

# “Hands on”

# Transverse variables

## References

- [1] A. J. Barr, T. J. Khoo, P. Konar, K. Kong, C. G. Lester, K. T. Matchev, and M. Park, “Guide to transverse projections and mass-constraining variables,” *Phys. Rev. D* **84** no. 9, (Nov., 2011) 095031, [arXiv:1105.2977 \[hep-ph\]](#).
- [2] A. Barr, C. Lester, and P. Stephens, “A variable for measuring masses at hadron colliders when missing energy is expected  $m_{T2}$ : the truth behind the glamour,” *Journal of Physics G Nuclear Physics* **29** (Oct., 2003) 2343–2363, [arXiv:hep-ph/0304226](#).
- [3] J. Smith, W. van Neerven, and J. Vermaseren, “The transverse mass and width of the W boson,” *Phys.Rev.Lett.* **50** (1983) 1738.
- [4] K. Agashe, D. Kim, D. G. E. Walker, and L. Zhu, “Using  $M_{T2}$  to distinguish dark matter stabilization symmetries,” *Phys. Rev. D* **84** no. 5, (Sept., 2011) 055020, [arXiv:1012.4460 \[hep-ph\]](#).
- [5] G. F. Giudice, B. Gripaios, and R. Mahbubani, “Counting dark matter particles in LHC events,” *Phys. Rev. D* **85** no. 7, (Apr., 2012) 075019, [arXiv:1108.1800 \[hep-ph\]](#).

# An old friend

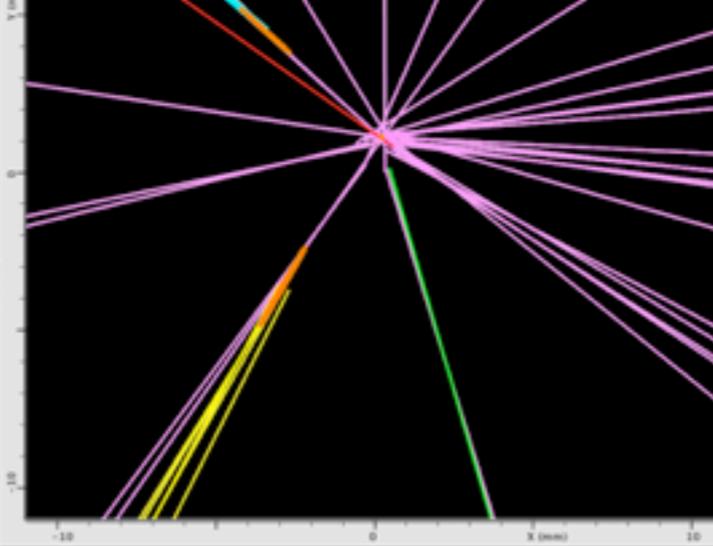
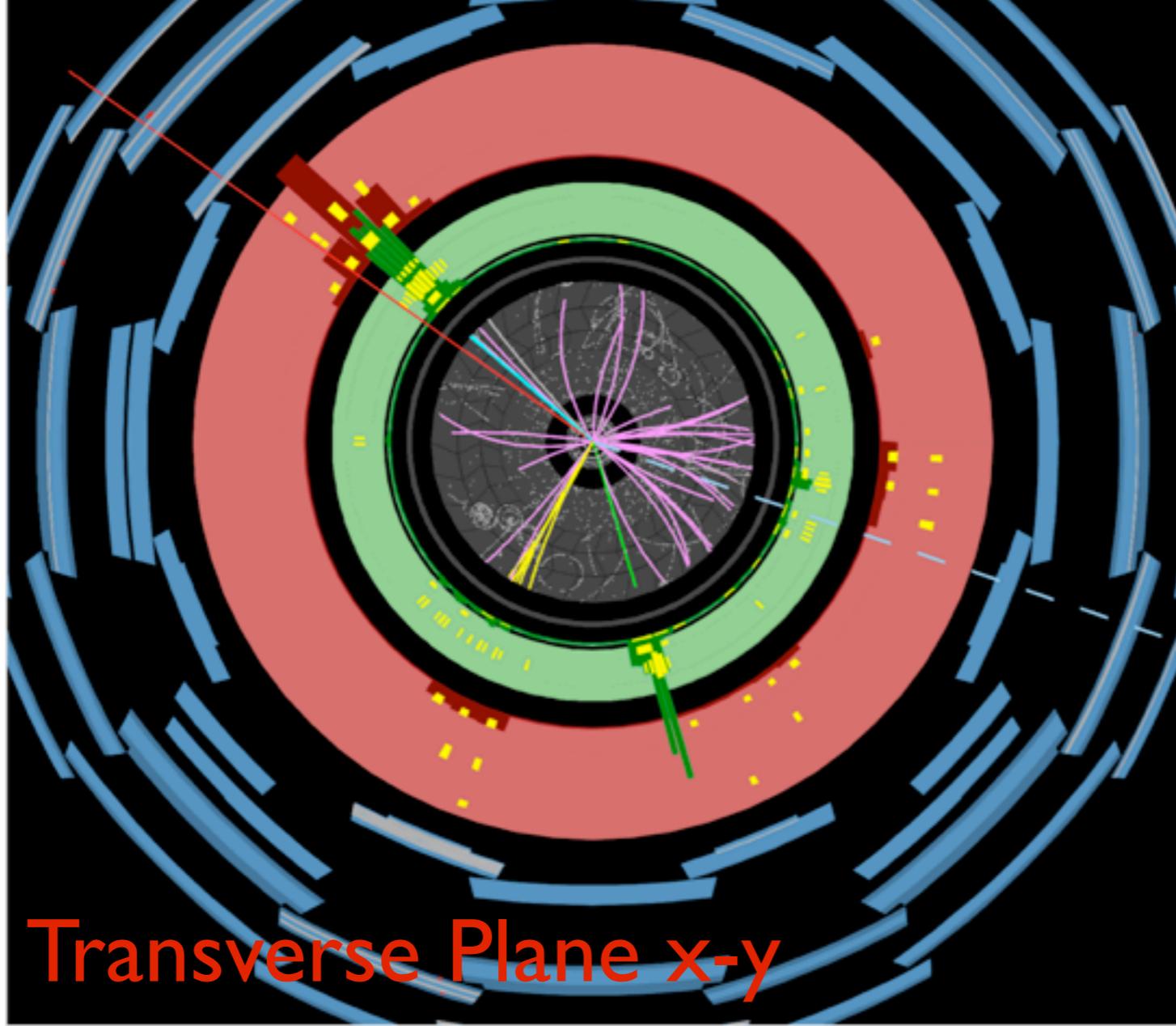
$$\begin{aligned} p_\mu &= \{E, p_x, p_y, p_z\} \\ &= \{E, p \sin \theta \sin \phi, p \sin \theta \cos \phi, p \cos \theta\} \end{aligned}$$

# A new (much better) friend

$$p_\mu(y, \phi, p_T, m) = \left\{ \cosh(y) \sqrt{m^2 + p_T^2}, p_T \sin(\phi), p_T \cos(\phi), \sinh(y) \sqrt{m^2 + p_T^2} \right\}$$

$$y = \frac{1}{2} \log \frac{E + p_z}{E - p_z} = \frac{1}{2} \log \frac{E + p \cos \theta}{E - p \cos \theta},$$

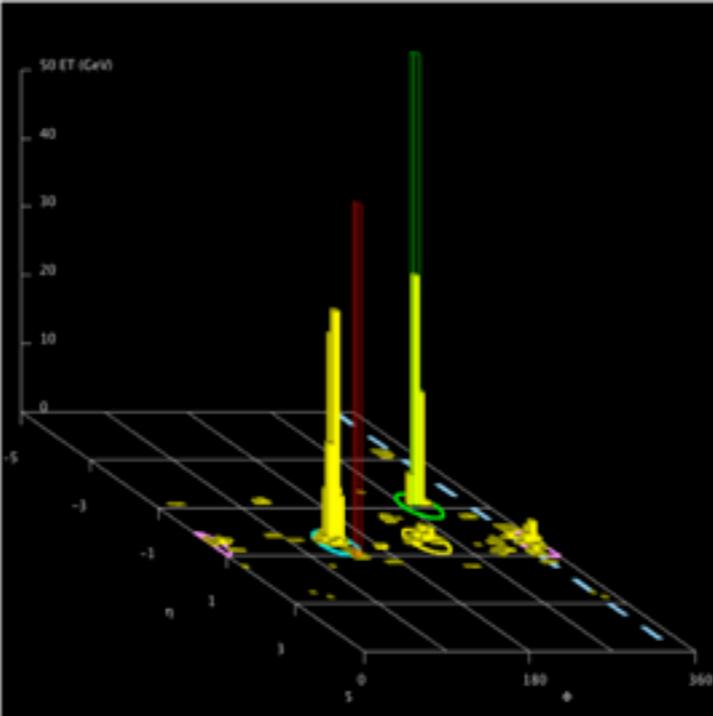
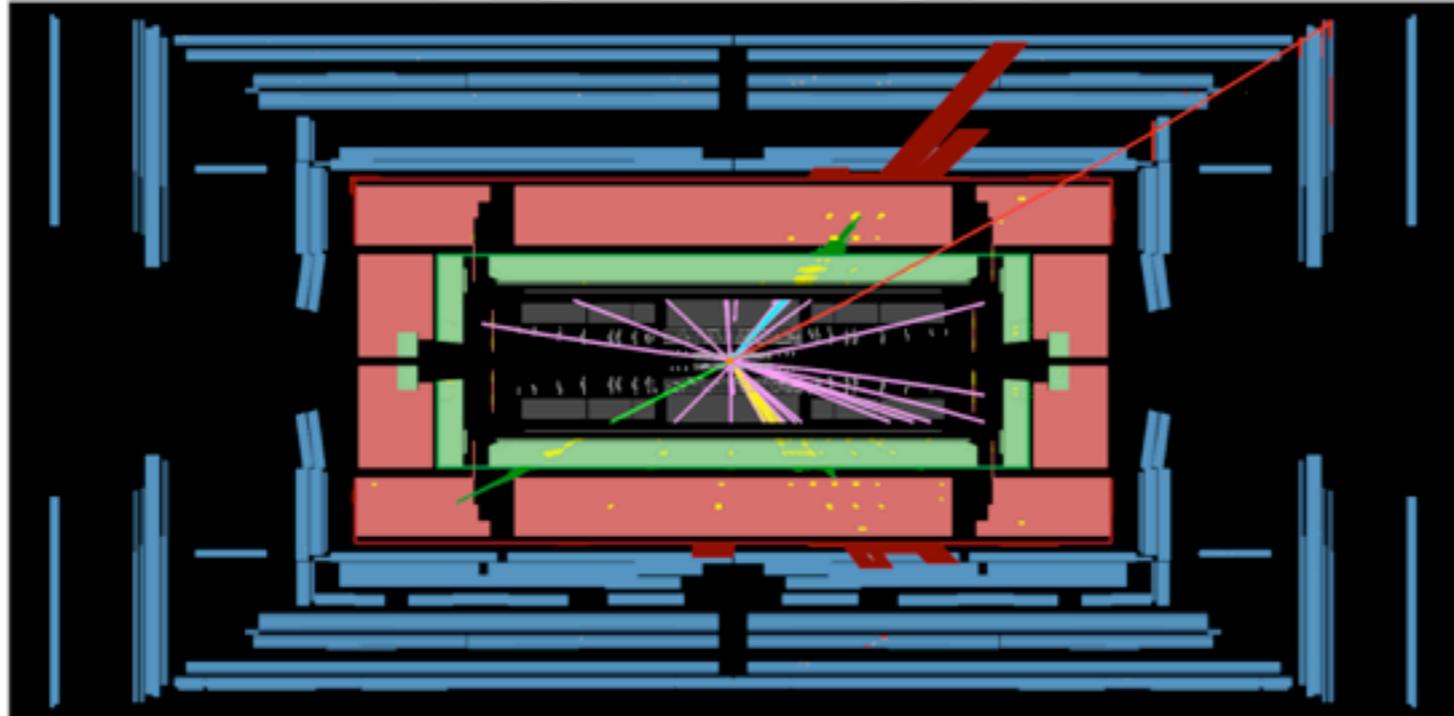
$$y \rightarrow y + y_{boost}, \text{ where } \cosh y_{boost} = \gamma, \sinh y_{boost} = \gamma\beta.$$




**ATLAS**  
**EXPERIMENT**

Run Number: 160958, Event Number: 9038972  
 Date: 2010-08-08 11:01:12 BST

Transverse Plane x-y



# A decay

$$a \rightarrow bc$$

$$p_{\mu}^{(a)} = p_{\mu}^{(b)} + p_{\mu}^{(c)}$$

$$\begin{aligned} m_a^2 &= p^{(a)} \cdot p^{(a)} = \left( p_{\mu}^{(b)} + p_{\mu}^{(c)} \right)^2 \\ &= 2 \cosh \Delta y \sqrt{p_{T,b}^2 + m_b^2} \sqrt{p_{T,c}^2 + m_c^2} - 2 \cos \Delta \phi p_{T,b} p_{T,c} + m_b^2 + m_c^2. \end{aligned}$$

# in presence of invisible particles

no complete longitudinal momentum information  
(partonic center of mass is not know)

zero transverse momentum for the initial state  
is still it is a good approximation

# Mass-preserving transverse projection

$$\{p_x, p_y, p_z\} \rightarrow \{p_x, p_y\}.$$

$$E = \sqrt{m^2 + p_x^2 + p_y^2 + p_z^2} \rightarrow E_T = \sqrt{m^2 + p_x^2 + p_y^2}$$

**(1+2)-vector**

$$\tilde{p}_\alpha(\phi, p_T, m) = \left\{ \sqrt{m^2 + p_T^2}, p_T \sin(\phi), p_T \cos(\phi) \right\}$$

$$\tilde{p} \cdot \tilde{p} = m^2$$

$$\eta_T = \{1, -1, -1\}$$

# A decay

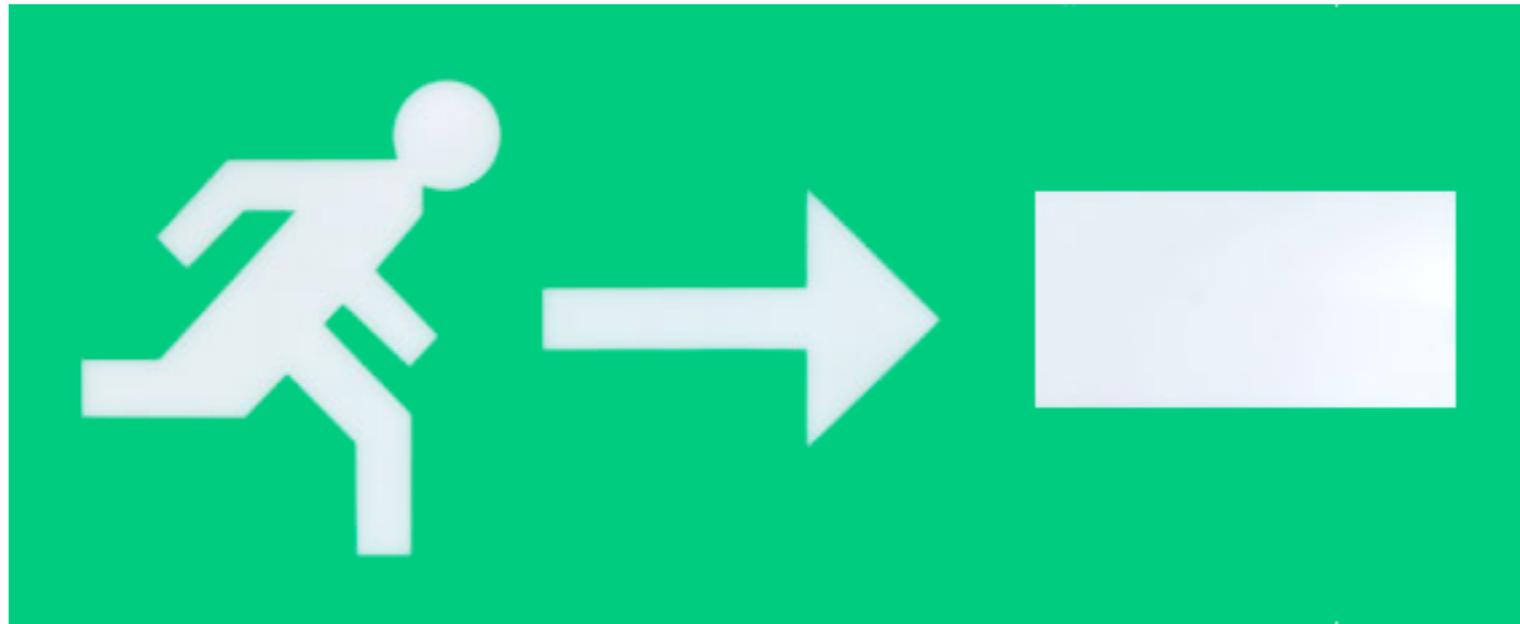
$$a \rightarrow bc$$

$$\begin{aligned}\tilde{m}_{bc}^2 &= \left( \tilde{p}_\alpha^{(b)} + \tilde{p}_\alpha^{(c)} \right)^2 \\ &= 2\sqrt{p_{T,b}^2 + m_b^2}\sqrt{p_{T,c}^2 + m_c^2} - 2\cos(\Delta\phi)p_{T,b}p_{T,c} + m_b^2 + m_c^2 \leq m_a^2\end{aligned}$$

$$\tilde{m}_{bc} = m_{bc} \quad \text{for } y_b = y_c$$

decay products have equal rapidity  
“ $y$ ”

# W decay



# rapidity alignment

rapidity is shifted under boosts

$$y \rightarrow y + y_{boost}$$

a boost along  $z$  brings the two particles to a frame where

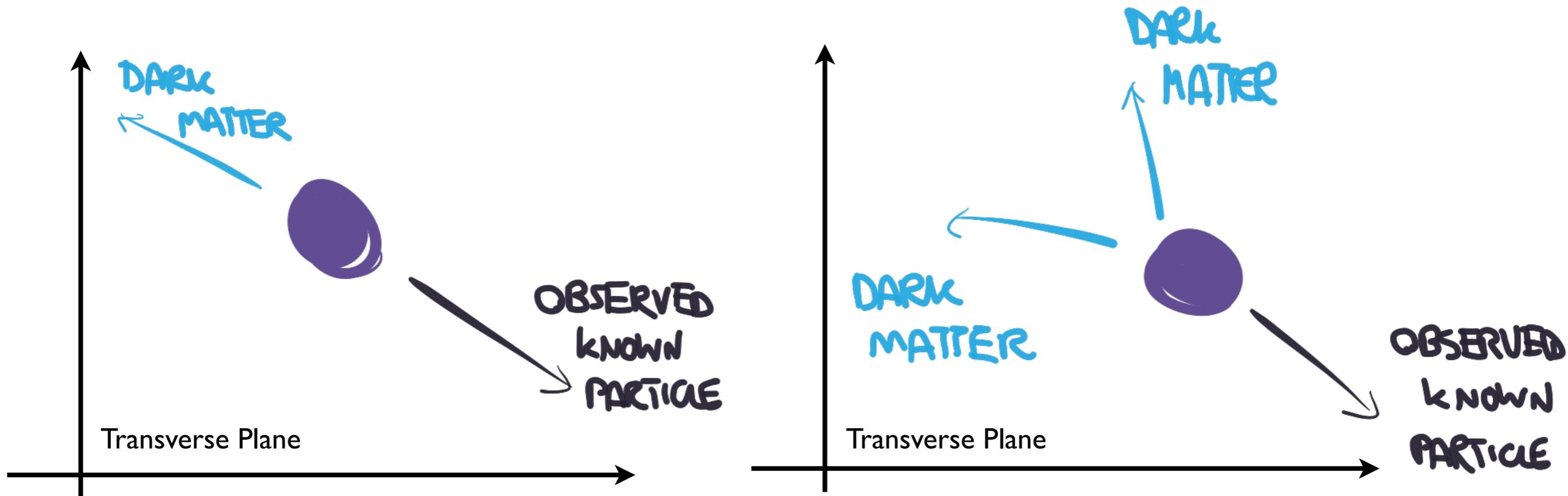
$$p_z = 0$$

*i.e.* they have momentum only along the transverse plane

this is clearly the case where the full invariant mass can be computed using only transverse quantities

# two invisible particles

## ... many troubles



$$\vec{i}_T = \sum_{i=\text{invisibles}} \vec{p}_{T,i} = - \sum_{v=\text{visibles}} \vec{p}_{T,v}$$

only the sum of the invisibles can be measured by the recoiling observable system

$$pp \rightarrow XX \rightarrow Y\chi Y\chi$$

**guess**

what you cannot measure

$$\bar{p}_{T,\chi_1} + \bar{p}_{T,\chi_2} = \dot{i}_T$$

**compute**

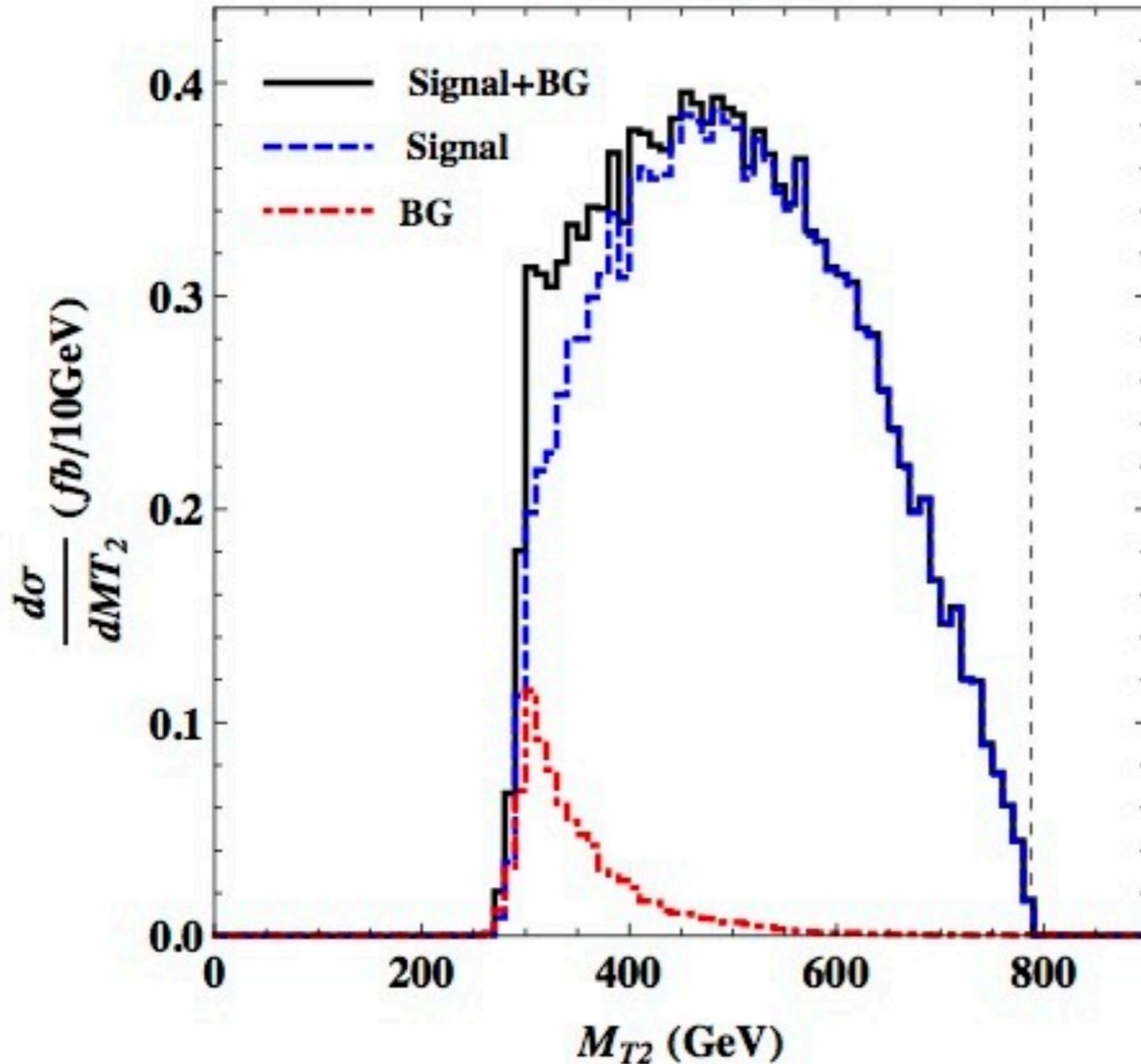
2 transverse masses per each event

$$\max(\tilde{m}_{Y_1\chi_1}, \tilde{m}_{Y_2\chi_2}) < m_X$$

**make the guess safe**

$$m_{T2} \equiv \min_{\text{ansatz on } p_{T,\chi}} (\max(\tilde{m}_{Y_1\chi_1}, \tilde{m}_{Y_2\chi_2}))$$

### Signal ( $Z_2$ )+BG



$$m_{T2}^{(max)} = \max_{events} m_{T2}$$

To compute transverse mass you need to know the mass of the DM ..

make a guess

$$m_{\chi,trial}$$

$$m_{T2}^{(max)}(m_{\chi,trial}) = C + \sqrt{C^2 + m_{\chi,trial}^2}$$

# cluster the invisibles

N invisible particles can always be thought as a single invisible object

$$P_{\mu,inv} = \sum_{i=invisibles} P_{\mu,i}$$

$$M_{cluster,inv} \geq \sum_{i=invisibles} m_i \quad \text{and} \quad M_{cluster,inv} < m_X$$

the minimal mass is attained for invisible particles at rest,  
hence they have the same rapidity in a boosted frame

$$m_{T2}^{(max)}(m_{\chi,trial}) = C + \sqrt{C^2 + m_{\chi,trial}^2}$$

the “C” parametrization holds for one as well as for many invisibles, though the meaning of “C” is different