Update on $B \to X_s \gamma$ Decay and Implications for New Physics

Matthias Neubert

Cornell University

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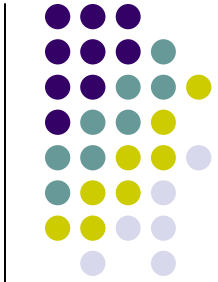
Outline

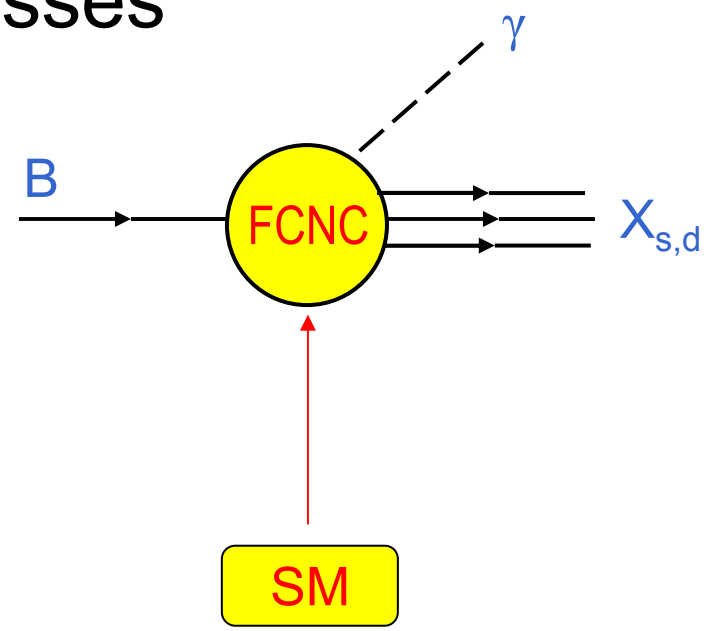


- Introduction
- Total decay rate and event fraction ($E_\gamma > E_0$) at NNLO
- Implications for New Physics
- Conclusions

Based on: T. Becher and MN, hep-ph/0512208, hep-ph/0603140 & in prep.
+ work by many others

Introduction

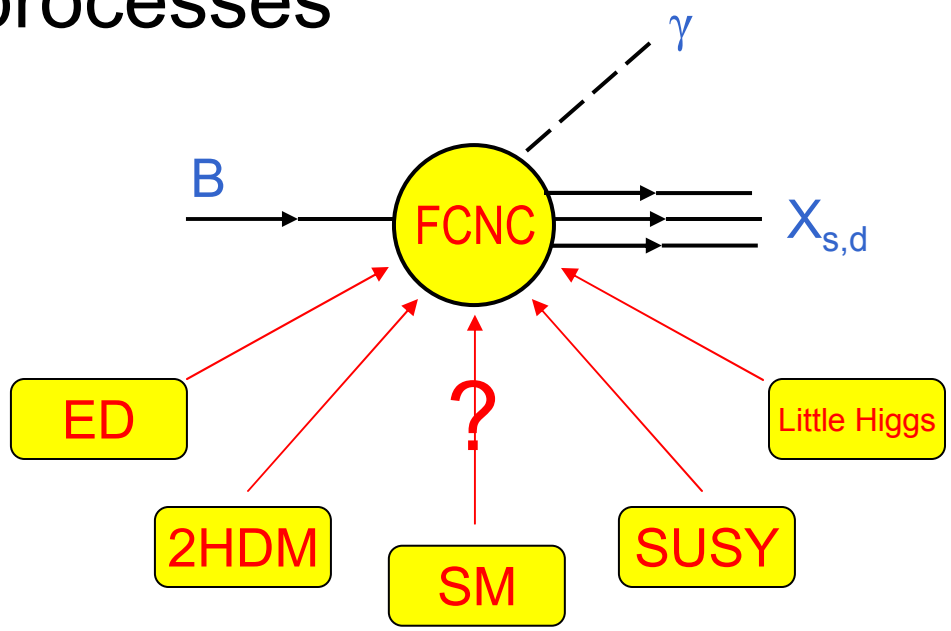
- Rare radiative decays $B \to X_{s,d}\gamma$ are prototype FCNC processes



- Precision test of flavor sector in and beyond SM

Introduction

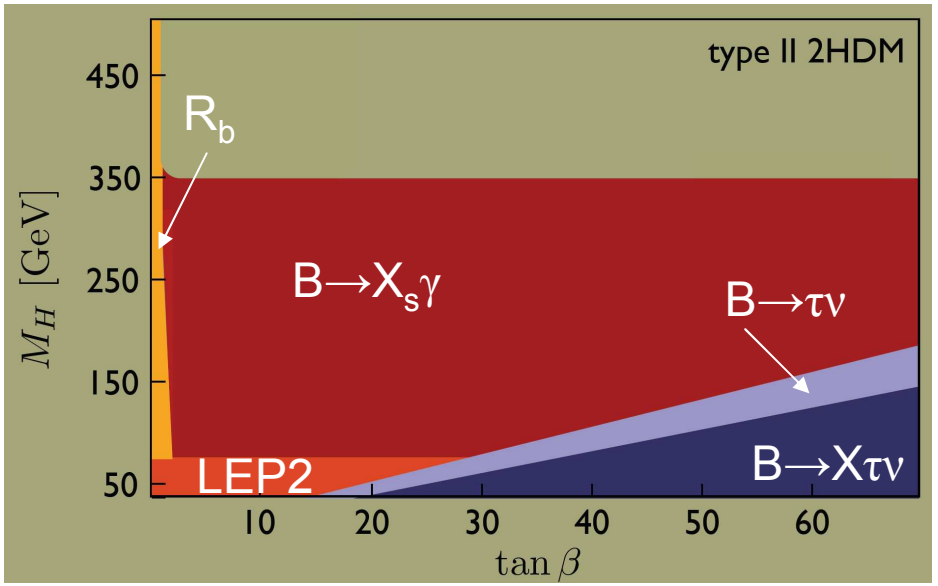
- Rare radiative decays $B \to X_{s,d}\gamma$ are prototype FCNC processes



- Precision test of flavor sector in and beyond SM

Introduction

- New Physics scales probed in $B \to X_s \gamma$ and other B decays are much higher than those accessible at colliders
- Constraints on New Physics crucially depend on theoretical accuracy of SM prediction



[Borzumati, Greub (1998)] [Gambino, Misiak (2001)]

- E.g., proper account of uncertainty due to photon-energy cut (E_γ>1.8 GeV or higher) significantly reduces Higgs-mass bound

[MN (2004)]

Introduction

- Theoretical results at NLO: *

$B_{SM}(E_\gamma>1.6\ \text{GeV}) = (3.21 \pm 0.32) \cdot 10^{-4}$ (m_c/m_b=0.29±0.02) [Kagan, MN (1998)]

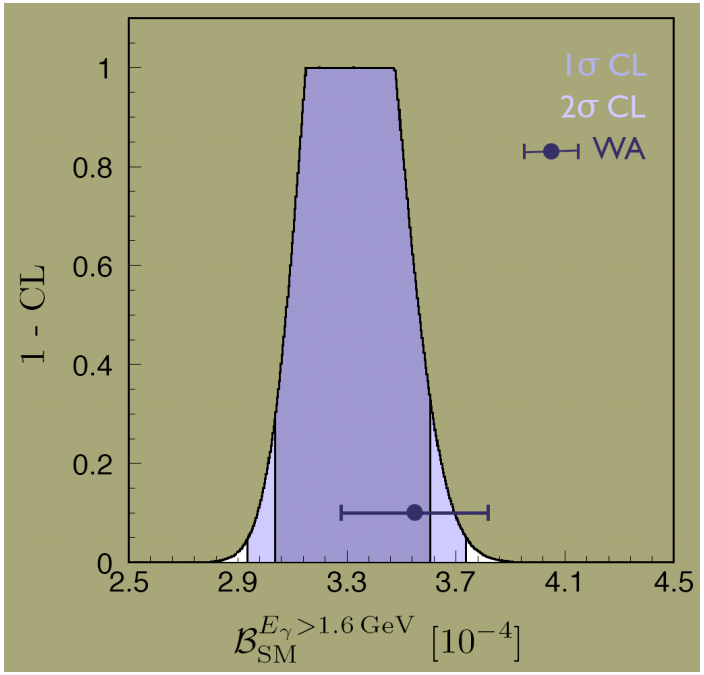
$B_{SM}(E_\gamma>1.6\ \text{GeV}) = (3.60 \pm 0.30) \cdot 10^{-4}$ (m_c/m_b=0.22±0.04) [Gambino, Misiak (2001)]

$B_{SM}(E_\gamma>1.6\ \text{GeV}) = (3.33 \pm 0.29) \cdot 10^{-4}$ (m_c/m_b=0.26±0.04) [Haisch (2006)]

- World-average experimental result, extrapolated to E_γ>1.6 GeV: *

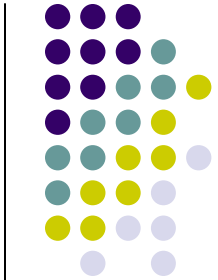
$B_{exp}(E_\gamma>1.6\ \text{GeV}) = (3.55 \pm 0.24 \pm 0.09 \pm 0.03) \cdot 10^{-4}$

[HFAG (2006)]



* assume *incorrectly* that event fractions with cuts $E_\gamma>E_0$ can be calculated reliably

Introduction



- Event fractions assumed by HFAG:

E_0	1.6 GeV	1.7 GeV	1.8 GeV	1.9 GeV	2.0 GeV
$F(E_0)_{model}$	0.965±0.002	0.951±0.004	0.933±0.006	0.903±0.010	0.863±0.015

[Buchmüller, Flächer (2005)]

  - Based on *model-dependent estimates* using shape functions with exponential fall-off (no tails)

- Model-independent calculation at NLO gives:

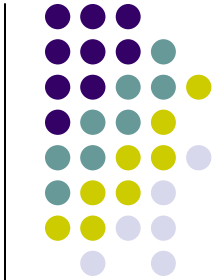
$F(E_\gamma>1.6\ \text{GeV}) = 0.93 \pm 0.07$ $F(E_\gamma>1.8\ \text{GeV}) = 0.89 \pm 0.07$

$B_{SM}(E_\gamma>1.6\ \text{GeV}) = (3.44 \pm 0.29 \pm 0.27) \cdot 10^{-4}$ [MN (2004)]

→ larger uncertainty!

scales parameters

Introduction



- Two most important areas of future (ongoing) theoretical improvements
  - Complete calculation of NNLO corrections to total $B \to X_s\gamma$ decay rate in SM
  - Complete model-independent analysis of photon-energy cut effect at NNLO

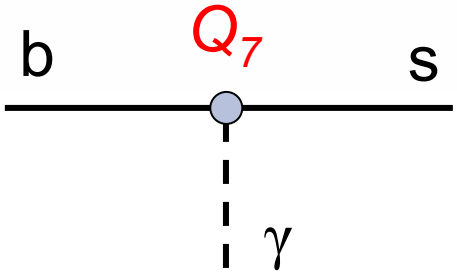
$$B_{SM}(E\gamma > E_0) = B_{SM}(\text{tot}) \cdot F(E_0)$$

total rate event fraction

Total decay rate at NNLO

$$\Gamma(B \to X_s\gamma) = \frac{G_F^2 \alpha}{32\pi^4} |V_{tb}V_{ts}^*|^2 \, m_b^5 \, C_{7\gamma}^2$$

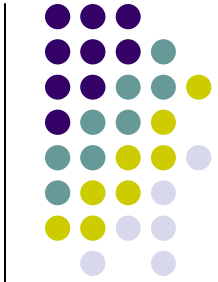
$$\times \left\{ 1 + O(\alpha_s) + O(\alpha_s^2) + O\left(\frac{\Lambda_{\rm QCD}^2}{m_b^2}\right) + \ldots \right\}$$



Calculable using OPE

- NLO calculation completed (1991-2001)
- NNLO calculation in progress (2003-…)

Total decay rate at NNLO



- Effective weak Hamiltonian:

$$\langle X_s\gamma|\,\mathcal{H}_{\rm eff}\,|B\rangle = \sum_{i=1,\ldots,8} C_i(\mu)\,\langle X_s\gamma|\,Q_i(\mu)\,|B\rangle$$

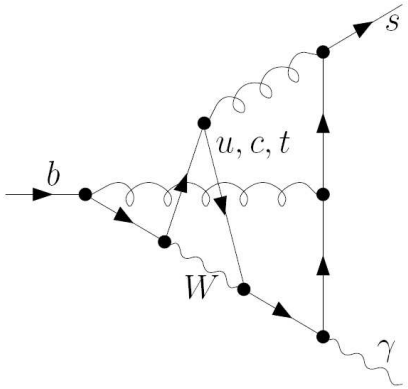
- RG evolution of Wilson coefficients:

$$C_i(\mu) = \sum_j C_j(M_W, m_t)\,U_{ji}(M_W, \mu)$$

matching running (anom. dims.)

Total decay rate at NNLO: Effective Weak Hamiltonian

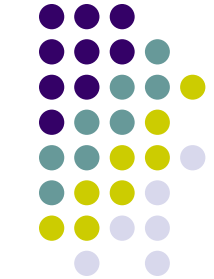
- Matching conditions
  - 2-loop corrections to $C_{1\text{-}6}(M_W)$ known
  - 3-loop corrections to $C_{7,8}(M_W)$ known [Bobeth, Misiak, Urban (1999)] [Misiak, Steinhauser (2004)]



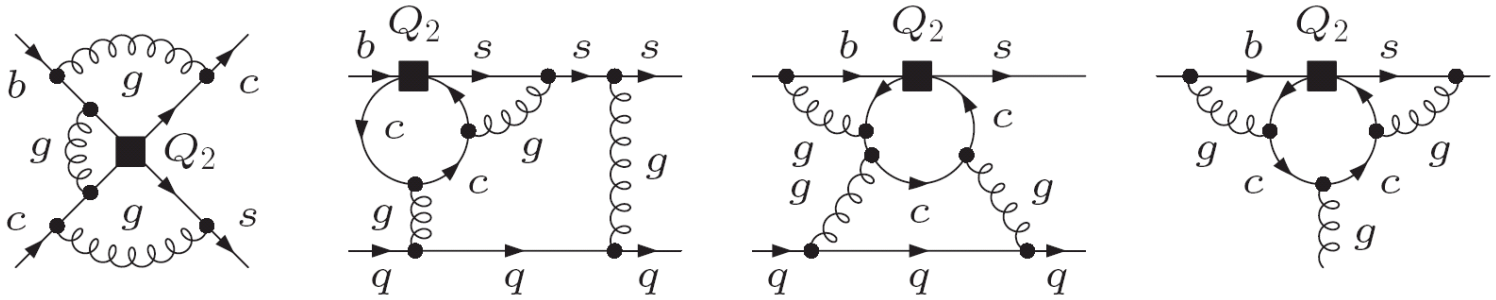
\+ O(1000) similar graphs

- In many models, this is where New Physics would enter!

Total decay rate at NNLO: Effective Weak Hamiltonian



- Anomalous dimensions
  - 3-loop mixing in $Q_{1\text{-}6}$ sector completed [Gorbahn, Haisch (2004)]
  - 3-loop mixing in $Q_{7,8}$ sector completed [Gorbahn, Haisch, Misiak (2005)]
  - 4-loop mixing of $Q_{1\text{-}6}$ into Q_7 completed and mixing into Q_8 almost finished [Czakon et al.]



- Largest effect on B_{SM} is estimated to be -2.4% (from 4-loop mixing)

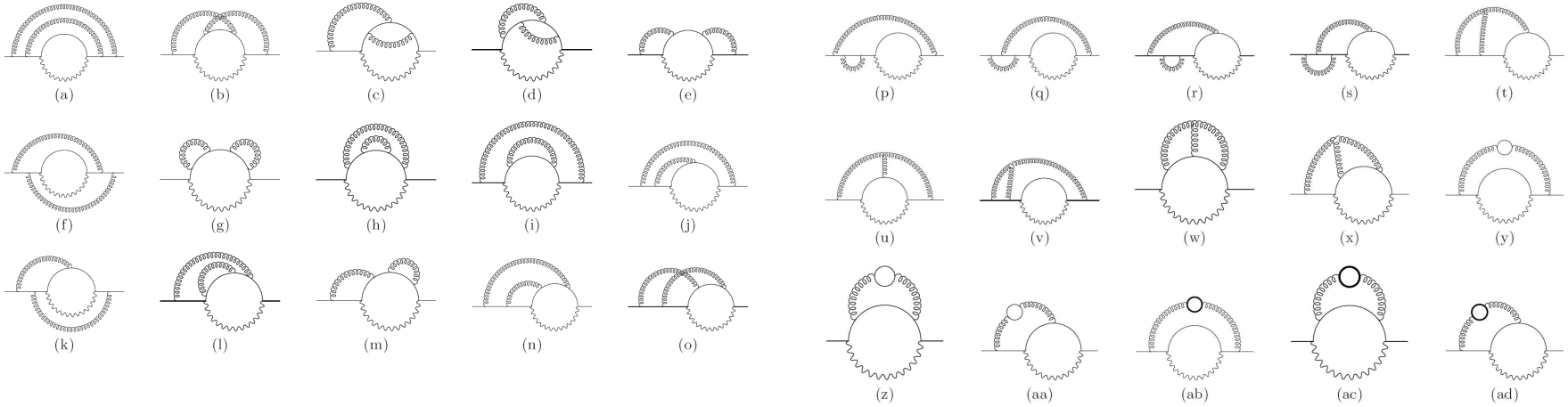
Total decay rate at NNLO: Matrix elements

- Large-β_0 limit of 2-loop matrix elements of $Q_{7,8}$ and 3-loop matrix elements of $Q_{1,2}$

[Bieri, Greub, Steinhauser (2003)]

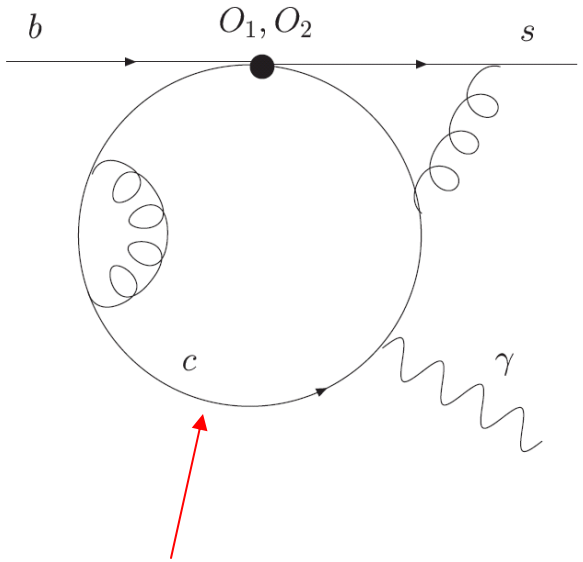
- 2-loop matrix elements of dipole operator Q_7

[Melnikov, Mitov (2005)]
[Blokland, Czarnecki, Misiak, Slusarczyk, Tkachov (2005)]
[Asatrian, Hoyhannisyan, Poghosyan, Ewerth, Greub, Hurth (2006)]



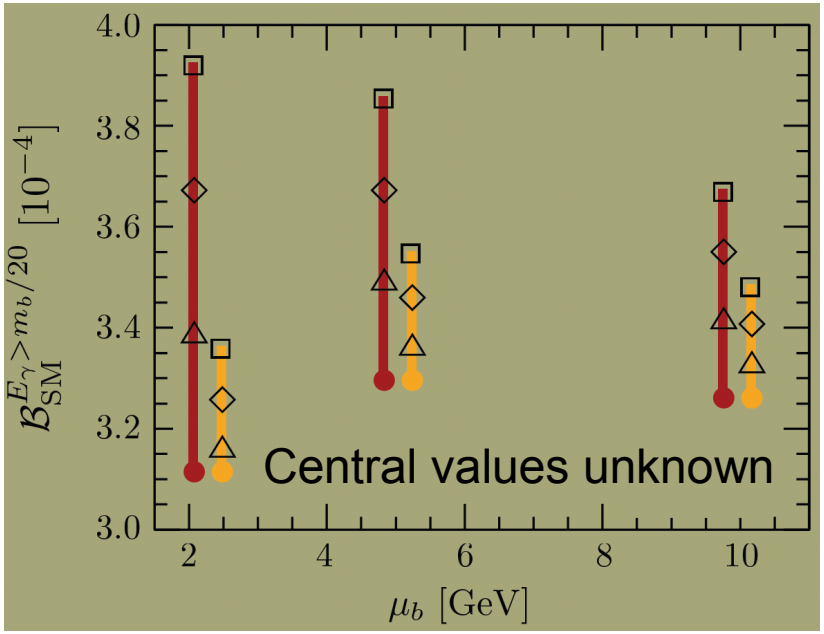
Total decay rate at NNLO: Matrix elements

- Interpolation in m_c of 3-loop matrix elements of $Q_{1,2}$ far advanced [Misiak, Steinhauser]



m_c dependence crucial!

- Estimates indicate that dominant uncertainty (m_c dependence) can be reduced from 6% to about 3%



[Bieri, Greub, Steinhauser (2003)]

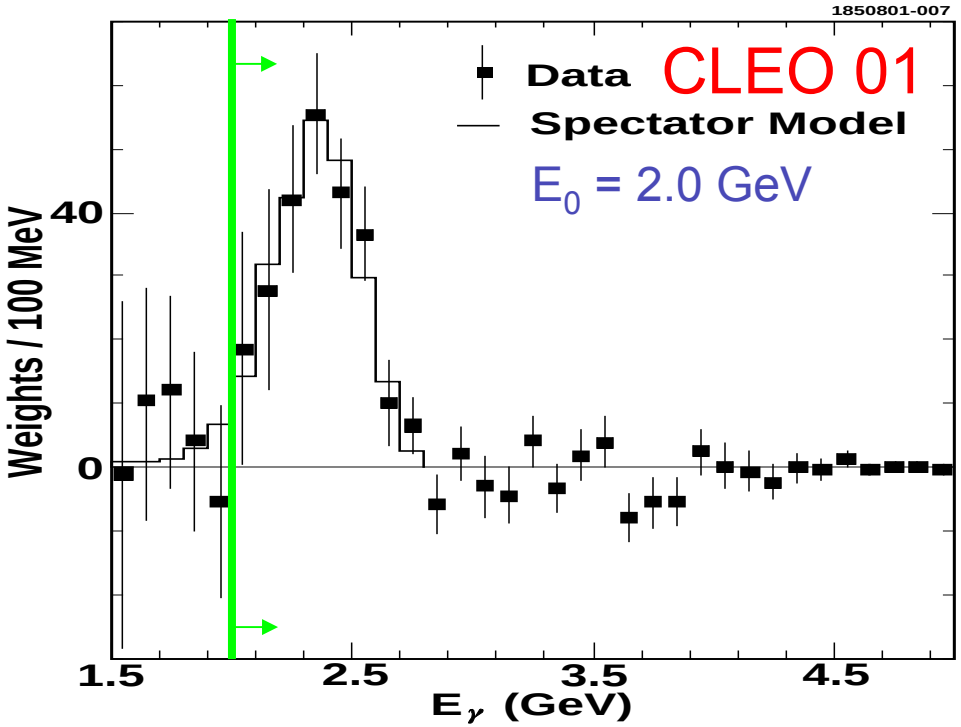
Total decay rate at NNLO: Theoretical error estimate

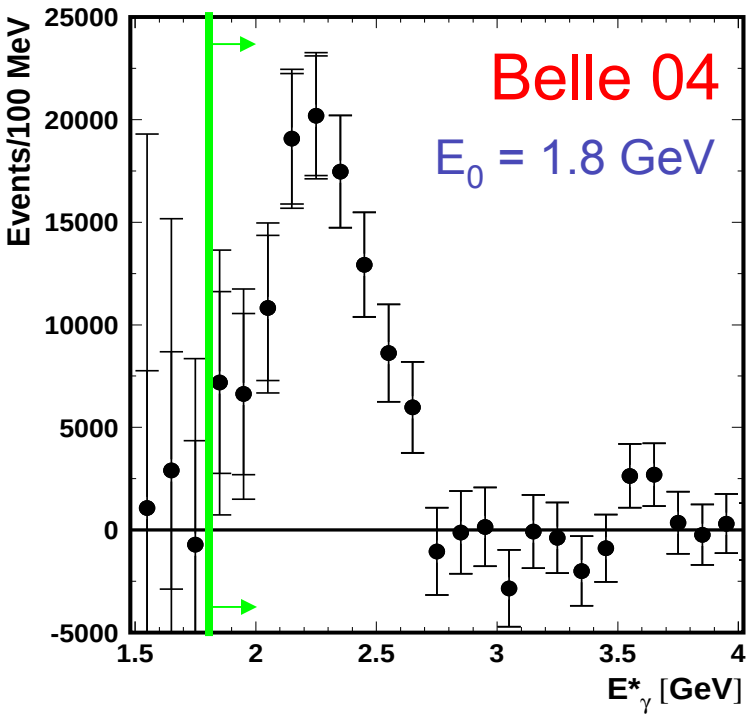
- Realistically (optimistically?), expect that after completion of NNLO calculation the dominant uncertainties are reduced to
  - Scales: ± 2%
  - m_c dependence: ± 3%
  - Others (α_s,CKM,m_t,etc.): ± 2%
- Total theoretical uncertainty:

$$\Delta B_{SM}(\text{tot}) \approx 4\%$$

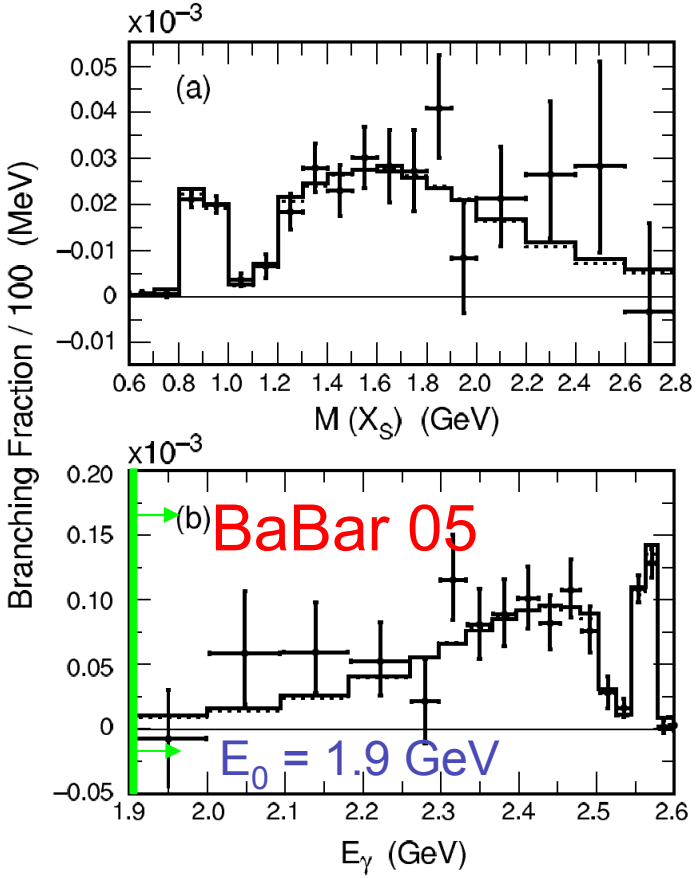
Event fraction at NNLO

- In practice, restriction to high-energy part of photon spectrum: $E_\gamma > E_0 = 1.8$ GeV or larger





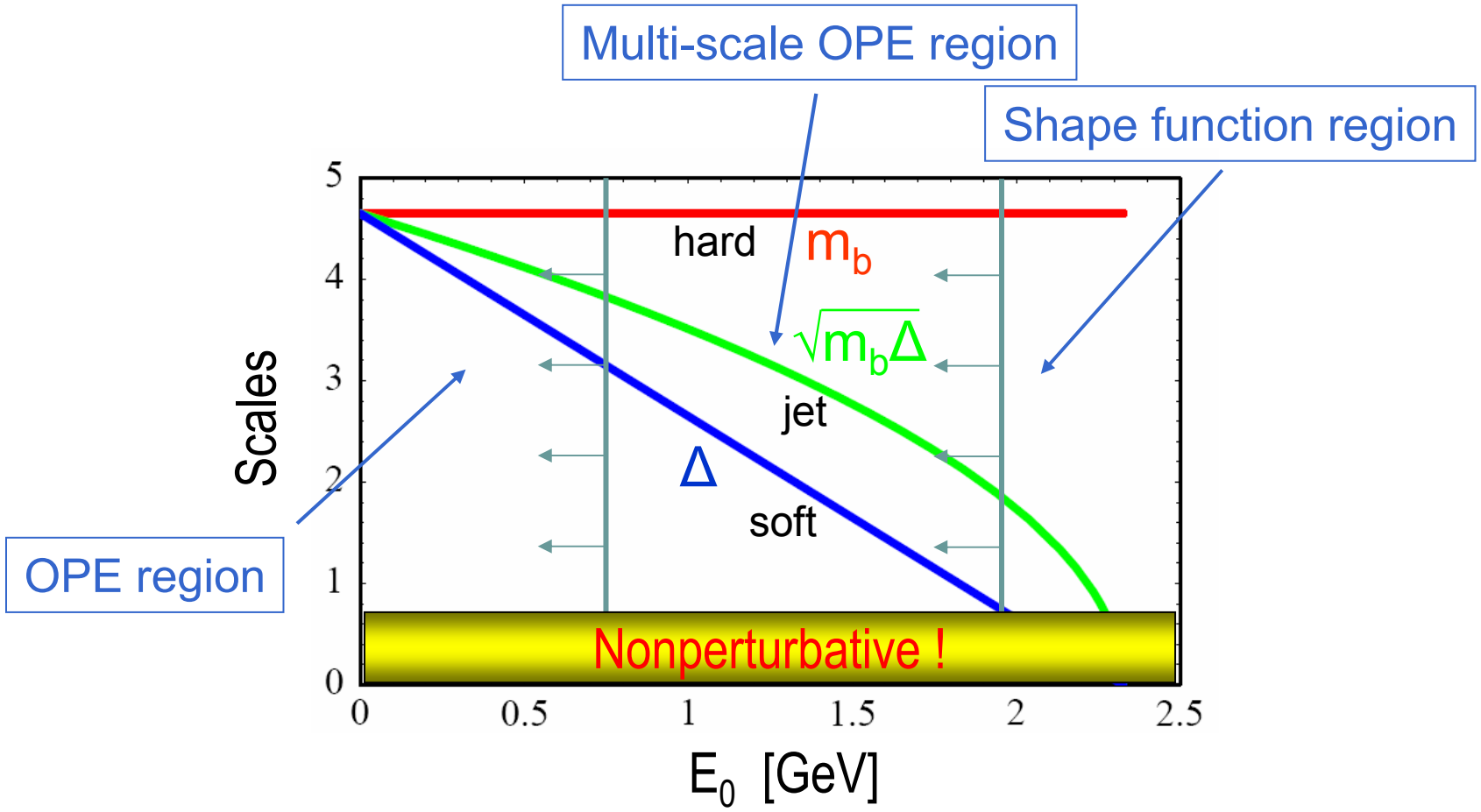
Event fraction at NNLO



- Presence of photon-energy cut leads to significant complications in theoretical analysis
- Sensitivity to low scale $\Delta = m_b - 2E_0 \approx 1$ GeV (barely perturbative)
- Effect controlled using multi-scale OPE; expansion in $(\Lambda_{QCD}/\Delta)^n$ and $\alpha_s(\Delta)$

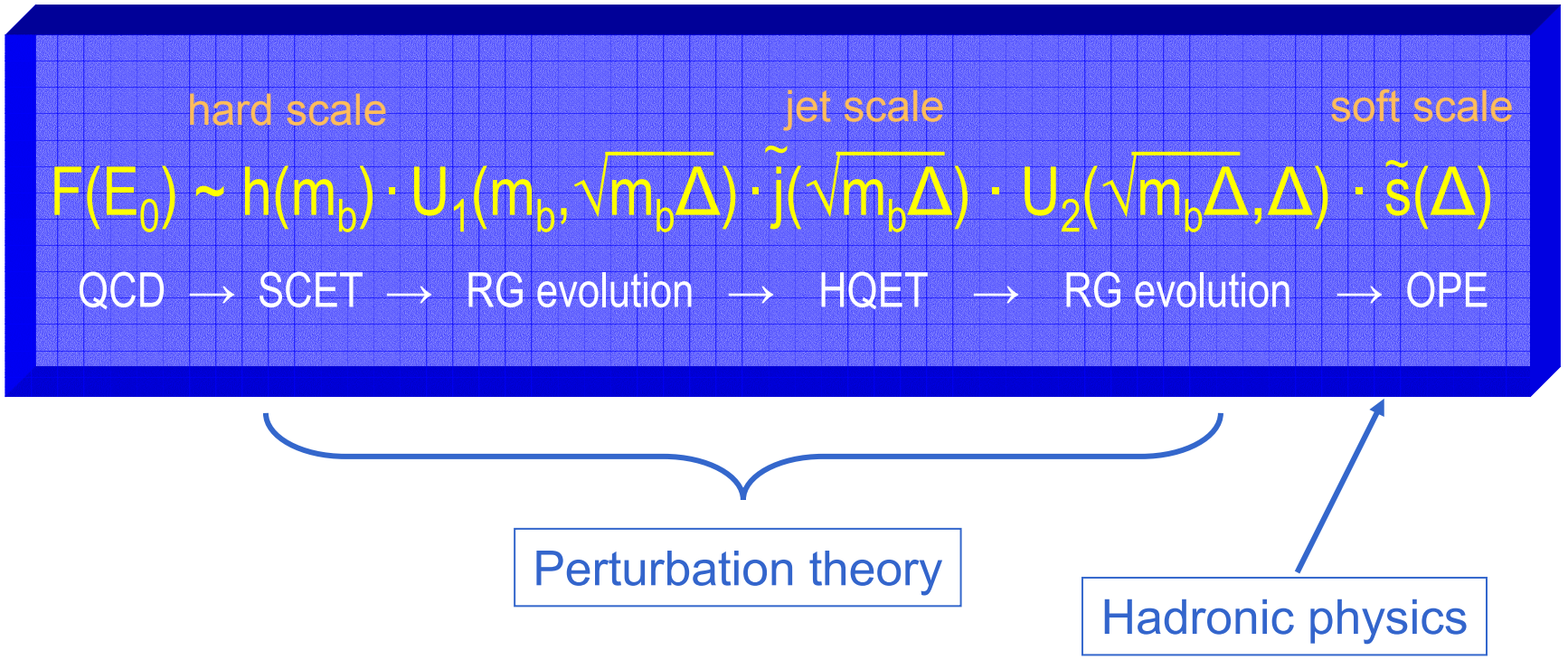
[MN (2004)]

Event fraction at NNLO



Event fraction at NNLO

- Master formula:



Event fraction at NNLO

- Exact all-order formula:

[MN (2005)]

$$F(E_0) = U_1(\mu_h,\mu_i)\,U_2(\mu_i,\mu_0)\left(\frac{m_b}{\mu_h}\right)^{-2a_\Gamma}\left(\frac{\Delta}{\mu_0}\right)^{\eta}$$
$$\times\, h\!\left(\ln\frac{m_b}{\mu_h}\right)\tilde{j}\!\left(\ln\frac{m_b\Delta}{\mu_i^2}+\partial_\eta\right)\tilde{s}\!\left(\ln\frac{\Delta}{\mu_0}+\partial_\eta\right)\frac{e^{-\gamma_E\eta}}{\Gamma(1+\eta)}\left[1-\frac{\eta(1-\eta)}{6}\frac{\mu_\pi^2}{\Delta^2}+\ldots\right]$$

Kinetic-energy parameter

Matching scales:

$\mu_h \sim m_b$
$\mu_i \sim \sqrt{m_b\Delta}$
$\mu_0 \sim \Delta = m_b - 2E_0$

RG functions:

U_1, U_2, a_Γ, η

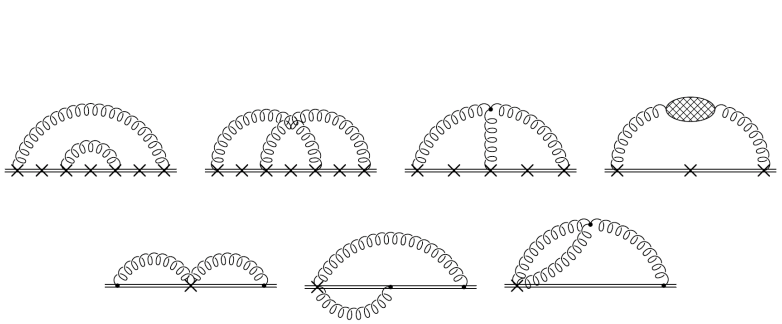
e.g.: $\eta = 2\int_{\mu_0}^{\mu_i}\frac{d\mu}{\mu}\,\Gamma_c[\alpha_s(\mu)]$

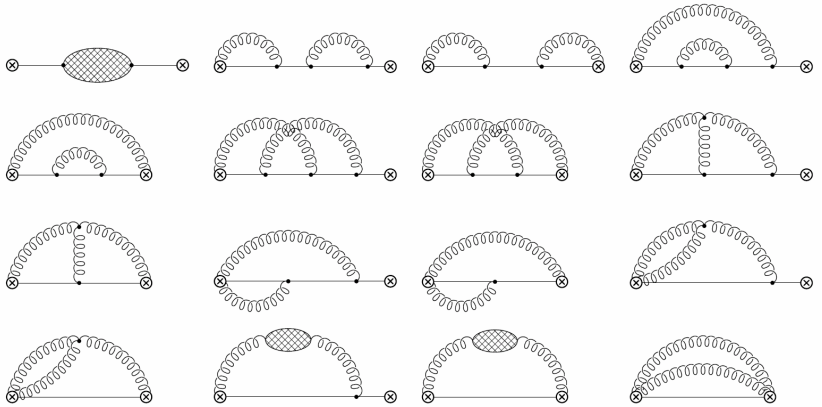
Event fraction at NNLO

- 2-loop soft and jet functions: [Becher, MN (2005, 2006)]

$$\widetilde{s}(L,\mu) = 1 + \left(-0.873 - 0.424L - 0.424L^2\right)\alpha_s + \left(-0.603 + 0.750L + 0.471L^2 + 0.368L^3 + 0.090L^4\right)\alpha_s^2 + \ldots$$

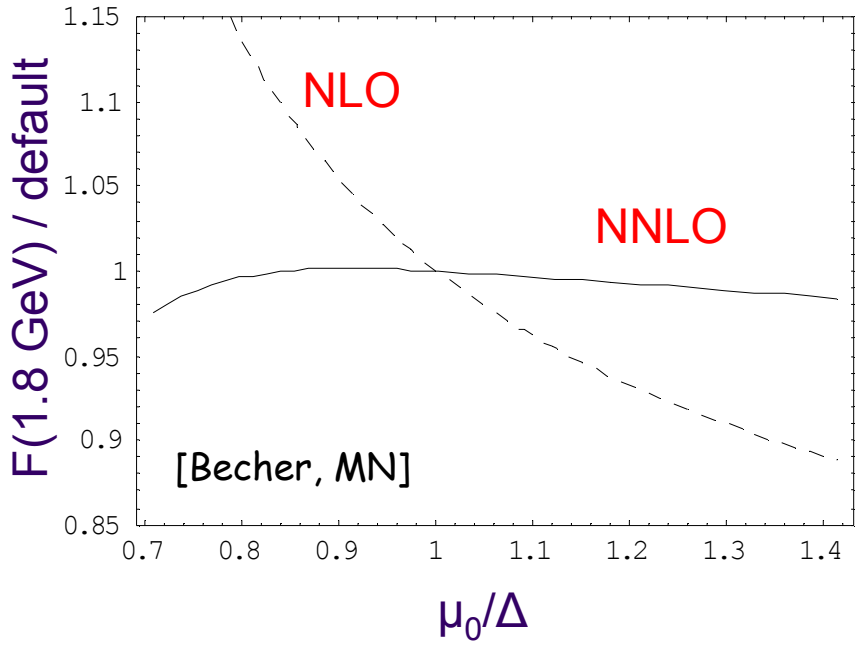
$$\widetilde{j}(L,\mu) = 1 + \left(0.045 - 0.318L + 0.212L^2\right)\alpha_s + \left(-0.185 + 0.145L + 0.301L^2 - 0.114L^3 + 0.023L^4\right)\alpha_s^2 + \ldots$$





Event fraction at NNLO

- 2-loop hard function h(L) can be extracted by combining effective field-theory results with 2-loop spectrum in QCD [Melnikov, Mitov (2005)]
- After resummation, largest uncertainties probed by variation of low scale $\mu_0 \sim 1$ GeV
  - Large uncertainty at NLO
  - Good stability at NNLO

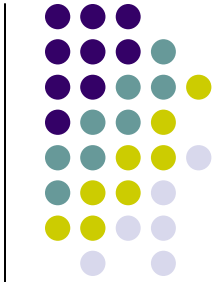


Event fraction at NNLO: Theoretical error estimate

- Realistically (optimistically?), expect that after completion of NNLO calculation the dominant uncertainties are reduced to
  - Scales: ± 3%
  - Hadronic parameters: ± 3%
- Total theoretical uncertainty:

$$\Delta F(1.8\ \text{GeV}) \approx 4\%$$

Implications for New Physics



- After completion of NNLO analysis, expect SM prediction:

$$B_{SM}(E_\gamma > 1.8\ \text{GeV}) = (3.30\ [?] \pm 0.13 \pm 0.13) \cdot 10^{-4}$$

total rate event fraction

  - Avoid interpolation to lower energy!
- To obtain similar precision in extensions of SM would require performing NNLO matching calculations including new particles in loops (not impossible)

Implications for New Physics

- Assuming improved experimental error of $0.2 \cdot 10^{-4}$ (B factories), combined uncertainty in difference between theory and experiment would be:

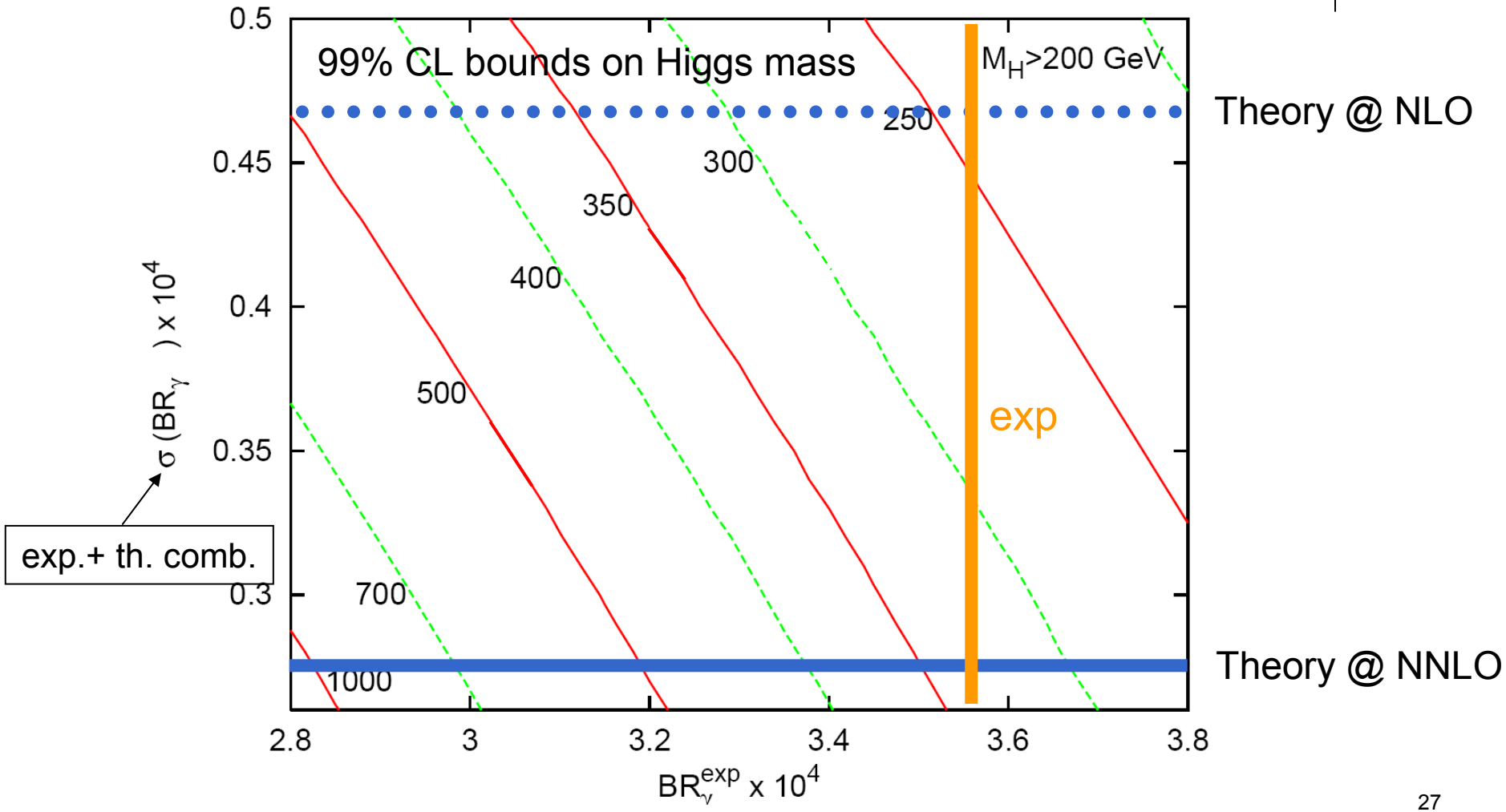
$$\Delta B_{NP} = B_{exp} - B_{SM} = (0.?? \pm 0.27) \cdot 10^{-4}$$

  - Much better than present error ± 0.47 (± 0.39 using HFAG'06)
  - Present 2σ deviation would become a 3.5σ effect

Implications for New Physics

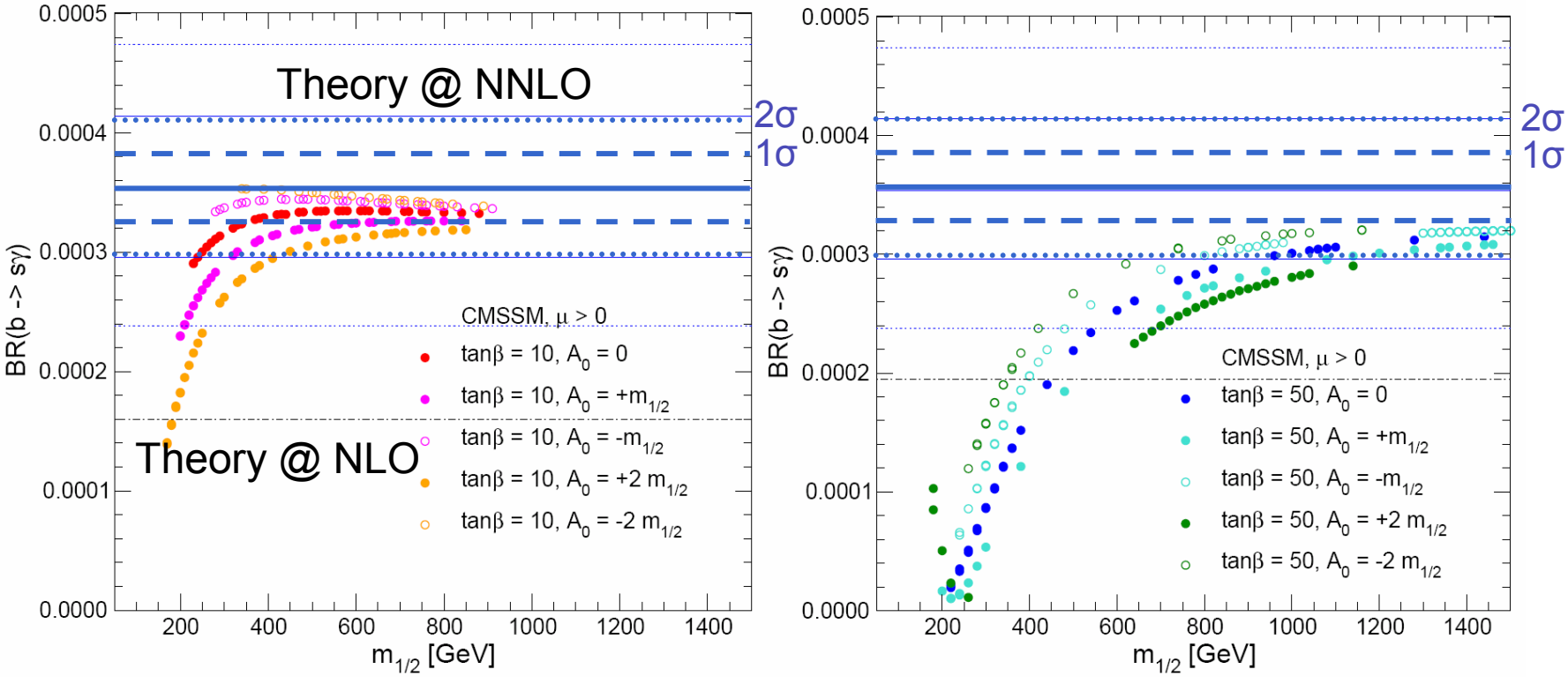
- Given present data, possibility of finding New Physics with $B \to X_s\gamma$ is essentially excluded (except for direct CP asymmetry) [Kagan, MN (1998)]
- However, reduced error will be very helpful in constraining parameters of New Physics models
- Discuss some examples …

Implications for New Physics: Type-II 2-Higgs-Doublet Model



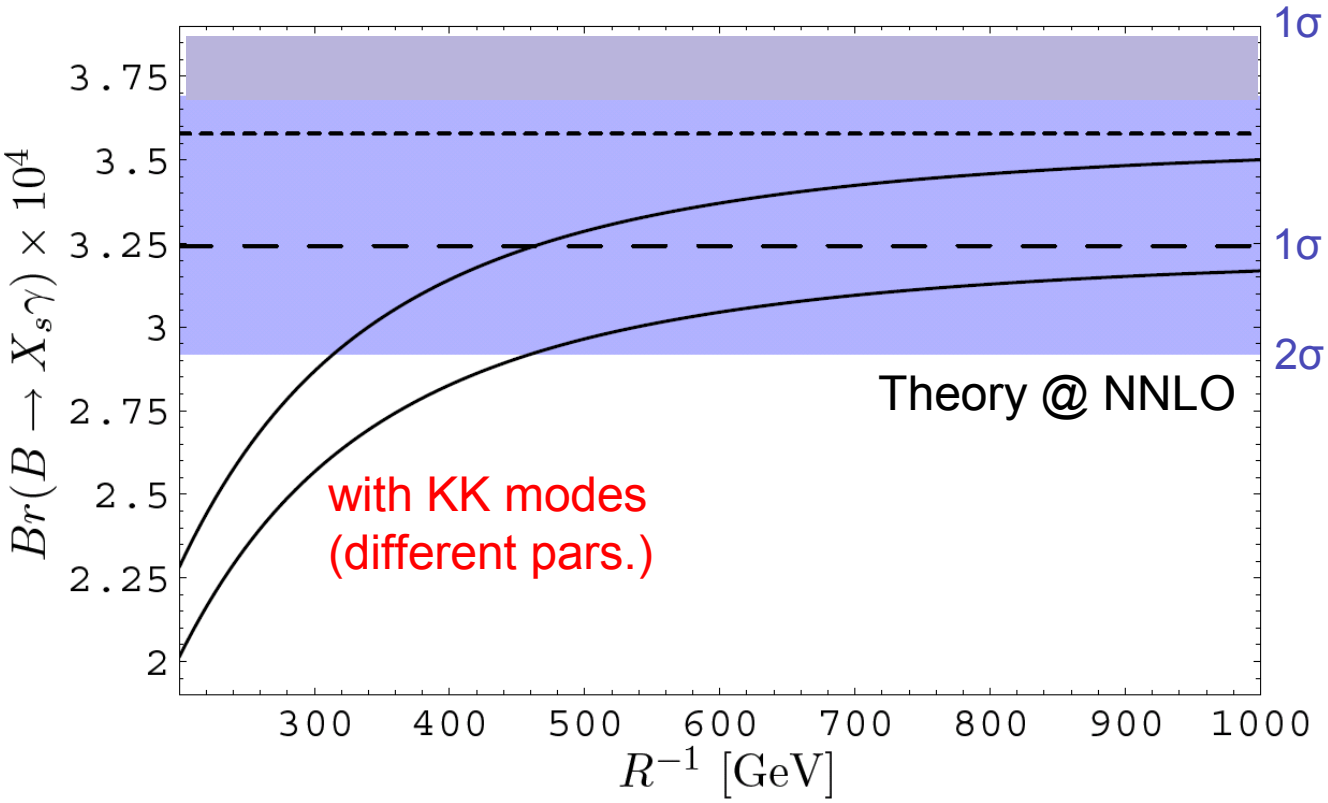
[Gambino, Misiak (2001); Becher, MN]

Implications for New Physics: Constrained MSSM



[Ellis, Heinemeyer, Olive, Weiglein (2004)]

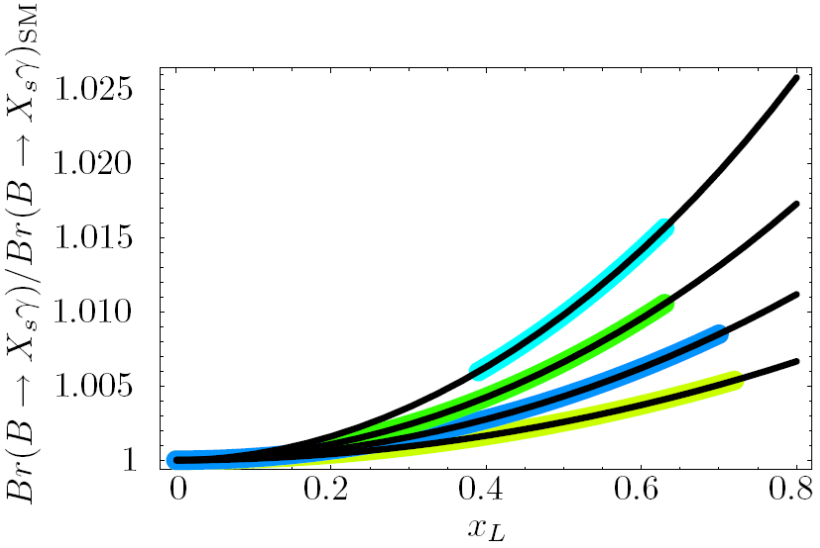
Implications for New Physics: Universal extra dimensions

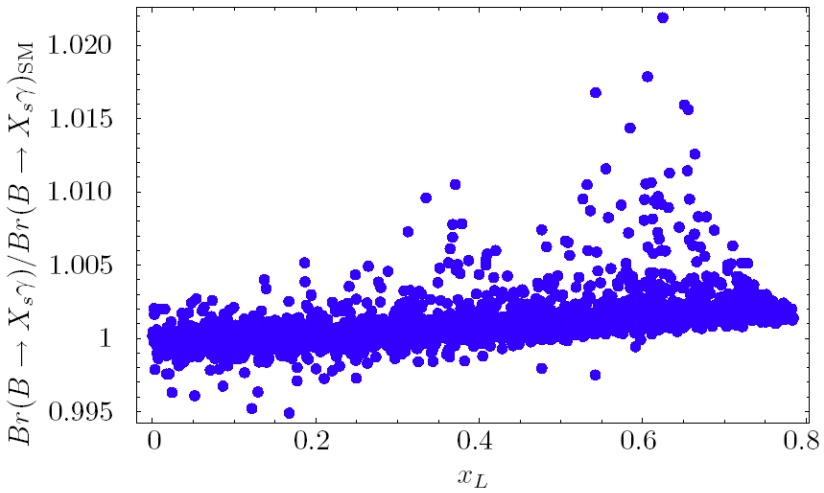


[Buras, Poschenrieder, Spranger, Weiler (2003)]

Implications for New Physics: Little Higgs with T parity

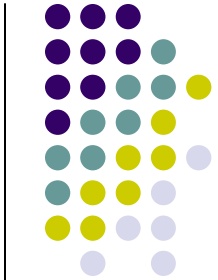
- Sometimes it's hopeless and New Physics effects are too small:





[Hubisz, Lee, Paz (2005)]
[Blanke, Buras, Poschenrieder, Tarantino, Uhlig, Weiler (2006)]

Conclusions



- $B \to X_s \gamma$ decay remains one of most sensitive probes of New Physics
- Strong motivation for NNLO calculation in Standard Model (many people involved!)
- Problem naturally splits up into two parts:
  - NNLO corrections to total decay rate
  - Effects of photon-energy cut at NNLO in multi-scale OPE
- Projected theoretical accuracy of 5-6% implies tight bounds on physics beyond SM