WRAPPING INTERACTIONS AND VIRTUAL CORRECTIONS

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1. INTRODUCTION AND MOTIVATION
2. RELATIVISTIC INTEGRABLE QFT ON A CYLINDER
   - LÜSCHER FORMULAS
   - THERMODYNAMIC BETHE ANSATZ
3. TBA MOTIVATED AdS EXPONENTIAL TERMS
   - WRAPPING
   - GIANT MAGNON
   - SPINNING STRING
4. PREFACTOR - GIANT MAGNON
5. DISCUSSION

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+ WORK IN PROGRESS

$\mathcal{N}=4$ SYM $\equiv$ SUPERSTRINGS IN $AdS_5 \times S_5$

GAUGE THEORY

ANOMALOUS DIMENSIONS

$\parallel\!|$

ENERGY LEVELS OF AN 'ASYMPTOTIC ALL-LOOP' SPIN CHAIN

ENERGIES OF STRING STATES

$\parallel\!|$

ENERGY LEVELS OF A QFT ON A CYLINDER

INTEGRABILITY

EX: SU(2) SECTOR

$$e^{i p_i L} = \prod_{k \neq i} S(p_i, p_k)$$

ASYMPTOTIC S-MATRIX

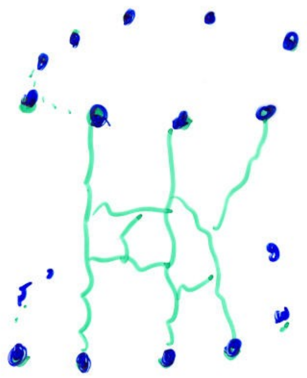
INCLUDING DRESSING FACTOR

INTERPOLATING BETWEEN WEAK AND STRONG COUPLING..

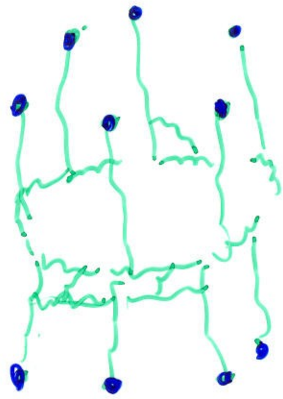
$$\Delta - J = E = \sum_{k=1}^{M} \sqrt{1 + 8 g^2 \sin^2 \frac{p_i}{2}}$$

$$g^2 = \frac{\lambda}{8\pi^2}$$

• CONJECTURALLY INCLUDES ALL CONTRIBUTIONS OF TYPE



BUT DOES NOT INCLUDE WRAPPING INTERACTIONS



← GENERICALLY APPEAR AT g^{2L}

→ BREAKDOWN OF ASYMPTOTIC BETHE ANSATZ

IS IT A BREAKDOWN OF INTEGRABILITY ?

• NO

– BETHE ANSATZ IS ONLY AN APPROXIMATION
GENERIC SITUATION IN INTEGRABLE QFT

→ NEED TO INTRODUCE A DIFFERENT
STRUCTURE INSTEAD OF BETHE ANSATZ

WHY UNDERSTANDING WRAPPING INTERACTIONS IS INTERESTING ?

EXAMPLES:

- FINITE SIZE CORRECTIONS TO THE MAGNON DISPERSION RELATION

ARUTYUNOV, FROLOV, ZAMAKLAR
ASTOLFI, FORINI, GRIGNIANI, SEMENOFF

$$E(p) = \sqrt{1 + \frac{\lambda}{\pi^2} \sin^2 \frac{p}{2}} \qquad J = \infty$$

- FINITE J (LARGE λ)

$$\delta E_{\text{MAGNON}} = -\frac{\sqrt{\lambda}}{\pi} \cdot \frac{4}{e^2} \cdot \sin^3 \frac{p}{2} \cdot e^{-\frac{2\pi J}{\sqrt{\lambda} \sin \frac{p}{2}}} + \dots$$

IN THE HUBBARD MODEL:

$$\delta E_{\text{HUBBARD}} = -\frac{\pi}{\sqrt{\lambda}} \cdot 2 \cdot \frac{1}{\sin \frac{p}{2}} \cdot e^{-\frac{2\pi J}{\sqrt{\lambda} \sin \frac{p}{2}}} + \dots$$

HOW TO UNDERSTAND THESE STRUCTURES ?

- EXPONENTIAL CORRECTIONS FOR SPINNING STRINGS IN $sl(2)$ SECTOR

SCHÄFER-NAMEKI, ZAREMBO, ZAMAKLAR

$$e^{-\frac{2\pi J}{\sqrt{\lambda}}}$$

- TWIST 2 OPERATORS IN THE $sl(2)$ SECTOR

KOTIKOV, LIPATOV, REJ, STAUDACHER, VELIZHANIN

- INCONSISTENCY WITH BFKL EXPECTATIONS AT 4-LOOP ORDER – FIRST ORDER WHERE WRAPPING INTERACTIONS SHOULD CONTRIBUTE (AT WEAK COUPLING)

MORE FUNDAMENTALLY

- UNDERSTAND INTEGRABLE STRUCTURE AT FINITE J

... HOW TO PROCEED?

TWO ROUTES:

GAUGE THEORY PERSPECTIVE

SPIN CHAIN

- XXX WELL DEFINED FOR FINITE LENGTH, BETHE ANSATZ IS EXACT

- HUBBARD REFORMULATION

RE), SERBAN, STAUDACHER

CURRENTLY SU(2) + NO DRESSING

????

STRING WORLDSHEET

INTEGRABLE 2D QFT ON A CYLINDER

FIND ENERGY LEVELS ON A CYLINDER

- BETHE ANSATZ IS ONLY AN APPROXIMATION
- BREAKDOWN OF THE BETHE ANSATZ FOLLOWS FROM THE THEORY

WORLDSHEET THEORY

$\downarrow$

INTEGRABLE 2D QUANTUM FIELD THEORY DEFINED ON A CYLINDER

- DIFFERENT FROM 'STANDARD' RELATIVISTIC INTEGRABLE FIELD THEORIES:
  - → FULL $psu(2,2|4)$ SYMMETRY IS NOT MANIFEST (LIGHT CONE QUANTIZATION)
  - → HAMILTONIAN PART OF SYMMETRY ALGEBRA
  - → NOT LORENTZ INVARIANT

DISPERSION RELATION

$$E(p) = \sqrt{1 + 8 g^2 \sin^2 \frac{p}{2}}$$

BUT E.G. ANALOG OF CROSSING EXISTS

→ EXAMINE GENERIC FEATURES OF A (RELATIVISTIC) 2D INTEGRABLE QFT ON A CYLINDER

RELATIVISTIC INTEGRABLE QFT

- SINGLE PARTICLE SPECIES OF MASS m
- SCATTERING DESCRIBED BY THE S MATRIX

$$S(p_i, p_j)$$

DATA FOR THE THEORY ON AN INFINITE PLANE

→ PUT THE THEORY ON A CYLINDER OF SIZE L

Q: WHAT ARE THE ENERGY LEVELS AS FUNCTIONS OF THE CIRCUMFERENCE L ?

1st GUESS

$$e^{i p_i L} = \prod_{j \neq i} S(p_i, p_j)$$

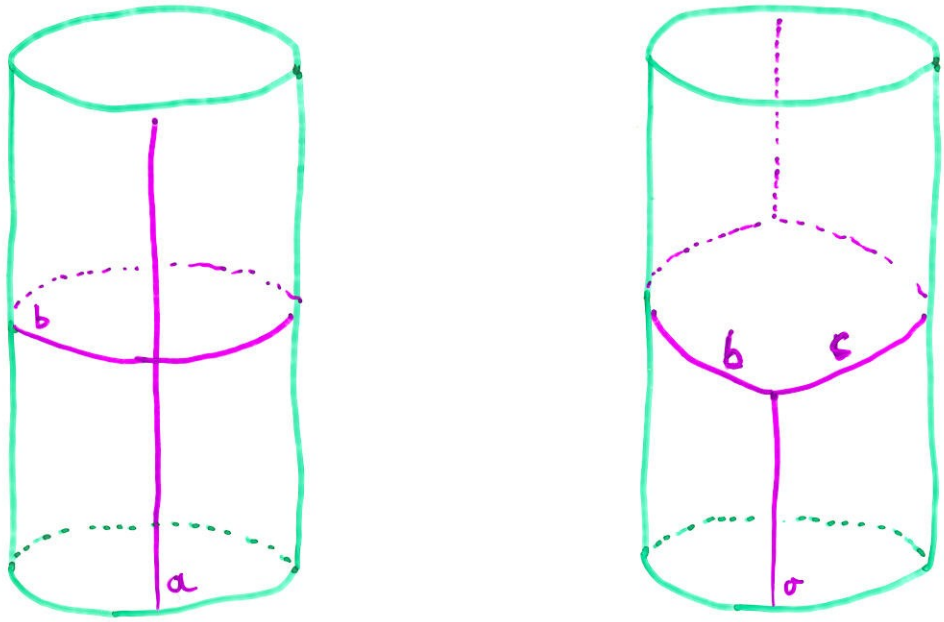
$$E = \sum E(p_i) = \sum \sqrt{m^2 + p_i^2}$$

⤳ THIS IS NOT EXACT !

• SINGLE PARTICLE STATE $(p=0)$

LÜSCHER '83 '86

$E \equiv m$ CHANGES DUE TO VIRTUAL CORRECTIONS



F-TERM

μ-TERM

LEADING ORDER CORRECTIONS

$$m(L) = m(L=\infty) + \Delta m_F(L) + \Delta m_\mu(L) + \dots$$

$$\Delta m_F(L) = -m\int_{-\infty}^{+\infty} \frac{d\theta}{2\pi}\, \underbrace{e^{-mL\cosh\theta}}_{e^{-LE(\theta)}} \cosh\theta \sum_b \left(\underbrace{S_{ab}^{ab}}_{\text{INFINITE VOLUME S-MATRIX}}\underbrace{\left(\theta + i\frac{\pi}{2}\right)}_{\text{COMPLEX SHIFT OF THE RAPIDITY } \theta} - 1\right)$$

$$\Delta m_\mu(L) = -\frac{\sqrt{3}}{2} m \sum_{b,c} M_{abc}\,(-i)\,\mathrm{res}_{\theta=2\pi i/3}\, S_{ab}^{ab}(\theta)\, e^{-\frac{\sqrt{3}}{2} L\cdot m}$$

1 IF c IS A BOUND STATE OF a AND b

0 OTHERWISE

- CORRECTIONS DEFINED IN TERMS OF INFINITE VOLUME DATA

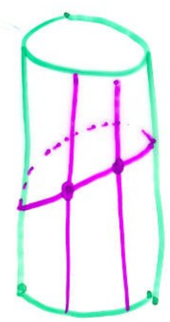
- MULTIPARTICLE STATES

$$e^{i p_i L} = \prod S(p_i, p_j)$$

$$E = \sum E(p_i)$$

— GETS MODIFIED BY VIRTUAL CORRECTIONS

E.G.



— BUT NO GENERAL FORMULAS HAVE BEEN OBTAINED SO FAR

• HOW TO OBTAIN THE EXACT SPECTRUM

– NO GENERAL RESULT

BUT IN SOME CASES METHODS EXIST

1. THERMODYNAMIC BETHE ANSATZ (TBA) FOR EXCITED STATES

DOREY, TATEO

2. NLIE (DESTRI-DE VEGA EQUATION)

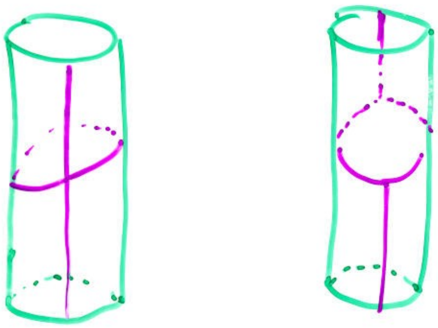
DESTRI, DE VEGA

3. FUNCTIONAL RELATIONS

BAZHANOV, LUKYANOV, ZAMOLODCHIKOV

FOR DEFORMED CFT'S VERY PRECISE TESTS USING TRUNCATED CONFORMAL SPACE APPROX (TCSA)

- IN AN INTEGRABLE QFT ON A CYLINDER, BETHE ANSATZ QUANTIZATION IS NOT EXACT
- BUT (IN SOME CASES AT LEAST) ALL CORRECTIONS ARE ENCODED IN A NLIE EXPRESSED IN TERMS OF THE S-MATRIX IN THE PLANE
- VIRTUAL CORRECTIONS



LOOK LIKE WRAPPING INTERACTIONS IN GAUGE THEORY!

⤳ EXPECT SIMILAR TYPE OF CORRECTIONS TO APPEAR FOR THE WORLDSHEET QFT OF THE $AdS_5 \times S^5$ SUPERSTRING

- UNFORTUNATELY WE CANNOT USE LÜSCHER FORMULAS DIRECTLY FOR THE $AdS_5 \times S^5$ WORLDSHEET THEORY

  ✗ LORENTZ INVARIANCE !!

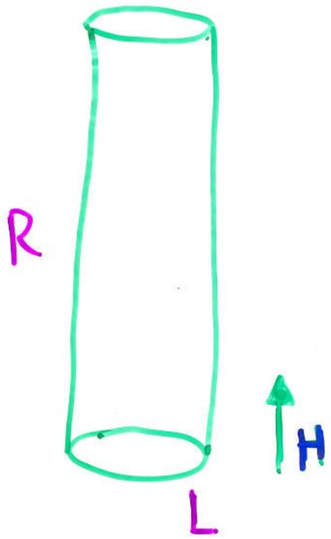
(i) GAIN INTUITION + ESTIMATE MAGNITUDE OF CORRECTIONS FROM THE THERMODYNAMIC BETHE ANSATZ (TBA) APPROACH

(ii) TRY TO GENERALIZE DIAGRAMMATIC DERIVATION

THERMODYNAMIC BETHE ANSATZ

- GROUND STATE ENERGY $E(L)$

PARTITION FUNCTION ON A CYLINDER OF LENGTH $R \to \infty$



$$Z = \mathrm{tr}\, e^{-RH} \underset{R\to\infty}{\to} e^{-RE(L)}$$

- INTERCHANGE SPACE AND TIME

THEORY ON A VERY BIG CIRCLE $R \to \infty$

$\equiv$ AT FINITE TEMPERATURE $T = \frac{1}{L}$

$\rightsquigarrow$

CALCULATE THE FREE ENERGY

- SINCE $R \to \infty$ BETHE ANSATZ QUANTIZATION BECOMES EXACT ! $\longrightarrow$ CONTINUOS DENSITIES OF ROOTS

- ONE GETS

ENERGY W.R.T. $\tilde{H}$

$$(*) \qquad \varepsilon(\theta) = L\, E_{TBA}(\theta) - \phi * \log\left(1 + e^{-\varepsilon(\theta')}\right)$$

WHERE

$$\phi(\theta) = -i \frac{d}{d\theta} \log S(\theta)$$

THEN THE GROUND STATE ENERGY IS

$$E_0 = \int_{-\infty}^{+\infty} \frac{d\theta}{2\pi}\, p'_{TBA}(\theta) \log\left(1 + e^{-\varepsilon(\theta)}\right)$$

- EXTENSION TO EXCITED STATES BY ANALYTIC CONTINUATION [DEFORMED CFT'S]

DOREY, TATEO '96

$$1 + e^{-\varepsilon(\theta)} = 0 \quad \text{FOR SOME} \quad \theta = \theta_0$$

$\downarrow$

DEFORM CONTOURS

$\downarrow$

NEW SOURCE TERMS IN $(*)$

$$\left.\begin{aligned} &\varepsilon(\theta) = E_{TBA}(\theta) + \sum_i i \log \frac{S(\theta - \theta_i)}{S(\theta - \bar{\theta}_i)} - \phi * \log\left(1 + e^{-\varepsilon(\theta')}\right) \\ &+ \text{EQUATIONS} \quad \varepsilon(\theta_i) = i\pi + 2k_i \pi i \end{aligned}\right\} \text{SOLVE BY ITERATION}$$

• 2-PARTICLE STATES IN SLYM (SCALING LEE-YANG MODEL)
($p_{TOT} = 0$)

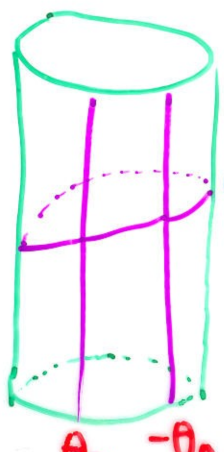
– 0^{TH} APPROXIMATION

$$E = 2\cosh\theta_B \qquad \text{WHERE} \qquad e^{iL\sinh\theta_B} = S(2\theta_B)$$

– FIRST CORRECTION

$$E = 2\cosh\theta_B + 2\cosh\theta_B\cdot\delta - \int_{-\infty}^{+\infty}\frac{d\theta}{2\pi}\cosh\theta\, e^{-L\cosh\theta}\cdot$$

$$\cdot\, S\left(\theta + i\tfrac{\pi}{2} - \theta_B\right) S\left(\theta + i\tfrac{\pi}{2} + \theta_B\right)$$



ANALOG OF μ-TERM

$$\delta = e^{-\frac{\sqrt{3}}{2}L\cosh\theta_B}\cdot\left(\ldots\ldots\right)$$

MAGNITUDE OF THE CORRECTION

$$e^{-L E_{TBA}(\theta)}$$

- MAIN FEATURES OF THE ANSWER

F-TERM

- MAGNITUDE OF THE CORRECTION

$$e^{-L\, E_{TBA}(\theta)}$$

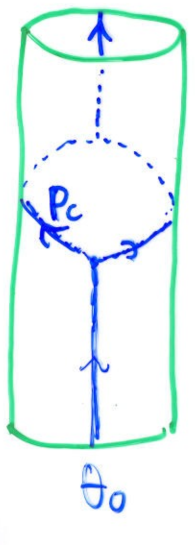
- VIRTUAL (ON-SHELL) PARTICLE HAS RAPIDITY

$$\theta + i\frac{\pi}{2}$$

μ-TERM

- MAGNITUDE OF THE CORRECTION

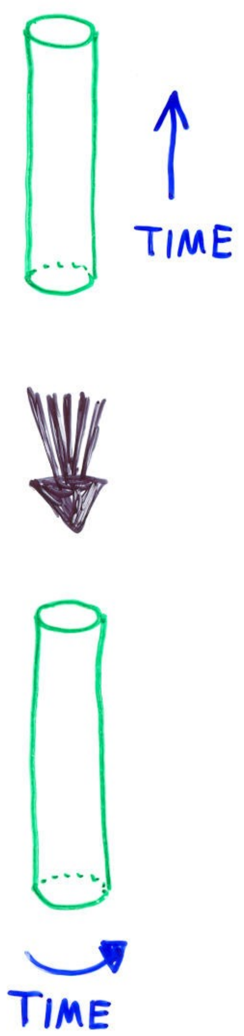
$$e^{-\frac{\sqrt{3}}{2} L \cosh\theta_0}$$



$$e^{-L\, \mathrm{Im}\, p_c}$$

HEURISTICS: APPLY TBA SETUP TO THE STRING WORLDSHEET THEORY

TBA ⤳ SPACE-TIME INTERCHANGE



$$E_{TBA} = -ip$$

$$p_{TBA} = -iE$$

IMPLEMENTED IN A RELATIVISTIC THEORY AS $\theta \to \theta + \frac{i\pi}{2}$

HENCE

$S\left(\theta + i\frac{\pi}{2} - \theta_B\right)$ IN F-TERM

- MAGNITUDE OF F-TERM CORRECTION

$$e^{-E_{TBA}(p_{TBA})L}$$

SPACE-TIME INTERCHANGE IN AdS STRING WORLDSHEET THEORY

$$E = \sqrt{1 + 8\, g^2 \sinh^2 \frac{p}{2}}$$

$$\Big\downarrow \quad \text{SPACE-TIME INTERCHANGE} \quad E = i\, p_{TBA} \qquad p = i\, E_{TBA}$$

$$i\, p_{TBA} = \sqrt{1 - 8\, g^2 \sinh^2 \frac{E_{TBA}}{2}}$$

$$\Big\downarrow$$

$$E_{TBA} = 2 \cdot \operatorname{arcsinh}\left(\frac{1}{2\sqrt{2}\, g} \cdot \sqrt{1 + p_{TBA}^2} \right)$$

- MAGNITUDE OF CORRECTIONS (F-TERM)

$$e^{-2L \cdot \operatorname{arcsinh}\left(\frac{1}{2\sqrt{2}\, g} \cdot \sqrt{1 + p_{TBA}^2} \right)}$$

USE THESE FORMULAS

TO ESTIMATE FINITE SIZE

CORRECTIONS IN THE AdS

CONTEXT

(a) WEAK COUPLING

F-TERM

$$e^{-LE_{TBA}} = e^{-L\cdot 2\cdot \underbrace{\operatorname{arcsinh}\left(\frac{\sqrt{1+p_{TBA}^2}}{2\sqrt{2}\,g}\right)}_{-\log g+\ldots}} \quad g\to 0$$

$$\sim g^{2L}$$

MAGNITUDE TYPICAL OF WRAPPING INTERACTIONS!

- SUPPORTS THE QUALITATIVE IDENTIFICATION BETWEEN WRAPPING INTERACTIONS AND VIRTUAL CORRECTIONS ON THE CYLINDER

- DOES NOT SPOIL ANY WEAK COUPLING CONSIDERATIONS

WORK IN PROGRESS

(b) STRONG COUPLING

F-TERM

$$e^{-L E_{TBA}} = \underbrace{e^{-L \cdot 2 \operatorname{arcsinh}\left(\frac{\sqrt{1+p_{TBA}^2}}{2\sqrt{2}\, g}\right)}}_{g \to \infty}$$

$$e^{-\frac{L}{\sqrt{2}\, g}\sqrt{1+p_{TBA}^2}}$$

$$\approx$$

$$e^{-\frac{L}{\sqrt{2}\, g}} \equiv e^{-\frac{2\pi L}{\sqrt{\lambda}}}$$

$$g = \sqrt{\frac{\lambda}{8\pi^2}}$$

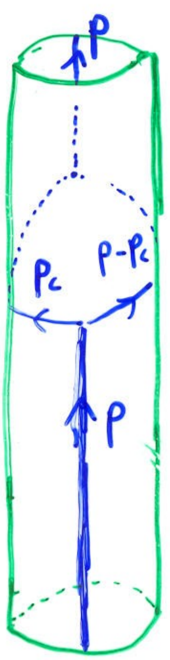
EXPONENTIAL CORRECTION FOR SPINNING $sl(2)$ STRING

IN SCHÄFER-NAMEKI, ZAMAKLAR, ZAREMBO

Ⓒ FINITE SIZE CORRECTION TO THE GIANT MAGNON

$$\cdots\cdot e^{-\frac{2\pi J}{\sqrt{\lambda}\sin\frac{p}{2}}}$$

• μ - TERM - COMES FROM A PARTICLE SPLITTING INTO ON-SHELL PARTICLES WHICH THEN RECOMBINE $\sim e^{-J \,\mathrm{Im}\, p_c}$



$$\sqrt{1+\frac{\lambda}{\pi^2}\sin^2\frac{p}{2}} = \sqrt{1+\frac{\lambda}{\pi^2}\sin^2\frac{p_c}{2}} + \sqrt{1+\frac{\lambda}{\pi^2}\sin^2\frac{p-p_c}{2}}$$

- SOLVE FOR p_c PERTURBATIVELY AT STRONG COUPLING

$$p_c = p + \frac{c\cdot\pi}{\sqrt{\lambda}}$$

$$\Downarrow$$

$$c = \frac{2\cdot i}{\sin\frac{p}{2}}$$

THEN $$e^{-J\,\mathrm{Im}\,p_c} = e^{-\frac{2\pi J}{\sqrt{\lambda}\sin\frac{p}{2}}}$$

- EXPONENTIAL TERM IS VERY GENERIC
  - DEPENDS JUST ON THE DISPERSION RELATION
  - SAME TERM CAN BE ALSO FOUND FROM F-TERM AT THE BPS BOUND STATE POLE

PREFACTORS ??

REPEAT DIAGRAMMATIC DERIVATION

WORK IN PROGRESS

- WANT TO HAVE COMPLETE EXPRESSION FOR THE μ-TERM

→ FOLLOW DIAGRAMMATIC DERIVATION WITH NEW DISPERSION RELATION $E = \varepsilon(p)$

HEURISTICS !

1. EUCLIDEAN EXPRESSION FOR SELF ENERGY
2. LEADING CORRECTION : FINITE SIZE MODIFICATION OF A SINGLE PROPAGATOR

$+ \ldots$

3. SHIFT CONTOUR OF $q \to q + i\kappa$
   - PICK A POLE OF $G(q)$ → THIS LINE BECOMES EFFECTIVELY ON-SHELL
4. RELATE INTEGRAND TO ON-SHELL S-MATRIX

F-TERM

$$\delta E^{(F)} = -\int \frac{dq^0_E}{2\pi} \left(1 - \frac{\varepsilon'(p)}{\varepsilon'(q)}\right) \cdot e^{\overbrace{i q_* L}^{-L E_{TBA}(q^0_E)}} \left(S(q,p) - 1\right)$$

- FOR RELATIVISTIC DISPERSION RELATION REDUCES TO

$$\delta E^{(F)} = \frac{-1}{\cosh\theta_p} \int \frac{d\theta}{2\pi} \cosh(\theta - \theta_p)\, e^{-L\cosh\theta} \left[S\left(\theta + i\frac{\pi}{2} - \theta_p\right) - 1\right]$$

KLASSEN, MELZER '90

- μ-TERM : RESIDUE ON BOUND STATE POLE

$$\delta E^{(\mu)} = -\underbrace{\left(1-\frac{\varepsilon'(p)}{\varepsilon'(\tilde{q})}\right)}_{\sin^2\frac{p}{2}} \cdot i \underbrace{\left(\operatorname*{Res}_{q=\tilde{q}} \sum_b S^{ab}_{ab}(q,p)\right)}_{\frac{4\sqrt{2}\, i}{g \sin^3\frac{p}{2}} \cdot \text{DRESSING} \cdot e^{ip}} \cdot \underbrace{e^{-L E_{TBA}(\tilde{q})}}_{e^{-\frac{2\pi L}{\sqrt{\lambda}\sin\frac{p}{2}}}}$$

PHASE FROM ARUTYONOV FROLOV ZAMAKLAR

- WE GET

$$\delta E^{(\mu)} = \frac{\sqrt{2}}{g}\, \frac{1}{\sin\frac{p}{2}} \cdot e^{-\frac{2\pi L}{\sqrt{\lambda}\sin\frac{p}{2}}} \cdot 4 \cdot \sigma^2(\tilde{q}, p) \cdot e^{ip}$$

g AND p SCALING AS FOR HUBBARD

COEFF. DOES NOT MATCH (OK)

$\longrightarrow$ CALCULATE THE DRESSING FACTOR

$g \to \infty$

$$\sigma^2_{AFS} = -\frac{g^2}{2} \cdot \underline{e^{-ip}} \cdot \sin^4\frac{p}{2}$$

CANCELS PHASE FROM STRING S-MATRIX FROM AFZ

$$\sigma^2_{HL} = \frac{1}{2}$$ (NUMERICALLY)

OTHER $\chi^{(2n)}(x,y)$ CONTRIBUTE AS NUMBERS (INDEPENDENT OF p AS $g \to \infty$

$\sigma^2_{REST} \approx 1.08\ldots$ ~ASYMPTOTIC SERIES ?

WORK IN PROGRESS

FINALLY

$$\delta E^{(\mu)} = -g \cdot \underbrace{\sqrt{2} \cdot 1.08\ldots}_{1.527} \cdot \sin^3\frac{p}{2} \cdot e^{-\frac{2\pi J}{\sqrt{\lambda}\sin\frac{p}{2}}}$$

EXPECTATION

$$\delta E = -g \cdot \underbrace{\frac{8\sqrt{2}}{e^2}}_{1.53114\ldots} \cdot \sin^3\frac{p}{2} \cdot e^{-\frac{2\pi J}{\sqrt{\lambda}\sin\frac{p}{2}}}$$

SUMMARY

- FOR AN INTEGRABLE QUANTUM FIELD THEORY ON A CYLINDER, BETHE ANSATZ QUANTIZATION IS NO LONGER EXACT DUE TO VIRTUAL CORRECTIONS
- CANNOT USE DIRECTLY LÜSCHER FORMULAS DUE TO LACK OF RELATIVISTIC INVARIANCE
- USE TBA + SPACETIME INTERCHANGE
- ESTIMATED MAGNITUDE OF CORRECTIONS (EXPONENTIAL TERM)
  - WEAK COUPLING g^{2L} (F-TERM)
    ~ WRAPPING INTERACTIONS
  - STRONG COUPLING → SPINNING STRING F-TERM
    → GIANT MAGNON μ-TERM
- COMPUTED PREFACTORS FOR GIANT MAGNON
  - SENSITIVE TO ALL ORDERS IN THE DRESSING PHASE
- POSSIBLE TEST OF MAGNON SUBSTRUCTURE / HIDDEN LEVELS
- BRANCH CUTS IN HL PHASE ????
- VERY INTERESTING TO UNDERSTAND THE INTEGRABLE STRUCTURE AT FINITE J