_\perp)$ is due to self-energy resummations required by gauge invariance

LPM effect (6/6)

- The collision kernel is:

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$\triangleright$ the LPM-resummed result depends only on $m_{\rm q}^2$ and $m_{\rm debye}^2$ $(= 3m_{\rm g}^2$ in the HTL framework)

Numerical resolution (1/3)

Arnold, Moore, Yaffe (2001)

- Variational method: the solution is an extremum of:

$$Q[\boldsymbol{f}] = \left\langle 2\boldsymbol{p}_\perp, \vec{\boldsymbol{f}}(\vec{\boldsymbol{p}}_\perp)\right\rangle - \frac{1}{2}\left\langle \vec{\boldsymbol{f}}(\vec{\boldsymbol{p}}_\perp), \widehat{\mathcal{O}}\vec{\boldsymbol{f}}(\boldsymbol{p}_\perp)\right\rangle$$

with: $\left\langle \vec{\boldsymbol{f}}_1(\vec{\boldsymbol{p}}_\perp), \vec{\boldsymbol{f}}_2(\vec{\boldsymbol{p}}_\perp)\right\rangle \equiv \int_{\vec{\boldsymbol{p}}_\perp} \vec{\boldsymbol{f}}_1(\vec{\boldsymbol{p}}_\perp) \cdot \vec{\boldsymbol{f}}_2(\vec{\boldsymbol{p}}_\perp)$

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- Problem: the extremum is a saddle point

  ▷ the accuracy is not guaranteed to increase when one enlarges the variational ansatz

Numerical resolution (2/3)

Aurenche, FG, Zaraket (2002)

- Fourier transform the integral equation:

$$\frac{\omega}{2p_0r_0}[m_{\rm q}^2 - \boldsymbol{\nabla}_\perp^2]\widetilde{\boldsymbol{f}}(\vec{\boldsymbol{x}}_\perp) = 2\vec{\boldsymbol{\nabla}}_\perp\delta(\vec{\boldsymbol{x}}_\perp) + i\alpha_s T D(x_\perp)\widetilde{\boldsymbol{f}}(\vec{\boldsymbol{x}}_\perp)$$

$$D(x_\perp) \equiv \frac{1}{2\pi}\left[\gamma_{_E} + \ln\left(\frac{m_{\rm debye}x_\perp}{2}\right) + K_0(m_{\rm debye}x_\perp)\right]$$

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- At the end, we need: $\mathrm{Im}\int_{\boldsymbol{p}_\perp}\vec{\boldsymbol{p}}_\perp\cdot\vec{\boldsymbol{f}}(\vec{\boldsymbol{p}}_\perp)=2\,\mathrm{Im}\,h(0^+)$

Numerical resolution (3/3)

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- Advantages:

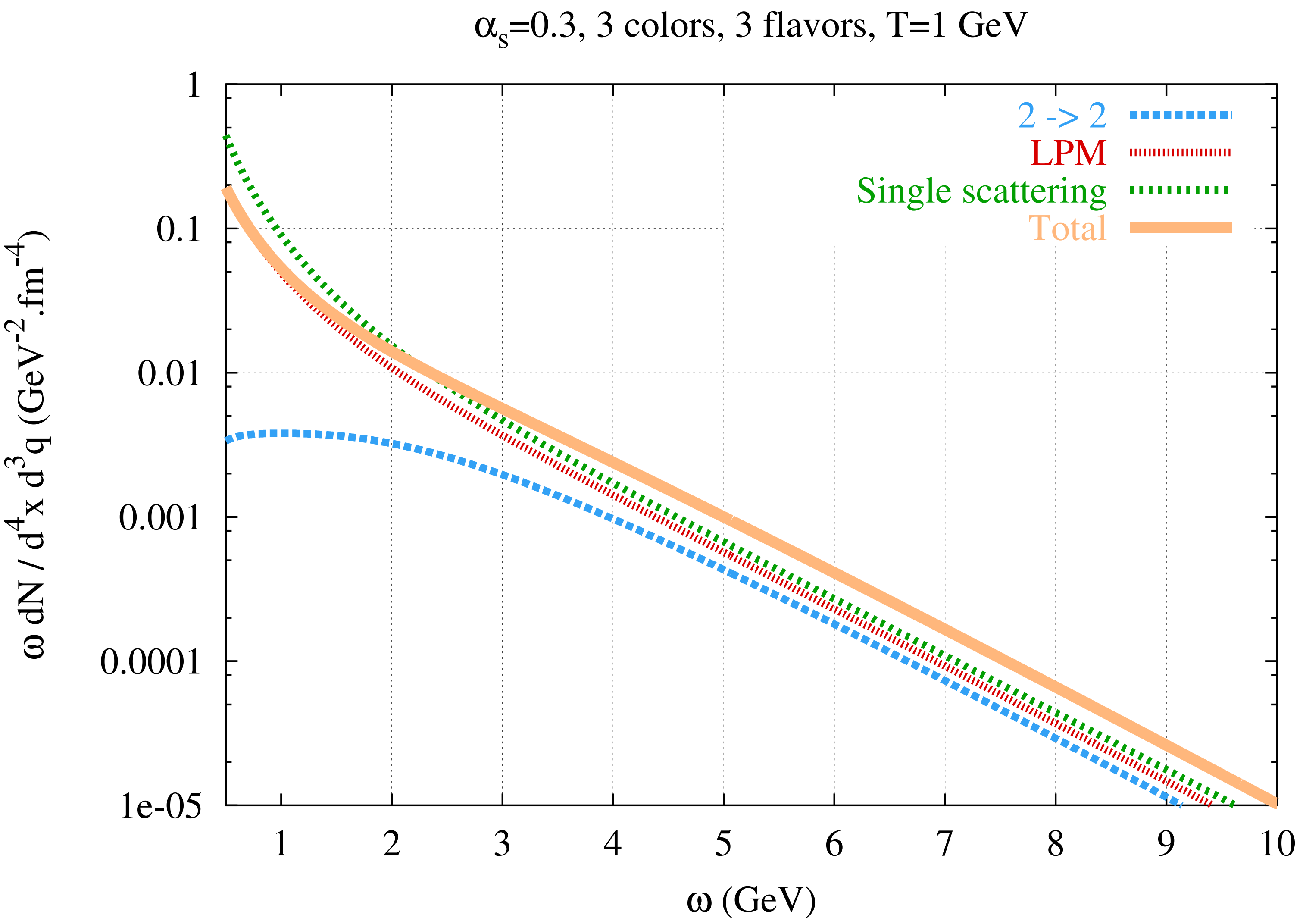
  ▷ no need to make an ansatz for the unknown function

  ▷ the accuracy is only limited by the numerical quadrature

  ▷ much faster than the variational method

LPM effect - Results

- Photon rate at $\mathcal{O}(\alpha\alpha_s)$



Dilepton production (1/4)

- What's different?

$$\triangleright \frac{dN_{l^+l^-}}{dtdV\,d^4Q} \propto \frac{\alpha}{Q^2}\frac{1}{e^{\omega/T}-1}\mathrm{Im}\,\Pi_{\mathrm{ret}}{}^{\mu}{}_{\mu}(Q)$$

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▷ The longitudinal mode of the γ^* contributes

Dilepton production (2/4)

Aurenche, FG, Moore, Zaraket (2002)

- Resummation of ladder diagrams

  ▷ Dyson equation:

$$\mathrm{Im}\,\Pi_{\mathrm{ret}}{}_{\mu}^{\mu} = \alpha\,\mathrm{Im}\int dp_0\, d^2\vec{\boldsymbol{p}}_\perp\,[\cdots]\,\left[2\vec{\boldsymbol{p}}_\perp\cdot\vec{\boldsymbol{f}}(\vec{\boldsymbol{p}}_\perp) + Q^2 g(\vec{\boldsymbol{p}}_\perp)\right]$$

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▷ Note: if $Q^2 > 4m_{\mathrm{q}}^2$, $1/t_F$ can vanish $\Rightarrow$ the solution starts at the order 0 in $\mathcal{C}$: Drell-Yan term $(q\bar{q}\to\gamma^*)$

Dilepton production (3/4)

- Scaling property:

$$\mathrm{Im}\,\Pi_{\mathrm{ret}}{}_{\mu}^{\mu}(\alpha,\alpha_s,T,\omega,Q^2)=\alpha^2\alpha_s T^2 F(\omega/T,Q^2/\alpha_s T^2)$$

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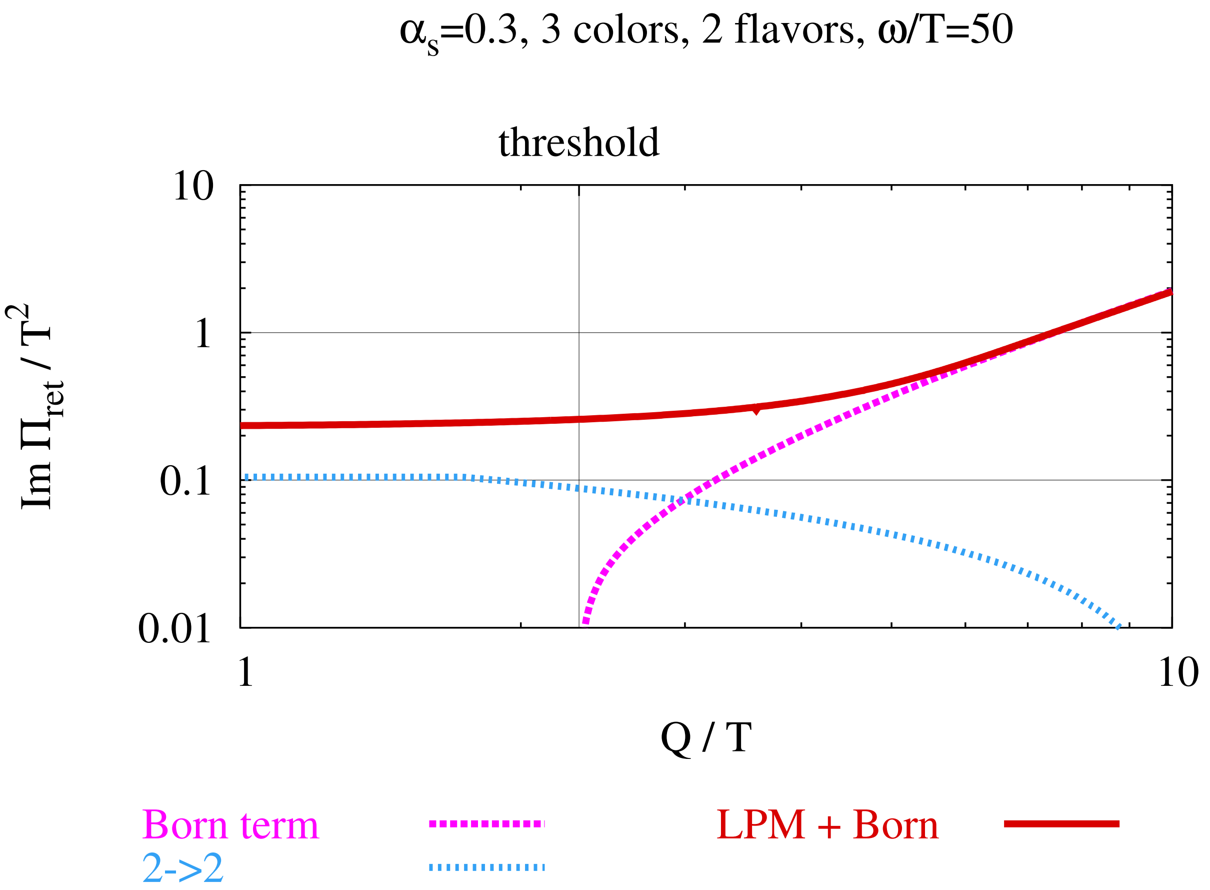
  $\mathrm{Im}\,\Pi_{\mathrm{ret}}{}_{\mu}^{\mu}(\alpha, \alpha_s, T, \omega, Q^2) = \alpha^2 \alpha_s T^2 F(\omega/T, Q^2/\alpha_s T^2)$

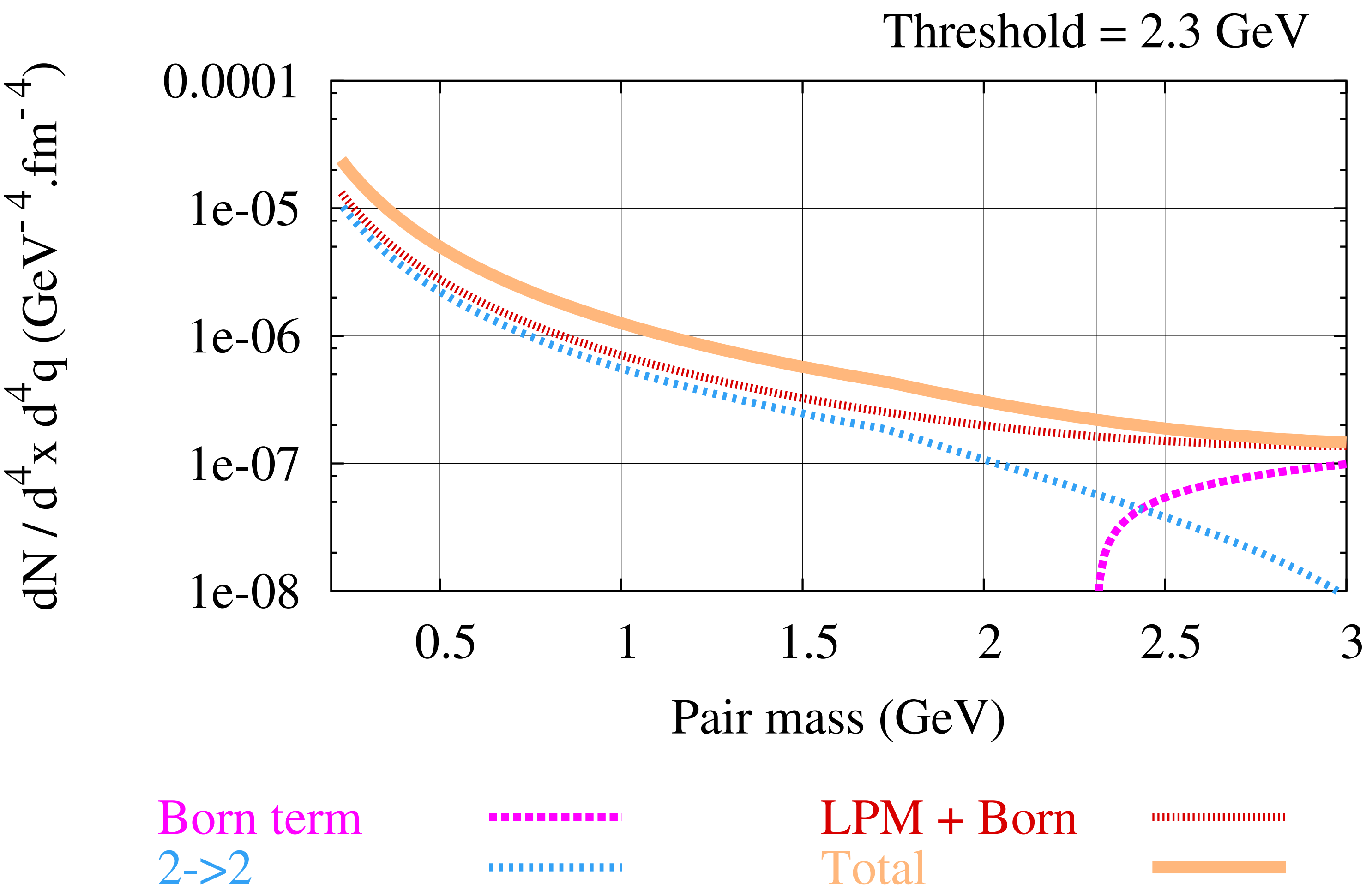
- All terms up to $\mathcal{O}(\alpha^2 \alpha_s)$



Dilepton production (4/4)

- Dilepton rate at $\mathcal{O}(\alpha^2\alpha_s)$

α_s=0.3, 3 colors, 2 flavors, T=1GeV, ω=5GeV



Non chemical equilibrium (1/2)

FG, Rasanen, Ruuskanen (2003)

- Phenomenological attempt in order to deal with the pre-equilibrium stage:

$$n(\omega) = \lambda\, n_{\rm eq}(\omega)\ ,\qquad 0 \le \lambda \le 1$$

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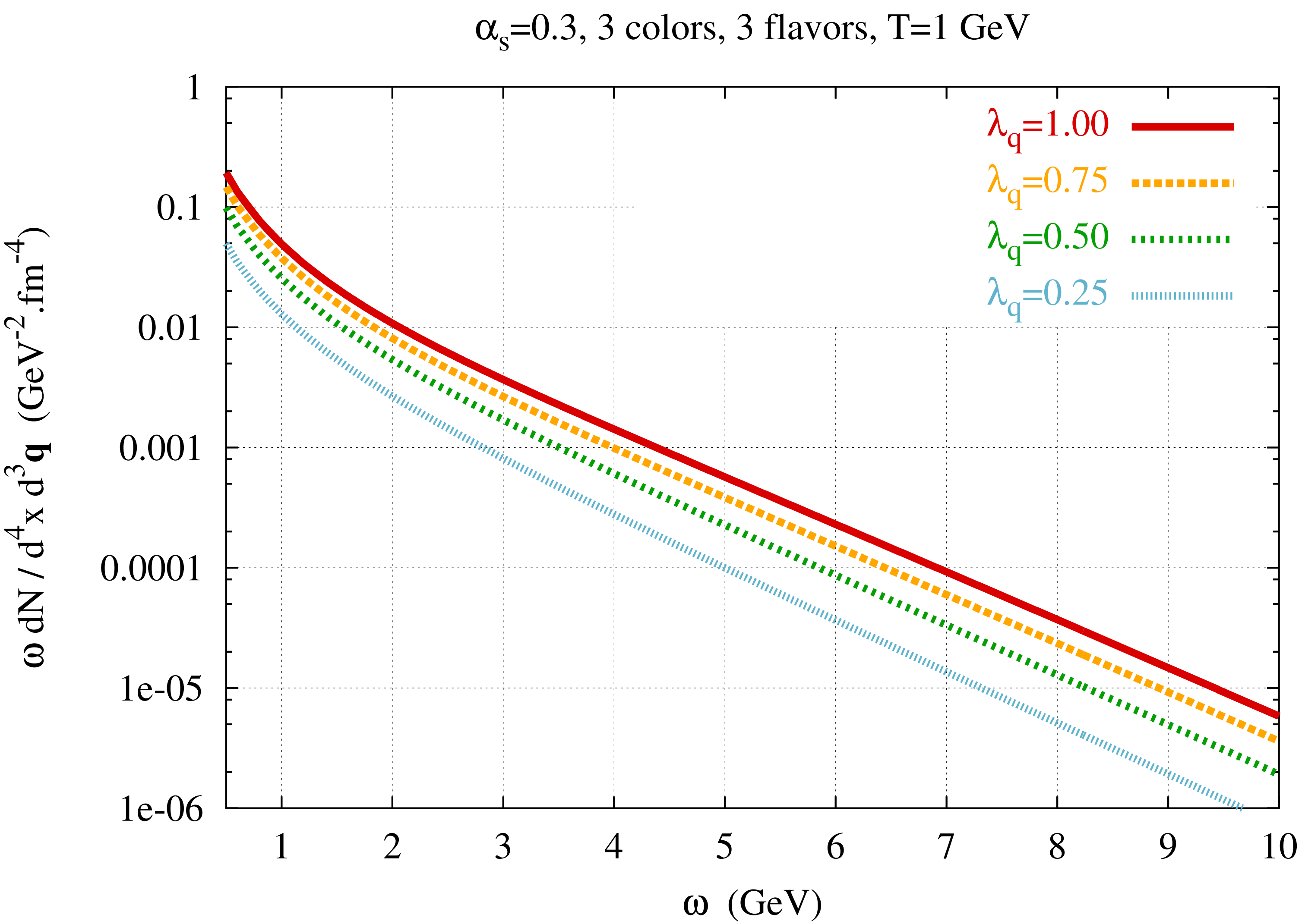
- The quark and gluon effective masses become:

$$m_{\rm q}^2 = \frac{8\pi}{9}\left(\lambda_{\rm g} + \frac{1}{2}\lambda_{\rm q}\right)\alpha_s T^2$$

$$m_{\rm g}^2 = \frac{4\pi}{3}\left(\lambda_{\rm g} + \frac{N_f}{6}\lambda_{\rm q}\right)\alpha_s T^2$$

Non chemical equilibrium (2/2)

- Example: effect of quark underpopulation



More on the Debye mass (1/2)

- The sum rule gives: $\mathcal{C}(-\boldsymbol{l}_\perp^2) = \frac{\pi}{2}\left[\frac{1}{\boldsymbol{l}_\perp^2+\Pi_T(0)} - \frac{1}{\boldsymbol{l}_\perp^2+\Pi_L(0)}\right]$
  - ▷ $\Pi_T(0) = 0, \quad \Pi_L(0) = m^2_{\rm debye}$
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- Photon/dilepton rates depend on $m_{\rm q}/m_{\rm debye}$
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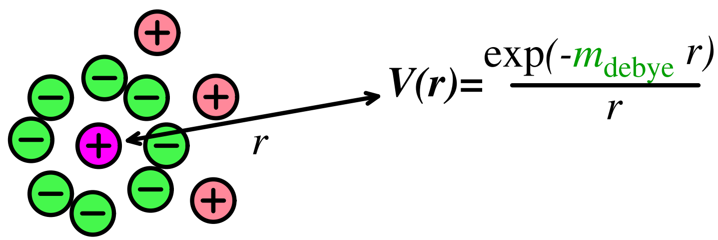
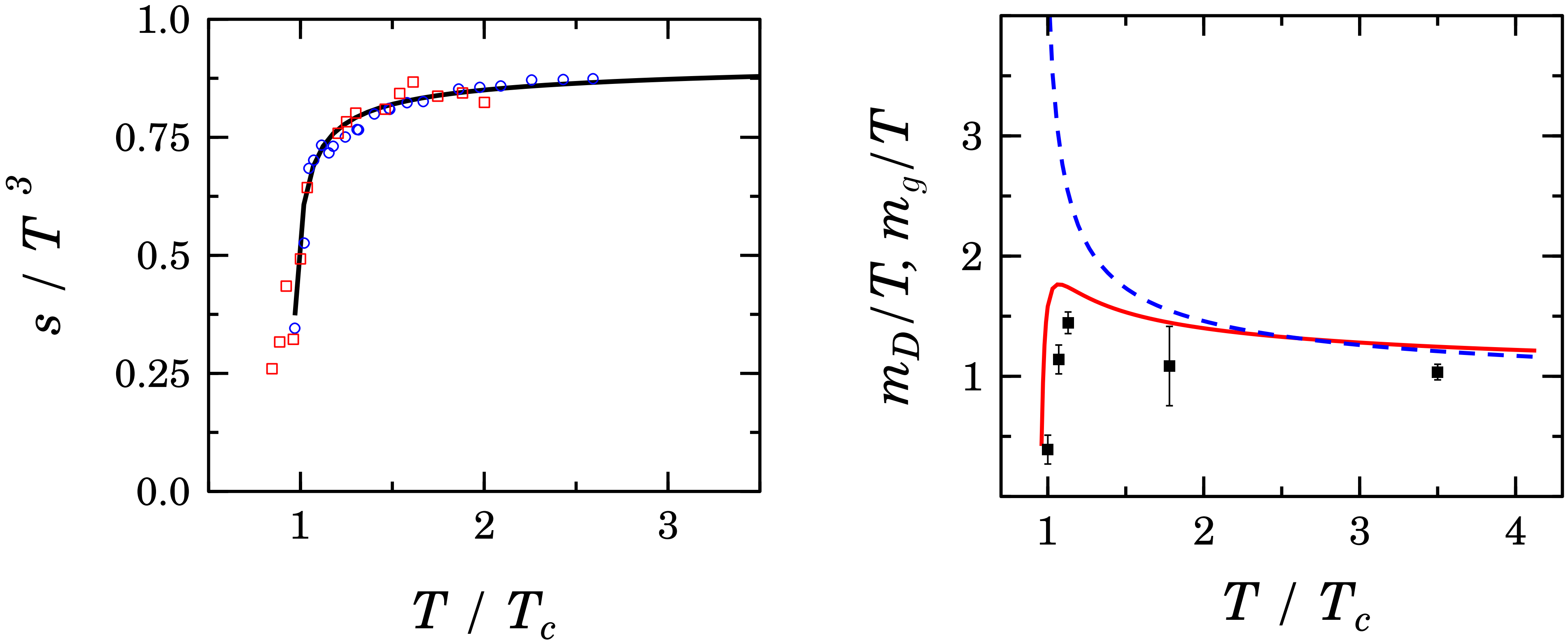
- On physical grounds, this cannot be exact:



Heavy quasiparticles (large $m_{\rm q}$ or $m_{\rm g}$)
▷ medium difficult to polarize $\Rightarrow$ small $m_{\rm debye}$

More on the Debye mass (2/2)

- Quasiparticle fits of the lattice entropy (or pressure) require large m_q and m_g near T_c
  $\triangleright$ m_debye computed in this model is small near T_c



Peshier, Kampfer, Soff (1996,2001,2002)

Conclusions

- Under control:
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- Under control:
  - All rates are known at leading order
  - Efficient methods to resum the LPM corrections
  - Rough estimates of effects due to non chemical equilibrium
- Needs more attention:
  - Size of higher order corrections?
  - What should be done in the regime of high gradients? (when the formation time is larger than the scale over which the temperature changes)
  - How to connect the initial production of prompt photons to the thermal stage? (equilibration...)