QUANTUM FIELD THEORY

From Basics to Modern Topics

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ERRATA/ADDENDUM

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(The elements affected by the modifications are highlighted in red.)

1. **Page 1:** In the third line of the second paragraph, the electron mass should read $m_e \approx 0.5$ MeV/c^2. (*suggested by Michael Ratz*)

2. **Generators of the Lorentz algebra:** There is an inconsistency between the sign convention used in Eq. (1.8) for the generators of the Lorentz algebra, and the explicit form of these generators in the spin-1/2 representation, in Eq. (3.5). The following changes in chapter 1 restore the consistency:

   - Eq. (1.8): $U(1+\omega) = 1 - \frac{i}{2}\omega_{\mu\nu}M^{\mu\nu} + \mathcal{O}(\omega^2)$.

   With the sign convention of this equation, the Lorentz algebra generators for the spin-1 and spin-1/2 representations take the following form:

   $$\text{spin-1/2:}\quad \big(M^{\mu\nu}\big)_{ab} = \tfrac{i}{4}\big[\gamma^\mu,\gamma^\nu\big]_{ab},\quad \text{spin-1:}\quad \big(M^{\mu\nu}\big)_{\alpha\beta} = i\big({\delta^\mu}_\alpha{\delta^\nu}_\beta - {\delta^\mu}_\beta{\delta^\nu}_\alpha\big).$$

   In addition, the following changes are necessary:

   - Eq. (1.10): $\big[M^{\mu\nu}, M^{\rho\sigma}\big] = -i(g^{\mu\rho}M^{\nu\sigma} - g^{\nu\rho}M^{\mu\sigma}) + i(g^{\mu\sigma}M^{\nu\rho} - g^{\nu\sigma}M^{\mu\rho})$,
   - Eq. (1.15): $\big[P^\mu, M^{\rho\sigma}\big] = -i(g^{\mu\sigma}P^\rho - g^{\mu\rho}P^\sigma)$,
   - Eq. (1.27): $U(\Sigma) \approx 1 + i\theta\, M^{12} + i\alpha_1\,(M^{10} + M^{31}) + i\alpha_2\,(M^{20} - M^{23})$,
   - Exercise **1.1**: $U^{-1}(\Lambda)M^{\mu\nu}U(\Lambda) \approx M^{\mu\nu} - \frac{i}{2}\omega_{\rho\sigma}\big[M^{\mu\nu}, M^{\rho\sigma}\big]$,
   - Exercise **1.2**: $U^{-1}(\Lambda)P^{\mu}U(\Lambda) \approx P^{\mu} - \frac{i}{2}\omega_{\rho\sigma}\big[P^{\mu}, M^{\rho\sigma}\big]$.
   - Page 248, below Eq. (9.74): $\Lambda^{\mu\nu} = \big[\exp(-\frac{i}{2}\omega^{\rho\sigma}M^{(1)}_{\rho\sigma})\big]^{\mu\nu}$

3. **Page 22:** Eq. (1.97) should read: $\big\langle 0_{\rm out}\big|a_{\mathbf{q},{\rm out}}a^\dagger_{\mathbf{p},{\rm out}}\big|0_{\rm in}\big\rangle = (2\pi)^3 2E_{\mathbf{p}}\,\delta(\mathbf{p}-\mathbf{q})\,\big\langle 0_{\rm out}\big|0_{\rm in}\big\rangle$.

4. **Page 24:** In the second line of Eq. (1.105), one should read: $e^{i\mathbf{q}_j\cdot\mathbf{y}_j}$.

Email me at francois.gelis@ipht.fr if you discover mistakes not yet listed here.

5. **Page 27:** In the second line of Eq. (1.117), one should read: $e^{iq_j \cdot y_j}$.
6. **Page 27:** In Eq. (1.121), the last factor should read $U(x_n^0, -\infty)$. (*suggested by Xinrui Liu*)
7. **Page 29:** Between eqs. (1.128) and (1.129), one should read: *[...] the annihilation operators are on the right and the creation operators are on the left [...]*. (*suggested by Michael Ratz*)
8. **Page 34:** In the second line of Exercice 1.11, one should read $M^{2-} = B^2$ instead of $M^{-2} = S^2$.
9. **Page 70:** In the sentence following Eq. (3.40), one should read: *[...] which is automatically satisfied for any A^μ thanks to the ~~antisymmetry~~ structure of $F^{\mu\nu}$.*
10. **Page 77:** Four lines before the bottom, one should read: *[...] various kinds of [...]*
11. **Page 81:** In the second line of Eq. (3.80), one should read: $\langle \beta_{out} | \cdots | \alpha_{in} \rangle$.
12. **Page 85:** In the line above Eq. (3.93), one should read: $F_r^\mu(k, p, q) \equiv S_r(-q)\big(-i\Gamma_r^\mu(k, p, q)\big)S_r(p)$.
13. **Page 85:** Footnote 11 should be completed as follows: *Note that this statement may be invalid with non-linear gauge fixing conditions (see the Exercise 8.1).*
14. **Page 97:** In the first line of section 4.1.3, one should read: *[...] under a ~~local~~ continuous [...]*
15. **Page 98:** A more accurate version of Footnote 3 reads: *This formula is known as Ky Fan's theorem. It is closely related to Courant-Fischer's min-max theorem.*
16. **Page 107:** Eq. (4.61) should read

$$\mathbb{F}^0_{a,i} = i\sigma_{a,i} \underbrace{\left|f(0)\right|^2 \int \frac{dp^-}{(4\pi)^2} \left|g(p^-)\right|^2}_{\text{constant indep. of } \lambda}.$$

17. **Page 155:** In Eqs. (6.28) and (6.29), all instances of $\mathfrak{S}_n$ should read $\mathfrak{S}_N$.
18. **Page 156:** Eq. (6.37) should read:

$$\int d\chi_N d\overline{\chi}_N \cdots d\chi_1 d\overline{\chi}_1 \;\exp\left(\overline{\chi}^T \mathbf{M}\chi + \overline{\eta}^T\chi - \overline{\chi}^T\eta\right) = \det\left(\mathbf{M}\right)\;\exp\left(+\overline{\eta}^T\mathbf{M}^{-1}\eta\right).$$

19. **Page 168:** The right hand side of Eq. (6.103) should not have a leading minus sign.
20. **Page 175:** First and third lines, one should read: $\exp(itX)$.
21. **Page 181:** Eq. (7.40) may be completed as follows: *If we parameterize $\Omega = \exp(ig\theta)$, a non-Abelian gauge transformation can also be written as*

$$A^\Omega_\mu(x) = \Omega^{-1}(x)\left(A_\mu(x) - \partial_\mu\theta(x)\right)\Omega(x).$$

Denoting $\theta = \theta_a t^a$, the components of the transformed field A^Ω_μ read

$$A^\Omega_{\mu\,a}(x) = \Omega^{-1}_{ab}(x)\left(A_{\mu\,b}(x) - \partial_\mu\theta_b(x)\right).$$

In this form, a non-Abelian gauge transformation appears to be the combination of an Abelian-like transformation applied independently to each component of the gauge field followed by a local adjoint rotation of the result.

22. **Page 181:** In Eq. (7.42), it is more appropriate to write f^{bca} instead of f^{abc} (this turns out to be the same in the case relevant for non-Abelian gauge theories, where the structure constants may be taken fully antisymmetric, but we have not yet established this fact at this point).
23. **Page 188:** The second of Eqs. (7.74) should read $m_a = 2\,\mathrm{tr}\,(M\,t^a_f)$.

24. **Page 189:** Two lines below Eq. (7.80), one should read: *the tracelessness of the generators.*
25. **Page 210:** The last equation of Exercise 7.6 should read $2(U-V) = U\otimes(igA^0)\otimes V$.
26. **Page 210:** The first equation of Exercise 7.7 should read $U(\boldsymbol{x}) \equiv T\,\exp\left(ig\int_{-\infty}^{+\infty} dx^0\; A^0_a(x^0,\boldsymbol{x})t^a_f\right)$.
27. **Page 210:** In Exercise 7.9, one should read C_{3f} instead of C_{3r}.
28. **Page 228:** Eq. (8.78) should read:

$$Q^2_{\text{BRST}}\chi_a = \tfrac{g^2}{2}\, f^{abc}f^{bde}\underbrace{\chi_c\chi_d\chi_e}_{\substack{\text{invariant under}\\ \text{cyclic permutations}}} = 0.$$

(Since $\chi_c\chi_d\chi_e$ is invariant under cyclic permutations, we can replace the prefactor $f^{abc}f^{bde}$ by one third of its sum over the circular permutations of (c,d,e), which is zero by Jacobi's identity.)

29. **Pages 232-233:** In Exercise 8.2, one should read: *[...] of the quark loop contribution to ~~the imaginary part of~~ the photon self-energy [...].*
30. **Page 238:** After Eq. (9.19), the transformation law of the measure can be justified as follows: *The transformation law of the measure is obvious for transformations that do not mix commuting and anticommuting variables, i.e., when the Jacobian matrix is block-diagonal. For mixed transformations, note that*

$$\begin{pmatrix} A & B \\ C & D\end{pmatrix} = \begin{pmatrix} 1 & BD^{-1} \\ 0 & 1\end{pmatrix}\begin{pmatrix} A-BD^{-1}C & 0 \\ 0 & D\end{pmatrix}\begin{pmatrix} 1 & 0 \\ D^{-1}C & 1\end{pmatrix},$$

and that the leftmost and rightmost matrices in the right hand side represent translations that preserve the measure.

31. **Page 249:** The footnote 3 should read: *For spin-0, $\frac{1}{2}$ and 1, [...].*
32. **Page 314:** The first of Eqs. (12.11) should read: $\widetilde{J}^\mu_5(q) \equiv \int d^4x\; e^{-iq\cdot x}\; J^\mu_5(x)$
33. **Page 336:** In the last sentence of the first paragraph of Section 12.5.3, one should read: *[...] the divergence of the dilatation ~~tensor~~ current is the trace [...].*
34. **Page 346:** In the caption of Figure 13.3, one should read: *Left: notations for the ~~polar~~ spherical coordinates local frame [...].*
35. **Page 347:** Three lines above Eq. (13.19), one should read: *See Figure 13.6.*
36. **Page 352:** Four lines before the bottom of the page, one should read: *[...] in Figure 13.6.*
37. **Page 353:** In the line after Eq. (13.39), one should read: *[...] in Figure 13.5.*
38. **Page 353:** The left hand side of Eq. (14.49) should read: $\mathcal{A}_5(1^-2^-3^+4^+5^+)$.
39. **Page 371:** In Exercice 13.10, the equation should read:

$$N = g^2\int\frac{d^4x}{64\pi^2}\,\epsilon_{ijkl}F^a_{ij}F^a_{kl} = \lim_{R\to\infty} g^2\int_{\mathcal{S}_R}\frac{d^3S_i}{32\pi^2}\,K^i.$$

40. **Page 391:** A closing parenthesis is missing at the end of the first trace in Eqs. (14.82) and (14.84).
41. **Page 409:** Three lines before the bottom of the page, one should read: *the pole at $z=0$ of [...].*
42. **Page 414:** Two lines above Eq. (14.179), one should read: $GL(2,\mathbb{C})$.
43. **Page 416:** In the second part of Exercise 14.10, one should read: $1^-2^+\cdots j^-\cdots n^+$.

44. **Page 433:** The second of Eqs. (15.78) should read: $\ddot{G}(\tau,\tau') \equiv \partial_\tau \partial_\tau G(\tau,\tau') = 2\,\delta(\tau-\tau') - \frac{2}{T}$.

45. **Page 439:** In Exercise 15.9, one should read: *Bernoulli* instead of *Bernouilli*.

46. **Page 454**: Eq. (16.45) should read:

$$\mathcal{F}_{\rm Landau}[U,\Omega] \equiv -2\,a^2 \sum_x \sum_\mu {\rm Re\ tr}\,\Big(\Omega^\dagger(x+\widehat{\mu})U_\mu(x)\Omega(x)\Big).$$

47. **Page 466:** Eq. (17.5) should read: $\lim_{T\to 0} \dfrac{{\rm tr}\,(\rho\,\mathcal{O})}{{\rm tr}\,(\rho)} = \big\langle 0\big|\mathcal{O}\big|0\big\rangle$.

48. **Page 472:** The left hand side of Eqs. (17.32) and (17.36) should read:

$$\begin{aligned}&{\rm tr}\,\big(e^{-\beta(\mathcal{H}_0+\mu\mathcal{Q})}\;a^\dagger_{\mathbf{p},{\rm in}}a_{\mathbf{p}',{\rm in}}\big) = \cdots\\ &{\rm tr}\,\big(e^{-\beta(\mathcal{H}_0+\mu\mathcal{Q})}\;b^\dagger_{\mathbf{p},{\rm in}}b_{\mathbf{p}',{\rm in}}\big) = \cdots\end{aligned}$$

49. **Page 474:** Eq. (17.42) should read:

$$(2\pi)^3 E_{\mathbf{p}}\frac{dN_\gamma}{dtd^3\mathbf{x}d^3\mathbf{p}} = \tfrac{1}{2}\,\Pi_{+-}(E_{\mathbf{p}},\mathbf{p}) = \frac{{\rm Im}\,\big(-i\Pi_{++}(E_{\mathbf{p}},\mathbf{p})\big)}{e^{\beta E_{\mathbf{p}}}+1} = \frac{{\rm Im}\,\big(-i\Pi_{_R}(E_{\mathbf{p}},\mathbf{p})\big)}{e^{\beta E_{\mathbf{p}}}-1}.$$

50. **Page 512:** The second of Eqs. (18.40) should read: $\lim_{x^0\to-\infty} \Phi(x) = \Phi_\chi(x)$.

51. **Page 527:** The third of Eqs. (18.100) should read: $G^0_s = \frac{1}{2}\big(G^0_{++}+G^0_{--}\big), \quad \mathcal{G}_s = \frac{1}{2}\big(\mathcal{G}^0_{++}+\mathcal{G}^0_{--}\big)$.

52. **Page 542:** Eq. (18.180) should read:

$$G^0_{22}(x,y) = \int \frac{d^3\mathbf{k}}{(2\pi)^3 2E_{\mathbf{k}}}\; e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}.$$

53. **Page 554:** Just above Eq. (19.16), one should read: *Generically, the loop integral is a linear combination of integrals, [...]*

54. **Page 563:** In the fifth line, one should read 19.6 instead of 1936.

55. **Page 575:** In the first paragraph, one should read: *[...] a very important object is its Gröbner basis* $\mathbb{G} = \{g_1,\cdots,g_p\}$*, defined as [...]*