Noncommutative Phenomenology: mm scale modification of gravity without mm scale extra dimensions

Steven Abel, IPPP

- w/ V.Khoze, J.Jaeckel, A.Ringwald, JHEP 0601:105, 2006
- w/ C-S.Chu, M.Goodsell to appear

Overview

$$[x^\mu, x^\nu] = i\,\theta^{\mu\nu} \;, \qquad M_{\rm NC}^2 \sim 1/|\theta|$$

Heisenberg
Snyder

- Noncommutative field theory - UV/IR mixing
- Summary of problems
- The UV completion, NC in string theory
- Resolving the singularities
- Phenomenology - birefringence and gravity

UV/IR mixing

- e.g. consider NC in 1\2 directions

$$\left.\begin{aligned} \Delta x_1 \Delta x_2 &\sim \theta^{12} \\ \Delta x_1 \Delta p_1 &\sim 1 \end{aligned}\right\} \Longrightarrow \Delta x_2 \sim \theta^{12} \Delta p_1$$

- Large momenta in 2 direction mixed with large distance in the 1 direction.

UV/IR in NC Field Theory

- The Moyal star product

e.g. Douglas, Nekrasov

$$(\phi * \varphi)(x) \equiv \phi(x)\ e^{\frac{i}{2}\theta^{\mu\nu}\overleftarrow{\partial_\mu}\overrightarrow{\partial_\nu}}\ \varphi(x)$$

leads to e.g.

$$S = -\int \frac{1}{4g^2} F_{\mu\nu} * F^{\mu\nu} d^4 x$$

- The vertices pick up a phase factor

$$e^{\frac{i}{2}p.\theta.k}$$

Model building constraints

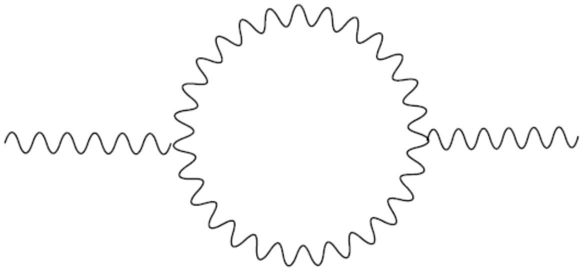
Matsubara
Armoni
Gracia Bondai et al
Terashima
Chaichian et al
Bonora et al

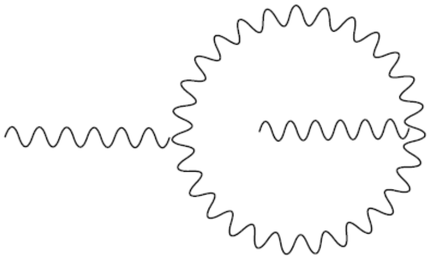
- Products of U(N) groups
- Fields in fundamental, bifundamental or adjoint
- Charges are 0, +1, -1
- Anomaly cancellation tricky -> vector-like theories preferred

Similar properties to D-branes at orbifold fixed points

E.g. planar and nonplanar diagrams in NC QED

Khoze,Travaglini
Alvarez-Gaume et al



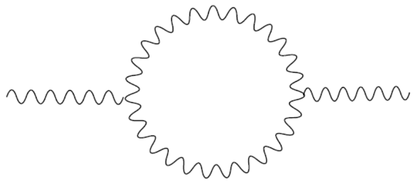


$$\tilde{k}_\mu := \theta_{\mu\nu} k^\nu$$

$$\Pi^{\text{4-dim}}_{\mu\nu}(k) = \Pi^{\text{4-dim}}_{\mu\nu}(k, \ell = 0) - \Pi^{\text{4-dim}}_{\mu\nu}(k, \ell = \tilde{k})$$

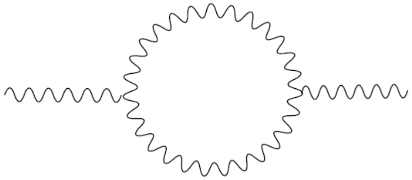
$$\Pi^{\text{4-dim}}_{\mu\nu}(k,\ell) \;\; = \;\; 4C(\mathbf{G}) \sum_j C_j \alpha_j \int \frac{d^4p}{(2\pi)^4} \frac{k^2 \delta_{\mu\nu} - k_\mu k_\nu}{(p^2 + m_j^2)[(p+k)^2 + m_j^2]} e^{ip\cdot\ell}$$

In a supersymmetric theory

 $|k| \ll \Lambda_{UV}$

$$\int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 - m^2)((p+k)^2 - m^2)} = \frac{1}{(4\pi)^2} \int_0^1 dx \int_0^\infty \frac{dt}{t}\, e^{-t(x(1-x)k^2 + m^2) - \frac{1}{4\Lambda_{UV}^2 t}}$$

$$\to -\frac{2}{(4\pi)^2} \int_0^1 dx\, \log\left(\frac{\sqrt{k^2 x(1-x) + m^2}}{\Lambda_{UV}}\right)$$



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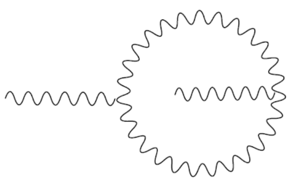
$$\int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2-m^2)((p+k)^2-m^2)} = \frac{1}{(4\pi)^2}\int_0^1 dx \int_0^\infty \frac{dt}{t}\, e^{-t(x(1-x)k^2+m^2)-\frac{1}{4\Lambda_{UV}^2 t}}$$

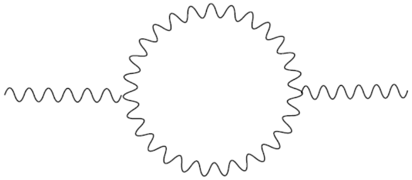
$$\rightarrow -\frac{2}{(4\pi)^2}\int_0^1 dx\, \log(\frac{\sqrt{k^2x(1-x)+m^2}}{\Lambda_{UV}})$$



$$|k| \gg \Lambda_{IR} = \frac{M_{NC}^2}{\Lambda_{UV}}$$

$$\int \frac{d^4p}{(2\pi)^4} \frac{e^{i\tilde{k}.p}}{(p^2-m^2)((p+k)^2-m^2)} = \frac{1}{(4\pi)^2}\int_0^1 dx \int_0^\infty \frac{dt}{t}\, e^{-t(x(1-x)k^2+m^2)-\frac{\tilde{k}^2}{4t}-\frac{1}{4\Lambda_{UV}^2 t}}$$

$$\rightarrow - \frac{2}{(4\pi)^2}\int_0^1 dx\, \log(|\tilde{k}|\sqrt{k^2x(1-x)+m^2})$$



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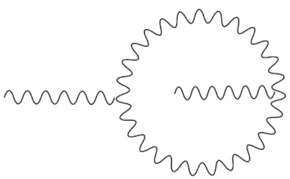

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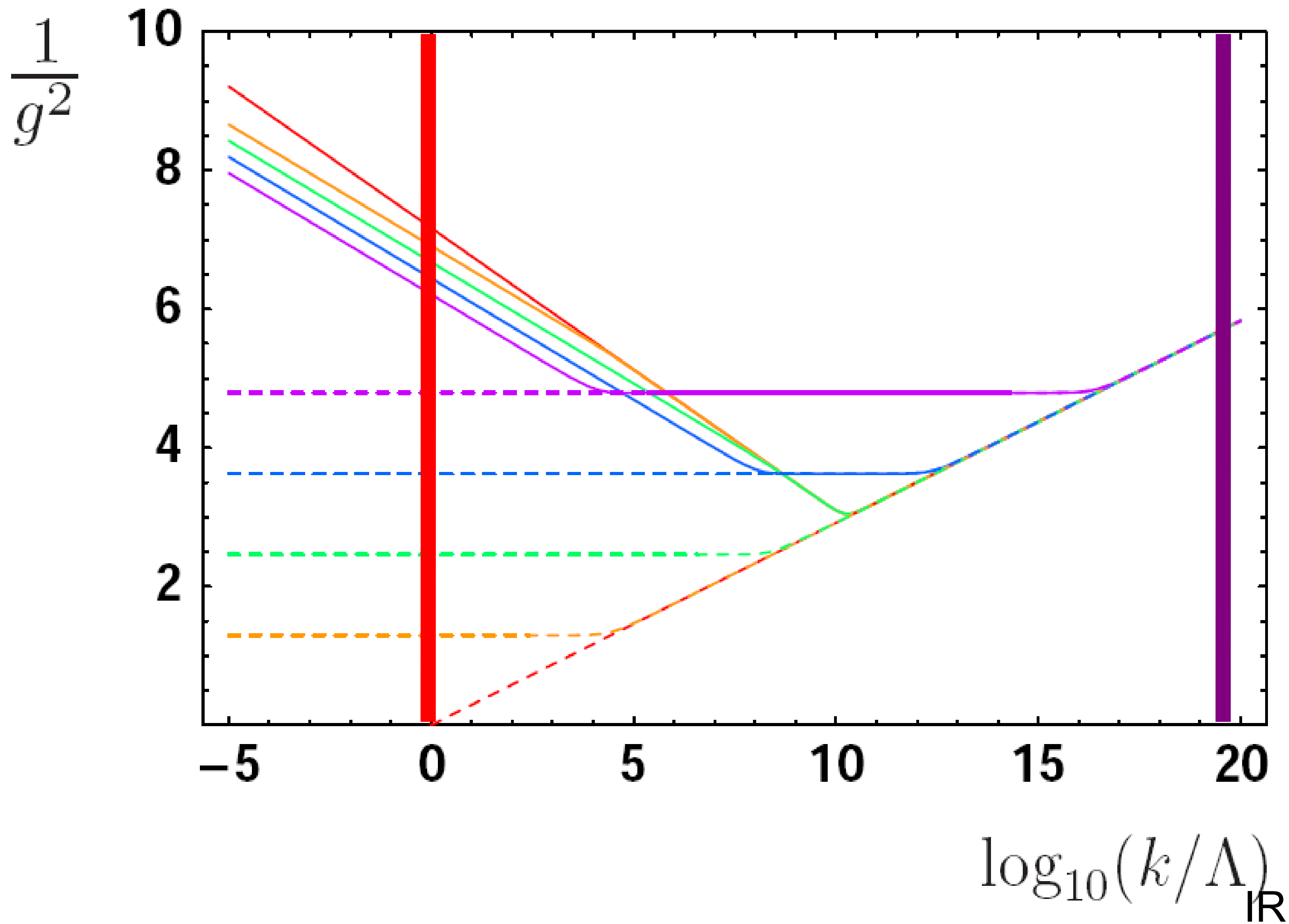
Matusis, Susskind, Toumbas

- UV modes do not decouple (UV/IR mixing)
- Physics when $\theta^{\mu\nu} \to 0$ is not the same as $\theta^{\mu\nu} = 0$
- Couplings run roughly symmetrically about $|k| = M_{NC}$
- Field theory only valid for

$$\Lambda_{UV} > |k| > \Lambda_{IR} = \frac{M_{NC}^2}{\Lambda_{UV}}$$

$$\left(\frac{1}{g^2(k)}\right)^{AB} = \left(\frac{1}{g_0^2}\right)^{AB} + 4\Pi_1^{AB}(k)$$

Jaeckel,Khoze,Ringwald



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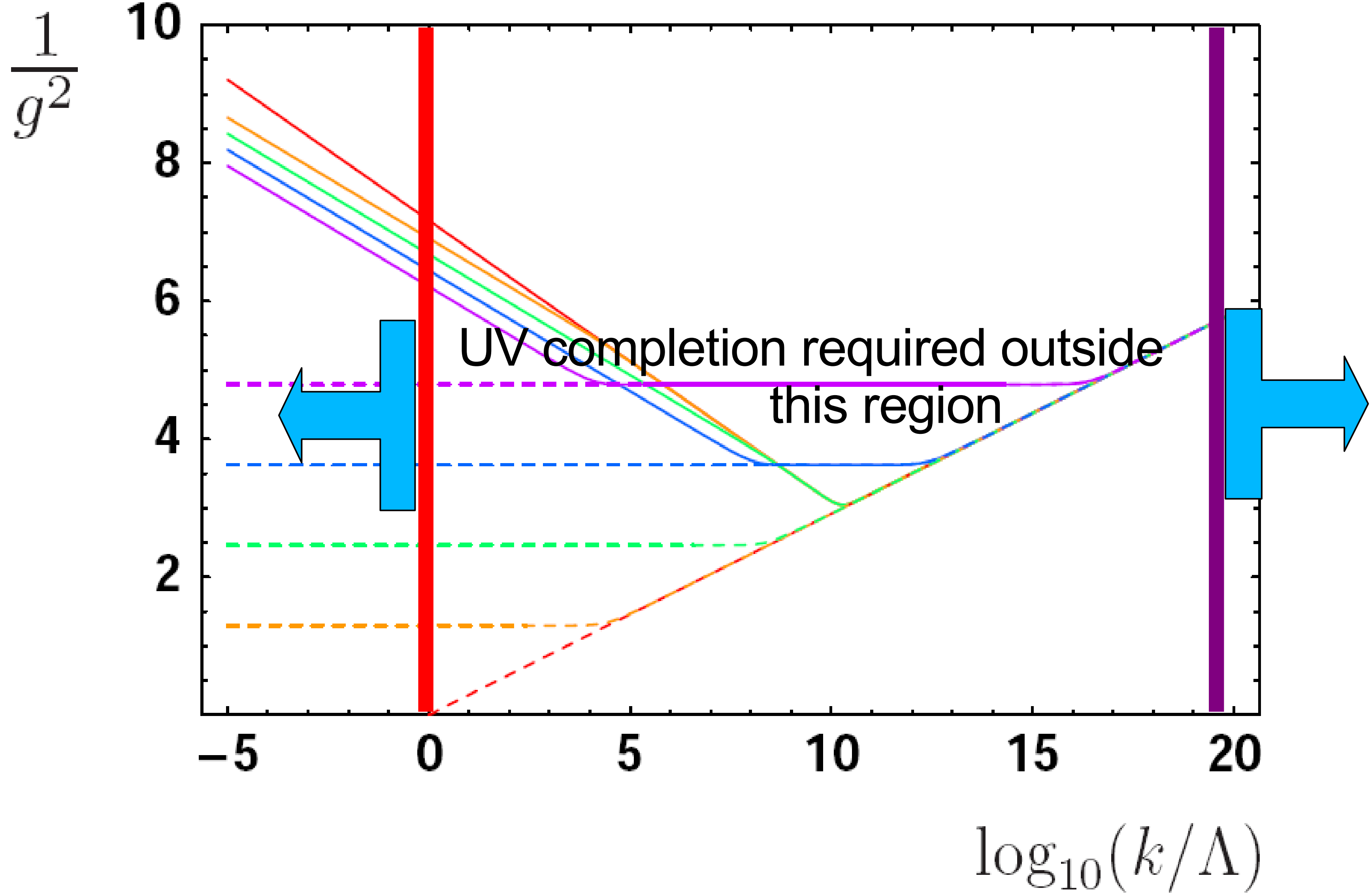
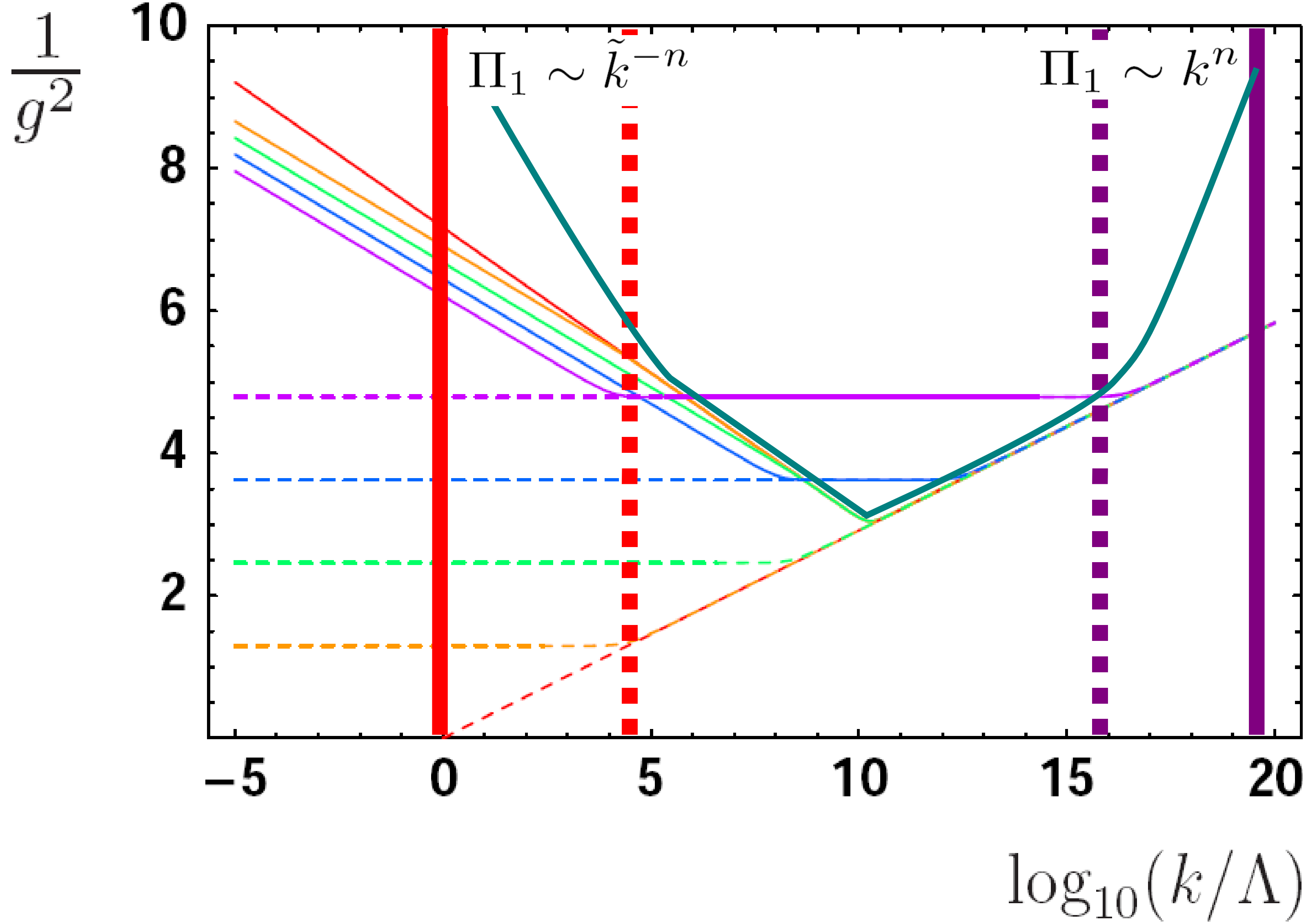


Power law running in the IR



But even worse, when SUSY broken Lorentz is violated

$$\Pi_{\mu\nu}(k) = \Pi_1(k^2, \tilde{k}^2)(k^2\delta_{\mu\nu} - k_\mu k_\nu) + \Pi_2(k^2, \tilde{k}^2)\frac{\tilde{k}_\mu \tilde{k}_\nu}{\tilde{k}^4}$$

- If SUSY softly broken then IR divergence is improved

$$\Pi_2 \sim M^2_{SUSY} |\tilde{k}|^2$$

- But still implies a mass for some polarizations of the photon!

e.g. Take a photon in the 3 direction $\theta^{13} = -\theta^{31} \neq 0$

$$k^\mu = (k^0, 0, 0, k^3)$$

equating coefficients ...

$$\begin{aligned} \Pi_1 k^2 - \Pi_2 \tilde{k}^{-2} &= 0 \;:\; \varepsilon^\mu = (0,1,0,0) \\ \Pi_1 k^2 &= 0 \;:\; \varepsilon^\mu = (0,0,1,0) \end{aligned}$$

$$m^2_{\gamma_3} \sim M^2_{SUSY}$$

c.f. Limits of $m_\gamma < 6 \times 10^{-17}\,\mathrm{eV}$

Summary of NCFT problems

- IR divergences and no decoupling
- Discontinuity in $\theta^{\mu\nu} \longrightarrow 0$ limit
- UV/IR coupling really requires UV completion in the IR
- Lorentz symmetry violated if supersymmetry broken

A string UV completion

Sheikh-Jabbari
Seiberg, Witten

- String theory with background B field in space-time

$$\langle x^i(\tau) x^j(\tau') \rangle = -\alpha' G^{ij} \log(\tau - \tau')^2 + \frac{i}{2}\theta^{ij} \epsilon(\tau - \tau')$$

where

$$G^{ij} = \left(\frac{1}{g + 2\pi\alpha' B} \right)_S^{ij} \qquad \theta^{ij} = 2\pi\alpha' \left(\frac{1}{g + 2\pi\alpha' B} \right)_A^{ij}$$

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$$G^{ij} = \left(\frac{1}{g+2\pi\alpha' B}\right)^{ij}_S \qquad \theta^{ij} = 2\pi\alpha'\left(\frac{1}{g+2\pi\alpha' B}\right)^{ij}_A$$

- Products of vertex operators will give the required phase

$$e^{ip\cdot x}(\tau)\cdot e^{iq\cdot x}(\tau') \sim (\tau-\tau')^{2\alpha' G^{ij}p_i q_j}\, e^{-\frac{1}{2}i\theta^{ij}p_i q_j}\, e^{i(p+q)\cdot x}(\tau') + \ldots$$

if we set $2\alpha' G^{ij} = 0$

The Seiberg-Witten limit

- Take the $\alpha' \to 0$ limit so as to keep the open string metric G_{ij} and noncommutativity θ^{ij} constant -> NC field theory

$$\alpha' \sim \epsilon^{\frac{1}{2}} \longrightarrow 0$$

$$g_{ij} \sim \epsilon \longrightarrow 0$$

$$G_{ij} = -(2\pi\alpha')^2(Bg^{-1}B)_{ij}$$

$$\theta^{ij} = \left(\frac{1}{B}\right)^{ij}$$

Resolving the singularities

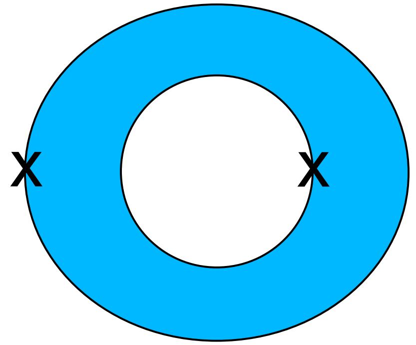
- *Do not* take the SW limit
- Keep the theory with nonzero α' and B-field as our UV completion
- Calculate polarization tensor in the string theory
- Find finite, continuous answers

Overview of amplitude

Bilal,Chu,Russo,Sciuto

- e.g. U(1) on Dp-branes in bosonic string

$$A_2(k,-k) \quad \sim \quad \int_0^\infty dt\,(4\pi t)^{-\frac{(p+1)}{2}}\eta(\frac{it}{2\pi\alpha'})^{-24} \times$$
$$\int_0^{\frac{t}{\alpha'}} dx\; e^{-2\alpha' k.G(x,x').k}\,(\varepsilon_1.G_{xx'}.\varepsilon_2 - 2\alpha'(\varepsilon_1.G_x.k)(\varepsilon_2.G_{x'}.k))\Big|_{x'=0}$$

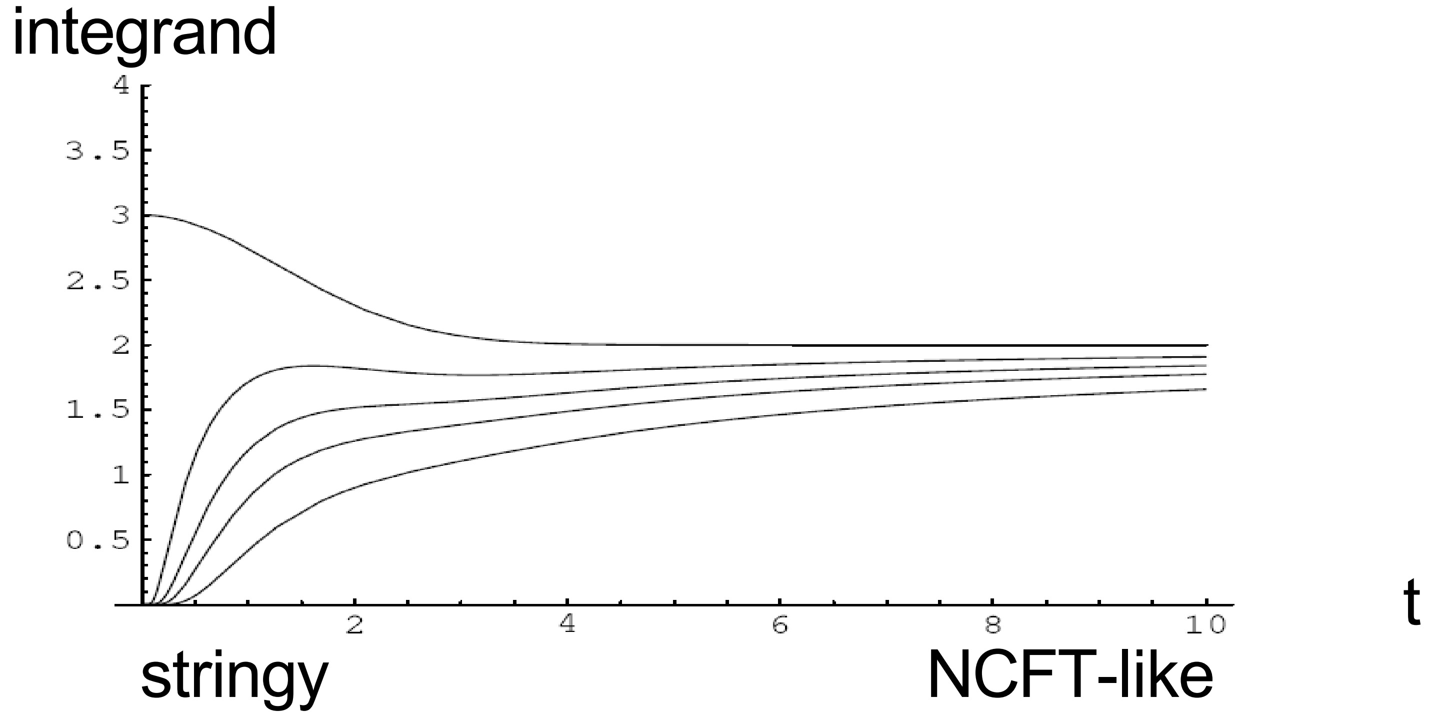


t->infinity "open channel"

t->0 "closed channel"

The effect of B is the same even before SW limit...

$$
\begin{aligned}
-2\alpha' k.G^{P}.k &= -2\alpha' k^2 I_0^{P} \\
-2\alpha' k.G^{NP}.k &= -2\alpha' k^2 I_0^{NP} - \frac{\tilde{k}^2}{4t}
\end{aligned}
$$



Behaviour in different limits

$$\tilde{k}^2/\alpha' \gg 1$$

$$\begin{aligned}
\Pi_1^{NP} &\sim \int_0^\infty dt\, t^{-\frac{(p-1)}{2}} \int_0^1 dx\, (1-2x)^2 e^{-k^2 t(x-x^2)-\frac{\tilde{k}^2}{4t}} \\
&\sim \frac{|\tilde{k}|^{3-p}}{\alpha'^{3-p}}
\end{aligned}$$

$$\Pi_2^{NP} \sim \frac{|\tilde{k}|^{3-p}}{\alpha'^{3-p}}$$

Behaviour in different limits

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$$\sim \frac{|\tilde{k}|^{3-p}}{\alpha'^{3-p}}$$

$$\Pi_2^{NP} \sim \frac{|\tilde{k}|^{3-p}}{\alpha'^{3-p}}$$

$$\tilde{k}^2/\alpha' \ll 1$$

$$\Pi_1^{NP} = \Pi_1^{NP}(\theta = 0) \times (1 + \mathcal{O}(\frac{\tilde{k}^2}{\alpha'}))$$

$$\Pi_2^{NP} \sim \frac{|\tilde{k}|^4}{\alpha'^2}$$

Amplitudes -> commutative ones in IR

Phenomenology

Carroll,Field,Jackiw
Kostelecky,Mewes
Anasimov,Banks,Dine,Graesser

- *Effect of Lorentz violation = birefringence...*

  e.g. in a SUSY model

$$\Pi_2 \sim M^2_{SUSY} \frac{|\tilde{k}|^4}{\alpha'}$$

if trace U(1) photon is hypercharge

$$\begin{aligned} \Pi_1 k^2 - \Pi_2 \tilde{k}^{-2} &\approx (k^2 - M^2_{SUSY} \frac{|\tilde{k}|^2}{\alpha'}) = 0 \\ \omega &\approx |\mathbf{k}| \left(1 - \frac{M^2_{SUSY} M^2_{string}}{M^4_{NC}}\right) \end{aligned}$$

$$M_{NC} > 10^{11} GeV$$

- *More detailed studies – consider "time-of-flight" of signals from pulsars*

Kostelecky,Mewes

$$\frac{M_{SUSY}^2 M_{string}^2}{M_{NC}^4} \quad < \quad 3 \times 10^{-16}$$

leads to $M_{NC} \quad > \quad 10^{14} GeV$

- *Similar to direct limits from atomic physics*

Mocioiu, Pospelov, Rioban

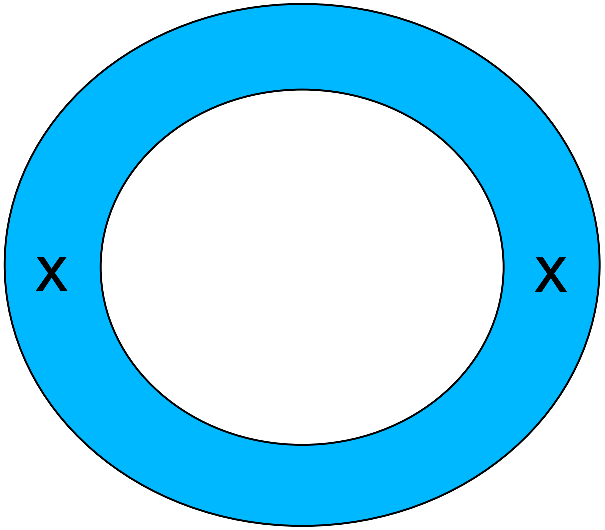
$$\Lambda_{IR} > 10^{10} GeV$$

- ***Modification of gravity****... if trace U(1) photon decoupled from hypercharge, then NC scale could be lower: guess*

$$S = -\int \frac{1}{4g^2} F_{\mu\nu} * F_{\rho\sigma} G^{\mu\rho} G^{\nu\sigma} \sqrt{-G} d^4 x$$

If Newton's constant has significant moduli dependence expect thresholds to be “settled” at Λ_{IR}

- *Graviton one loop diagram has a field theory limit but doesn't correspond to planar and nonplanar and doesn't look like naïve field theory result*



- *Calculate force law in usual way*

$$
\begin{aligned}
V_{one-loop}(r) &= \int\int \frac{d^4p}{(2\pi)^4} e^{ik.x} \tilde{G}\, dt \\
&\sim \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}.\mathbf{x}} \frac{1}{M_P^2 \mathbf{k}^2} \, \frac{1 + \lambda(1 + \frac{\tilde{\mathbf{k}}^2}{\alpha'})^{p-3}}{1+\lambda}
\end{aligned}
$$

- *Calculate force law in usual way*

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$$\sim \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}.\mathbf{x}} \frac{1}{M_P^2 \mathbf{k}^2} \frac{1+\lambda(1+\frac{\tilde{\mathbf{k}}^2}{\alpha'})^{p-3}}{1+\lambda}$$

-> gravitation "gets its thresholds" at Λ_{IR}

$$V(r) \sim \frac{1}{8\pi M_P^2 r}(1+\lambda(1-e^{-\frac{\Gamma(p-\frac{7}{2})}{\sqrt{\pi}\Gamma p-3)}\frac{r}{r_c \cos\vartheta}})) \qquad r > r_c = \frac{M_{string}}{M_{NC}^2}$$

- *e.g. Take*

$$
\begin{aligned}
M_{string} &= 10^{18} GeV \\
M_{NC} &= 1 TeV \\
r_c &\sim M_{string}/M_{NC}^2 \sim 0.1 mm
\end{aligned}
$$

Summary

- *NCFT limit already ruled out* $\Lambda_{IR} > 10^{10} GeV$
- *String theory with non-zero B field -> class of theories with arbitrarily small Lorentz violating terms*
- *BUT: no more UV/IR mixing – IR theory has normal Wilsonian behaviour*
- *Experimental signatures, PVLAS?*
- *If trace U(1) decoupled, then long range gravity modification possible*
- *cosmological implications of explicit Lorentz violation in graviton tensor-structure?*