OPEN STRINGS

AND

SPIN CHAINS

WITH BOUNDARIES

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&
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"INTEGRABILITY IN GAUGE & STRING THEORY"

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• PLANAR LIMIT OF $\mathcal{N}=4$ SYM

GIVES RISE TO A SPIN CHAIN

SPECTRUM OF OPERATORS = SPECTRUM OF THE SPIN CHAIN

$\Delta = E$

$\updownarrow$ GAUGE/STRING DUALITY

STRINGS IN $AdS_5 \times S^5$

CLOSED STRINGS $\leftrightarrow$ SPIN CHAINS WITH PERIODIC BOUNDARY CONDITIONS

OPEN STRINGS

D-BRANES

- D7 BRANES ↔ EXTRA FLAVORS.

HERE →
- D3 BRANES on $S^3 \subset S^5$

  " GIANT GRAVITONS

- WILSON LOOPS

OPERATORS
ON
WILSON LINES

- HOW DO WE DESCRIBE THESE SPIN CHAINS & THEIR BOUNDARIES?

LARGE LITERATURE ON ONE LOOP COMPUTATIONS IN GAUGE THEORY
(2)
AND ALSO ON THE PLANE WAVE & NEAR PLANE WAVE REGIME

Open strings and pp-waves, etc

D. Berenstein, E. Gava, J. Maldacena, K. S. Narain and H. Nastase, Open strings on plane waves and their Yang-Mills duals, arXiv:hep-th/0203249. A. Dabholkar and S. Parvizi, Dp branes in pp-wave background, arXiv:hep-th/0203231. K. Skenderis and M. Taylor, Branes in AdS and pp-wave spacetimes, arXiv:hep-th/0204054 C. S. Chu and P. M. Ho, Noncommutative D-brane and open string in pp-wave background with B-field, arXiv:hep-th/0203186; A. Kumar, R. R. Nayak and Sanjay, D-brane solutions in pp-wave background, arXiv:hep-th/0204025.

Open strings from giant gravitons

D. Berenstein, C. P. Herzog and I. R. Klebanov, Baryon spectra and AdS/CFT correspondence, arXiv:hep-th/0202150. S. Das, A. Jevicki, S. Mathur, Vibration modes of giant gravitons Phys. Rev. D63 (2001) 024013, hep-th/0009019. Open strings from N=4 superYang-Mills. Vijay Balasubramanian, Min-xin Huang, Thomas S. Levi, Asad Naqvi JHEP 0208:037,2002. hep-th/0204196 D. Berenstein, Shape and holography: Studies of dual operators to giant gravitons, Nucl. Phys. B 675 (2003) 179 [arXiv:hep-th/0306090]. V. Balasubramanian, D. Berenstein, B. Feng and M. x. Huang, D-branes in Yang-Mills theory and emergent gauge symmetry, [arXiv:hep-th/0411205] Integrable open spin chains from giant gravitons. David Berenstein , Samuel E. Vazquez HEP 0506:059,2005. hep-th/0501078 Quantizing open spin chains with variable length: An Example from giant gravitons. David Berenstein , Diego H. Correa , Samuel E. Vazquez Phys.Rev.Lett.95:191601,2005. hep-th/0502172 A Study of open strings ending on giant gravitons, spin chains and integrability. David Berenstein , Diego H. Correa , Samuel E. Vazquez, JHEP 0609:065,2006. e-Print: hep-th/0604123

Defect CFTs and D-branes

O. DeWolfe and N. Mann, Integrable open spin chains in defect conformal field theory, JHEP 0404, 035 (2004) [arXiv:hep-th/0401041]. B. Chen, X. J. Wang and Y. S. Wu, Integrable open spin chain in super Yang-Mills and the plane-wave / SYM duality, JHEP 0402, 029 (2004) [arXiv:hep-th/0401016]. B. Chen, X. J. Wang and Y. S. Wu, Open spin chain and open spinning string, Phys. Lett. B 591, 170 (2004) [arXiv:hep-th/0403004]. Open spinning strings and AdS/dCFT duality. Keisuke Okamura , Yastoshi Takayama , Kentaroh Yoshida JHEP 0601:112,2006. hep-th/0511139

Open strings and operators on Wilson loops

Small deformations of supersymmetric Wilson loops and open spin-chains. Nadav Drukker , Shoichi Kawamoto JHEP 0607:024,2006. hep-th/0604124

Two loops

A. Agarwal, Open spin chains in super Yang-Mills at higher loops: Some potential problems with integrability, arXiv:hep-th/0603067. K. Okamura and K. Yoshida, Higher Loop Bethe Ansatz for Open Spin-Chains in AdS/CFT, arXiv:hep-th/0604100.

Near pp-wave

Open string integrability and AdS/CFT. Tristan McLoughlin, Ian J. Swanson Nucl.Phys.B723:132-162,2005. e-Print: hep-th/0504203

Classical integrability

N. Mann and S. E. Vazquez, "Classical open string integrability," JHEP **0704**, 065 (2007) [arXiv:hep-th/0612038].

REVIEW

- OPERATORS WITH LARGE CHARGE

BERENSTEIN
JM
NASTASE

$$\mathrm{Tr}[Z^J] \qquad J \in SO(2) \subset SO(6)$$

$$\mathrm{Tr}[W Z^{l} Y Z^{J-l}]$$

$$\mathrm{Tr}[\partial_1 Z Z^{l} \partial_2 Z Z^{m} \dots]$$

STAUDACHER

- INFINITE J LIMIT ; $J \to \infty$
- FINITE # OF "IMPURITIES"

$$\dots\dots ZZWZZ\dots\dots$$
$$\xrightarrow{p}$$

- SYMMETRY

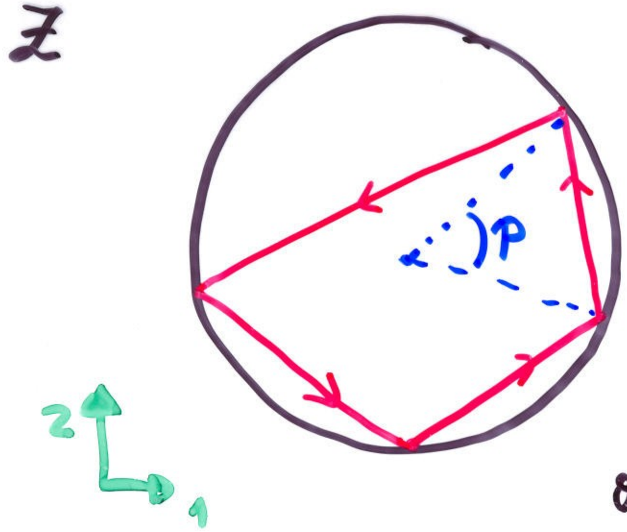
$$SU(2|2)^2 \subset PSU(2,2|4)$$

BEISERT

$$\Big\downarrow \text{ 2 EXTRA CENTRAL CHARGES}$$

$$SU(2|2)^2 \ltimes \mathbb{R}^2$$

. SEMICLASSICAL PICTURE



HOFMAN
J.M.
BERENSTEIN
CORREA
VAZQUEZ

CENTRAL CHARGES
$\approx$
WINDING CHARGES

$$\mathcal{O} \sim \mathrm{Tr}\left[\underset{\vec{p}_1}{\phi_1} Z^{J_1} \underset{\vec{p}_2}{\phi_2} Z^{J_2} \underset{\vec{p}_3}{\phi_3} Z^{J_3} \underset{\vec{p}_4}{\phi_4} Z^{J_4}\right]$$

. SINGLE IMPURITY IS IN A BPS REPRESENTATION

BEISERT
DIPPEL
STAUDACHER

BEISERT

$$E(p) = \sqrt{1 + \frac{\lambda}{\pi^2} \sin^2 \frac{p}{2}}$$

. SCATTERING AMPLITUDE (ASYMPTOTIC)

$$Z Z Z \underset{\vec{p}_1}{W} Z Z Z \underset{\overleftarrow{p}_2}{\psi} Z Z Z \ldots$$

$$S \cdot Z Z Z \underset{\overleftarrow{p}_2}{\partial Z} Z \cdots Z \underset{\vec{p}_1}{\psi} Z \cdots Z$$

- SINGLE IMPURITY

8 BOSONS $\begin{cases} 4 \text{ SCALARS} \\ 4\ \partial_i Z \end{cases}$

\+ 8 FERMIONS

$\Delta - J = 1$

- 2 IMPURITIES WITH GENERIC MOMENTA

$\#$: $16 \times 16 = (2^4)^2$

DIM OF SMALLEST NON BPS REPRESENTATION OF $SU(2|2)^2$ IS 16^2

$\Rightarrow$ MATRIX STRUCTURE OF S MATRIX IS FIXED

BEISERT

$$S_{AB}{}^{CD}(p_1, p_2) = \underbrace{S_0(p_1, p_2)}_{\text{CROSSING EQUATION}} \hat{S}_{AB}{}^{CD}(p_1, p_2)$$

JANIK

BEISERT - HERNANDEZ - LOPEZ - EDEN - STAUDACHER -

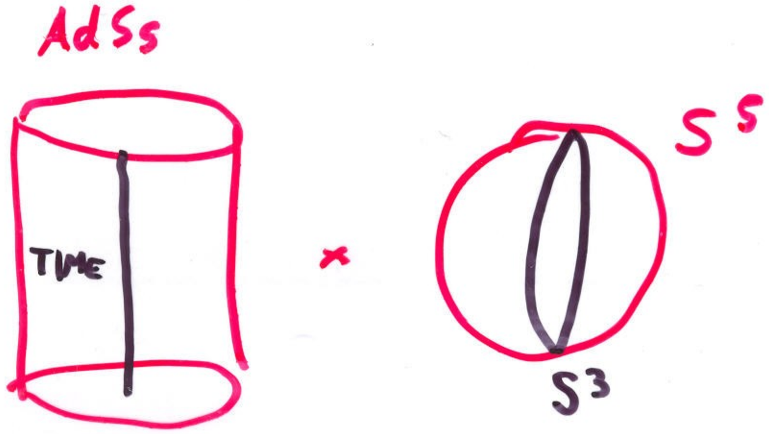
. DO THE SAME FOR OPEN STRINGS.

. WE WILL HAVE BOUNDARIES

. Integrable ?

. REFLECTION MATRICES.

. WE WILL STUDY THIS FOR (MAXIMAL) GIANT - GRAVITON D3 BRANES



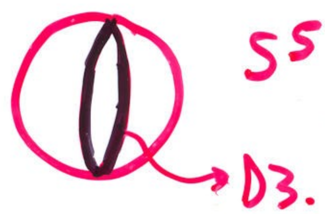
. GIANT GRAVITON D3 BRANES

S^5: $|Z|^2+|W|^2+|Y|^2=1$

— D3 ON S^3 GIVEN BY $W=0$

$\updownarrow$

OPERATOR $\det W$

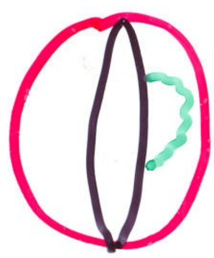


BALASUBRAMANIAN
BERKOOZ
NAQVI
STRASSLER

HUANG
LEVI

COREY
JEVICKI
RANGOOLAM

— D-3 BRANE PLUS OPEN STRINGS



$$\mathcal{O}=\varepsilon^{j_1\cdots j_{N-1}\,j}_{i_1\cdots i_{N-1}\,i}\,W^{i_1}_{j_1}W^{i_2}_{j_2}\cdots W^{i_{N-1}}_{j_{N-1}}\left(ZZ\cdots WZ\cdots ZY\cdots Z\right)^{i}_{j}$$

. CONSIDER J LARGE

$(\text{BOUNDARY}|\,Z\cdots Z\,Y\cdot ZZ\;YZZ\,|\,\text{BOUNDARY})$

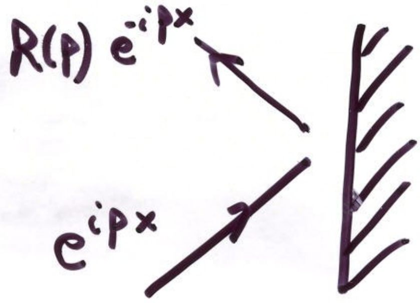
. LARGE N COUNTING $\Rightarrow$ BEHAVES AS A CHAIN WITH BOUNDARIES.

BERENSTEIN
HERZOG KLEBANOV
- BALASUBRAMANIAN, HUANG, LEVI, NAQVI - BERENSTEIN - CORREA - VAZQUEZ

- SYMMETRIES

$$SU(2|2)^2 \ltimes \mathbb{R}^2 \longrightarrow SU(1|2)^2$$

- BOUNDARY S-MATRIX



- BERENSTEIN
  VASQUEZ.
  CORREA
- AGARWAL
- OKAMURA
  YOSHIDA

- ASSUME : - NO BOUNDARY DEGREE OF FREEDOM
  - $1 \to 1$ SCATTERING

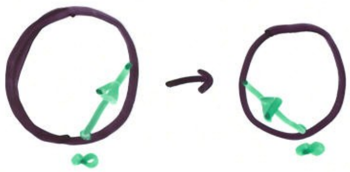
- $16 = \left(\text{DIMENSION OF SMALLEST NON BPS REPRESENTATION OF } SU(1,2)\right)^2$

- EACH $SU(2|2)$ & $SU(1|2)$ FACTOR

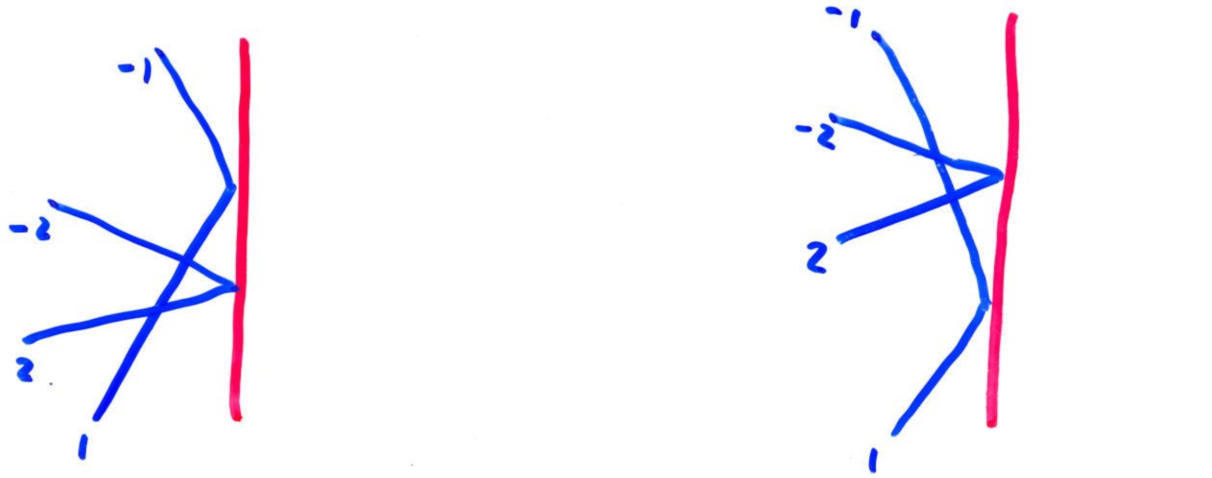
$$\begin{pmatrix} \phi^{\pm} \\ \psi^{\pm} \end{pmatrix} : 4 \text{ STATES}$$

SYMMETRY FIXES:

$$R(p) = R_0(p) \begin{array}{c} \begin{array}{cccc} \phi^+ & \phi^- & \psi^+ & \psi^- \end{array} \\ \begin{pmatrix} -e^{-ip} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array} \begin{array}{c} \phi^+ \\ \phi^- \\ \psi^+ \\ \psi^- \end{array}$$



. OBEYS THE BOUNDARY YANG BAXTER EQUATION:



USE $S_0(1,2) = \frac{1}{S_0(2,1)} = \frac{1}{S_0(-1,-2)}$

CONSISTENT WITH INTEGRABILITY

$$\cdot\ R_0^2 = \begin{cases} -e^{2ip} & \text{UP TO 2 LOOPS} \\ \sim e^{i\, 2\frac{\sqrt{2}}{\pi}\cos\frac{p}{2}\log\left[\cos\frac{p}{2}\right]} & ;\ \lambda \gg 1 \end{cases}$$

fixed AGARWAL'S PAPER

- CLASSICAL σ-MODEL WITH BDY → INTEGRABLE

MANN VAZQUEZ

CROSSING EQUATION

$$\mathbb{1} = \underset{p\,:\,\bar{p}}{\bullet\;\bullet}$$

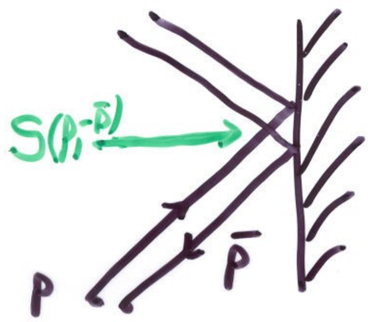
- SCATTER THIS FROM ANOTHER PARTICLE

BEISERT & DEMAND THAT PHASE IS ONE

(JANIK) $\Rightarrow$ BULK CROSSING EQUATION FOR S_0

- DO THE SAME SCATTERING FROM THE BOUNDARY

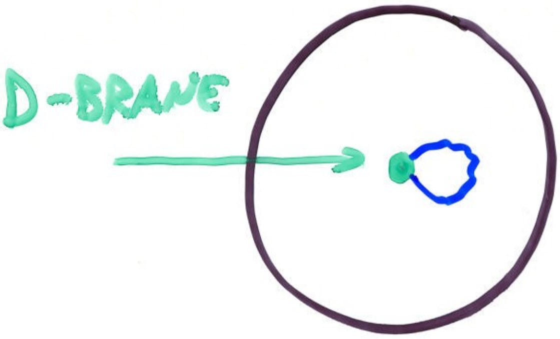
$$R_0(1)\,R_0(\bar{1})\;S_0(1,-\bar{1}) = \frac{x^-}{x^+}\,\frac{(1+x^+)^2}{(1+x^-)^2}$$



ANOTHER GIANT GRAVITON

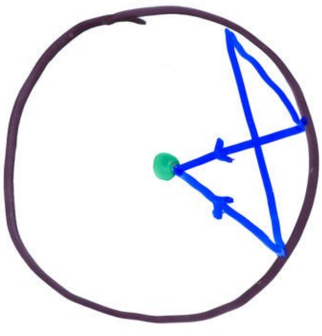
$$|Z|^2 + |W|^2 + |Y|^2 = 1$$

SET $Z = 0$ $\longleftrightarrow$ $\det Z$



$$\varepsilon^{\cdots a}_{\cdots b} Z^{\cdots}_{\cdots} (W Z^J Y)_a{}^b$$

LARGE J:

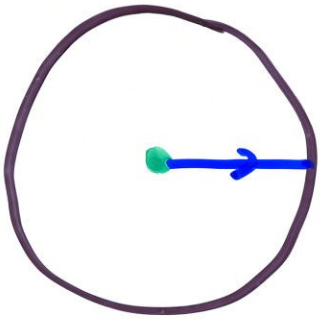


• WE HAVE BOUNDARY DEGREES OF FREEDOM

. SYMMETRY

$$SU(2|2)^2$$

. BOUNDARY DEGREE OF FREEDOM



EXTRA CENTRAL CHARGE

$|\vec{k}| =$ FIXED.

. ENERGY

(VALID ONLY FOR LARGE J)

$$E = \sqrt{1 + \frac{\lambda}{4\pi^2}}$$

✓ 2 LOOPS

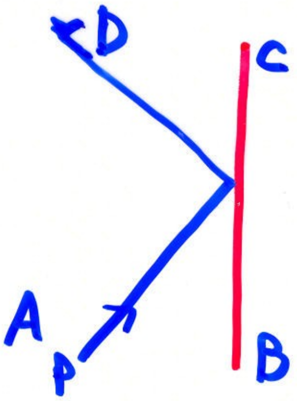
✓ STRONG COUPLING

DOREY

. BPS BOUND STATES

$$E = \sqrt{M^2 + \frac{\lambda}{4\pi^2}}$$

S-MATRIX



$$R_{AB}^{CD}(p) = R_0 \times \hat{R}_{AB}^{CD}$$

. MULTIPLETS → SAME AS IN THE BULK CASE

→ BOUNDARY PARTICLE → SPECIAL CENTRAL CHARGE

⇝ SIMILAR FORM FOR THE S-MATRIX.

. SHOULD BE RELATED TO THE PREVIOUS ONE BY :

$$\det Z\, W^L \xrightarrow[\text{FILL } Z\text{'s}]{} \det Z\,(W Z^L \ldots)$$

CONCLUSIONS

- BOUNDARIES CAN BE ADDED
- NO PROBLEM WITH INTEGRABILITY
- SYMMETRIES DETERMINE A GREAT DEAL

FUTURE

- OTHER CASES, WILSON LOOPS
- SOLVE CROSSING EQUATION & FIX $\mathcal{R}_0(p)$
- NON-MAXIMAL GIANTS? BERENSTEIN-CORREA-VAZQUEZ

• STRONG COUPLING

$$ds^2_{AdS_5} = \cosh^2\rho \, ds^2_{AdS_3} + \sinh^2\rho \, d\varphi^2 + d\rho^2$$

$$ds^2_{AdS_3} = -du^2 + dx^2 + d\sigma^2 + 2\,\mathrm{sh}\,2\sigma \, du\, dx$$

STRING AT $\sigma = 0$, $\rho = 0$

$$\text{ACTION} \approx \frac{R^2}{2\pi\alpha'} \int du\, dx$$

→ GIVES DIRECTLY THE STRONG COUPLING ANSWER.

1-LOOP

- BREAK SUSY
  ⇒ 8 FERMIONS OF MASS $m = 1$.
- BREAK SOME ISOMETRIES OF AdS_3
  1 BOSON OF MASS $m = 2$
- DIRECT COMPUTATION FOR $\rho e^{i\varphi}$
  $m^2 = 2$.
- 5 MASSLESS DIRECTIONS IN S^5.