$\mathbb{C}$-DEFORMATION in SUPERSYMMETRY and SUPERGRAVITY

SVK

Dept. Physics, Tokyo Metropolitan University

based on

hep-th/0502026, 0504191, 0506071, 0602066, 0602115

all published in NPB, PLB, CQG

with T. Hatanaka, Y. Kobayashi (TMU), S. Sasaki (TIT)

BASIC IDEAS

Seiberg, Witten (1999)
Ooguri, Vafa (2003)
Seiberg (2003)
Ferrara, Lledo (2000)
Klemm, Tamassia, Penati (2001) ... etc.

spacetime non-commutativity

eff. field theory ← from strings

↓

superspace non-ANTI-commutativity

susy field theory ← from (nonperturbative) superstrings and branes

- gravi-photon $\langle C^{+}_{\mu\nu} \rangle \neq 0$!
- partial susy breaking to $N = 1/2$

NON-COMMUTATIVITY

Motivation

- quatum spacetime
- controlled non-locality in quantum field theory
- signatures of underlying theory (strings and branes)
- abelian solitons
- singularity resolution

Technicalities

$$[x^\mu, x^\nu]_- = i B^{\mu\nu} = \text{const.} \neq 0$$

Moyal-Weyl star product:

$$f(x) \star g(x) = f(x) \exp\left(\frac{i}{2} \overleftarrow{\partial}_\mu B^{\mu\nu} \overrightarrow{\partial}_\nu\right) g(x)$$

Problems

- violation of fundamental symmetries, e.g. Lorentz inv.
- UV/IR mixing $\rightarrow$ questionable renormalization
- infinitely many derivatives $\rightarrow$ unsolvable equations

etc.

N=1 non-anticommutative superspace

Graviphoton background ➔ N=1 non(anti)commutative superspace
(Ooguri and Vafa (2003), Seiberg (2003))

$$\left\{\theta^{\alpha}, \theta^{\beta}\right\} = C^{\alpha\beta} \neq 0$$

<u>Chiral part only</u>

$$\left\{\bar{\theta}^{\dot{\alpha}}, \bar{\theta}^{\dot{\beta}}\right\} = \left\{\bar{\theta}^{\dot{\alpha}}, \theta^{\beta}\right\} = \left[\bar{\theta}^{\dot{\alpha}}, x^{\mu}\right] = 0$$

<u>On four dimensional N=1 Euclidean superspace</u>

$$f(\theta) \star g(\theta) = f(\theta) \exp\left[-\frac{C^{\alpha\beta}}{2}\frac{\overleftarrow{\partial}}{\partial\theta^{\alpha}}\frac{\overrightarrow{\partial}}{\partial\theta^{\beta}}\right] g(\theta)$$

star product

$$= f(\theta)\left[1 - \frac{C^{\alpha\beta}}{2}\frac{\overleftarrow{\partial}}{\partial\theta^{\alpha}}\frac{\overrightarrow{\partial}}{\partial\theta^{\beta}} + \det C \frac{\overleftarrow{\partial}}{\partial\theta\theta}\frac{\overrightarrow{\partial}}{\partial\theta\theta}\right] g(\theta)$$

<u>*Preserves only N=1/2 supersymmetry*</u>

NAC deformation—Lower star power example

NAC deformed Wess-Zumino model (Seiberg(2003))

$$\boxed{\begin{aligned}\mathcal{L} &= \int d^4\theta\ \overline{\Phi}\star\Phi + \int d^2\theta\left(\frac{1}{2}m\Phi\star\Phi + \frac{1}{3}g\Phi\star\Phi\star\Phi\right) \\ &\qquad + \int d^2\overline{\theta}\left(\frac{1}{2}\overline{m}\overline{\Phi}\star\overline{\Phi} + \frac{1}{3}\overline{g}\overline{\Phi}\star\overline{\Phi}\star\overline{\Phi}\right) \\ &= \mathcal{L}(c=0) - \frac{1}{3}g\,\det cF^3 + (\text{total derivative})\end{aligned}}$$

Quantum property and renormalizability

R.Britto,B.Feng, S.J.Rey (2003) , R.Britto, B.Feng (2003)

S.Terashima J.Yee (2003), A.Romagnoni (2003)

O.Lunin, S.J.Rey (2003), D.Berenstein, S.J.Rey (2003)

A.T.Banin, I.L.Buchbinder, N.G.Pletney (2004)

Etc…

4 a

NAC deformed gauge theory (Seiberg(2003))

Modified Wess-Zumino gauge

$$V_c(y,\theta,\bar{\theta}) = V(y,\theta,\bar{\theta}) - \frac{i}{4}\bar{\theta}^2\theta^{\alpha}\varepsilon_{\alpha\beta}C^{\beta\gamma}\sigma^{\mu}_{\gamma\dot{\gamma}}\{\bar{\lambda}^{\dot{\gamma}}, A_{\mu}\}$$

$$\boxed{\begin{aligned}\int d^2\theta\ \mathrm{Tr}W^{\alpha}W_{\alpha}|_{\star} &= \int d^2\theta\ \mathrm{Tr}W^{\alpha}W_{\alpha}(C=0) - iC^{\mu\nu}\mathrm{Tr}F_{\mu\nu}\bar{\lambda}\bar{\lambda} + \frac{|C|^2}{4}\mathrm{Tr}\left(\bar{\lambda}\bar{\lambda}\right)^2 \\ \int d^2\theta\ \mathrm{Tr}\overline{W}_{\dot{\alpha}}\overline{W}^{\dot{\alpha}}\Big|_{\star} &= \int d^2\theta\ \mathrm{Tr}W^{\alpha}W_{\alpha}(C=0) - iC^{\mu\nu}\mathrm{Tr}F_{\mu\nu}\bar{\lambda}\bar{\lambda} + \frac{|C|^2}{4}\mathrm{Tr}\left(\bar{\lambda}\bar{\lambda}\right)^2 \\ &\quad +(\text{total derivative})\end{aligned}}$$

Various works

T.Araki, K.Ito, A.Ohtsuka (2003,2004,2005)
M.Alishahiha, A.Ghodsi, N.Sadooghi (2004)
I.Jack, D.R.T.Jones, L.A.Worthy (2004), A.Imaanpur (2003)
P.A.Grassi, R.Ricci, D.Robles-Llana (2003)
M.Billo, M.Frau, I.Pesando and A.Lerda (2004)
Etc…

4 в

4D single chiral model deformation

(T.Hatanaka, S.V.Ketov, Y.Kobayashi and S.S, (hep-th/0502026))

Superpotential part $\Phi = \phi + i\sqrt{2}\theta\psi + \theta^2 M$

$$\int d^2\theta \; W_\star(\Phi) = \frac{1}{2c}\left[W(\phi + cM) - W(\phi - cM)\right] - \frac{\chi^2}{4cM}\left[W_{,\phi}(\phi + cM) - W_{,\phi}(\phi - cM)\right]$$

$$c \equiv \sqrt{-\det C}$$

Non(anti)commutativity induces splitting effect in the superpotential

Generally, to calculate Kahler part deformation is very complicated
→ *Chiral reduced model* is proposed

Result is

$$
\begin{aligned}
L_{\mathsf{b}} \;=\;& \frac{1}{2}\partial^\mu\bar\phi\partial_\mu\bar\phi\left[K,_{\bar\phi\bar\phi}(\phi+cM,\bar\phi)+K,_{\bar\phi\bar\phi}(\phi-cM,\bar\phi)\right]\\
&+\frac{1}{2}\partial^2\bar\phi\left[K,_{\bar\phi}(\phi+cM,\bar\phi)+K,_{\bar\phi}(\phi-cM,\bar\phi)\right]\\
&+\frac{\bar M}{2c}\left[K,_{\bar\phi}(\phi+cM,\bar\phi)-K,_{\bar\phi}(\phi-cM,\bar\phi)\right]\\
&+\frac{1}{2c}[W(\phi+cM)-W(\phi-cM)]+\bar M\frac{\partial\overline{W}}{\partial\bar\phi}
\end{aligned}
$$

$$
\begin{aligned}
L_{\mathsf{f}} \;=\;& -\frac{1}{4c}\bar\chi^2\left[K,_{\bar\phi\bar\phi}(\phi+cM,\bar\phi)-K,_{\bar\phi\bar\phi}(\phi-cM,\bar\phi)\right]\\
&-\frac{i}{2cM}(\chi\sigma^\mu\bar\chi)\partial_\mu\bar\phi\left[K,_{\bar\phi\bar\phi}(\phi+cM,\bar\phi)-K,_{\bar\phi\bar\phi}(\phi-cM,\bar\phi)\right]\\
&-\frac{i}{2cM}(\chi\sigma^\mu\partial_\mu\bar\chi)\left[K,_{\bar\phi}(\phi+cM,\bar\phi)-K,_{\bar\phi}(\phi-cM,\bar\phi)\right]\\
&-\frac{\bar M}{4cM}\chi^2\left[K,_{\phi\bar\phi}(\phi+cM,\bar\phi)-K,_{\phi\bar\phi}(\phi-cM,\bar\phi)\right]\\
&\dots
\end{aligned}
$$

Multi chiral deformation

Two dimensional result (L.A-Gaume and M.A.V-Mozo (2005))

Four dimensional chiral reduced model (HKKS(hep-th/0506071))

Chiral reduced multi-chiral NAC deformed NLSM Lagrangian

$$\boxed{\begin{aligned}\mathcal{L} \ = \ & \frac{1}{2}M^iY_{,i}+\frac{1}{2}\partial^\mu\bar{\phi}^{\bar{p}}\partial_\mu\bar{\phi}^{\bar{q}}Z_{,\bar{p}\bar{q}}+\frac{1}{2}\partial^2\bar{\phi}^{\bar{p}}Z_{,\bar{p}}-\frac{1}{4}(\chi^i\chi^j)Y_{,ij}\\ & -\frac{1}{2}i(\chi^i\sigma^\mu\bar{\chi}^{\bar{p}})\partial_\mu\bar{\phi}^{\bar{q}}Z_{,\bar{p}\bar{q}}-\frac{1}{2}i(\chi^i\sigma^\mu\partial_\mu\bar{\chi}^{\bar{p}})Z_{,i\bar{p}}\end{aligned}}$$

$$Z(\phi,\bar{\phi},M) \ = \ \int_{-1}^{1}d\xi\ K^\xi=\int_{-1}^{1}d\xi\ K(\phi^i+\xi cM^i,\bar{\phi}^{\bar{j}})$$

$$Y(\phi,\bar{\phi},M,\bar{M})=\bar{M}^{\bar{p}}Z_{,\bar{p}}-\frac{1}{2}(\bar{\chi}^{\bar{p}}\bar{\chi}^{\bar{q}})Z_{,\bar{p}\bar{q}}+c\int_{-1}^{1}d\xi\ \xi\left[\partial^\mu\bar{\phi}^{\bar{p}}\partial_\mu\bar{\phi}^{\bar{q}}K^\xi{}_{,\bar{p}\bar{q}}+\partial^2\bar{\phi}^{\bar{p}}K^\xi{}_{,\bar{p}}\right]$$

$$\mathcal{L}_{\text{pot.}}=\frac{1}{2}M^i\widetilde{W}_{,i}-\frac{1}{4}(\chi^i chi^j)\widetilde{W}_{,ij}+\bar{M}^{\bar{p}}\overline{W}_{,\bar{p}}-\frac{1}{2}(\bar{\chi}^{\bar{\phi}}\bar{\chi}^{\bar{q}})\overline{W}_{,\bar{p}\bar{q}}$$

$$\widetilde{W}(\phi,M)=\int_{-1}^{1}d\xi\ W(\phi^i+\xi cM^i)$$

7

EXAMPLE

Deformed CP^1 metric with superpotential

$$M = -\frac{\alpha - \sqrt{\alpha^2 + \left(2c\bar{\phi}(1+\kappa^{-2}\phi\bar{\phi})\overline{W}_{,\bar{\phi}}\right)^2}}{2c^2\kappa^{-2}\bar{\phi}^2\overline{W}_{,\bar{\phi}}}$$ On-shell auxiliary field

$$\boxed{\begin{aligned}
g_{\phi\bar{\phi}} &= \frac{\alpha\kappa^{-2}c^2\bar{\phi}^2(\overline{W}_{,\bar{\phi}})^2}{\left(\alpha - \sqrt{\alpha^2 + \left(2c\bar{\phi}(1+\kappa^{-2}\phi\bar{\phi})\overline{W}_{,\bar{\phi}}\right)^2}\right)\sqrt{\alpha^2 + \left(2c\bar{\phi}(1+\kappa^{-2}\phi\bar{\phi})\overline{W}_{,\bar{\phi}}\right)^2}} \\
g_{\phi\phi} &= 0 \\
g_{\bar{\phi}\bar{\phi}} &= \frac{-2\alpha^{-1}c^2(1+\kappa^{-2}\phi\bar{\phi})\overline{W}_{,\bar{\phi}}}{\left(\alpha - \sqrt{\alpha^2 + \left(2c\bar{\phi}(1+\kappa^{-2}\phi\bar{\phi})\overline{W}_{,\bar{\phi}}\right)^2}\right)\sqrt{\alpha^2 + \left(2c\bar{\phi}(1+\kappa^{-2}\phi\bar{\phi})\overline{W}_{,\bar{\phi}}\right)^2}} \\
&\quad \times \Big[4c^2\bar{\phi}^2(\overline{W}_{,\bar{\phi}})^3(1+\kappa^{-2}\phi\bar{\phi}) \\
&\quad +\alpha\left(\alpha - \sqrt{\alpha^2 + \left(2c\bar{\phi}(1+\kappa^{-2}\phi\bar{\phi})\overline{W}_{,\bar{\phi}}\right)^2}\right)(2\overline{W}_{,\bar{\phi}} + \bar{\phi}\overline{W}_{,\bar{\phi}\bar{\phi}})\Big]
\end{aligned}}$$

$\Rightarrow$ **Metric is controlled by the superpotential**

FURTHER APPLICATIONS

- C-deformation of effective potentials in susy gauge theories (hep-th/0502026)

C-deformed Veneziano-Yankielowicz superpotential:

(ii) unbroken gauge invariance

(iii) no further dynamical susy breaking

✓ (i) non-vanishing gluino condensate

- C-deformation of $N=2$ extended supergravity (hep-th/0602115):

(i) partial local susy breaking (to $N=1/2$)

(ii) relation to gravitino condensate:

$$< C^+_{\mu\nu} > = \frac{\varkappa}{2} < \bar{\Psi}_{[\mu} \chi_{\nu]} >^+$$

and much more...

CONCLUSION

C-deformation ~ simple window into some nonperturbative physics of susy gauge theories and superstrings.