Hitchin Functionals in Supergravity

Mariana Graña
Ecole Normale Supérieure
Ecole Polytechnique

Collaborations with

Ruben Minasian, Michela Petrini, Alessandro Tomasiello hep-th/0406137
Jan Louis, Dan Waldram hep-th/0503…

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Type II sugra on $\mathcal{M}_{10} = \mathcal{M}_4 \times \mathcal{M}_6$

Minimal supersymmetry

⬇

$\mathcal{M}_6$ is CY ⟶ $\mathcal{M}_6$ ~~CY~~ unless T = 0

SU(3) holonomy ⟶ SU(3) structure

$\nabla_m \eta = 0$ —more general⟶ $\nabla_m^{(T)} \eta = 0$

SU(3) invariant spinor

Compactifications on CY ⟶ Compactifications on $\mathcal{M}_6$?

↓

4D $\mathcal{N} = 2$ sugra

- ⟶ complex structure def: special Kähler
- ⟶ complexified Kähler def: special Kähler

⟶ very similar…

all given by Hitchin functionals

Turn on fluxes on CY ⟶ gauged/massive sugra

Outline

- SU(3) structure
- Generalized complex geometry
- Hitchin functionals
- Special Kähler structures
- N = 2 superpotentials
- N =1 superpotentials
- Conclusions

To get action similar to 4D N = 2 $\mathcal{M}_{10} = \mathcal{M}_4$ x Y

Q_α must be well-defined on Y $\Rightarrow$ Y must admit globally well defined spinors

↓

reduced structure

- Structure group of a manifold:

group of transformations required to patch the orthonormal frame bundle

Y is Riemannian	➡	SO(d)
Y is Spin	➡	Spin(d)
reduced structure	➡	G ½ SO(d)

A manifold has G-structure if (equivalently)

- The structure group reduces to G
- 9 globally defined non-vanishing G-invariant tensors
- 9 globally defined non-vanishing G-invariant spinor

SU(3) structure in 6 dimensions

	SO(6)	!	SU(3)	
Vector	6	!	3 + $\bar{3}$	
A_2	15	!	8 + 3 + $\bar{3}$ + 1	← J_{mn}
A_3	10	!	6 + 3 + 1	← Ω_{mnp}
Spinor	4	!	3 + 1	← η

$$\eta^\dagger \gamma_{mn} \gamma \eta = i\, J_{mn}$$

$$\eta^\dagger \gamma_{mnp} (1+\gamma) \eta = i\, \Omega_{mnp}$$

as in CY

in CY (SU(3) holonomy): r_m η =0

- SU(3) structure: r_m^T η =0) r η ≠ 0 → dJ ≠ 0, dΩ ≠ 0

Torsion: $dJ = \mathrm{Im}\,(W_1\,\Omega) + W_4$ Æ $J + W_3$

$\quad\quad 1 \oplus 1 \quad\quad 3 \oplus \bar{3} \quad\quad 6 \oplus \bar{6}$

$d\,\Omega = W_1\, J^2 + W_5$ Æ $\Omega + W_2$ Æ J

$\quad\quad 1 \oplus 1 \quad\quad 3 \oplus \bar{3} \quad\quad 8 \oplus 8$

- We used J and Ω, but one can define the SU(3) structure in terms of real forms

  $(J\,,\,\rho) \quad : \rho = \mathrm{Re}\,\Omega$

- Must be stable: live in open orbit under action of GL(6,R) — Hitchin 00

  each element in a neighborhood of ρ is GL(6,R)-equivalent to ρ

Natural in the context of Generalized Complex Geometry — Hitchin, Gualtieri

- Usual differential geometry → T: tangent bundle, sections vector fields X | T*: cotangent bundle, sections are 1-forms ζ

- Want differential geometry on T © T* sections are X+ ζ

Natural metric I on T © T* : (X+ ζ, Y+η) = $\iota_X\, \eta + \iota_Y\, \zeta$ $\quad\quad I = \begin{pmatrix} 0 & 1_d \\ 1_d & 0 \end{pmatrix}$

Structure group of T © T* : SO(6,6) (or Spin(6,6))

	T	T © T*
	Clifford 6	Clifford (6,6)
Algebra	{γ^m,γ^n}=g^{mn}	{Γ^m, Γ^n}={Γ_m, Γ_n}=0, {Γ^m, Γ_n}= $\delta^m{}_n$
Representation in terms of forms	γ^m=dz^mÆ + g^{mn} $\iota_{\partial n}$	Γ^m=dx^mÆ and Γ_m = $\iota_{\partial m}$ Γ^Σ=(Γ_m,Γ^m)
Clifford vacuum	η_+	$\eta_+{}^y$ - η_- = Ω $\eta_+ \otimes \eta_\pm^\dagger = \sum_{k=0}^{6} \frac{1}{k!} \eta_\pm^\dagger \gamma_{i_1 \ldots i_k} \eta_+ \gamma^{i_1 \ldots i_k}$
Pure spinor	γ^i η_+ =0	Γ^i Ω=$\Gamma_{\bar{i}}$ Ω =0
Basis	η_+ $\gamma^{\bar{i}}$ η_+ $\gamma^{\bar{i}\bar{j}}$ η_+ $\overline{\gamma^{\bar{i}\bar{j}k}}$ η_+	Ω : (3,0) Ω : (3,0) $\Gamma^{\bar{i}}$ Ω : (3,1) $\Gamma_i\Omega$: (2,0) $\Gamma^{\bar{i}\bar{j}}$ Ω : (3,2) Γ_{ij} Ω : (1,0) $\Gamma^{\overline{ijk}}$ Ω : (3,3) Γ_{ijk} Ω : (0,0) Spinors $ (p,q) forms
	Weyl spinors $\eta_§$ η_-	$S^\pm$ = $\Lambda^{ev,odd}$ T* Ω 2 S^- $\eta_+{}^y$ - η_+ =e^{iJ} 2 S^+ $e^{iJ} = 1 + iJ - \frac{1}{2}J^2 - i\frac{1}{3!}J^3$ (Γ_m + i J_{mn} Γ^n) e^{iJ}=0
9 pure spinor	reduces str. group SU(3)	reduces str. group SU(3,3)
Inner product	η^y χ	$(\bar{\phi}, \rho)\epsilon = \langle \phi, \rho \rangle$ (volume form) $\langle \phi^+, \rho^+ \rangle = \phi_6 \wedge \rho_0 - \phi_4 \wedge \rho_2 + \phi_2 \wedge \rho_6 - \phi_0 \wedge \rho_6$ $\langle \phi^-, \rho^- \rangle = \phi_5 \wedge \rho_1 - \phi_3 \wedge \rho_3 + \phi_1 \wedge \rho_5$ (Muk)
9 2 pure spinors	reduces str group to SU(2) if η_1 never k η_2	SU(3) £ SU(3) if $\bar{\phi}^- \Gamma^\Sigma \phi^+ = 0$ → J Æ Ω =0

- We want to perform a compactification on a manifold with SU(3) structure

→ Get an effective theory of light modes

Problem:

$$dJ = \mathrm{Im}\,(W_1\,\Omega) + W_4 \text{ Æ } J + W_3$$
$$d\,\Omega = W_1\, J^2 + W_5 \text{ Æ } \Omega + W_2 \text{ Æ } J$$

→ $r^2 J$ » $W^2 J$ torsion gives masses to deformations of J

Distinction between light and heavy modes not clear

- Alternative route: do not compactify
  truncate the spectrum

~~$\mathcal{M}_{10} = \mathcal{M}_4 \times Y$~~

SO(6) vector bundle admits SU(3) structure

SO(1,3) vector bundle

But: Spin(1,9) → Spin(1,3) £ Spin(6) ⇒ $T\mathcal{M}^{1,9} = T^{1,3}$ © F

Q_α 2 $16 + \overline{16}$ IIA
16 + 16 IIB

$$16 \to (2,4) \oplus (\bar{2},\bar{4}) \to (2,1) \oplus (2,3) \oplus (\bar{2},1) \oplus (\bar{2},\bar{3})$$
$$16 \to (2,4) \oplus (\bar{2},\bar{4}) \to (2,1) \oplus (2,3) \oplus (\bar{2},1) \oplus (\bar{2},\bar{3})$$

8 supercharges are singled out

- Original type II theory formulated on $M^{1,9\,|\,16+16}$

- Demanding: $TM^{1,9} = T^{1,3}$ © F and F admits SU(3) structure ⇒ 9 $N^{1,9\,|\,4+4}$ ½ $M^{1,9\,|\,16+16}$

Reformulate the theory on N

The theory on $N^{1,9|4+4}$ ½ $M^{1,9|16+16}$

- Has 8 supercharges
- Spin(1,3) preserved

Want to show that although it is 10D

- Has the same structures as 4D $\mathcal{N}$=2 sugra

→ Special Kähler manifolds for
  - complex structure deformations
  - complexified "Kähler" deformations

→ $\mathcal{N}$ = 2 superpotential

Deformations of SU(3) structure

Metric deformations

$$\delta g^{10}_{MN} : \quad \underset{\mathbf{1_2}}{\delta g^{10}_{\mu\nu}} \quad \underset{(\mathbf{3}+\bar{\mathbf{3}})_\mathbf{1}}{\delta g^{10}_{\mu m}} \quad \underset{\mathbf{1_0}+(\mathbf{6}+\bar{\mathbf{6}})_\mathbf{0}+\mathbf{8_0}}{\delta g^{10}_{mn}}$$

$\mathbf{1_0}+(\mathbf{6}+\bar{\mathbf{6}})_\mathbf{0}+\mathbf{8_0}$: 21 = dim GL(6,**R**) / SO(6)

Consider instead deformations of the SU(3) structure

$$\delta J = \underset{1}{\lambda J} + \underset{3\oplus\bar{3}}{v \perp \rho} + \underset{8}{K}$$

$$\delta\rho = \frac{3}{2}\lambda\rho + \underset{1}{\gamma\hat{\rho}} - v\wedge J + \underset{6\oplus\bar{6}}{M}$$

28 = dim GL(6,**R**) / SU(3)

γ and v defs of SU(3) leave metric invariant

Indeed: $\delta g^{10}_{mn} = \lambda g^{10}_{mn} - J_m{}^p K_{pn} - \frac{1}{2}\rho_m{}^{pq} M_{pqn}$

Consider deformations of ρ, J and then both together

ρ, J: special Kähler manifolds

both together: Hitchin

	F	F © F*	
	η_+	ϕ§ 2 S§ $\cong \Lambda^{\text{even/odd}}$ F* real	
	SU(3) structure	Each ϕ: SU(3,3) structure if stable	each element in a neighborhood of ϕ is GL(6,R)-equivalent to ϕ
2-form	$\eta_+^\dagger \gamma_{mn} \eta_+ = i\, J_{mn}$	$\bar{\phi}\, \Gamma_{\Sigma\Pi}\, \phi = \mathcal{J}_{\Sigma\Pi}$	
	$J_{mn}\, J^{mn} = 6$	$\mathcal{J}_{\Sigma\Pi}\, \mathcal{J}^{\Sigma\Pi} \neq 12$	

F © F*   Hitchin

$$\lambda(\phi) \equiv \frac{1}{12} \mathcal{J}_{\Sigma\Pi} \mathcal{J}^{\Sigma\Pi}$$

Hitchin: ϕ is stable if $\lambda > 0$

$$U = \{\phi \in S^{\pm} \ st \ \lambda(\phi) > 0\}$$

space of stable Spin(6,6) spinors

↑ dim $\Lambda^{\text{even/odd}}$ = 32

↑ reduce structure of F© F* to SU(3,3)

Want to show that there is a special Kähler structure on U

- → complex structure
- → symplectic structure
- → prepotential

$$\lambda(\phi) \equiv \frac{1}{12}\mathcal{J}_{\Sigma\Pi}\mathcal{J}^{\Sigma\Pi}$$

$$\left\langle \phi^+, \rho^+ \right\rangle = \phi_6 \wedge \rho_0 - \phi_4 \wedge \rho_2 + \phi_2 \wedge \rho_6 - \phi_0 \wedge \rho_6$$

$$\left\langle \phi^-, \rho^- \right\rangle = \phi_5 \wedge \rho_1 - \phi_3 \wedge \rho_3 + \phi_1 \wedge \rho_5$$

Hitchin functional

$$h(\phi) = \sqrt{\underbrace{\frac{1}{12}\underbrace{\langle \phi, \Gamma_{\Pi\Sigma}\phi\rangle}_{\mathcal{J}_{\Pi\Sigma}\,\epsilon}\left\langle \phi, \Gamma^{\Pi\Sigma}\phi\right\rangle}_{\lambda\ \epsilon\otimes\epsilon}} \in \Lambda^6 F^* \quad \Rightarrow \quad h(\phi) = \sqrt{\lambda}\,\epsilon$$

Ex: for $\phi = \rho$ $\Rightarrow$ $\langle \phi, \Gamma_{\Pi\Sigma}\phi\rangle \left\langle \phi, \Gamma^{\Pi\Sigma}\phi\right\rangle = (\rho\wedge e^m \wedge i_n\rho)(\rho e^n \wedge i_m \rho)$

$$= \rho_{a[i_1 i_2}\,\rho_{i_3 i_4 i_5}\rho_{i_6][j_1 j_2}\,\rho_{j_3 j_4 j_5}\,\delta_{j_6]}{}^{a}$$

$\Downarrow$

$$h = \sqrt{\epsilon^{i_1 i_2 i_3 i_4 i_5 i_6}\epsilon^{j_1 j_2 j_3 j_4 j_5 j_6}\rho_{j_6 i_1 i_2}\,\rho_{i_3 i_4 i_5}\rho_{i_6 j_1 j_2}\,\rho_{j_3 j_4 j_5}}\ \epsilon$$

Define $\hat{\phi} \equiv \frac{Dh}{D\phi} = \frac{1}{6\sqrt{\mathcal{J}^2/12}}\mathcal{J}^{\Pi\Sigma}\Gamma_{\Pi\Sigma}\phi \quad \in S^{\pm}$

$\hat{\phi} = \frac{Dh}{D\phi}$ and *h* homogeneous of degree 2 in ϕ

$\Downarrow$

$$h = \frac{1}{2}\left\langle \phi, \hat{\phi}\right\rangle$$

Combine ϕ and $\hat{\phi}$ into complex Spin(6,6) spinor

$\Phi = \phi + i\hat{\phi}$ Pure spinor ! Ex: $\phi = \rho = Re\Omega$) $\hat{\phi} = *\phi = Im\Omega$ and then

$$\Phi = \Omega$$

Want to show that there is a special Kähler structure on U

$$U = \{\phi \in S^{\pm} \ st \ \lambda(\phi) > 0\}$$ space of stable Spin(6,6) spinors

$S^{\S} \cong \Lambda^{\text{even/odd}}$: 32 dim vector space) $T_{\phi} U \cong S^{\S}$

ϕ: coordinate on U
vector field on U

ϕ^{α} ← 1, …, 32

• Symplectic structure:

Spinor norm

$$\langle \phi, \rho \rangle \equiv w(\phi, \rho)\epsilon \quad \Rightarrow \quad w(\phi, \rho) = \bar{\phi}\rho = w_{\alpha\beta}\,\phi^{\alpha}\rho^{\beta}$$

↑ symplectic structure

• Complex structure

$$I^{\alpha}{}_{\beta} = -\frac{\partial \hat{\phi}^{\alpha}}{\partial \phi^{\beta}} = -\partial_{\beta}\hat{\phi}^{\alpha} = -(w^{-1})^{\alpha\beta}\partial_{\alpha}\partial_{\beta}h$$

Using $\hat{\hat{\phi}} = \frac{Dh}{D\hat{\phi}} = -\phi$ can show that $I^2 = -1_{32\times 32}$

Symplectic and complex structures are integrable

Metric $$G = (w_{\alpha\gamma}I^{\gamma}{}_{\beta})d\phi^{\alpha}\,d\phi^{\beta} = \partial_{\alpha}\partial_{\beta}h\,d\phi^{\alpha}d\phi^{\beta}$$
$$= \partial_{\alpha}\partial_{\bar{\beta}}h\,d\Phi^{\alpha}d\bar{\Phi}^{\bar{\beta}}$$

h: Kähler potential

$$K = h \;\; = \frac{1}{2} w(\phi, \hat{\phi}) = i w(\Phi, \bar{\Phi}) = i(\bar{Z}^A F_A - Z^A \bar{F}_A)$$

$$F_A = \frac{\partial F}{\partial Z^A} \quad \text{prepotential}$$

$$Z^A = x^A + i \frac{\partial h}{\partial y_A}$$

Darboux coordinates

$$\phi^\alpha = (x^A, y_B)$$

$$w = dx^A \wedge dy_A$$

• Space of stable forms is special Kähler

- complex structure
- symplectic structure
- prepotential

• Kähler potential is Hitchin functional

$$K = h \;\; = i w(\Phi, \bar{\Phi})$$

Local special Kähler geometry : mod out by ϕ ! λ ϕ , λ 2 **C***

$$K = -\ln h = -ln\left(i w(\Phi, \bar{\Phi})\right)$$

For ϕ= ρ 2 S^-

Special Kähler structure for complex structure deformations

Hitchin functional: $h = \sqrt{\epsilon^{i_1 i_2 i_3 i_4 i_5 i_6} \epsilon^{j_1 j_2 j_3 j_4 j_5 j_6} \rho_{j_6 i_1 i_2} \, \rho_{i_3 i_4 i_5} \rho_{i_6 j_1 j_2} \, \rho_{j_3 j_4 j_5}} \;\; \epsilon$

Pure spinor: $\hat{\phi} \equiv \frac{Dh}{D\rho} = *\rho = Im\Omega \quad \Rightarrow \quad \Phi = \phi + i\hat{\phi} = \Omega$

↑ pure spinor

Kähler potential: $K = -\ln h = -ln\,(iw(\Omega, \bar{\Omega}))$

↑

$w(\Omega, \bar{\Omega})\,\epsilon = \Omega \wedge \bar{\Omega}$

Exactly as for CY !

For ϕ= Re (c e^{iJ}) 2 S^+

Special Kähler structure for “Kähler” deformations

Hitchin functional: $h = \frac{1}{3}|c|^2 J \wedge J \wedge J$

Pure spinor: $\hat{\phi} \equiv \frac{Dh}{D\rho} = *\phi = Im(ce^{iJ}) \quad \Rightarrow \quad \Phi = \phi + i\hat{\phi} = c\,e^{iJ}$

↑ pure spinor

Kähler potential: $K = -\ln h = -ln\, iw(c\,e^{iJ}, \bar{c}\,e^{-iJ})$

↑

$iw(e^{iJ}, e^{-iJ})\epsilon = J \wedge J \wedge J$ Exactly as for CY !

The theory on N $^{1,9\,|\,4+4}$ ½ M $^{1,9\,|\,16+16}$ has the same structures as 4D $\mathcal{N}$=2 sugra

internal spinor is SU(3) singlet

Demanding $TM^{1,9} = T^{1,3}$ © F and F admits SU(3) structure → J, ρ

Moduli space of deformations of J and ρ are special Kähler √

$\mathcal{N}$= 2 superpotential

$\mathcal{N}$=2 superpotential

Torsion and fluxes generate scalar potential

$\mathcal{N}$=2 $$V \sim |\delta\psi|^2 + |\delta\lambda|^2 + |\delta\xi|^2$$

gravitino gaugino hyperino

Gravitino transformation contains all the information about V

A=1,2

$$\delta\psi_\mu^A = D_\mu\epsilon^A + i\gamma_\mu S^{AB}\epsilon_B$$

S^{AB} gravitino mass matrix

$$\mathcal{L}_{mass} = S_{AB}\bar{\psi}_\mu^A\gamma^{\mu\nu}\psi_\nu^B$$

$\mathcal{N}$=2 superpotential

D-term

$$S^{AB} = \frac{i}{2}e^{\frac{1}{2}K_V}\sigma^x_{AB}\mathcal{P}^x = -\frac{i}{2}e^{\frac{1}{2}K_V}\begin{pmatrix} -\mathcal{P}^1 + i\mathcal{P}^2 & \mathcal{P}^3 \\ \mathcal{P}^3 & \mathcal{P}^1 + i\mathcal{P}^2 \end{pmatrix}$$

Killing prepotentials (which isometries are gauged)

$\mathcal{N}$=1 superpotential

⇨ We want to compute $\delta\,\psi_\mu$ on N $^{1,9\,|\,4+4}$

Under SU(3)

η

Spinor : 4 ! 3 + 1

$$\tilde{\Psi}_\mu = \psi_\mu \otimes \eta + \text{ triplets}$$

$1_{3/2}$ $3_{3/2}$

"4D" gravitino

Subtletly: kinetic term not diagonal in Ψ_μ and Ψ_m

$\Rightarrow$ to diagonalize, need redefinition:

$$\tilde{\Psi}_\mu = \Psi_\mu + \tfrac{1}{2}\gamma_\mu{}^m \Psi_m$$

$$\boxed{\delta\psi_\mu^A = D_\mu \epsilon^A + i\gamma_\mu S^{AB}\epsilon_B}$$

Given susy transformations

$$\Pi\, \delta\tilde{\Psi}_\mu \sim \left(D_\mu + \frac{1}{2}\gamma_\mu{}^m D_m + \frac{1}{8} H_{mnp}\gamma_\mu{}^{mnp}\sigma^3 + \frac{1}{16} e^\phi \sum_n \not{F}_n \gamma_\mu \sigma^{1,2} \right)\epsilon$$

torsion and NSNS flux ~ S^{11}

RR flux ~ S^{12}

To extract $\delta\psi_\mu$ apply projector $\Pi = \mathbf{1} \otimes (\eta_+ \otimes \eta_+^\dagger)$

Get $H_{mnp}\eta_+^\dagger \gamma^{mnp}\eta_- = H_{mnp}\Omega^{mnp}$

Notice $\langle H_3, \Omega \rangle = i\, H_{mnp}\Omega^{mnp}\, \epsilon_V$

All terms in $\Pi\delta\tilde{\Psi}_\mu$ can be written in terms of Mukai pairs

Compare to: $\delta\psi_\mu^A = D_\mu\epsilon^A + i\gamma_\mu S^{AB}\epsilon_B$

In 10D theory, S^{AB} is naturally a 6-form

$$S^{AB} = -\frac{i}{2} e^{\frac{1}{2}K_V} \begin{pmatrix} -\mathcal{P}^1 + i\mathcal{P}^2 & \mathcal{P}^3 \\ \mathcal{P}^3 & \mathcal{P}^1 + i\mathcal{P}^2 \end{pmatrix}$$

IIA	IIB
$e^{\frac{1}{2}K_J}(\mathcal{P}^1 + i\mathcal{P}^2) = \frac{1}{4}\left\langle de^{B+iJ}, \Omega \right\rangle$	$e^{\frac{1}{2}K_\Omega}(\mathcal{P}^1 + i\mathcal{P}^2) = \frac{1}{4}\left\langle de^{B+iJ}, \Omega \right\rangle$
$e^{\frac{1}{2}K_J}\mathcal{P}^3 = \frac{1}{4}\, e^{\phi}\left\langle F_{2n}, e^{B+iJ} \right\rangle$	$e^{\frac{1}{2}K_\Omega}\mathcal{P}^3 = \frac{1}{4}\, e^{\phi}\left\langle F_{2n+1}, \Omega \right\rangle$

Depend on $e^B e^{iJ}$ → B-transform   If Φ pure) e^B Φ pure

$$\mathcal{P}^3 \sim \left\langle F^{\pm}, \Phi^{\pm} \right\rangle \qquad \mathcal{P}^1 + i\mathcal{P}^2 \sim \left\langle d\Phi^{+}, \Phi^{-} \right\rangle$$

RRsector

IIA $ IIB

F^+ $ F^-

Φ^+ $ Φ^-

NSNS sector

$$\left\langle d\Phi^{+}, \Phi^{-} \right\rangle$$

sym under exchange Φ+ $ Φ-
when integrated

exchange of two pure spinors -- action of mirror symmetry

more details in Minasian & Tomasiello's talks

$\mathcal{N}$=1 superpotential

- Want to break $SU(2)_R$ into U(1)
- Look at $L^{1,9|2+\bar{2}} \subset N^{1,9|4+\bar{4}} \subset M^{1,9|16+16}$
- In $\mathcal{N}$= 1 language $S^{AB} = -\frac{i}{2} e^{\frac{1}{2}K_V} \begin{pmatrix} -\mathcal{P}^1 + i\mathcal{P}^2 & \overbrace{\mathcal{P}^3}^{\text{D-term}} \\ \mathcal{P}^3 & \underbrace{\mathcal{P}^1 + i\mathcal{P}^2}_{\mathcal{N}=1 \text{ superpotential}} \end{pmatrix}$

But this assumes a particular breaking of $SU(2)_R$

(a particular way of embedding L in N)

Gauge invariant statement: $*$ *P* k U(1) gives D-term.

$*$ Other *P*'s combine into $\mathcal{N}$=1 superpotential

Parameterize gauge choice by two complex numbers a and b $\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} = \begin{pmatrix} a\,\epsilon \\ b\,\epsilon \end{pmatrix}$ (arrow to ϵ: $\mathcal{N}$=1 susy parameter)

Look at what SU(2) generator leaves this spinor invariant ! U(1) ½ SU(2)

$\mathcal{N}$=1 superpotential

IIA

$$e^K \mathcal{W} = e^{-i\beta} \sin^2\alpha(\bar{H}+i\bar{W}) - e^{i\beta} \cos^2\alpha(H-iW) + \sin(2\alpha)\, \bar{F}_A$$

IIB

$$e^K \mathcal{W} = (e^{i\beta} \cos^2\alpha - e^{-i\beta} \sin^2\alpha)\, H + i(e^{i\beta} \cos^2\alpha + e^{-i\beta} \sin^2\alpha)\, W - \sin(2\alpha)\, F_B$$

$$W = i\left\langle de^{iJ}, \Omega\right\rangle = W_1\, \epsilon_V \qquad H = \langle H_3, \Omega\rangle \qquad F_A = e^{\phi}\left\langle F_A, e^{B+iJ}\right\rangle \qquad F_B = e^{\phi}\langle F_B, \Omega\rangle$$

$$\tan\alpha = \frac{|a|}{|b|} \qquad e^{i\beta} = \frac{a\,\bar{b}}{|ab|}$$

- Depend on two angles → parameterize breaking $SU(2)_R$! $U(1)_R$
- Contains known cases so far: In IIB α= π/4 , β=-π/2 : $\left\langle F_3 - ie^{-\phi} H_3, \Omega\right\rangle$ GVW
  Gukov, Vafa, Witten, Taylor

  In IIA α=β=0 $\langle H + idJ, \Omega\rangle$ Heterotic
  IIB
  Becker, Becker, Dasgupta, Green

General $\mathcal{N}$= 1 superpotential for manifolds with SU(3) structure

Conclusions

- Type II theories on manifolds admiting SU(3) structure → special Kähler moduli spaces; → $\mathcal{N} = 2$ superpotential

- Kähler potential is Hitchin functional

- Superpotential given by inner products of pure spinors and flux / torsion

- N = 1 superpotential given by inner products of pure spinors and flux / torsion
  depend on two angles which define $U(1)_R$ ½ $SU(2)_R$

Hitchin functionals (generalized complex geometry) at core of $\mathcal{N}$=2 / $\mathcal{N}$=1 "compactifications" of type II

More on this in upcoming talks…

IIA:

$$(\mathcal{P}^1 + iP^2) = \frac{1}{4} e^{\phi} e^{\frac{1}{2}K_\Omega} X^A (e_{AK} Z^K + m_A^K F_K)$$

$$\mathcal{P}^3 = -\frac{1}{4} e^{2\phi} \left(X^A (e_{AK} \xi^K + m_A^K \tilde{\xi}_K) + (X^A e_{RR\,A} + F_A m_{RR}^A) \right)$$

IIB:

$$(\mathcal{P}^1 + i\mathcal{P}^2) = \frac{i}{4} e^{\phi} e^{\frac{1}{2}K_J} (Z^K e_{AK} X^A + F_K m_A^K X^A)$$

$$\mathcal{P}^3 = -\frac{1}{4} e^{2\phi} [Z^K (e_{RR\,K} - e_{AK} \xi^A) + F_K (m_{RR}^K - m_A^K \xi^A)]$$

$$\Omega = Z^K \alpha_K - F_L \beta^L \qquad B + iJ = t^a w_a \qquad X^A = (1, t^a)$$

$$dw^a = m_a{}^K \alpha_K + e_{aL} \beta^L \quad d\alpha_K = e_a{}^K \tilde{w}^a \quad d\beta_L = -m_a{}^K \tilde{w}^a$$

$$F_2 = m_{RR}^a w_a$$

$$e_{AK} = (e_{0\,K}, e_{a\,K})$$

$$F_4 = e_{a\,RR} \tilde{w}^a$$

$$m_A{}^K = (m_0\,K, m_a\,K)$$

$$F_3 = m_{RR}^K \alpha_K + e_{K\,RR} \beta^K$$

$$H_3 = m_0^K \alpha_K + e_{K\,0} \beta^K \qquad \xi^A = (l, c^a - l b^a) \qquad A_3 = \xi^K \alpha_K + \tilde{\xi}_L \beta^L$$