Localizing fermions on thick branes

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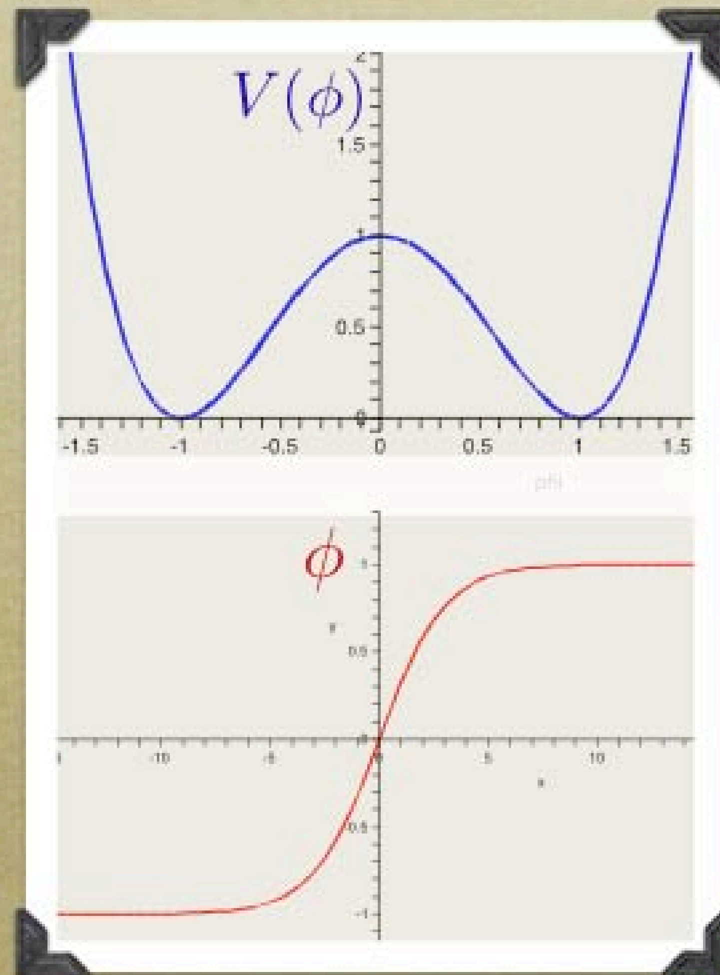
Living in a domain wall

Rubakov + Shaposhnikov (1983)

D-1 topological defect in extra dimension ξ

$$V = \lambda(\phi^2 - \phi_0^2)^2$$ *Symmetry breaking potential*



Solve field equations

$$\partial_A \partial^A \phi + \frac{dV}{d\phi} = 0$$

$$\phi(\xi = \pm\infty) = \pm\phi_0$$

Topologically stable solution

$$\phi = \phi_0 \tanh(\xi/\delta)$$

Ψ = *5-dimensional fermion*

$$\Gamma^\mu = \gamma^\mu$$

$$\Gamma^\xi = \gamma^5$$

$$i\gamma^\mu \partial_\mu \psi(x^\mu) = 0$$

Dirac eq. $$i\Gamma^A \partial_A \Psi + h\,\phi\Psi = 0$$

$$\Psi = f_L(\xi)\,\psi_L(x^\mu) + f_R(\xi)\,\psi_R(x^\mu)$$

$$f_L(\xi) = e^{-h\int \phi d\xi} = [\cosh(\xi/\delta)]^{-h\delta\phi_0}$$

$$f_R(\xi) = e^{+h\int \phi d\xi} = [\cosh(\xi/\delta)]^{+h\delta\phi_0}$$

L-fermions get confined to the wall

But: add gravity

Fermions on the branes

Bajc + Gabadadze 1999

Write Dirac eq. in a warped 5-D spacetime

$$g_0 = N_g e^{2A(\xi)}$$

$$ds^2 = e^{2A(\xi)} d\eta^2 + d\xi^2$$

$$\Psi = f_L(\xi)\,\psi_L(x^\mu) + f_R(\xi)\,\psi_R(x^\mu)$$

$$i\sqrt{g}\,\Gamma^A \partial_A \Psi = 0 \Longrightarrow (\partial_\xi + A')f(\xi) = 0$$

$$f(\xi) = N_f\, e^{-2A}$$

- *Fermions are not confined by the warp factor*
- *Add a Yukawa interaction with ϕ : thick wall*

can we use this?

RS + RS

Thick self-gravitating domain wall

Solve the coupled system of Einstein-Klein-Gordon equations

$$ds^2 = e^{2A(\xi)} d\eta^2 + d\xi^2 \qquad\qquad D_\mu D^\mu \phi = -\frac{dV}{d\phi}$$

Add a coupling $\quad h\,\bar{\Psi}\Psi\phi$

can confine one chiral mode $\quad f_L(\xi) = N_f\, e^{-2A+h\int \phi d\xi}$

E-K-G eqs. are quite involved in general

simpler if spacetime on the wall is Minkowski

Fermion localization on thick walls

Does the internal structure affect localization?

A.M. +Pantoja+Tempo 2006

We have studied several examples

- *Asymmetric walls*

  Different cosmological constants on both sides

  Castillo-Ramírez+A.M. +Pantoja 2005

- *Dynamic walls*

$$ds^2 = e^{2A(r)}\left[-dt^2 + e^{2\alpha t}d\vec{x}^2 + dr^2\right]$$

  De-Sitter expansion on the wall -no localization

- *Double walls*

  Interesting effects

BPS walls

Skenderis + Townsend 1999

For conformally flat walls, E-KG system can be written in terms of a function $W(\phi)$

$$A' = -W \qquad \phi' = 3\frac{dW}{d\phi}$$

$$V(\phi) = \frac{3}{2}\left[3\left(\frac{dW}{d\phi}\right)^2 - 4W^2\right]$$

Gremm 2000: thick version of RS scenario

$$W = \alpha \sin(\phi/\phi_0)$$

Single BPS wall

Gremm 2000

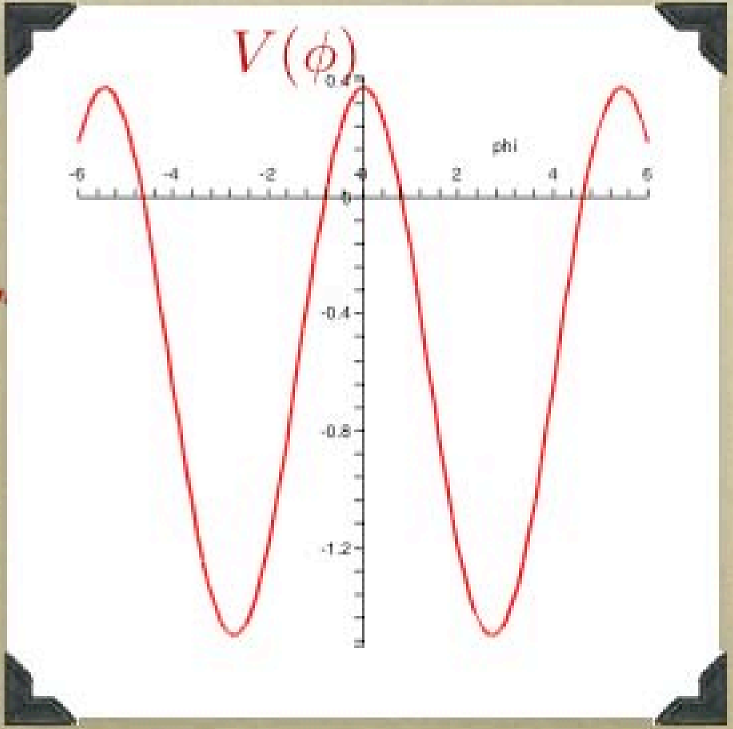
$$W(\phi) = \alpha \left[\sin\left(\frac{\phi}{\phi_0} \right) \right]$$

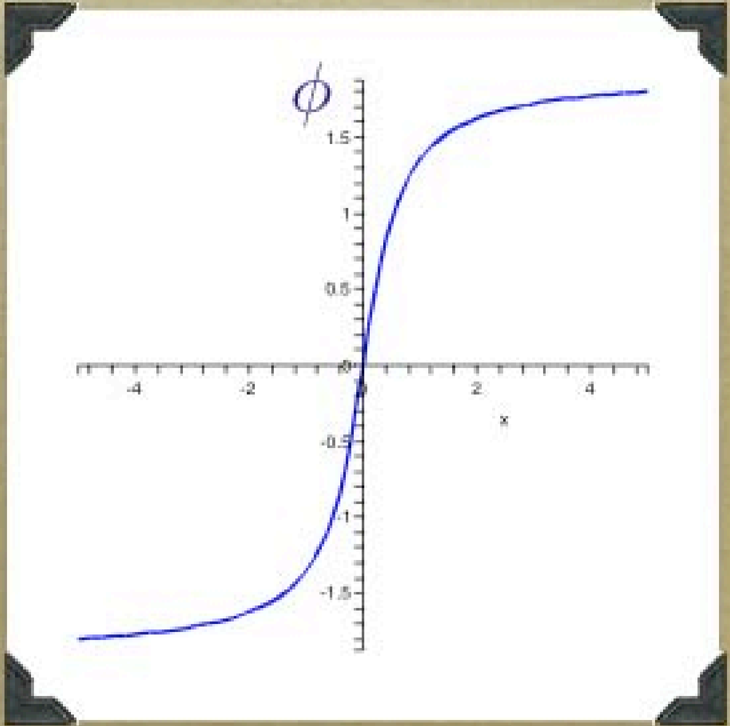
$$ds^2 = e^{A(\xi)} d\eta^2 + d\xi^2$$

$$A' = -W$$

$$\phi' = 3 \frac{dW}{d\phi}$$

- $V(\phi) = \frac{3}{2} \left[3 \left(\frac{dW}{d\phi} \right)^2 - 4W^2 \right]$





$$Q_T = 2\alpha$$

Double BPS walls

A.M.+Pantoja+Skirzewski 2003

$$W(\phi) = \alpha \left[\sin\left(\frac{\phi}{\phi_0} \right) \right]^{2-\epsilon}$$

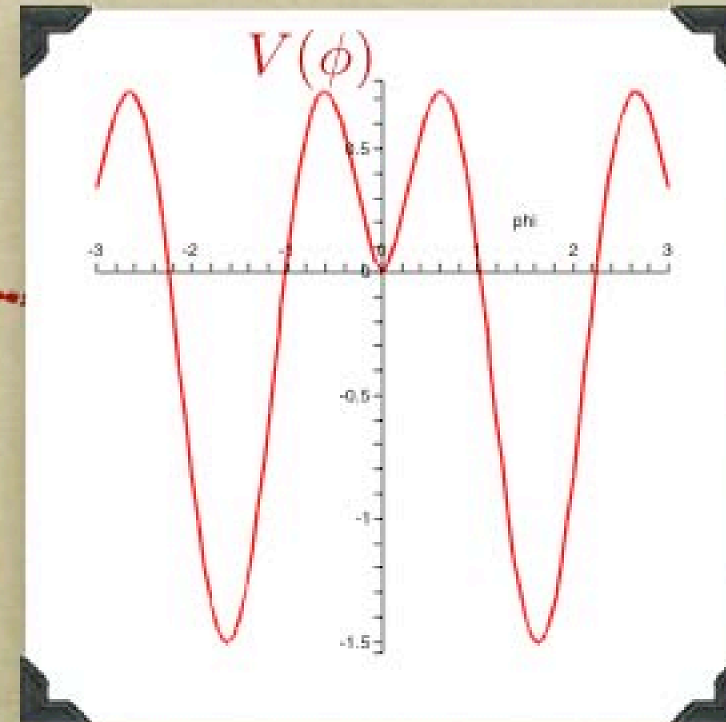
$$ds^2 = e^{A(\xi)} d\eta^2 + d\xi^2$$

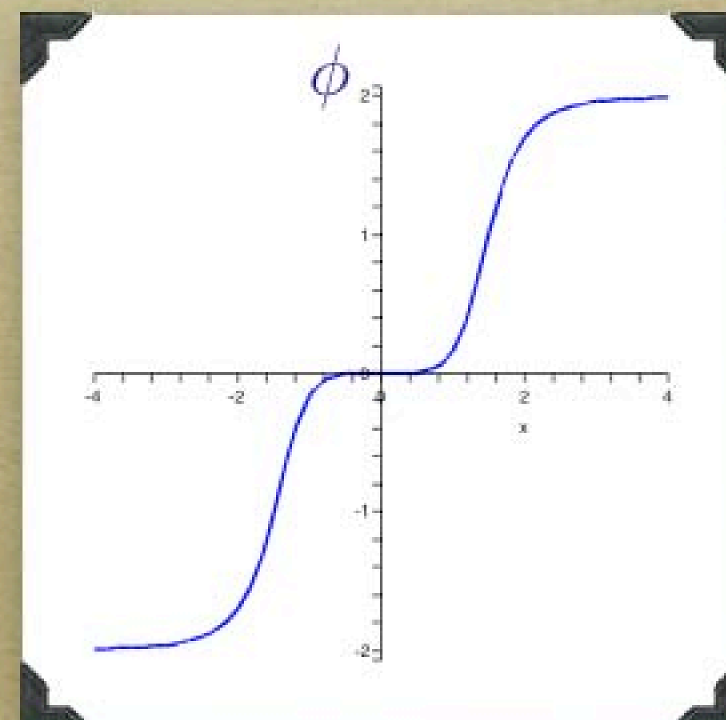
$$A' = -W$$

$$\phi' = 3 \frac{dW}{d\phi}$$

- $$V(\phi) = \frac{3}{2} \left[3 \left(\frac{dW}{d\phi} \right)^2 - 4W^2 \right]$$

Solved for $\epsilon = 1/(2n+1)$





$Q_T = 2\,\alpha$

In coordinates $ds^2 = e^{2A(r)} d\eta^2 + e^{2H(r)} dr^2$

$$H(r) = -\frac{\epsilon}{2} \ln[1 + (\frac{\alpha r}{\delta})^{2/\epsilon}] \qquad A(r) = \delta H(r)$$

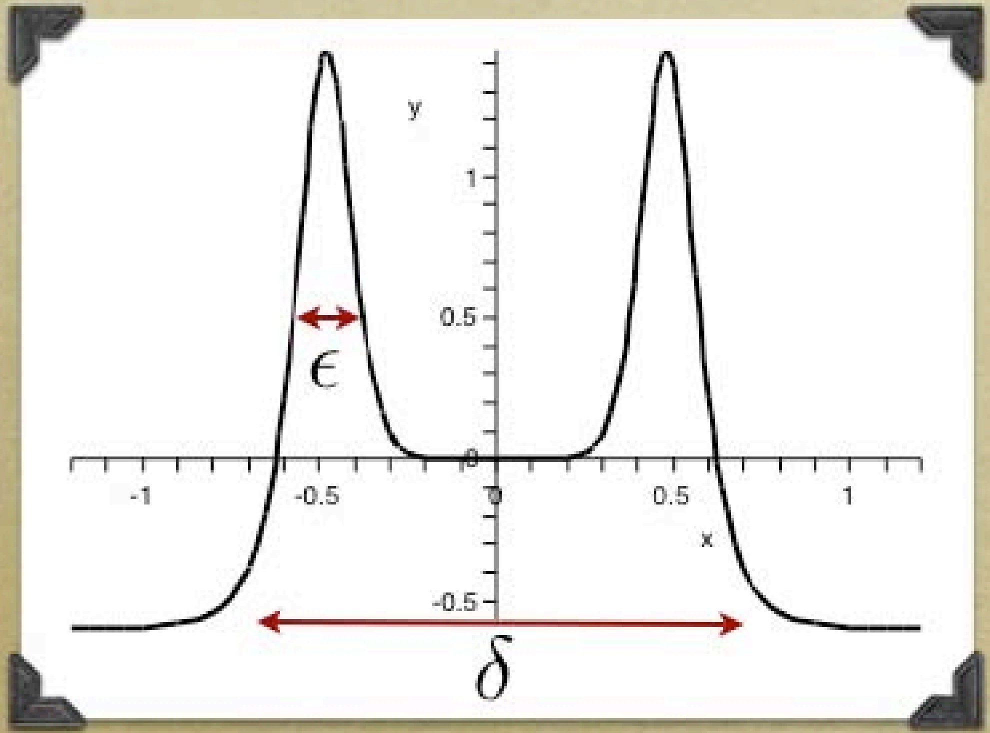
$$\phi(r) = \phi_0 \arctan(\frac{\alpha r}{\delta})^{1/\epsilon} \qquad \phi_0 = \sqrt{3\,\delta\epsilon(2-\epsilon)}$$

$$V(\phi) = 3\alpha^2 \sin(\phi/\phi_0)^{2(1-\epsilon)} \left[\frac{2/\epsilon + 4\delta - 1}{2\delta} \cos^2(\phi/\phi_0) - 2\right]$$

3 parameters

- *sub-wall thickness* ϵ
- *system thickness* $\sim \delta$
- *cosmological constant* $\Lambda = -6\alpha^2$

$$\Delta = \frac{2}{\alpha} \left(\frac{1-\epsilon}{1+\epsilon\delta} \right)^{\epsilon} \delta$$

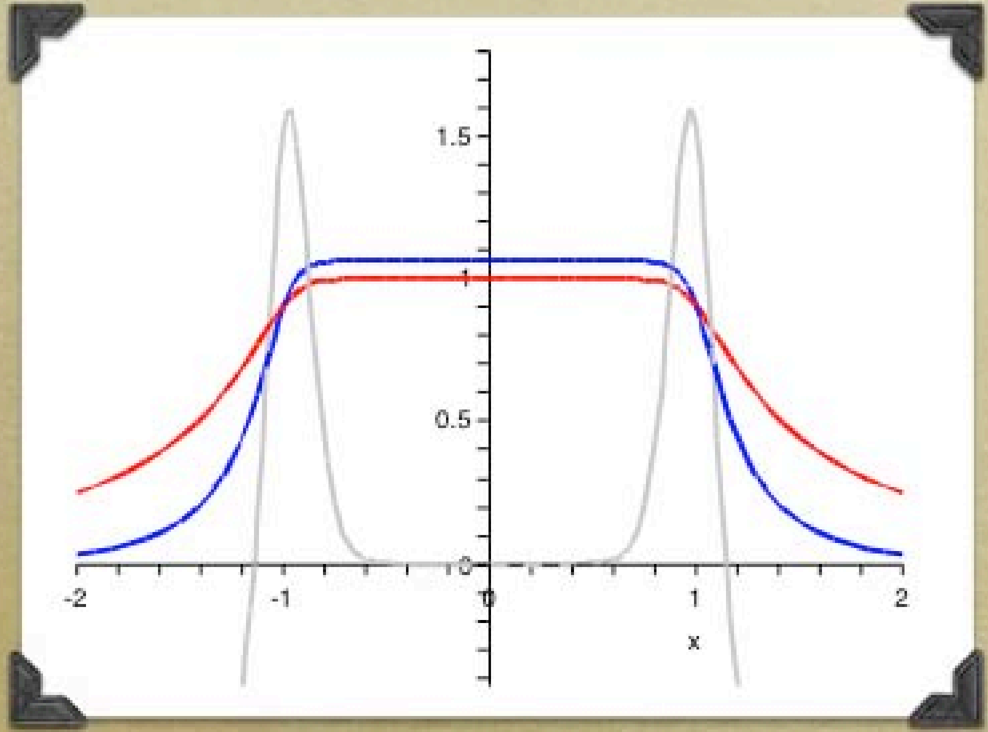


$$\Lambda = -6\alpha^2$$

Energy density of the double wall

Gravitational and fermionic zero modes

— $g_0 = N_g\, e^{2A(\xi)}$ —$f_0 = N_f\, e^{-2A(\xi) - h\int \phi(\xi) d\xi}$



For $\delta \to 0$ *, the RS wall*

Double walls -shifted

hep-th/0605160

Guerrero + AM + Pantoja + Rodríguez

$$W(\phi) = \alpha \left[\sin\left(\frac{\phi}{\phi_0}\right)\right]^{2-\epsilon} + \beta$$

$$\beta < \alpha$$

$$ds^2 = e^{A(\xi)} d\eta^2 + d\xi^2$$

$$Q_T = 2\,\alpha$$

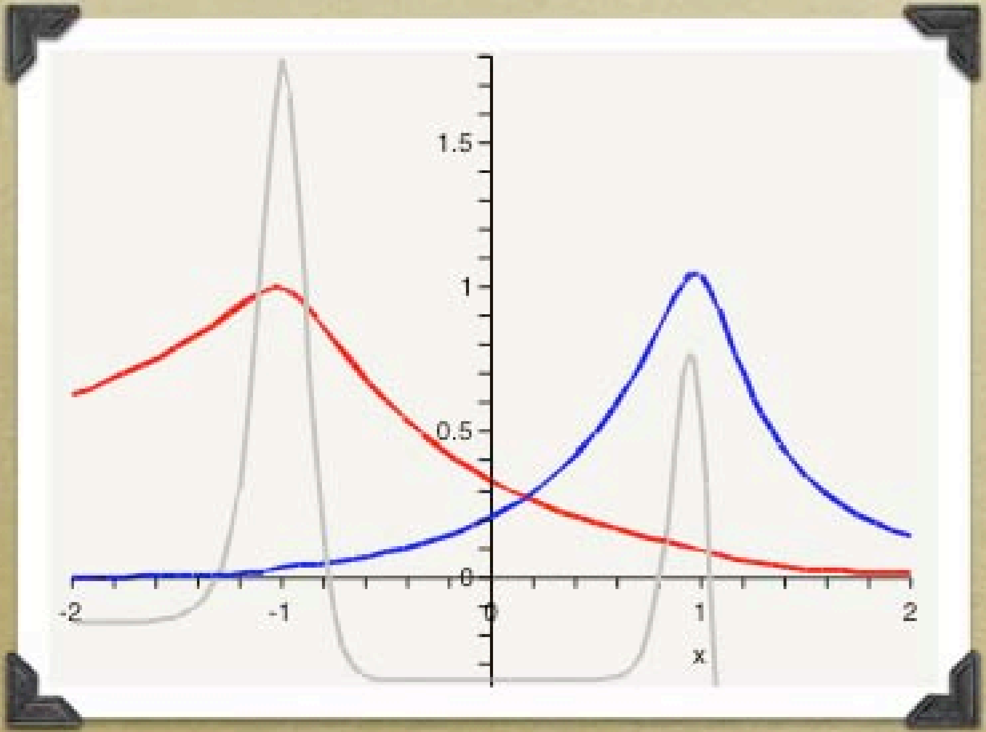
$$A' = -W - \beta$$

$$\phi' = 3\frac{dW}{d\phi}$$

- $$V = \frac{3}{2}\left[3\left(\frac{dW}{d\phi}\right)^2 - 4(W+\beta)^2\right]$$

Gravitational and fermionic zero modes

$$— g_0 = N_g \, e^{2A(\xi)} e^{-\beta \, \xi} \qquad — f_0 = N_f \, e^{-2A(\xi) - h \int \phi(\xi) d\xi} e^{+\beta \, \xi}$$

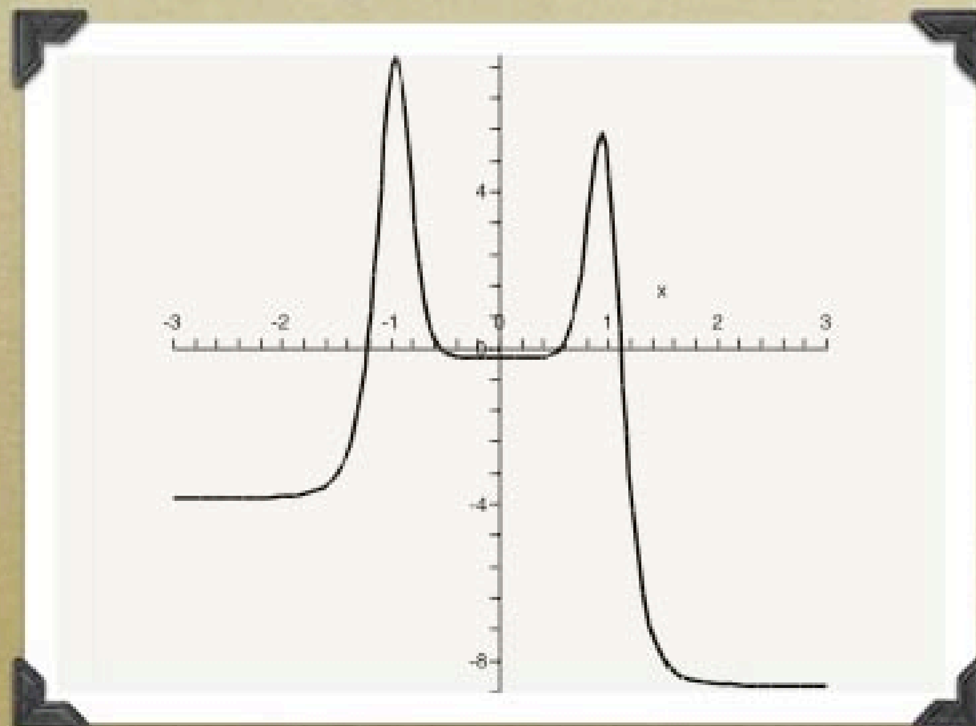


$\beta < \alpha$

Located in opposite walls

Asymmetric double wall

$$\Lambda_{+} = -6(\alpha - \beta)^2 \ , \ \ \Lambda_{+} = -6(\alpha + \beta)^2 \ , \ \ \Lambda_{\text{in}} = -6\beta^2$$



$\beta < \alpha$

Gravitons $\Rightarrow$ *smaller* Λ

Fermions $\Rightarrow$ *larger* Λ

Graviton wave function suppressed at the + wall

$$g_0 \to g_0 e^{-2\beta\xi}$$

Δ = inter-wall distance

$$g_0(\xi = \xi_+) \simeq e^{-2\beta\Delta} g_0(\xi = \xi_-)$$

$$\beta\Delta = 2\frac{\beta}{\alpha}\left(\frac{1-\epsilon}{1+\delta\epsilon}\right)^{\epsilon/2}\delta$$

$\alpha \sim \beta \qquad \epsilon \to 0 \qquad \delta \sim 20 \quad \Rightarrow$

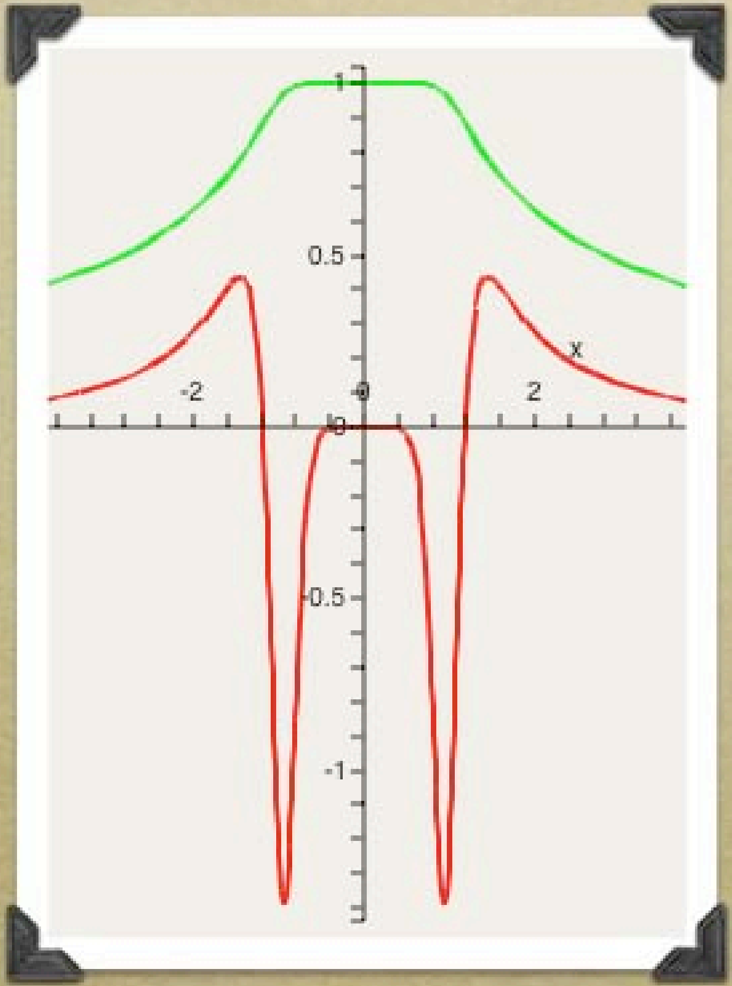
Supression

$$V_N(\xi_+) \sim 10^{-32} V_N(\xi_-)$$

$$M_p(\xi_+) \sim 10^{16} M_p(\xi_-)$$

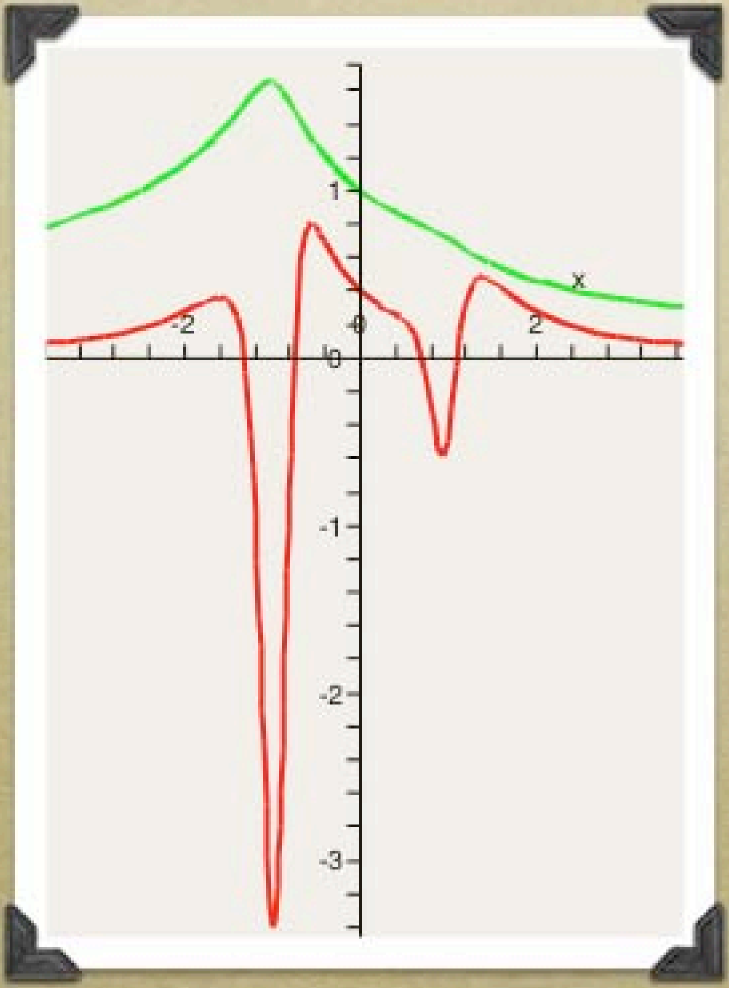
$$W(\phi) = \alpha \left[\sin\left(\frac{\phi}{\phi_0}\right) \right]^{2-\epsilon}$$

Equivalent QM potential and graviton zero mode



$$W(\phi) = \alpha \left[\sin\left(\frac{\phi}{\phi_0} \right) \right]^{2-\frac{1}{s}} + \beta$$

Equivalent QM potential and graviton zero-mode for the shifted case



Newtonian potential

- *Weak brane*

$$V_+(r) = \frac{G^{\mathrm{W}} m_1 m_2}{r}\left[1 + \kappa_+^2 \left(\frac{r_c}{r}\right)^6\right] \qquad G_N^{\mathrm{W}} = G_5 (N_0^g)^2 e^{-3\beta\delta/\alpha}$$

- *Planck brane*

$$V_-(r) = \frac{G^{\mathrm{P}} m_1 m_2}{r}\left[1 + \kappa_-^2 \left(\frac{r_c}{r}\right)^6\right] \qquad G_N^{\mathrm{P}} = G_5 (N_0^g)^2 e^{3\beta\delta/\alpha}$$

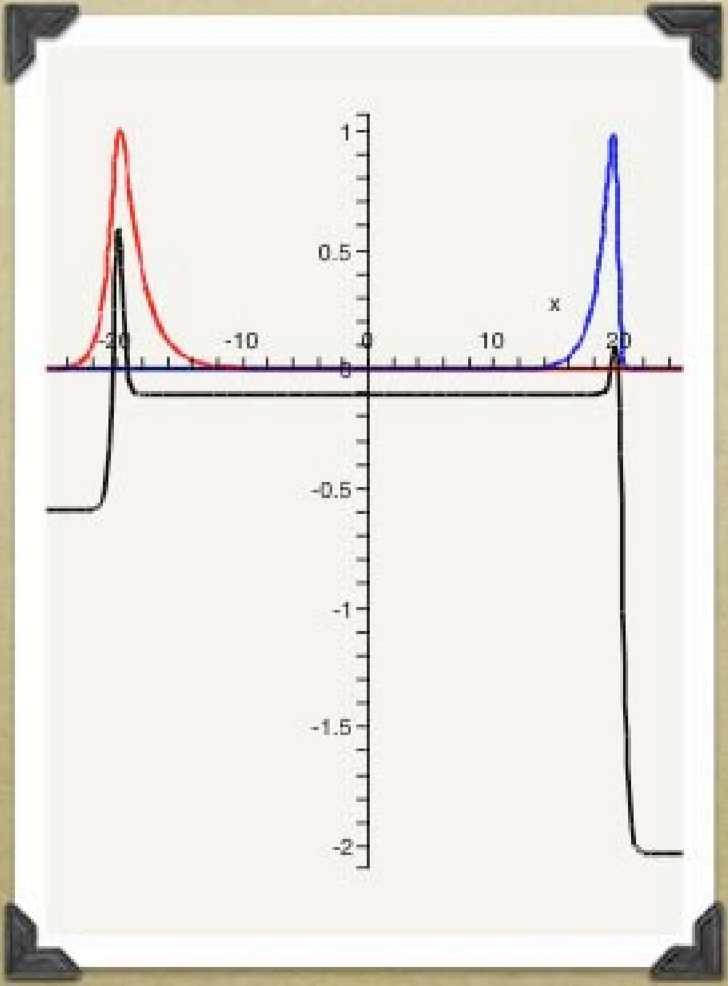
$$r_c = \alpha^{-1} \exp\left(5\beta\delta/3\alpha\right)$$

zero mode

$$\psi_0^g(z) = N_0^g e^{3A(z)/2}$$

Fermion confinement requires:

$$h > \frac{\sqrt{\Lambda_+}}{\phi_0^2} = \sqrt{\Lambda_+}\frac{1}{3(2-\epsilon)}\,\frac{1}{\delta\,\epsilon}$$



- *widely separated walls: large δ*
- *thin sub-walls: small ϵ*

$$\frac{h}{\sqrt{\Lambda_+}} < 1$$

Other double walls: similar effect

Asymmetric double walls can provide

- *Localization of gravity in 4-d*
- *Large hierarchy between gravitational constants*
- *Matter confined to the weak brane*
- *Stable solution - extension of the simplest BPS brane*

Very interesting effect