Looking for the RATIONALE in Fermion Masses

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Flavour issue in and beyond SM: an **OPEN PROBLEM**

Some bottom-up ATTEMPTS in coll. with *C.A.Savoy*

A) Viable SIMPLE TEXTURES (quarks mainly)

Status of the Art & Ours [hep-ph/0603101]

...underlying structures (hopefully) reminiscent of some

B) Broken Flavour Symmetry

a GUT Model with a rationale for quark hierarchies [hep-ph/0605xxx]

QUARK TEXTURES: the status

Goal: SIMPLE patterns with least numb. of par's (=rationale) for

$$T_d = \frac{m_d}{y_b v_d} \qquad T_u = \frac{m_u}{y_t v_u} \qquad \text{(3° eigenv =1)}$$

Difficulty: $(4_{m.rat}+3_{mix}+1_{CPV}=)$ **8 observ << texture elem's**

ASSUMPTIONS: e.g. **textures 0's** to relate CKM angles and quark mass ratios

Since GST '68

$$\begin{pmatrix} 0 & \lambda \\ \lambda & 1 \end{pmatrix} \Leftrightarrow \lambda \equiv \sin\theta_c \simeq \sqrt{\frac{m_d}{m_s}}$$

~~Georgi-Jarlskog '79~~ '93

~~U(2)-insp'd [Barbieri et al] '96 (symm & 11=31=0)~~ '01 ~~...unless fill 31~~ [Roberts et al, Kim Raby Shradin] '06 [IM Savoy]

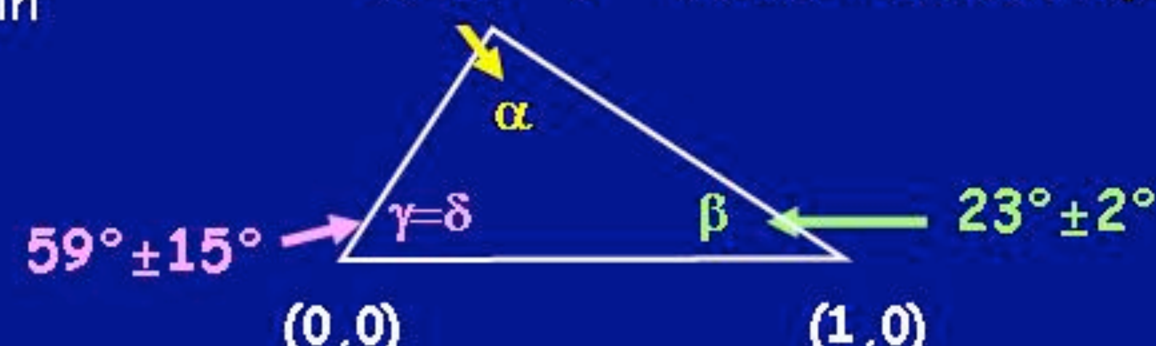
etc(ν)...

Exp. data: light q masses still uncertain

but progr. in V_{cb} V_{ub} and UT

-> STRONG TEXT. SELECTION

90°? To be TESTED! (<2yrs?)



Q: More&more complicated? Where are we?

Looking for textures...

[M&Savoy,ph/0603101]

2 non-rem CPV ph

standard par (=PDG)

$$V_{CKM} = U^{s\,\dagger}_{u_L} \begin{pmatrix} e^{-i(\phi_{12}+\phi_{23})} & & \\ & e^{-i\phi_{23}} & \\ & & 1 \end{pmatrix} U^{s}_{d_L}$$

ASSUME $s^{u_L,d_L}_{ij} \ll 1$ & $s^{u_L,d_L}_{13} \simeq 0$ ← suggested by $V_{ub} \simeq V_{us} V_{cb}$

Then $\underbrace{\alpha \simeq \phi_{12} \qquad e^{i\beta}|V_{us}| \simeq s^{d_L}_{12} - e^{-i\phi_{12}} s^{u_L}_{12}}_{\text{UT form controlled by } \phi_{12}}$ $\qquad |V_{cb}| \simeq |s^{d_L}_{23} - e^{-i\phi_{23}} s^{u_L}_{23}|$

Looking for textures...

[M&Savoy, ph/0603101]

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Then $\alpha \simeq \phi_{12}$ $\quad e^{i\beta}|V_{us}| \simeq s^{d_L}_{12} - e^{-i\phi_{12}} s^{u_L}_{12}$ $\quad |V_{cb}| \simeq |s^{d_L}_{23} - e^{-i\phi_{23}} s^{u_L}_{23}|$

FOR $\boxed{\phi_{23} = 0 \quad \phi_{12} = \alpha = \frac{\pi}{2}}$

RECOGNIZE relations among mass ratios and mix's (@m_Z)

$$\boxed{s^{d_L}_{12} \simeq \sqrt{\frac{m_d}{m_s}} \quad s^{d_L}_{23} \simeq \frac{m_s}{m_b} \quad s^{u_L}_{12} \simeq \sqrt{\cancel{3}\,\frac{m_u}{m_c}} \quad s^{u_L}_{23} \simeq \sqrt{\frac{m_c}{m_t}}}$$

m_u too big

OR V_{ub}/V_{cb} and β too small

...what kills simple U(2)-insp!

[Roberts et al, Kim Raby Shradin; '01]

... plug in !

The textures
(up to trivial sign changes):

$\lambda = \sin\theta_C$, $a,b,c,g,j = \mathbb{R}_+$ and $O(1)$

$$T_d = \begin{pmatrix} 0 & g\lambda^3 & 0 \\ g\lambda^3 & j\lambda^2 & 0 \\ 0 & j\lambda^2 & 1 \end{pmatrix} \qquad T_u = \begin{pmatrix} -c\lambda^8 & a\lambda^6 & 0 \\ a\lambda^6 & 0 & b\lambda^2 \\ 0 & b\lambda^2 & 1 \end{pmatrix}$$

source of CPV
giving $\alpha \approx$ **90°**

gives **3** if **c**$\approx -2a^2/3b^2$

$\mathbb{R}$ or $\mathbb{I}$ elements!

Par. count.: $(2)_d + (2+1)_u = 5$ $+\, i$ < 8 observables -> PREDICTIVE!

Naturally defined at M_{GUT} and embedded in SU(5):

if j from 45_H -> $T_e = T_d^T |_{j \to -3j}$ -> $m_e = m_d/3$ $m_\mu = 3m_s$ $m_\tau = m_b$ @M_{GUT}

An 'updated' GJ!

Take textures @M_{GUT} and RGE-run down to m_Z

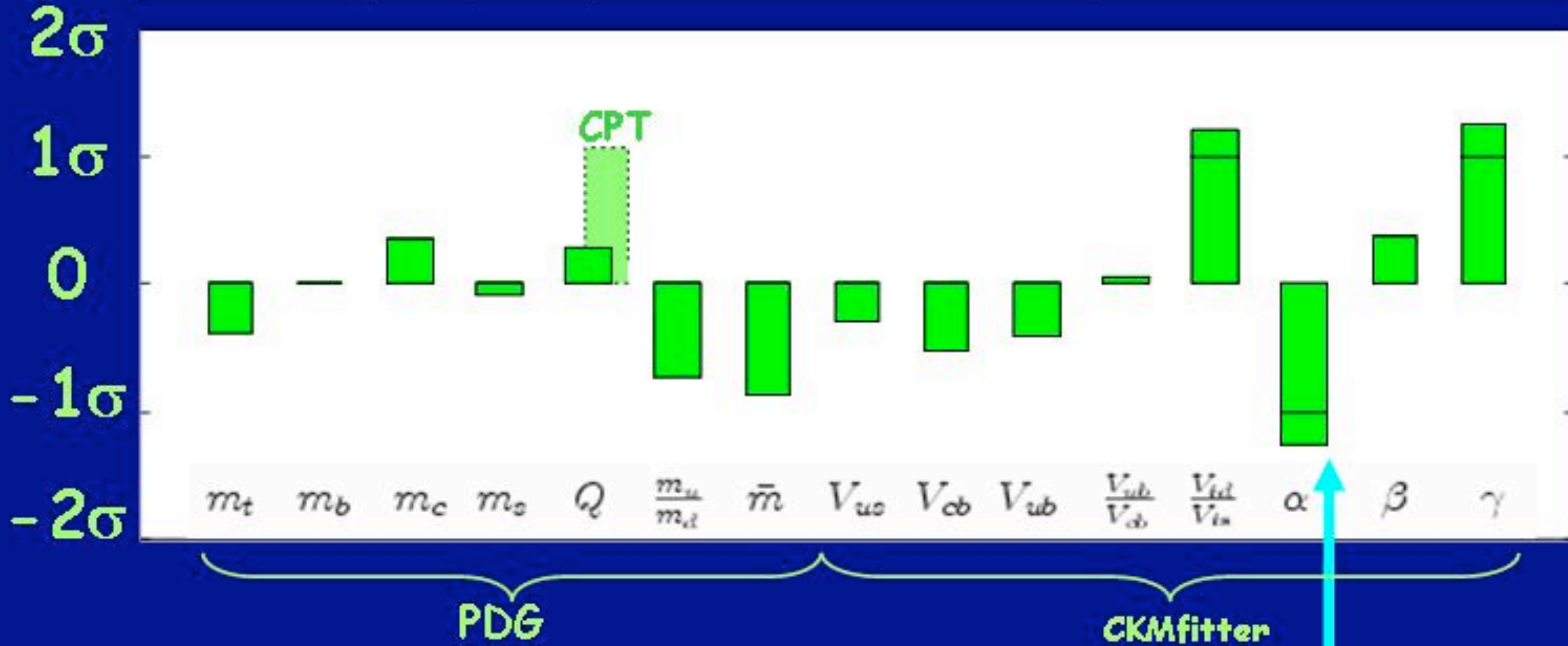
How GOOD is the fit?

EXAMPLE

large tanβ slightly preferred by fit

$\tan\beta = 35$	y_t	y_b	$\mathbf{T}_u$	$\mathbf{T}_d$
M_{GUT}	0.61	0.233	$\begin{pmatrix} -3.60\lambda^8 & 1.77\lambda^6 & 0 \\ 1.77\lambda^6 & 0 & 1.01\lambda^2 \\ 0 & 1.01\lambda^2 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0.32\lambda^3 & 0 \\ 0.32\lambda^3 & i\,0.33\lambda^2 & 0 \\ 0 & 0.33\lambda^2 & 1 \end{pmatrix}$

All ≈ within 1σ!



Now α=98°±16° while prediction is $\alpha \approx 90°$

To be TESTED in future (<2yrs?)

... what if α=98° ?

EXAMPLE

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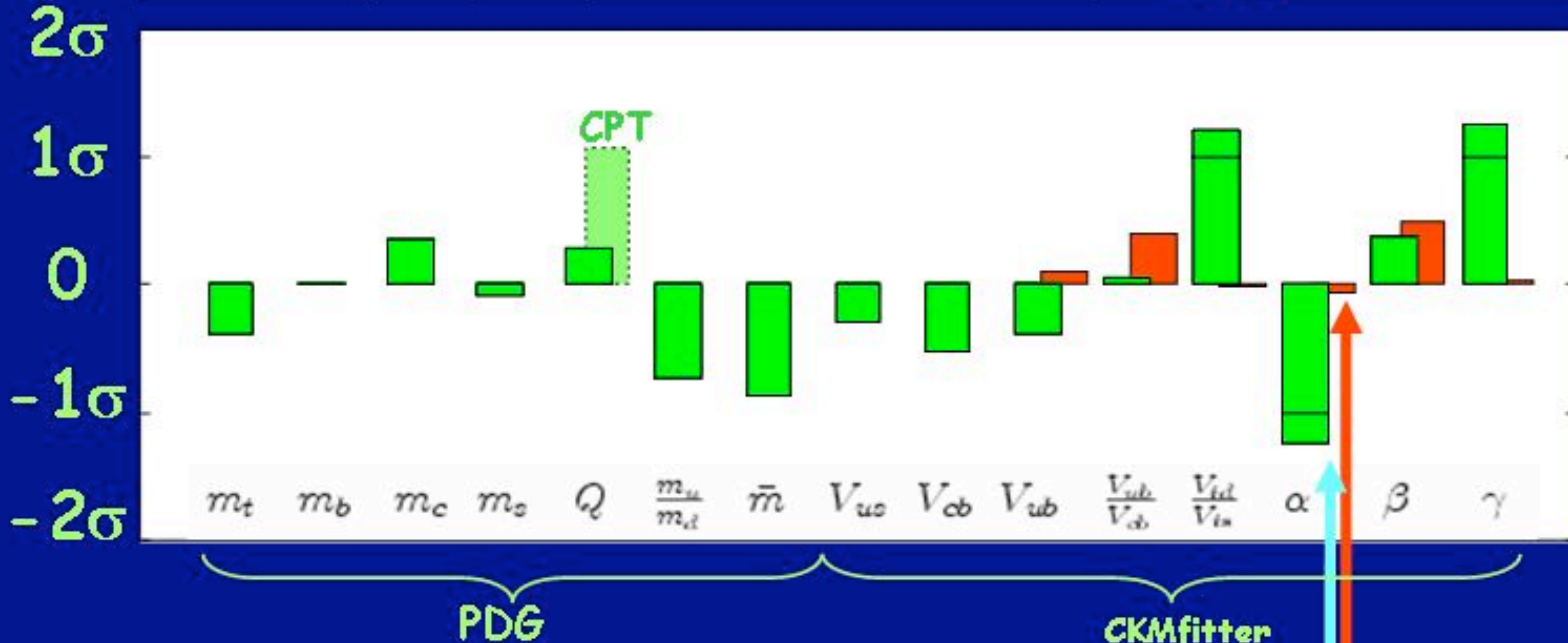


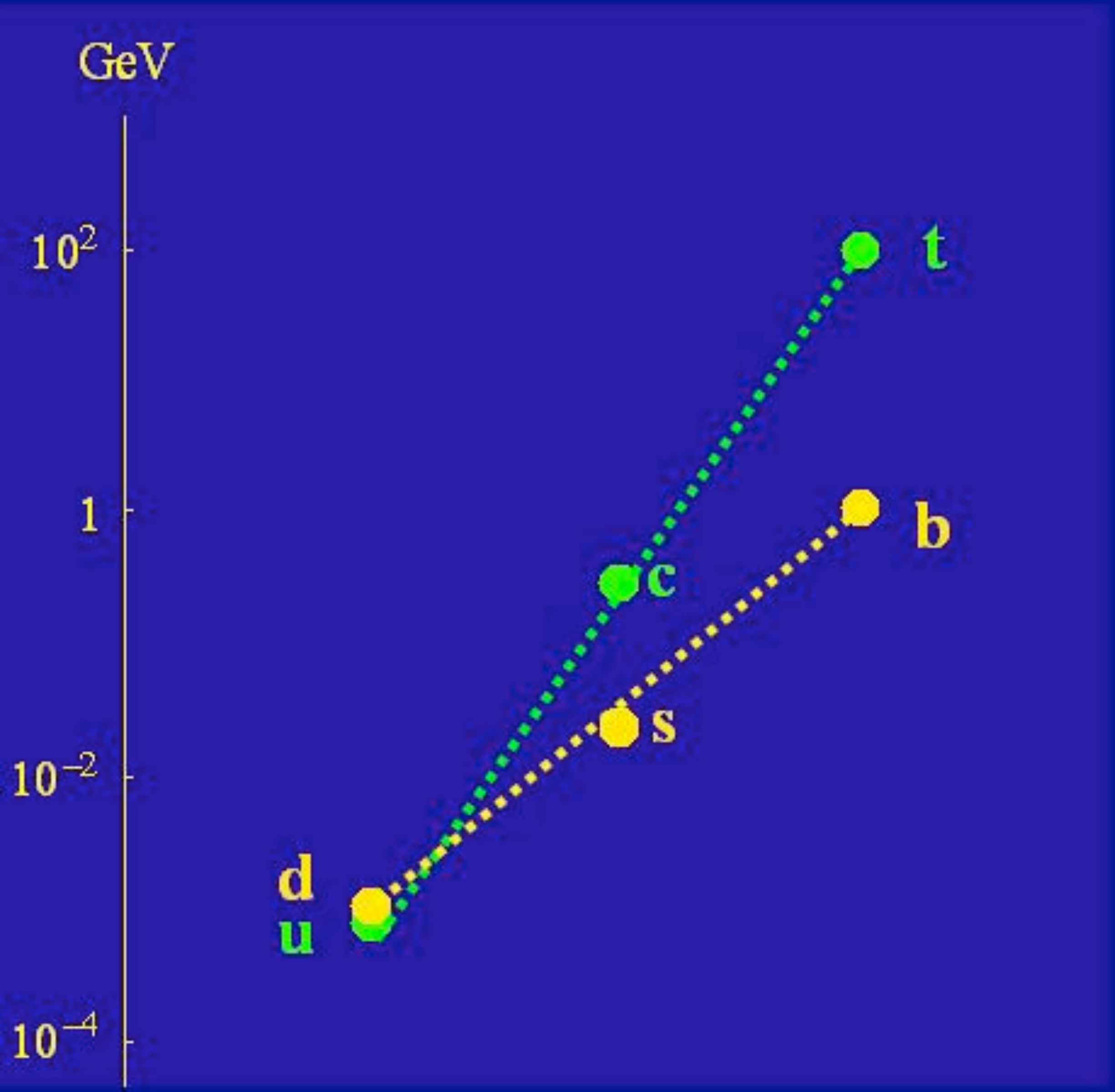
Now $\alpha = 98° \pm 16°$ while prediction is $\alpha \approx 90°$

To be TESTED in future (<2yrs?)

... what if $\alpha = 98°$?

Speculations

5 $(+i)$ par's: could be further reduced by relating



$$T_d \rightleftharpoons T_u$$

Latent GUIDELINES for flavour model building ($@M_{GUT}$):

$$\frac{m_s}{m_b} \approx \frac{m_d}{m_s} \approx \sqrt{\frac{m_c}{m_t}} \approx \sqrt{\frac{m_u}{m_c}} \approx \frac{M_{GUT}}{M_{Pl}}$$

Speculations

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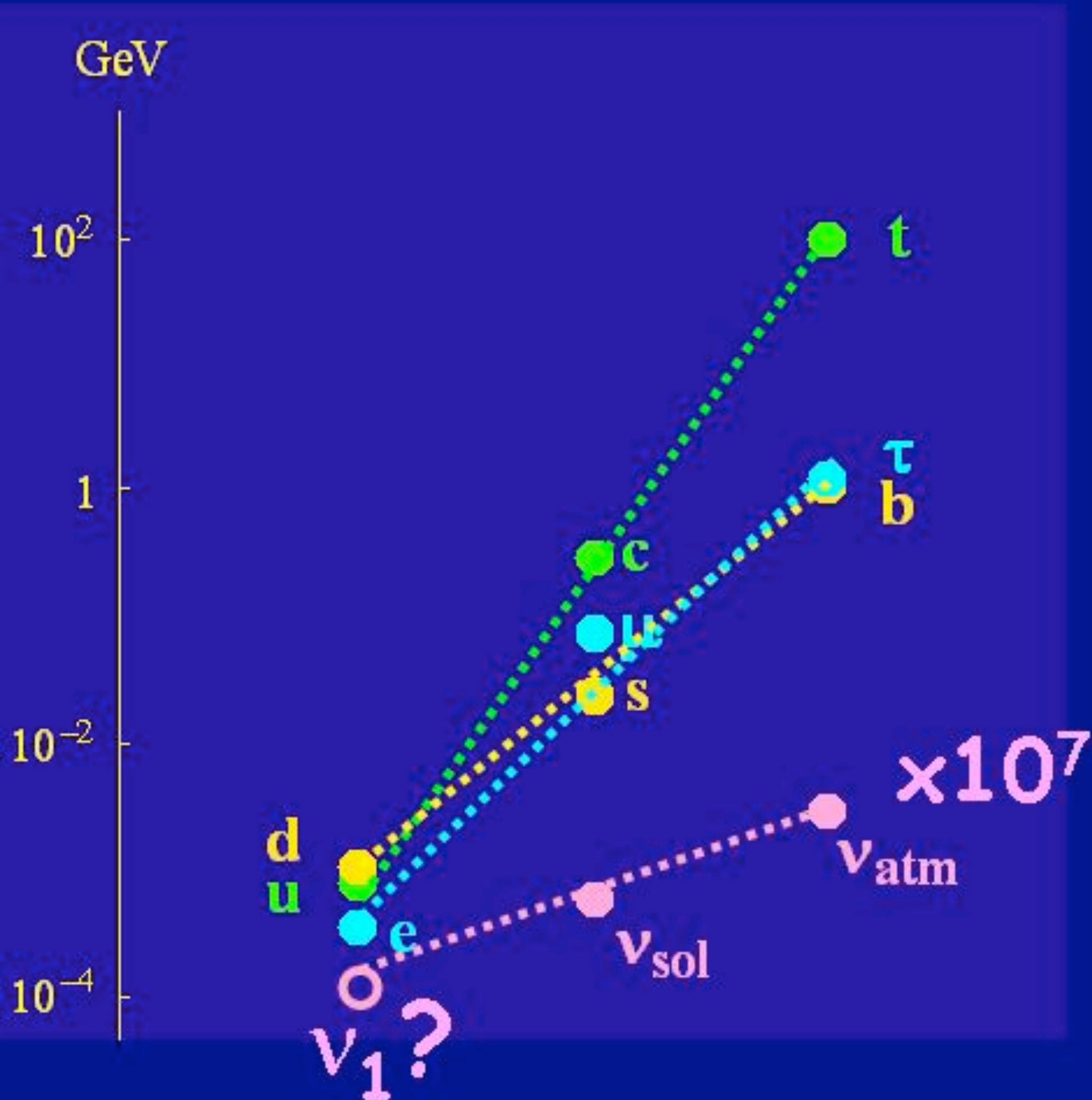
$$T_e^T \sim T_d \rightleftharpoons T_u$$



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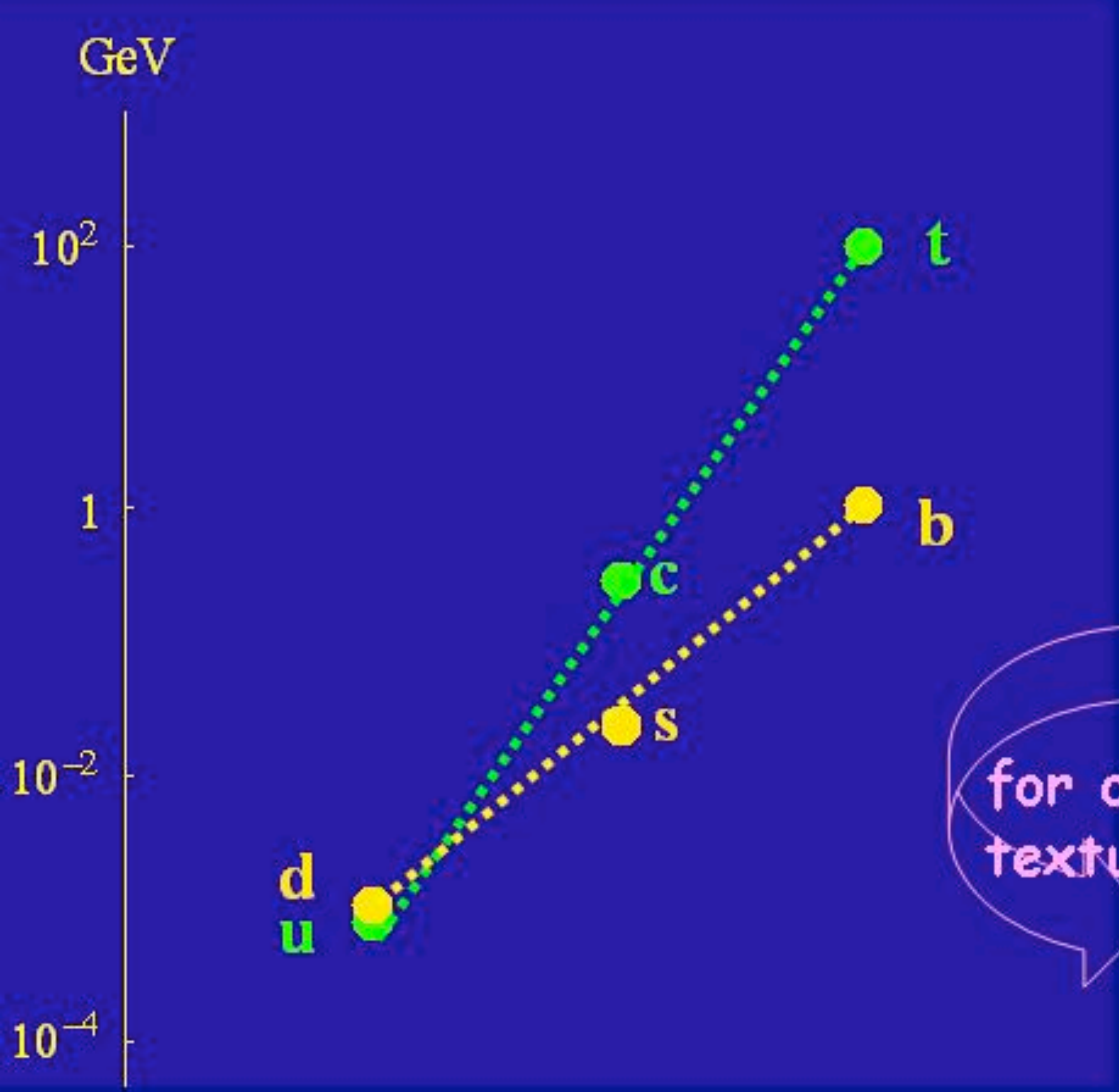

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for our textures

$$j\lambda^2 \approx \left(\frac{g\lambda^3}{j\lambda^2}\right)^2 \approx b\lambda^2 \approx \frac{a\lambda^6}{b\lambda^2}$$

O(3)!

$$\begin{pmatrix} \overset{O(\lambda^8)}{\boxed{g^2\lambda^6}} & \overset{O(\lambda^6)}{i\,\boxed{gj\lambda^5}} & 0 \\ i\,gj\lambda^5 & g^2\lambda^6 & j\lambda^2 \\ 0 & j\lambda^2 & 1 \end{pmatrix} = T_d^{T} \circ T_d \approx T_u = \begin{pmatrix} c\lambda^8 & a\lambda^6 & 0 \\ a\lambda^6 & 0 & b\lambda^2 \\ 0 & b\lambda^2 & 1 \end{pmatrix}$$

A Model

[M&Savoy,ph/0605xxx]

gauge flavour (chiral)

$$\underbrace{SU(5)}_{} \times \underbrace{O(3)_{\bar{5}} \times O(3)_{10}}_{\subset\, U(3)_{\bar{5}} \times U(3)_{10}}$$

the maximal fl symm in guage SU(5)

A Model

[M&Savoy,ph/0605xxx]

gauge — flavour (chiral!)

$$SU(5) \times O(3)_{\bar{5}} \times O(3)_{10}$$

$$m_{down} \sim (\bar{5}\times 10, 3, 3) \qquad m_{up} \sim (10\times 10_{s(a)}, 1, 3\times 3_{s(a)})$$

From eff Yuk coupl with light 2's in fl-singlet $H_{\bar{5}}$ H_5 resp (2-3 splitt assumed)
invariant under fl symm -> functions of the vev's of BIFUNDAMENTAL FLAVONS

$$P,\ J,\ G \sim (1,3,3)$$

To begin

$$\langle P\rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Lambda \qquad \langle J\rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & i & 0 \\ 0 & 1 & 0 \end{pmatrix} j\,\lambda^2 \Lambda \qquad \langle G\rangle = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} g\,\lambda^3 \Lambda$$

F-break scale $\sim M_{Pl}$

scale for GUT breaking

O(1) to be det'd from m_{down}

$$m_{down} = (P + J + G)\, v_d / M_{Pl}$$

If $JH_{\bar{5}} \sim \overline{45}$ $\Rightarrow$ $m_e^T = (P - 3J + G)\, v_d / M_{Pl}$

A Model

[M&Savoy,ph/0605xxx]

gauge — flavour (chiral)

$$SU(5) \times O(3)_{\bar{5}} \times O(3)_{10}$$

$$m_{down} \sim (\bar{5}\times 10, 3, 3) \qquad m_{up} \sim (10\times 10_{s(a)}, 1, 3\times 3_{s(a)})$$

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Proj'r: $O(3)_5 \times O(3)_{10} \to O(2)_5 \times O(2)_{10}$ -> nothing

Are sol's of these eq of motions:

$$P^T P = \Lambda P \qquad \mathrm{Tr}P = \Lambda \qquad P J^T = 0$$

$$J^T J = 0 \qquad P^T G = 0 \qquad J^T G + G^T J = i\, j \lambda^2 \Lambda G$$

$$m_{up} \sim (10\times10_{s(a)}, \underbrace{1, 3\times3_{s(a)}}_{\to\ \mathrm{flav}^T \circ\ \mathrm{flav}}) = (\overline{5}_s + \overline{50}_s, 1, \cancel{1}_s + 5_s) + (\overline{45}_a, 1, 3_a)$$

Z_4

$p, q', q, \ldots = O(1)$

$$m_{up} = [\ p\ P^TP + q''\ (P^TJ + J^TP) + q'\ (P^TJ - J^TP) + r''\ (J^TG + G^TJ) + r'\ (J^TG - G^TJ) + s\ G^TG\ ]\ \frac{v_u}{M_{Pl}^2}$$

75 or 24w/o5^{eff}

If $JH_5 \sim 45 \Rightarrow q'' = r'' = 0$

If $r' = ir$, $p, q, r, -s = \mathbb{R}_+$

$$T_u = \begin{pmatrix} sg^2\lambda^6 & rgj\lambda^5 & 0 \\ -rgj\lambda^5 & sg^2\lambda^6 & -qj\lambda^2 \\ 0 & qj\lambda^2 & p \end{pmatrix}$$

CPV by hand ...

derived from T_d !

NB: $\tan\beta \sim m_t/m_b \gg 1$

Z_4: 10, flav $\sim i$ $\quad \overline{5} \sim -1$

Neutrinos

$(\bar{5}\times\bar{5}_{s(a)}, 3\times 3_{s(a)}, 1) \sim m_\nu^{eff} \sim O(v_u^2/M_{Pl}) << m_{atm}$!

Seesaw: $O(3)_{\bar{5}} \times O(3)_{10} \times O(3)_1$

$P', J', G' \sim (1,3,1,3)$ of the **same form** of P, J, G

$(5,3,1,3) \sim m_D \sim T_d^T v_u$

Up to $O(1)$ coeff in each elem't

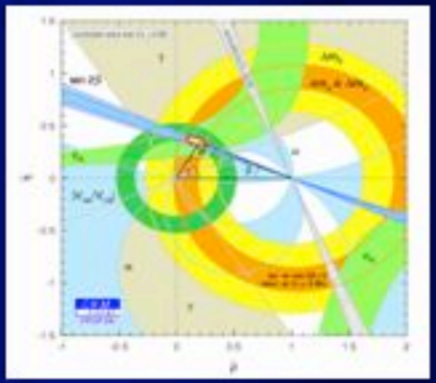
$(1,1,1,\cancel{1}+5) \sim m_M \sim T_u \langle X_{B-L}\rangle \sim T_d^T T_d \langle X_{B-L}\rangle$

$Z_4: 1 \sim i$

$m_\nu^{eff} \sim T_d (T_d^T T_d)^{-1} T_d^T v_u^2/\langle X_{B-L}\rangle \sim m_{atm}$

SMALL HIERARCHY! and also **large mixings**

Outlook & Conclusions



Quark masses and mixings:

could hide these SIMPLE TEXTURES

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with characteristic TESTABLE prediction: $\alpha \approx 90°$

and suggesting $\boxed{T_d^T \circ T_d \approx T_u}$ reminiscent of

gauge flavour symm

$$SU(5) \times O(3)_5 \times O(3)_{10}$$

An explicit model with a rationale for $\dfrac{m_s}{m_b} \approx \dfrac{m_d}{m_s} \approx \sqrt{\dfrac{m_c}{m_t}} \approx \sqrt{\dfrac{m_u}{m_c}} \approx \dfrac{M_{GUT}}{M_{Pl}}$