String Theory Compactifications with Background Fluxes

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Journées Physique et Mathématique – IHES -- Novembre 2005

Motivation

- One of the most important unanswered question in string theory:

  What is the structure of the vacuum we live in?

  $\longrightarrow$ One possibility being intensively explored: compactifications with background fluxes

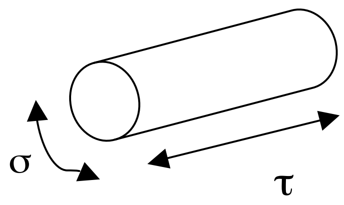
Structure of the talk

- Introduction to string theory
- Traditional compactifications
- Flux compactifications
- Conclusions and open problems

Introduction to string theory. Why do we need to compactify?

- String: 1d object moving in D space-time dimensions

World-sheet



Its evolution is given by 2d theory on the world-sheet

$$S = -\frac{1}{4\pi\alpha'}\int \mathrm{d}\sigma \mathrm{d}\tau \sqrt{\gamma}\,\gamma^{\alpha\beta}\eta_{MN}\partial_\alpha X^M \partial_\beta X^N$$

X^M (σ,τ): space-time coordinates of the string

$\gamma^{\alpha\beta}$: world-sheet metric

η_{MN} : Minkowski metric in space-time

$\frac{1}{2\pi\alpha'} = \frac{1}{2\pi l_s^2} = T$: string tension

- Action gives equations of motion

$$\left(\frac{\delta^2}{\delta\sigma^2} - \frac{\delta^2}{\delta\tau^2}\right) X^M(\tau,\sigma) = 0 \quad \longrightarrow \quad X^M(\tau,\sigma) \sim \underbrace{\sum_n \tilde{\alpha}_n^M e^{2in\overset{\tau-\sigma}{\sigma^-}}}_{X_R^M(\sigma^-)} + \underbrace{\sum_n \alpha_n^M e^{2in\overset{\tau+\sigma}{\sigma^+}}}_{X_L^M(\sigma^+)}$$

+ bdy cond: $X^M(\tau, 2\pi)=X^M(\tau,0)$

- Action can be quantized

α_n^M : creation operators

α_{-n}^M: anihilation operators

$$\longrightarrow \quad [\alpha_n^P, \alpha_m^Q] = n\delta_{m+n}\eta^{PQ}$$

- Quantized states of mass

$$M^2 = \frac{2}{\alpha'}\left(\sum_{n=1}^{\infty} \alpha_n \cdot \alpha_{-n} + \tilde{\alpha}_n \cdot \tilde{\alpha}_{-n} - 2\right)$$

- Massless states

center of mass momentum -- $\vec{k}^2 = 0 \Rightarrow \exists\,\text{frame s.t. } \vec{k} = (k, k, 0, ..., 0)$

$\zeta_M \tilde{\zeta}_N \alpha_1^M \tilde{\alpha}_1^N |0; k>$ Positive norm if $\zeta \cdot k = \tilde{\zeta} \cdot k = 0$: states classified by SO(D-2) representation

$$2\,\zeta_M \tilde{\zeta}_N = \underbrace{\zeta^0_{\{MN\}}}_{\text{graviton!}} + 2\,\underbrace{\zeta_t}_{\text{dilaton}}\,\eta_{MN} + \underbrace{\zeta_{[MN]}}_{\text{B -field}}$$

$$S = -\frac{1}{4\pi\alpha'}\int \mathrm{d}\sigma\mathrm{d}\tau\sqrt{\gamma}\,\gamma^{\alpha\beta}\eta_{MN}\partial_\alpha X^M\partial_\beta X^N$$

$$\eta_{MN} \to \eta_{MN} + \underbrace{h_{MN}}_{\propto\, \zeta_{MN}}$$

Insert in path integral

$$Z = \int \mathcal{D}X\mathcal{D}\gamma e^{-S} = \int \mathcal{D}X\mathcal{D}\gamma e^{-S_0}\left(1 + \frac{1}{4\pi\alpha'}\int \mathrm{d}\sigma\mathrm{d}\tau\sqrt{\gamma}\,\gamma^{\alpha\beta}h_{MN}\partial_\alpha X^M\partial_\beta X^N + \ldots\right)$$

coherent state of gravitons = curved background

• Consider the "σ-model" action

$$S = -\frac{1}{4\pi\alpha'}\int \mathrm{d}\sigma\mathrm{d}\tau\sqrt{\gamma}\,\left(\gamma^{\alpha\beta}\underbrace{g_{MN}}_{\text{metric}}(X) + i\epsilon^{\alpha\beta}\underbrace{B_{MN}}_{\text{B-field}}(X)\right)\partial_\alpha X^M\partial_\beta X^N + \alpha'\underbrace{\Phi}_{\text{dilaton}}R$$

Conformal invariance

$$T^\alpha_\alpha = \frac{1}{\sqrt{-\gamma}}\frac{\delta S}{\delta\gamma_{\alpha\beta}}\gamma_{\alpha\beta} = 0$$

$$-2\alpha' T^{\alpha}{}_{\alpha} = \beta^g_{MN}\gamma^{\alpha\beta}\partial_\alpha X^M \partial_\beta X^N + i\beta^B_{MN}\epsilon^{\alpha\beta}\partial_\alpha X^M \partial_\beta X^N + \beta^\Phi R$$

where...

$$\beta^g_{MN} = \alpha' \left(R_{MN} + 2\nabla_M \nabla_N \Phi - \frac{1}{4} H_{MPQ} H_N{}^{PQ} \right) + \mathcal{O}(\alpha'^2)$$ → EOM for metric: Einstein's eq.

$$\beta^B_{MN} = \alpha' \left(-\frac{1}{2} \nabla^P H_{PMN} + \nabla^P \Phi H_{PMN} \right) + \mathcal{O}(\alpha'^2)$$ → EOM for B-field: general. of Maxwell's eq.

$$\beta^\Phi = \alpha' \left(\frac{D-26}{\alpha'} - \frac{1}{2}\nabla^2 \Phi + \nabla_P \nabla^P \Phi - \frac{1}{24} H_{MNP} H^{MNP} \right) + \mathcal{O}(\alpha'^2)$$ → EOM for dilaton: D = 26 !

$H_3 = dB_2$ is the field-strength of the B-field

- Conformal invariance of 2d theory ⇒

→ Fixes space-time dimension: D = 26

→ Gives EOM for massless fields to lowest order in α'

→ Can be derived from effective space-time action

$$S = \frac{1}{\kappa_0} \int d^{26}X \sqrt{-g} e^{-2\Phi} \left(R + 4\nabla_M \Phi \nabla^M \Phi - \frac{1}{12} H_{MNP} H^{MNP} + \mathcal{O}(\alpha') \right)$$

- Gravity is described by coherent state of massless closed strings
- Gauge fields are described by coherent states of massless open strings:

$X^M(\tau,\sigma) \sim \sum_n \alpha_n^M e^{in\tau} cos(n\sigma)$ (Neumann bdy cond $\partial_\sigma X^M(\tau,0) = \partial_\sigma X^M(\tau,\pi) = 0$)

Massless state $\zeta_M \alpha_{-1}^M |0; k> \rightarrow$ gauge field A_M

- Where is the matter? (electrons, quarks...) → space-time fermions

$\Downarrow$

need world-sheet fermions

Ψ^M superpartner of X^M

supersymmetry ➡ superstring

World-sheet action → EOM + 2 possible bdy conditions

Ramond -R- (integer modes)

$$\Psi^M(\tau,0) = \pm\Psi^M(\tau,2\pi) \qquad \{\psi_r^M, \psi_s^N\} = \{\tilde\psi_r^M, \tilde\psi_s^N\} = \eta^{MN}\delta_{r+s}$$

Neveu - Schwarz -NS- (half integer modes)

$\boxed{\text{Conformal anomaly cancellation} \Rightarrow D = 10}$

- Quantized states of mass

0 for R, 1/2 for NS

$$M^2 = \frac{1}{\alpha'}\left(\sum_{n=1}^{\infty} \alpha_n \cdot \alpha_{-n} + \sum_r r\psi_r \cdot \psi_{-r} - a + \text{same with tilde}\right)$$

- Massless states

$$\underbrace{\Psi_0^M \tilde{\Psi}_0^N}_{\substack{\text{R}\otimes\text{R} \\ \text{boson}}} |0, k> \qquad \underbrace{\Psi_{1/2}^M \tilde{\Psi}_0^N}_{\substack{\text{NS}\otimes\text{R} \\ \text{fermion}}} |0, k> \qquad \underbrace{\Psi_{1/2}^M \tilde{\Psi}_{1/2}^N}_{\substack{\text{NS}\otimes\text{NS} \\ \text{boson}}} |0, k>$$

$\{\psi_r^M \psi_s^N\} = \eta^{MN} \delta_{r+s} \Rightarrow$ ψ_0 obeys Clifford algebra $\Rightarrow \psi_0{}^M \cong \Gamma^M$

$\zeta_M \cdot k = 0$

Ramond ground states form a representation of gamma matrix algebra $|\mathrm{s}> = |\pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}\rangle$

space-time fermion

$\mathbf{16} \rightarrow \mathbf{8}_s + \cancel{\mathbf{8}_c}$

to get space-time SUSY (GSO)

NS: $\Psi_{1/2}^M |0, k>$ transforms in $\mathbf{8}_v \longrightarrow$ space-time boson

Closed strings massless spectrum

$$\underbrace{\Psi_0^M \tilde{\Psi}_0^N}_{\substack{\text{R}\otimes\text{R} \\ \text{boson}}} |0, k> \qquad \underbrace{\Psi_{1/2}^M \tilde{\Psi}_0^N}_{\substack{\text{NS}\otimes\text{R} \\ \text{fermion}}} |0, k> \qquad \underbrace{\Psi_{1/2}^M \tilde{\Psi}_{1/2}^N}_{\substack{\text{NS}\otimes\text{NS} \\ \text{boson}}} |0, k>$$

R⊗R : type IIB $\quad \mathbf{8}_s \otimes \mathbf{8}_s = [0] \oplus [2] \oplus [4]_+ = \mathbf{1} \oplus \mathbf{28} \oplus \mathbf{35}_+ = C_0 \oplus C_2 \oplus C_{4+}$

type IIA $\quad \mathbf{8}_s \otimes \mathbf{8}_c = [1] \oplus [3] = \mathbf{8_v} \oplus \mathbf{56_t} = C_1 \oplus C_3$

type IIB even RR potentials

type IIA odd RR potentials

NS⊗R: type IIB $\quad \mathbf{8}_v \otimes \mathbf{8}_s = \mathbf{8}_s \oplus \mathbf{56}_s = \underbrace{\lambda}_{\text{dilatino}} \oplus \underbrace{\Psi^M}_{\text{gravitino}}$

type IIA $\quad \mathbf{8}_v \otimes \mathbf{8}_c = \mathbf{8}_c \oplus \mathbf{56}_c$

R⊗NS type IIB $\quad \mathbf{8}_s \otimes \mathbf{8}_v = \mathbf{8}_s \oplus \mathbf{56}_s$

type IIA $\quad \mathbf{8}_s \otimes \mathbf{8}_v = \mathbf{8}_s \oplus \mathbf{56}_s$

type IIB two dilatinos & gravitinos of same chirality

type IIA two dilatinos & gravitinos of opposite chirality

NS⊗NS types IIA & IIB $\quad \mathbf{8}_v \otimes \mathbf{8}_v = [0] \oplus [2] \oplus \{2\} = \mathbf{1} \oplus \mathbf{28} \oplus \mathbf{35} = \underbrace{\Phi}_{\text{dilaton}} \oplus \underbrace{B_2}_{\text{B-field}} \oplus \underbrace{G_{MN}}_{\text{metric}}$

Dilaton, B-field and metric for type IIB and IIA

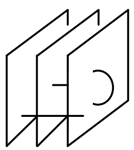
Open string spectrum

Massless state $\psi^M_{1/2}|0;k>$ → gauge field A_M

$\psi^M_0|0;k>$ → fermion $|\mathbf{s}>$

Dirichlet boundary conditions -> fixed extrema: attached to D-brane

Open strings: can have MSSM spectrum:



Open strings D3

→ U(N) gauge field & gaugino ($\psi^\mu{}_{1/2}$, $\psi^\mu{}_0$)
(excitations along brane)

→ Matter (boson & fermion) in adjoint of U(N): ($\psi^m{}_{1/2}$, $\psi^m{}_0$)
(excitations orthogonal to brane)

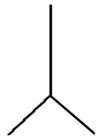
Stacks of D-branes
D-branes at singularity

➡ $U(N) \rightarrow SU(3) \times SU(2) \times U(1)$
chiral matter of MSSM

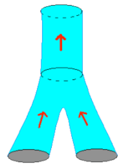
Superstring theory

- Gravity: ∃ graviton, interaction at low energy reduces to general relativity
- Consistent theory of quantum gravity: interaction of 1 dim object is smeared out
  no nonrenormalizable divergencies

point particle interaction



string interaction



stringy loop expansion

- Supersymmetry
- String coupling constant: VEV of dilaton
- Chiral matter & Grand unification: MSSM comes from $U(N) \rightarrow SU(3) \times SU(2) \times U(1)$
- Extra dimensions
- Uniqueness

Consider $M_{10}=M_9 \times S^1$

Momentum along $S^1 = n/R$

Bdy condition $X^M(\tau, 2\pi)=X^M(\tau,0)+ 2\pi mR$

$$M^2 = \frac{n^2}{R^2} + \frac{m^2R^2}{\alpha'^2} + \frac{2}{\alpha'}(N+\tilde{N}-2)$$

symmetric under

$n \leftrightarrow m$

$R \leftrightarrow \frac{\alpha'}{R}$!

T duality

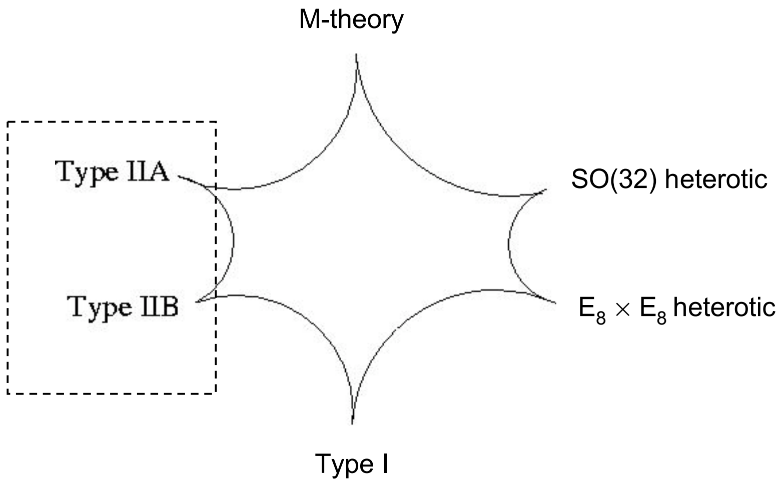
$$X_L + X_R \leftrightarrow X_L - X_R$$

$\mathbf{8}_s \leftrightarrow \mathbf{8}_c$ for right movers

IIA and IIB are related by T duality

IIA ⇔ IIB

Other “theories” -- all related by dualities



Compactifications of IIA and IIB in the low energy limit ($\alpha' \to 0$)

- Massless states

NSNS
→ Graviton g_{MN}
→ B-field B_{MN}
→ Dilaton ϕ

RR
→ $C_1 + C_3$ in type IIA
→ $C_0 + C_2 + C_{4+}$ in type IIB

NSR
RNS
→ $\psi^{1M}_{\alpha}, \psi^{2M}_{\dot\alpha}, \lambda^1_{\alpha}, \lambda^2_{\dot\alpha}$ in type IIA
→ $\psi^{1M}_{\alpha}, \psi^{2M}_{\alpha}, \lambda^1_{\alpha}, \lambda^2_{\alpha}$ in type IIB

- ~~Tower of massive states~~ -- $M^2 = \frac{n}{\alpha'}$

- Conformal anomaly → space-time equations of motion
  → effective space-time action : $\mathcal{N} = 2$ supersymmetry in 10D

- Look for solutions to the equations of motion

  → $M_{10} = M_4 \times M_6$
  → preserve some supersymmetry: guaranteed to be stable
  → have background internal fluxes

Where do these fluxes come from?

- In 4d: A_μ ($\equiv A_1$) potential for the EM field.
  $F_{\mu\nu}=\partial_{[\mu}A_{\nu]}$ ($F_2=dA_1$) field strength.

  Electric and magnetic sources for A_1 are 0-dimensional

Magnetic flux: $\int_{S^2} F_2 = m$

Electric flux: $\int_{S^2} *F_2 = e$

surrounds source

Consider A_2 : $dA_2=F_3= * F_1= * dA_0$

- In type II theories, 10d

$C_0 \to F_1$
$C_1 \to F_2$
$C_2 \to F_3$ IIBA
$C_3 \to F_4$
$C_4 \to F_5$
RR

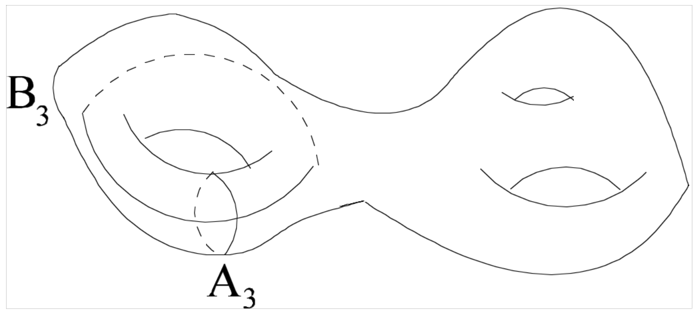
$F_3 \to$ electric source D1-brane, magnetic source D5-brane

$H_3 \to$ electric source F1-string, magnetic source NS5-brane   IIA and IIB
NSNS

Fluxes for F_3 and H_3: $\int_{A_i^3} F_3 = m_i^F \qquad \int_{B_i^3} F_3 = e_i^F$

i=0,…,$h^{2,1}$

$h^{2,1}$: topological data of manifold



Consistent compactifications with $(m^i,e^i) \neq 0$

Supersymmetric solutions with fluxes

SUSY vacuum: $< 0 \mid \{Q_\alpha, \chi_\beta\} \mid 0 > \; = 0 = \; < 0 \mid \delta_{\varepsilon_\alpha} \chi_\beta \mid 0 >$

supercharge

fermionic field

Fermionic fields: • gravitino $\psi^i_M \rightarrow \; < \delta_\varepsilon \psi^i_M > = 0$

• dilatino $\lambda^i \rightarrow \; < \delta_\varepsilon \lambda^i > = 0$

$$\delta_\epsilon \psi_M = \nabla_M \epsilon + H_{Mnp} \gamma^{np} \epsilon + e^\phi \sum_n \not{F}_n \gamma_M \epsilon \,, \qquad \not{F}_n = F_{a_1 \ldots a_n} \gamma^{a_1 \ldots a_n}$$

n=1,3,5 for IIB

• Linear in fields and $< 0 \mid \{Q,Q\} \mid 0 >$ are quadratic)

• Without fluxes $\delta \psi_M = 0 \Rightarrow \nabla_M \epsilon = 0$

Candelas, Horowitz, Strominger, Witten 85

Sol to SUSY + Bianchi + EOM for the fluxes ($dF = d{*}F = 0$) $\rightarrow$ sol EOM

$\mathcal{M}_{10} = \mathcal{M}_4 \times \mathcal{M}_6 \rightarrow g_{MN} = \begin{pmatrix} \eta_{\mu\nu} & 0 \\ 0 & g_{mn} \end{pmatrix}$

$\epsilon = \theta \otimes \eta \rightarrow \nabla_M \epsilon = 0$

- $\partial_\mu \theta = 0 \,, \; \theta$ is a constant spinor in 4d
- $\nabla_m \eta = 0 \,, \; \eta$ is a covariantly constant spinor on $\mathcal{M}_6$

$\Downarrow$

$\mathcal{M}_6$ has reduced holonomy: SU(3)

$\mathcal{M}_6$ is Calabi-Yau

Hol 6d manifold Riemannian $\subseteq$ SO(6)

Smaller holonomy group, more special manifold

SUSY $\rightarrow$ Hol $\mathcal{M}_6 \subseteq$ SU(3)

	SO(6)	$\rightarrow$	SU(3)	
vector	6	$\rightarrow$	$3+\bar{3}$	
spinor	4	$\rightarrow$	3+1	η
A_2	15	$\rightarrow$	8+3+3+1	J
A_3	10	$\rightarrow$	6+3+1	Ω

$$\begin{array}{l} \epsilon^1 = \theta^1 \otimes \eta \\ \epsilon^2 = \theta^2 \otimes \eta \end{array} \rightarrow \mathcal{M}_{10} = \mathcal{M}_4 \times CY_6 : \mathcal{N} = 2 \text{ in 4d}$$

Moduli space of CY compactifications

- $\int_{A_i^3} \Omega_3 = z^i$

  i=0,..,$h^{2,1}$

  $h^{2,1}$: topological data of manifold

  $\Omega \rightarrow \lambda\Omega$
  $z^i \rightarrow \lambda z^i$ ➡
  $g_{mn} \rightarrow |\lambda|^2 g_{mn}$

  z^i : i=1,..,$h^{2,1}$
  complex structure moduli
  "shape" moduli
  (i=0: overall volume)

- $\int_{w_a^2} J_2 = \int_{w_a^2} \sqrt{g} = v^a$

  a =1,..,$h^{1,1}$

  $h^{1,1}$: topological data of manifold

  v^a volume of 2-cycles ➡

  v^a : a =1,…,$h^{1,1}$
  Kähler moduli
  "size" moduli

- Dilaton $\phi \rightarrow \langle e^{\phi} \rangle = g_s$

- $h^{2,1}$ shape moduli - $h^{1,1}$ size moduli - 1 dilaton

Typical $h^{2,1}$, $h^{1,1}$ ~ 100 $\Rightarrow$ 200 massless scalars in 4d effective theory!

Need to understand mechanism of moduli fixing

Turn on fluxes

$$\delta\psi_M = \nabla_M\epsilon + H_{Mnp}\gamma^{np}\epsilon + e^{\phi}\, \not{F}_1\gamma_M\epsilon + e^{\phi}\, \not{F}_3\gamma_M\epsilon + e^{\phi}\, \not{F}_5\gamma_M\epsilon$$

Contributions from H_3 and F_3 cancel if

MG, Polchinski 00

$$e^{\phi} *_6 F_3 = H_3$$
$$\Downarrow$$
$$e^{\phi} e_i^F = m_i^H$$

and $e_0^F = m_0^H = 0$

If $F_1 = F_5 = 0$ → $\nabla_M\epsilon = 0$ → CY

But Bianchi id for F_5: d F_5 = d $*F_5$ = $H_3 \wedge F_3 \neq 0$ → Fluxes act like electric-magnetic source
EOM (effective D3-charge)

Einstein's eq : $R_{\mu\nu} = g_{\mu\nu} H_{mnp} F^{mnp}$ → Fluxes have effective tension

↓

$$F_5 = (1+*)Vol_4 \wedge dA \quad \text{and} \quad g_{MN} = \begin{pmatrix} e^{2A(y)}\eta_{\mu\nu} & 0 \\ 0 & e^{-2A(y)}\tilde{g}_{mn}(y) \end{pmatrix}$$

$\mathcal{M}_{10} = \mathcal{M}_4 \times_w \mathcal{M}_6$
↓
$\mathcal{M}_6$ conformal CY

SUSY $\begin{pmatrix} \delta\psi_M^1 \\ \delta\psi_M^2 \end{pmatrix} = \nabla_M \begin{pmatrix} \epsilon^1 \\ \epsilon^2 \end{pmatrix} + \not{F}_5\gamma_M \begin{pmatrix} \epsilon^2 \\ \epsilon^1 \end{pmatrix} = 0$ → $\epsilon^1 = \epsilon^2$
↓
$\mathcal{N}$=2 → $\mathcal{N}$=1

↓

$\epsilon = \theta \otimes e^{A/2}\eta$ → $\tilde{\nabla}_m\eta = 0$ → $\tilde{g}_{mn}$ is CY !

Easiest solution with fluxes
Apply powerful tools of CY

Turning on $e_0^F = m_0^H$ → Break SUSY completely in a stable way (solution to EOM)

Applications

Moduli stabilization

Giddings, Kachru, Polchinski 01

Moduli fixing from susy conditions:

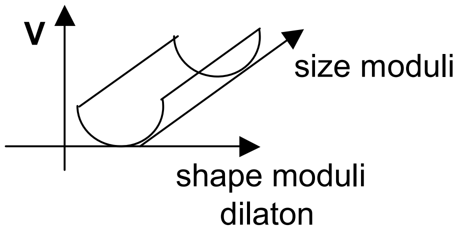
Given $H_3\,, F_3$ $\rightarrow$ $\exists A_i\,, B_i$ such that $e^{\phi} e_i^F = m_i^H$ is satisfied ?

$$\int_{A_i^3} H_3 = m_i^H$$
$$\int_{B_i^3} F_3 = e_i^F$$

- Fixes relative size and orientation of $A_i\,, B_i$ $\rightarrow$ complex structure fixed "shape" moduli

- Fixes dilaton

Fluxes induce potential for moduli



➡ dilaton and shape moduli stabilized
size moduli unfixed
"no-scale" solution

Size moduli can be fixed by non-perturbative effects

Most general solution with fluxes

• GP class ($e^{\phi} *F_3 = H_3$)correponds to "minimal" back-reaction of fluxes on $\mathcal{M}_6$ (CY $\rightarrow$ conformal CY)

What is the most general susy solution with fluxes ?

• Susy requires topological condition on $\mathcal{M}_6$

Gauntlett, Martelli, Pakis, Waldram 02

Only H_3: $\delta\psi_m = \nabla_m \eta + H_m \eta = 0 \quad (H_m = H_{mnp}\gamma^{np})$

$\nabla' = \nabla + H$

η is covariantly constant
in a connection with torsion
torsion $\leftrightarrow$ flux

∇' has SU(3) holonomy

➡ $\boxed{\mathcal{M}_6 \text{ has SU(3) structure}}$

SU(3) invariant
spinor η not cov const in LC ∇
2- form J ➝ cov const in ∇'
3-form Ω $\nabla J \sim W J$
$d\Omega \neq 0$, $dJ \neq 0$

W and H are decomposed in SU(3) representations

SUSY allows to turn on only certain SU(3) representations of H ~ W

Fluxes:

Only flux in certain SU(3) representations turned on. For ex. IIB no singlets for F_3 and H_3: $e_0 = m_0 = 0$

Manifold:

SU(3) structure is a necessary condition (topological requirement). Can we get sufficient conditions (differential) ?

Most general solution with fluxes (conditions on $\mathcal{M}_6$)

No fluxes: $\mathcal{M}_6$ CY
- complex (define complex coordinates in patches,or almost complex structure. If integrable -> complex)
- symplectic ($\exists$ nondegenerate closed 2-form: $dJ_2=0$ - Integrable symplectic structure)

$$\delta_\epsilon \psi_M = \nabla_M \epsilon + H_{Mnp}\gamma^{np}\sigma^3\epsilon + e^\phi \sum_n \not{F}_n \gamma_M \mathcal{P}_n \epsilon \,,$$

$$\sim W\epsilon + \quad H\,\epsilon \quad + \quad F\,\epsilon \quad = 0$$

SU(3) Torsion:

$dJ = \mathrm{Im}\,(W_1\,\Omega) + W_4 \wedge J + W_3$

$d\,\Omega = W_1\,J^2 + W_5 \wedge \Omega + W_2 \wedge J$

Torsion + fluxes = 0

Cancelation works representation by representation

	$1 \oplus 1$	$3 \oplus 3$	$6 \oplus 6$	$8 \oplus 8$
Torsion	1 (W_1)	2 (W_4, W_5)	1 (W_3)	1 (W_2)
H_3	1	1	1	0
IIA: F_{2n}	2 (F_0, F_2, F_4)	2 (F_2, F_4)	0	1 (F_2, F_4)
IIB: F_{2n+1}	1 (F_3)	3 (F_1, F_3, F_5)	1 (F_3)	0

In IIB $W_2 = 0$ (integrability of complex structure)

In IIA $W_3 \sim H^{(6)}$ (symplectic geometry)

If also $W_1=0$ → IIB: $d\Omega = W_5 \wedge \Omega$ — $\mathcal{M}_6$ is complex

(true in all susy vacua)

IIA: $dJ = W_4 \wedge J + H^{(6)}$ — $\mathcal{M}_6$ is "twisted symplectic"

Is there a mathematical construction that extends complex and symplectic geometry?

Most general solution with fluxes (conditions on $\mathcal{M}_6$)

MG, Minasian, Petrini, Tomasiello 04

No fluxes (or GP solution): $\mathcal{M}_6$ is (conformal) CY

CY
- Complex: define complex coordinates in patches: almost complex structure. If integrable $\Rightarrow$ complex manifold
- Symplectic: $\exists$ nondegenerate closed 2-form: $dJ_2=0$

Found that in most general SUSY solution:

IIB $\rightarrow$ $\mathcal{M}$ is complex
IIA $\rightarrow$ $\mathcal{M}$ is symplectic

➜ To describe IIA and IIB vacua on the same footing need mathematical construction that contains complex and symplectic geometry

Generalized Complex Geometry

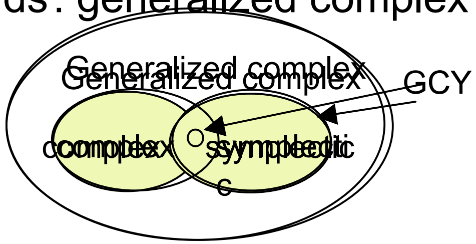
Hitchin 02
Gualtieri 03

tangent bundle   cotangent bundle

Define complex coordinates in $T\mathcal{M} \oplus T^*\mathcal{M}$:12 dimensional space
$X + \xi$

Generalized complex structure

Generalized complex manifolds: generalized complex structure is integrable



Generalized Calabi-Yau is a generalized complex M with additional constraint

All susy vacua are **generalized Calabi-Yau's** !

(Also restrictions on the allowed fluxes)

Generalized Complex Geometry

Hitchin 02
Gualtieri 03

- In almost complex manifolds $\exists$ almost complex structure (ACS)

$\tilde{J}$: $T\mathcal{M} \to T\mathcal{M}$, $\tilde{J}^2 = -1_{d\times d}$ $\to$ $\exists$ basis $\tilde{J} = \begin{pmatrix} +i & 0 \\ 0 & -i \end{pmatrix}$ Projectors: $\pi_{\pm} = (1 \pm i\,\tilde{J})$ project onto holo/antiholo

holomorphic vectors

antiholomorphic vectors

Integrability of ACS (condition for ACS $\to$ CS): $\forall$ X,Y $\in T\mathcal{M}$: $\pi_-[\pi_+ X, \pi_+ Y]=0$ $\Rightarrow$ $\mathcal{M}$ is complex

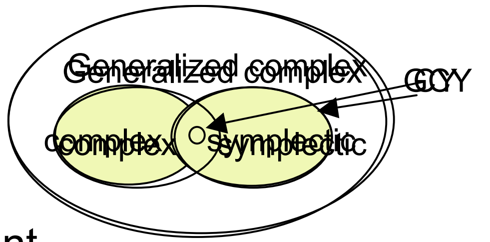
- In Generalized Complex Geometry

$\mathcal{J}: T\mathcal{M} \oplus T^*\mathcal{M} \to T\mathcal{M} \oplus T^*\mathcal{M}$, $\mathcal{J}^2 = -1_{2d\times 2d}$ $\to$ Projectors: $\Pi_{\pm} = (1 \pm i\,\mathcal{J})$

X + ξ

Integrability: $\forall$ $X+\xi$, $Y+\zeta \in T\mathcal{M} \oplus T^*\mathcal{M}$: $\Pi_-[\Pi_+ X+\xi, \Pi_+ Y+\zeta]_C = 0 \Rightarrow \mathcal{M}$ is Generalized Complex

- If $\mathcal{M}$ admits integrable $\tilde{J}$ $\to$ integrable $\mathcal{J} = \begin{pmatrix} \tilde{J} & 0 \\ 0 & -\tilde{J}^t \end{pmatrix}$
- If $\mathcal{M}$ has closed 2-form J $\to$ integrable $\mathcal{J} = \begin{pmatrix} 0 & -J^{-1} \\ J & 0 \end{pmatrix}$



Generalized Calabi-Yau is a generalized complex M with additional constraint

All susy vacua are twisted **generalized Calabi-Yau's** !

Fluxes
- NSNS: twists the differential operator $d \to d + H\wedge$ -- $[\,,\,]_C \to [\,,\,]_{\text{twisted}}$
- RR: act as a defect for integrability of second generalized complex structure

Summary / Open problems

String theory is beautiful and unique (all “theories” are connected by dualities)

It has many solutions!!

Which solution is relevant for us??

one is relevant for us??

Discussed supersymmetric solutions with fluxes

Fluxes
- Break part or all susy
- Stabilize some moduli (in conformal CY solution fluxes fix shape and dilaton moduli)
- Used to construct dS solutions / cosmological solutions
- Applications in AdS/CFT correspondance

Is there a way to fix size moduli perturbatively ?

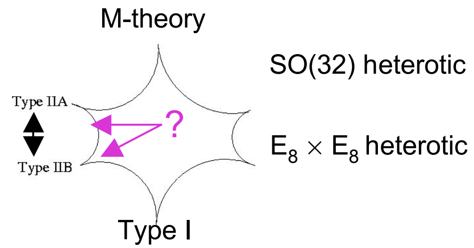
Explicit examples?

Do we have control over corrections ?

Conditions on general susy solution : $\mathcal{M}_6$
- complex IIB
- symplectic IIA

} generalized complex manifolds

Generalized complex geometry is the right tool for systematic decription of flux backgrounds



what are the moduli spaces of GCY ?
adding extra fluxes stabilizes them ?
consistent compactifications ?
susy breaking ?

> Long term goal: understand how Einstein gravity and SM emerge as low-energy limits of string theory
> Flux compactifications seems to be a necessary ingredient in the answer !