

# QUANTUM PARAMETER ESTIMATION: FROM FUNDAMENTALS TO TECHNOLOGY

Manuel Gessner

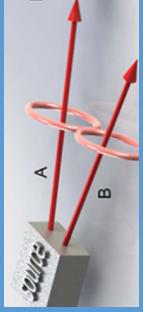
École Normale Supérieure, Paris

Colloquium, IPhT, CEA-Saclay

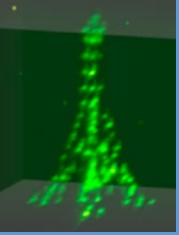
26/05/2020

# QUANTUM INFORMATION THEORY AND EXPERIMENTS

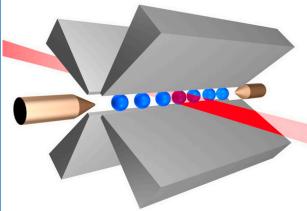
## Controlled multipartite quantum systems



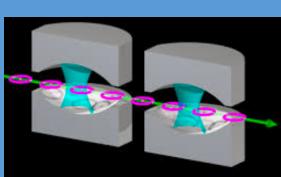
Photons



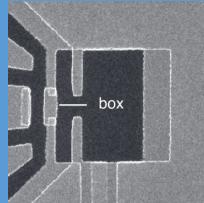
Cold atoms



Trapped ions



Cavity QED

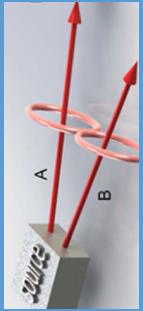


Circuit QED

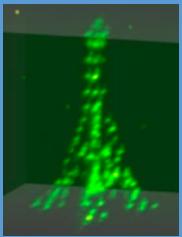
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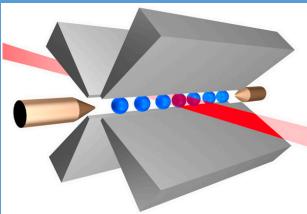
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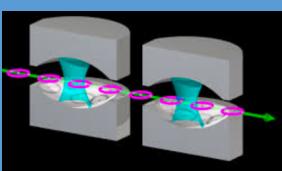
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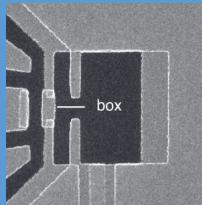
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## QUANTUM TECHNOLOGIES

Metrology & sensing

Simulations

Communication

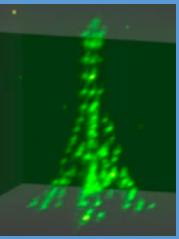
Computation

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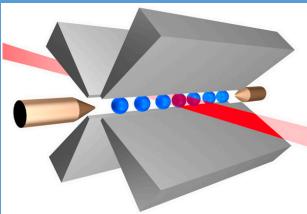
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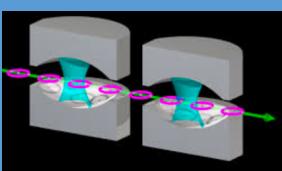
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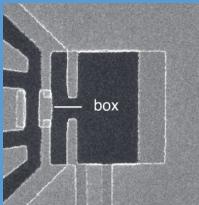
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## Theory and physics of quantum systems

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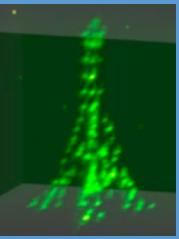
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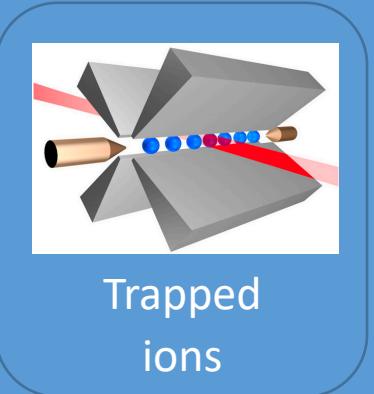
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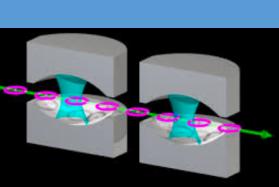
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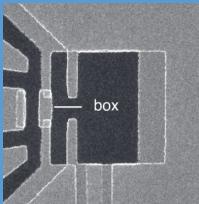
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## Theory and physics of quantum systems

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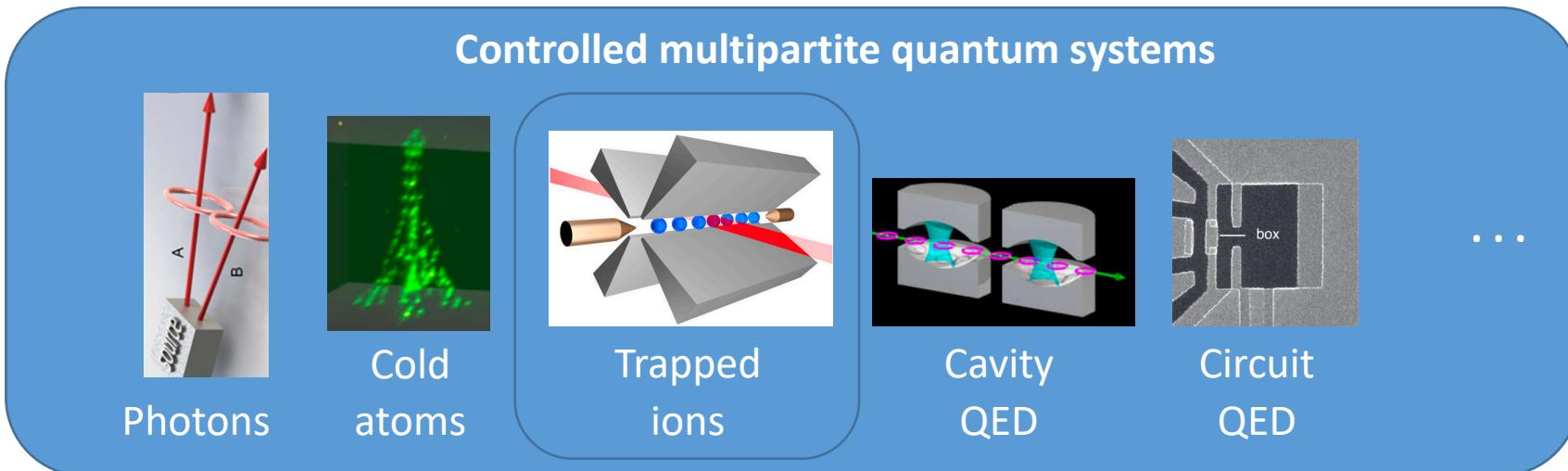
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# QUANTUM INFORMATION THEORY AND EXPERIMENTS



## Theory and physics of quantum systems

Single trapped ion



## QUANTUM TECHNOLOGIES

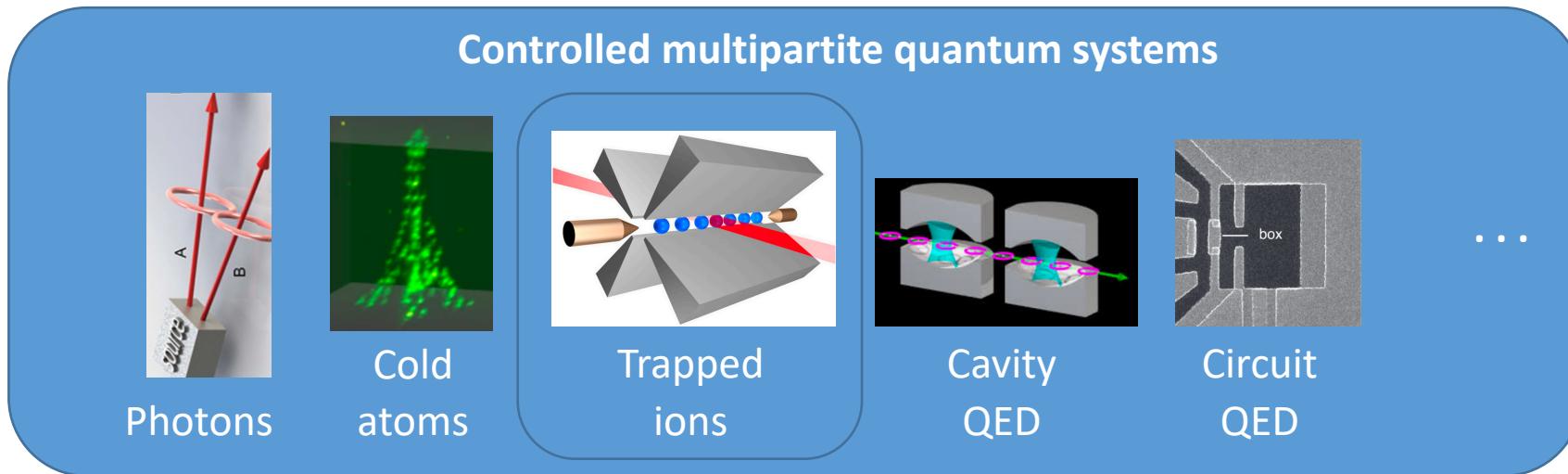
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## Theory and physics of quantum systems

Single trapped ion



Spins:  
Discrete



## QUANTUM TECHNOLOGIES

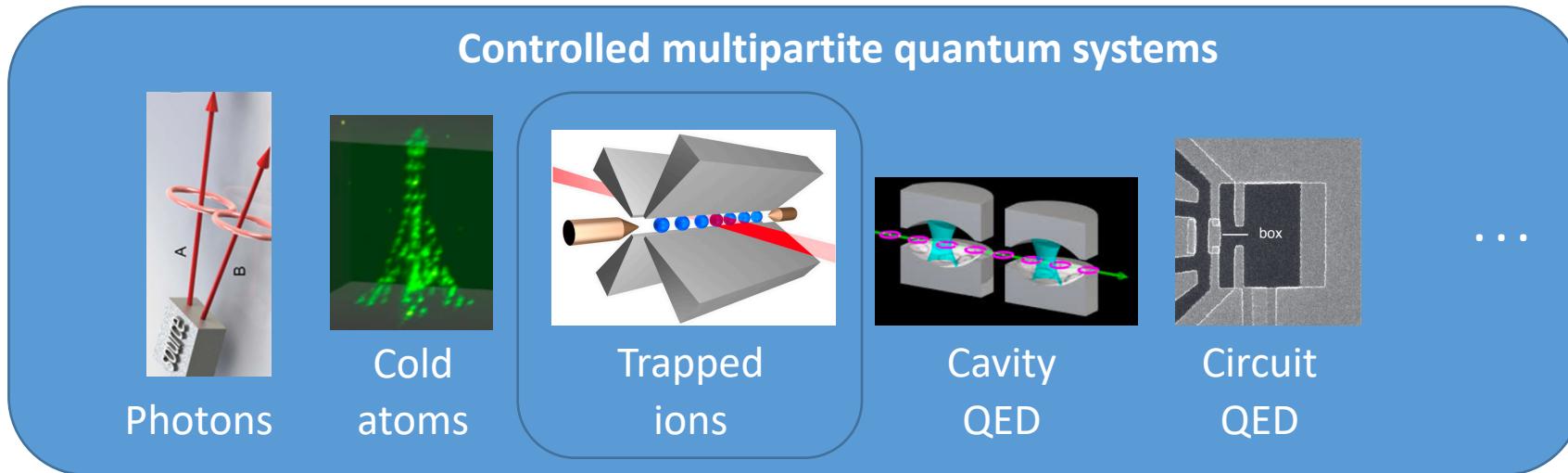
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## Theory and physics of quantum systems

Spins:

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Phonons:

Continuous



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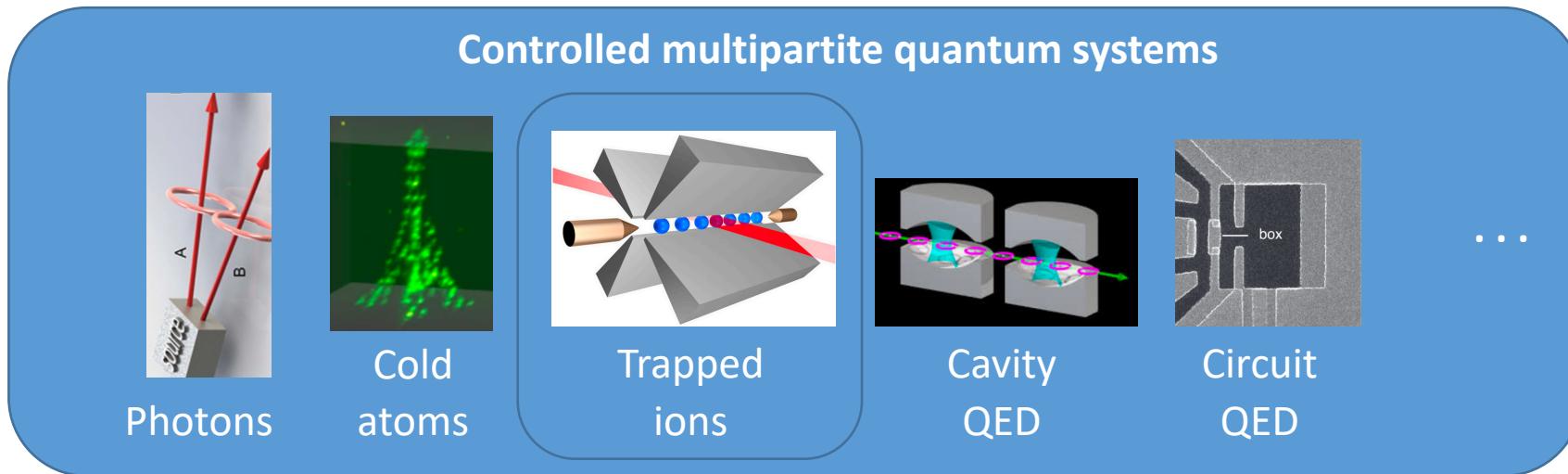
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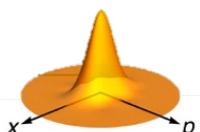
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Spins:  
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Quantum information  
& correlations

## QUANTUM TECHNOLOGIES

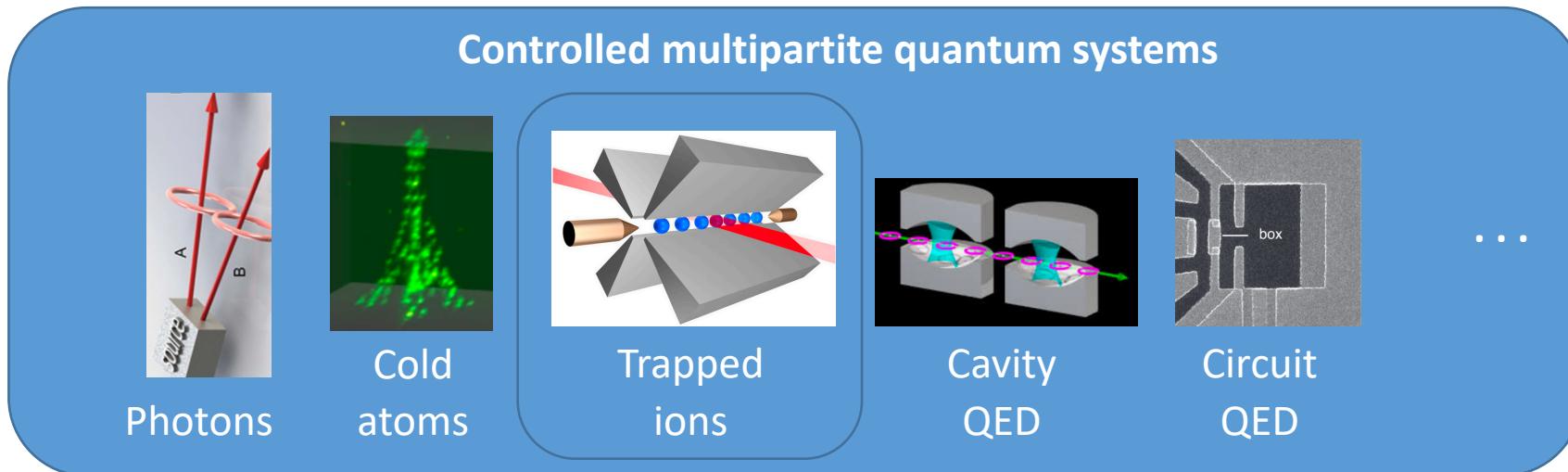
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## Theory and physics of quantum systems

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Laser coupling

Quantum information  
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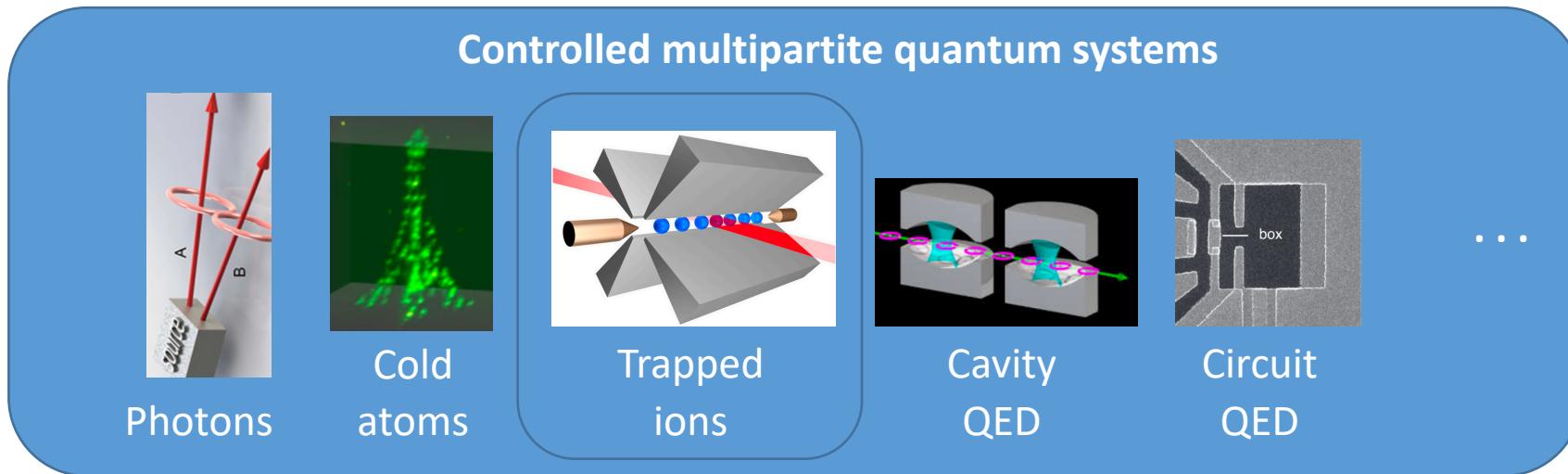
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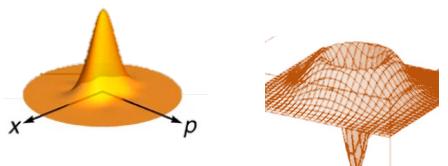
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Precision measurements with  
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Nat. Commun. **10**, 2929 (2019)

Quantum information  
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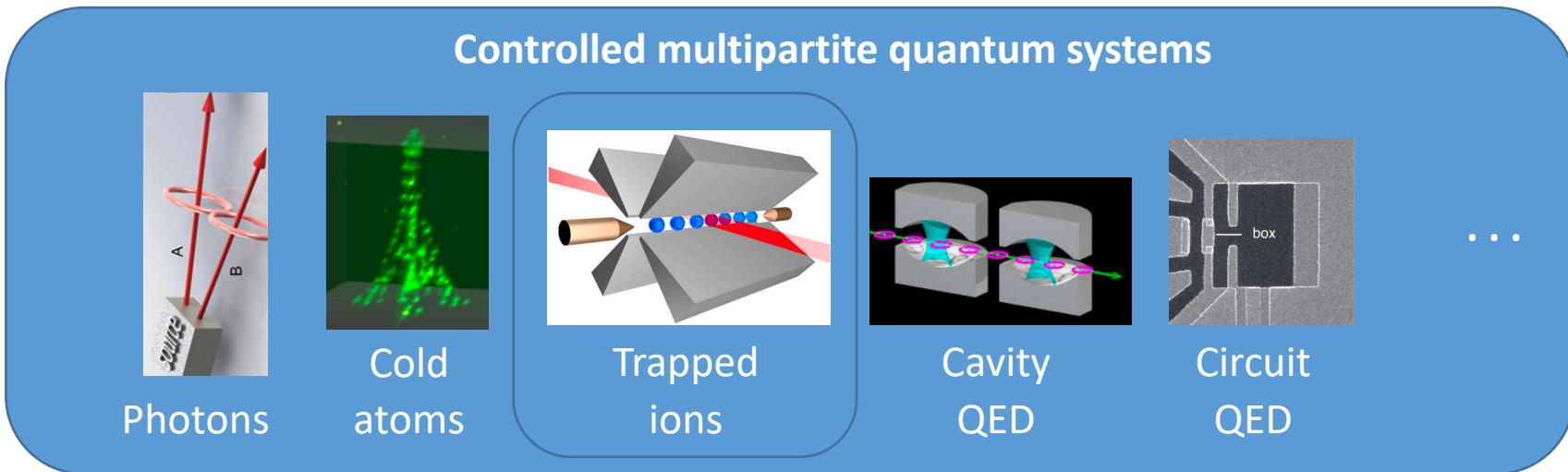
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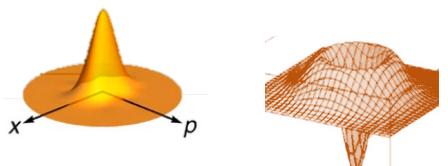
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Electron transfer  
in molecular systems  
arXiv:2004.02925

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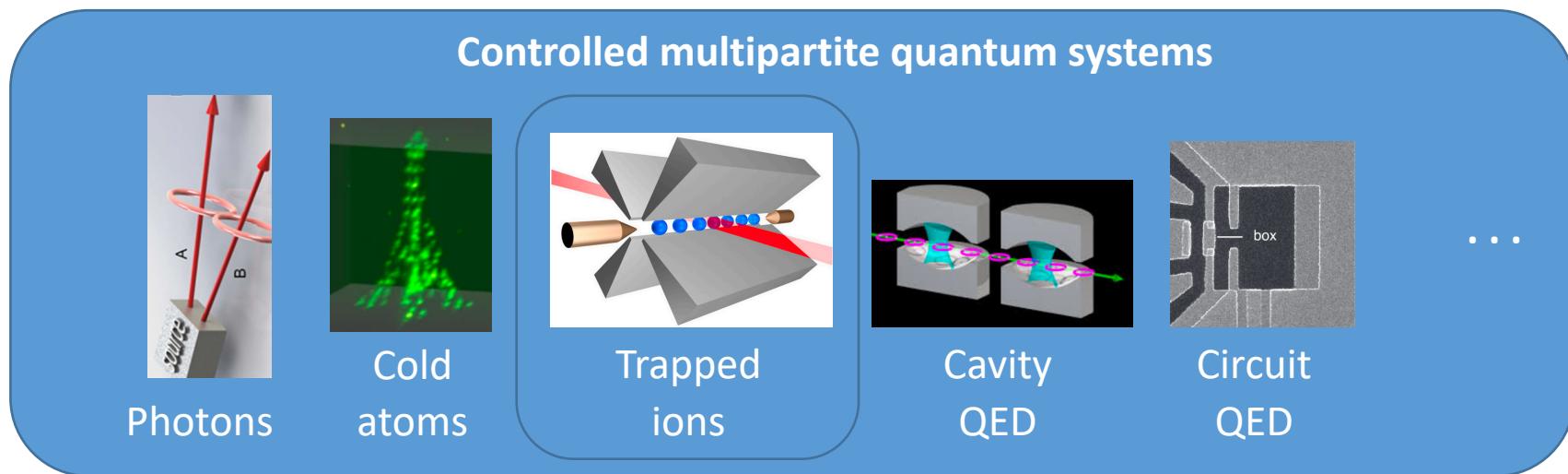
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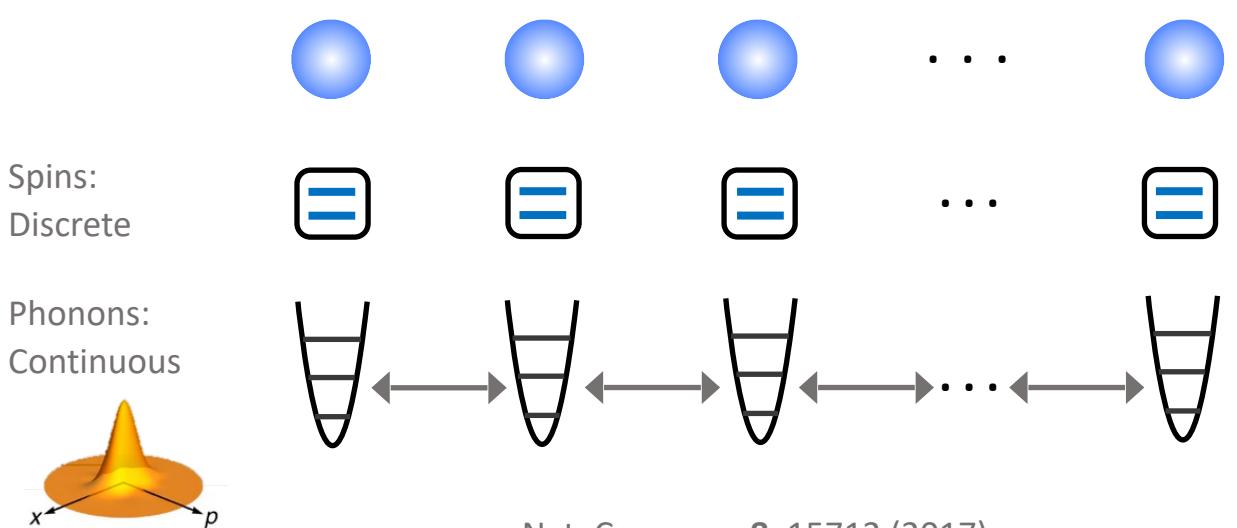
Communication

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# QUANTUM INFORMATION THEORY AND EXPERIMENTS



## Theory and physics of quantum systems



Quantum information & correlations

Many-body physics

## QUANTUM TECHNOLOGIES

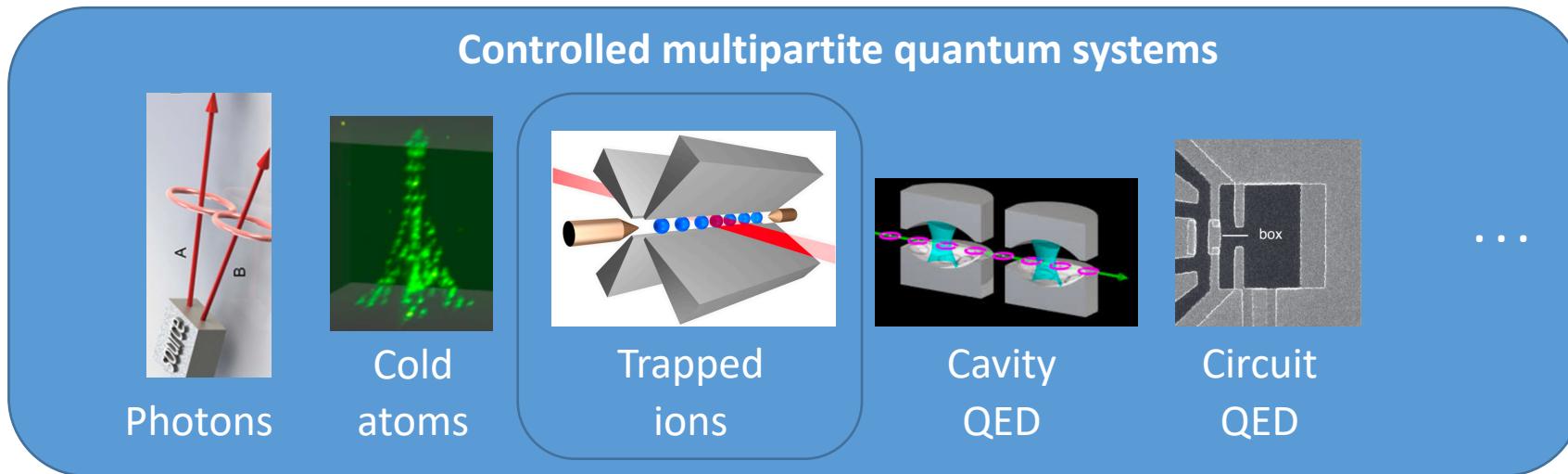
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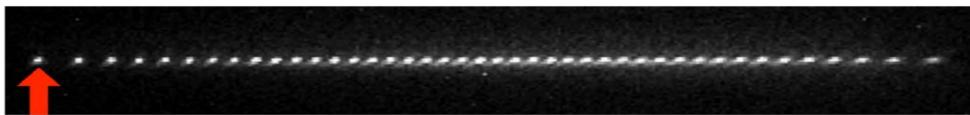
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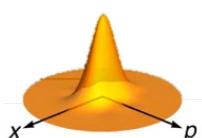
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Phonons:  
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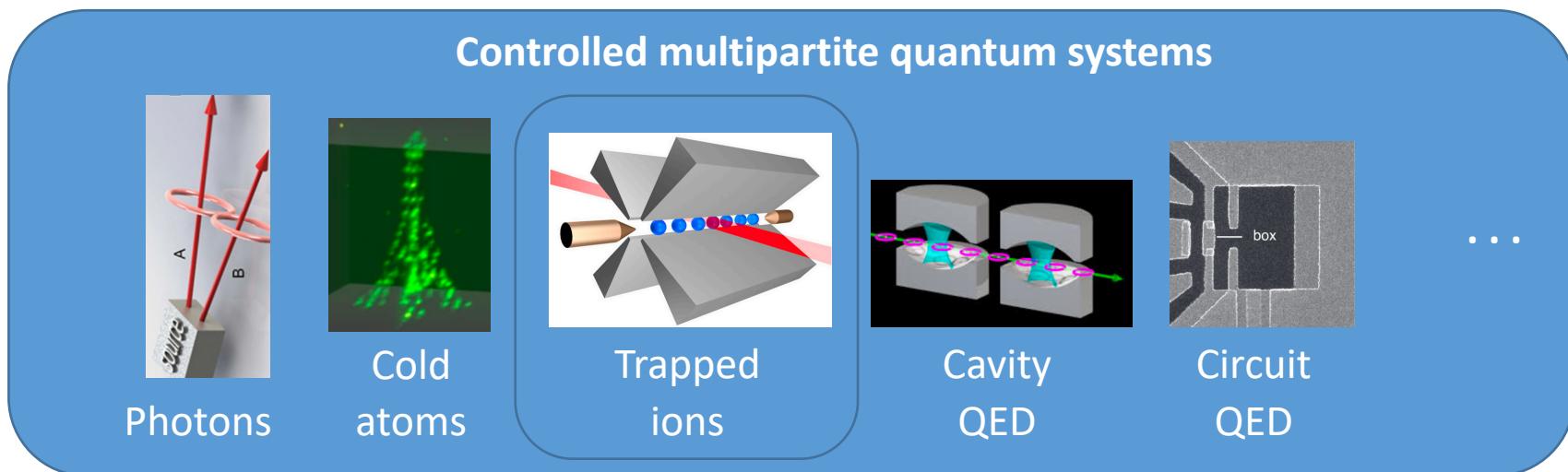
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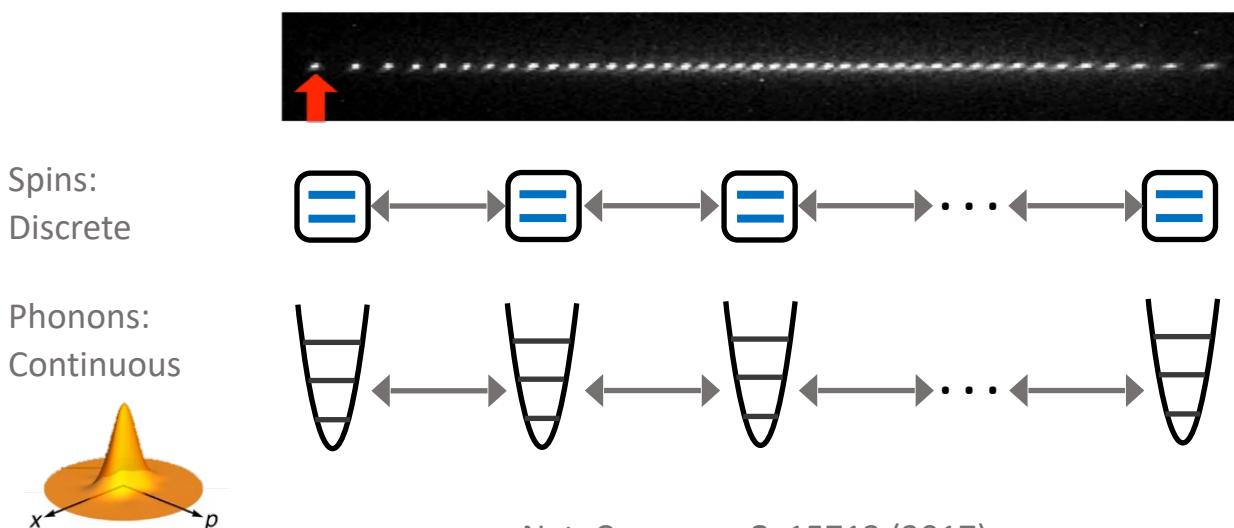
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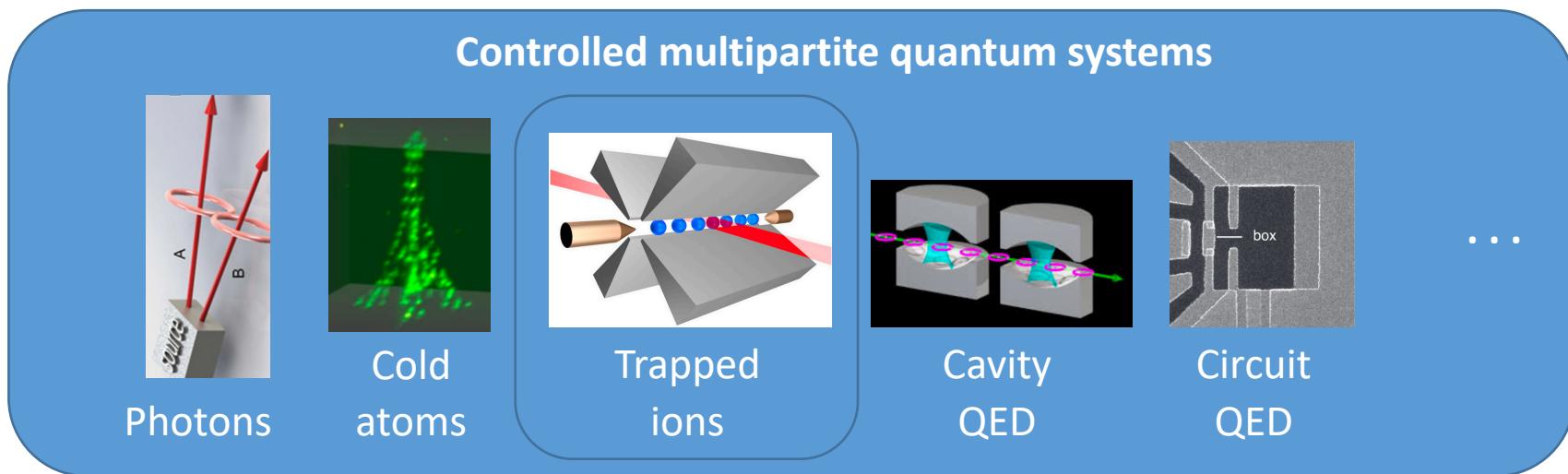
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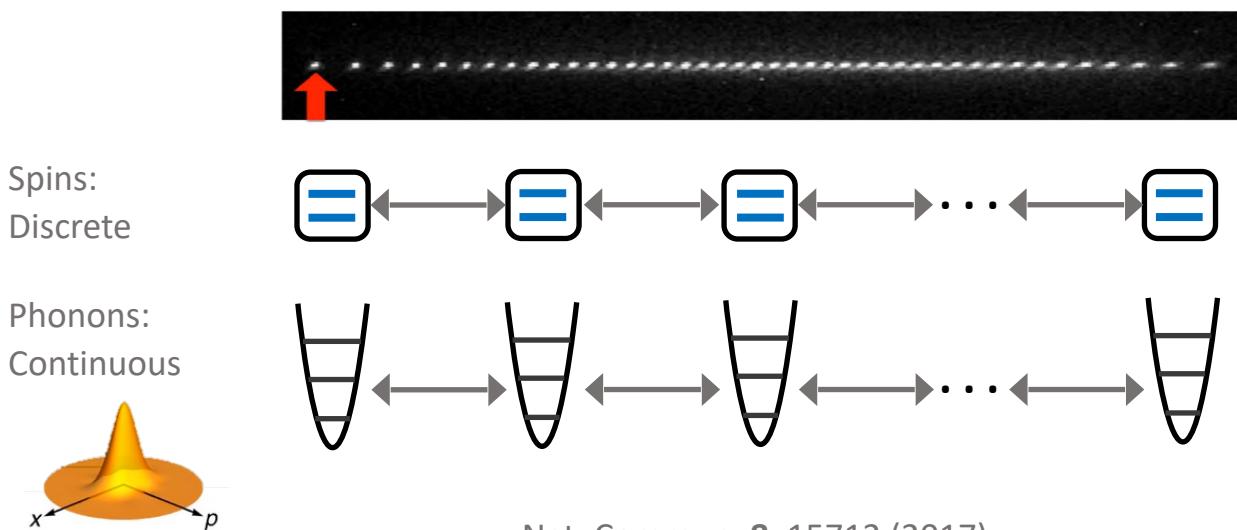
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Spin chain models

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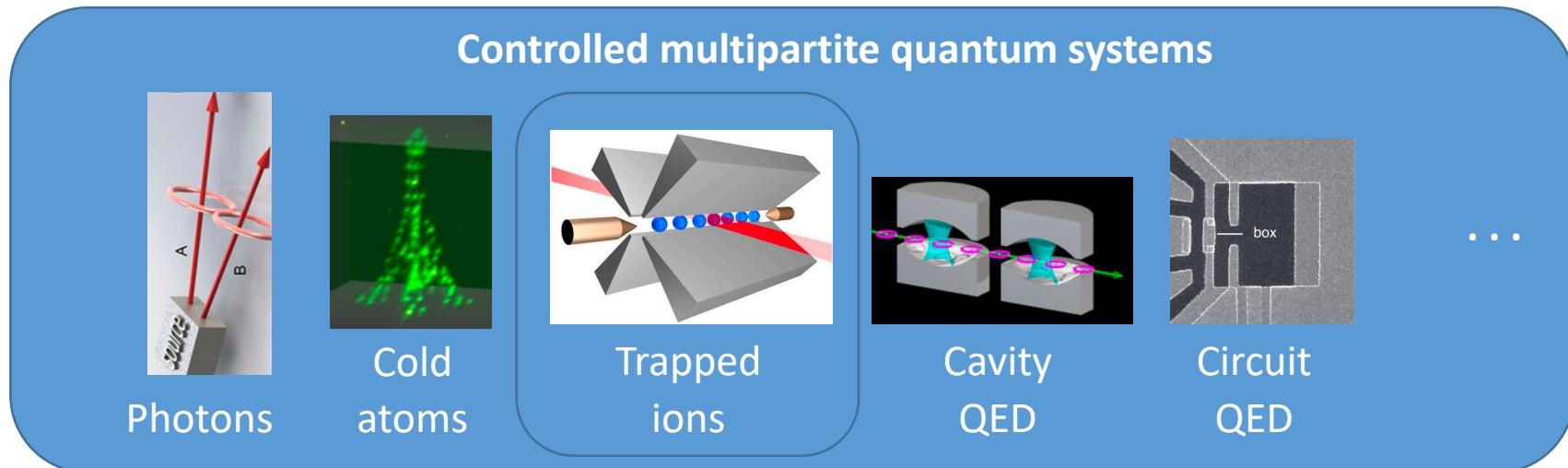
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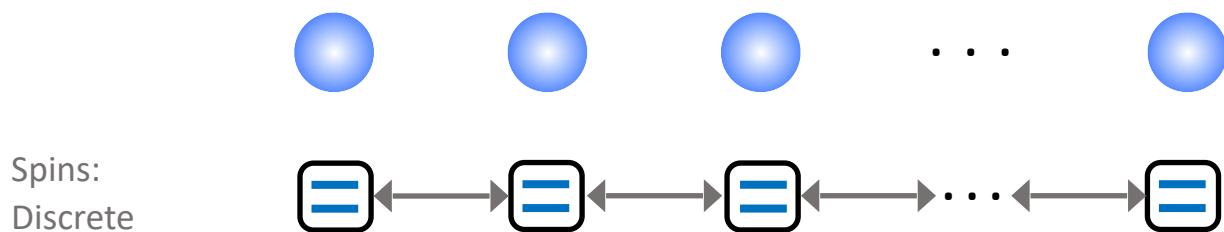
Communication

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# QUANTUM INFORMATION THEORY AND EXPERIMENTS



## Theory and physics of quantum systems



D. Porras & J. I. Cirac, PRL '04

Ising model with long-range interactions

Quantum information  
& correlations

Many-body physics

Spin chain models

## QUANTUM TECHNOLOGIES

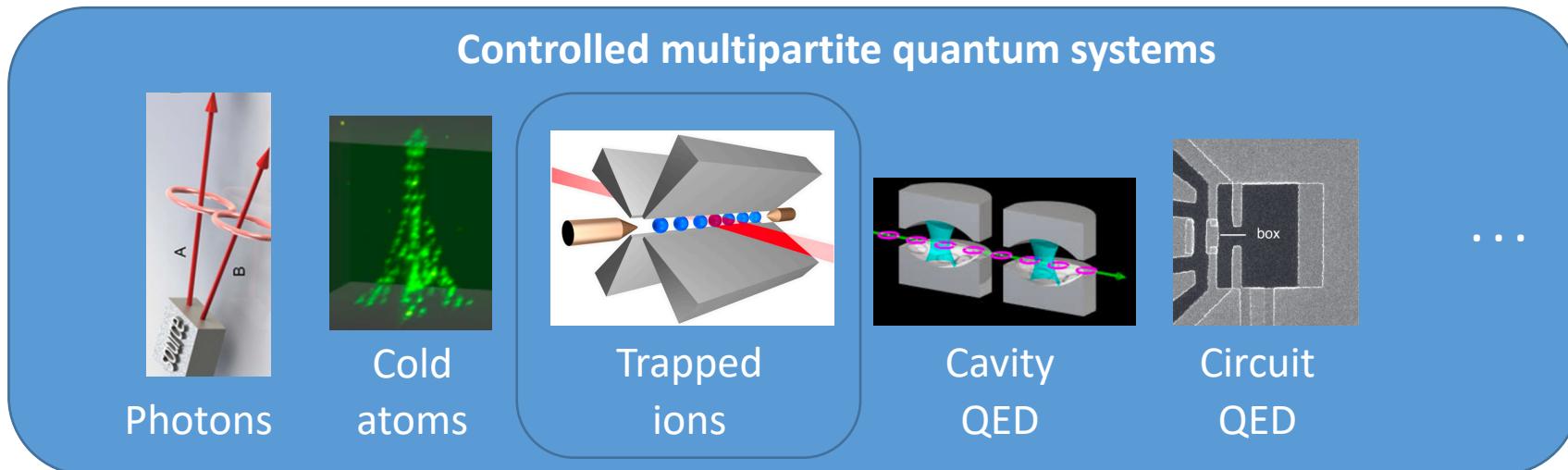
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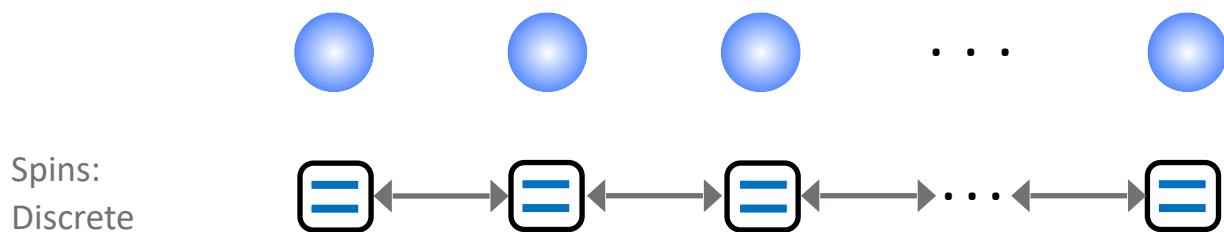
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Semiclassics and spin-wave theory  
PRB 93, 155153 (2016)

## QUANTUM TECHNOLOGIES

Metrology & sensing

Simulations

Communication

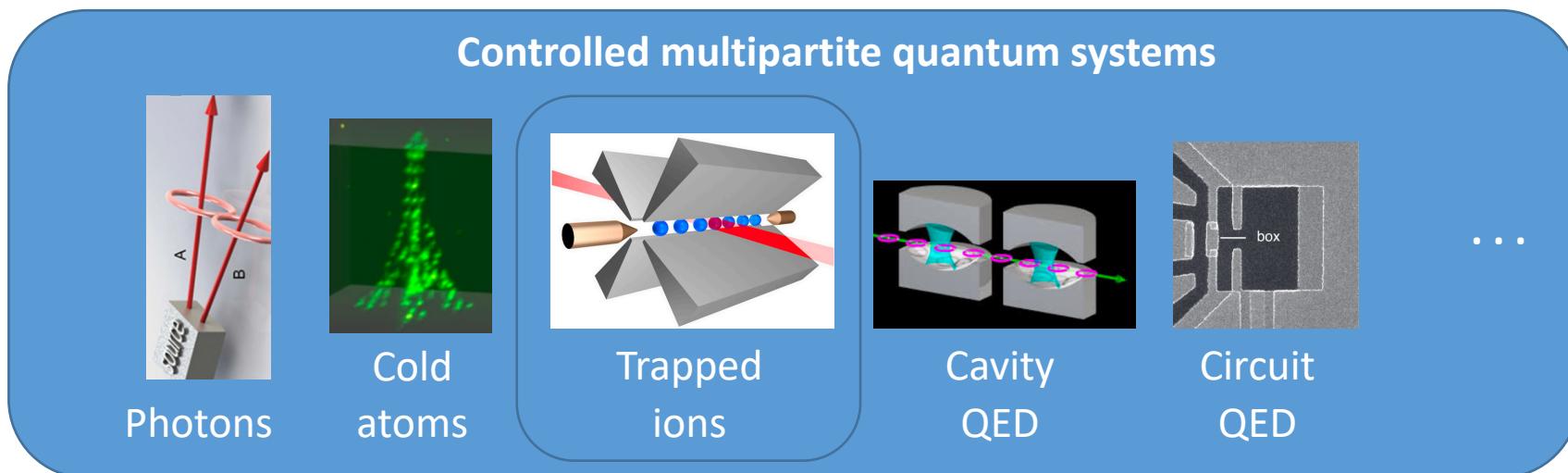
Computation

Quantum information  
& correlations

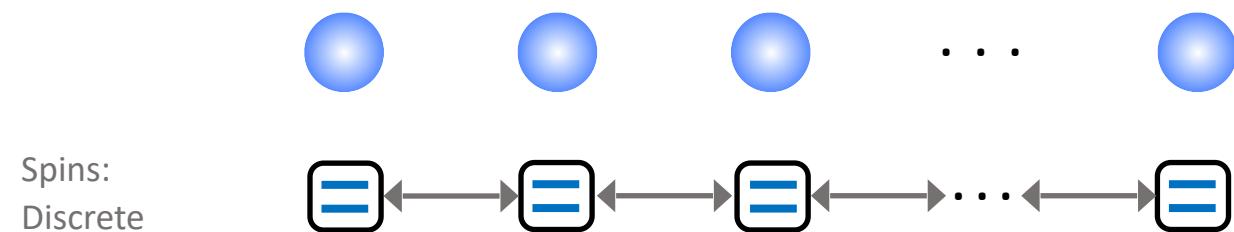
Many-body physics

Spin chain models

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D. Porras & J. I. Cirac, PRL '04

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Spin-spin  
interaction

Transverse field

Quantum information  
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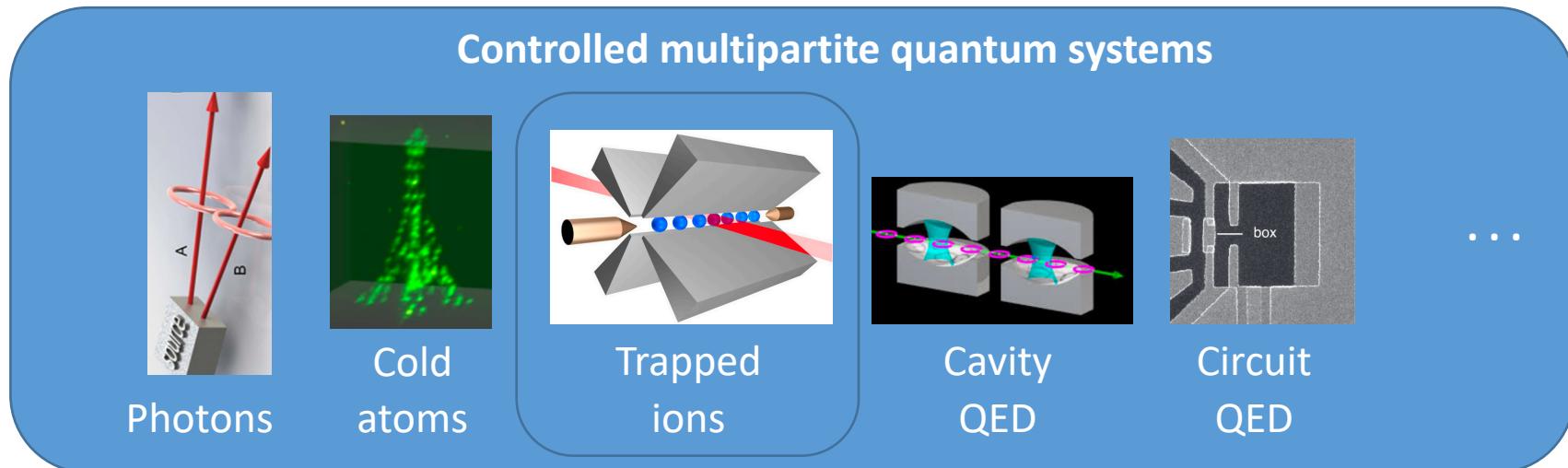
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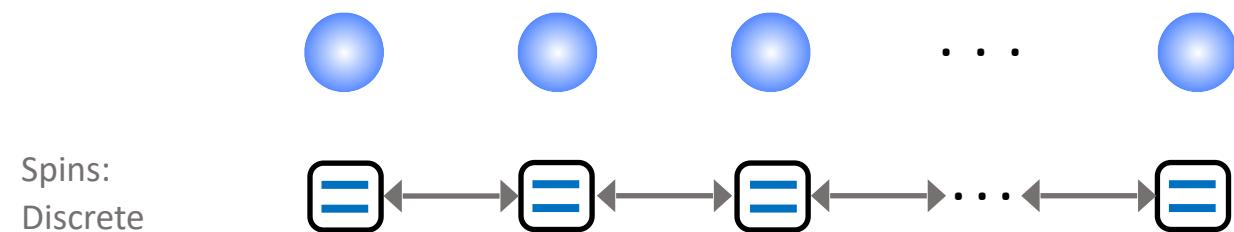
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Semiclassics and spin-wave theory  
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Spin-spin interaction  
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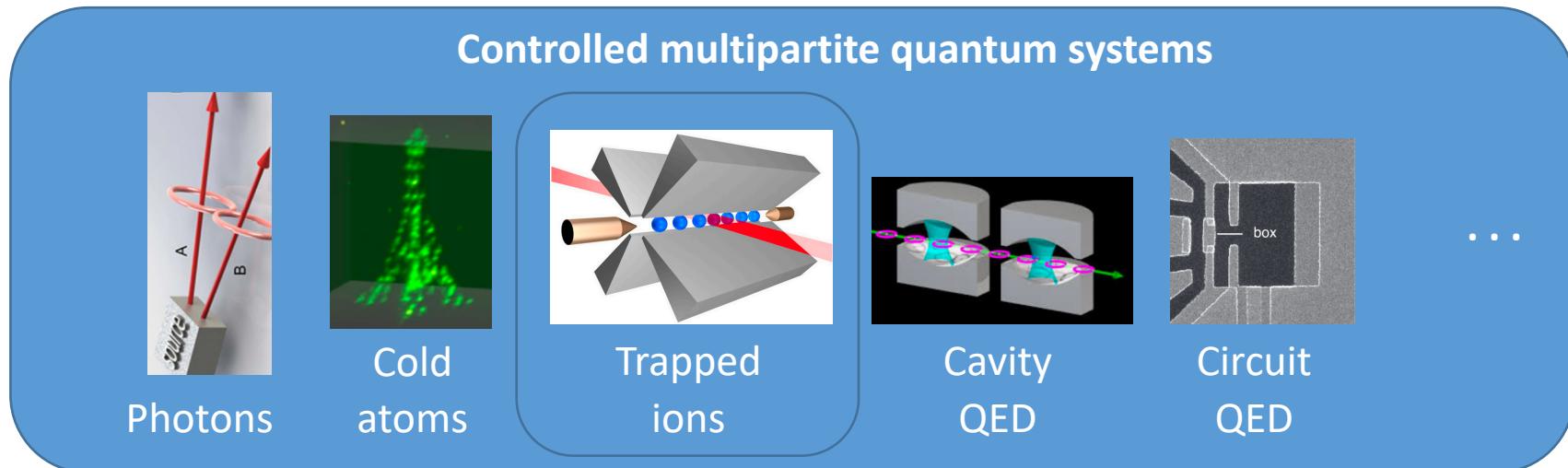
Metrology & sensing

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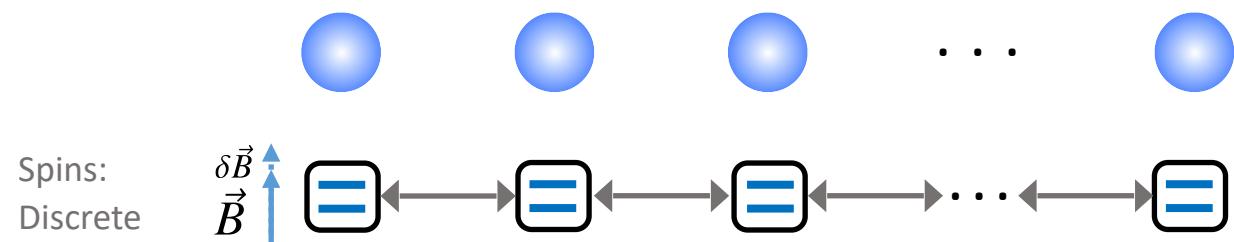
Communication

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# QUANTUM INFORMATION THEORY AND EXPERIMENTS



## Theory and physics of quantum systems



Quantum information  
& correlations

Many-body physics

Spin chain models

Quantum phase transitions

### Collective dephasing:

Exploiting symmetries to protect quantum correlations

PRL 115, 010404 (2015)

## QUANTUM TECHNOLOGIES

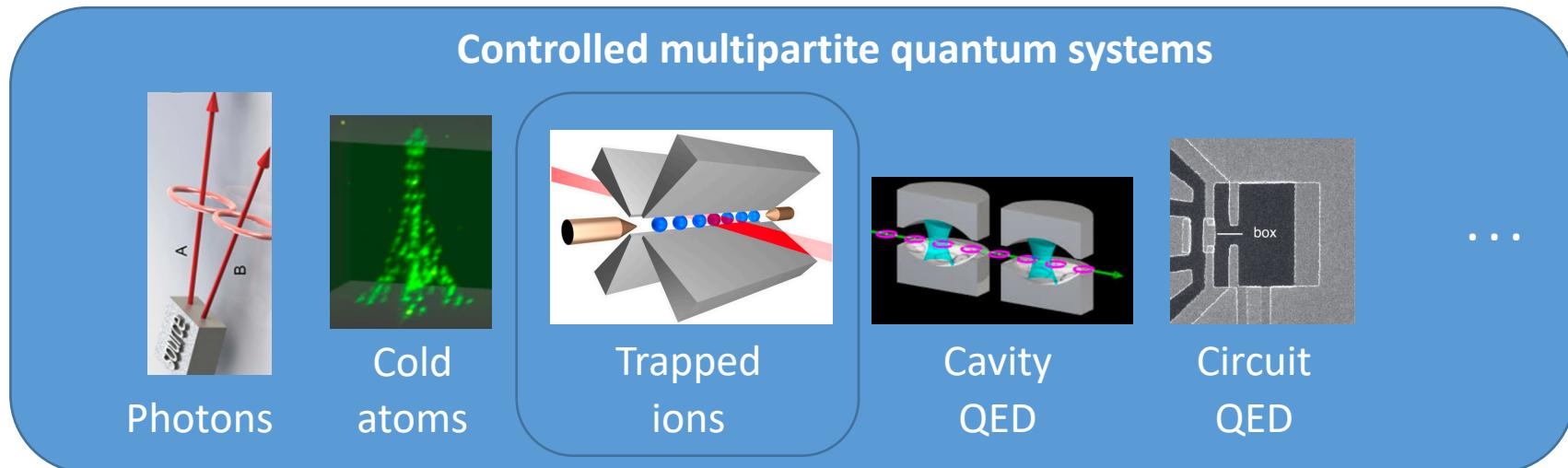
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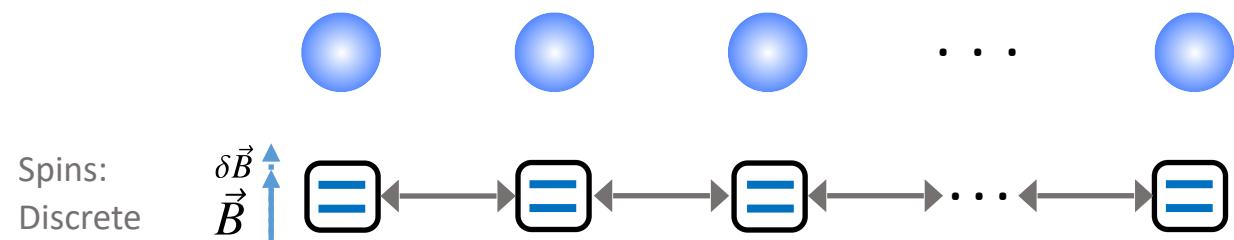
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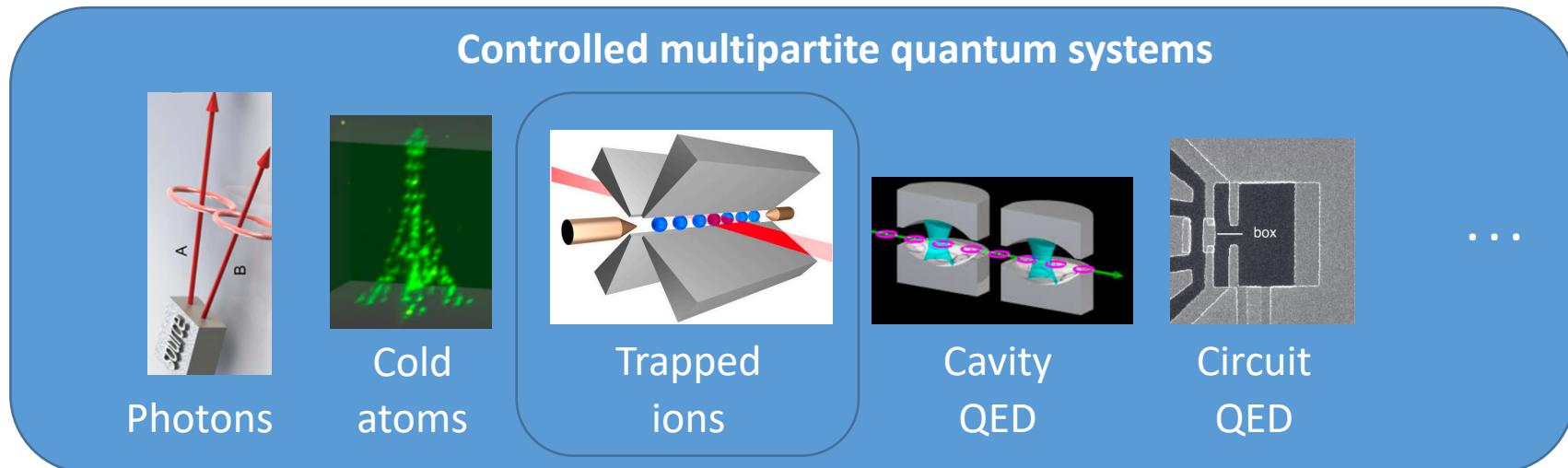
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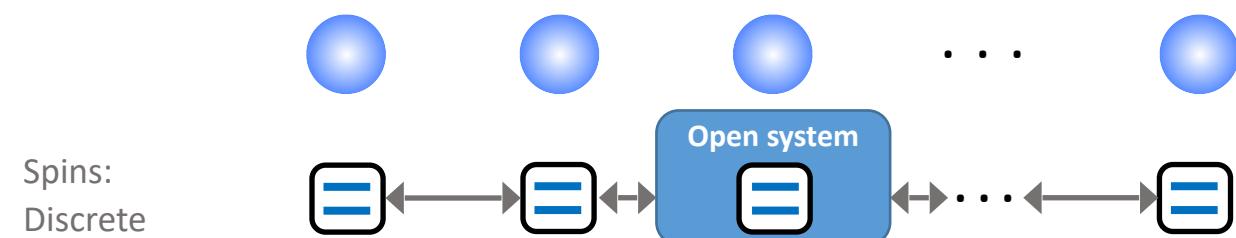
Communication

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## Theory and physics of quantum systems



Information flow & system-environment correlations

PRL 107, 180402 (2011)

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Quantum information  
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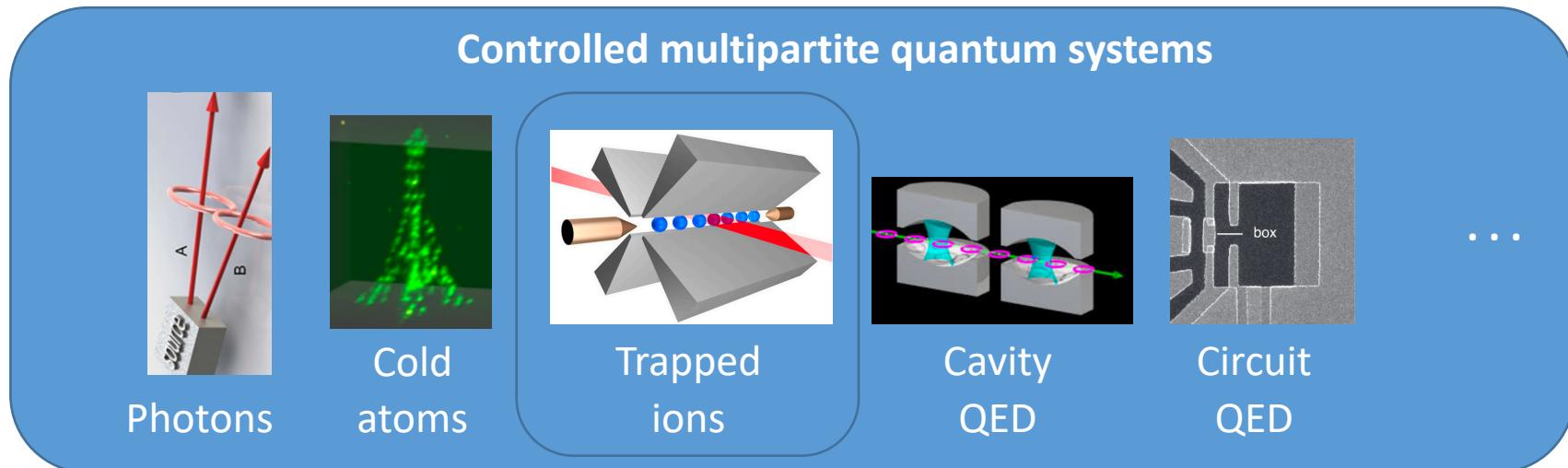
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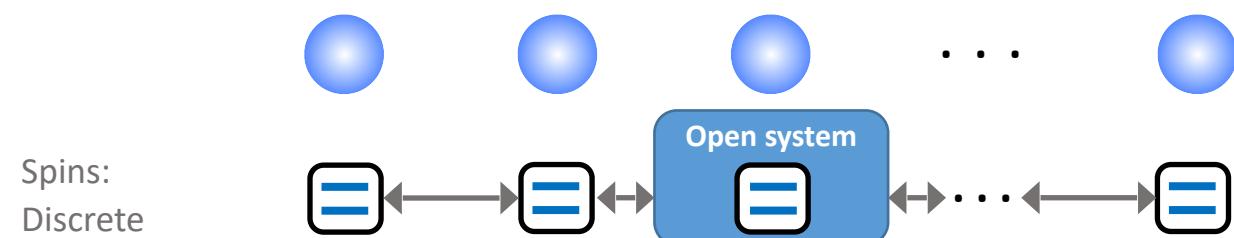
Communication

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Information flow & system-environment correlations

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Statistical approach  
PRE **87**, 042128 (2013)

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Complex quantum systems

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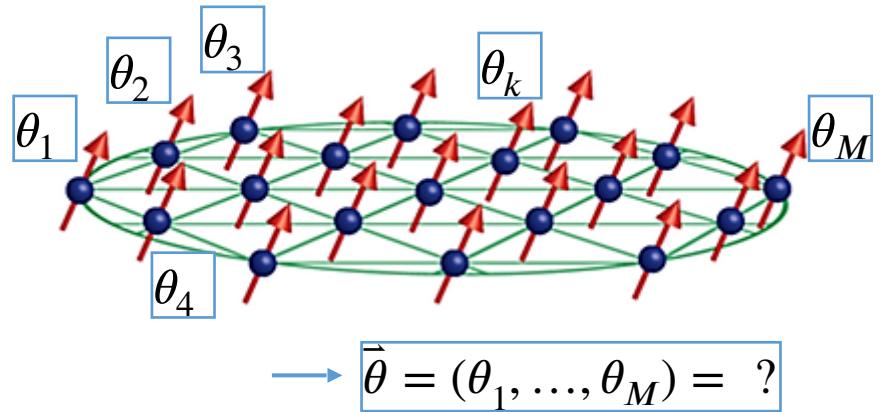
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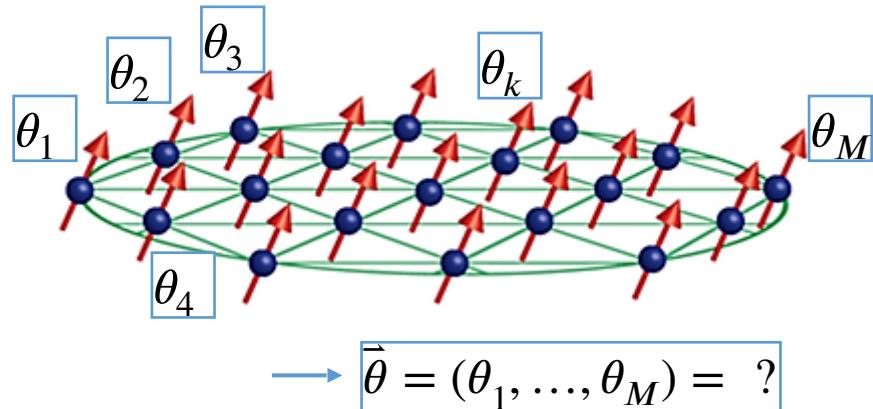
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Quantum system



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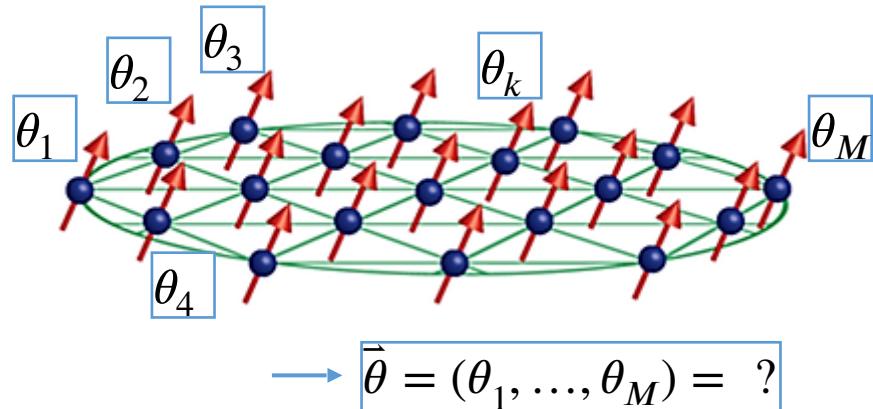
Quantum system



What are the fundamental limits  
on the precision of  $\vec{\theta} = (\theta_1, \dots, \theta_M)$  ?

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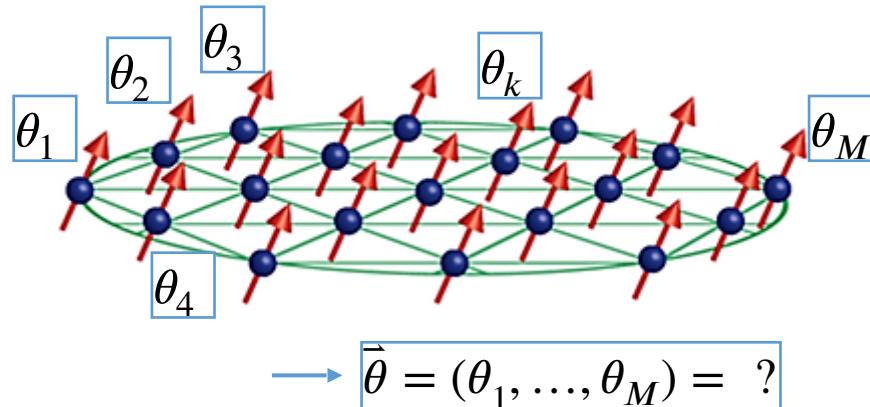
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How to choose:

Observables

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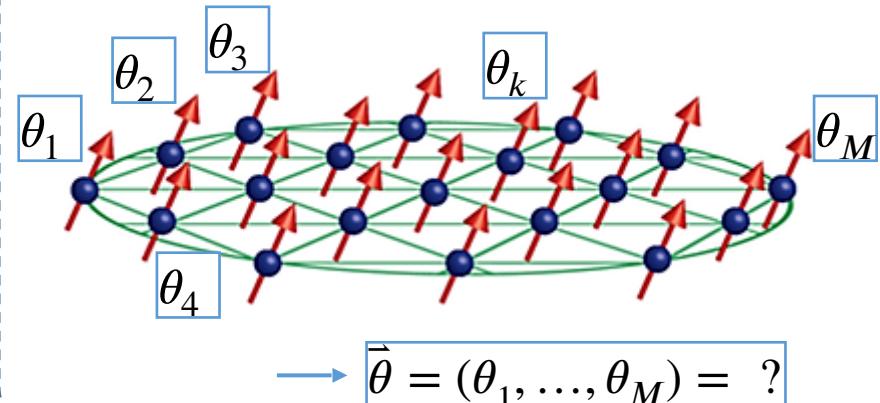
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Quantum system



Quantum  
information

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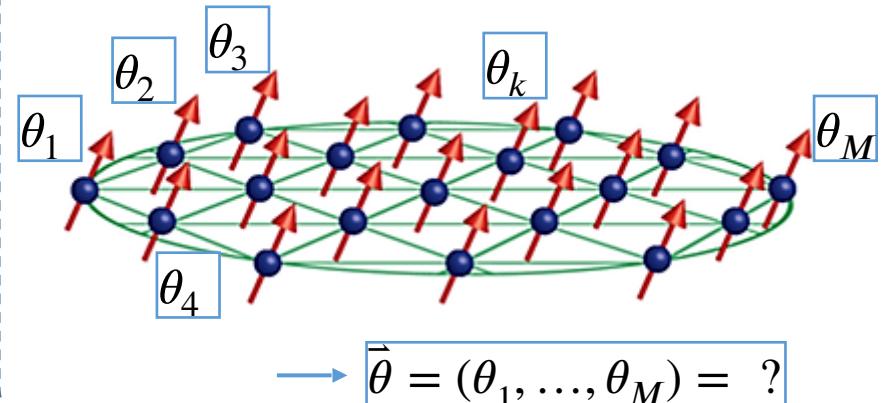
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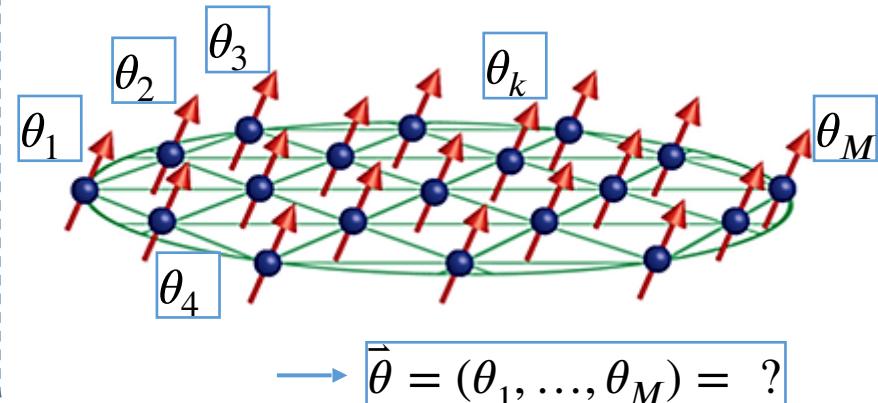
Quantum states

Motivation

Precision measurements

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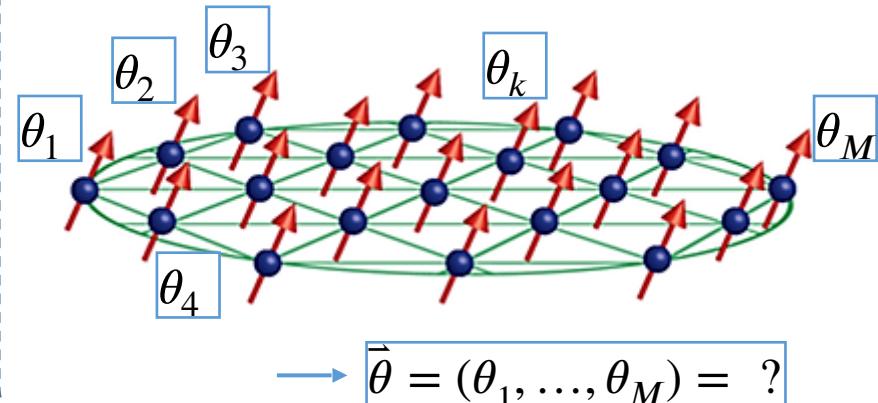
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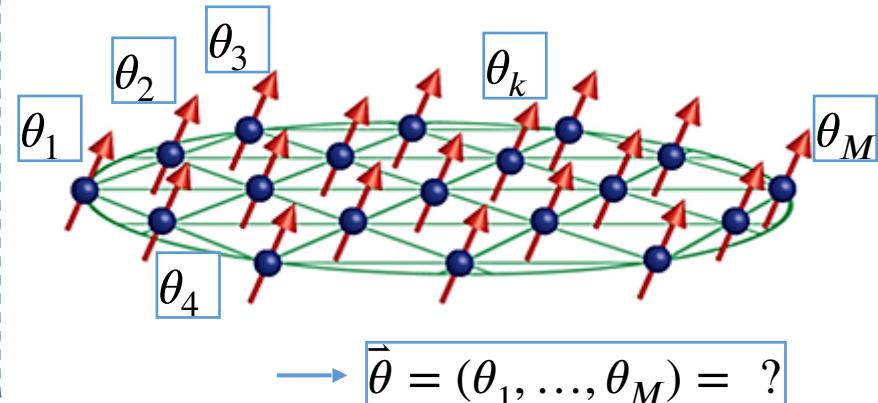
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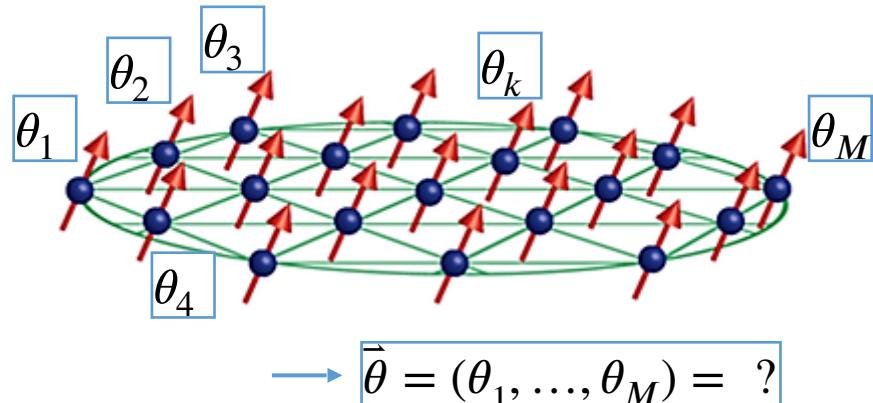
Quantum  
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Fundamental tests of physics

Characterization of models and states

# QUANTUM METROLOGY

Quantum system



Quantum  
information

What are the fundamental limits  
on the precision of  $\vec{\theta} = (\theta_1, \dots, \theta_M)$  ?

How to choose:

Observables

Quantum states

## Motivation

Precision measurements

Quantum  
many-body physics

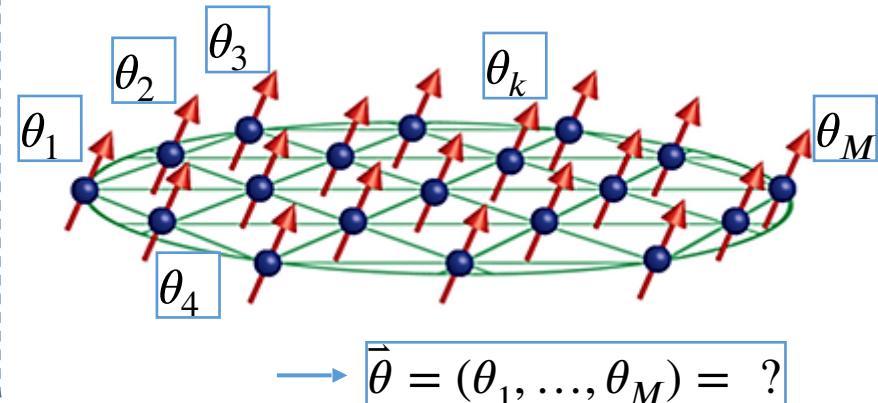
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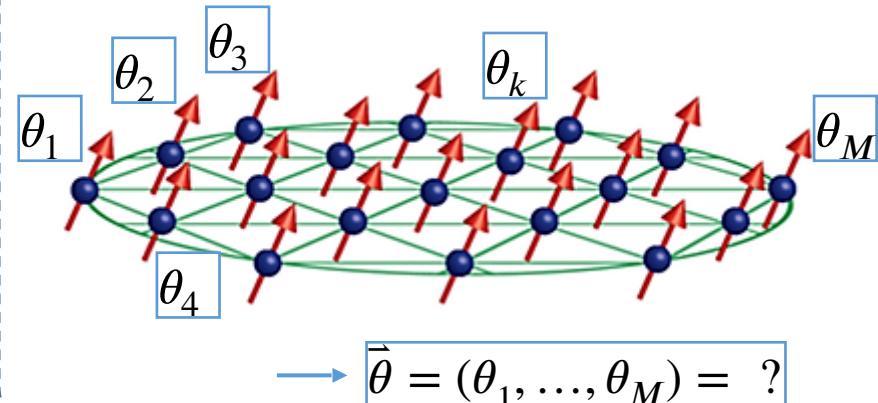
Implementation of quantum information

## Challenges

Quantum fluctuations

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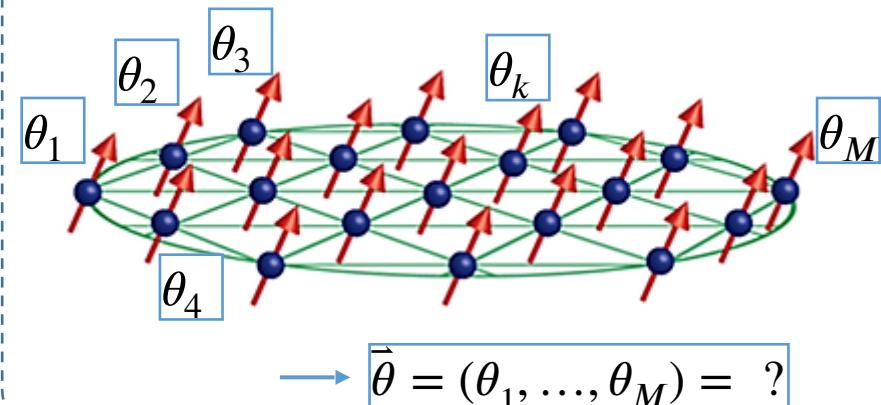
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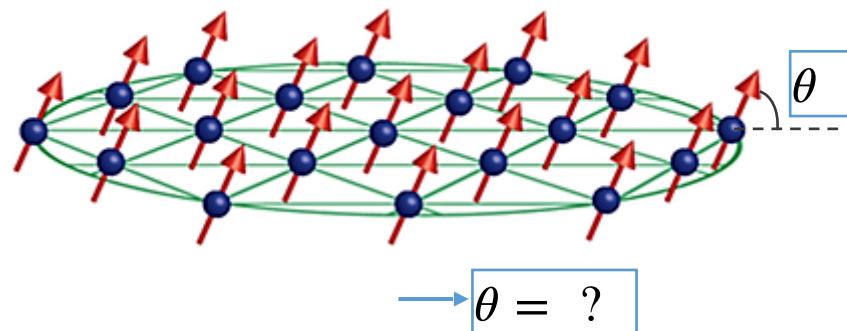
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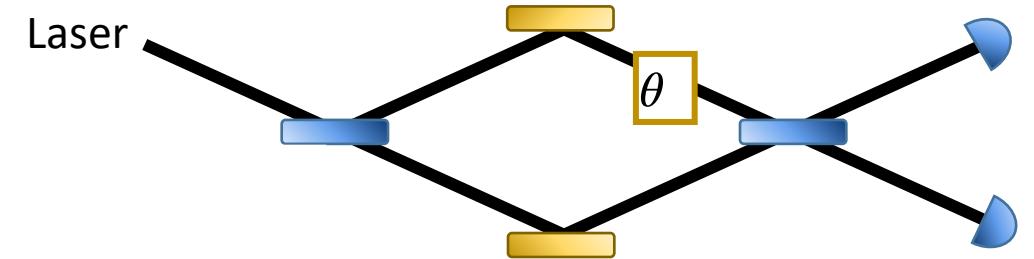
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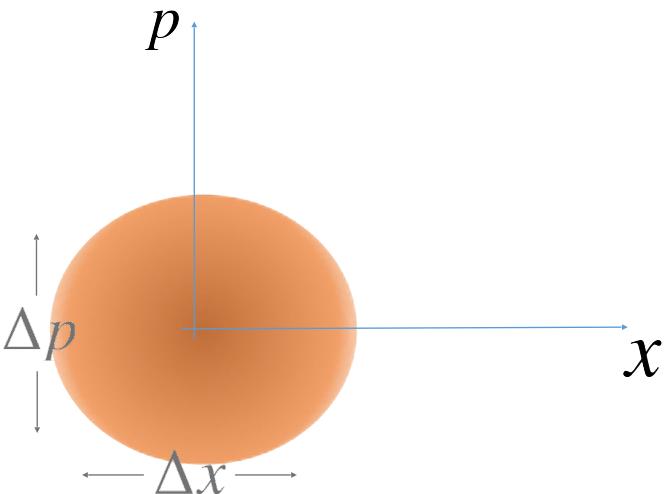
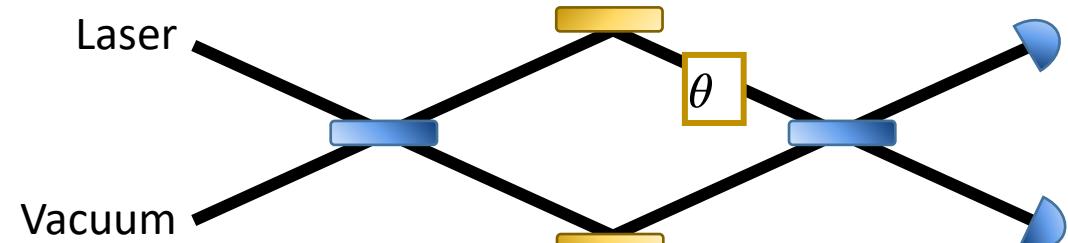
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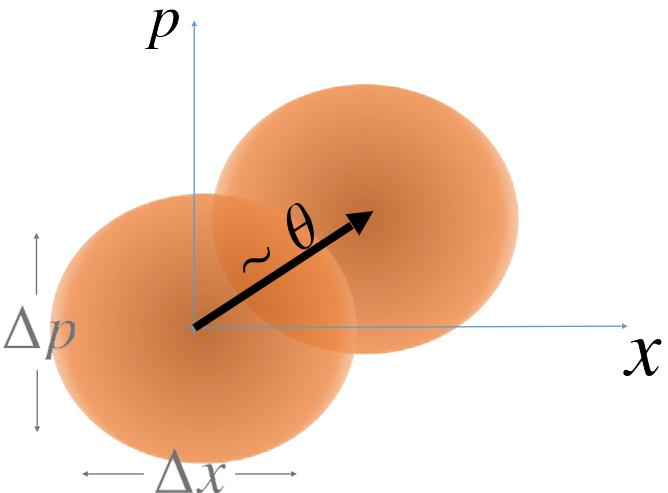
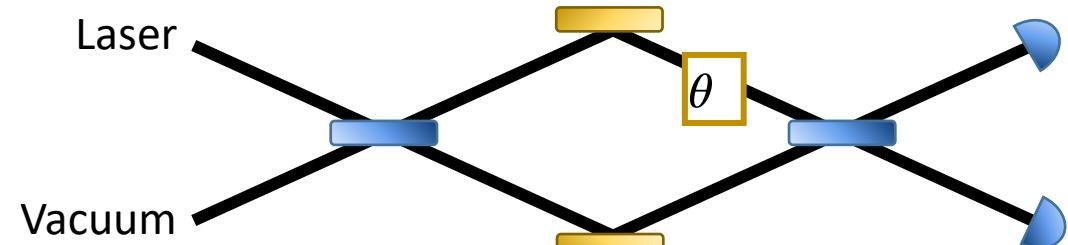
# SQUEEZING IN INTERFEROMETRY



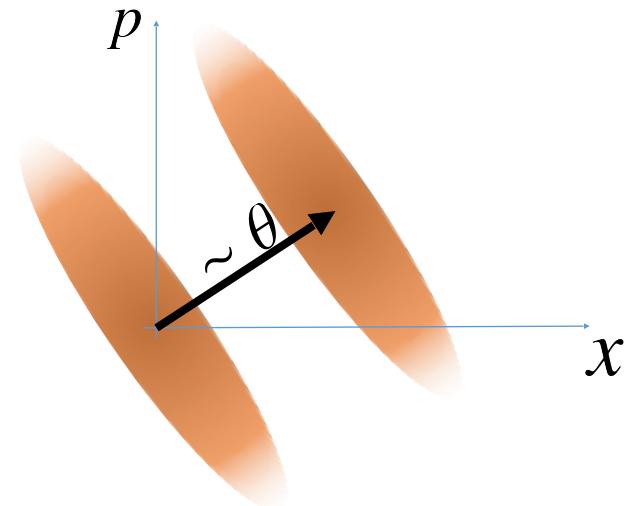
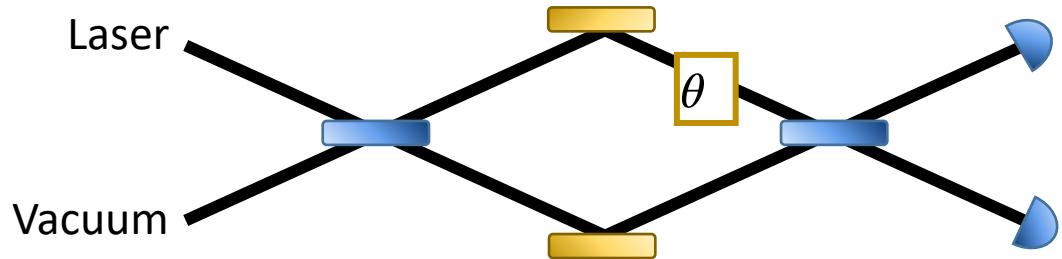
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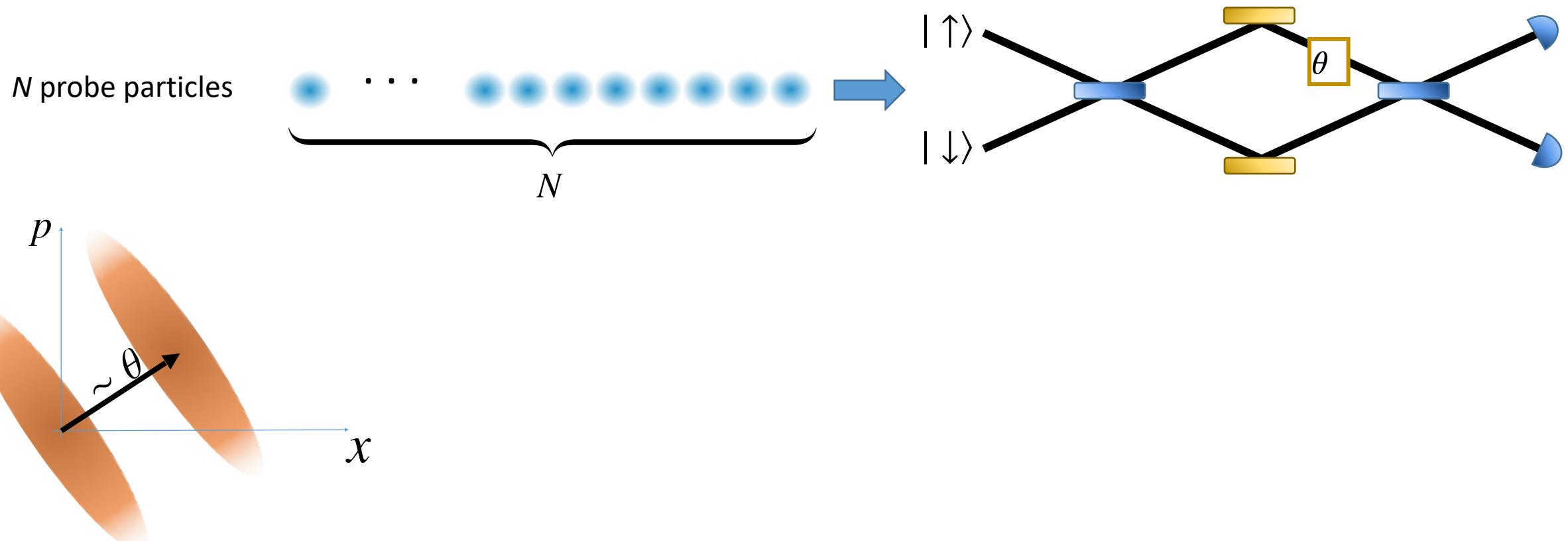


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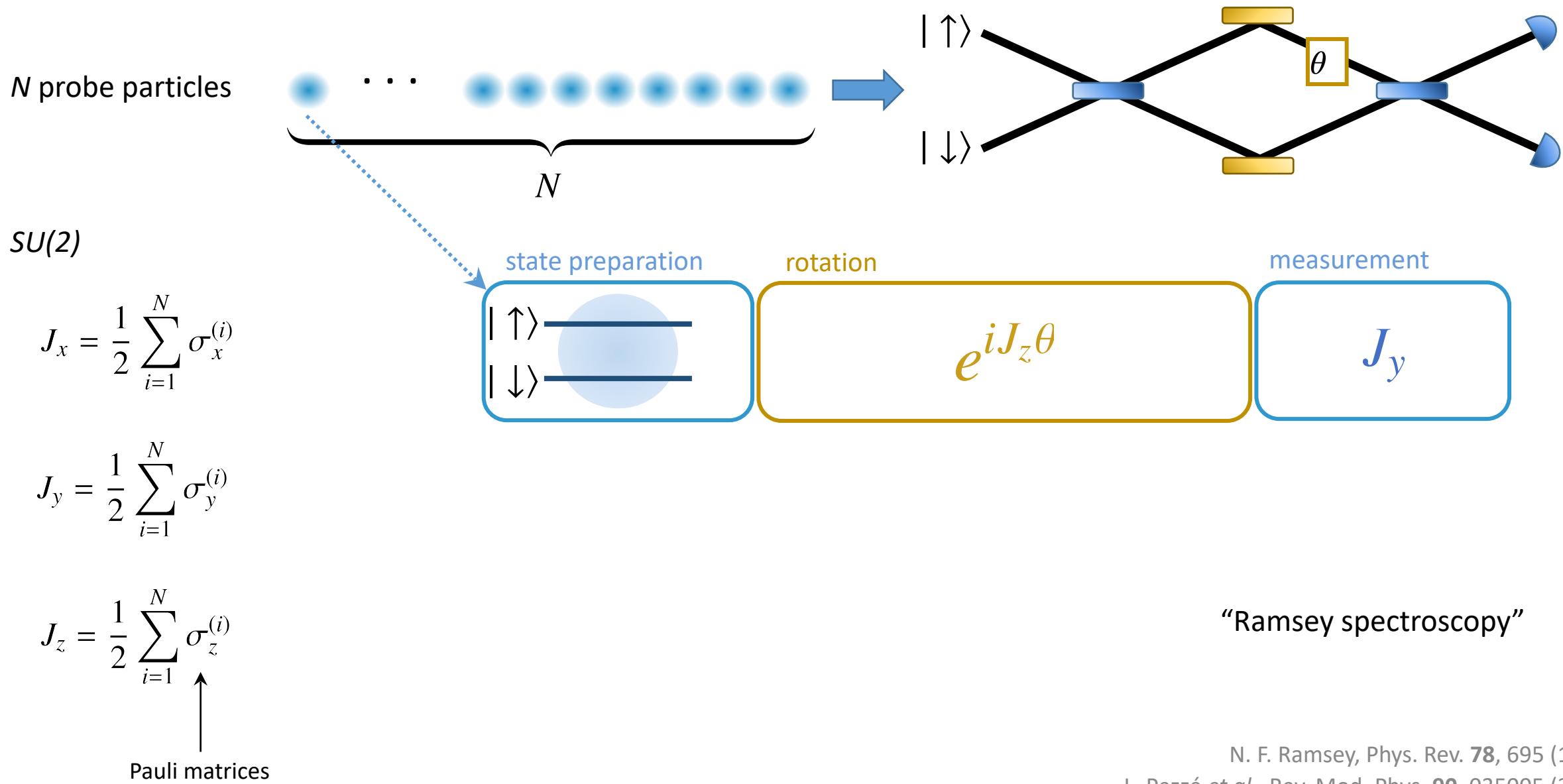
C.M. Caves, Phys. Rev. D 23, 1693 (1981)

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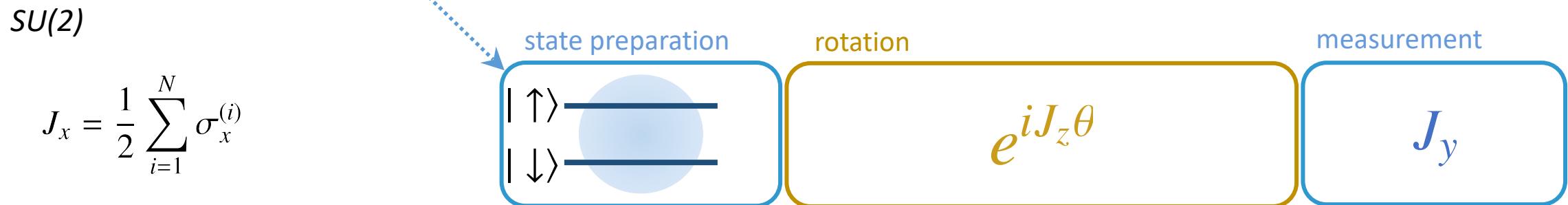
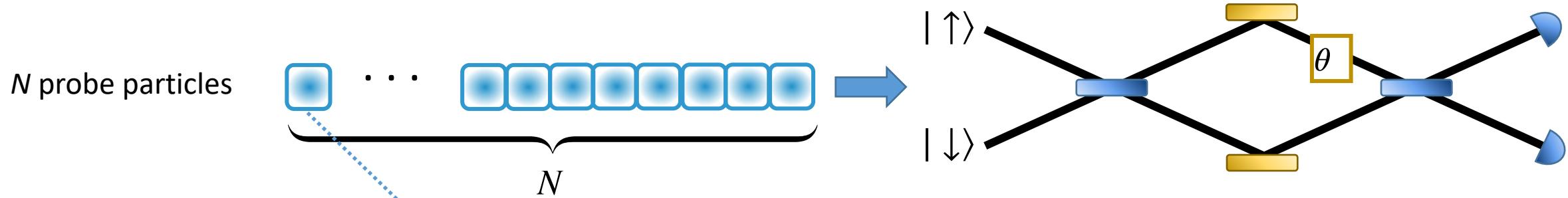


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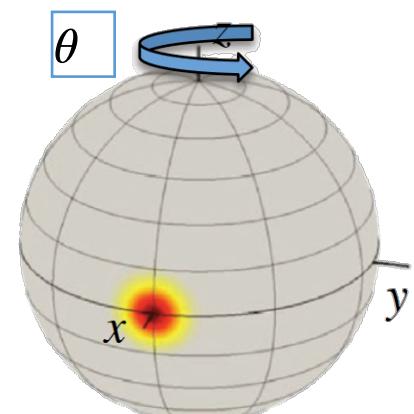
# SQUEEZING IN INTERFEROMETRY



$$J_y = \frac{1}{2} \sum_{i=1}^N \sigma_y^{(i)}$$

$$J_z = \frac{1}{2} \sum_{i=1}^N \sigma_z^{(i)}$$

↑  
Pauli matrices

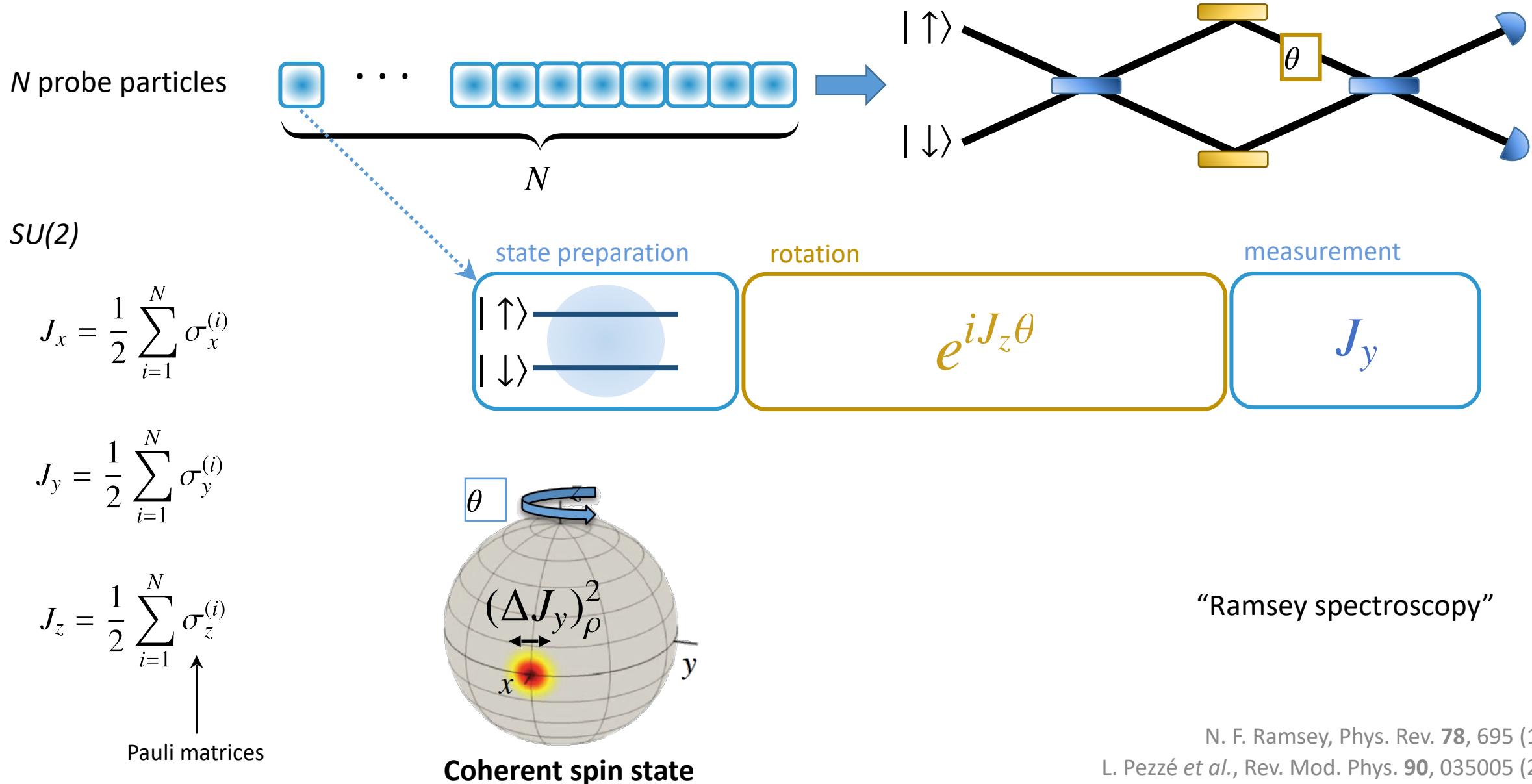


Coherent spin state

“Ramsey spectroscopy”

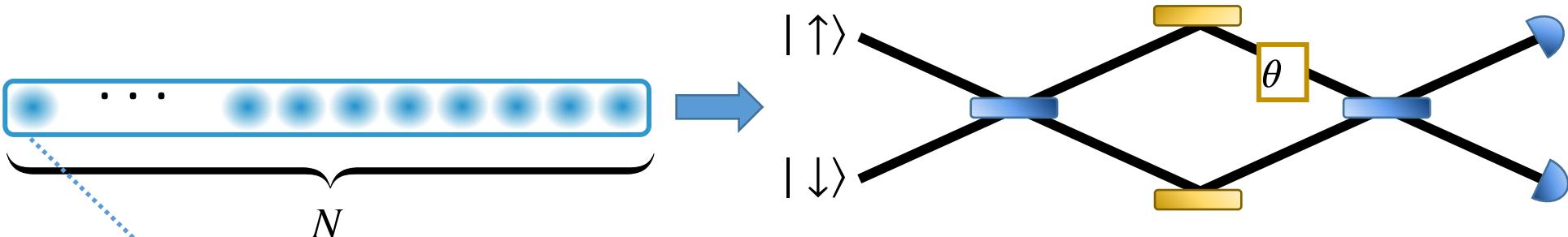
N. F. Ramsey, Phys. Rev. **78**, 695 (1950)  
L. Pezzé *et al.*, Rev. Mod. Phys. **90**, 035005 (2018)

# SQUEEZING IN INTERFEROMETRY



# SQUEEZING IN INTERFEROMETRY

$N$  probe particles



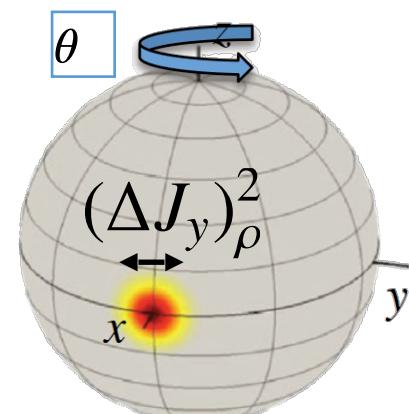
$SU(2)$

$$J_x = \frac{1}{2} \sum_{i=1}^N \sigma_x^{(i)}$$

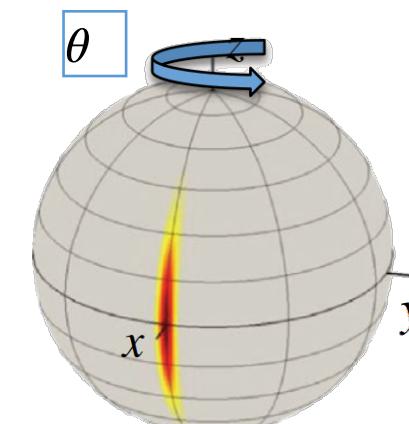
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Coherent spin state



Squeezed spin state

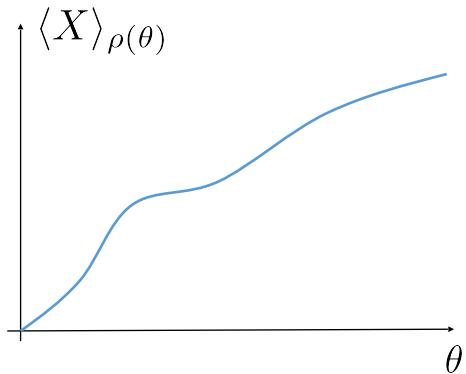
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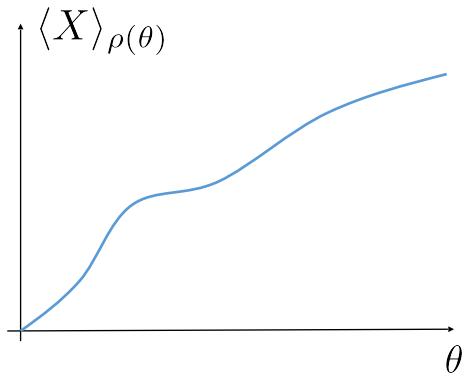
# QUANTUM NOISE AND SQUEEZING

## Calibration



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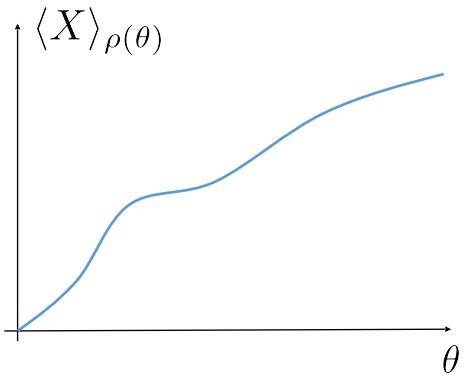


## Measurement

$$x_1, \dots, x_\mu$$

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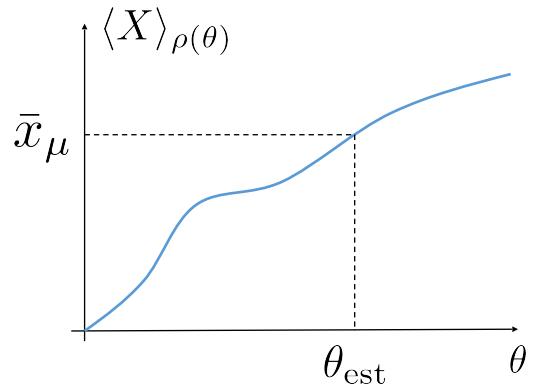


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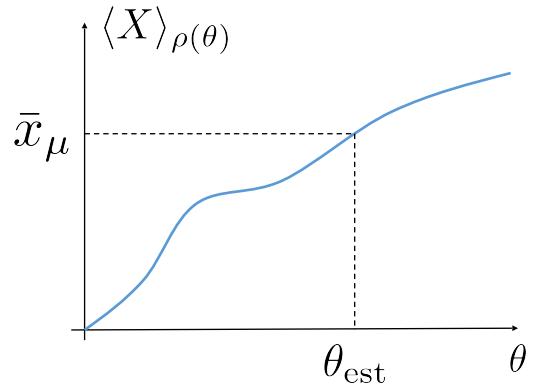


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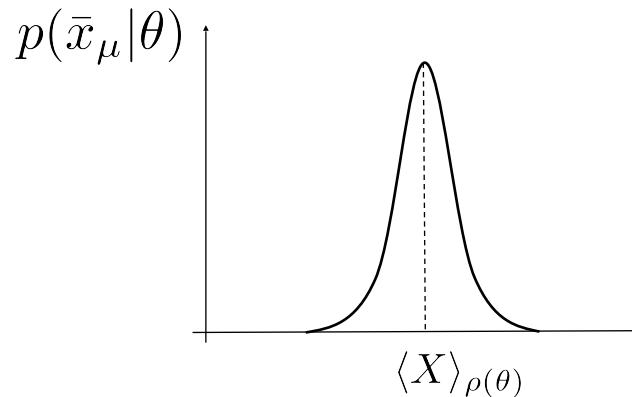
# QUANTUM NOISE AND SQUEEZING

**Calibration**



**Uncertainty**

In the central limit,  $\mu \gg 1$

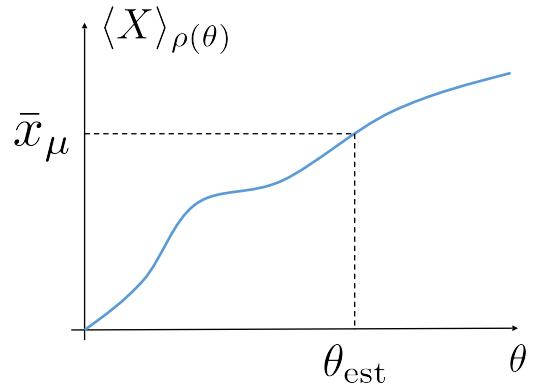


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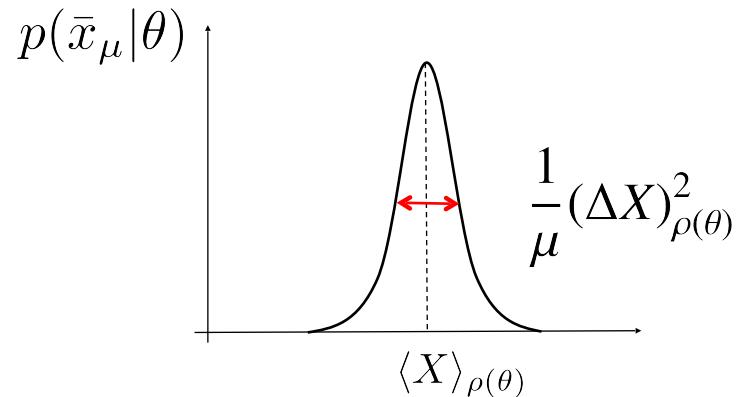
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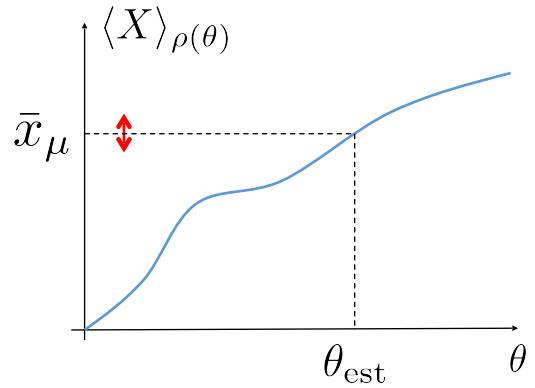


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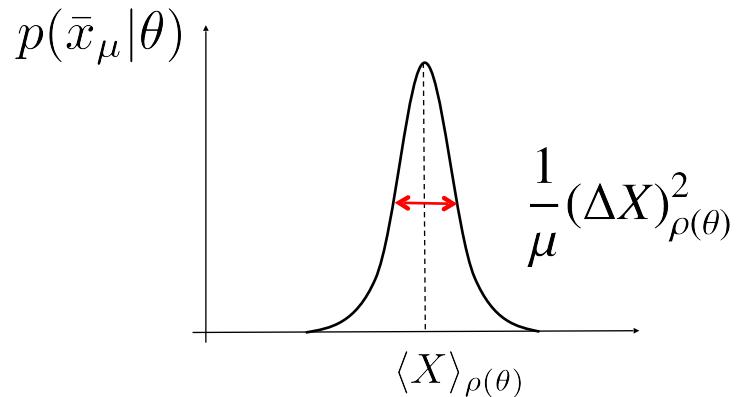
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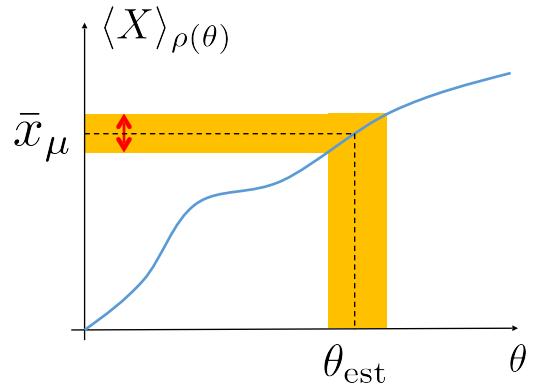


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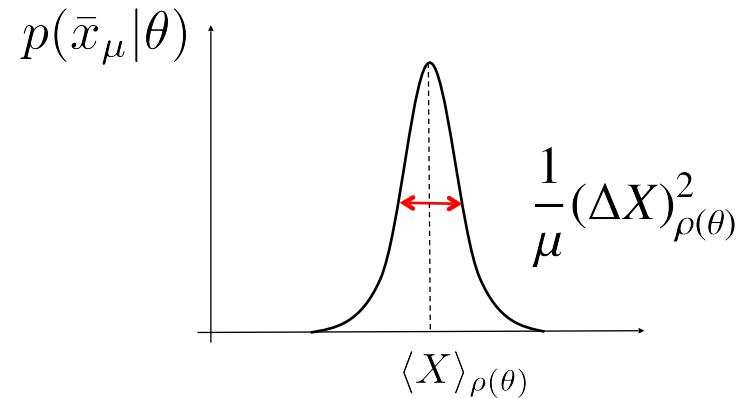
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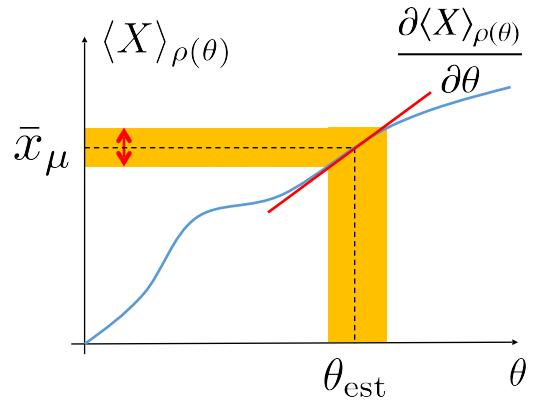


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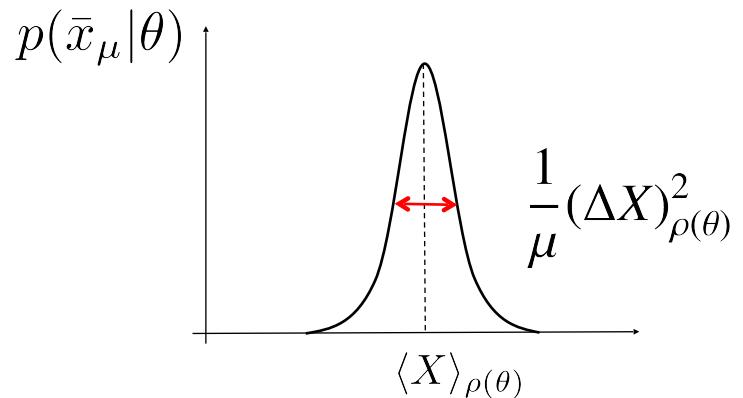
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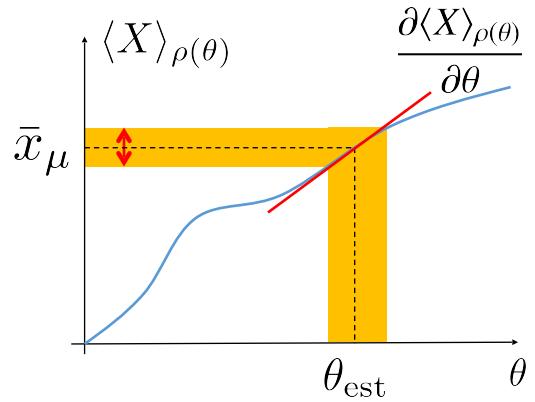


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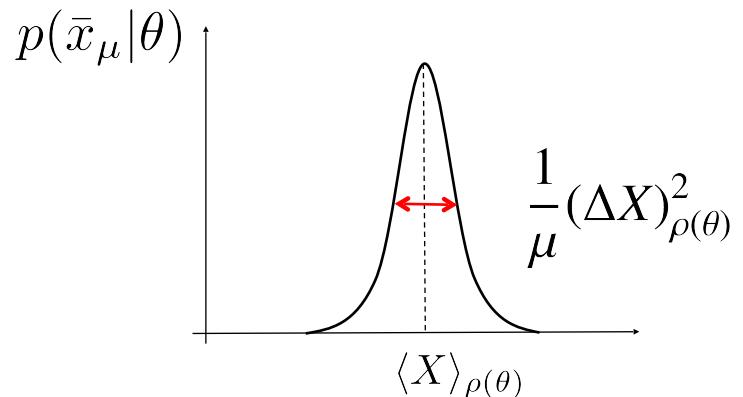
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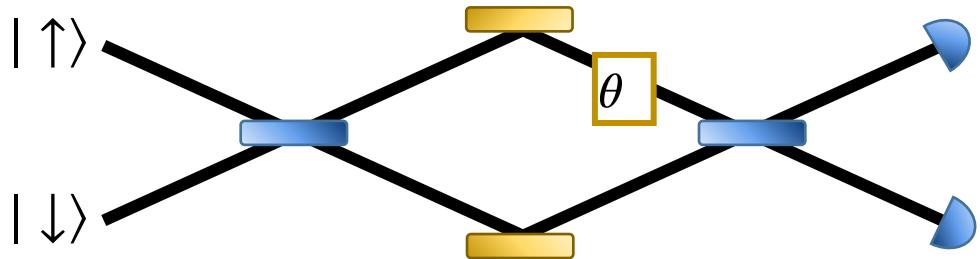


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$$(\Delta \theta_{\text{est}})^2 = \frac{1}{\mu} \frac{(\Delta X)_{\rho(\theta)}^2}{\left| \frac{\partial \langle X \rangle_{\rho(\theta)}}{\partial \theta} \right|^2}$$

# QUANTUM NOISE AND SQUEEZING



Ramsey spectroscopy

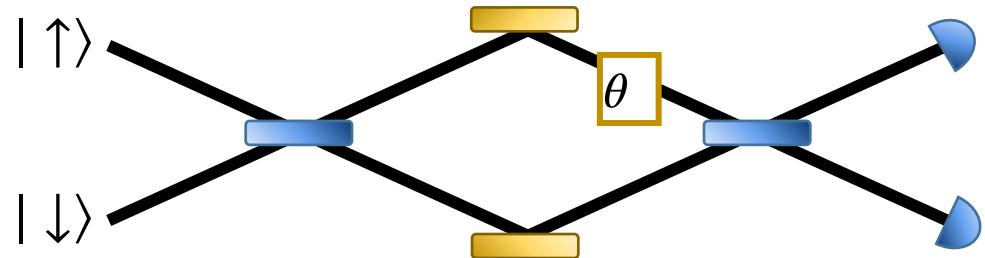
$$X = J_y$$

Schrödinger's equation

$$\frac{\partial \langle X \rangle_\rho}{\partial \theta} = i \langle [J_z, J_y] \rangle_\rho = i \langle J_x \rangle_\rho$$

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# QUANTUM NOISE AND SQUEEZING



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„Spin Squeezing“

D. J. Wineland *et al.*,  
Phys. Rev. A **46**, R6797 (1992)

state preparation

$$| \uparrow \rangle$$
  
$$| \downarrow \rangle$$

rotation

$$e^{iJ_z\theta}$$

measurement

$$J_y$$

Ramsey spectroscopy

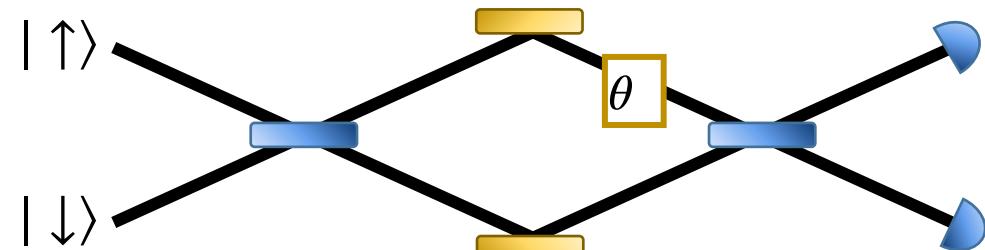
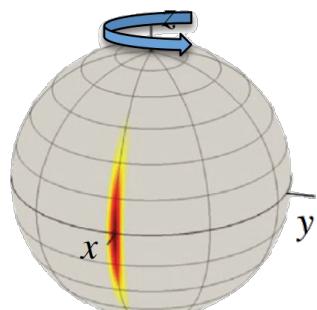
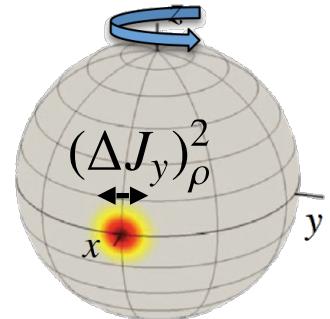
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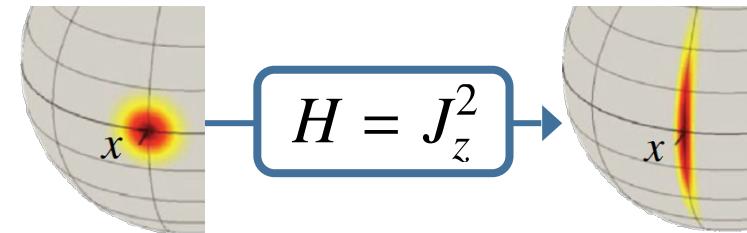
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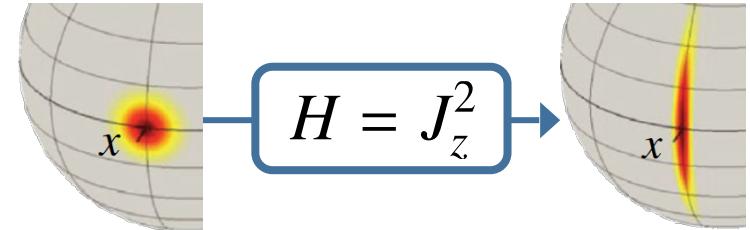
# QUANTUM NOISE AND SQUEEZING: EXPERIMENTS

Kitagawa & Ueda, PRA **47** 5138 (1993)



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Proposals for implementation

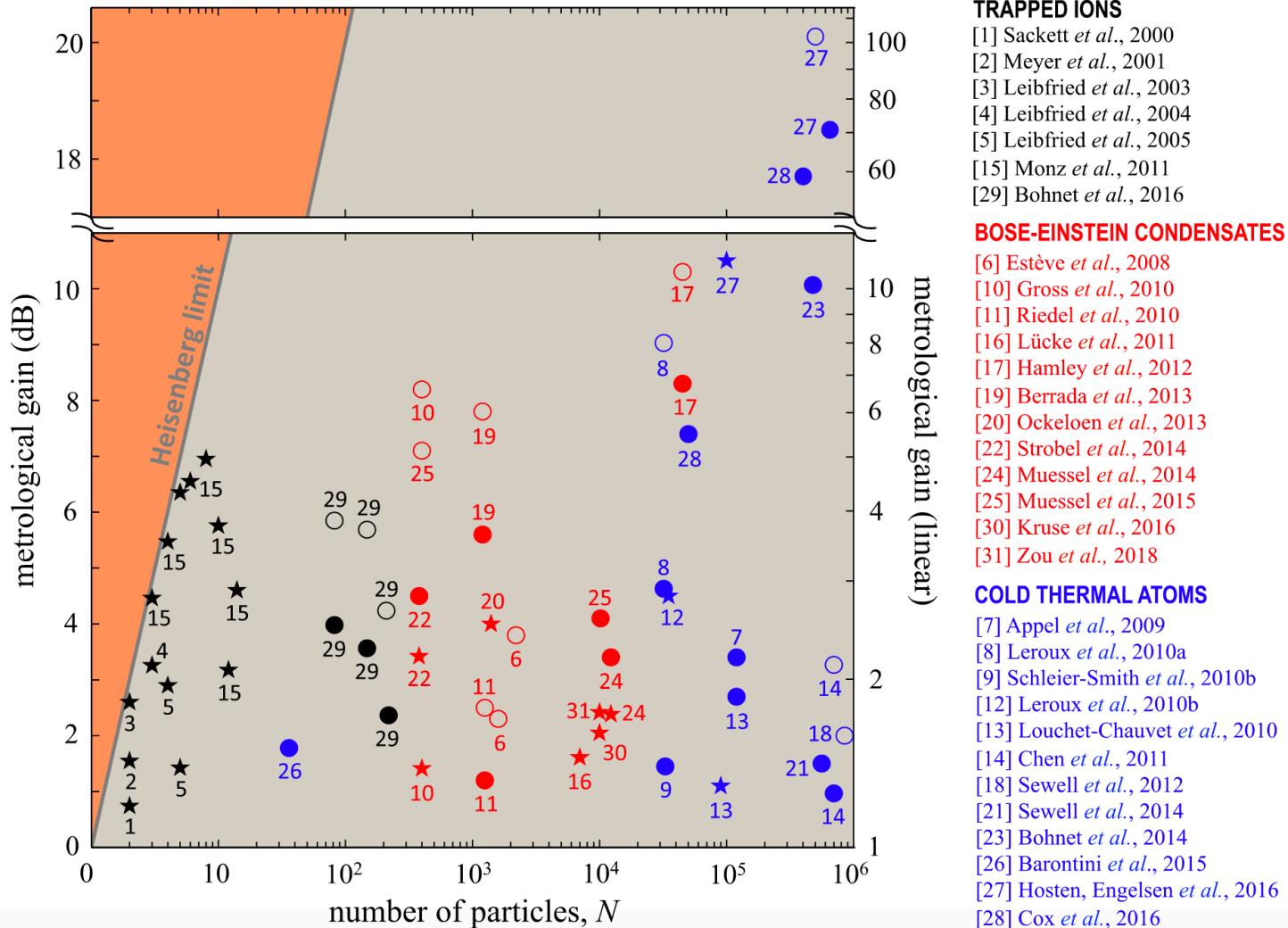
**Trapped ions:**

Mølmer & Sørensen PRL **82** 1835 (1999)

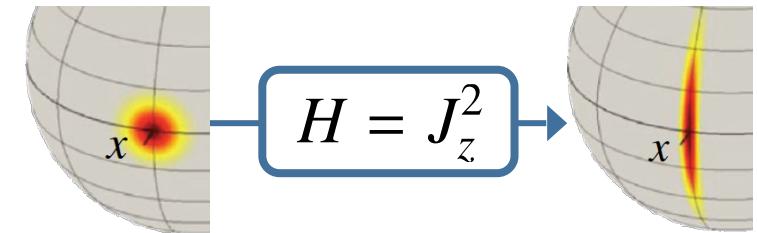
**Bose-Einstein condensates:**

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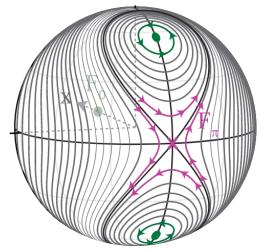
**Review:**

Pezzè, Smerzi, Oberthaler, Schmied, Treutlein  
Rev. Mod. Phys. **90**, 035005 (2018)

# QUANTUM GAIN: GAUSSIAN SQUEEZING AND BEYOND

Sensitivity

$$\chi^{-2} = \frac{|\langle [J_{\mathbf{n}}, J_{\mathbf{m}}] \rangle|^2}{(\Delta J_{\mathbf{n}})^2}$$



$$H_{\text{NL}} = \frac{1}{N} J_z^2 + \lambda J_x$$

PRA 99, 022329 (2019)

state preparation

$$\begin{array}{c} |\uparrow\rangle \\ |\downarrow\rangle \end{array}$$

rotation

$$e^{-iJ_{\mathbf{m}}\theta}$$

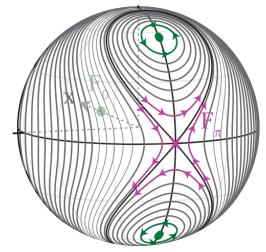
measurement

$$J_{\mathbf{n}}$$

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PRA 99, 022329 (2019)

state preparation

$$e^{-iH_{\text{NL}}\tau}$$

Nonlinear evolution

rotation

$$e^{-iJ_{\mathbf{m}}\theta}$$

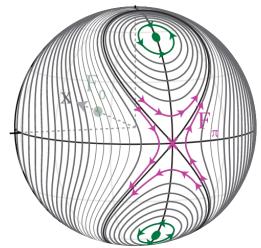
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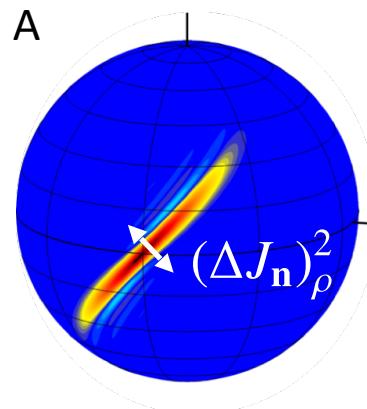
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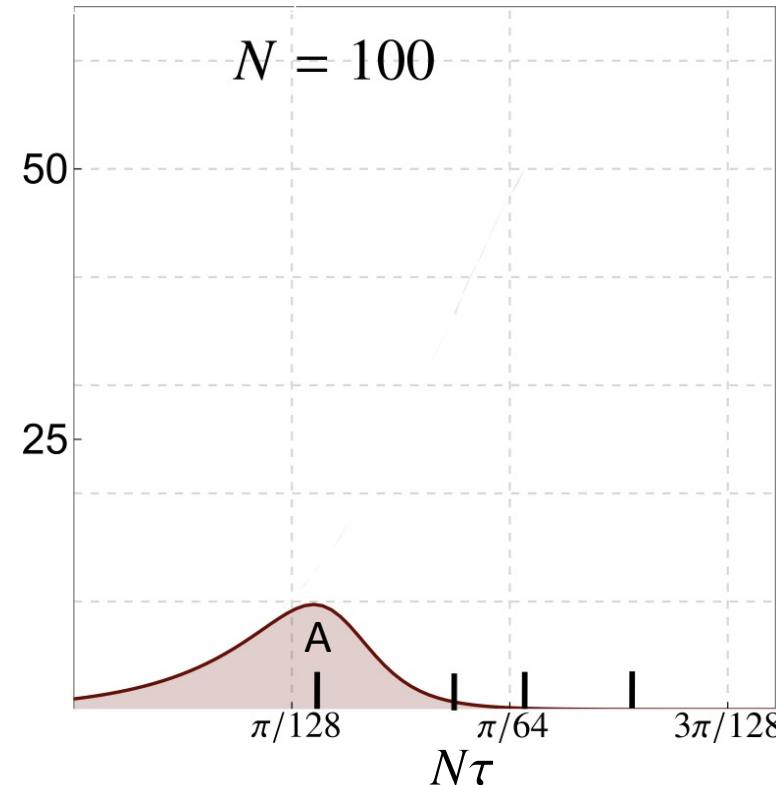
measurement

$$J_{\mathbf{n}}$$

Nonlinear evolution



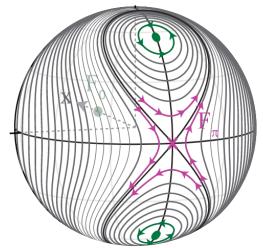
Sensitivity



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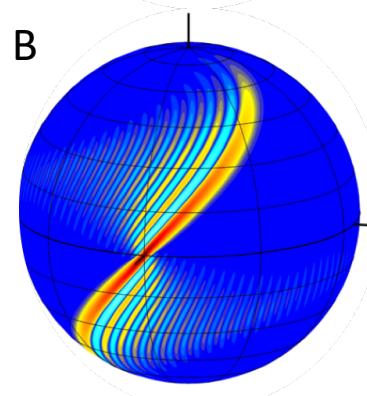
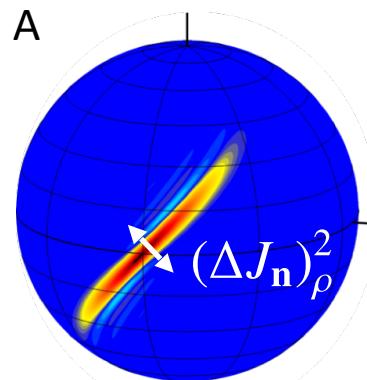
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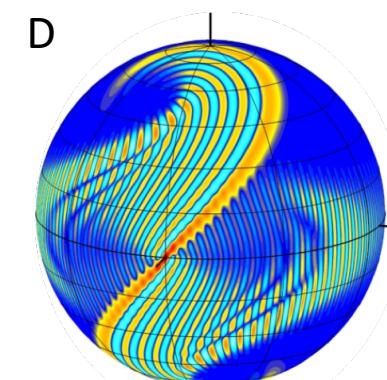
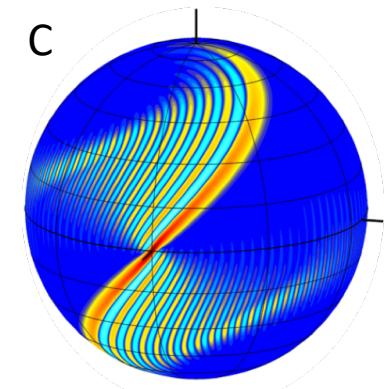
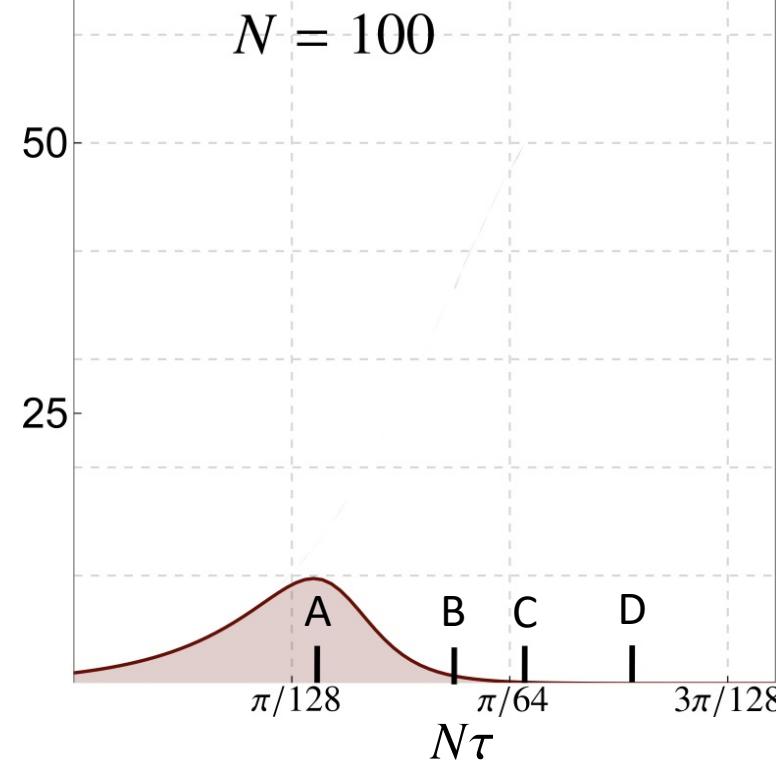
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$$J_{\mathbf{n}}$$

Nonlinear evolution



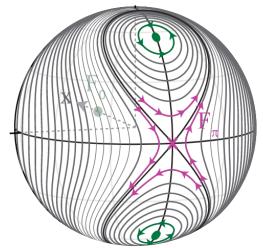
Sensitivity



# QUANTUM GAIN: GAUSSIAN SQUEEZING AND BEYOND

Sensitivity

$$\chi^{-2} = \frac{|\langle [J_{\mathbf{n}}, J_{\mathbf{m}}] \rangle|^2}{(\Delta J_{\mathbf{n}})^2}$$



$$H_{\text{NL}} = \frac{1}{N} J_z^2 + \lambda J_x$$

PRA 99, 022329 (2019)

state preparation

$$e^{-iH_{\text{NL}}\tau}$$

Nonlinear evolution

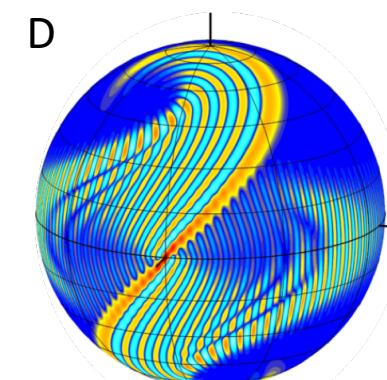
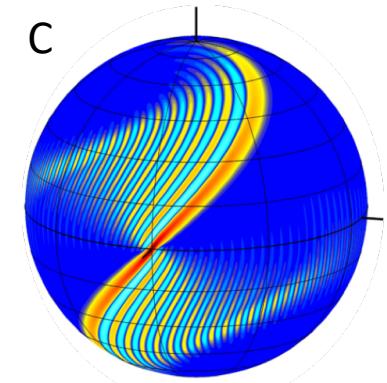
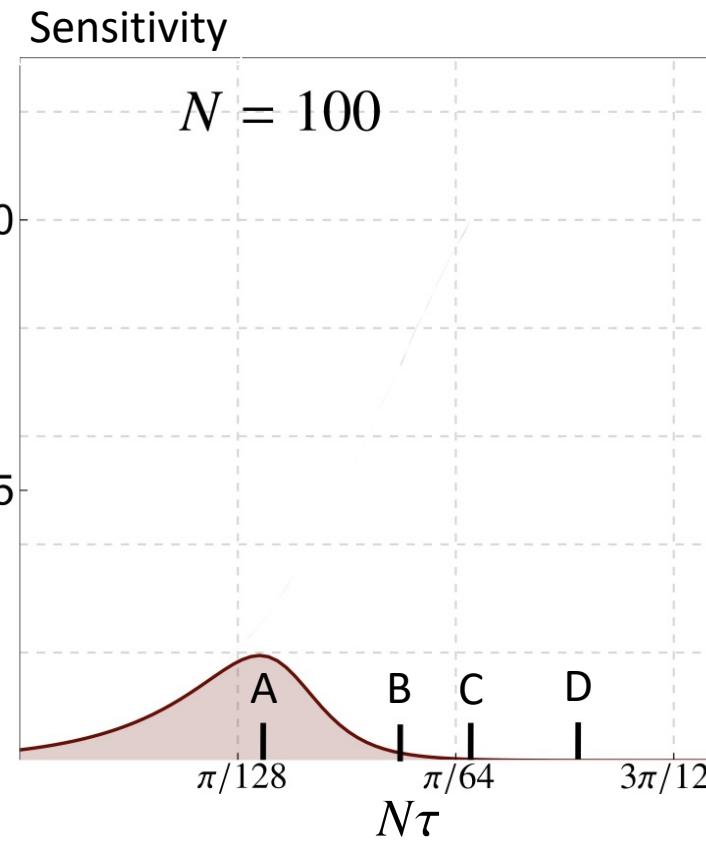
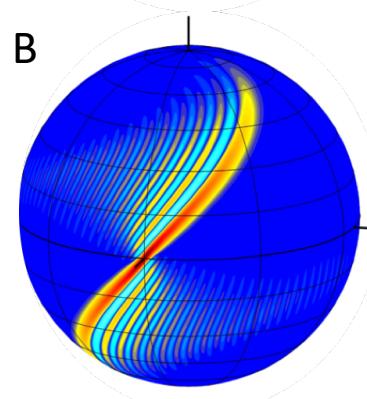
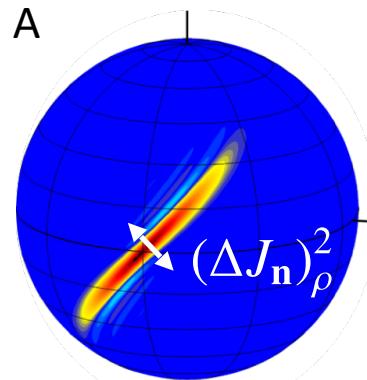
rotation

$$e^{-iJ_{\mathbf{m}}\theta}$$

measurement

$$J_{\mathbf{n}}$$

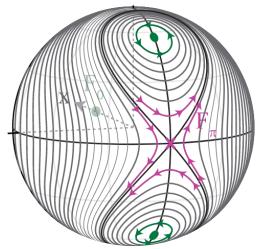
Nonlinear observable



# QUANTUM GAIN: GAUSSIAN SQUEEZING AND BEYOND

Sensitivity

$$\chi^{-2} = \frac{|\langle [X, J_m] \rangle|^2}{(\Delta X)^2}$$



$$H_{\text{NL}} = \frac{1}{N} J_z^2 + \lambda J_x$$

PRA **99**, 022329 (2019)

M. Gessner, A. Smerzi, and L. Pezzè  
Phys. Rev. Lett. **122**, 090503 (2019)

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Optimized *nonlinear* observables  
as a function of accessible operators

state preparation

$$e^{-iH_{\text{NL}}\tau}$$

Nonlinear evolution

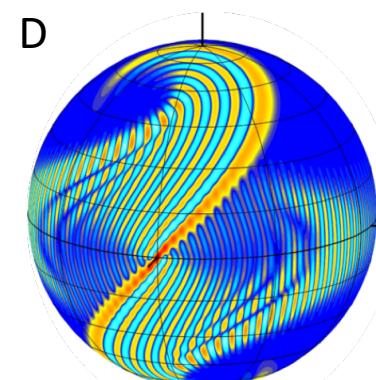
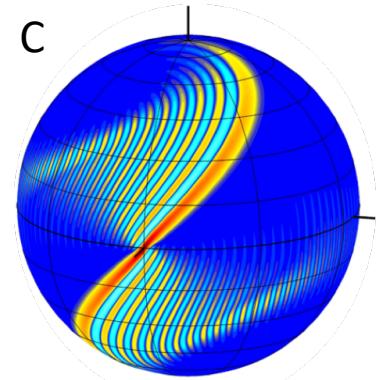
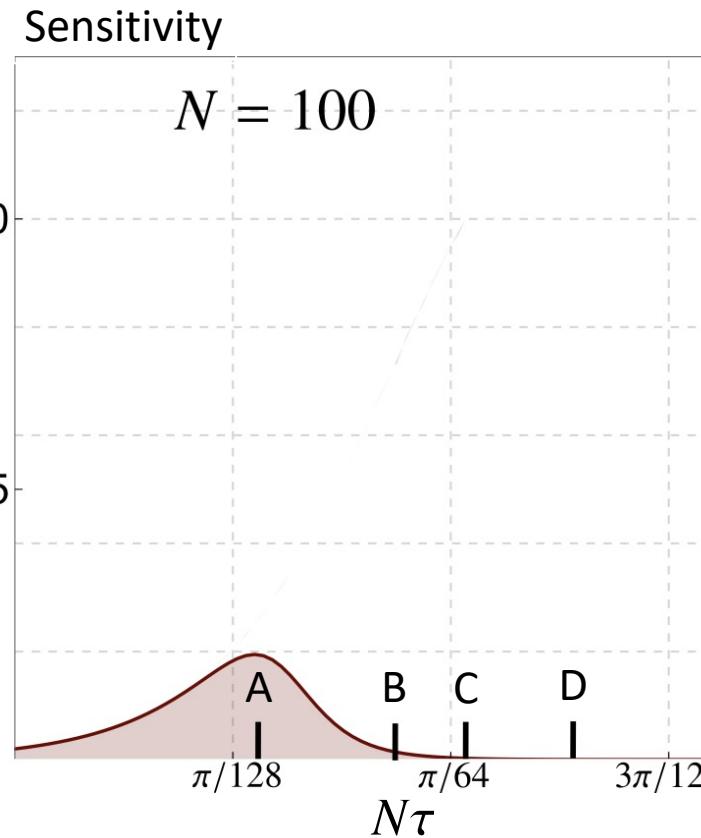
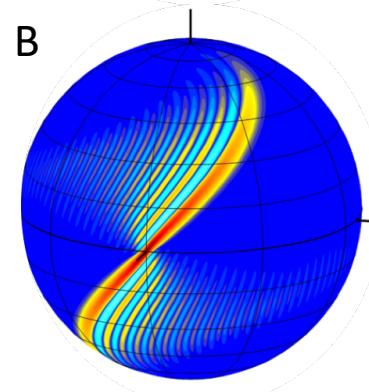
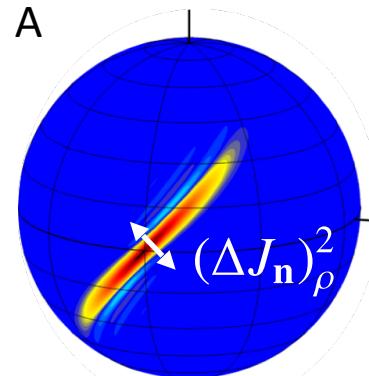
rotation

$$e^{-iJ_m\theta}$$

measurement

$$X$$

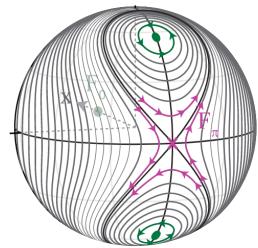
Nonlinear observable



# QUANTUM GAIN: GAUSSIAN SQUEEZING AND BEYOND

Sensitivity

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state preparation

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Nonlinear evolution

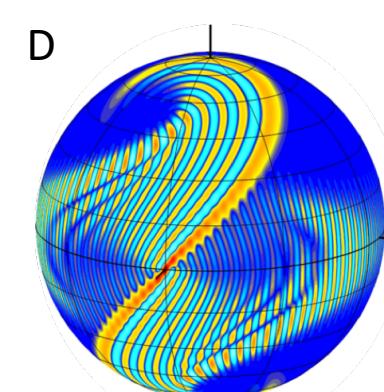
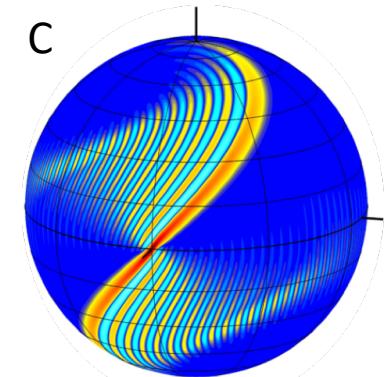
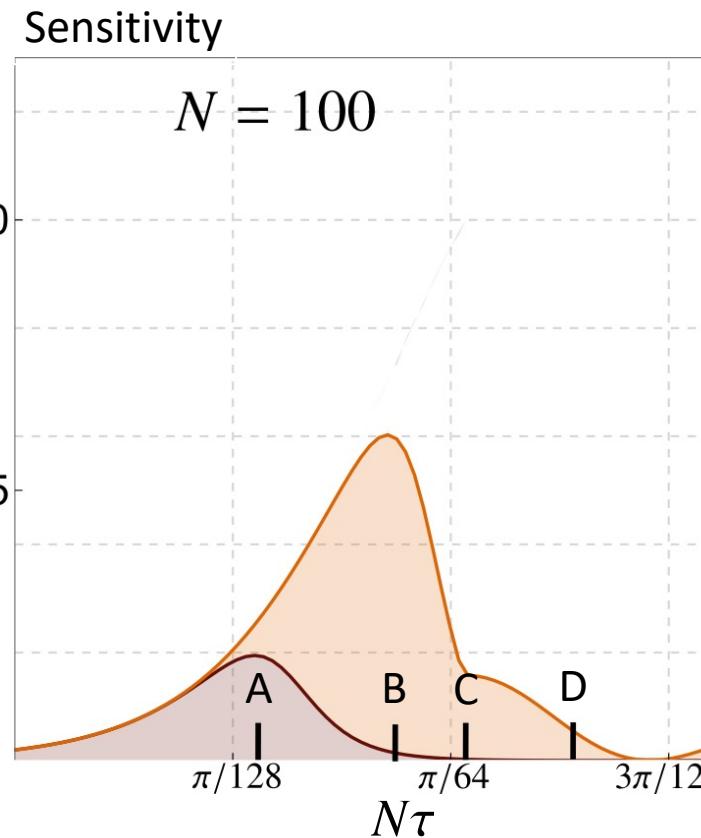
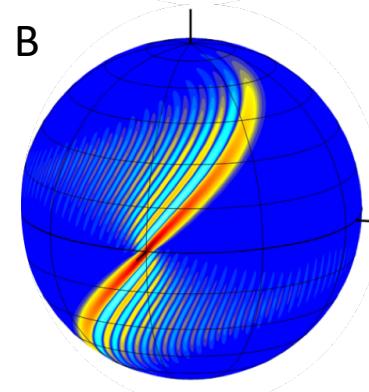
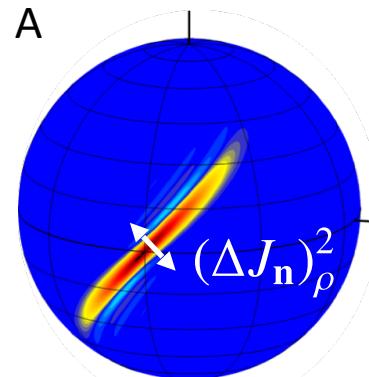
rotation

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$$X$$

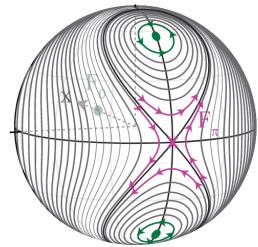
Nonlinear observable



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state preparation

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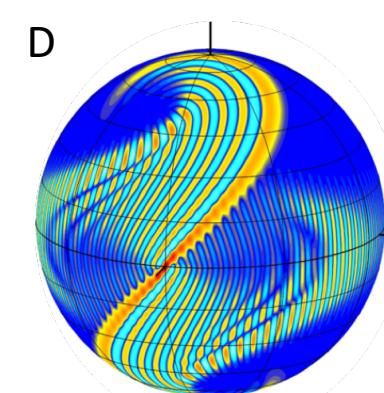
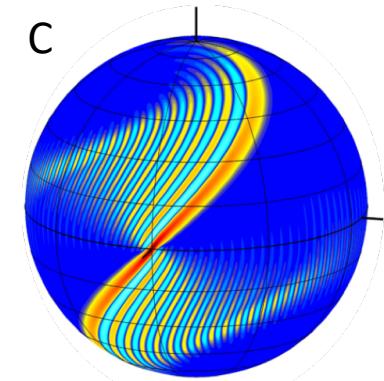
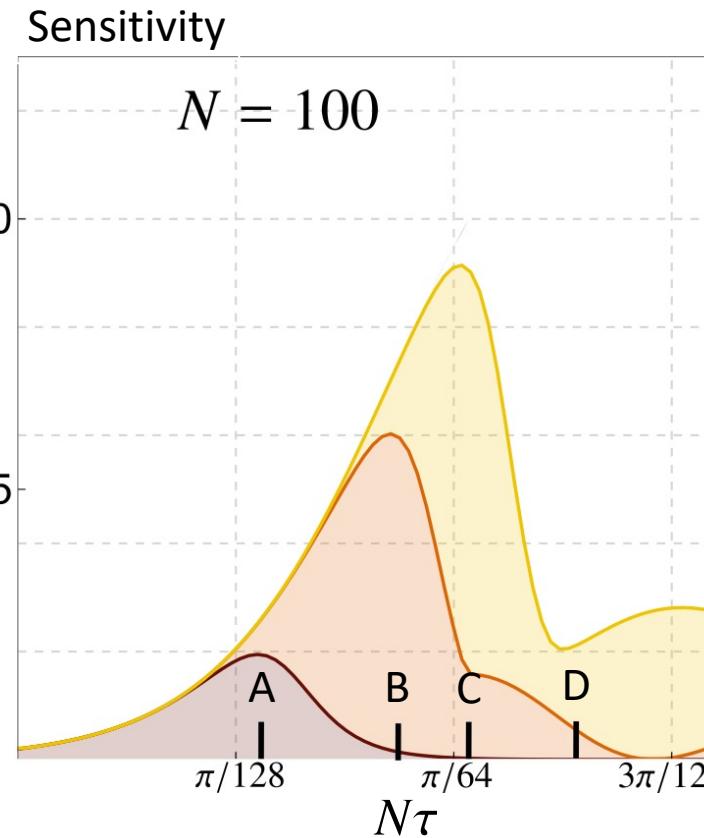
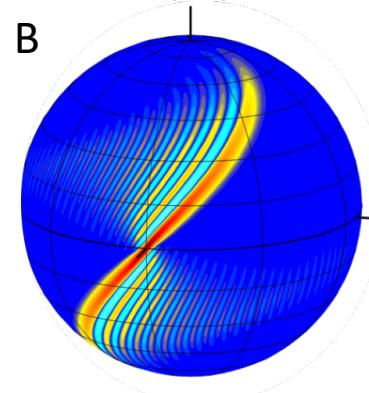
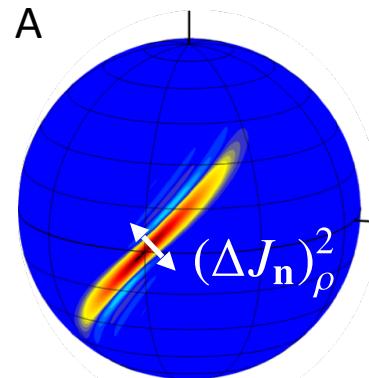
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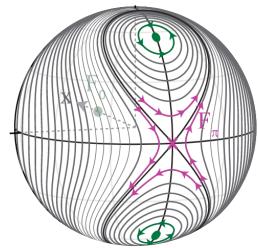
Nonlinear observable



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state preparation

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Nonlinear evolution

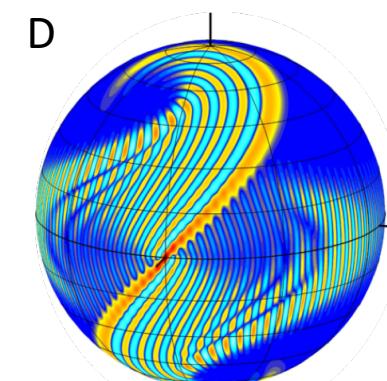
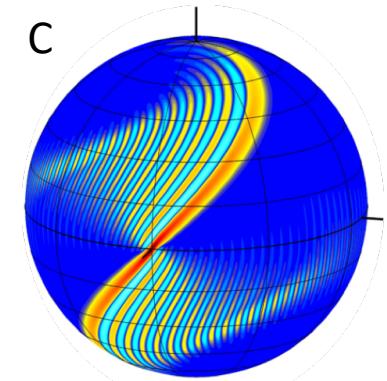
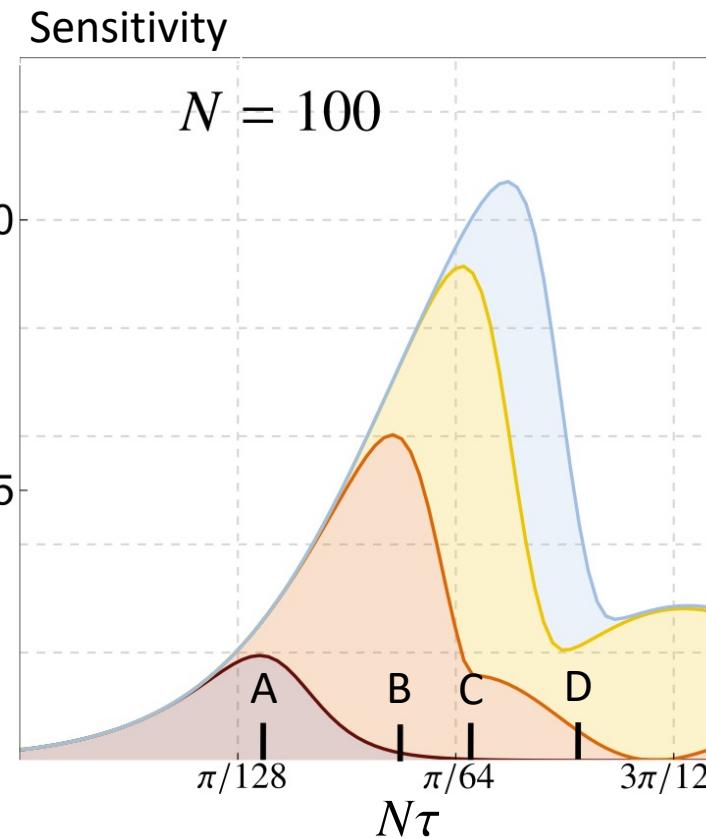
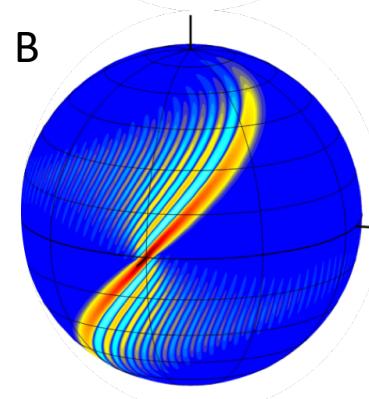
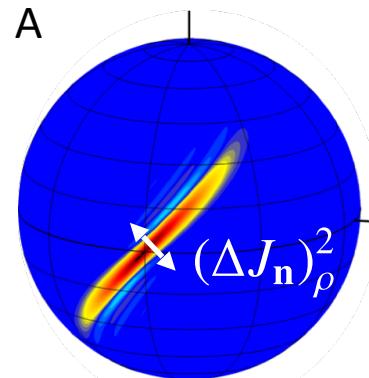
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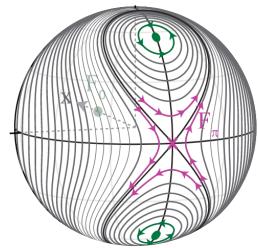
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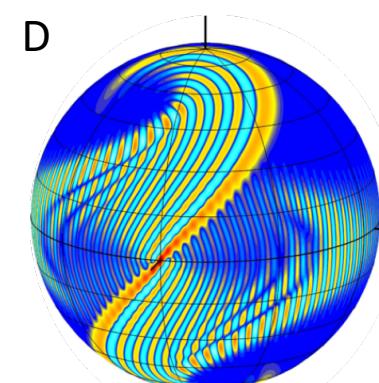
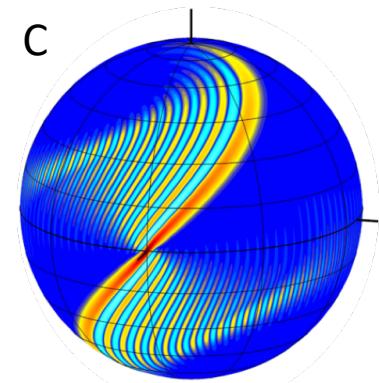
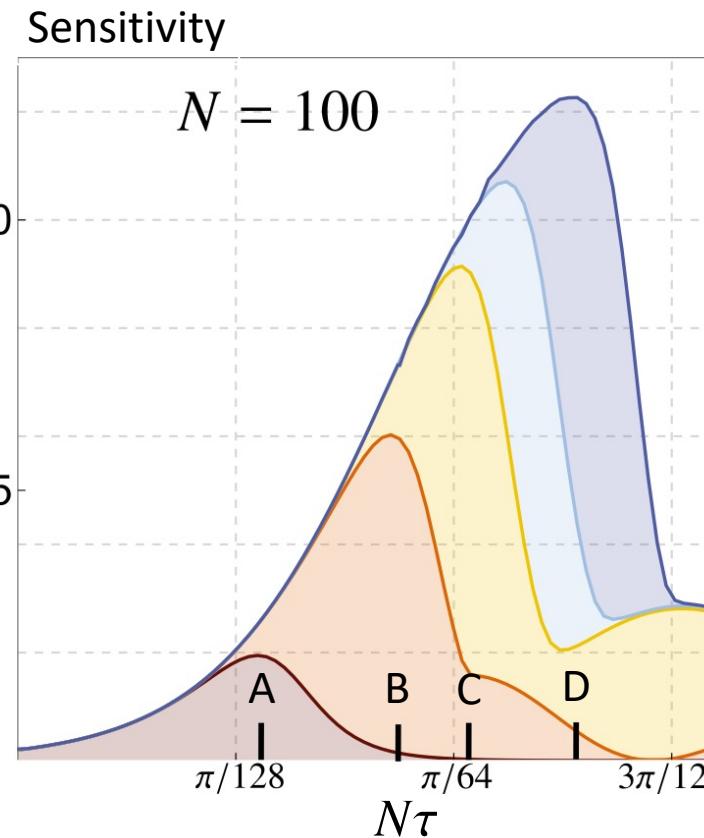
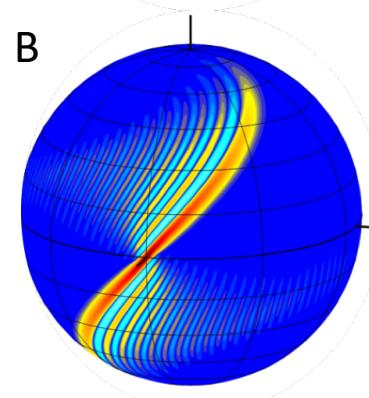
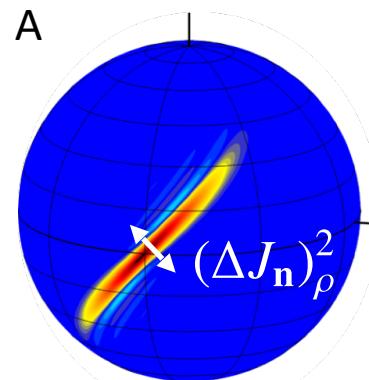
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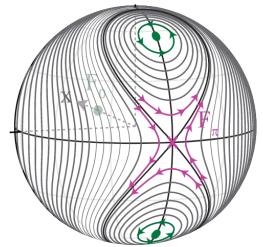
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- Optimize over *all* observables

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Nonlinear evolution

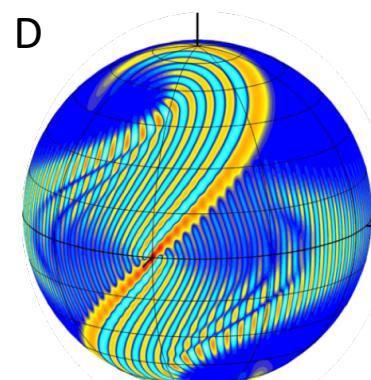
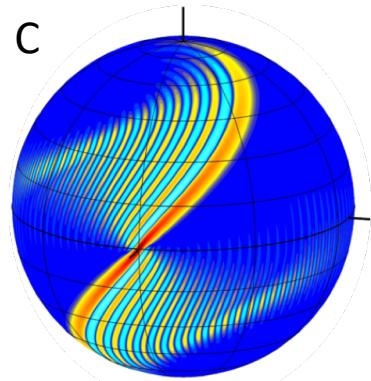
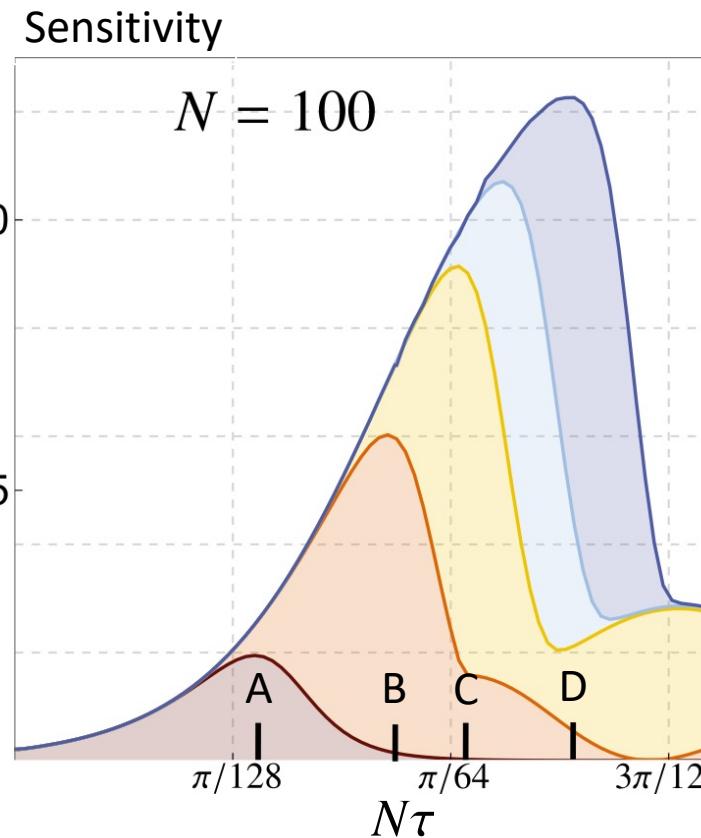
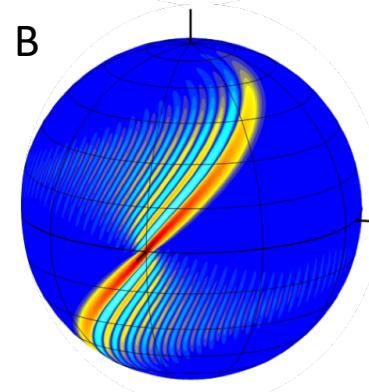
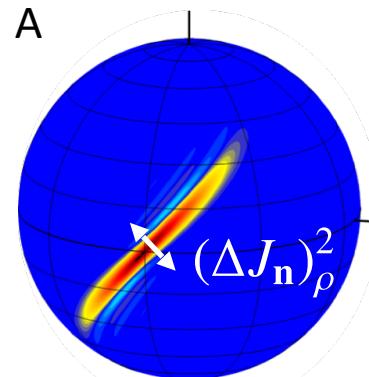
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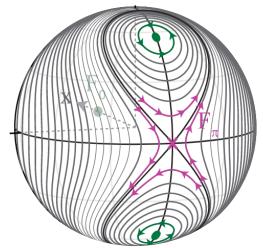
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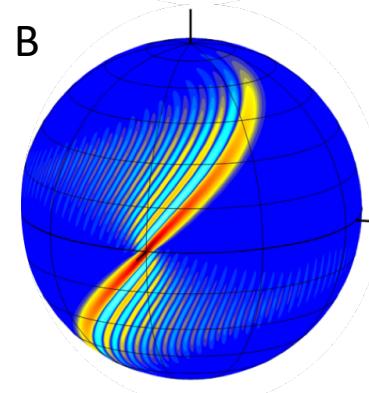
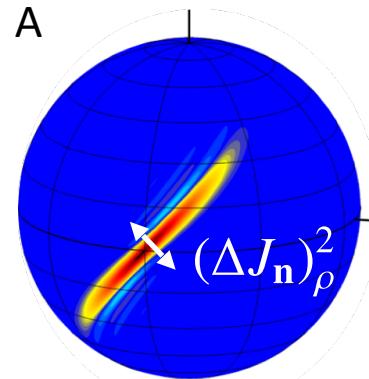
rotation

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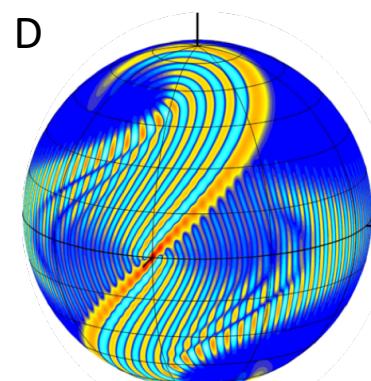
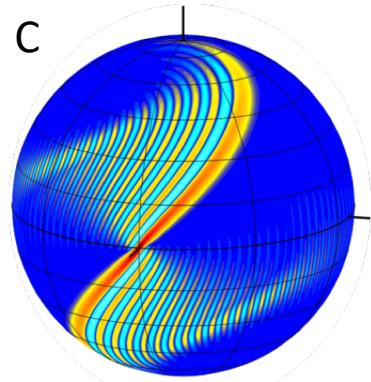
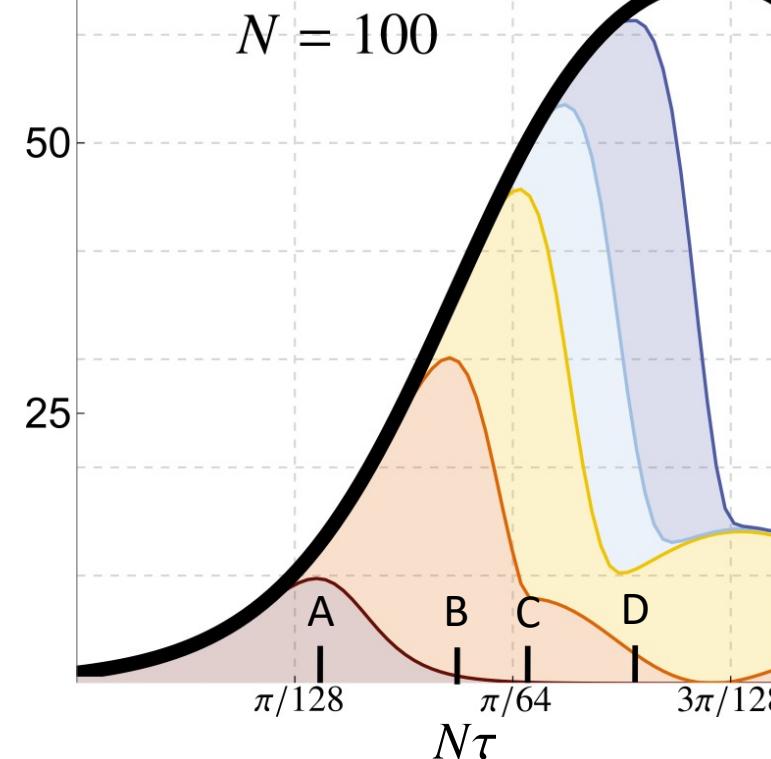
measurement

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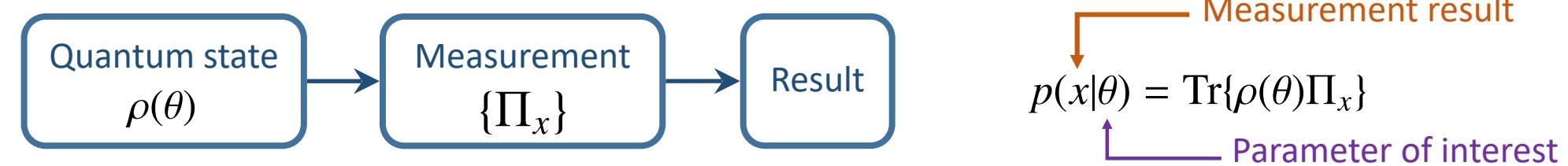
Nonlinear observable



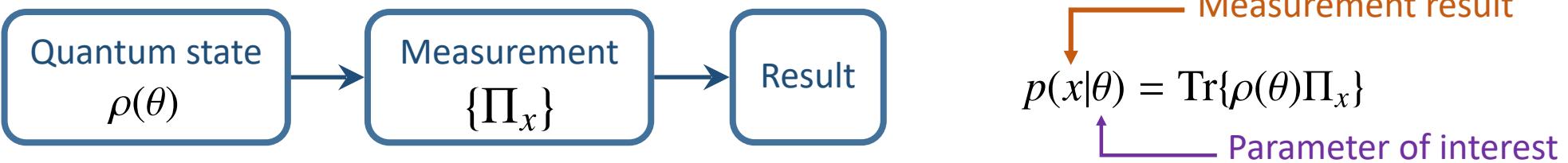
Sensitivity



# QUANTUM PARAMETER ESTIMATION



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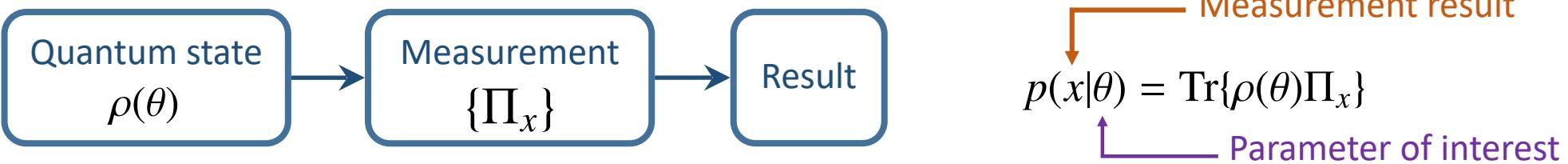


**Sensitivity limit:**  
Cramér-Rao bound

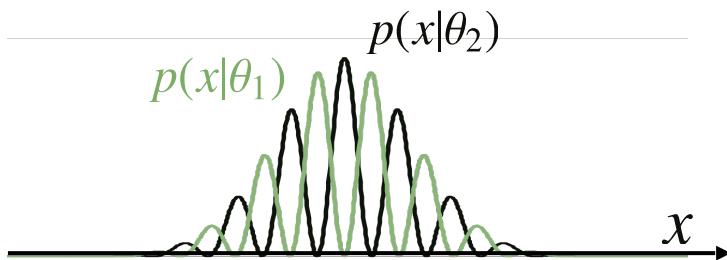
$$\Delta\theta_{\text{est}} \geq \frac{1}{\sqrt{F(\theta)}}$$

$$F(\theta) = \sum_x \frac{1}{p(x|\theta)} \left( \frac{\partial p(x|\theta)}{\partial \theta} \right)^2$$

# QUANTUM PARAMETER ESTIMATION



Interpretation of the Fisher information?

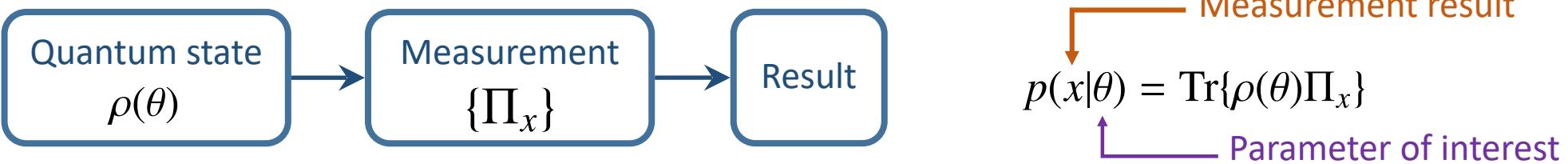


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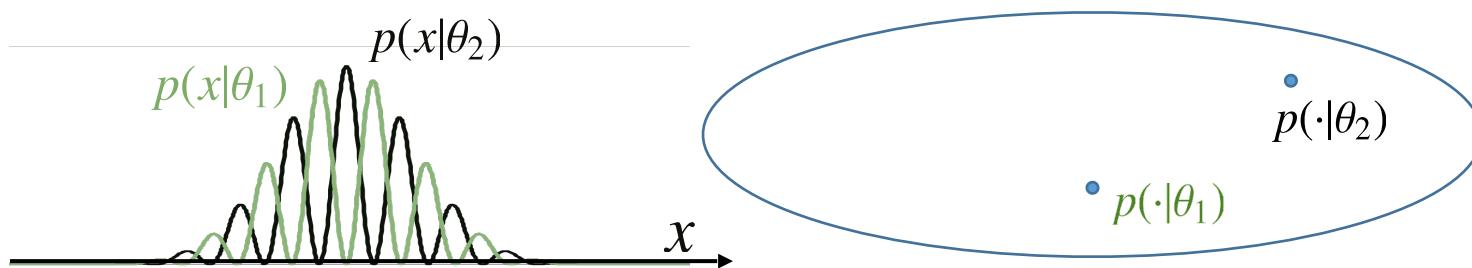
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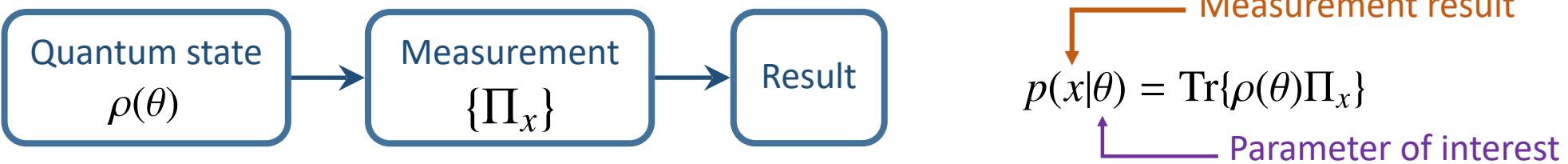


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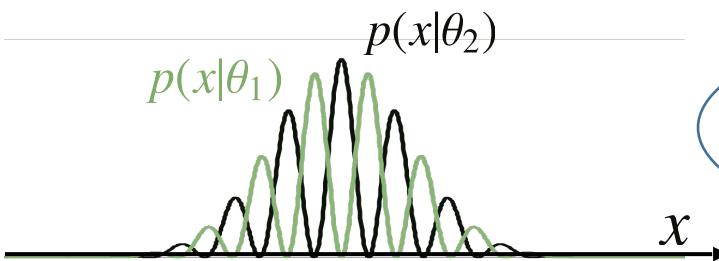
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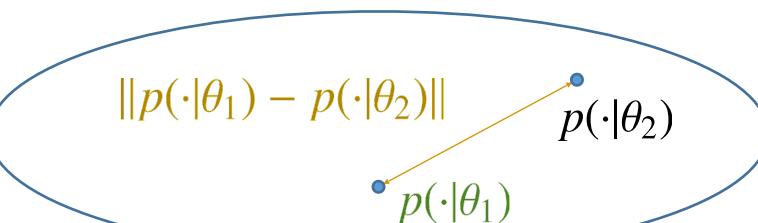
# QUANTUM PARAMETER ESTIMATION



Interpretation of the Fisher information?



Statistical distance

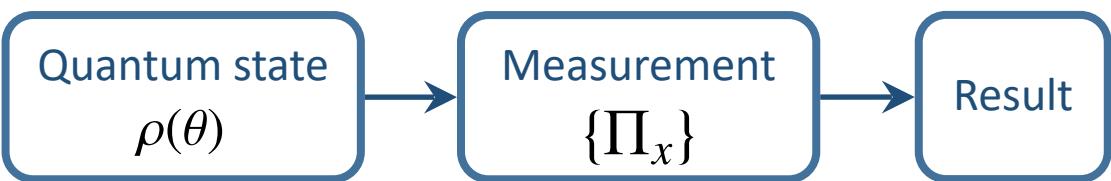


Sensitivity limit:  
Cramér-Rao bound

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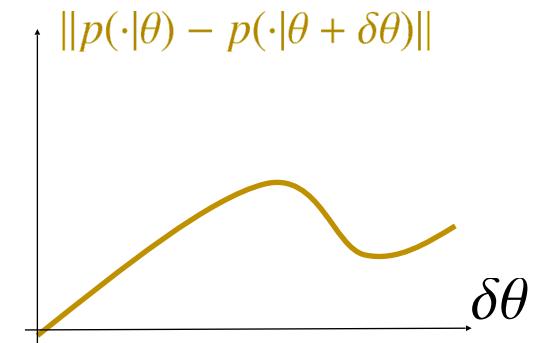
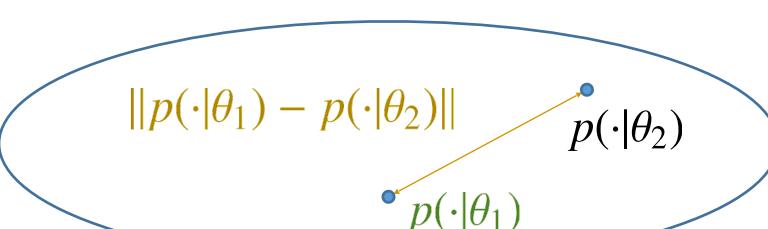
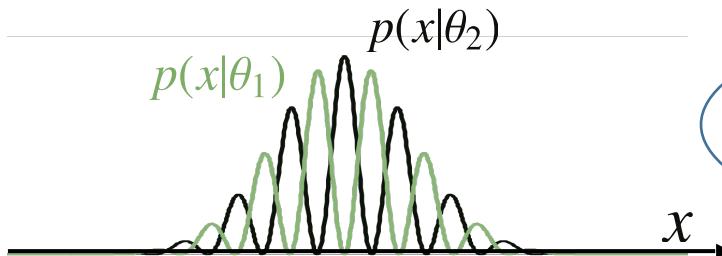
# QUANTUM PARAMETER ESTIMATION



Measurement result  
 $p(x|\theta) = \text{Tr}\{\rho(\theta)\Pi_x\}$

Parameter of interest

Interpretation of the Fisher information?

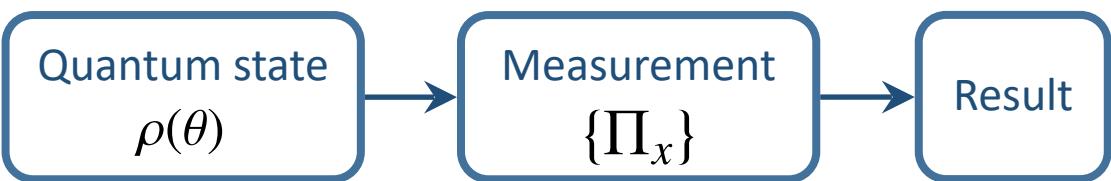


Sensitivity limit:  
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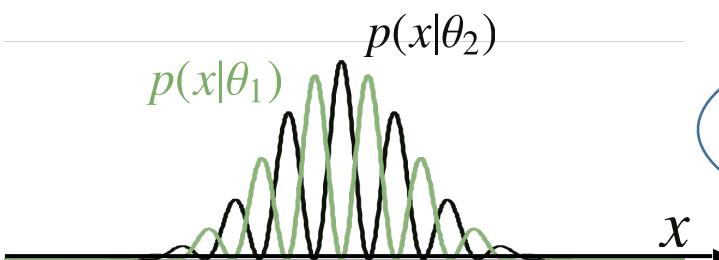


Measurement result

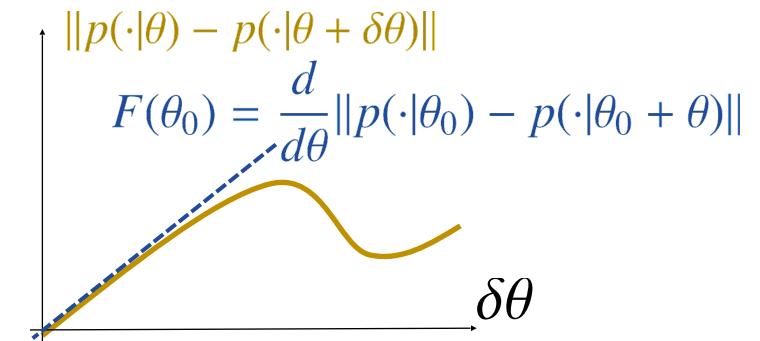
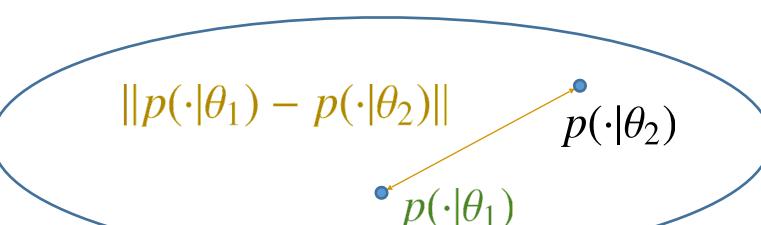
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Interpretation of the Fisher information?



Statistical distance

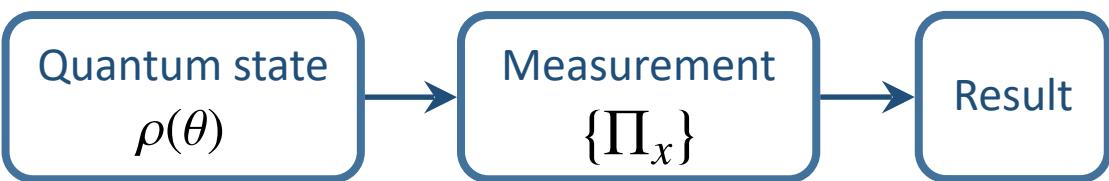


**Sensitivity limit:**  
Cramér-Rao bound

$$\Delta\theta_{\text{est}} \geq \frac{1}{\sqrt{F(\theta)}}$$

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# QUANTUM PARAMETER ESTIMATION

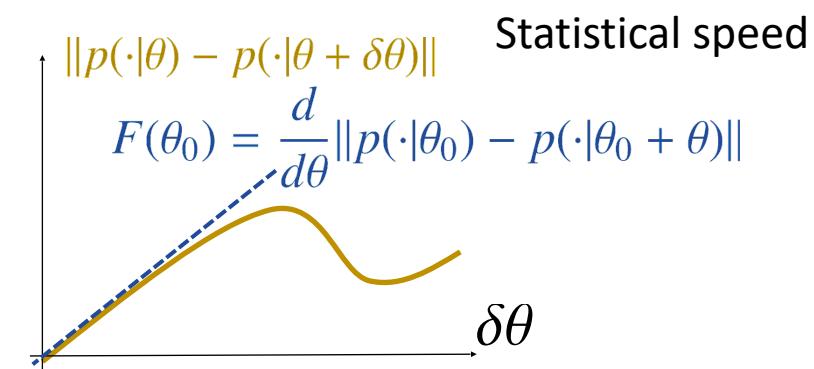
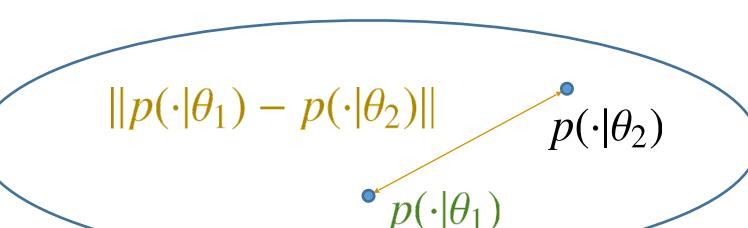
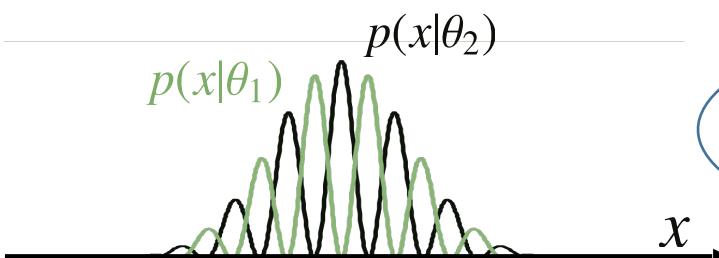


Measurement result

$$p(x|\theta) = \text{Tr}\{\rho(\theta)\Pi_x\}$$

Parameter of interest

Interpretation of the Fisher information?

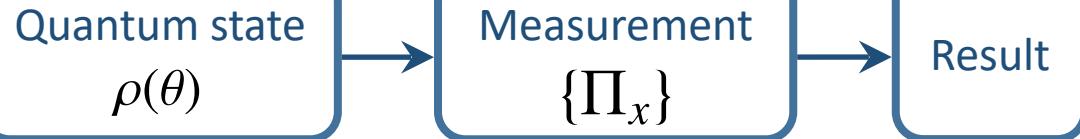


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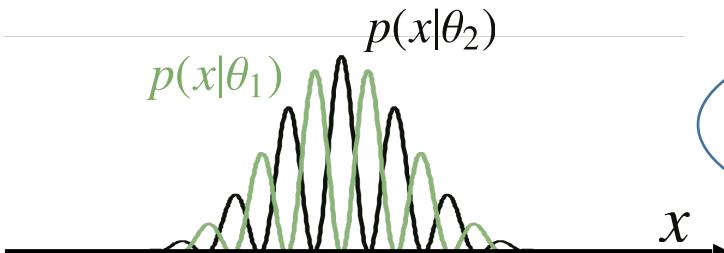
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$$\|p(\cdot|\theta_1) - p(\cdot|\theta_2)\|$$

$$p(\cdot|\theta_1)$$

$$p(\cdot|\theta_2)$$

**Optimal measurement**

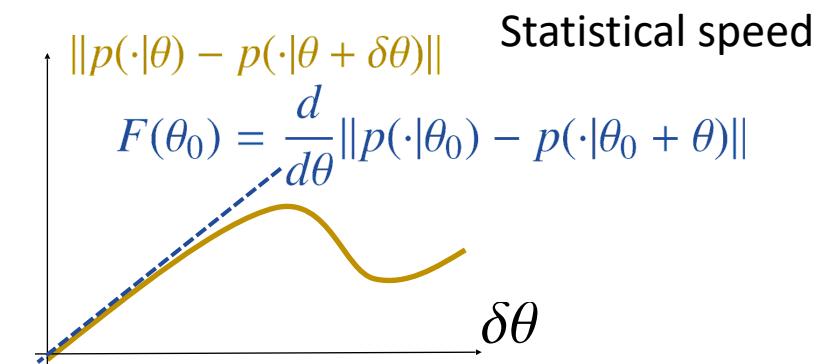
Maximizes distance and speed

S. L. Braunstein and C. M. Caves  
 Phys. Rev. Lett. **72**, 3439 (1994)

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$$\rho(\theta_2)$$

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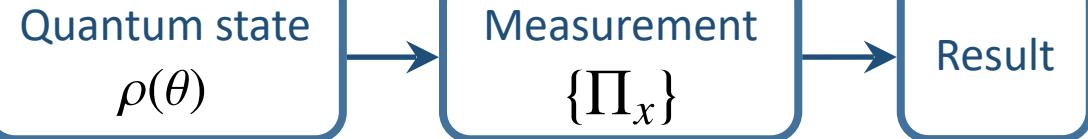


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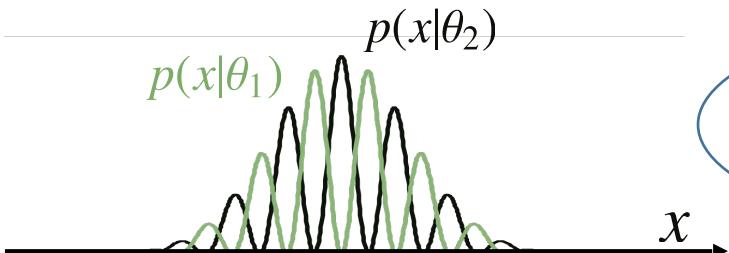
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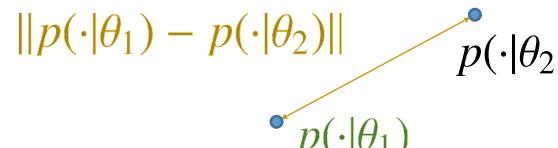
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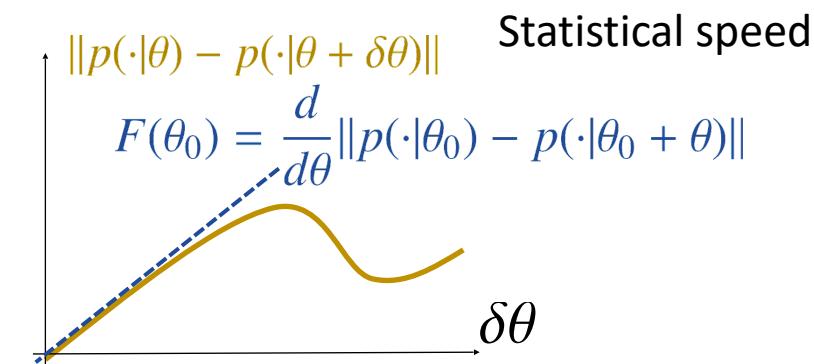
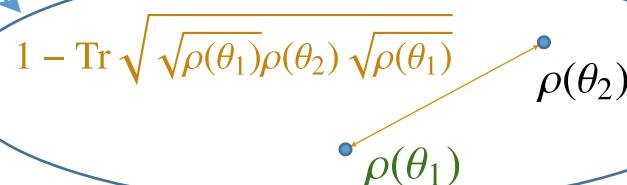


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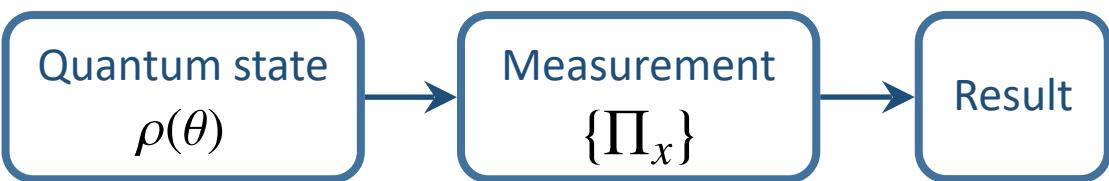


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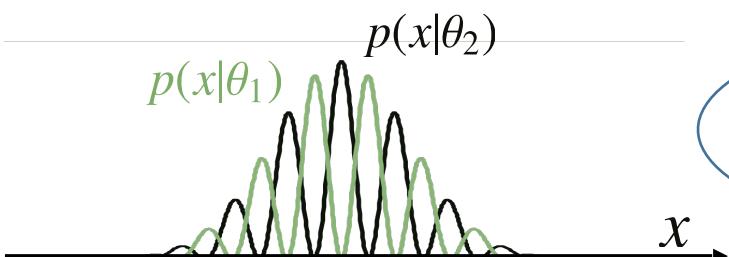


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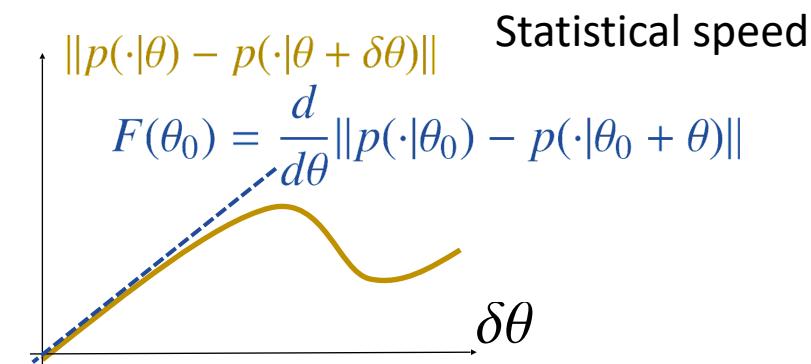
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Quantum statistical speed

$$F_Q[\rho(\theta)] = \max_{\{\Pi_x\}} F(\theta)$$

# METROLOGICAL SENSITIVITY OF QUANTUM STATES

“Squeezing”

$\leq$

Metrological sensitivity

$$\frac{|\langle [H, X] \rangle_{\rho}|^2}{(\Delta X)_{\rho}^2} \leq F_Q[\rho, H]$$

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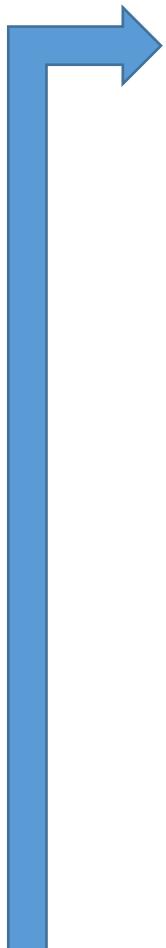
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$$F_Q[\rho(\theta)]$$

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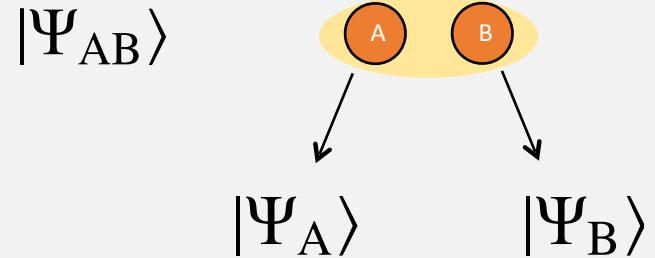
EINSTEIN, PODOLSKI, ROSEN (1935)

$|\Psi_{AB}\rangle$



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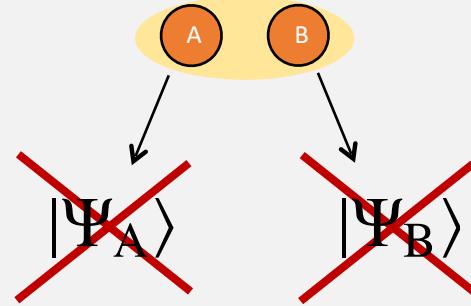
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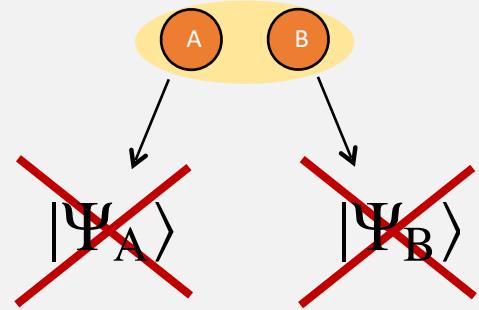


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Strong (beyond classical) correlations!

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Entangled

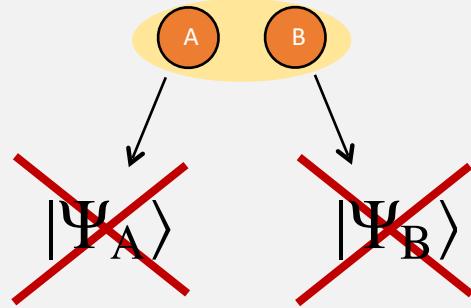


otherwise

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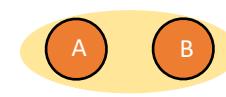
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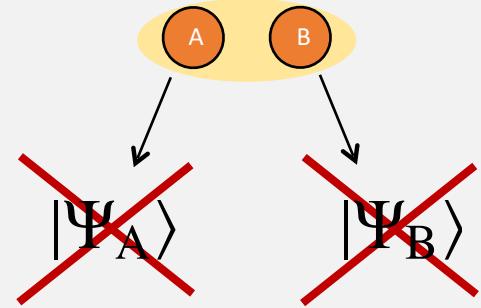
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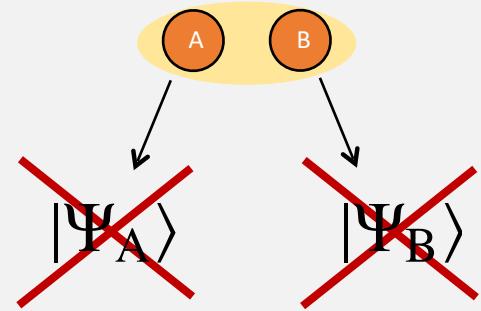
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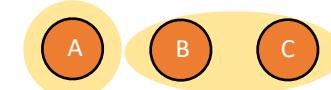


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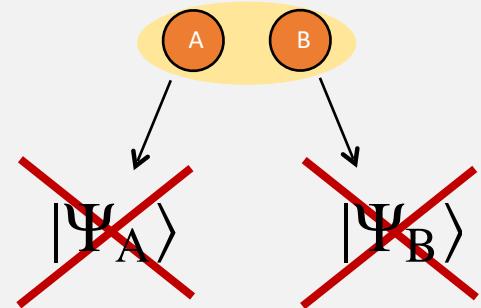
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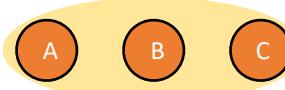


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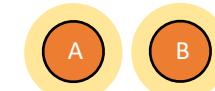


“fully separable”



3-partite entanglement

Separable



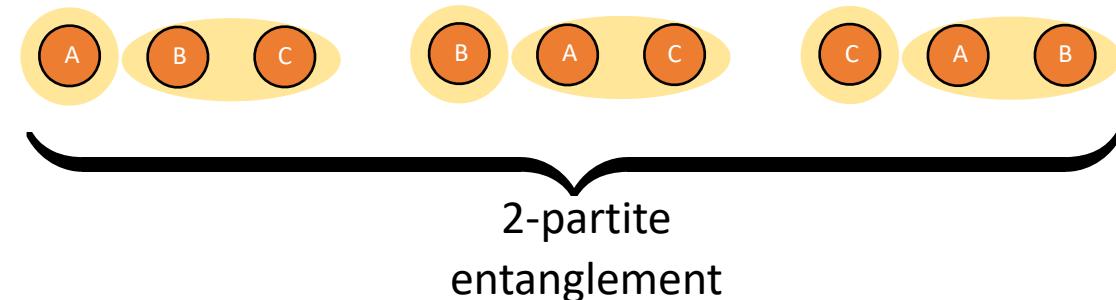
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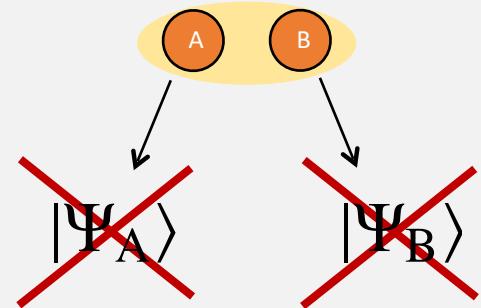
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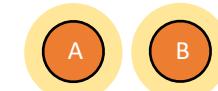
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$N$ -partite system

Separable



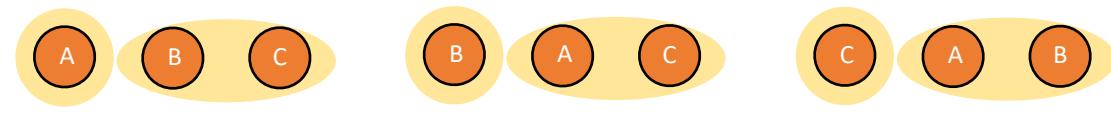
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2-partite entanglement

$$\rho = \sum_k p_k \rho_1^{(k)} \otimes \cdots \otimes \rho_N^{(k)}$$

# METROLOGICAL ENTANGLEMENT DETECTION

Violation of this bound: **metrological entanglement witness**

“Squeezing”

$\leq$

Metrological sensitivity

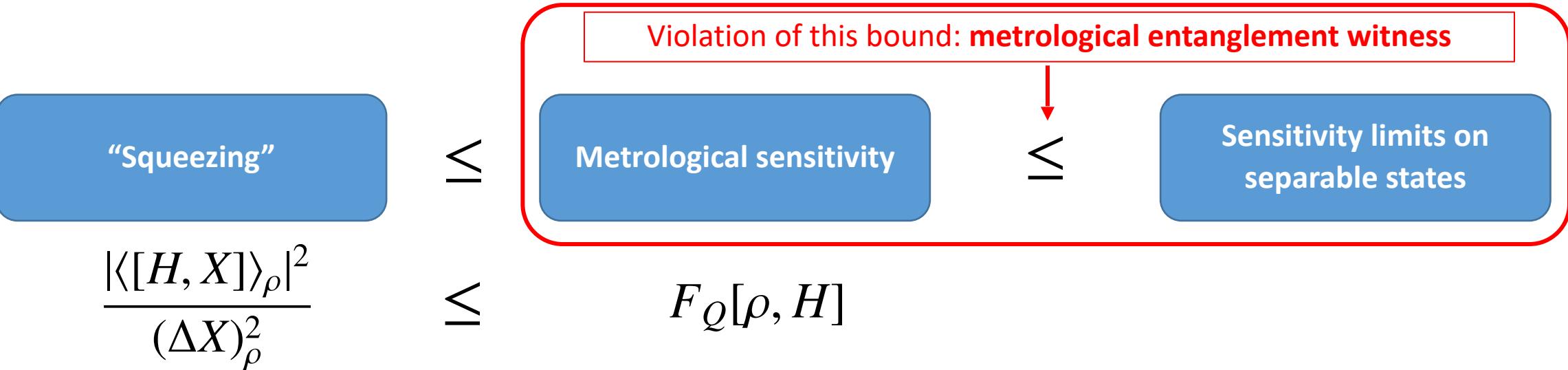
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Sensitivity limits on  
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# SENSITIVITY LIMITS FOR SEPARABLE STATES

$$F_Q[\rho, H] \leq$$

## Properties

- Convexity
- Additivity

**Sensitivity limit:  
Cramér-Rao bound**

$$\Delta\theta_{\text{est}} \geq \frac{1}{\sqrt{F_Q[\rho, H]}}$$

# SENSITIVITY LIMITS FOR SEPARABLE STATES

N particles, two states

$$F_Q[\rho, H] \leq N \quad \text{fully separable states}$$

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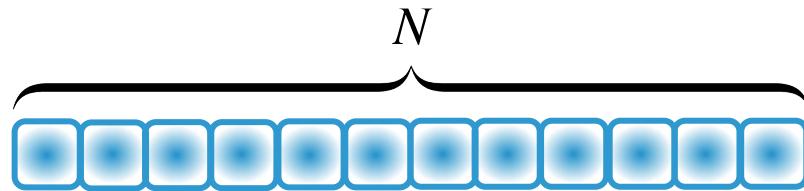
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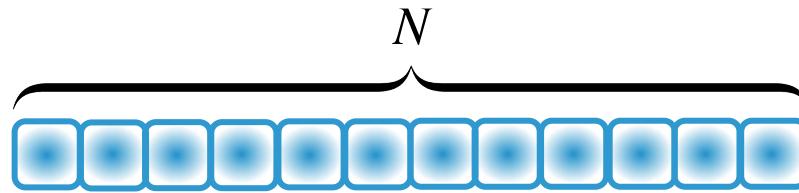
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$k$  entangled particles



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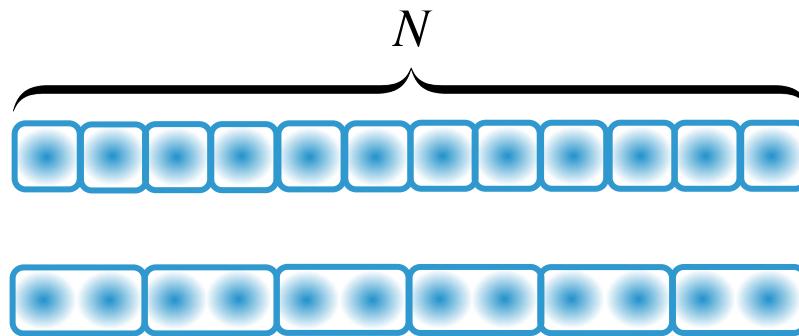
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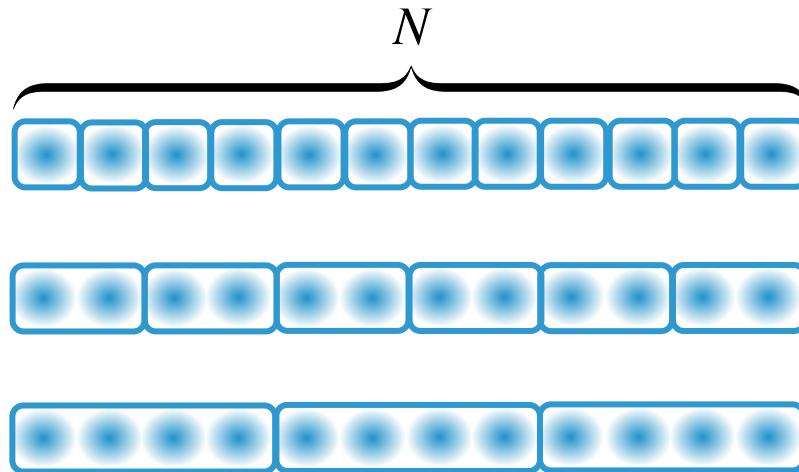
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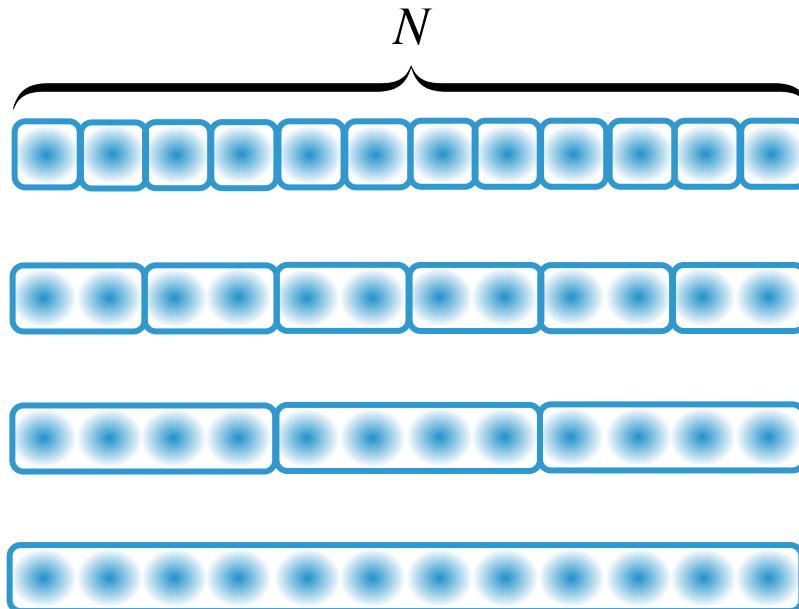
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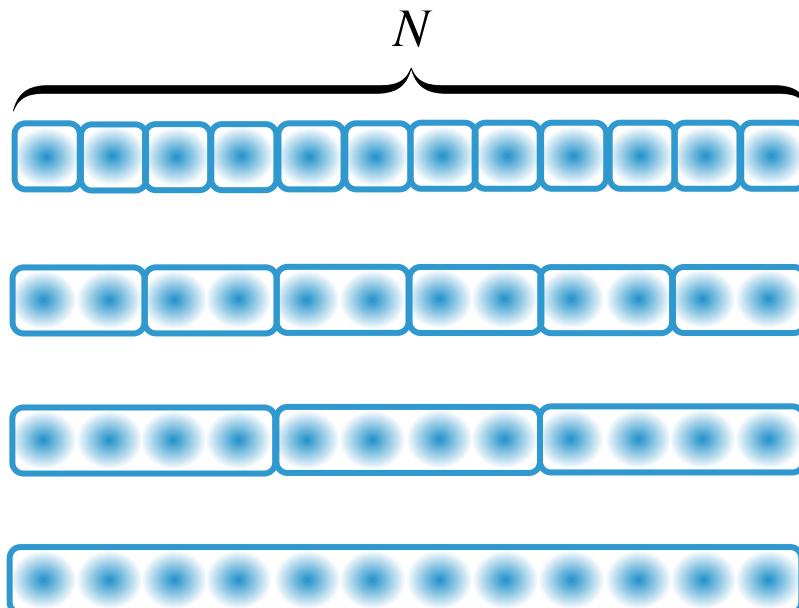
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Phys. Rev. Lett. **102**, 100401 (2009)

P. Hyllus et al. Phys. Rev. A **85**, 022321 (2012)  
G. Tóth, Phys. Rev. A **85**, 022322 (2012)

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Quantum correlations

enhance

Sensitivity

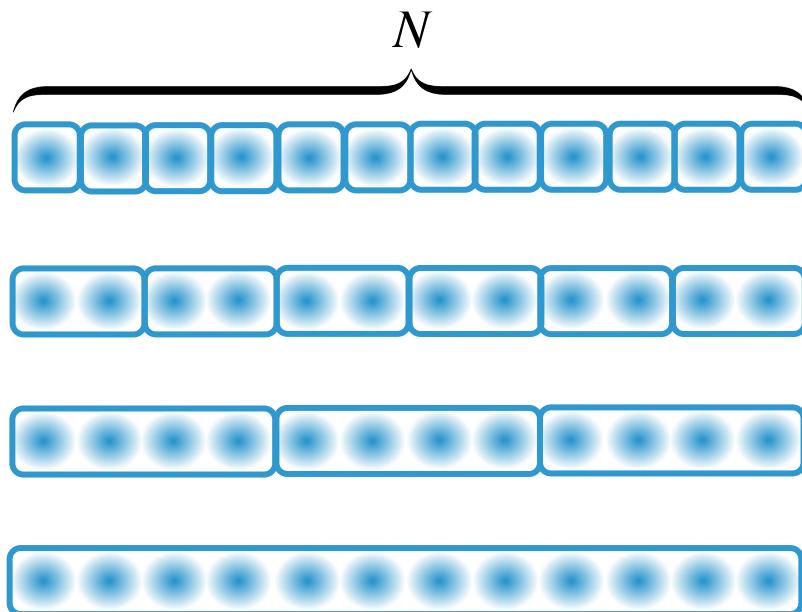
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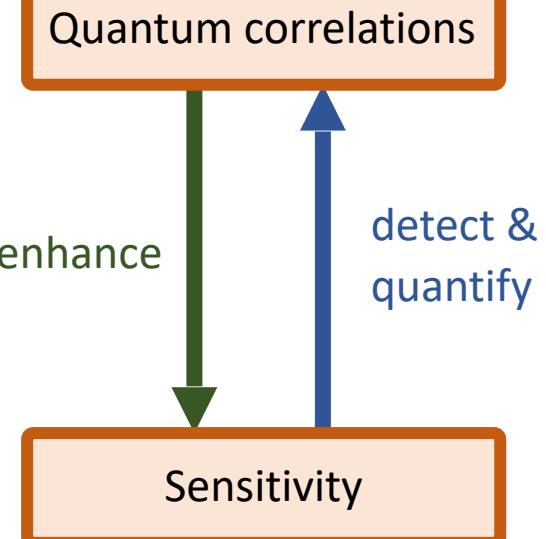
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Phys. Rev. Lett. **102**, 100401 (2009)

P. Hyllus et al. Phys. Rev. A **85**, 022321 (2012)  
G. Tóth, Phys. Rev. A **85**, 022322 (2012)

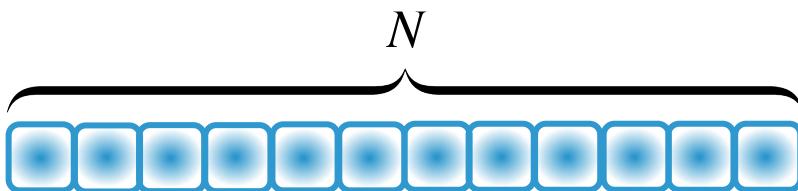
# SENSITIVITY LIMITS FOR SEPARABLE STATES

$$F_Q[\rho, H] \leq$$

N particles, two states

fully **separable** states

$k$  entangled particles



$k = 1$

$k = N$

- Properties
- Convexity
  - Additivity

**Sensitivity limit:**  
Cramér-Rao bound

$$\Delta\theta_{\text{est}} \geq \frac{1}{\sqrt{F_Q[\rho, H]}}$$

Quantum correlations

enhance

detect & quantify

Sensitivity

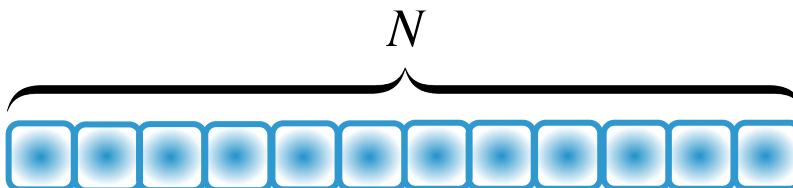
# SENSITIVITY LIMITS FOR SEPARABLE STATES

N particles, two states

$$F_Q[\rho, H] \leq kN$$

fully **separable** states  
 $k$  entangled particles

- Properties
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  - Additivity



$k = 1$

$$\Lambda(\rho) = \rho_{ABC} \otimes \rho_D \otimes \rho_{EF} \otimes \rho_{GHIJ} \otimes \rho_{KL}$$



$k = N$

Arbitrary-dimensional systems  
If  $\rho$  is separable:

$$F_Q[\rho, H] \leq 4(\Delta H)_{\Lambda(\rho)}^2$$

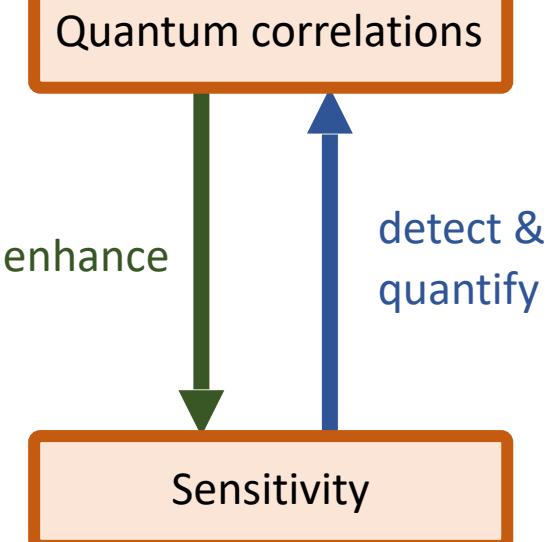
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M. Gessner, L. Pezzé, and A. Smerzi,  
Phys. Rev. A **94**, 020101(R) (2016);  
Phys. Rev. A **95**, 032326 (2017).

**Sensitivity limit:**  
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# METROLOGICAL ENTANGLEMENT DETECTION

Violation of this bound: **metrological entanglement witness**

“Squeezing”

$\leq$

Metrological sensitivity

$\leq$

Sensitivity limits on  
separable states

$$\frac{|\langle [H, X] \rangle_{\rho}|^2}{(\Delta X)_{\rho}^2} \leq F_Q[\rho, H] \leq 4(\Delta H)_{\rho_1 \otimes \dots \otimes \rho_N}^2$$

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**Uncertainty-based metrological entanglement witness**

Separable states

$$(\Delta H)_{\rho_1 \otimes \dots \otimes \rho_N}^2 (\Delta X)_{\rho}^2 \geq \frac{|\langle [H, X] \rangle_{\rho}|^2}{4}$$

M. Gessner, L. Pezzè, and A. Smerzi  
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Modified uncertainty relation detects entanglement

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$\leq$

$$F_Q[\rho, H]$$

$\leq$

$$4(\Delta H)_{\rho_1 \otimes \dots \otimes \rho_N}^2$$

$\leq N$   
“worst-case”  
for  $N$  spin-1/2

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**Generalizes spin squeezing:**

- Stronger entanglement condition
- Tells us which parts of the system are entangled
- Applicable to continuous-variable systems

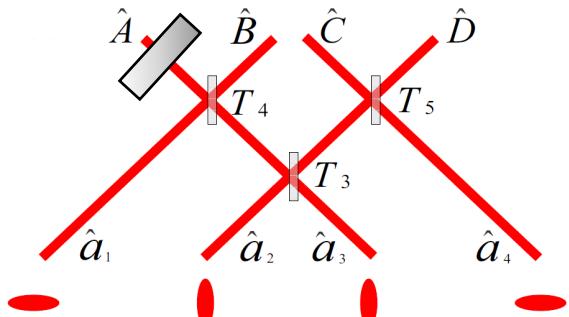
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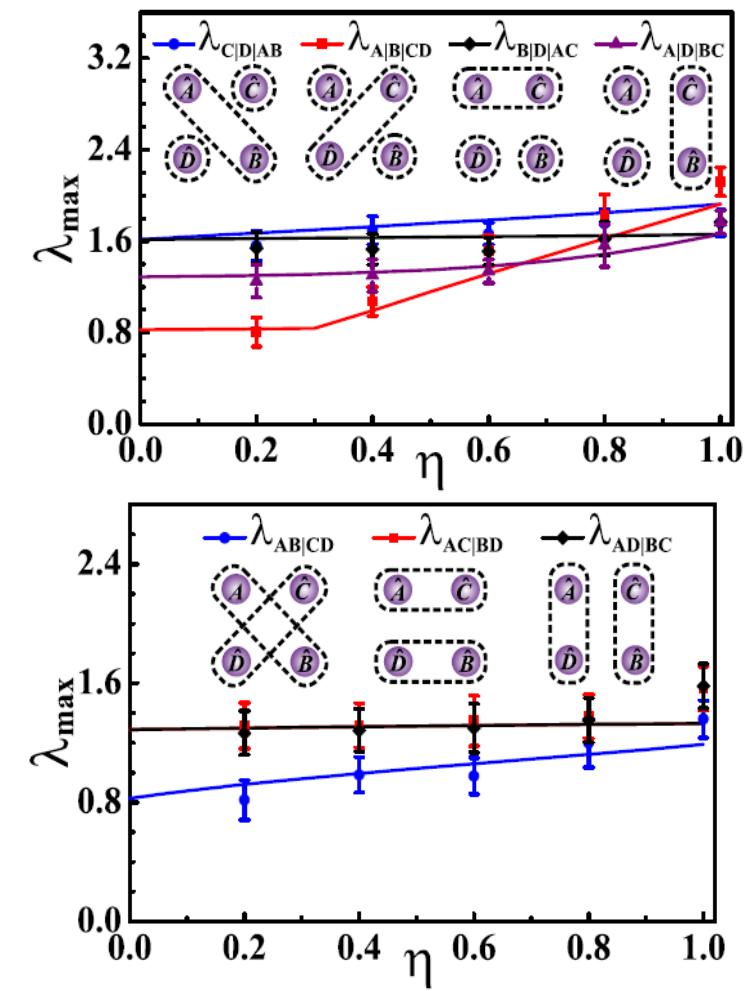
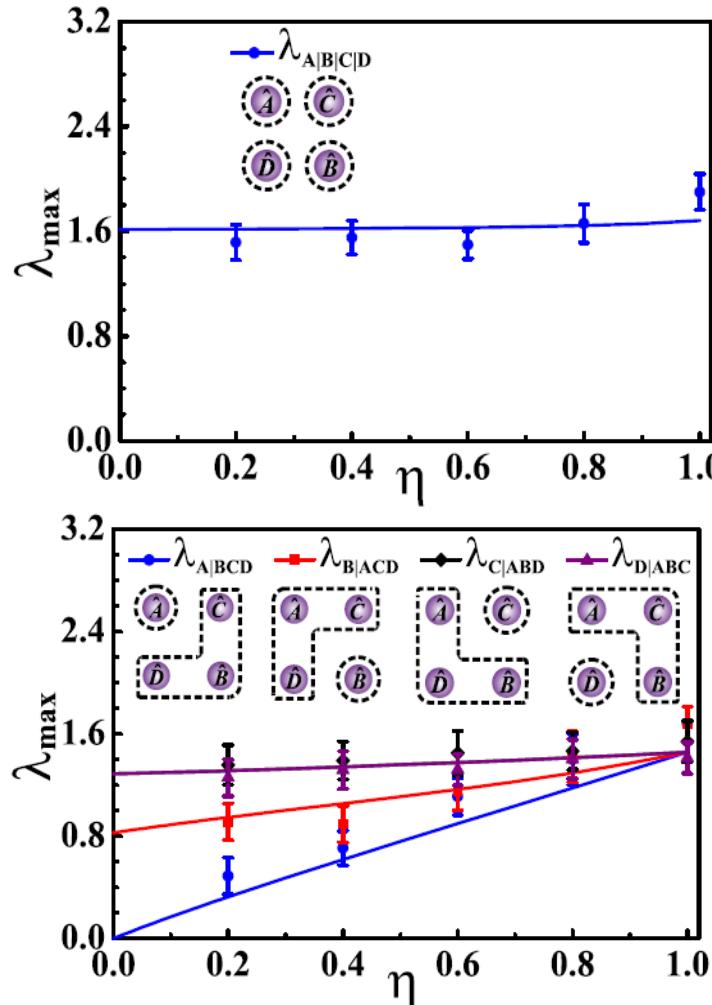
# MULTIMODE ENTANGLEMENT STRUCTURE IN CONTINUOUS VARIABLES



Multimode squeezed vacuum states  
(Cluster states)

Z. Z. Qin, M. Gessner, Z. Ren, X. Deng,  
D. Han, W. Li, X. Su, A. Smerzi, and K. Peng,  
npj Quant. Inf. 5, 3 (2019)

CV Entanglement in all (multi-)partitions detected



# SENSITIVITY OF QUANTUM STATES

## Precision limits for quantum measurements

$$\Delta\theta \geq \frac{1}{\sqrt{F_Q[\rho(\theta)]}}$$

- Quantum-enhanced interferometry
- Applicable to arbitrary measurements

How to choose:

Observables

Quantum states

Sensitivity of quantum states

$$F_Q[\rho(\theta)]$$

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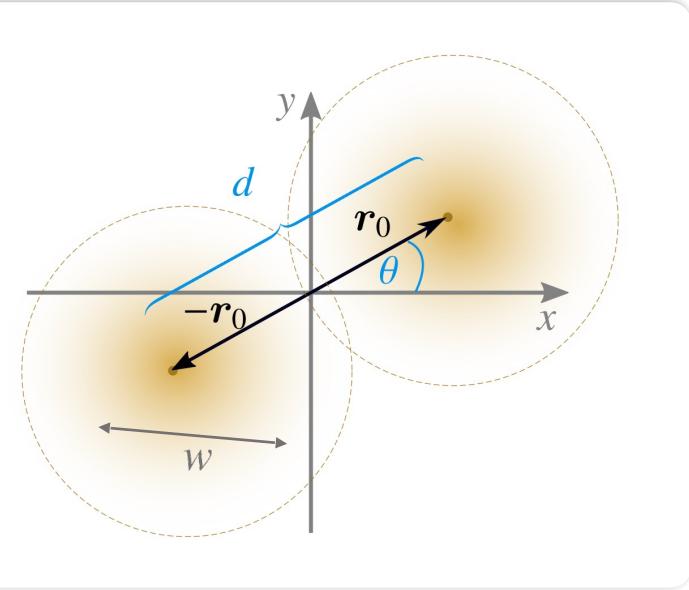
Observables

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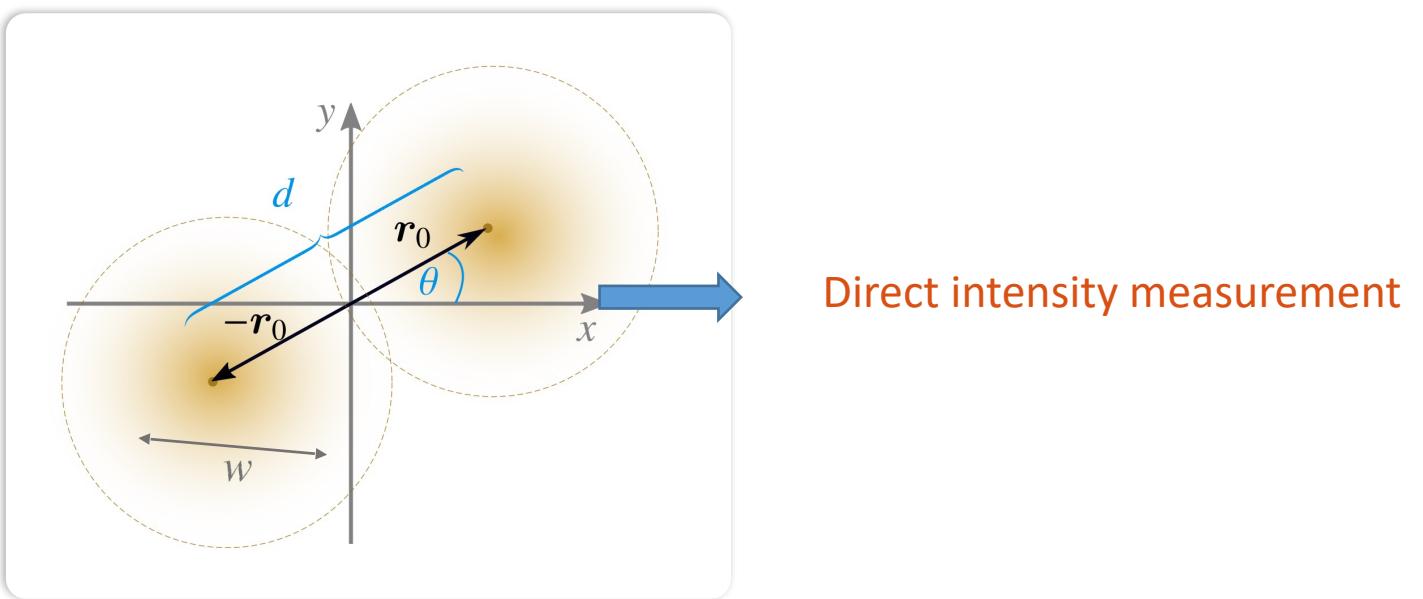
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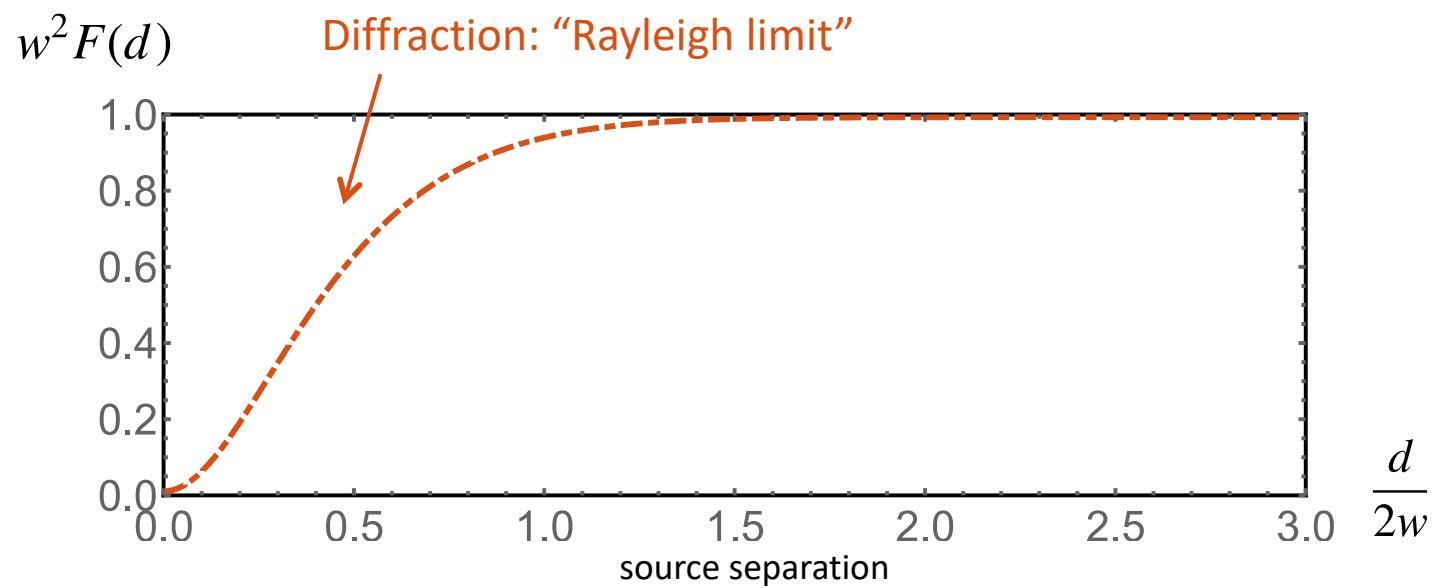
# SUPERRESOLUTION IN ASTRONOMY AND MICROSCOPY



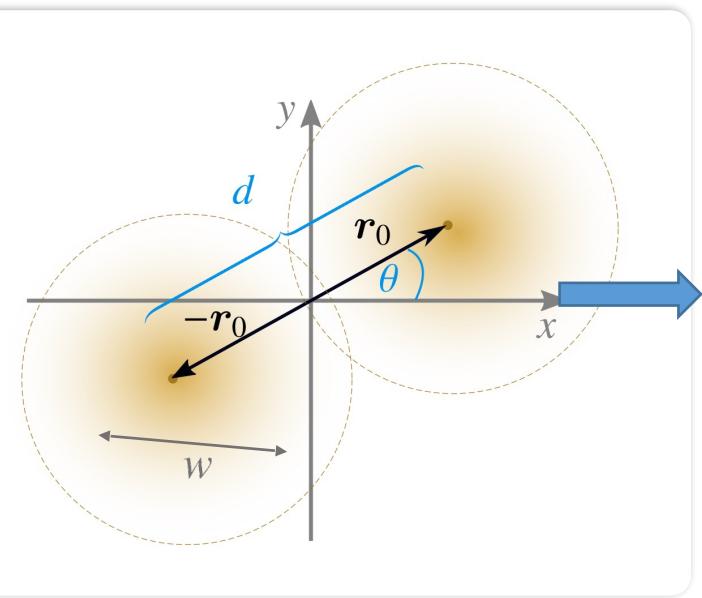
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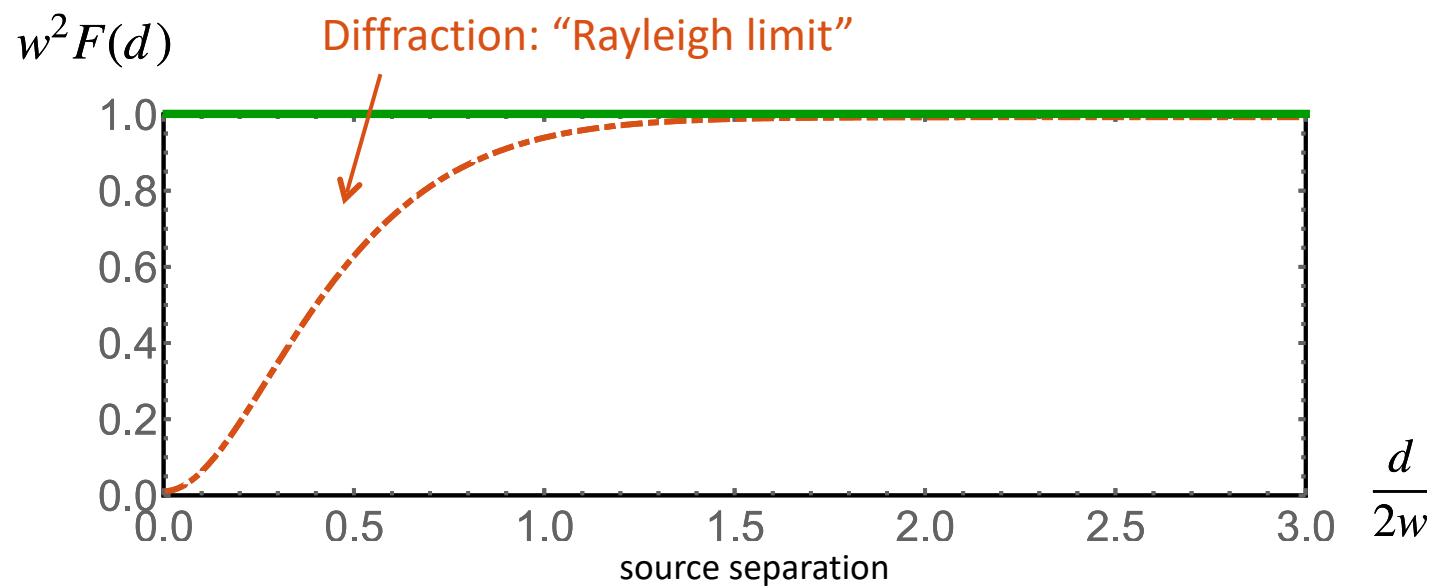
Direct intensity measurement



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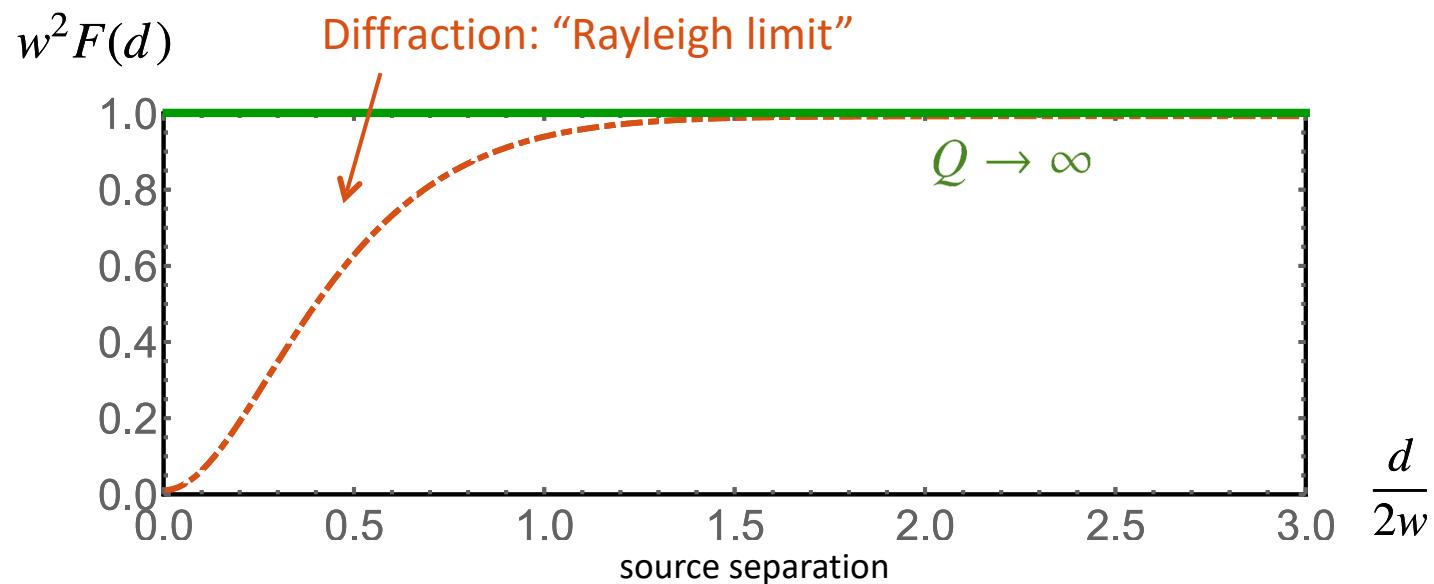
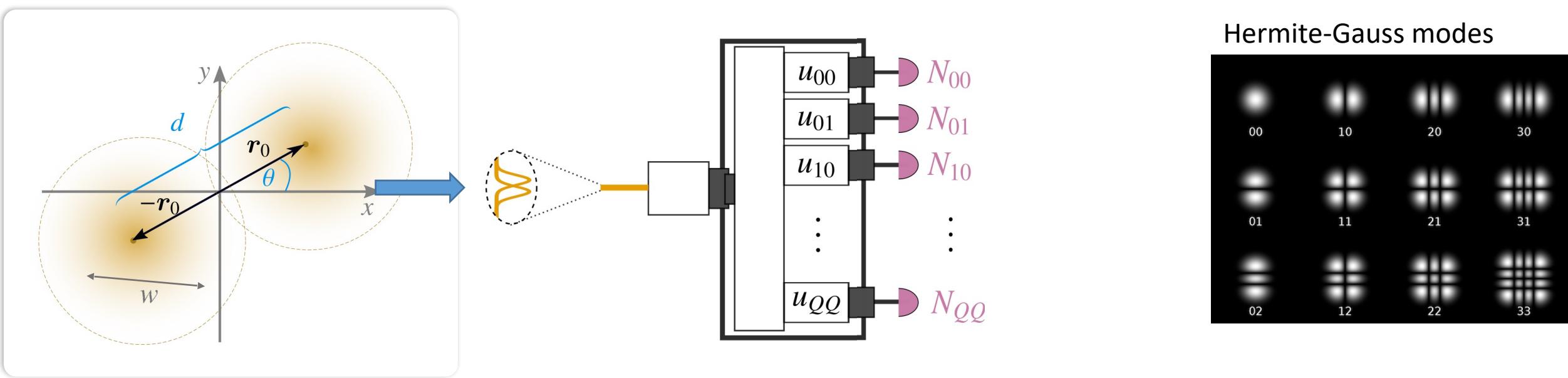
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M. Tsang, R. Nair, X.-M. Lu, PRX 6, 031033 (2016)  
Quantum Fisher information

$$F_q(d) = w^{-2}$$

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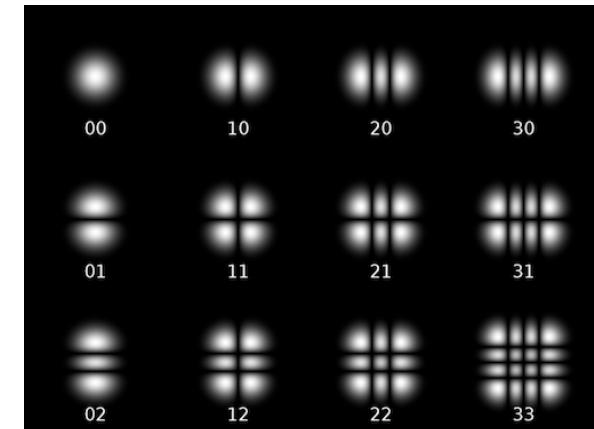


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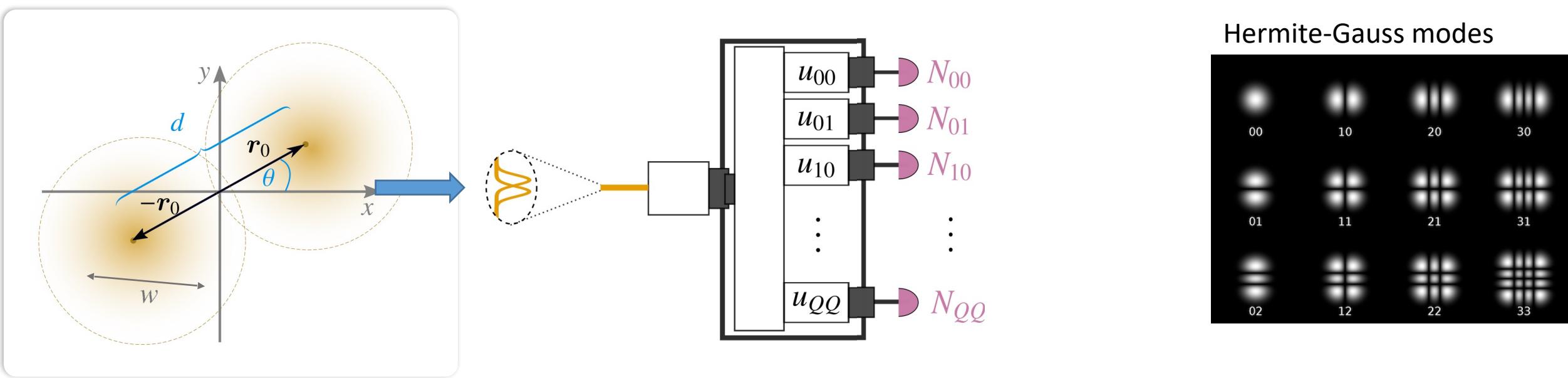
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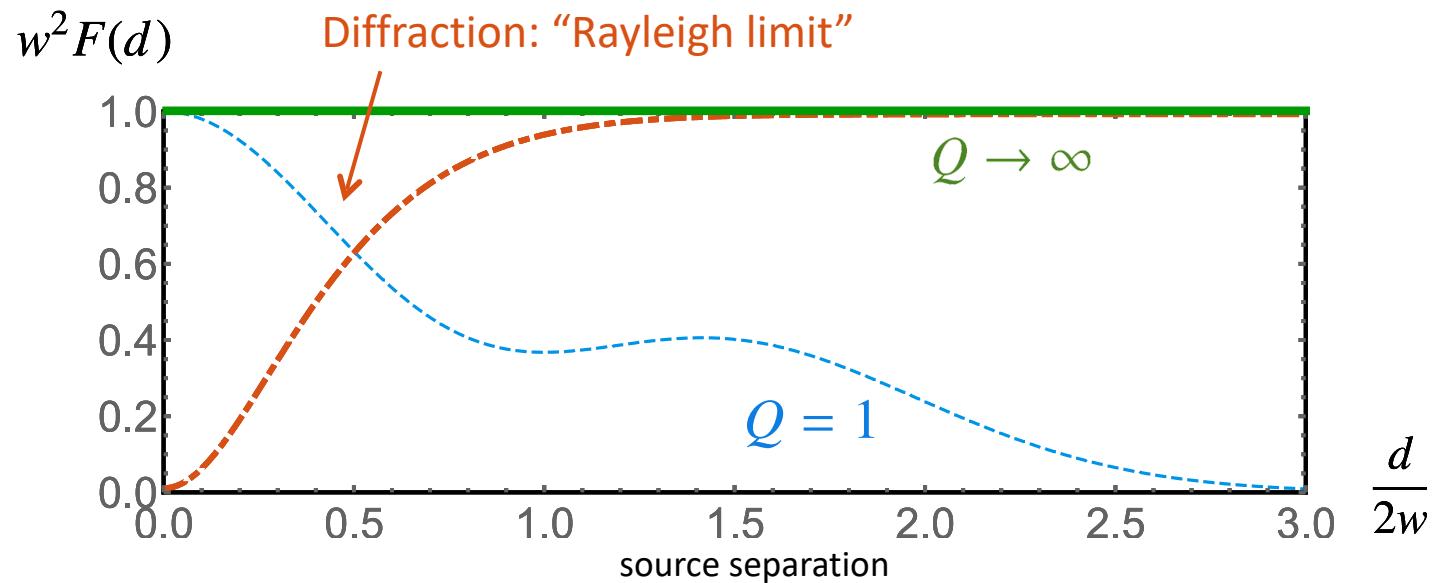
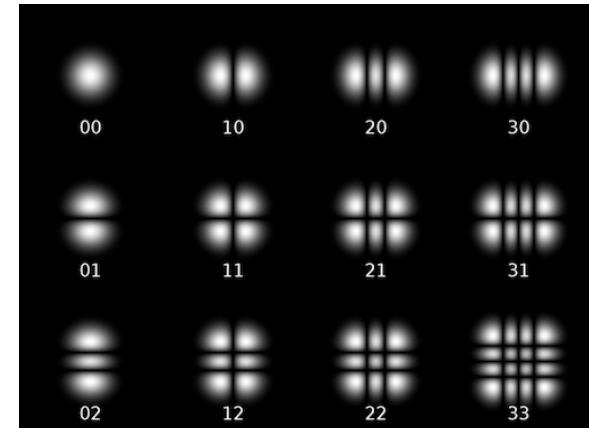
Hermite-Gauss modes



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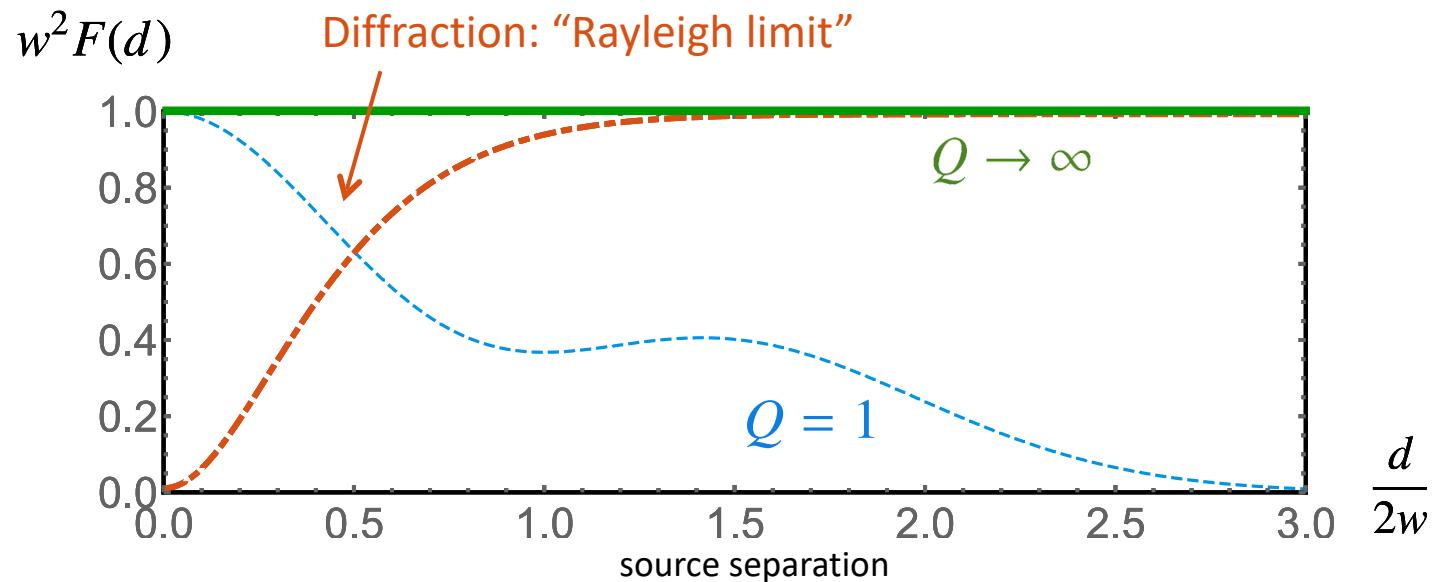
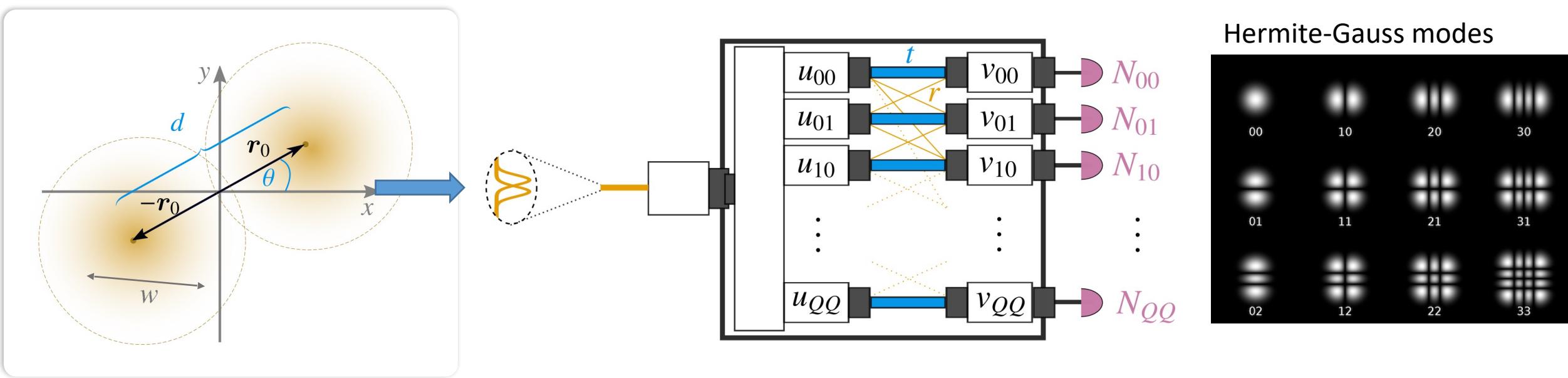


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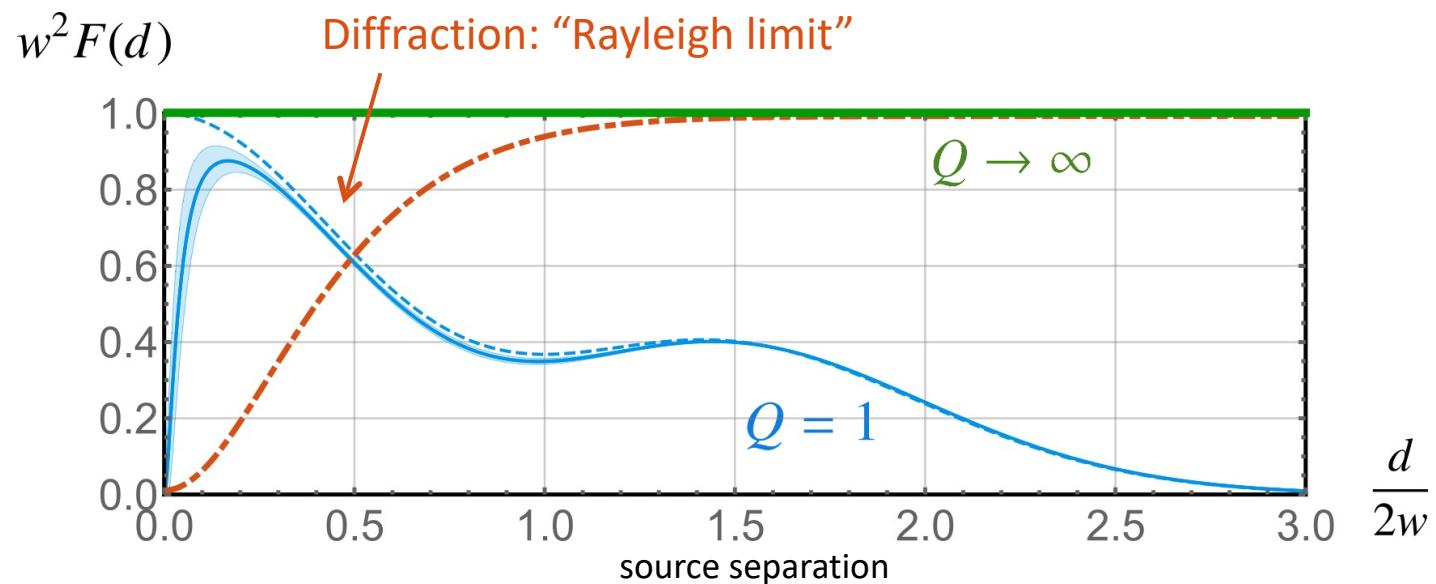
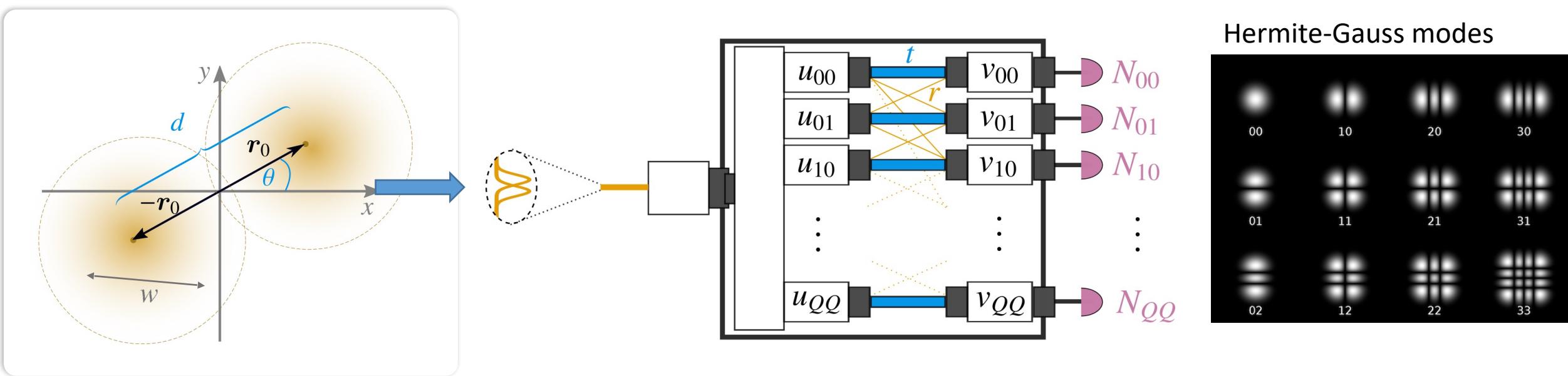
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arxiv:2004.07228

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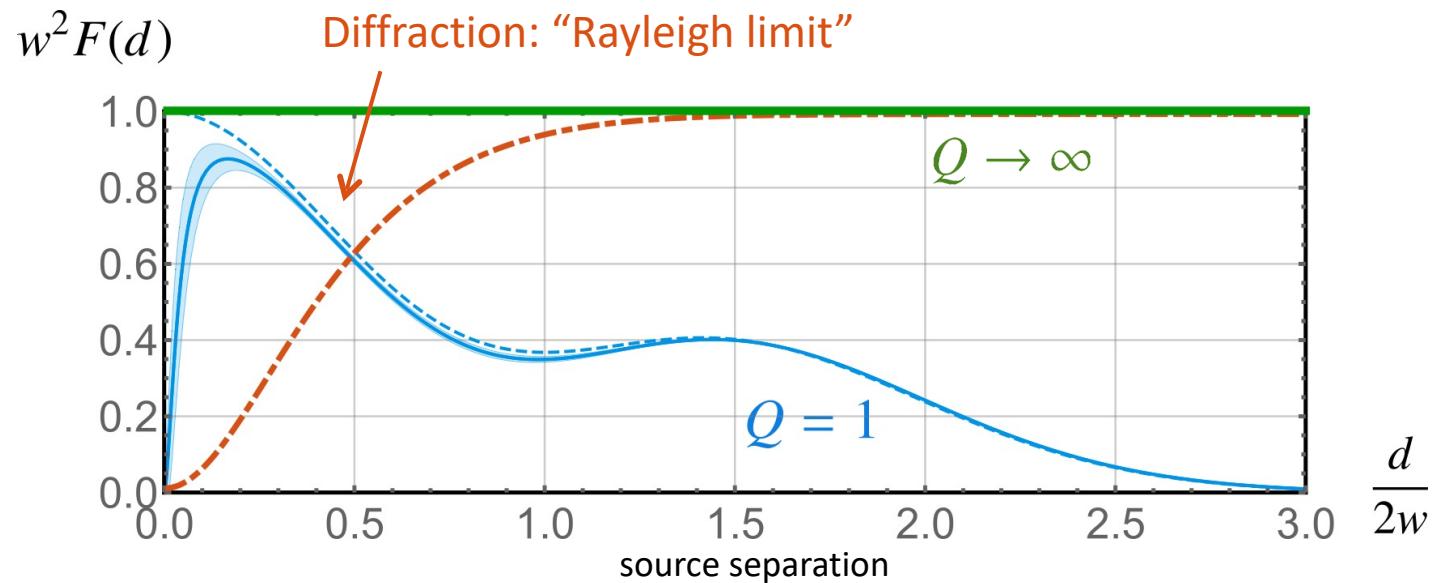
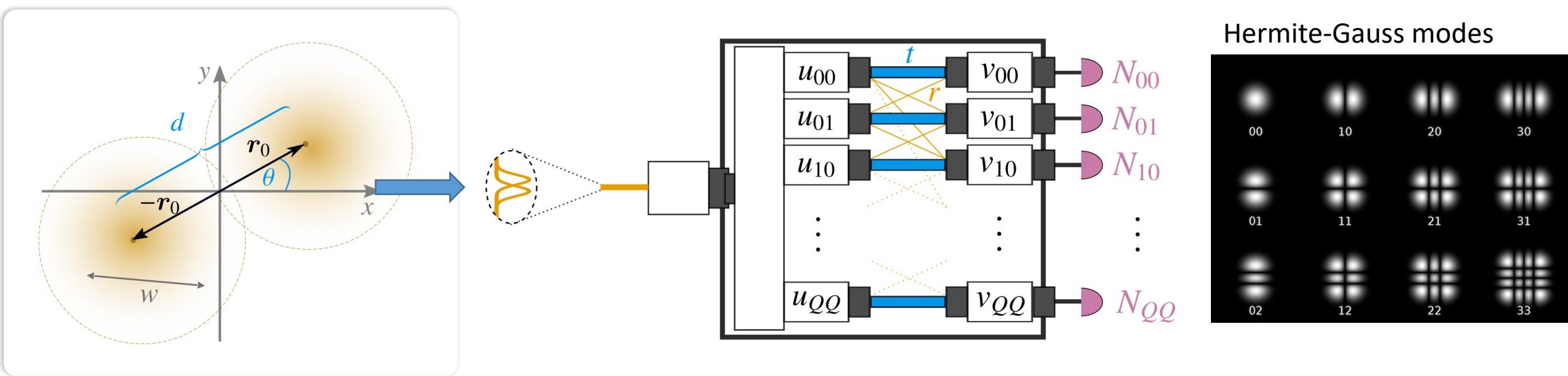
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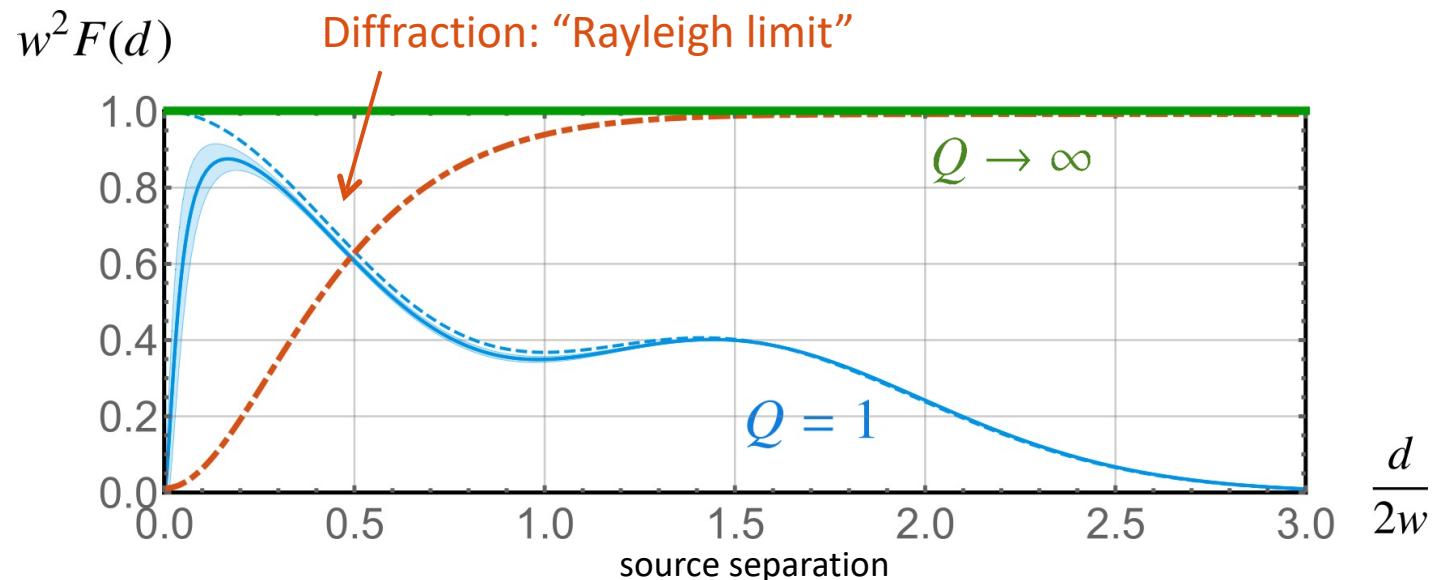
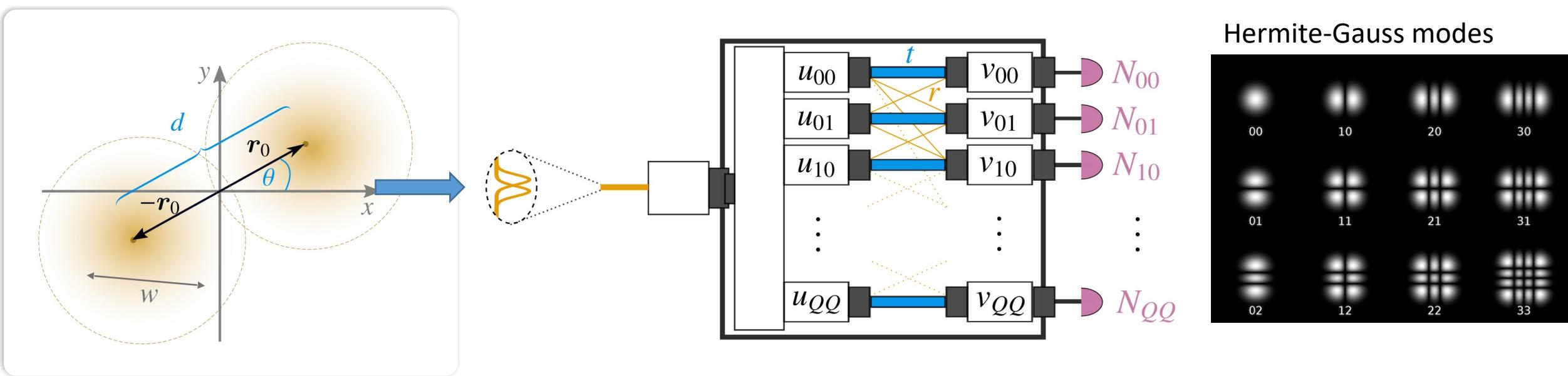
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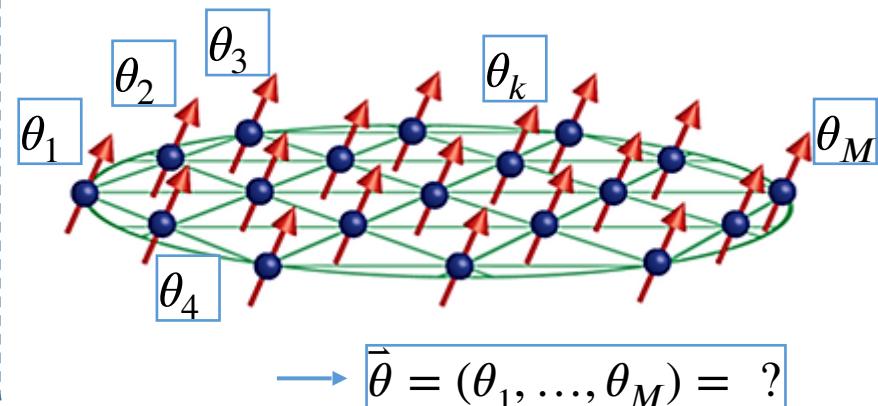
M. Gessner, C. Fabre, and N. Treps  
arxiv:2004.07228

Any nonzero crosstalk:

$$d_{\min} \sim N^{-1/4}$$

# MULTIPARAMETER QUANTUM METROLOGY

Quantum system



Quantum  
information

What are the fundamental limits  
on the precision of  $\bar{\theta} = (\theta_1, \dots, \theta_M)$  ?

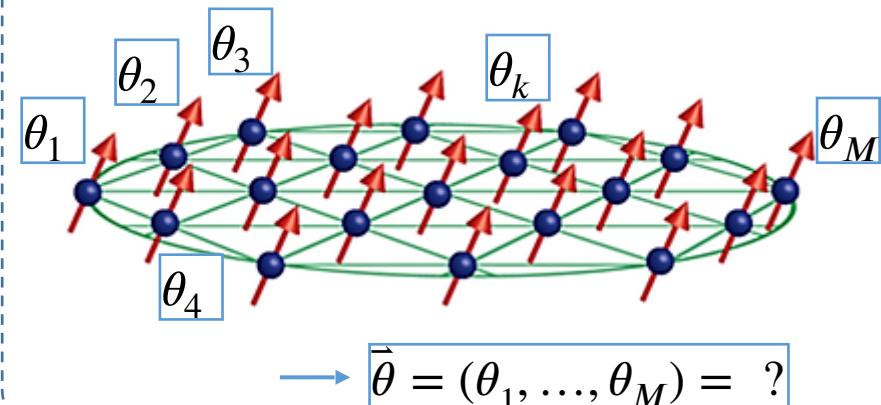
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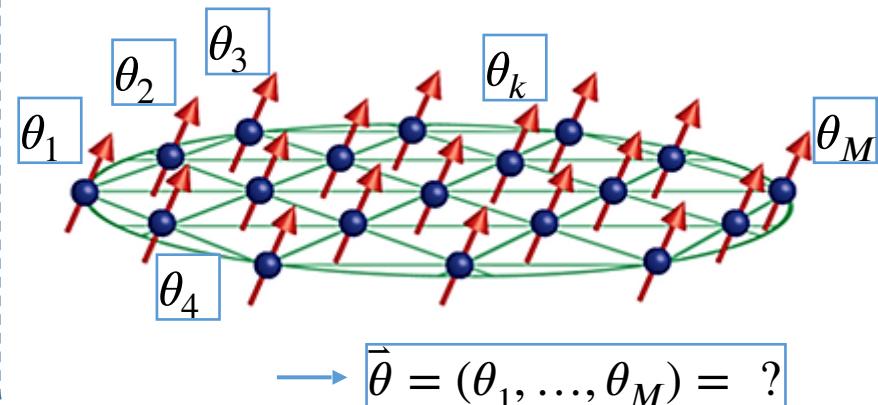
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Covariance matrix

$$\Sigma \geq F^{-1} \geq F_Q^{-1}$$

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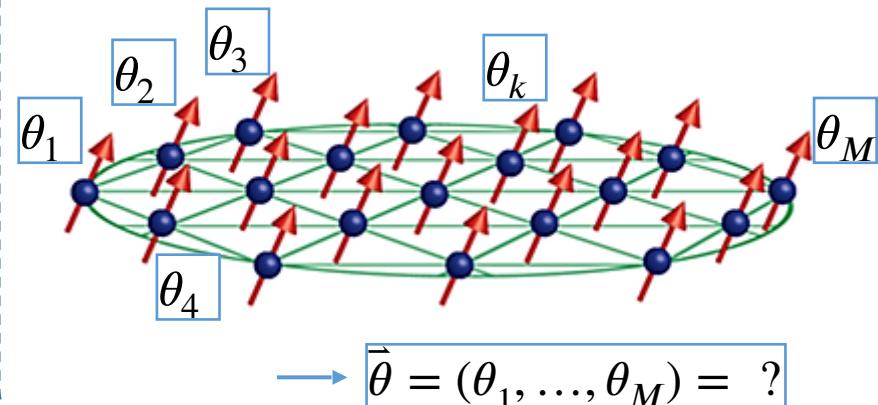
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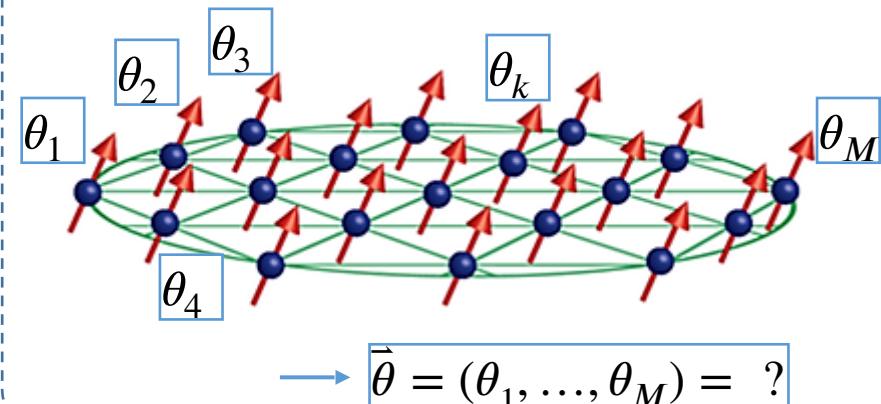
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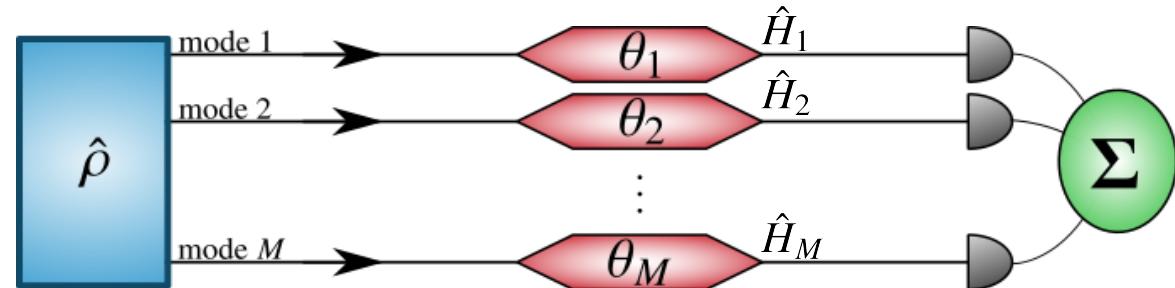
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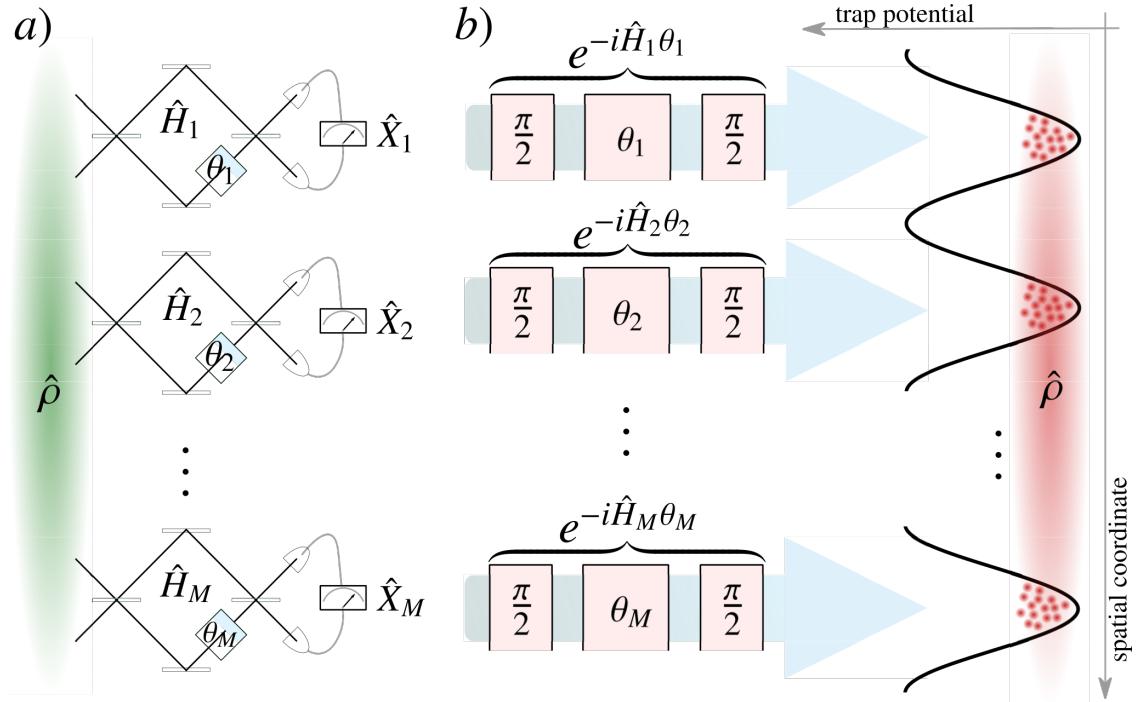


M. Gessner, L. Pezzè, and A. Smerzi, Phys. Rev. Lett. **121**, 130503 (2018)

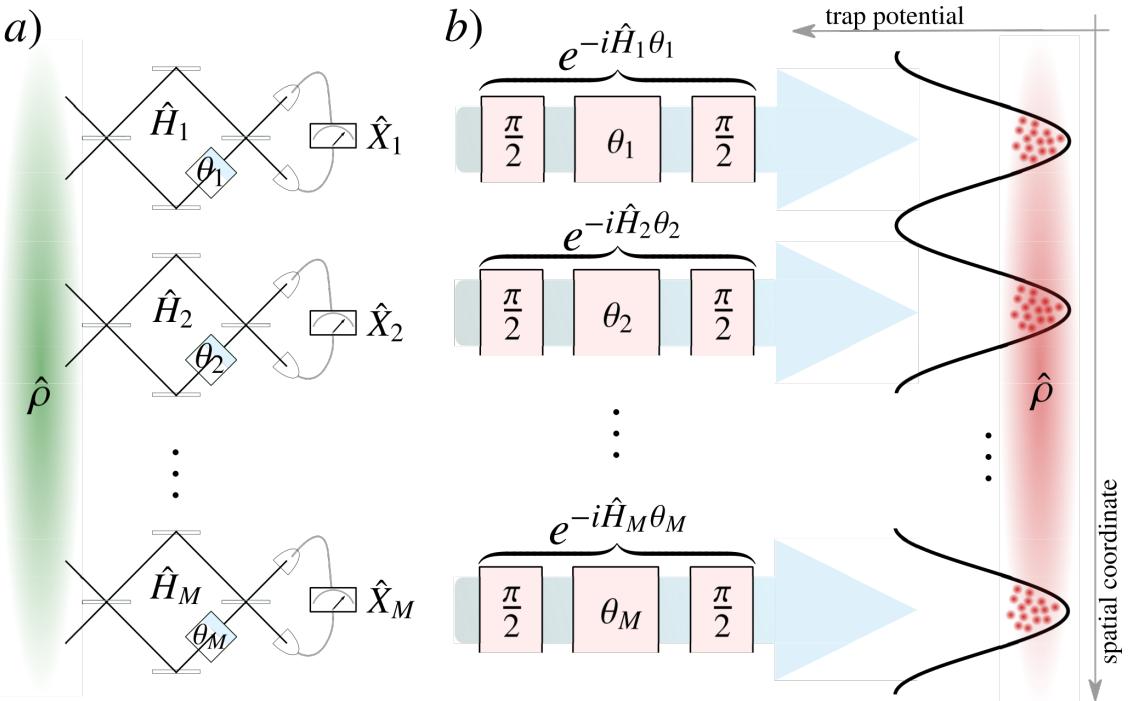
Sensitivity limits of multiparameter estimation

- Entanglement: Probe particles vs parameter-encoding modes
- Collective quantum enhancement

# MULTIPARAMETER SQUEEZING

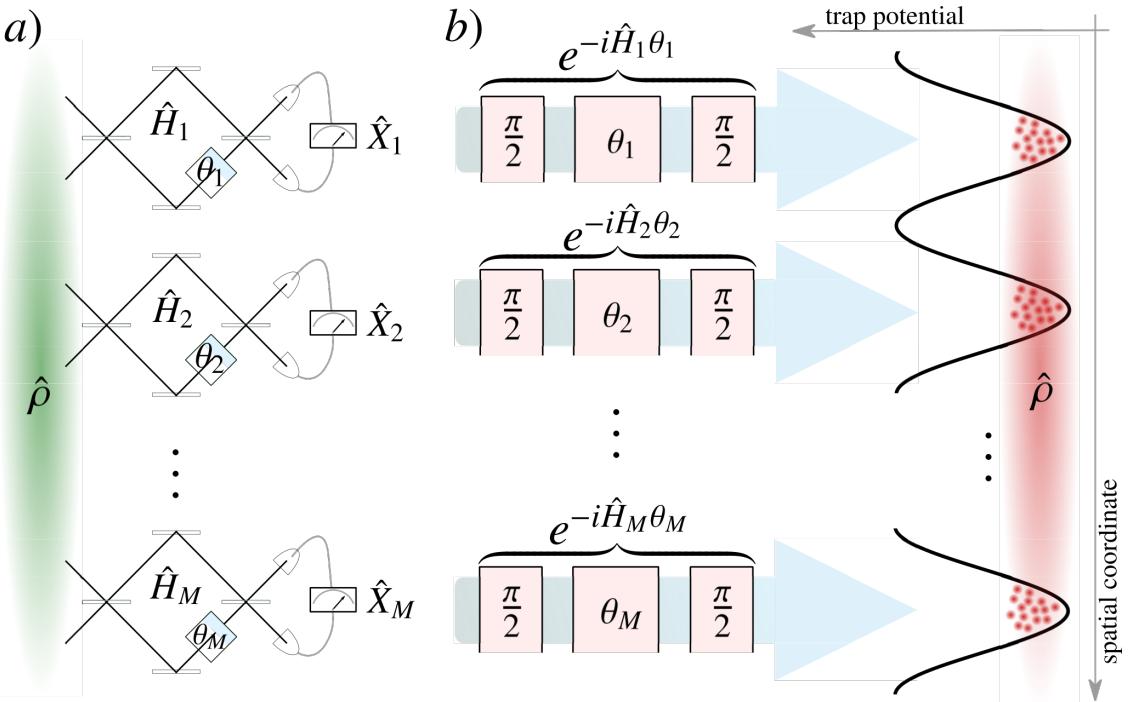


# MULTIPARAMETER SQUEEZING



M. Fadel *et al.*, Science **360**, 409 (2018)  
P. Kunkel *et al.*, Science **360**, 413 (2018)  
K. Lange *et al.*, Science **360**, 416 (2018)

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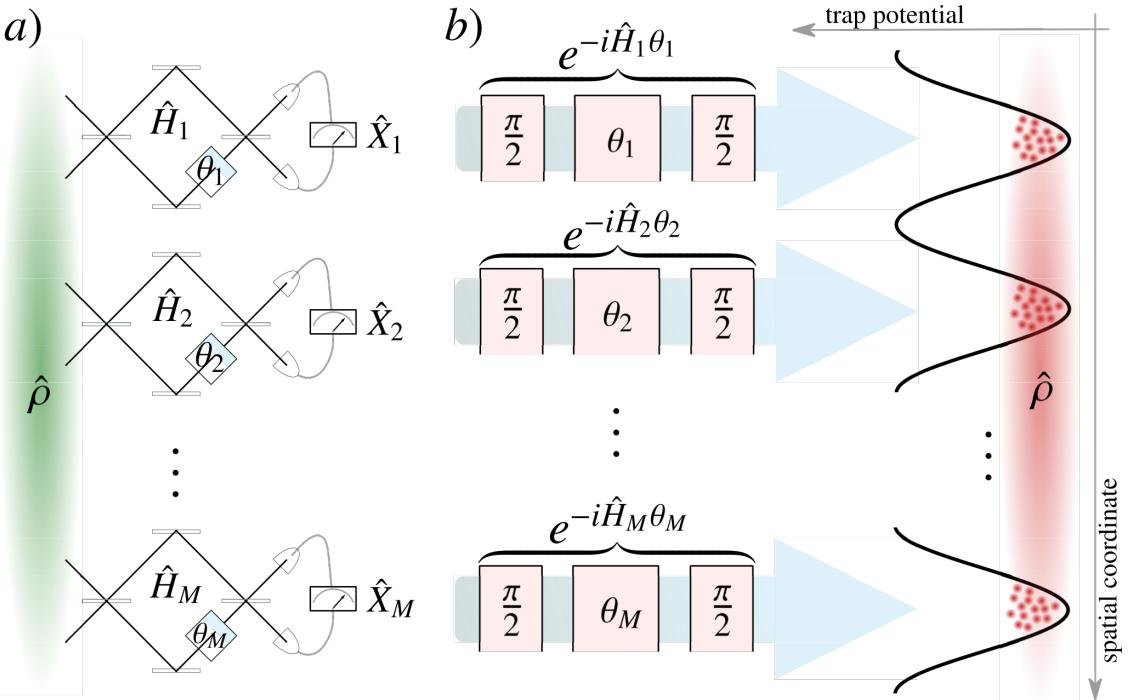


M. Fadel *et al.*, Science **360**, 409 (2018)  
P. Kunkel *et al.*, Science **360**, 413 (2018)  
K. Lange *et al.*, Science **360**, 416 (2018)

## Applications:

- Sensing electromagnetic field distributions
- Quantum imaging problems

# MULTIPARAMETER SQUEEZING



M. Gessner, A. Smerzi and L. Pezzè, arXiv:1910.14014

## Saturable lower bounds on multiparameter sensitivity

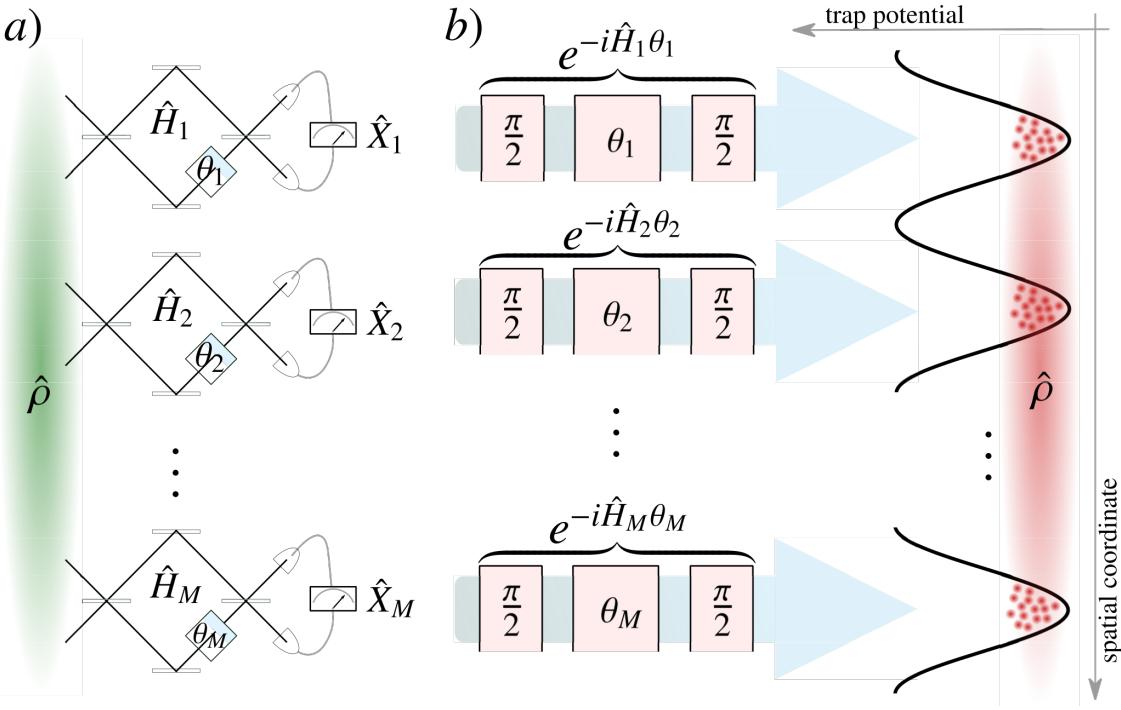
- Collective squeezing of a set of observables
- Sensitivity  $\sim \Gamma^{-1}$  (Covariance matrix)

M. Fadel *et al.*, Science **360**, 409 (2018)  
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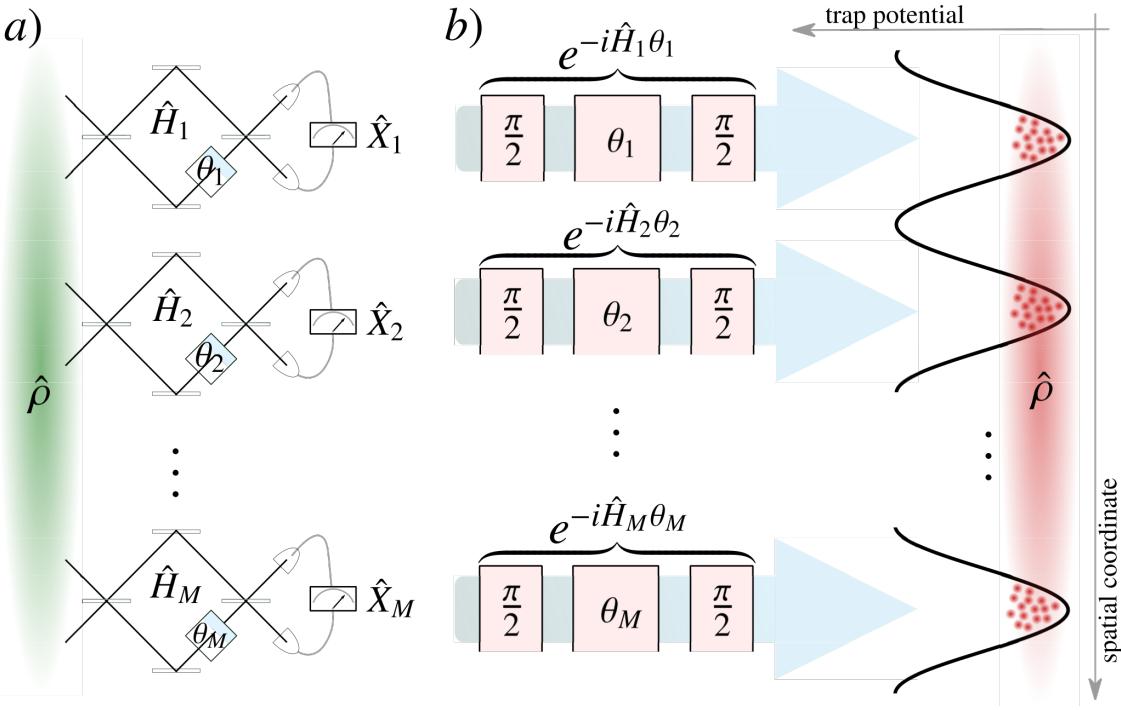
## Spin squeezing matrix

$$\xi^2[\rho] = \begin{pmatrix} \frac{N_1(\Delta J_y^{(1)})_\rho^2}{\langle J_z^{(1)} \rangle_\rho^2} & \frac{\sqrt{N_1 N_2} \text{Cov}(J_y^{(1)}, J_y^{(2)})_\rho}{\langle J_z^{(1)} \rangle_\rho \langle J_z^{(2)} \rangle_\rho} \\ \frac{\sqrt{N_1 N_2} \text{Cov}(J_y^{(1)}, J_y^{(2)})_\rho}{\langle J_z^{(1)} \rangle_\rho \langle J_z^{(2)} \rangle_\rho} & \frac{N_2(\Delta J_y^{(2)})_\rho^2}{\langle J_z^{(2)} \rangle_\rho^2} \end{pmatrix}$$

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M. Fadel *et al.*, Science **360**, 409 (2018)  
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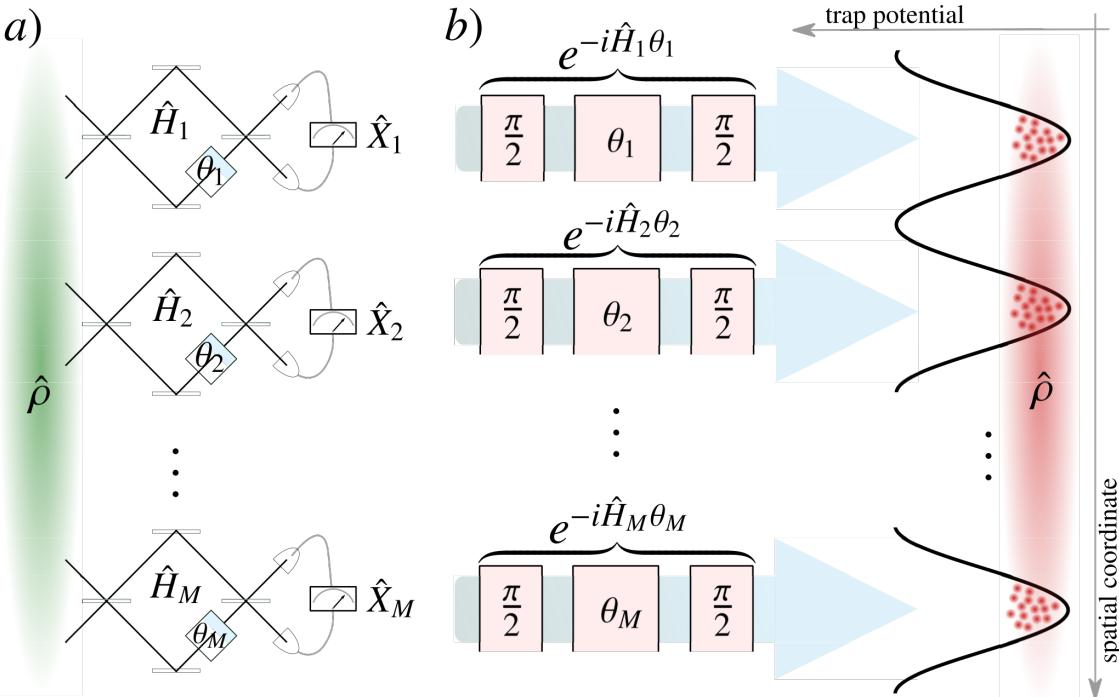
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Violation of this bound:  
**metrological entanglement witness**

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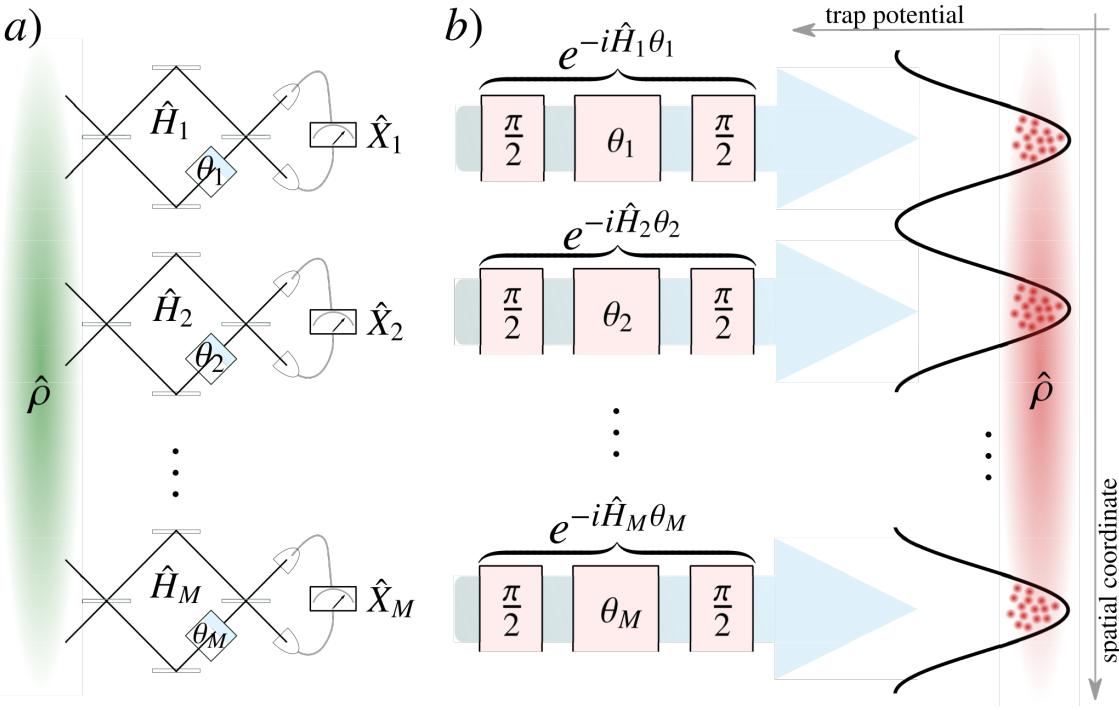
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Combines:

- Single parameter sensitivity gain (diagonal) **local squeezing, particle entanglement**

Violation of this bound:  
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M. Fadel *et al.*, Science **360**, 409 (2018)  
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M. Gessner, A. Smerzi and L. Pezzè, arXiv:1910.14014

### Saturable lower bounds on multiparameter sensitivity

- Collective squeezing of a set of observables
- Sensitivity  $\sim \Gamma^{-1}$  (Covariance matrix)

### Spin squeezing matrix

$$\xi^2[\rho] = \left( \begin{array}{c} \frac{N_1(\Delta J_y^{(1)})_\rho^2}{\langle J_z^{(1)} \rangle_\rho^2} \\ \frac{\sqrt{N_1 N_2} \text{Cov}(J_y^{(1)}, J_y^{(2)})_\rho}{\langle J_z^{(1)} \rangle_\rho \langle J_z^{(2)} \rangle_\rho} \\ \frac{\sqrt{N_1 N_2} \text{Cov}(J_y^{(1)}, J_y^{(2)})_\rho}{\langle J_z^{(1)} \rangle_\rho \langle J_z^{(2)} \rangle_\rho} \end{array} \right) \geq 1$$

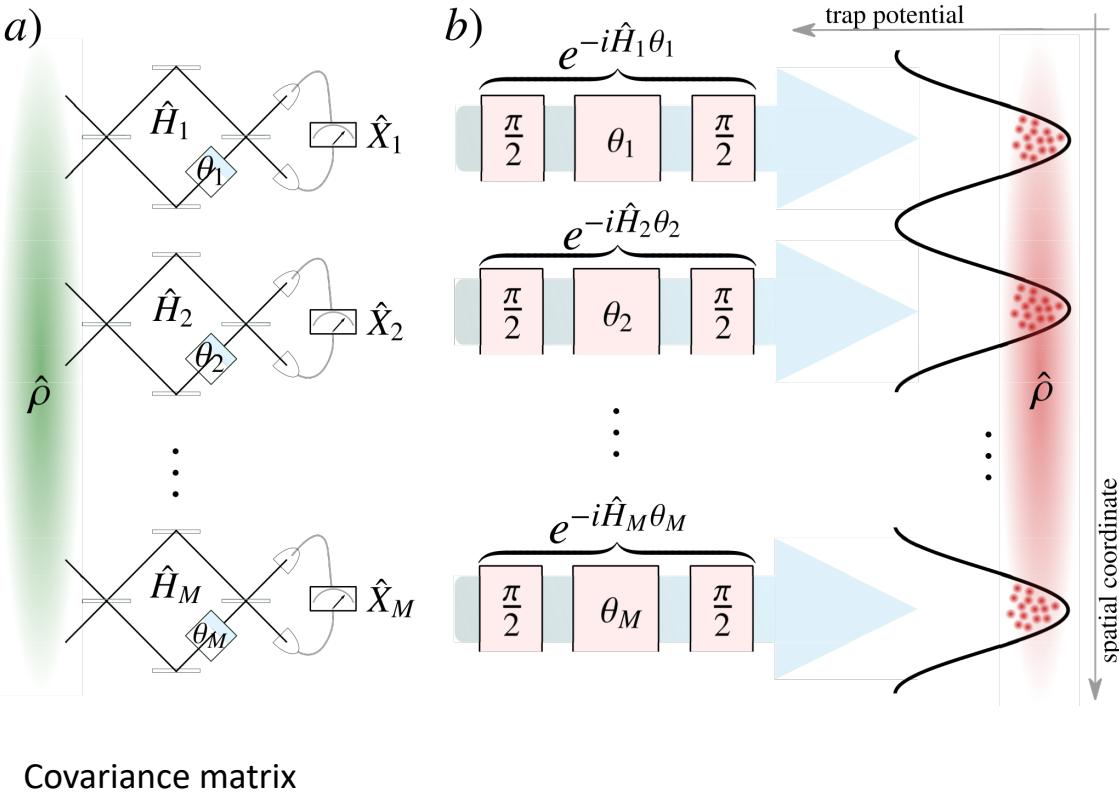
Combines:

- Single parameter sensitivity gain (diagonal) **local squeezing, particle entanglement**
- Multiparameter gain (off-diagonal) **nonlocal squeezing, mode entanglement**

Violation of this bound:  
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1

# MULTIPARAMETER SQUEEZING



$$\Sigma \geq F^{-1} \geq F_Q^{-1}$$

Not saturable in general!

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## General scenarios (non-commuting generators)

- Cramér-Rao bounds are of limited use (not saturable)
- New approach required!
- So far, no satisfactory answer is known!

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### Role of stronger quantum correlations than entanglement

- “Einstein-Podolski-Rosen steering”
- Violation of Bell’s inequalities

# CONCLUSIONS

## QUANTUM PARAMETER ESTIMATION

Quantum correlations

Metrological sensitivity  
of quantum systems

Observables

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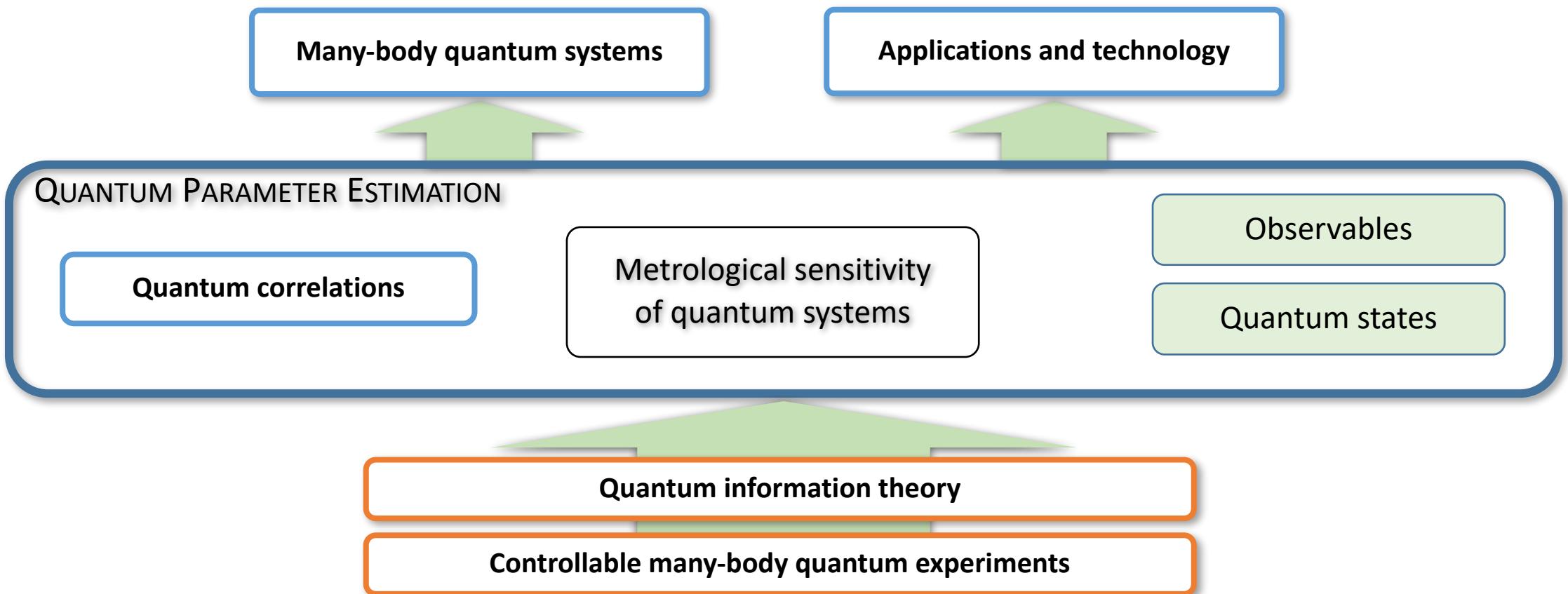
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