Institut de Physique théorique (IPhT), CEA Saclay
M2 Internship proposal
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Title: **The Haldane-Shastry spin chain at root of unity**

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Quantum spin chains are usually studied under the assumption that only nearest-neigbouring spins interact, such as for the Heisenberg spin chain. However, in reality spins that are further apart may interact as well. Long-range interactions are for example important in cold-atom experiments, and possibly for quantum computing. The special class of exactly-solvable / quantum-integrable models with long-range interactions offer the exciting opportunity to test which paradigms for nearest-neighbour models survive in the presence of long-range interactions. These models moreover possess a beautiful mathematical structure.

An important example of a long-range spin chain was found independently by Nobel laureate Haldane and by Shastry in 1988. The energy spectrum of this Haldane--Shastry spin chain is remarkably simple, and the corresponding wave functions can be given exactly, in terms of so-called Jack polynomials. From a physical point of view this is one of the simplest models that exhibits fractional statistics in one dimension. This model is quantum integrable, with an elegant underlying algebraic structure, the Yangian, that resembles the one for the Heisenberg XXX spin chain up to some crucial differences.

Recently progress was made in solving the XXZ-like version of the Haldane--Shastry spin chain. Its hamiltonian is more complicated, but in such a way that most of the special properties of the Haldane--Shastry spin chain are preserved. The goal of this project is to study the XXZ-like Haldane--Shastry spin chain for the case where the parameter q, which sets the anisotropy, is a root of unity. This regime is expected to be most interesting physically. Here the model simplifies significantly, and the eigenvalues appear to exhibit a neat combinatorial structure. This project seeks to investigate this in more detail, with special emphasis on the values $q=i$, which should be related to free fermions, and $q = e^{2\pi/3}$, which might exhibit supersymmetry on the lattice.