Unification in Intersecting Brane Models

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University of Bonn

based on

- KSC, J. E. Kim, JHEP **0511** (2005) 043
- KSC, hep-th/0603186
- KSC, to appear

Planck 06, Paris, June 1, 2006

The Standard Model

$SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge theory with

q	$(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$	u^c	$(\overline{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}}$	d^c	$(\overline{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}$
l	$(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$	e^c	$(\mathbf{1},\mathbf{1})_1$	ν	$(\mathbf{1},\mathbf{1})_0$ (?)

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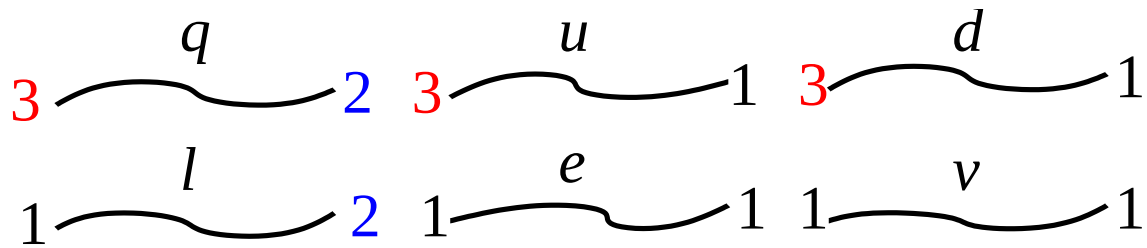
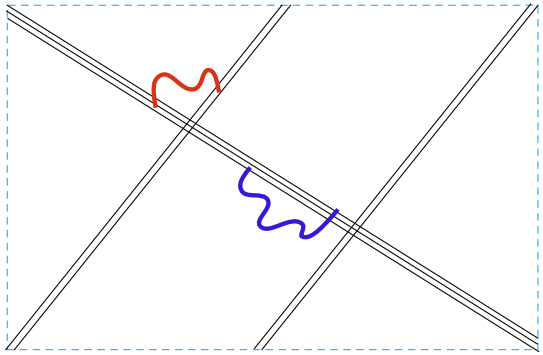
bifundamental

Explanation: attaching charges on open string endpoints



Intersecting brane world

Typical setup: D6s on T^6 wrapping 3-cycle = 3 + (3+1)D



2 families of quarks $(\mathbf{3},\mathbf{2})$ under $SU(3)\times SU(2)$

- # families = # intersections
- Yukawa hierarchy
- T-dual: D6 with constant flux $\quad A_1 = 0, A_2 = F_{12}X^1 = X'^2$
- gauge coupling

$$-\frac{1}{4g^2}\int d^{p+1}xF^2 \rightarrow$$

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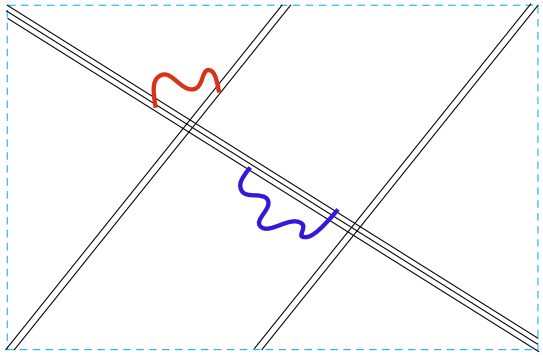

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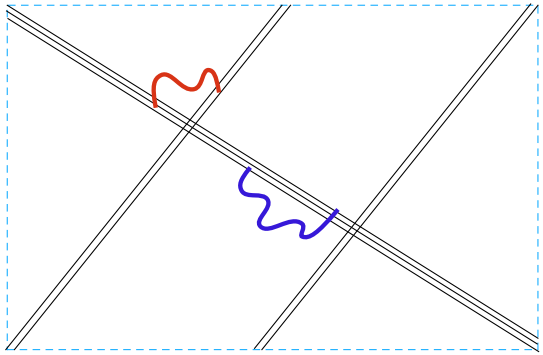

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- running coupling
- spontaneously broken gauge symmetry

Origin of bifundamental

24 of SU(5)

$$\mathbf{25} = \begin{pmatrix} \mathbf{9} & (\mathbf{3},\mathbf{2}) \\ (\overline{\mathbf{3}},\mathbf{2}) & \mathbf{4} \end{pmatrix}$$

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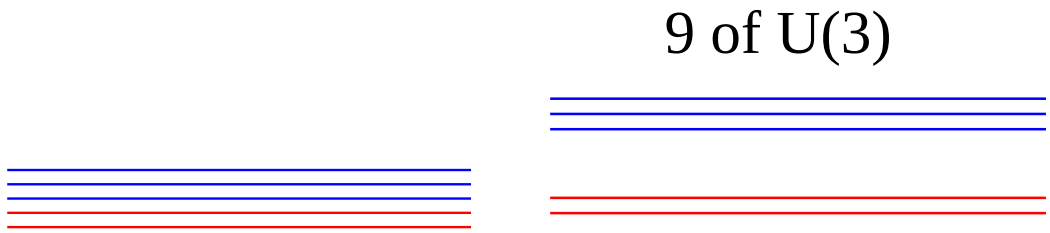

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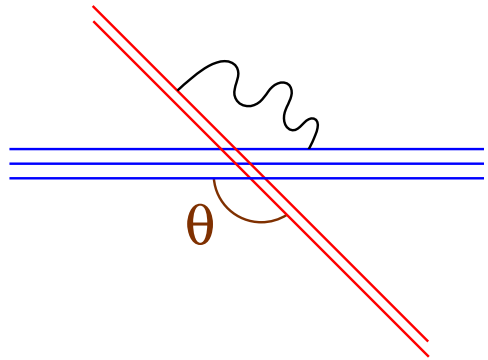

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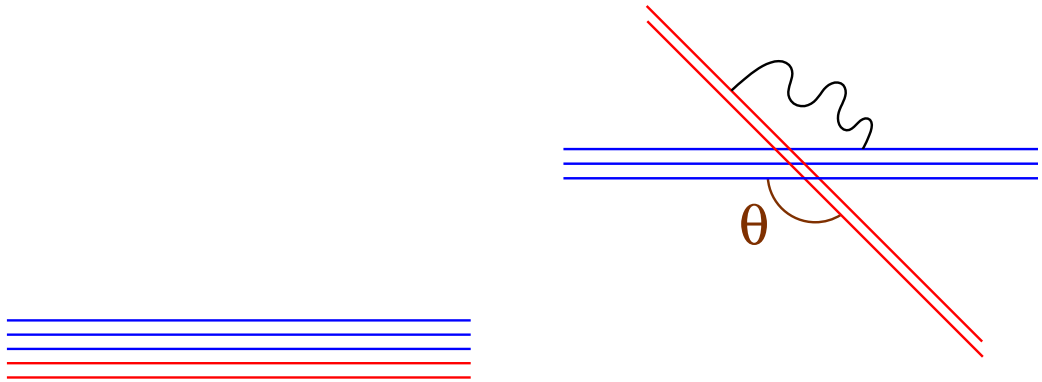

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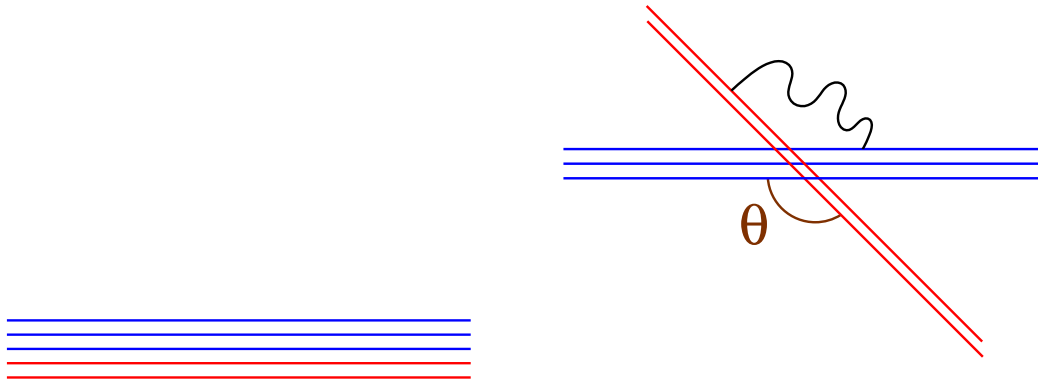

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gauge symmetry breaking

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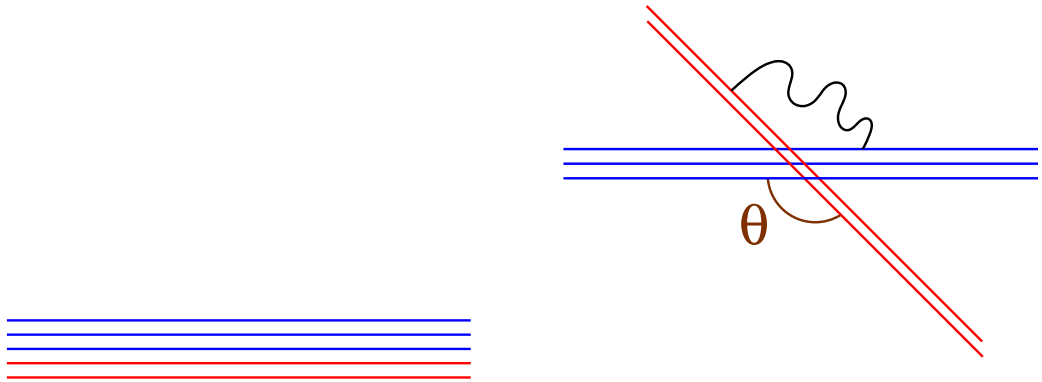

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gauge symmetry breaking

- parallel separation
- rotation ☹ angle quantized in a torus ['t Hooft]
- further broken by orbifold projection

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D-brane $\leftrightarrow$ gauge theory
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Tadpole cancellation guarantees anomaly freedom

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- $\Omega R(-1)^{F_L}$ fixed plane R: refl, $\Omega : X_L \leftrightarrow X_R$
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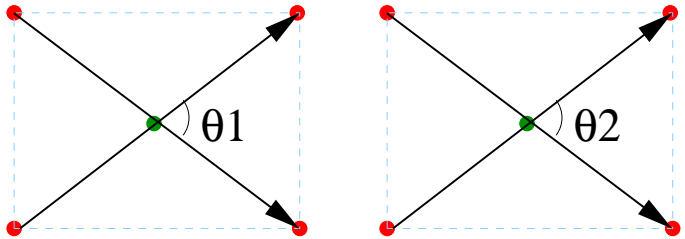
☺ 16 D9s + 1 O9 all coincident = Type I $SO(32)$ theory

- broken by parallel separation and "rotation"

Recombination of 1/4-BPS cycles



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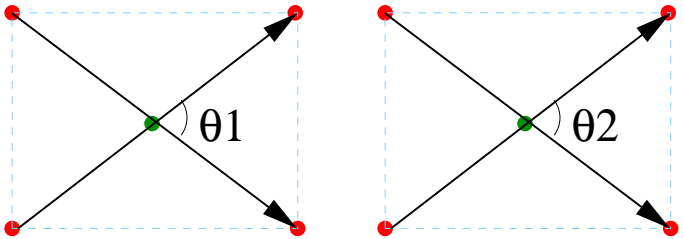

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- $\theta_1 = \theta_2 \leftrightarrow F_{12} = F_{34}$ $\delta\chi = \Gamma^{mn} F_{mn} = (\Gamma^{12} F_{12} + \Gamma^{34} F_{34})\epsilon = 0$

  local equation $\frac{\partial y_1}{\partial x_1} = -\frac{\partial y_2}{\partial x_2}$ Cauchy–Riemann

  1/4 BPS = stable SUSY = minimal vol. (special Lagrangian)

  Theorem: Any complex curves in $\mathbf{C}^2$ is sLag

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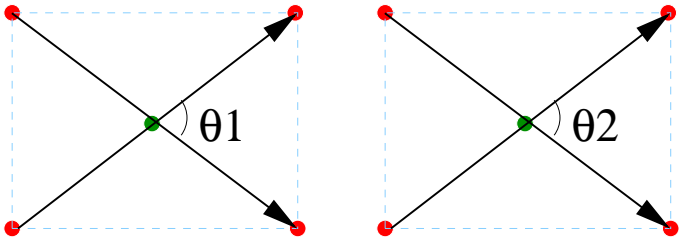

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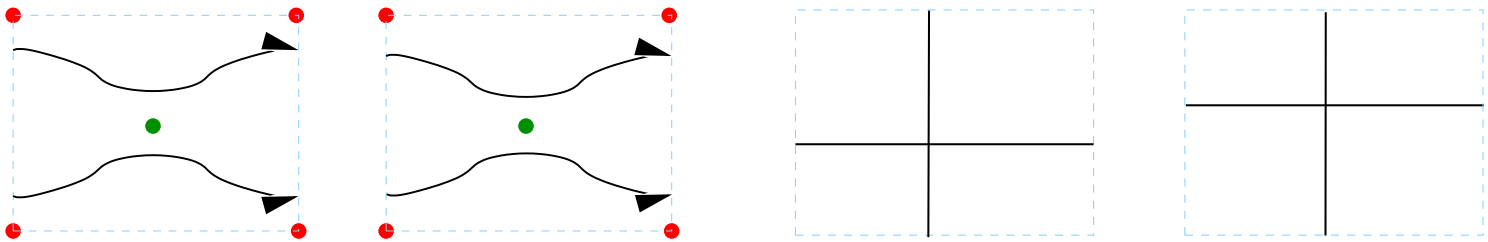

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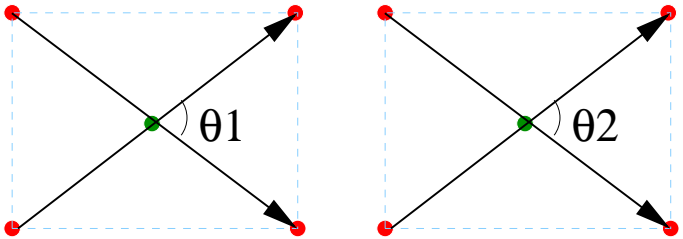

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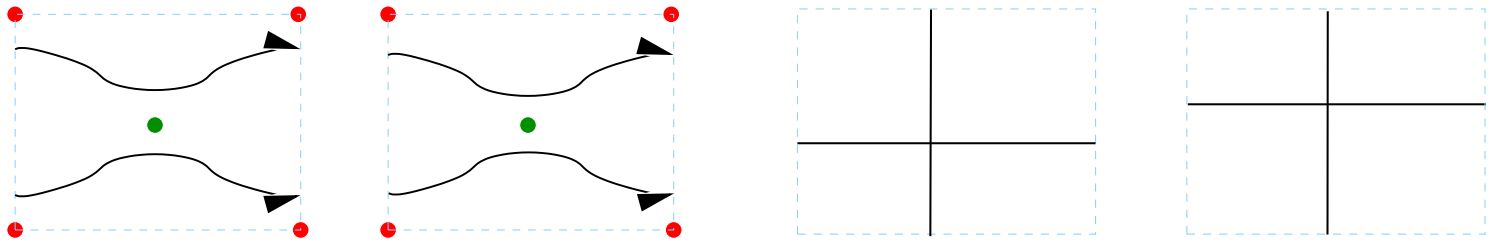

- marginal deformation, same volume [CKS, Kim] [Erdminger et al] [Douglas and Zhou]
- T-dual to D4-D0 bound state: 1/4 BPS toron
- U-dual to (F,Dp) bound state = string junction [CKS, Kim], [Dasgupta, Mukhi]

Coupling unification

Dirac–Born–Infeld action

$$S = -T_p \int_{p+1} e^{-\Phi} \sqrt{-\det(G_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})}$$

Expansion around ‘D-brane’ background ‘A_μ’ $= \langle A_{\mu,0} \rangle + A_\mu$

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$$-\frac{1}{4{g_{p+1}}^2}\int_{p+1} F^2 \quad \longrightarrow \quad -\frac{V_{p-3}}{4{g_{p+1}}^2}\int_4 F^2.$$

4D coupling

$$g_4^2 \propto \frac{1}{V_{p-3}}.$$

V_{p-3} volume of the cycle D-branes wrap

previous example: naive volume dependence fails

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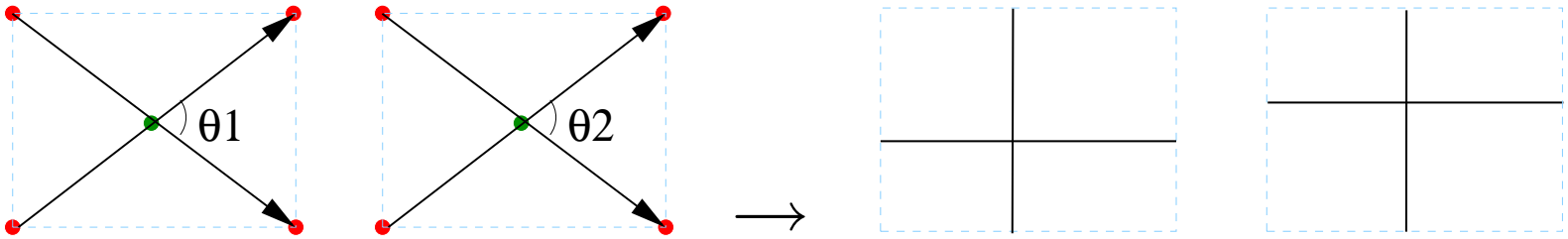
tilted and nonabelian — Full DBI!

Correct volume and coupling [Choi] [Choi, Kim]

Again D6 and O-image of it

$$\big[(1,1)(1,1)+(1,-1)(1,-1)\to 2(1,0)(1,0)+2(0,1)(0,1)\big](1,0)$$



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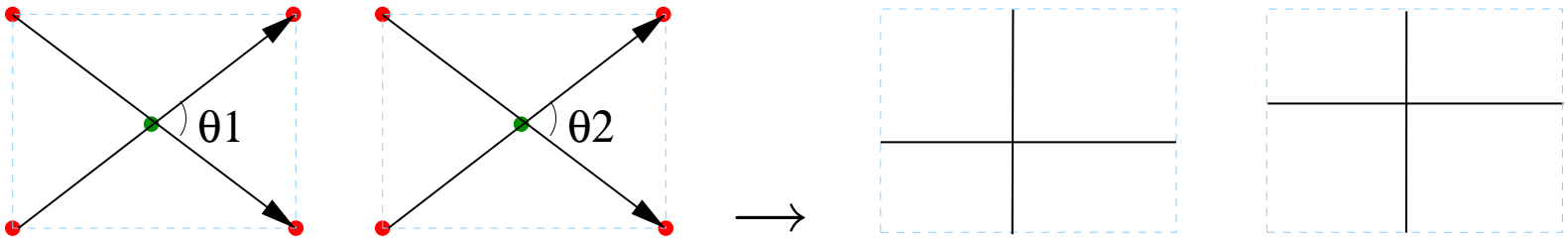

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volume actually the same at every stage: full DBI action

$$\mathrm{Tr}\sqrt{(1+(2\pi\alpha F_{12})^2)(1+(2\pi\alpha F_{12})^2)(1+0^2)}$$
$$=2\tau_9+2\tau_5=N\tau_9+C_0\tau_5$$

nonabelian, tilted geometry cannot be caught by YM

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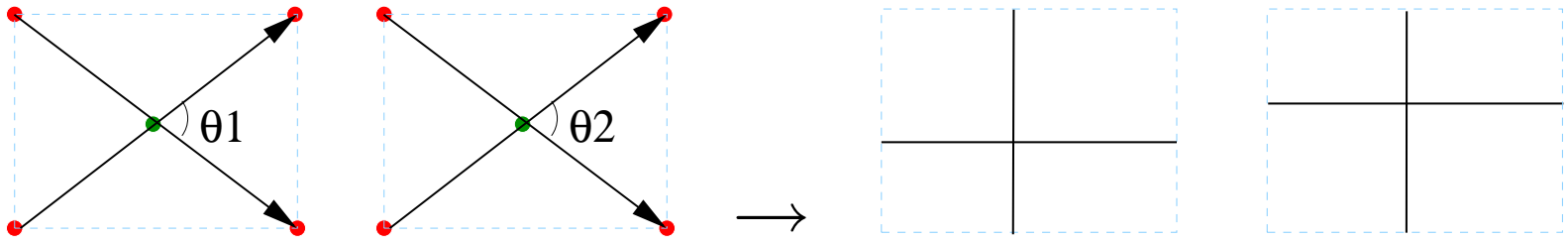

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same volume = same energy = same coupling constant 'A_μ' $=\langle A_{\mu,0}\rangle+A_\mu$

coupling unification

no trace ambiguity $F_{12}=F_{34}\propto\sigma_3$, others $\propto I_2$

Example [Cvetic, Shiu, Uranga]=[Berkooz, Leigh]

[Cvetic, Shiu, Uranga 01]

- IIA on T^6/P orbifold, $P = \mathbf{Z}_2 \times \mathbf{Z}_2 = \{\theta, \omega\}$
- orientifold action $R : (z_1, z_2, z_3) \leftrightarrow (\bar{z}_1, \bar{z}_2, \bar{z}_3)$
  4 O6s $\Omega R, \theta\Omega R, \omega\Omega R, \omega\theta\Omega$ R
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Deformable

- By brane recombinations and parallel translation

all can be on top of O6s

- Final group $Sp(8)^4 \in SO(32)^4$, branching $\mathbf{496} \to \mathbf{136} + 3 \cdot \mathbf{120}$
- T_{579}-dual: 4 O6s $\to$ 1 O9 and 3 O5s $\Omega, \theta\Omega, \omega\Omega, \omega\theta\Omega$,
  orbifold group P invariant

☺ type I compactification on P [Berkooz, Leigh 96]

All is type I SO(32) compactification

For every Dp

- Op required by tadpole cancellation
- All the Dp branes on top of Op planes
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More than one: ODps and OD$(p-4)$s since #ND$=4$ for SUSY

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- Every Op, we have 16 Dp (fixed pts $\times$ tension) = SO(32)
  OD9 and OD5 exchange symmetry
- embedded into SO(32) adjoints

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SO(32) adoints = # O-planes

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Also,

- always exists a T-dual dir.: OD9s and/or OD5s $\rightarrow$ OD6s
- M-theory on G_2 manifold

Conclusion

All SUSY intersecting brane model is type I compactification

- Symmetry breaking: brane recombinations
- D-branes and O-mirror images: preserving 1/4 SUSY
  physics dual to D0-D4 bound state: toron
- Gauge group and coupling unify
  Correct $\sin^2\theta_W = 3/8$ possible
- Compactification specified by orbifold group
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Outlook

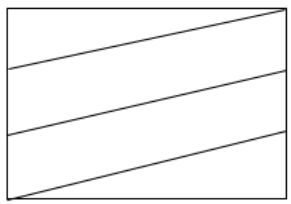
- Calabi-Yau, flux?
- Finding MSSM vacua by top-down
- Nonabelian DBI action: intersecting brane physics
- Moduli stabilization
- More understanding on D0-D4 torons
- Duality

Toron quantization ['t Hooft] [Guralnik, Ramgoolam]

T-dual picture: constant magnetic field
$U(N) \simeq SU(N) \times U(1)/\mathbf{Z}_N$
On a torus, field periodicity up to $\mathbf{Z}_N$ twist allowed $\psi \to e^{2\pi i n_{mn}\mathbf{1}}\psi$

$$(0,0) \to (2\pi R_1, 0) \to (2\pi R_2, 2\pi R_2) \to (0, 2\pi R_1) \to (0,0)$$

Quantization $F_{12} = 2\pi \begin{pmatrix} \frac{m^{(1)}}{n^{(1)}} I_{n^{(1)}} & & \\ & \ddots & \\ & & \frac{m^{(k)}}{n^{(k)}} I_{n^{(k)}} \end{pmatrix}.$



$U(1)$ embedded in $U(3)$

broken phase of $U(n^{(1)}) \times \cdots \times U(n^{(k)}) \in U(N), N = \sum n^{(i)}$

Every quantity represented by Chern numbers $c_1 = \sum_1^k m_a$