Simulating the Universe in a computer

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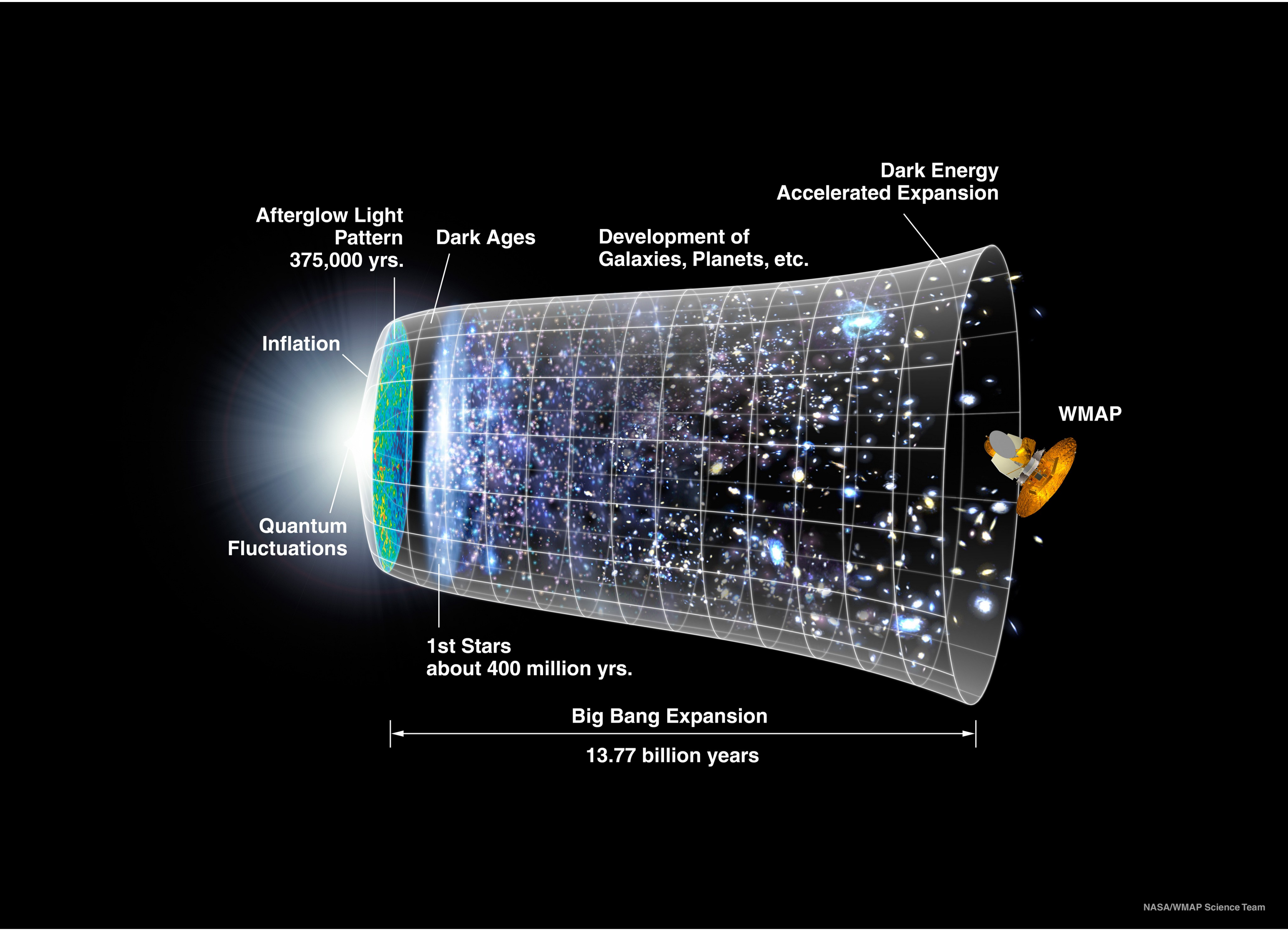
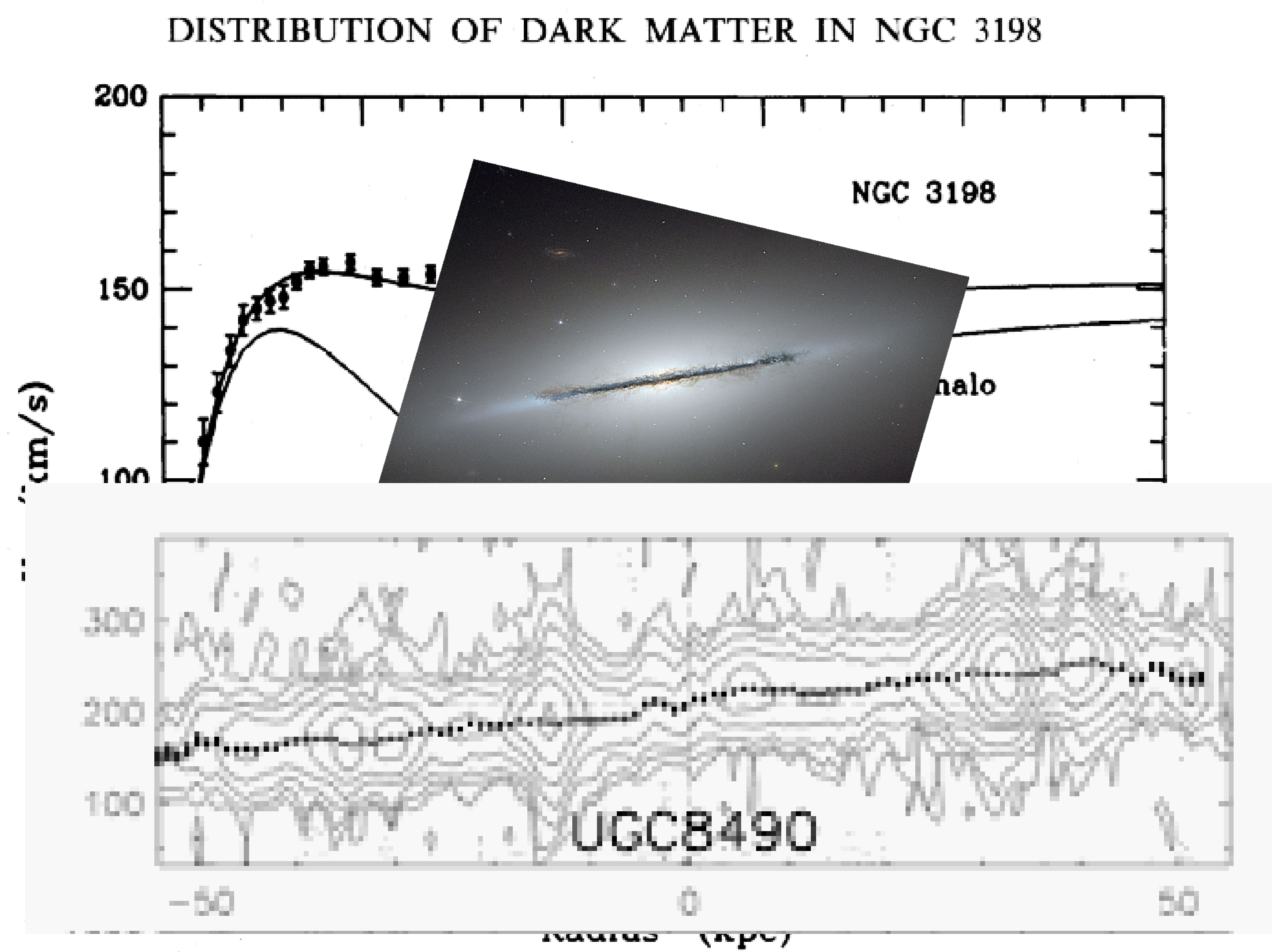
Saclay – September 15th 2014

The Dark Matter Universe

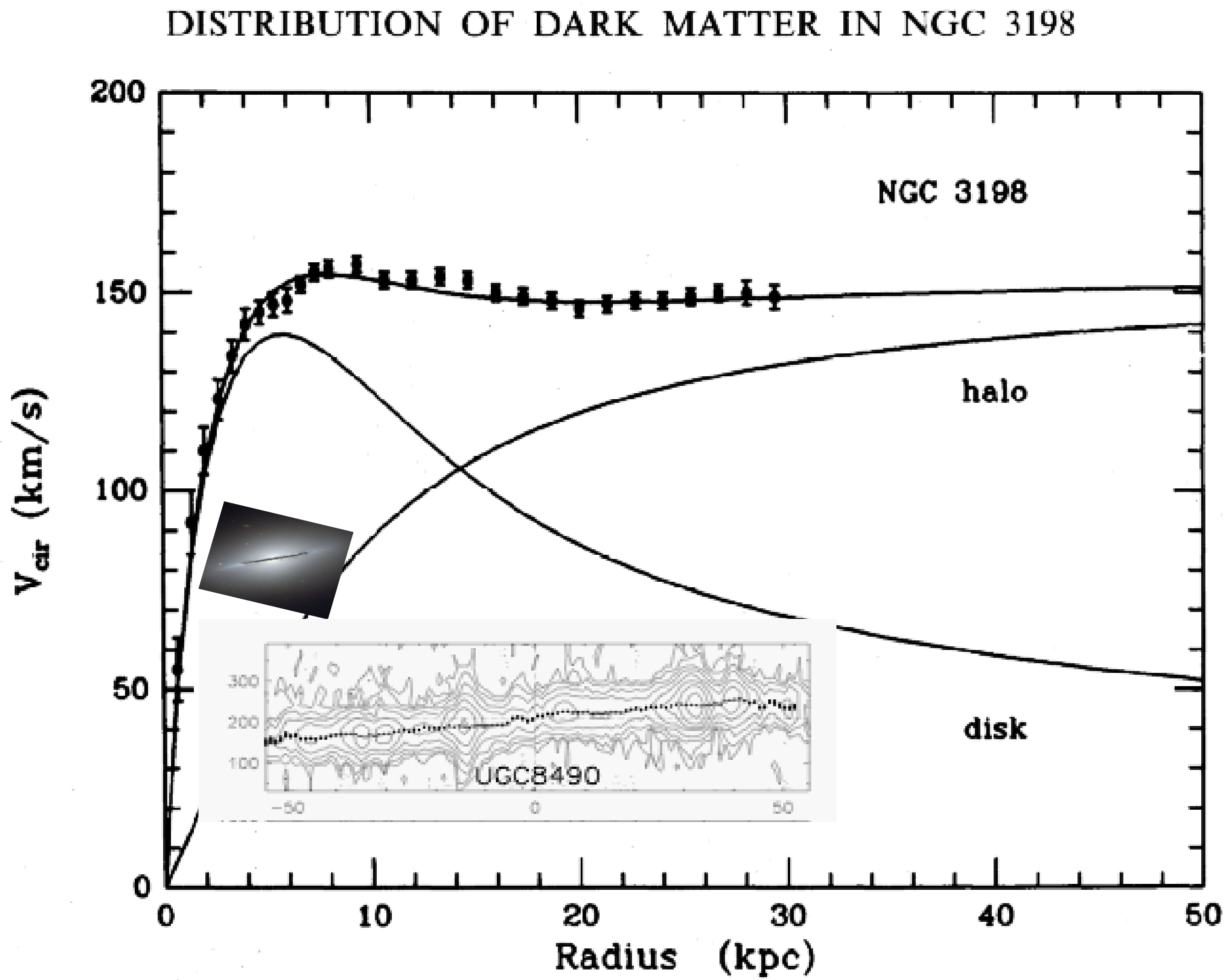
- Why Dark Matter
- Numerical Simulations
  - Initial conditions
  - Nbody technique
- Large Scale Structure
- DM halo – Galaxies connection
- DM halo structure
- Tension with observations


Afterglow Light Pattern 375,000 yrs.
Dark Ages
Development of Galaxies, Planets, etc.
Dark Energy Accelerated Expansion
Inflation
Quantum Fluctuations
WMAP
1st Stars about 400 million yrs.
Big Bang Expansion
13.77 billion years
NASA/WMAP Science Team

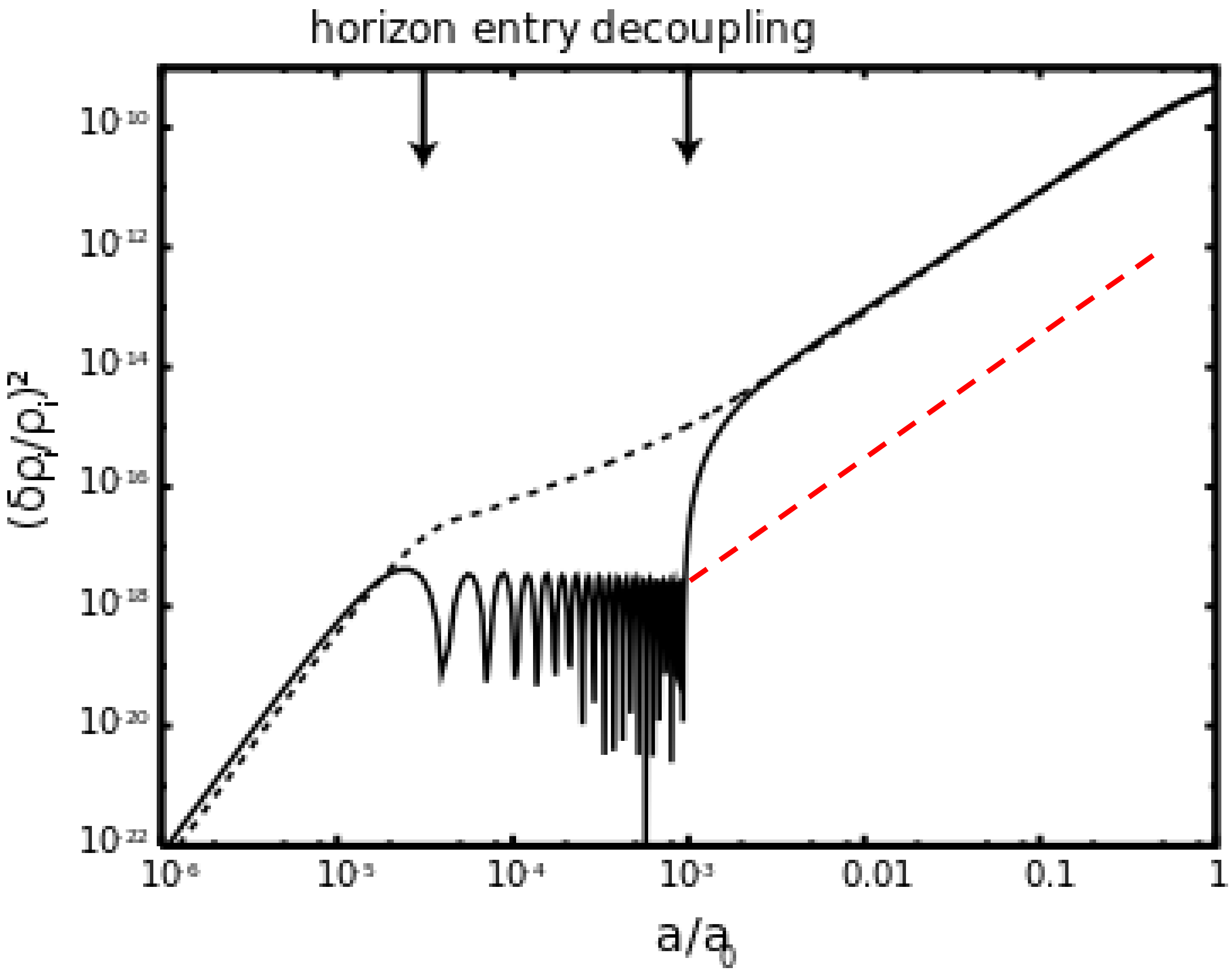
Rotation Curves



Rotation Curves



Why Dark Matter



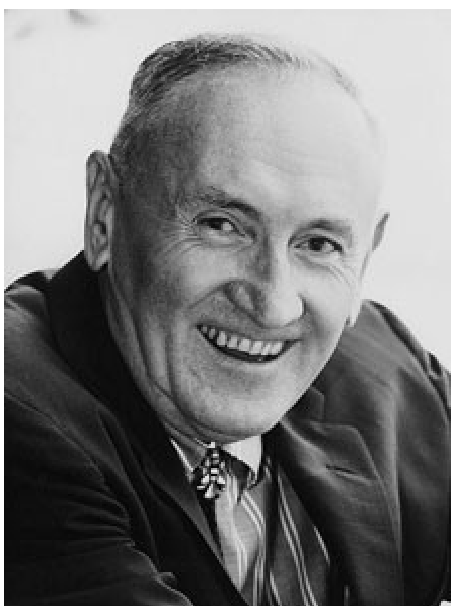
δCMB ~ 10-5
aCMB ~ 1000

δtoday 0.1

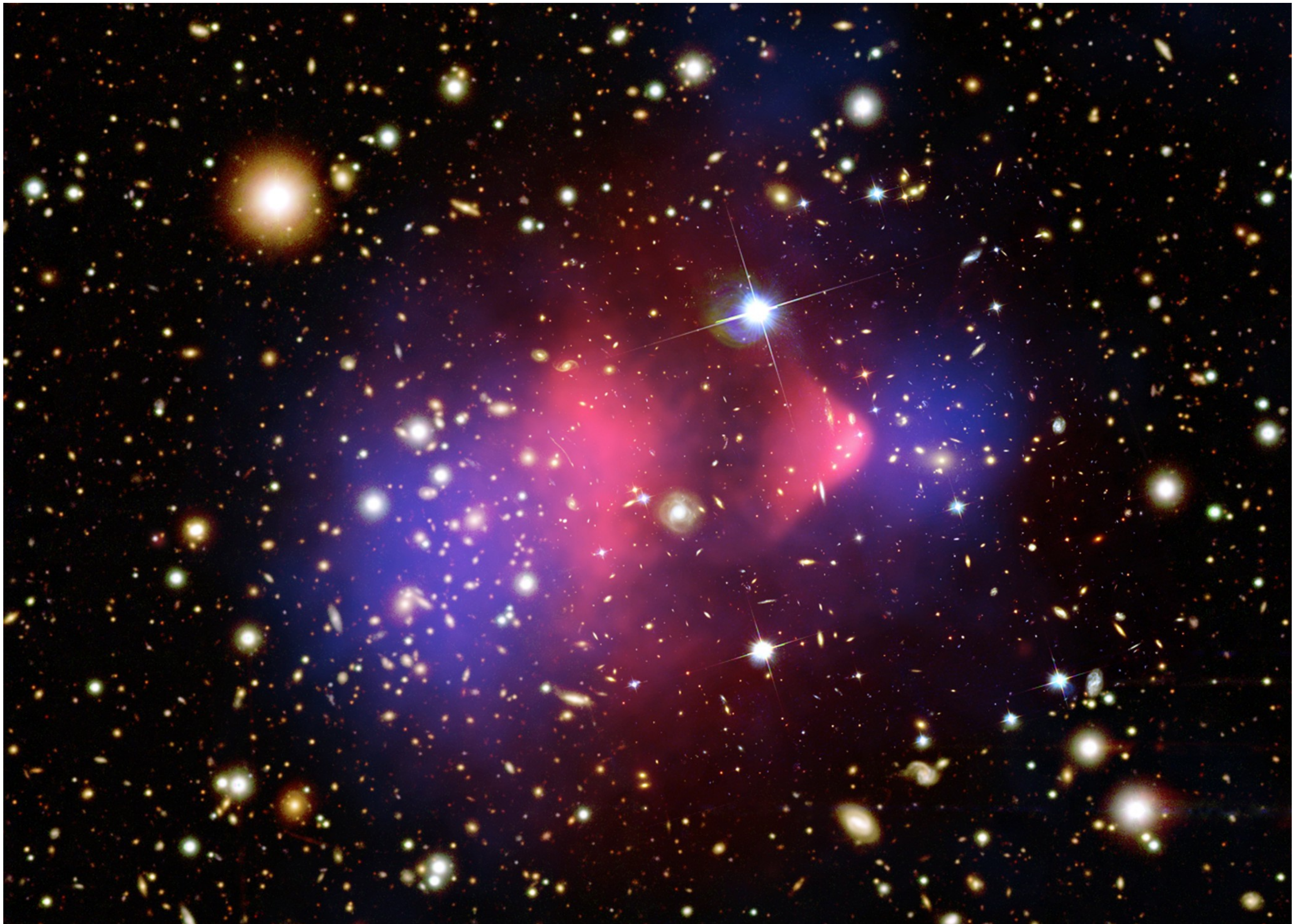
Virial Theorem in Galaxy Clusters

$$\frac{GM}{R} \approx \sigma^2$$

Mvir~500 M*



Lensing and Dark Matter



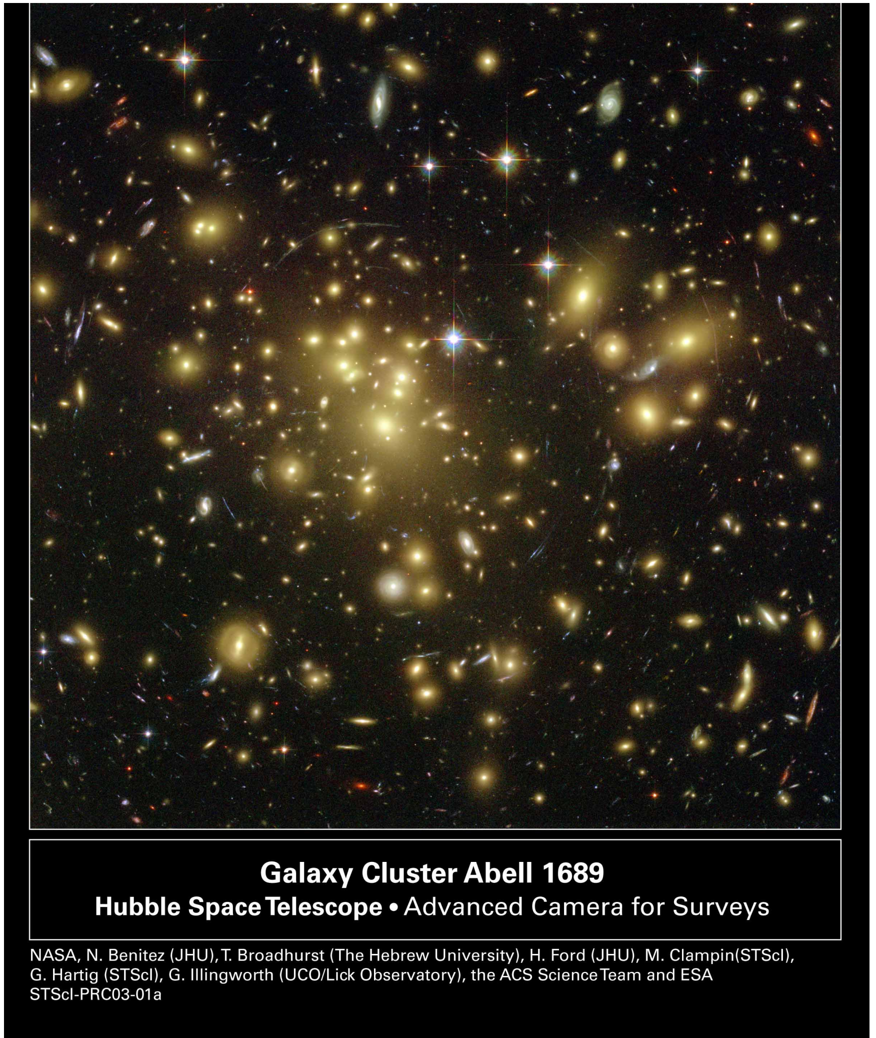
Cosmological Simulations

Why?

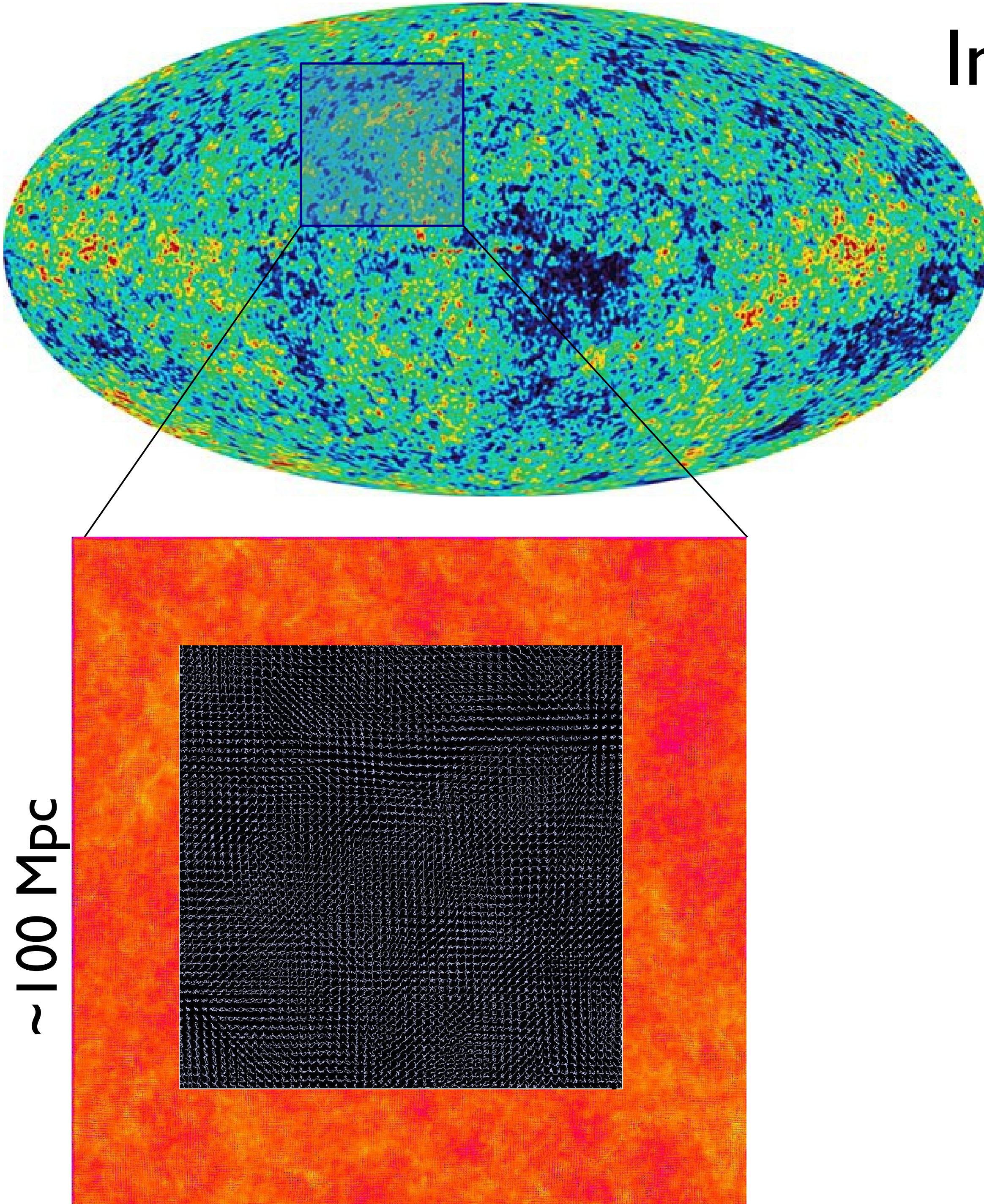
CMB $$\frac{\partial T}{T} \approx \frac{\partial \rho}{\rho} \approx 10^{-5}$$

$$\frac{\partial \rho}{\rho}(cluster\ center) \approx 10^{5}$$

10 orders of magnitude
Highly non linear problem

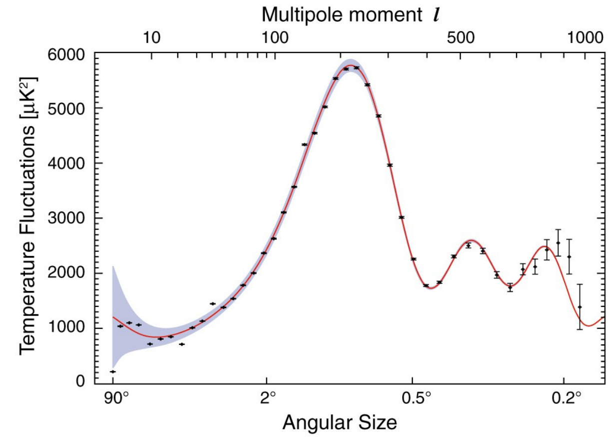


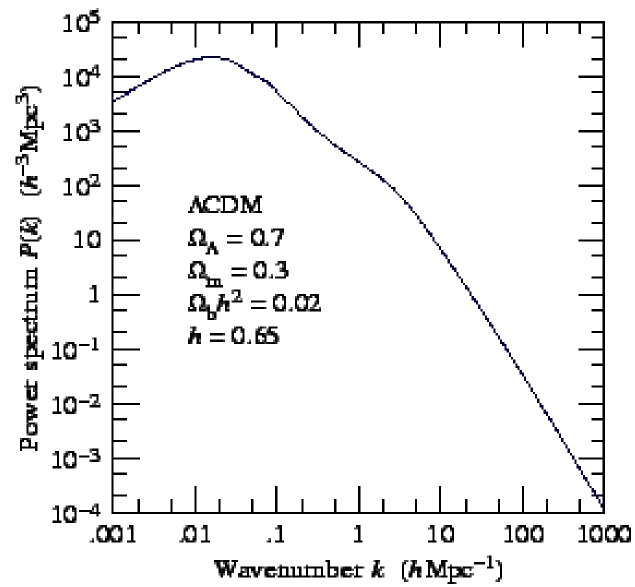
Initial Conditions



~100 Mpc

T ⟷ density

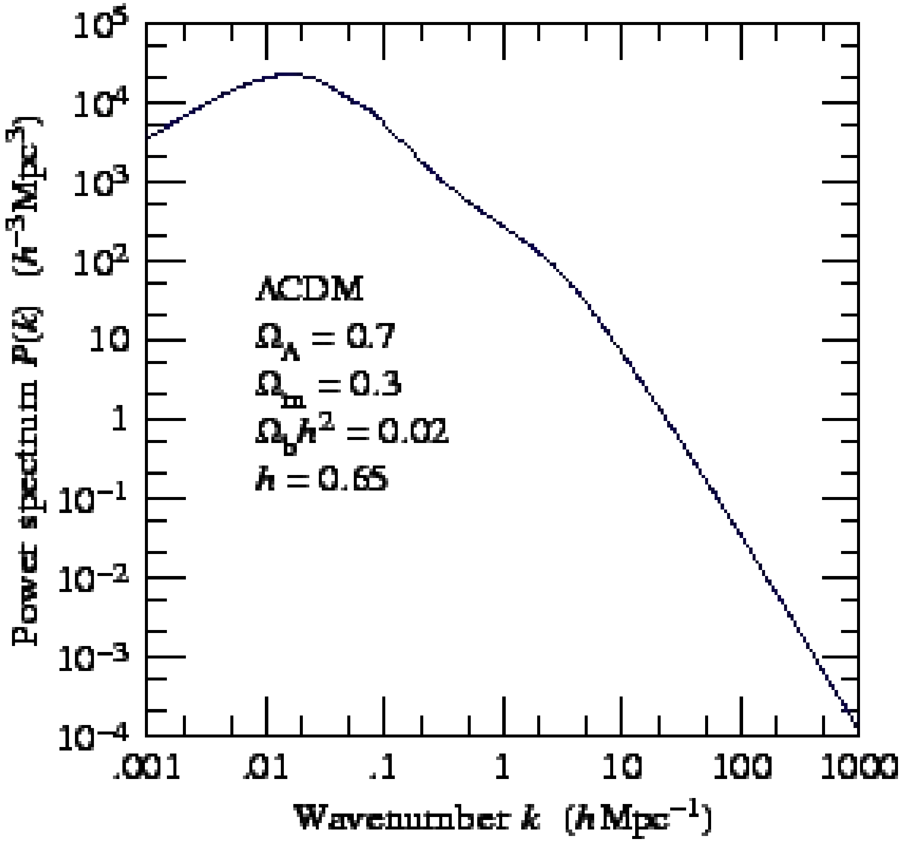


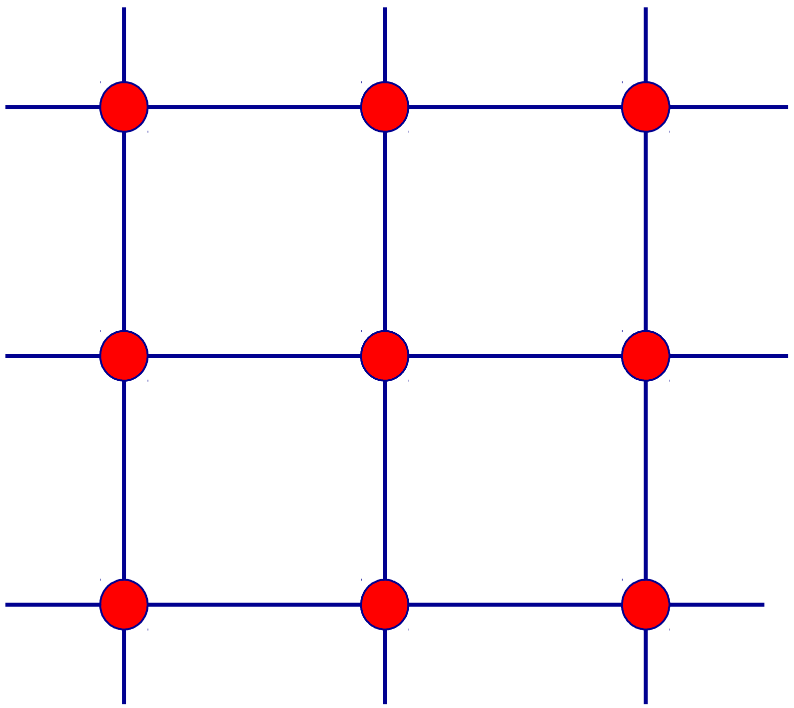


Cosmological Simulations

- CMB provides the initial power spectrum
- Evolve the PS using linear theory till $\delta \sim 1$
- Make a realization of the matter density field
- Use mass elements (particles) to describe the field
- Evolve the density field forward in time into the non linear regime
- Results: Number, position and structure of collapsed objects.

The initial conditions





The initial conditions

The Power Spectrum is defined as:

$$P(k) = Ak^n T^2(k,z)$$

T(k,z) provided by linear theory

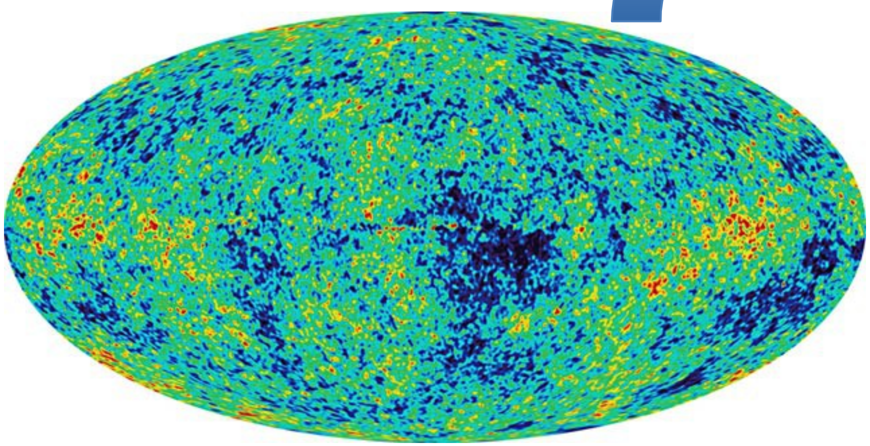
n = directly from CMB

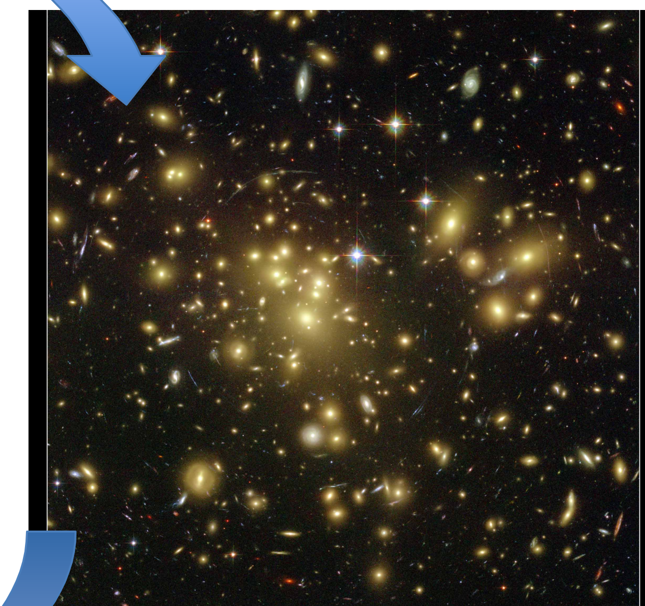
A = Normalization from CMB or σ8

$$\sigma_L(z) = \int Ak^n T^2(k,z) W_L(k) dk$$

The initial conditions - Normalization

CMB quadrupole

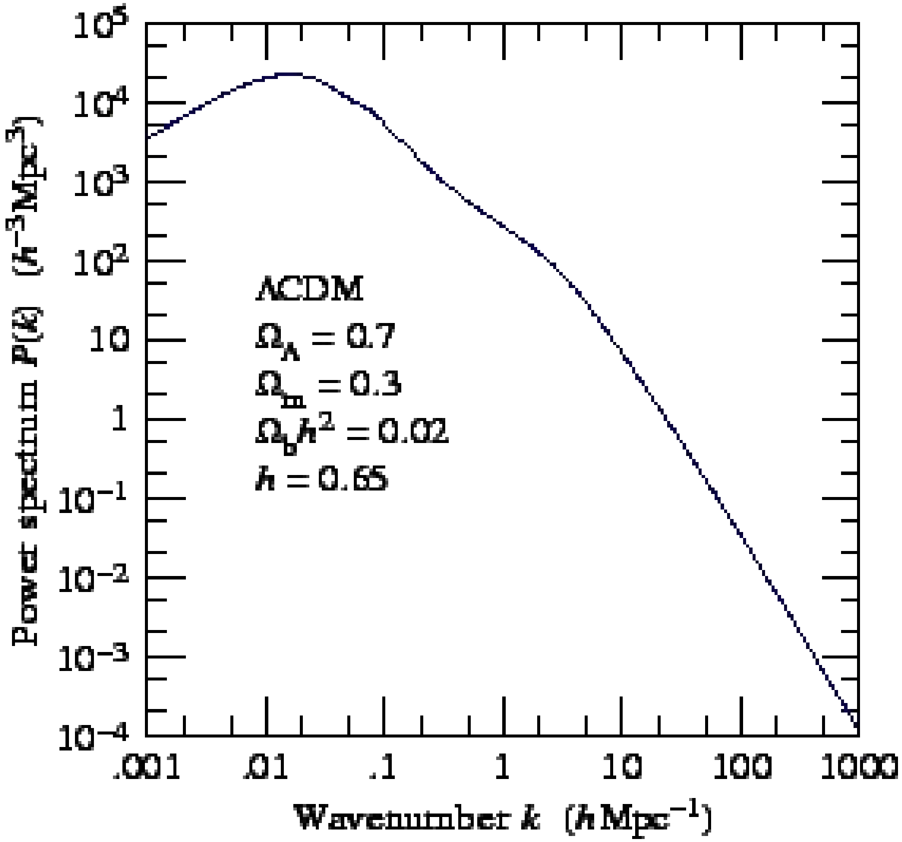


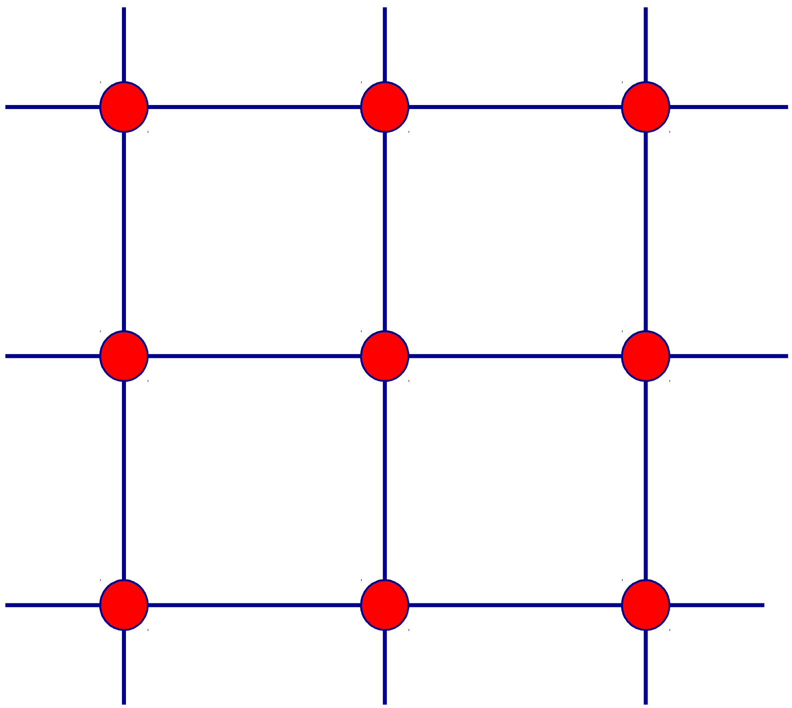


σ8 today

Linear growth factor D(Ωm, ΩL)

The initial conditions





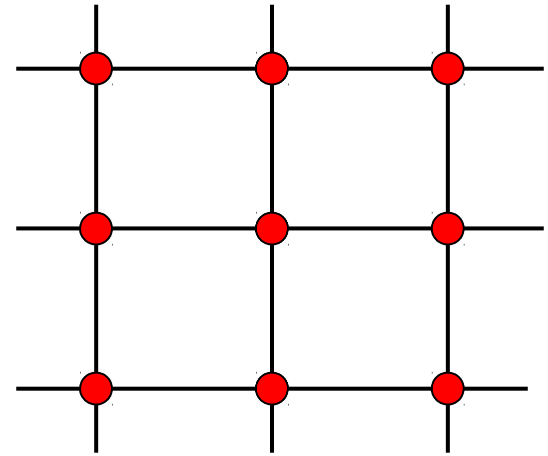
We want to imprint the PS on the initial distribution of particles

The Zeldovich approximation

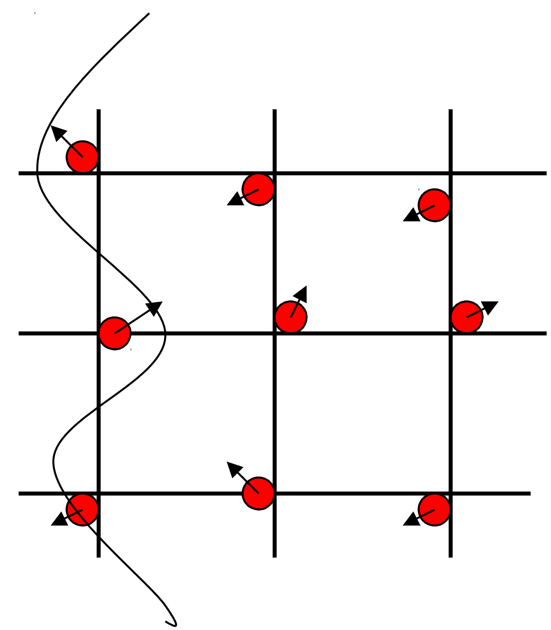
$$r(q,t) = a(t)\left[q + D(t)s(q)\right]$$

$$s(q) = \nabla\varphi_0(q)$$

$$\varphi_0(q) = \sum_k a_k\cos(kq) + b_k\sin(kq)$$



$$a_k, b_k = \sqrt{P(|k|)}\,\frac{Gauss(0,1)}{|k|^2}$$

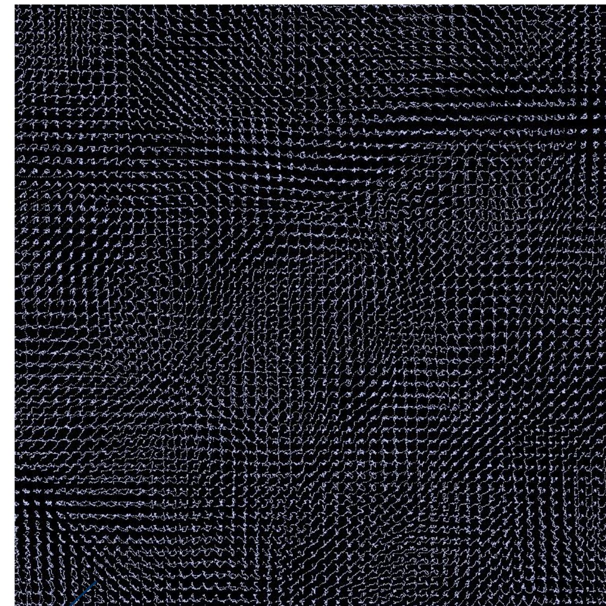


“Particles” and simulations

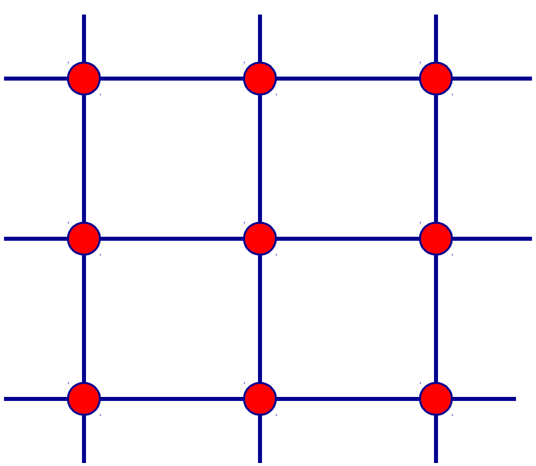
Modern computer can handle more than 109 particles/elements

$$V = (200\, h^{-1}\, Mpc)^3$$

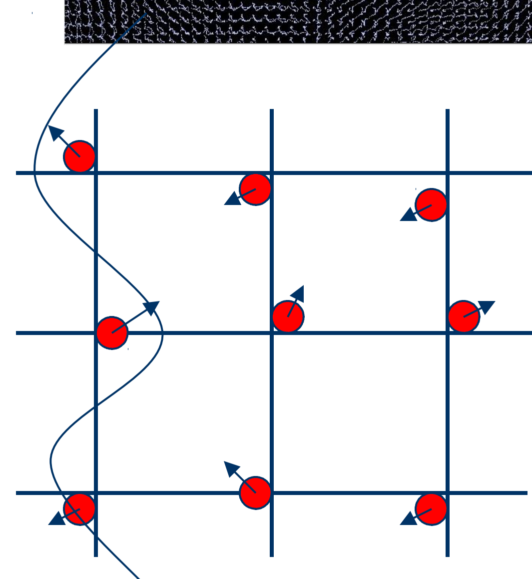
$$m_p = \frac{V}{N_p} \times \rho_{cr} \times \Omega_m = 6.66 \times 10^8\, M_{sun}$$

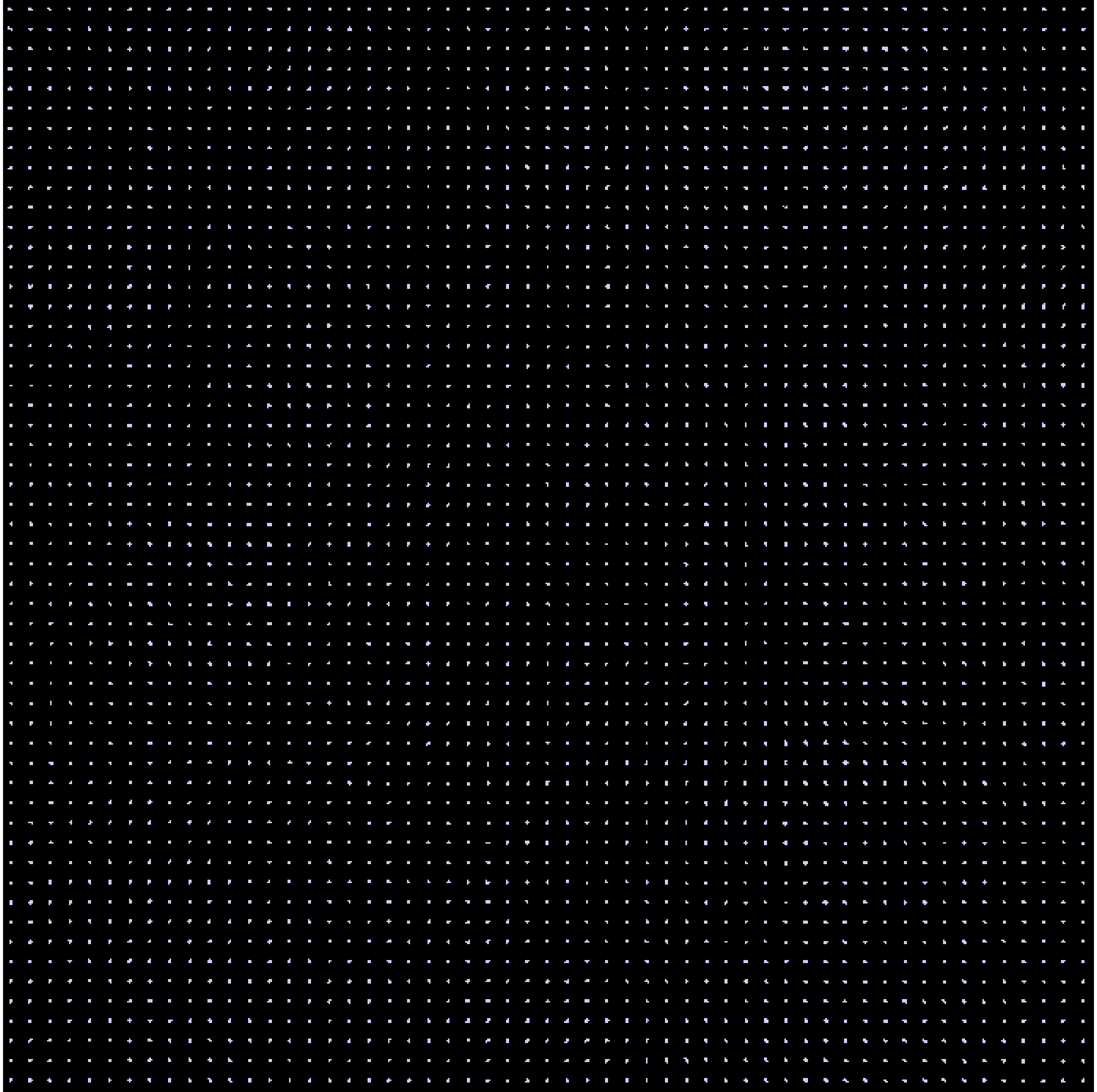


Our particles have the same mass of a dwarf galaxy…

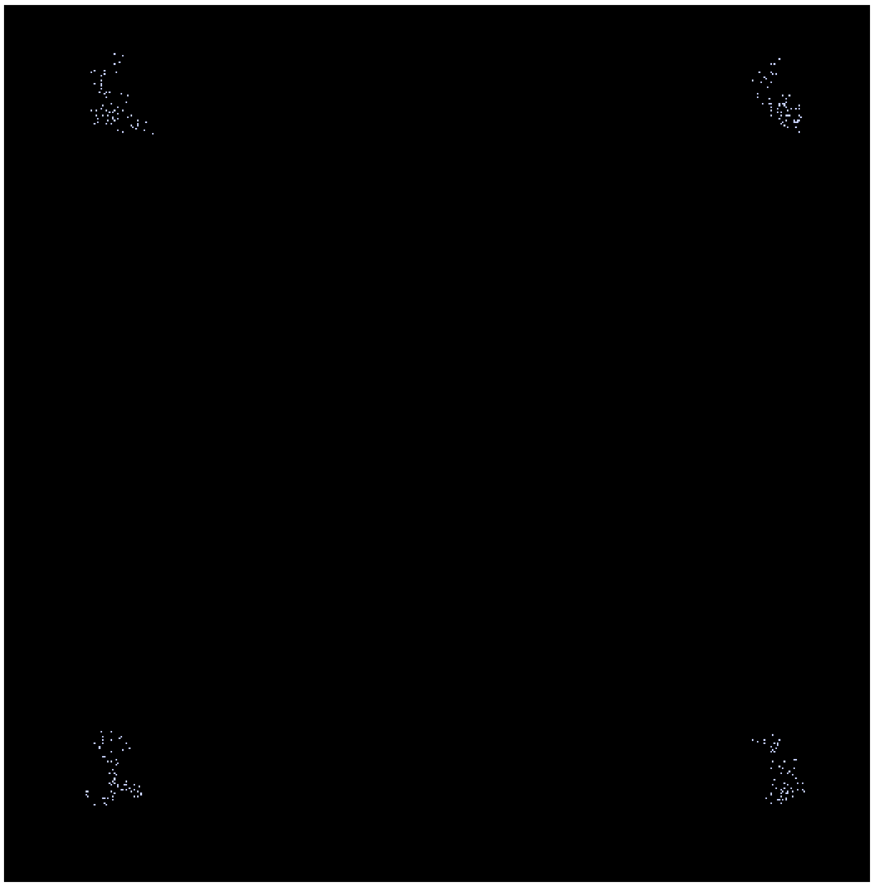


+ P(k,z) =

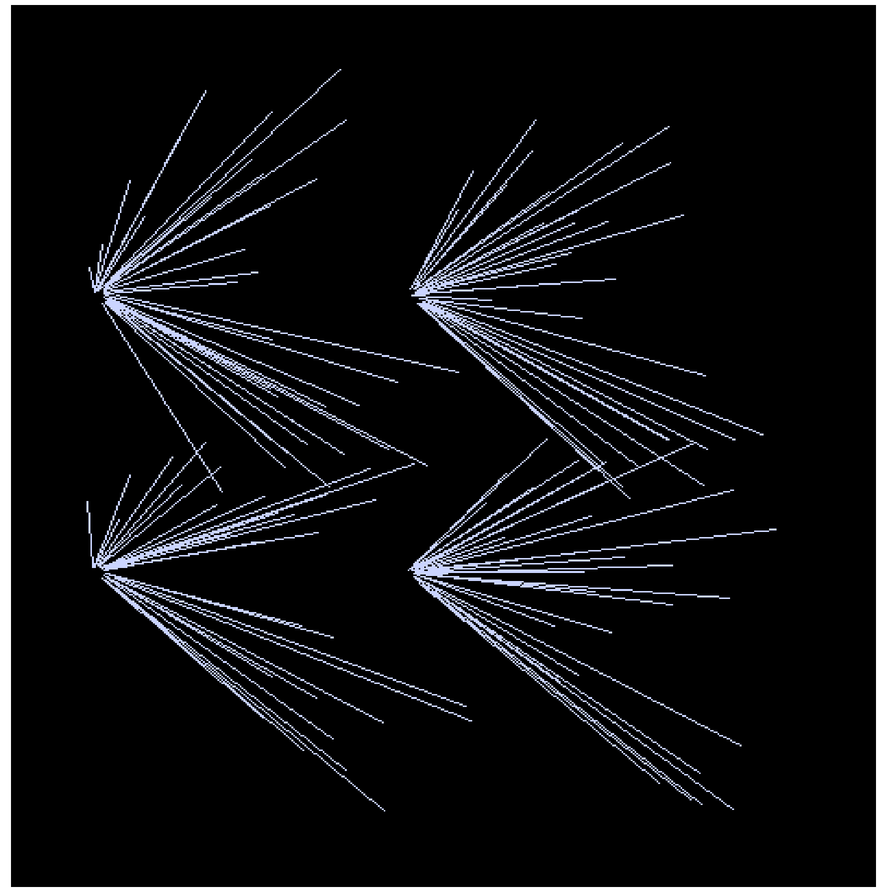




The Zeldovich approximation



Space



Velocity

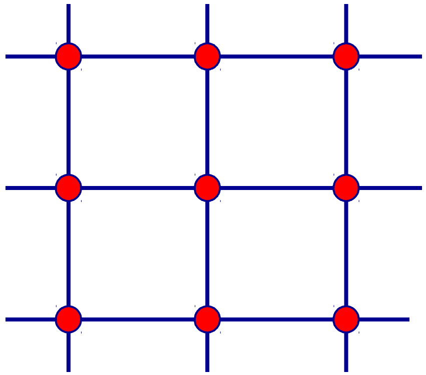
Public Codes

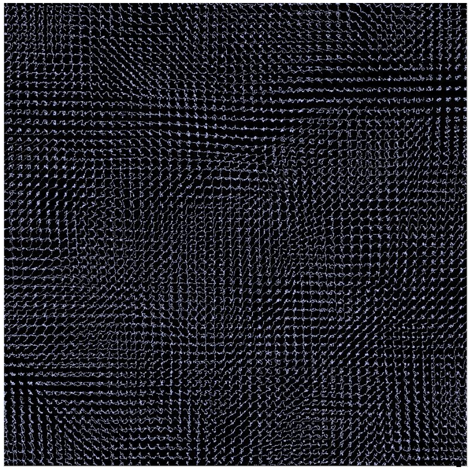
MUSIC (Oliver Hahn)
http://www.phys.ethz.ch/~hahn/MUSIC/

Grafic++ (Doug Potter)
https://hpcforge.org/projects/grafic/

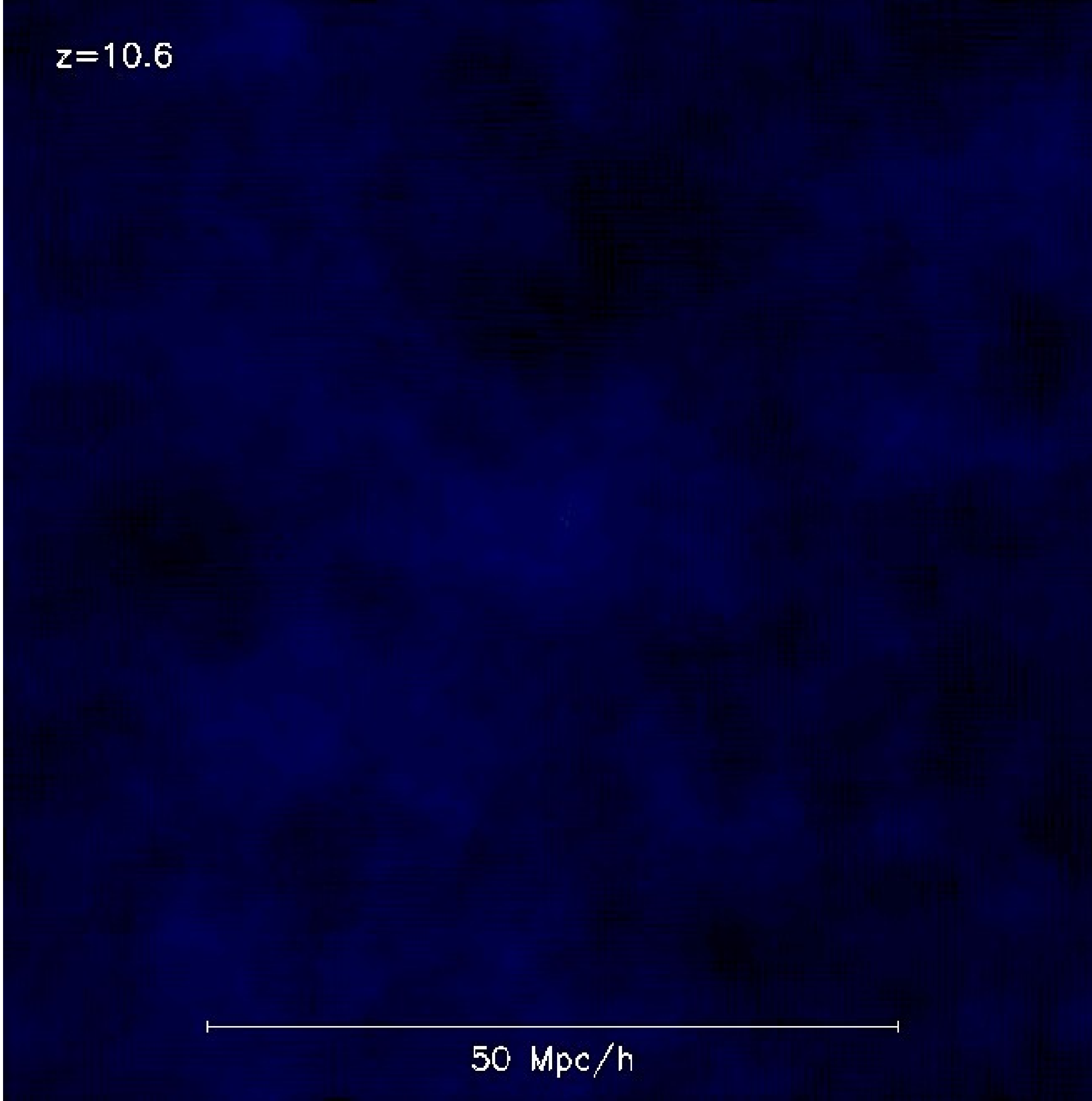
N-genic (Volker Springel)
http://www.mpa-garching.mpg.de/gadget/

2LPT (Scoccimarro)
http://cosmo.nyu.edu/roman/2LPT/





+
GRAVITY

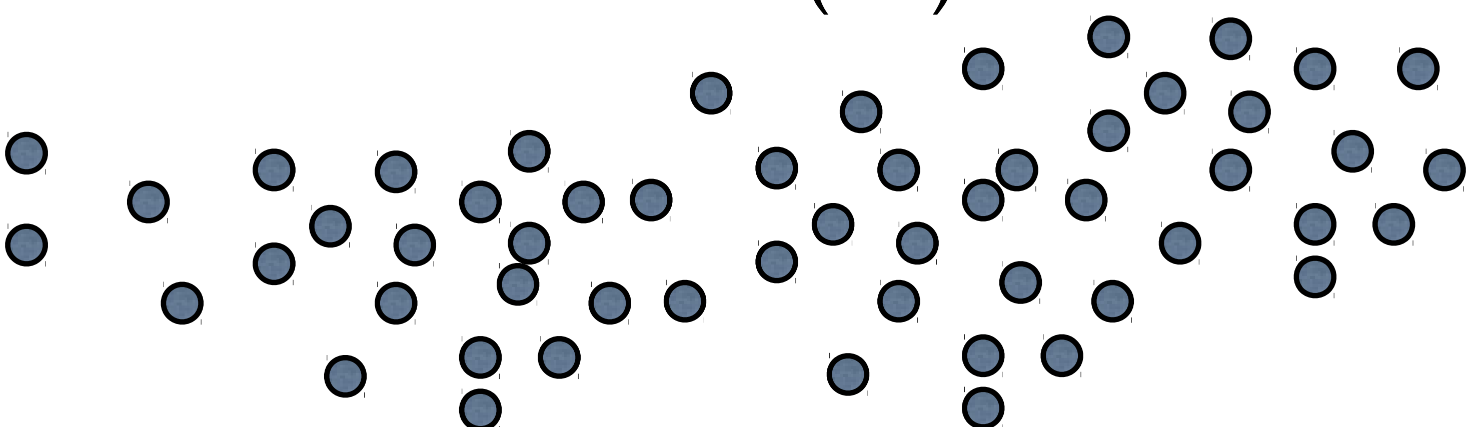


How to evolve the Initial Conditions

Newtonian Gravity

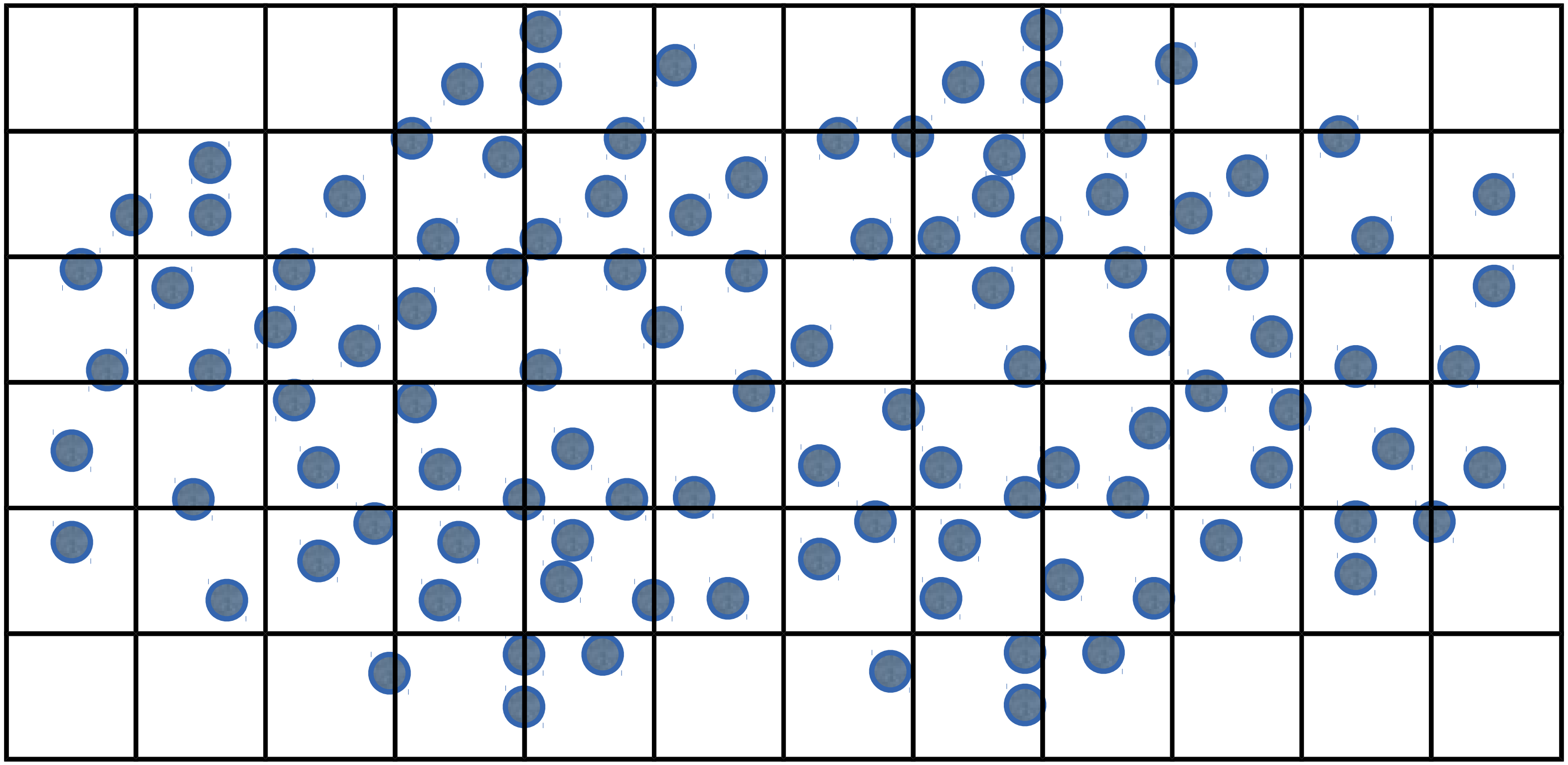
$$F_g = \frac{GMm}{r^2}$$

- Solving once or twice is not very hard, but we now have "N" bodies O(N2).



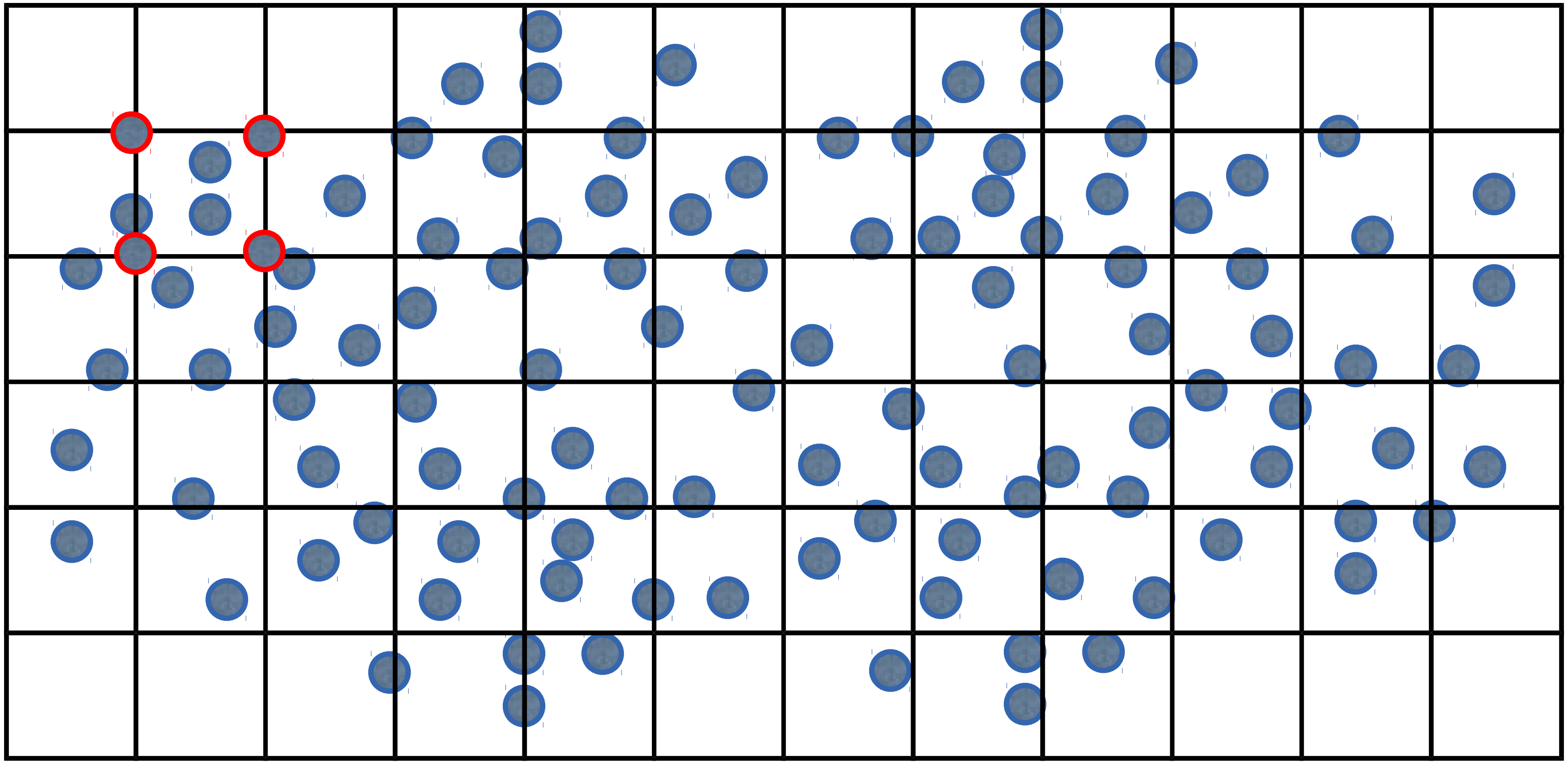
so we need a clever trick . . .

Mesh means Grid



Calculate density in each cell

Mesh means Grid



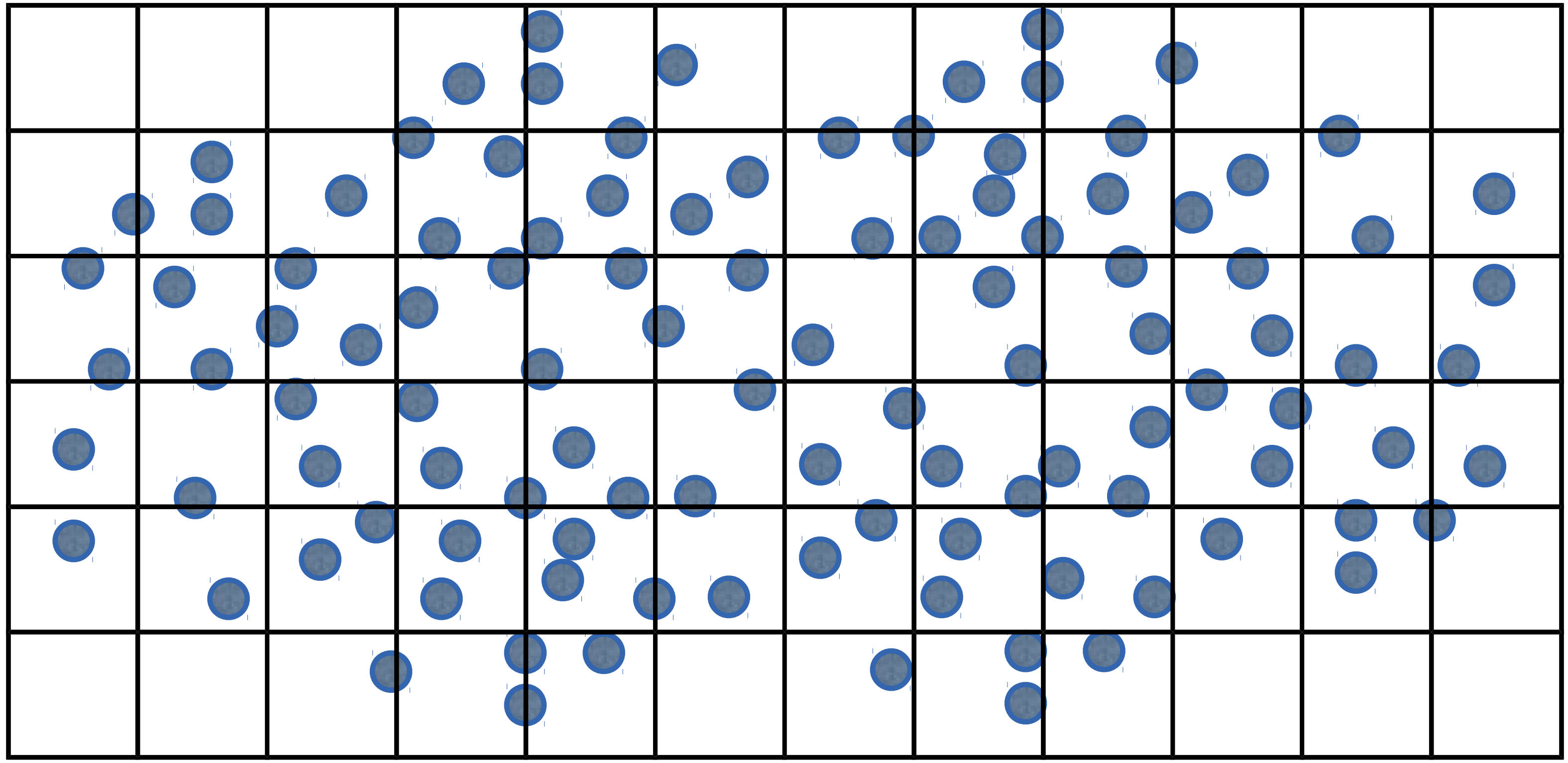
Calculating Large Scale Forces

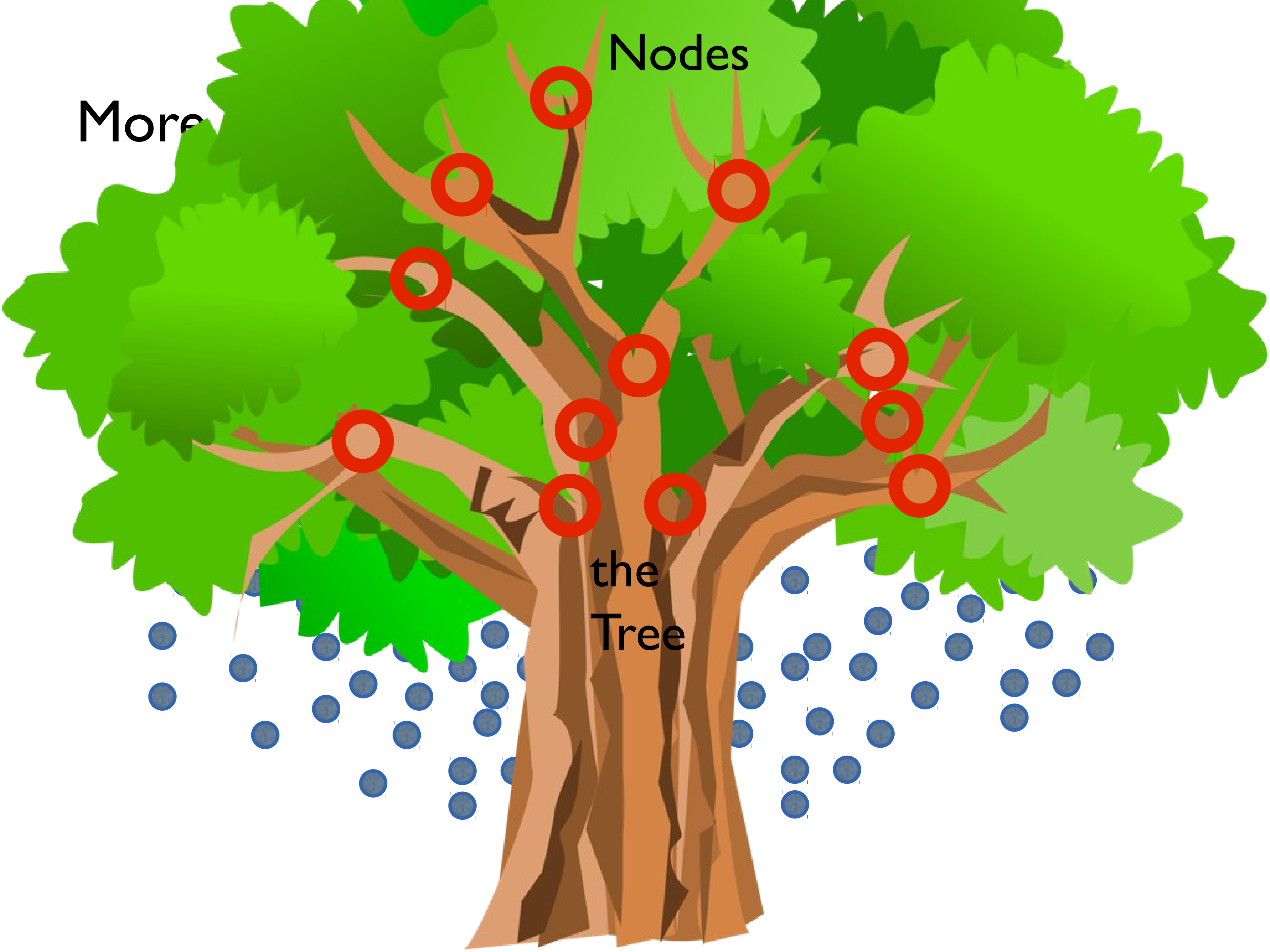
- Take (Fast) Fourier Transform of density field
- Calculate forces on each cell
- Interpolate forces to particle positions
- Grid can only be so fine due to memory constraints (2563? 5123?)

What about small scale forces?

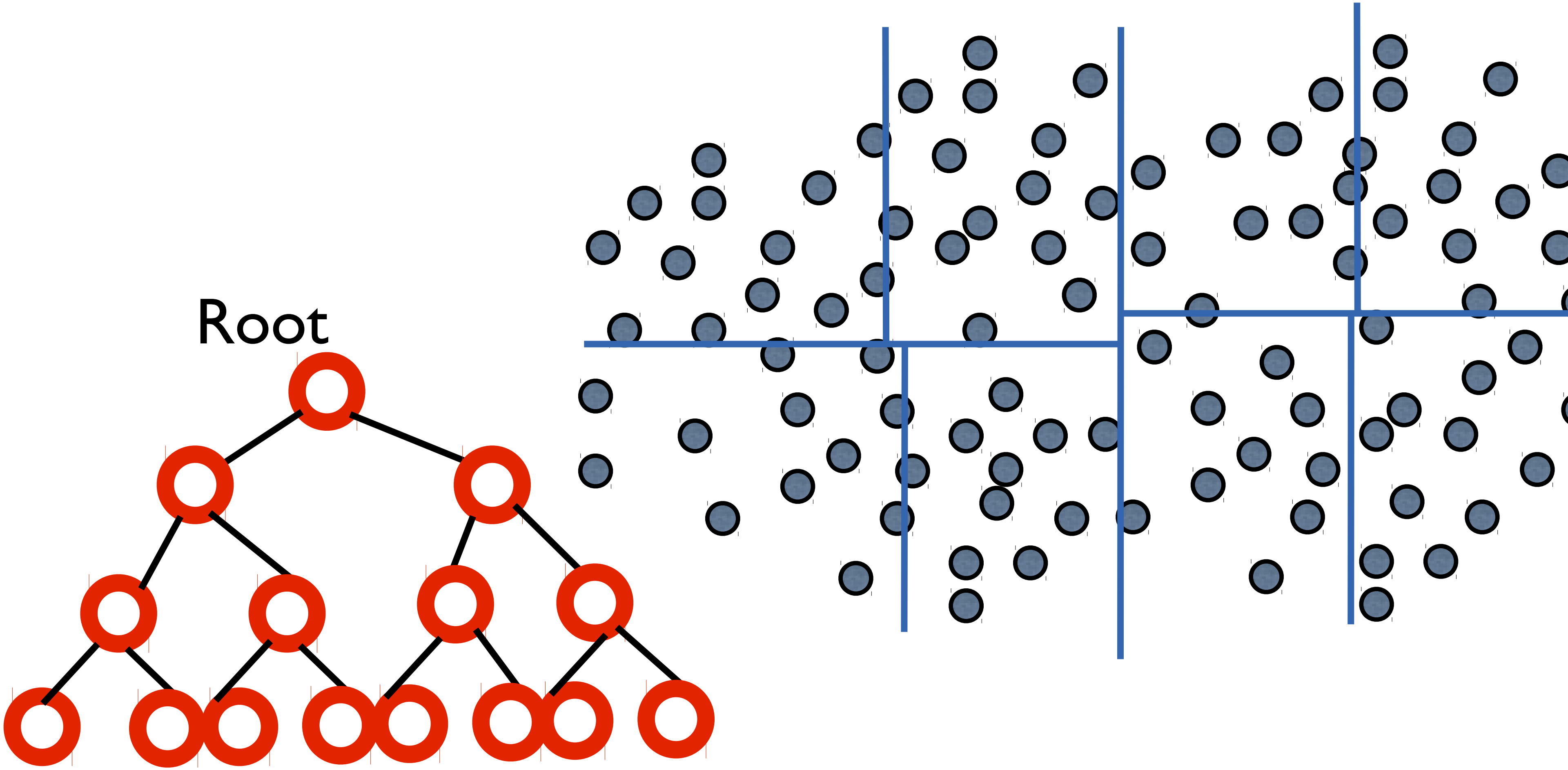
- If your simulation volume is 100 Mpc, then 5123 gives you ~200 kpc resolution, the mean interparticle separation (IPS)
- You typically want 1/40 IPS resolution
- so you have to calculate individual particle-particle forces: P3M "particle-particle, particle-mesh"

Particle-Particle Particle-Mesh (P3M)



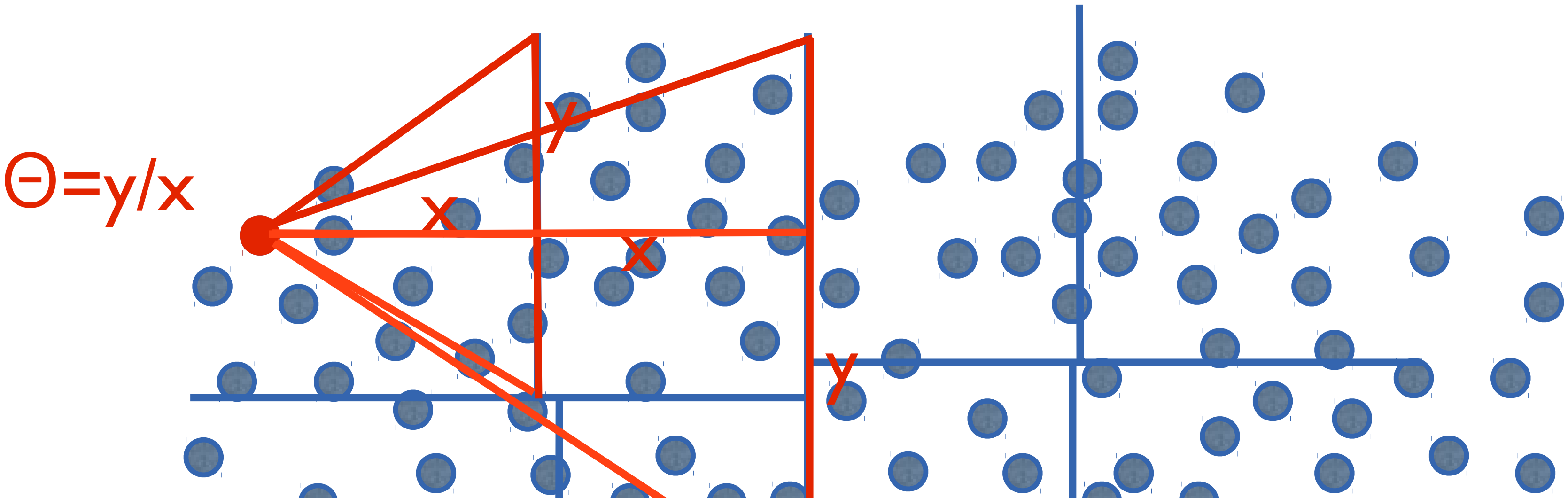

Nodes
More
the
Tree

Growing the Tree



The Power of Nodes

- You only need to calculate gravity between each particle and a node if the node is not too close
- *Opening Angle* criterion
- When Θ<0.55, gravity is correct if you expand node gravity field to enough (16) moments



Force of Gravity

- For each particle, you add the forces of gravity from every massive source (particle or node)
- This is fairly straightforward to parallelize
  - divide root nodes between processors
  - add local contributions and parallel contributions

Stadel, J (PhD Thesis 2001)

Spatial Resolution

- How close should you allow 2 particles to get to each other?

$$F_g = \frac{GMm}{r^2}$$

  - could get very big, so it is "softened"

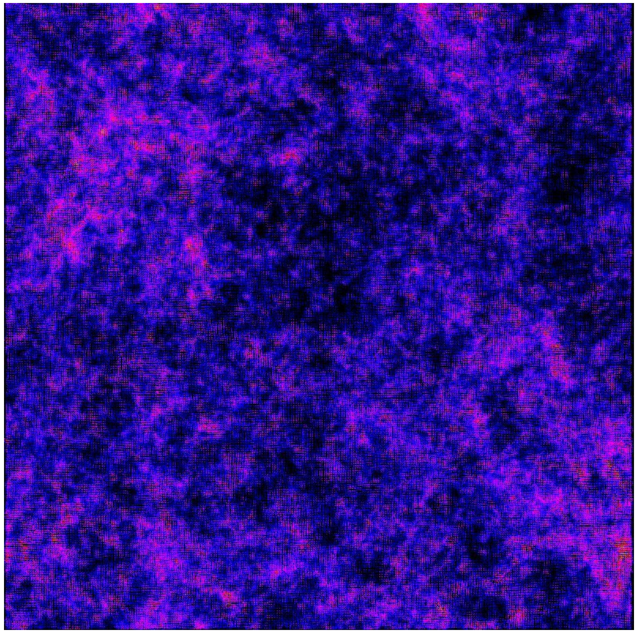
$$F_g = \frac{GMm}{(r+\epsilon)^2}$$

  - where ε is the "softening length"

Spatial Resolution

- Power et al (2003) gives guidelines for how long softening length should be in cosmological simulations
  - 1/40 interparticle separation
    - 512^3, 100 Mpc $\Rightarrow$ 5 kpc
  - $r_{200} / \sqrt{N_{200}}$
    - 2500 particles in 10^{12} $M_\odot$ halo $\Rightarrow$4 kpc

Where is cosmology?



$$P(k,z) = A\,k^{n}\;T^{2}(k,z)$$

$$\nabla^{2}\Phi(x,t) = 4\pi G\;a(t)^{2}\;\rho(x,t)$$

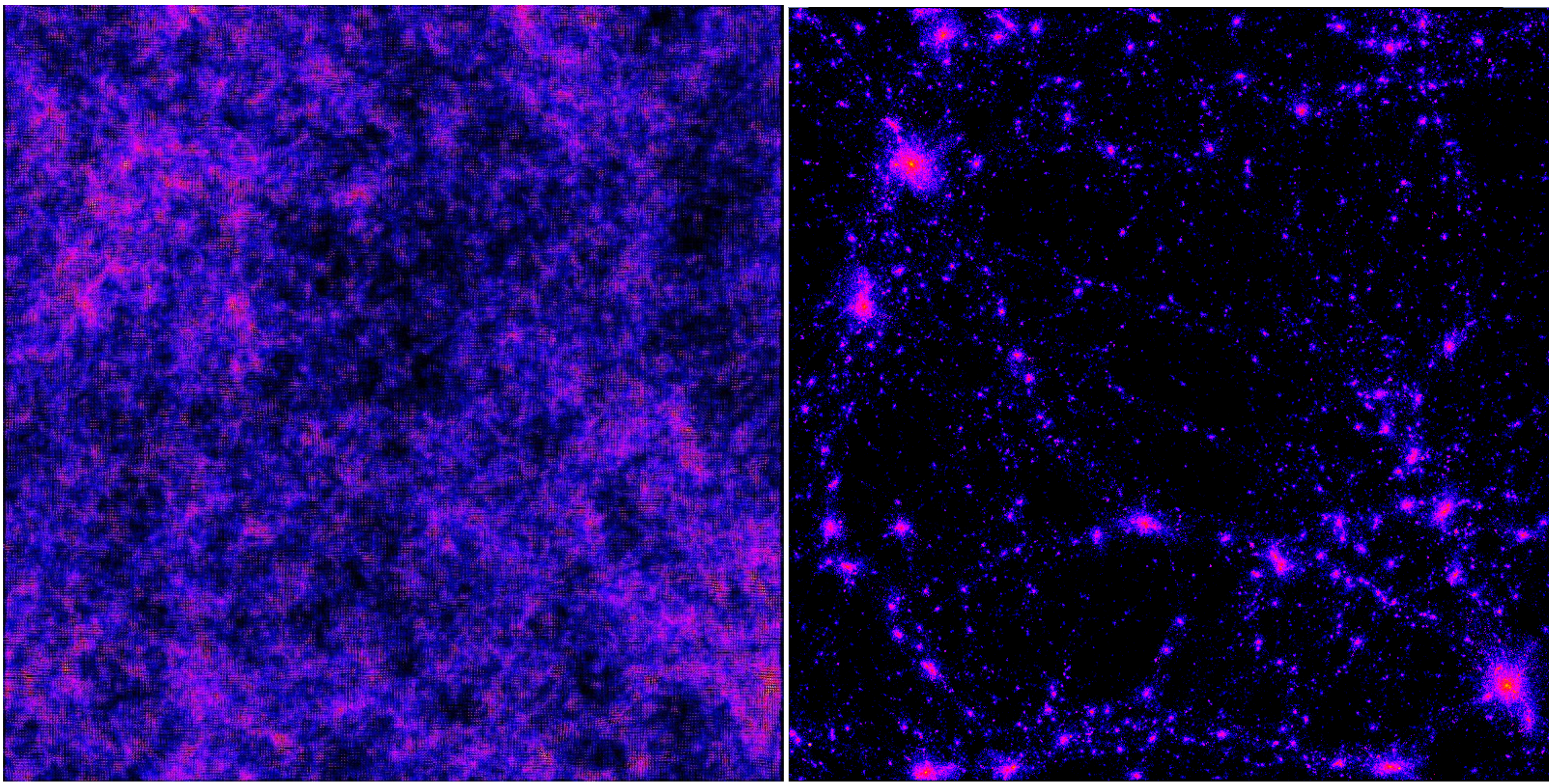
$$\frac{dx}{da} = \frac{dx}{dt}\frac{dt}{da}$$

Expansion velocity

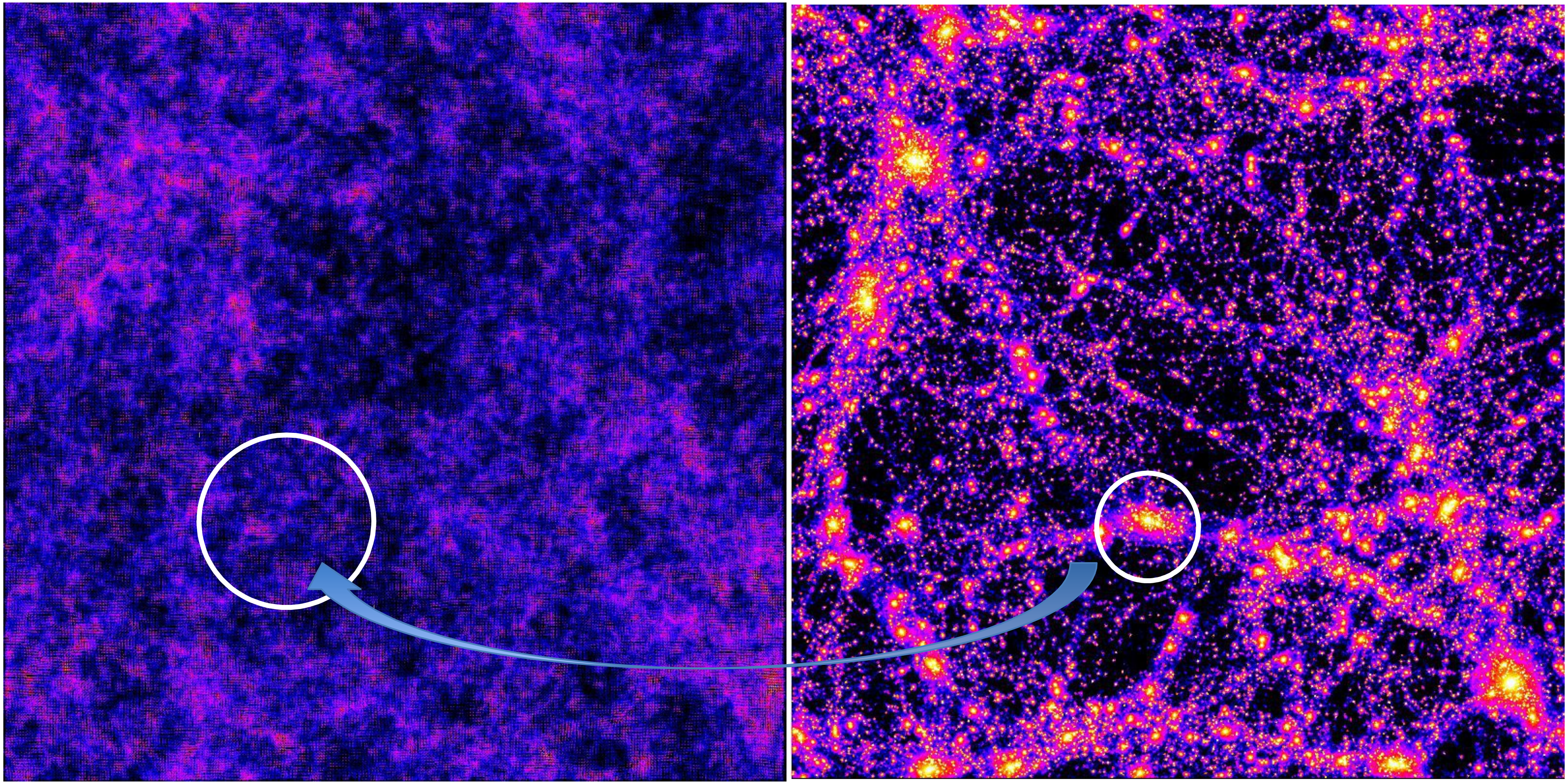
"Normal gravity"

1) Initial conditions: Power Spectrum
2) Background evolution: a(t)

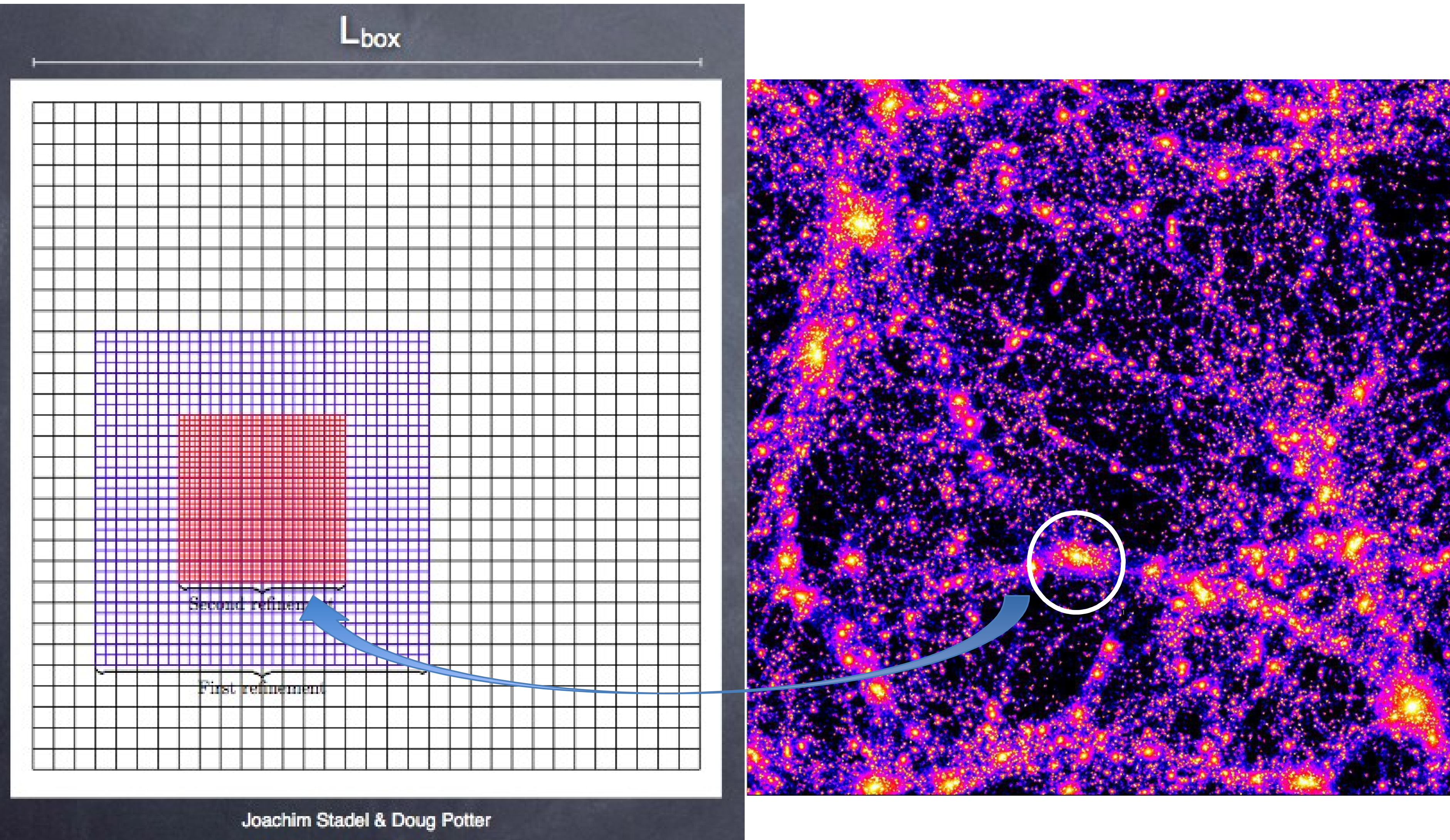
IC and final outcome



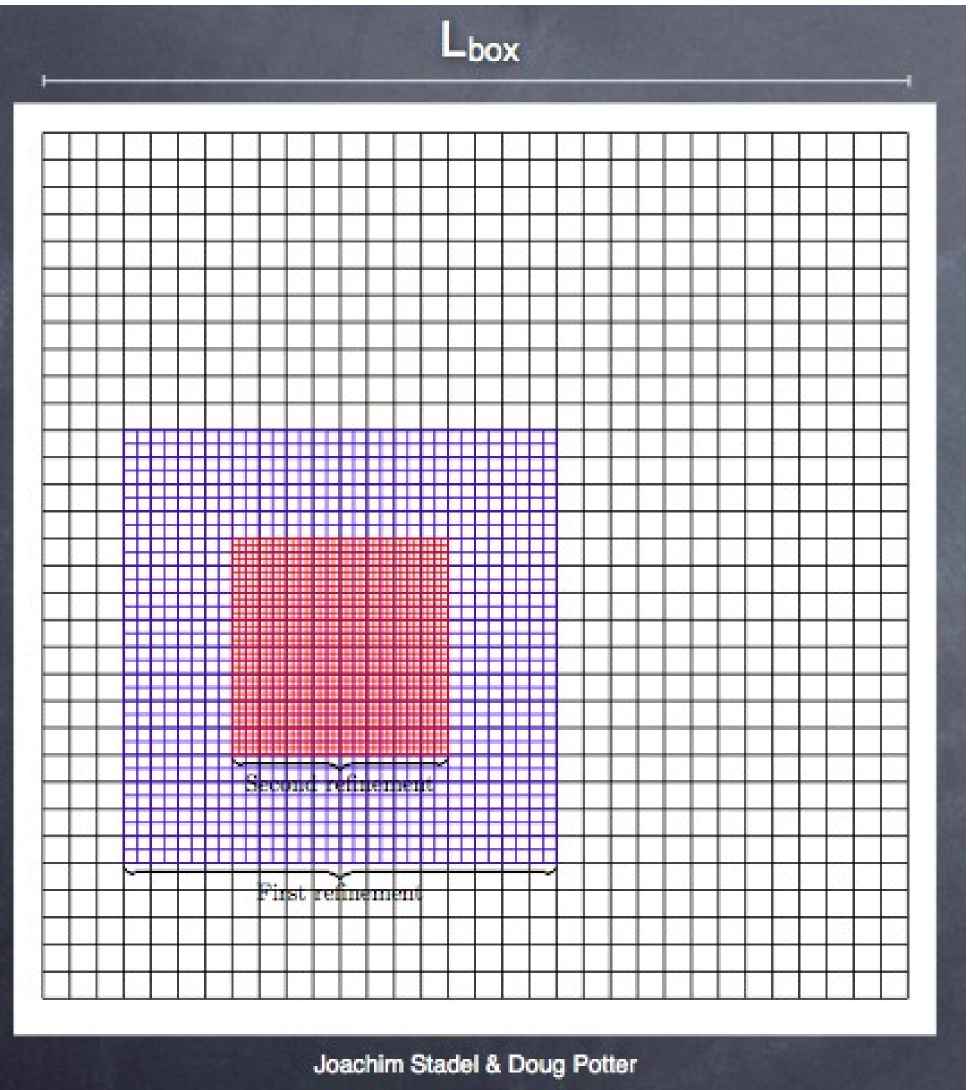
Zoom simulations

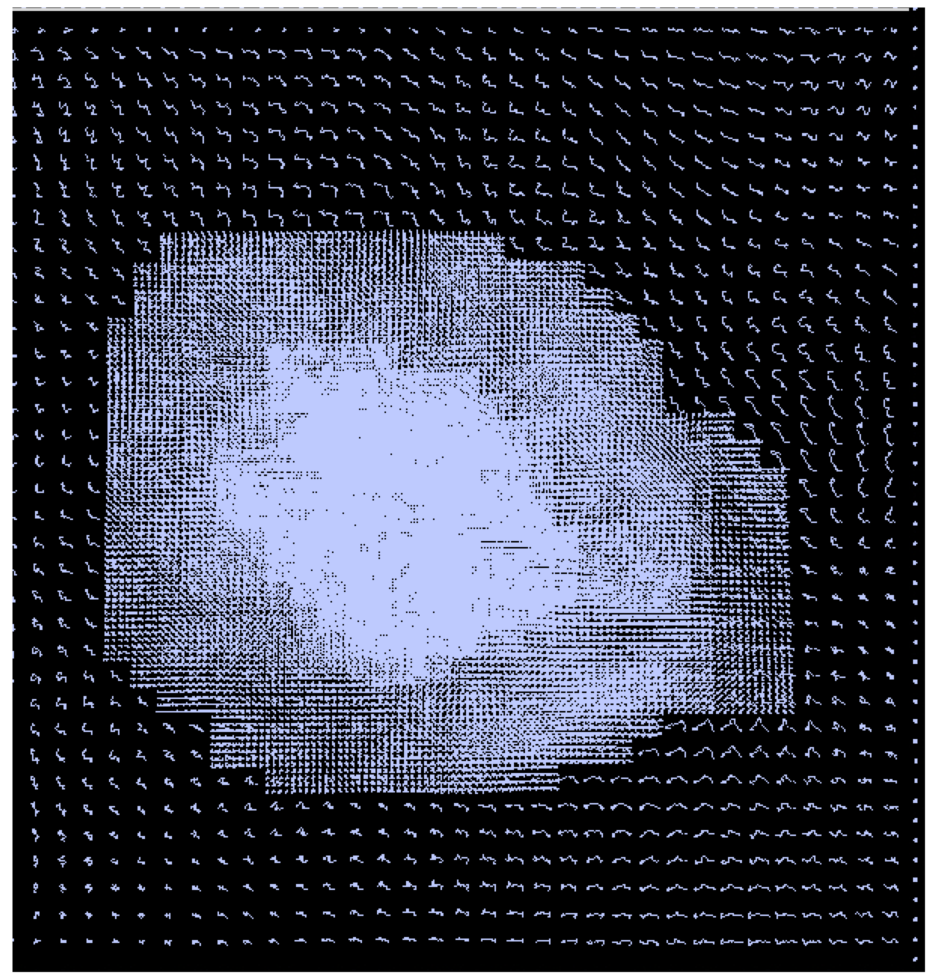


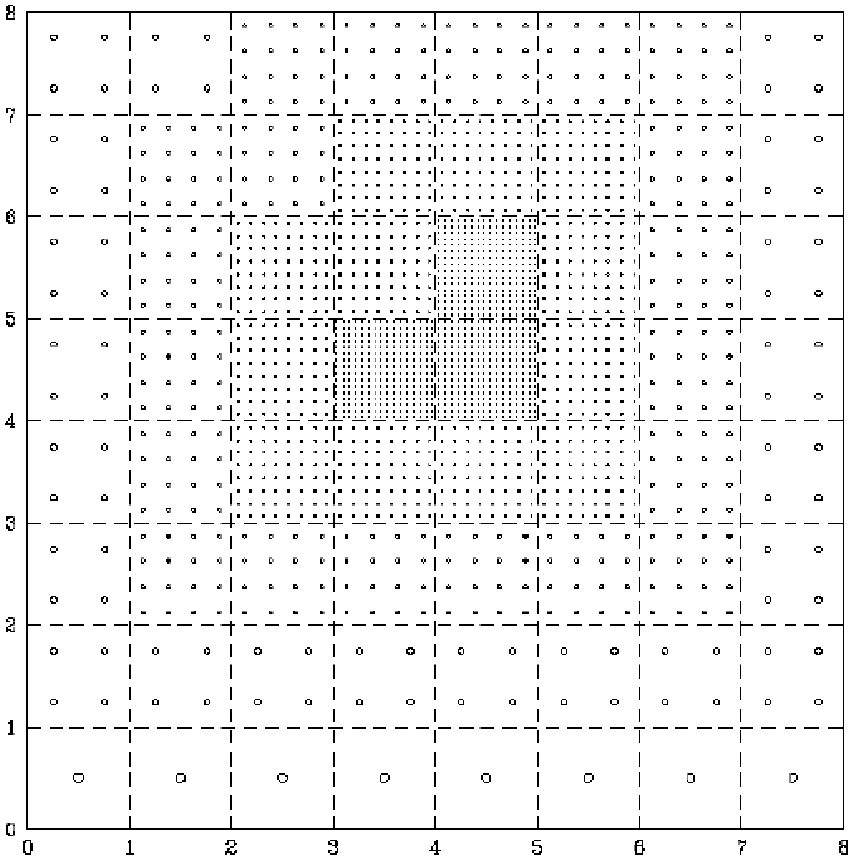
Zoom simulations



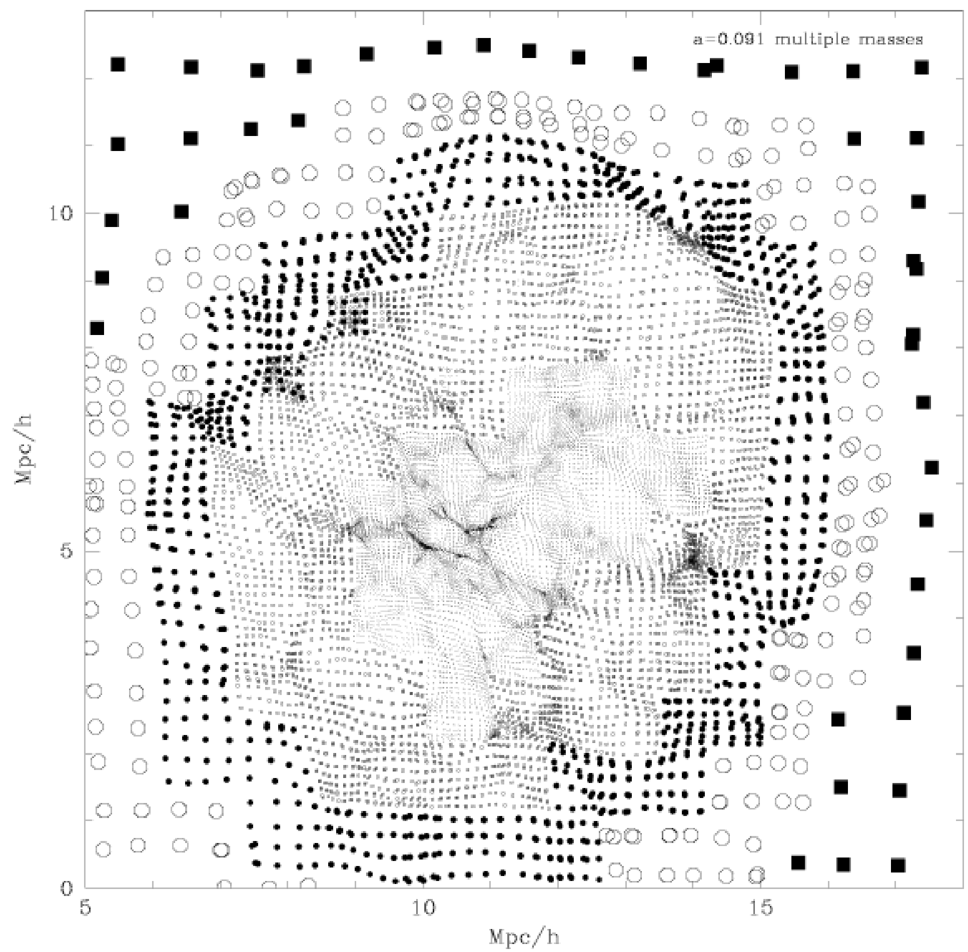
Zoom simulations







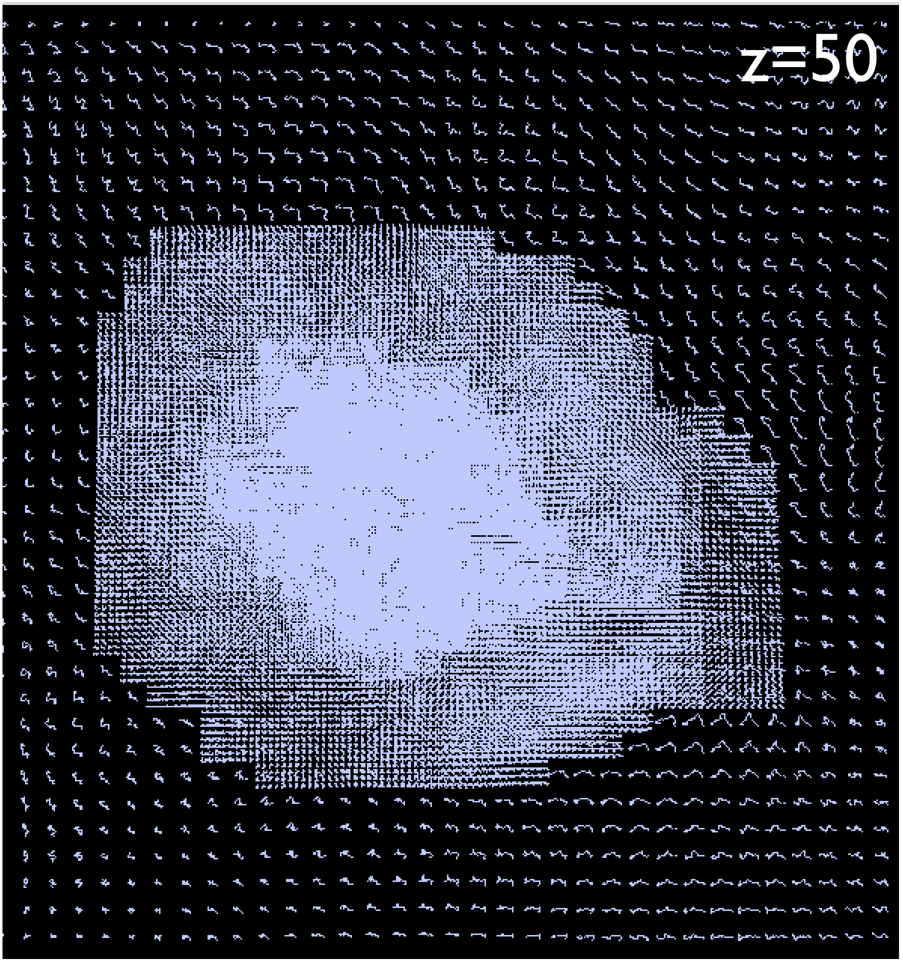
Three central blocks were marked for highest mass resolution.
(from Klypin et al. 2001)

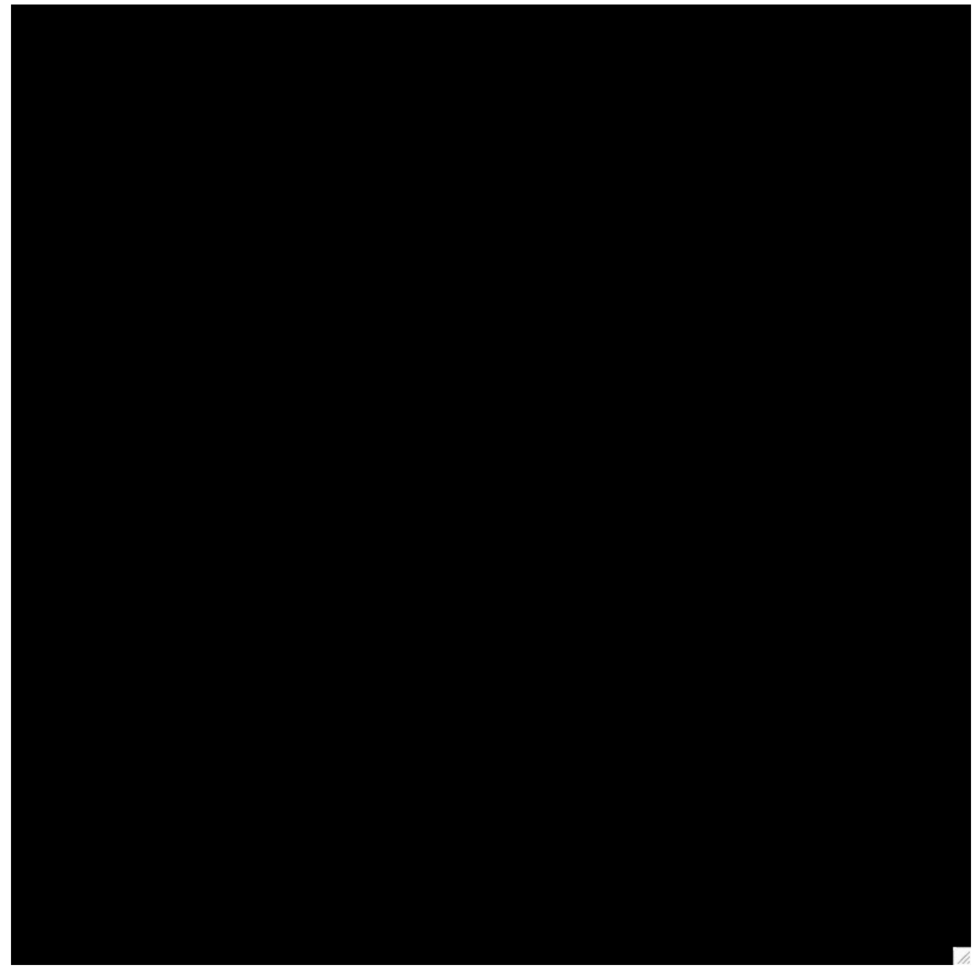


Distribution of particles of different masses (i.e. different symbols) at redshift 10.
(from Klypin et al. 2001)

High resolution Cosmological Nbody simulations

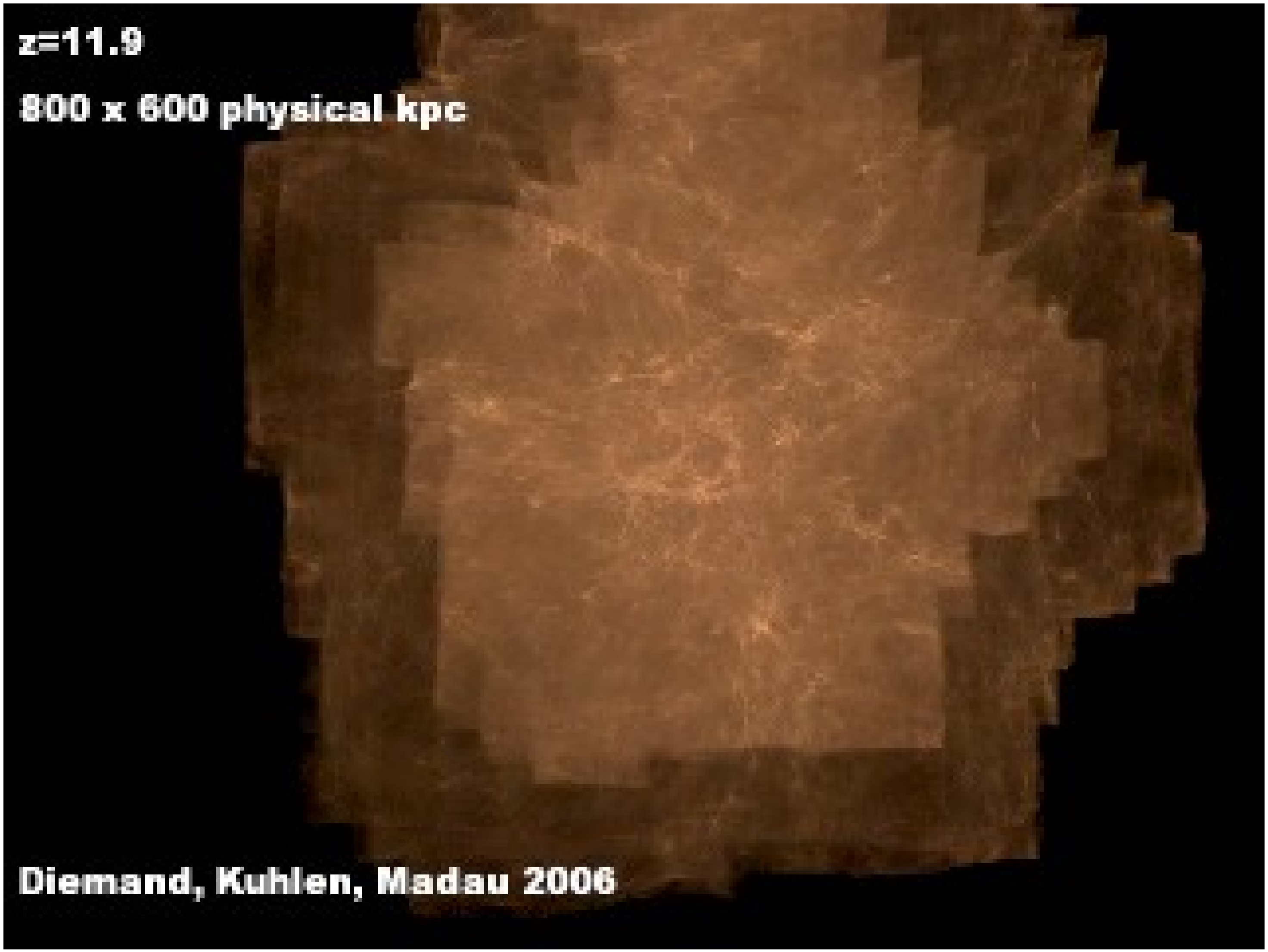
mp=104 Msun



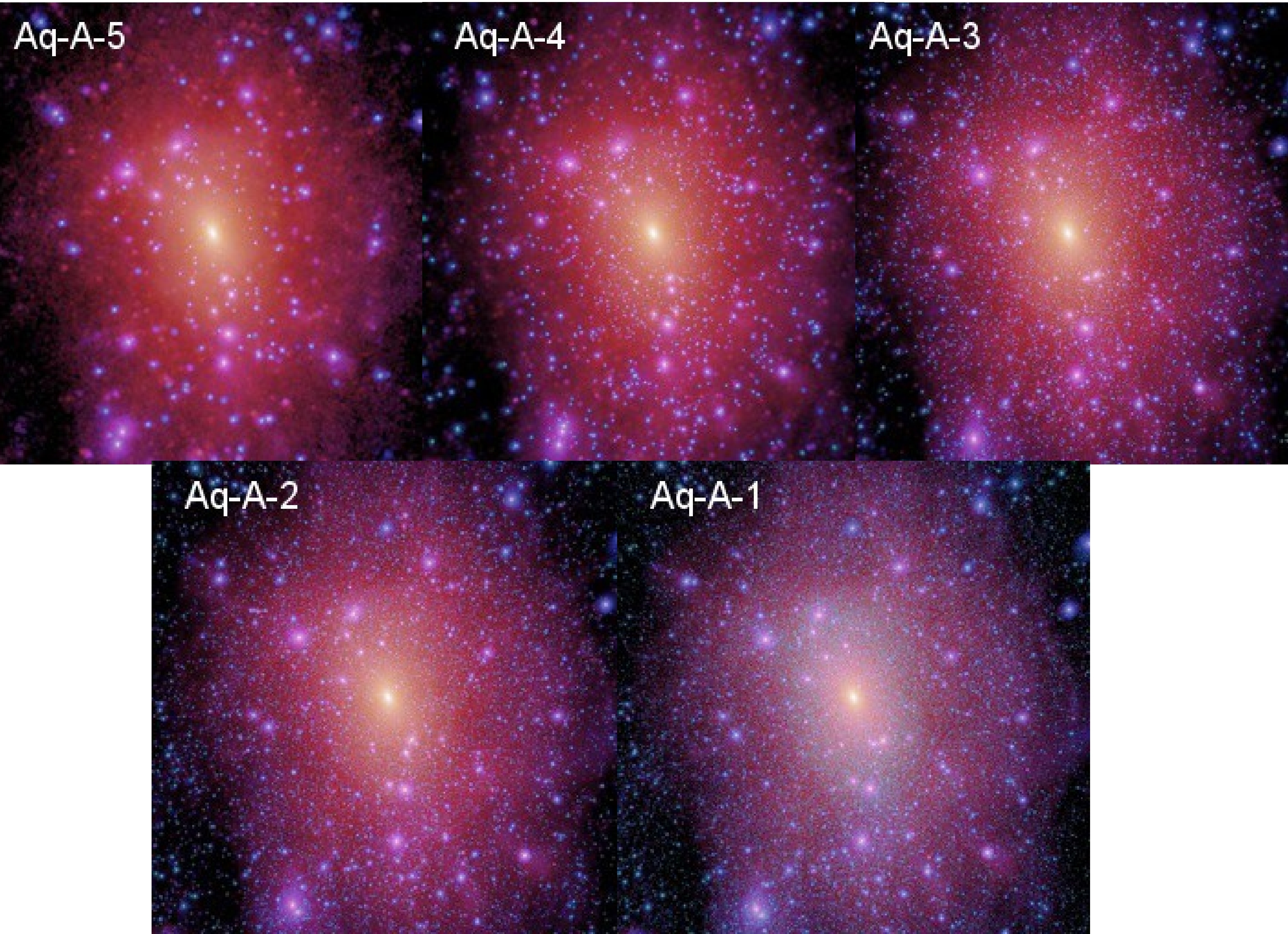


400 kpc comoving

Physical Coordinates



Resolution matters



Analysis

- How many objects of which mass?
- How is mass distributed?
  - object to object (clustering)
  - inside objects (density profiles)
- Depends on finding groups of particles:
  - Halos

What are Halos?

- Bound objects: *Virialized*
  - -2K < U
  - ~200 ςc   (ςc=3 H02 / 8ρG)

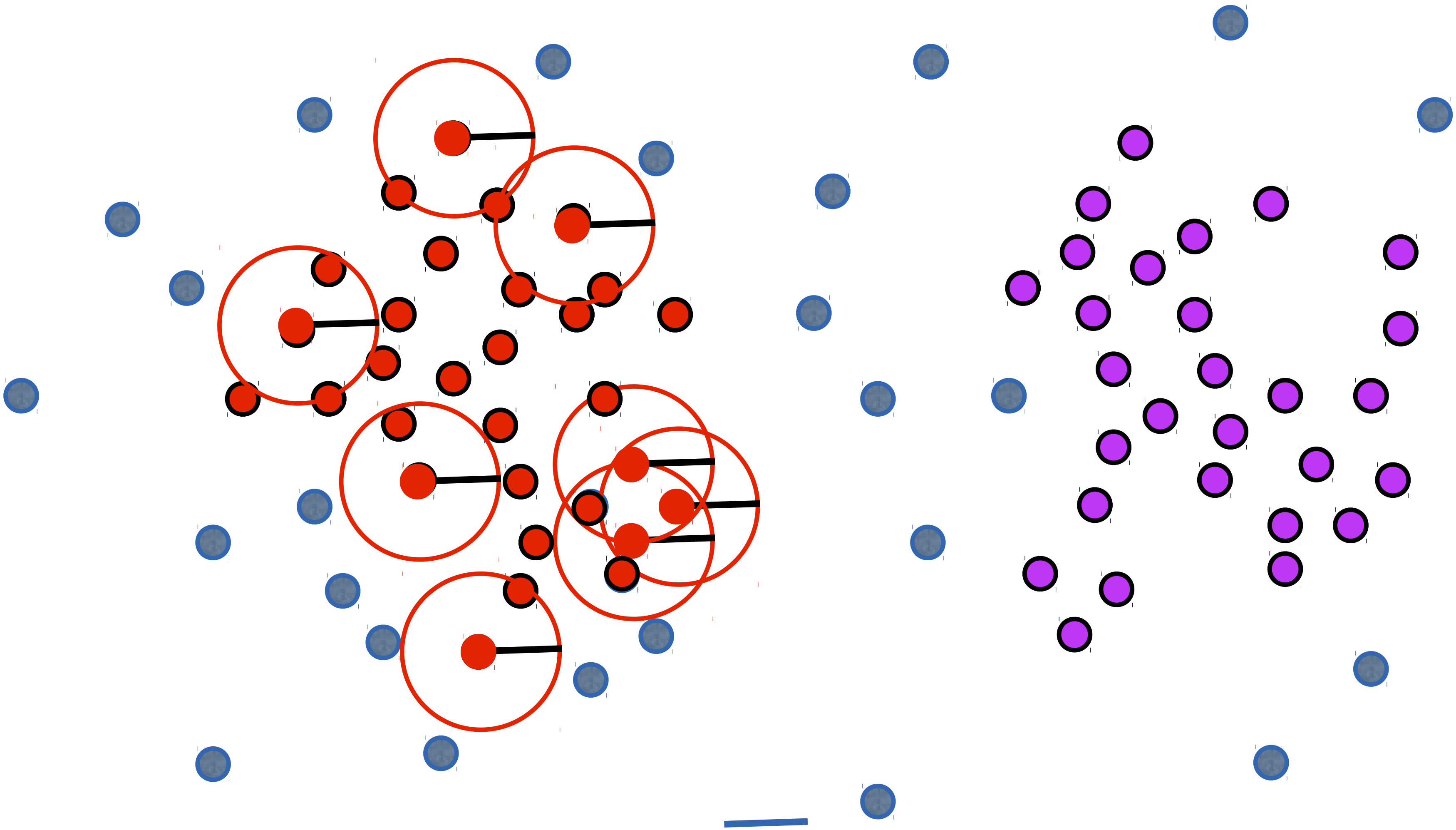
- How do we find halos?

Halo Finding

Friends-of-friends

Spherical Overdensity

Friends-of-Friends



Define linking length

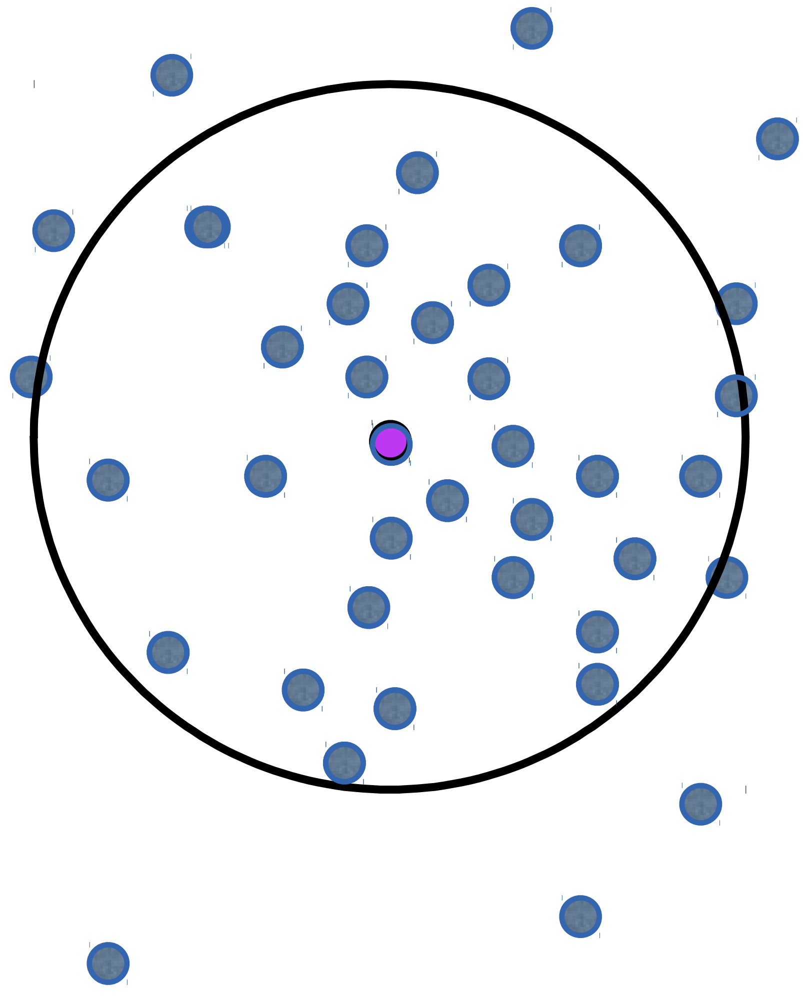
How to set linking length

- We want object with overdensity of ~200
- Mean density is related to IPS
- To get 125 times mean density, particles need to be 5 times closer (53=125)
- So, IPS / 5 is convenient choice

Publicly available codes

- Rockstar (Peter Behroozi)
  - Uses 6D (position + velocity) to find substructures
  - http://code.google.com/p/rockstar/
- FOF (Joachim Stadel, N-body shop)
  - http://www-hpcc.astro.washington.edu/tools/fof.html

Spherical Overdensity



Dark Matter haloes

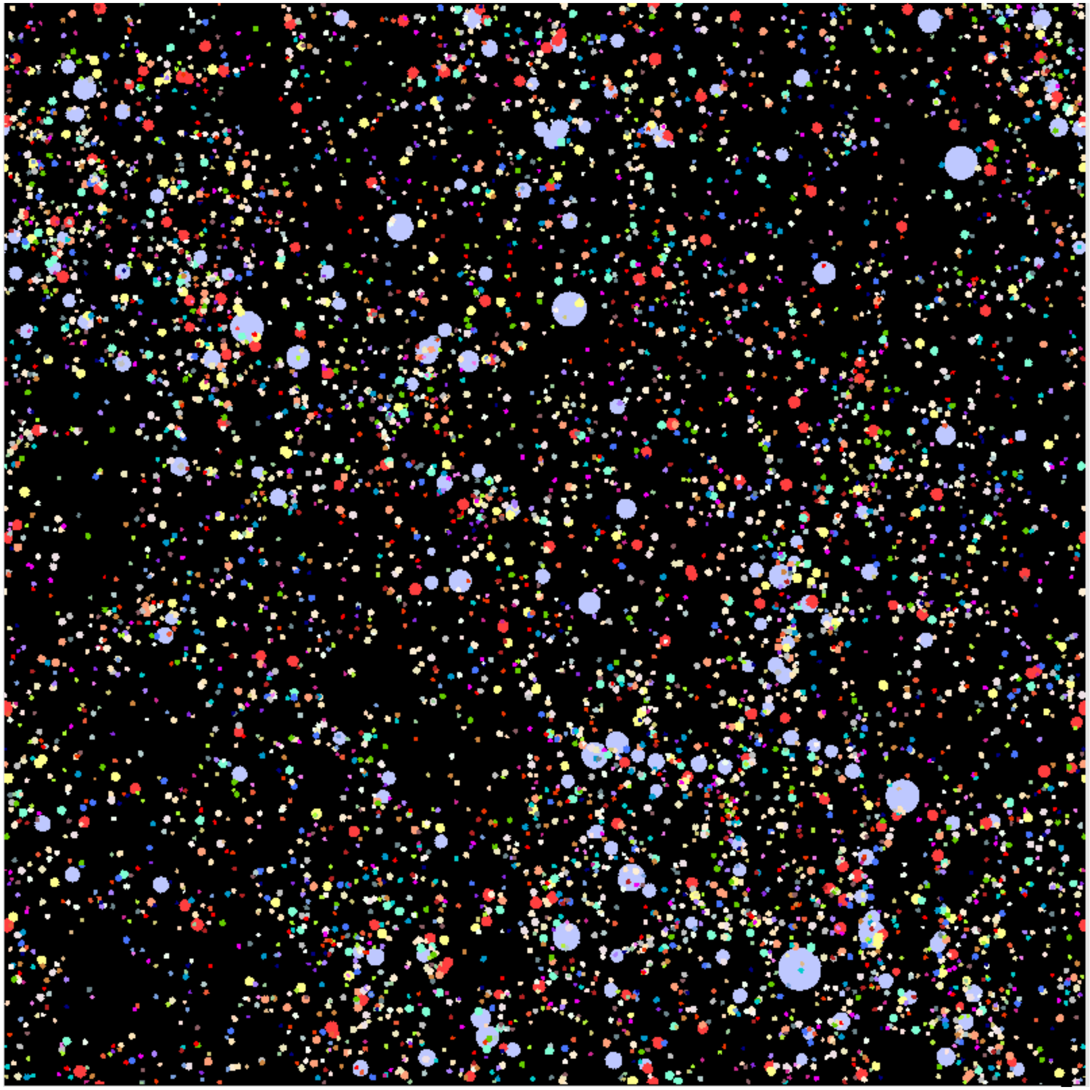
ςvir/ς0 =Δvir = 95

Rvir , Mvir

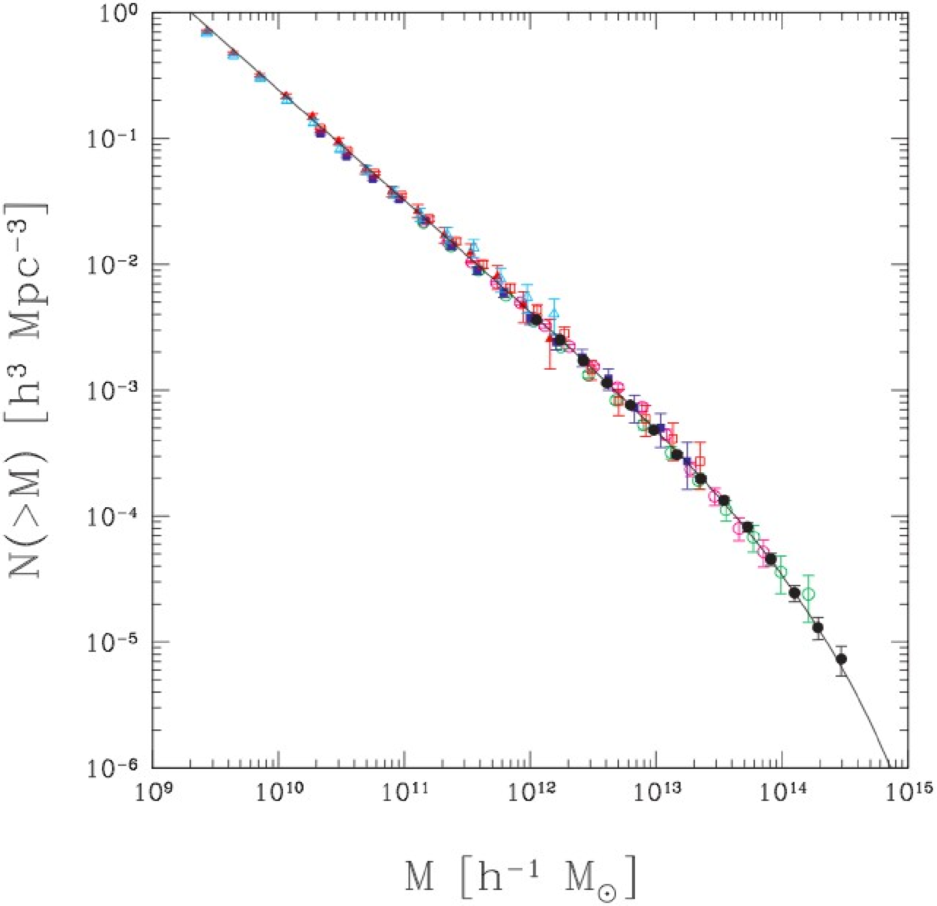
ς200/ς0 =Δvir = 200

R200 , M200

Macciò+2008



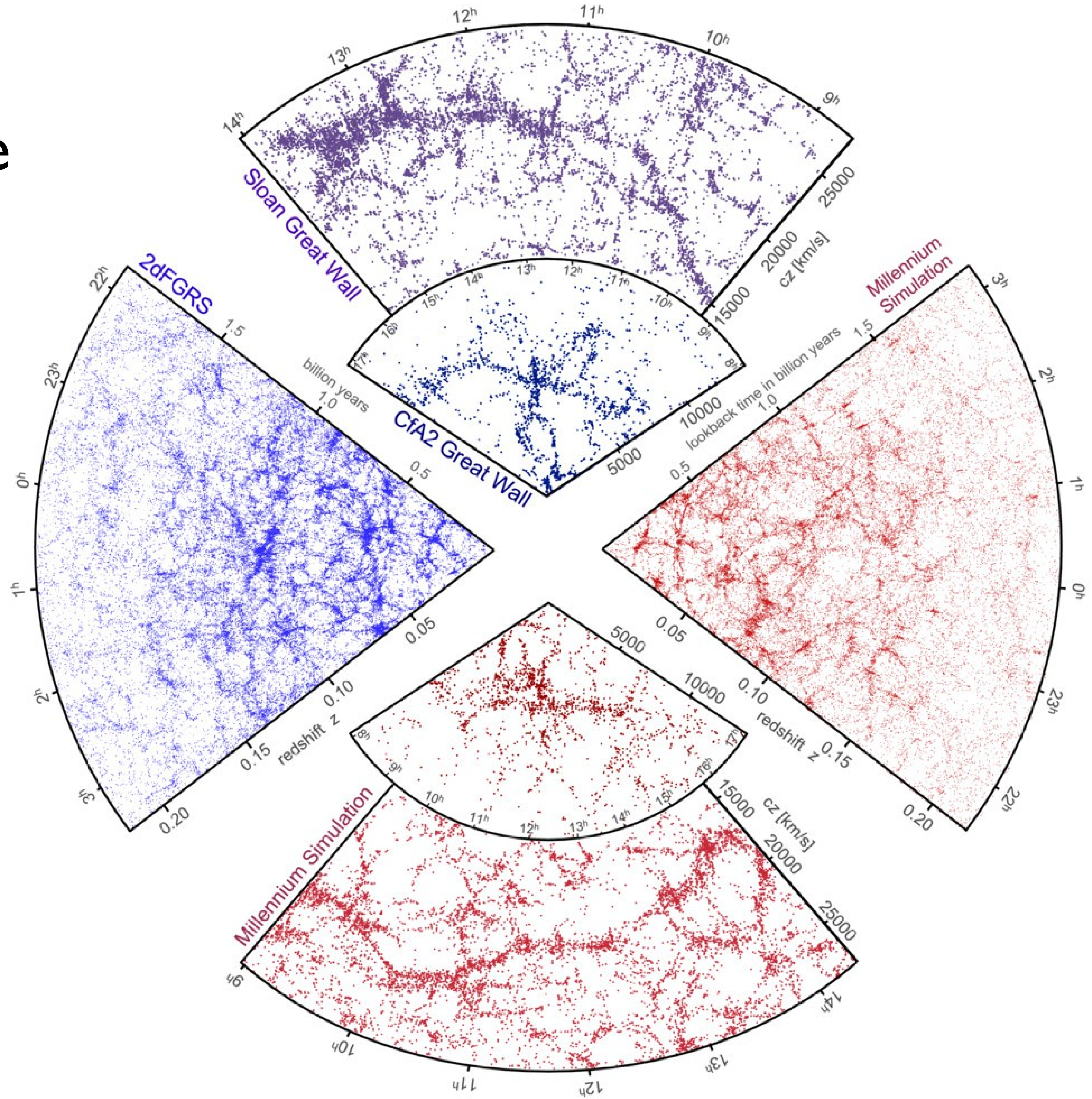
The halo mass function



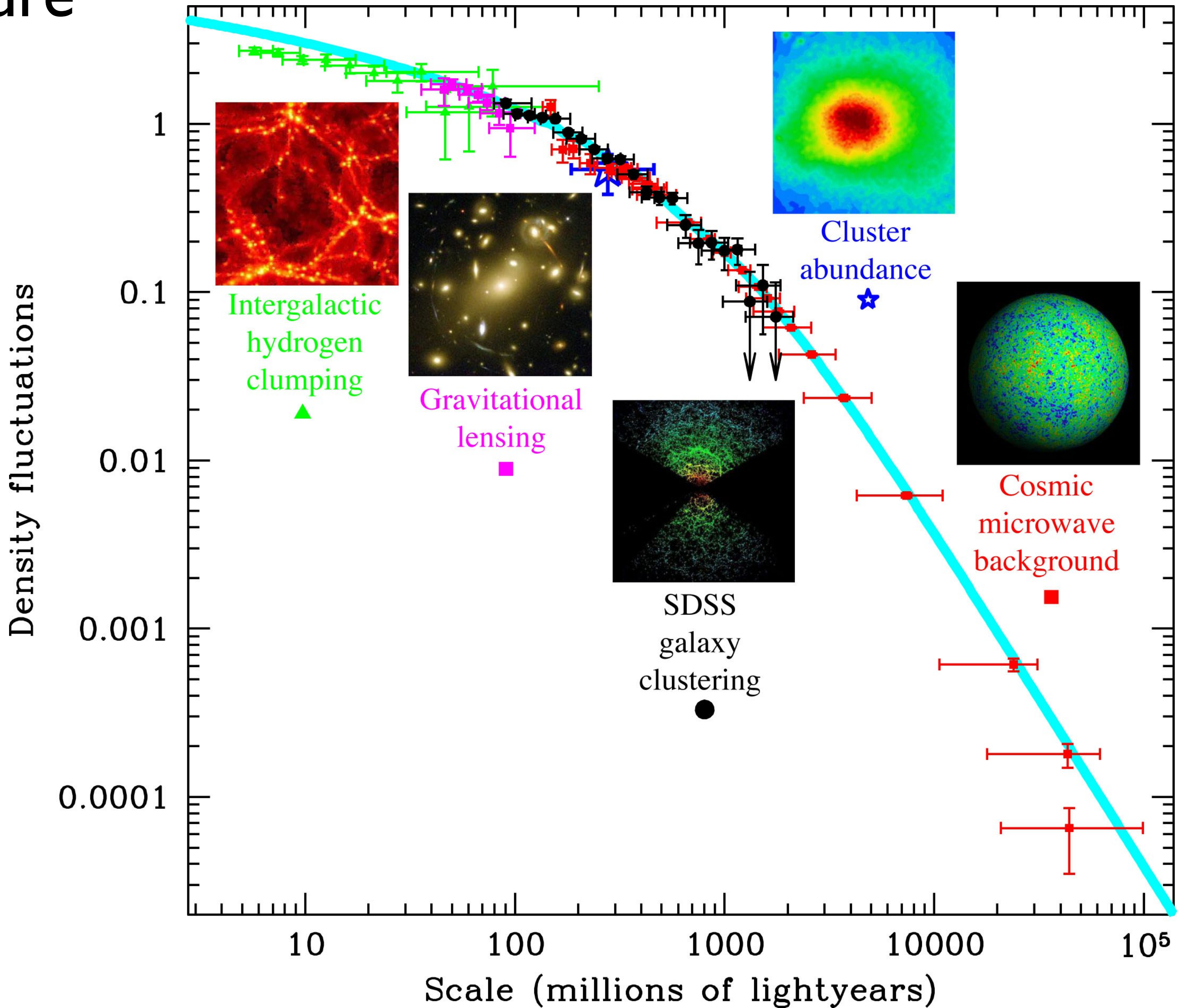
Macciò+07

$$n(M)\,dM = \frac{\alpha\bar{\rho}}{\pi^{1/2}}\,\frac{1}{{M_\star}^2}\left(\frac{M}{M_\star}\right)^{\alpha-2} e^{-(M/M_\star)^{2\alpha}}\,dM$$

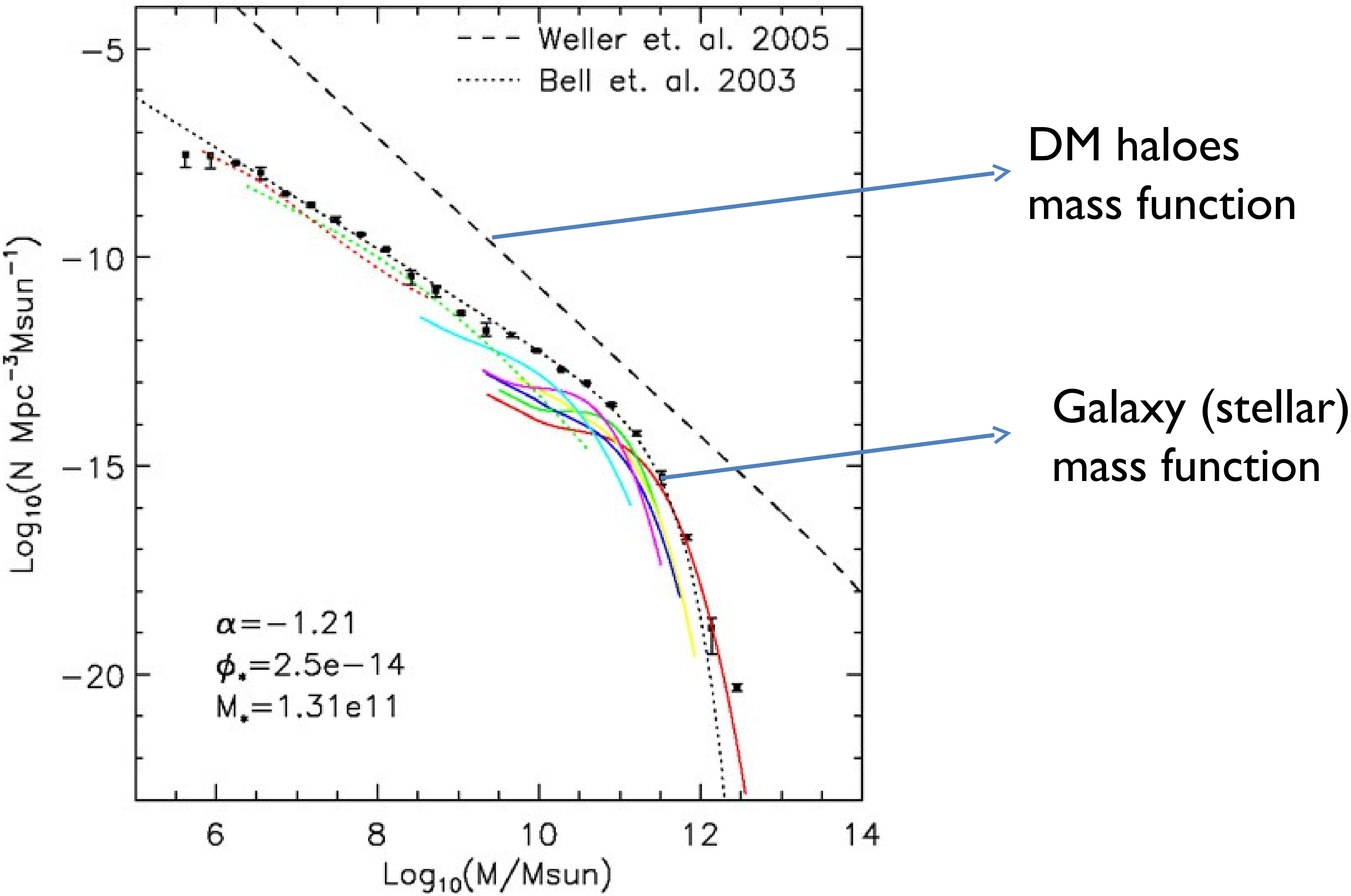
The large scale structure success



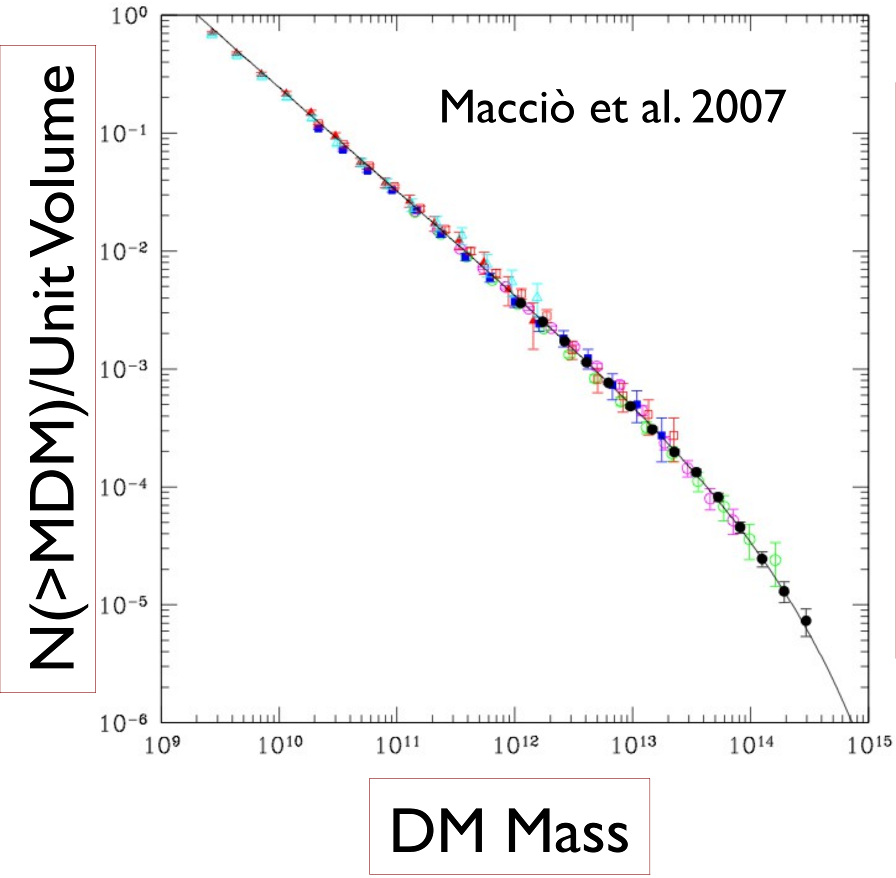
The large scale structure success

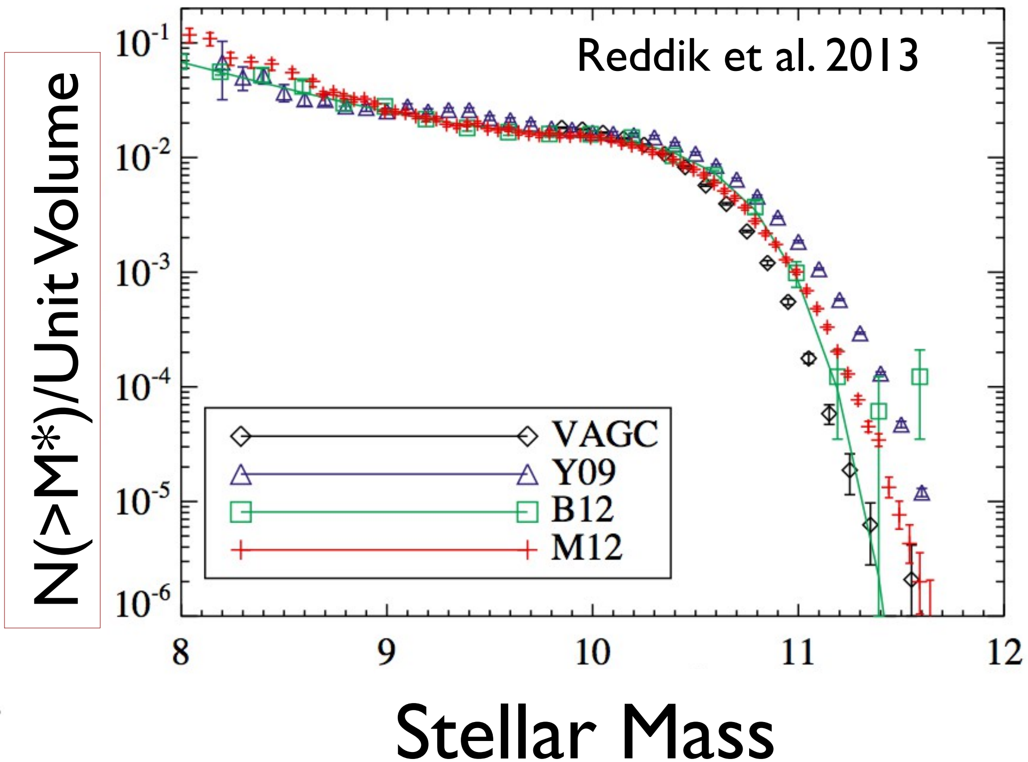


Haloes and Galaxies



Haloes and Galaxies

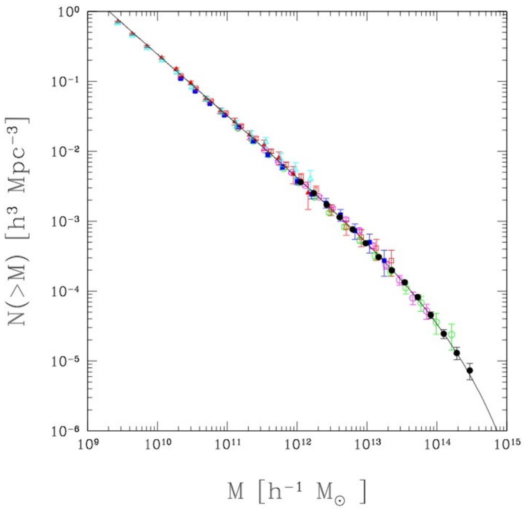


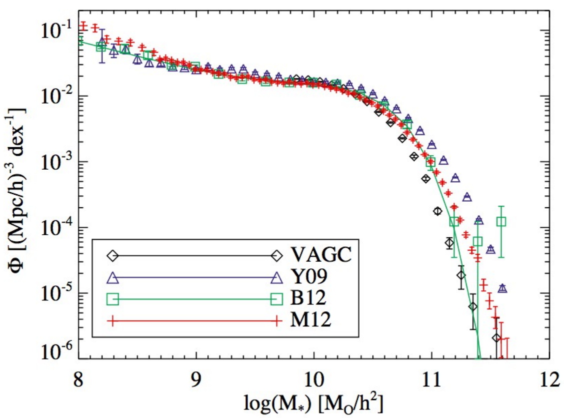


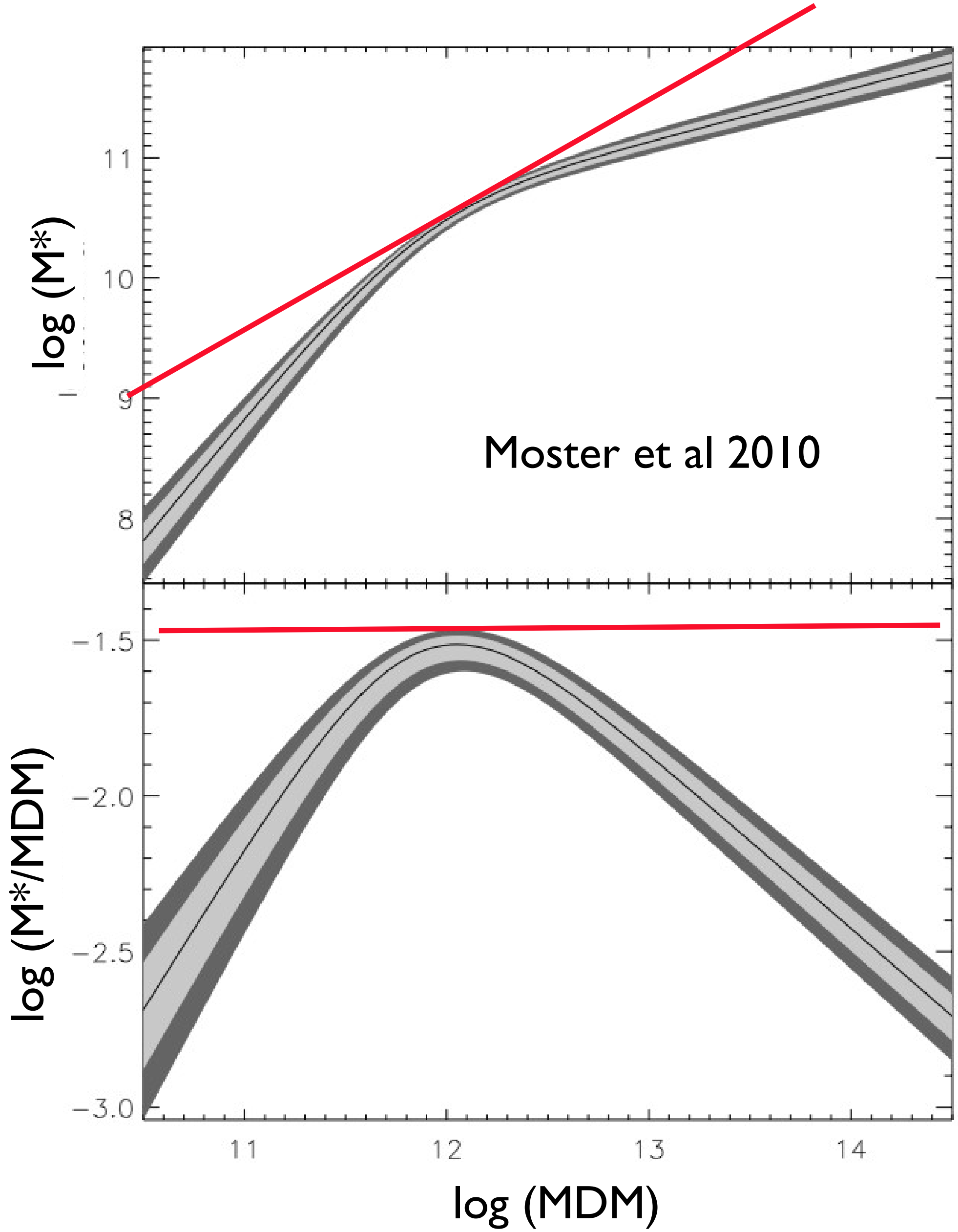
Dark Matter haloes mass function
PREDICTED

Galaxy Stellar Mass Function
OBSERVED

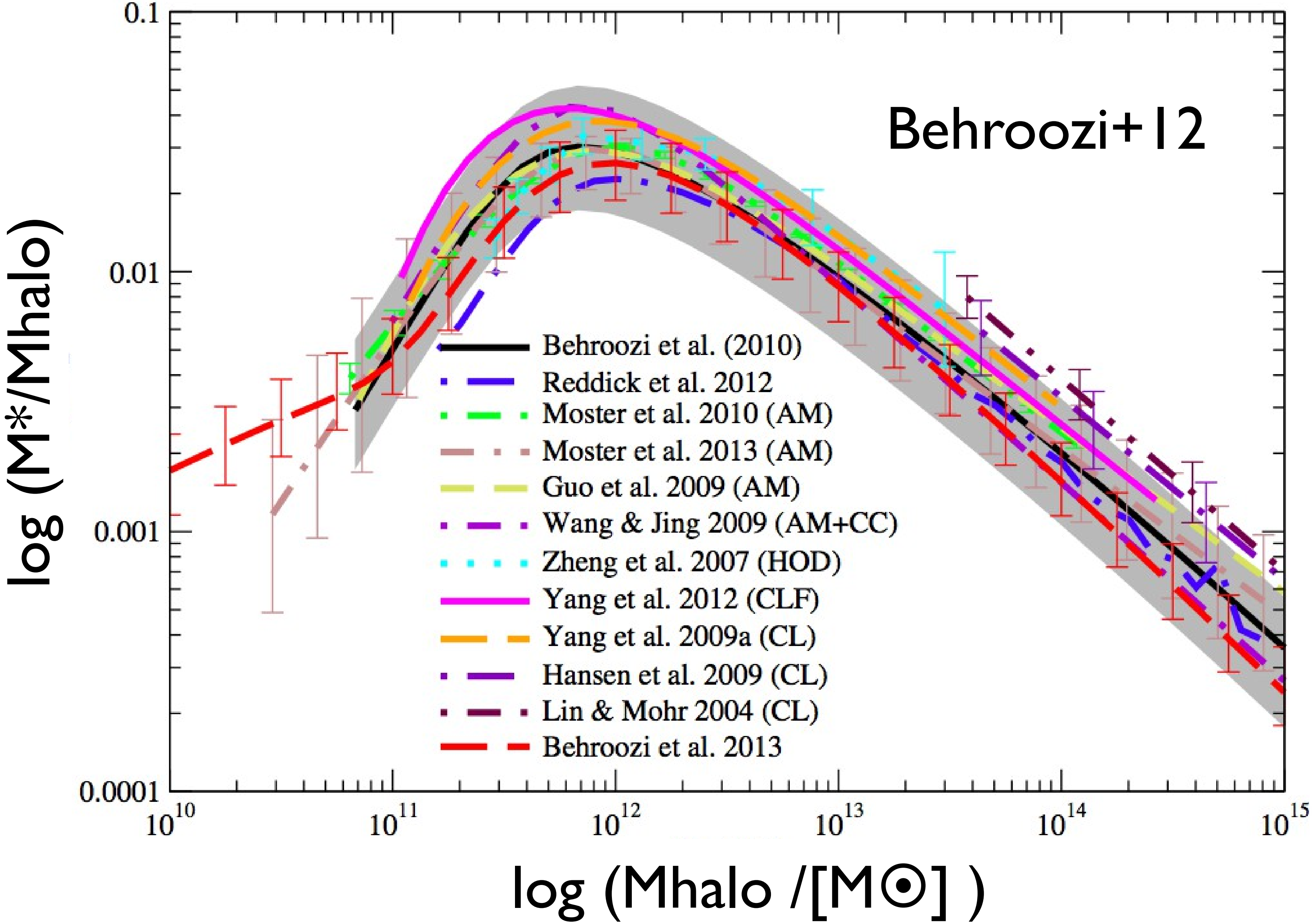
The halo mass – stellar mass relation







Halo mass – stellar mass relation



End of the first part

Next: Structure of DM haloes

Questions?