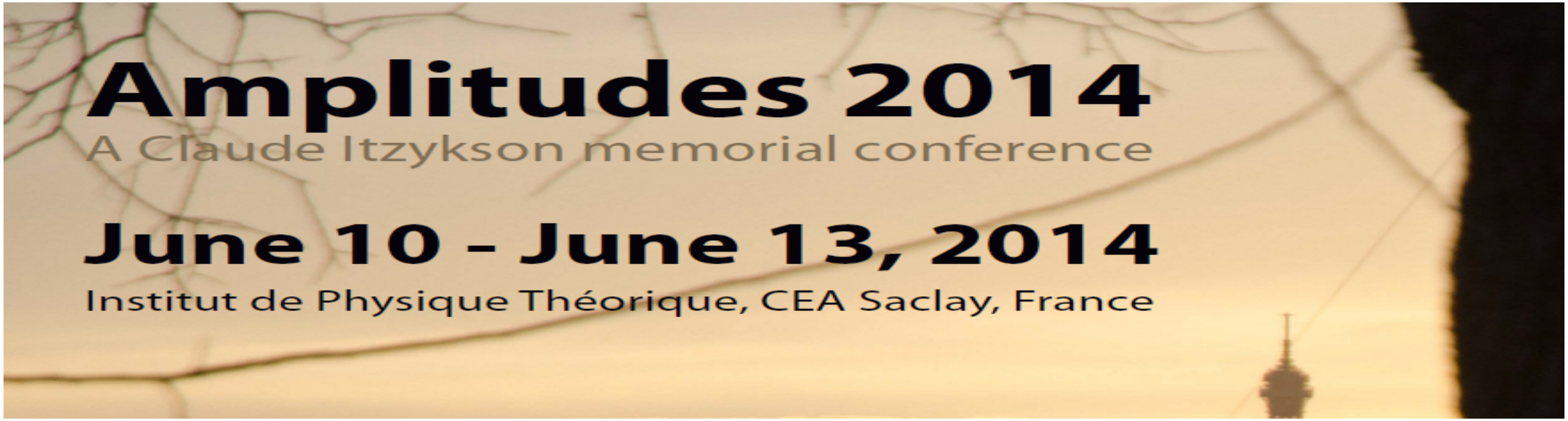


Polygon Amplitudes at High Energies

Volker Schomerus
DESY, Hamburg

GATIS
Gauge Theory as an Integrable System

Motivation and Programme

Goal: Interpolation of scattering amplitudes in N = 4 SYM theory from weak to strong coupling

$$\mathcal{A}_n = \mathcal{A}_n^{(0)} e^{F_n^{\mathrm{BDS}}(s,t;\epsilon,\lambda)+R_n(s,t;\lambda)}$$

Only remainder function $R_n = R_n$ (u) remains to be found

cross ratios

1. From successful interpolation of anomalous dimensions

→ String theory in AdS can provide decisive input

2. In perturbative N = 4 SYM amplitudes simplify in HE limit

BFKL,BKP integrability

Still very interesting: physically relevant, global information, boundary data [→ Dixon's talk]

→ Understand *R* in HE limit within string theory on AdS

Main Results

<u>Main Insight</u>: HE limit of remainder *R* at a = ∞ is

determined by IR limit of 1D q-integrable system

Simplification: TBA integral eqs → algebraic BA eqs

n = 6: [Bartels,Kotanski,VS] **n > 6: [Bartels,VS,Sprenger]**

Exploit this insight to compute remainder fct R$^\infty$ in HE limit

n = 6 [Bartels,Kotanski,VS,Sprenger]

$$R_6^{\infty,\mathrm{MRL}}(u,w) \sim \mathcal{R}^\infty(u,w) := \frac{\sqrt{\lambda}}{2\pi} e_2 \left(\ln(1-u) + \frac{1}{2} \frac{\ln|w|^2}{\ln|1+w|^4} \right)$$

n = 7 [Bartels,VS,Sprenger]

$$R_{7,\varrho_3}^{\infty\,\mathrm{MRL}} \sim \mathcal{R}^\infty(u_1,w_1) + \mathcal{R}^\infty(u_2,w_2)$$

↔ [Bartels,Kormilitzin,Lipatov, Prygarin] ; [Golden,Spradlin]

Plan

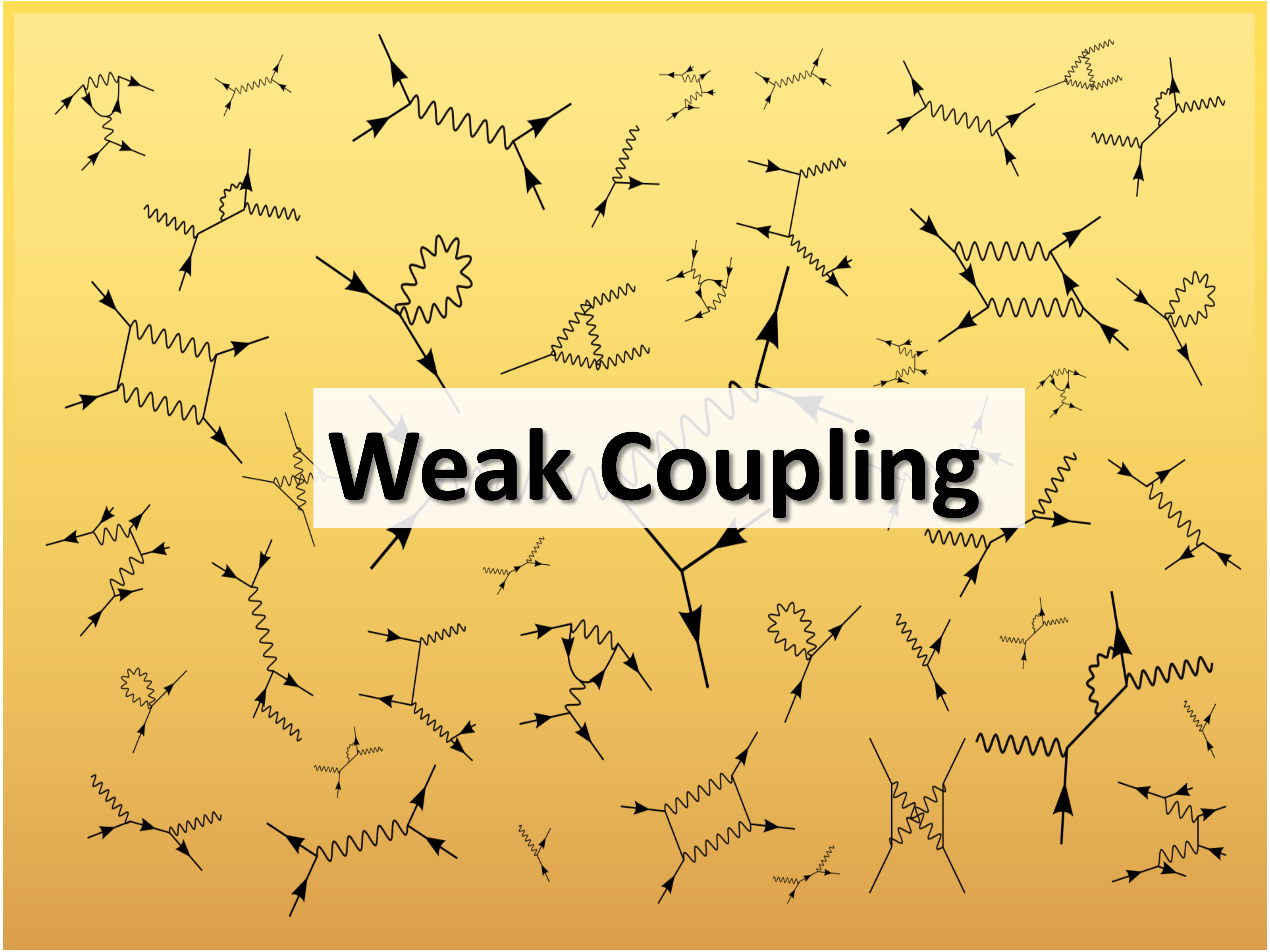
1. Multi-Regge kinematics & regions
2. The remainder function *R* in MRL — brief review
3. MRL of Thermodynamic Bubble Ansatz — = Bethe Ansatz
4. Results for n = 6,7 gluons
5. Conclusions and outlook

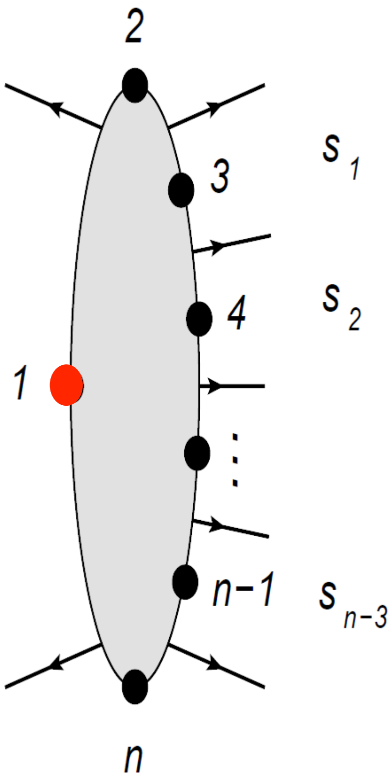
Based on work with Jochen Bartels, (Jan Kotanski), Martin Sprenger

arXiv:1009.3938,arXiv:1207.4204,arXiv:1311.1512,arXiv:1405.3658

and on discussions with Yasuyuki Hatsuda, Andrey Kormilitzin

Weak Coupling





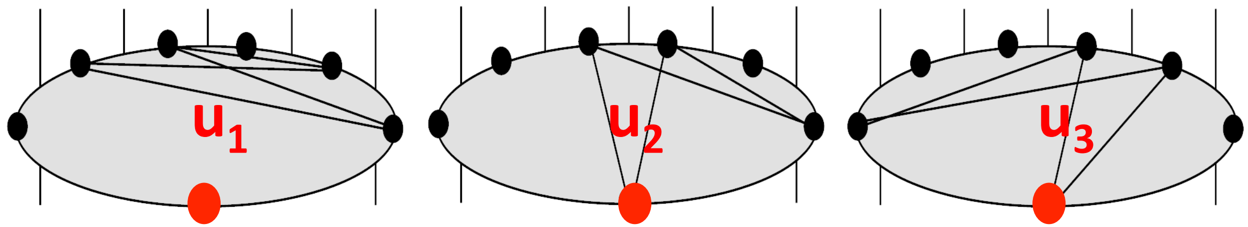
2 → n-2 Multi-Regge limit

Is the limit $s_r \to \infty$ such that

$$- x^2_{1,r+2} =: t_r \qquad \frac{x^2_{r+1,r'+3}}{s_r \cdots s_{r'}} =: \eta_{rr'}$$

remain finite $\quad r = 1, \ldots, n-3$

$\sigma = 1, \ldots, n-5$



In terms of 3n-15 cross ratios: $u_{1\sigma} \to 1 \quad u_{2\sigma}, u_{3\sigma} \to 0$

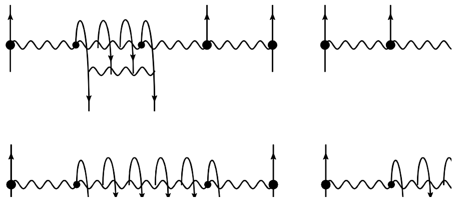
$$\left[\frac{u_{2\sigma}}{1-u_{1\sigma}}\right]^{\mathrm{MRL}} =: \frac{1}{|1+w_\sigma|^2} \qquad \left[\frac{u_{3\sigma}}{1-u_{1\sigma}}\right]^{\mathrm{MRL}} =: \frac{|w_\sigma|^2}{|1+w_\sigma|^2}$$

Multi-Regge Regions

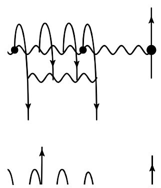
BDS formula produces correct MRL in Eucl. region

$$R(u_{1\sigma}, u_{2\sigma}, u_{3\sigma})^{\mathrm{MRL}}_{\varrho_e} = 0 \qquad s_r > 0 \qquad \varrho_e = (+ + \cdots +)$$

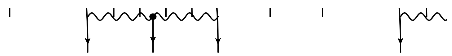
Multi-Regge regions are labeled by $\varrho = (\mathrm{sign}\, s_r)$



$\varrho_1 = (- + - +)$



$\varrho_2 = (+ - + -)$



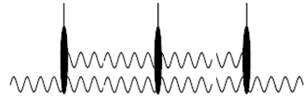
$\varrho_3 = (- + + -)$

Upon analytic continuation to $\varrho \neq \varrho_e$, $u_{1\sigma}$ winds

around $u_{1\sigma} = 1$ → *R* may pick up cut contributions

$$Li_2\left(1 - \frac{1}{u_1'}\right) R(u_{1\sigma}, u_{2\sigma}, u_{3\sigma})^{\mathrm{MRL}}_{\varrho} \neq 0 \qquad \pi i \log(1-u_1) + 2\pi^2$$

Cut contributions in MRL - form



Cut contributions are labeled by $P=(p_1,p_2,\dots,p_m)$

$m=n-5$ $\qquad$ $|p_\sigma - p_{\sigma+1}| \le 1 \quad , \quad p_\sigma \ge 1$

BFKL eigenvalue

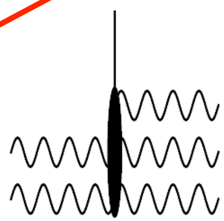
$$\mu(\nu,n;w)=(-1)^n\left(\frac{w}{\bar{w}}\right)^{\frac{n}{2}}|w|^{2i\nu} \qquad F_p(n,\nu;u,w)=\omega_p(\nu,n)\left(\log(1-u)+\frac{1}{2}\log\frac{|w|^2}{|1+w|^4}\right)$$

$$C_P(u_\sigma,w_\sigma) = i\frac{a}{2}\sum_{n_\sigma}\int\frac{d^m\nu_\sigma}{(2\pi)^m}\prod_{\sigma=1}^{m}\mu(\nu_\sigma,n_\sigma;w_\sigma)e^{-F_{p_\sigma}(\nu_\sigma,n_\sigma;u_\sigma,w_\sigma)}\times$$

$u_\sigma = u_{1\sigma}$

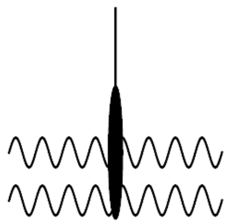
$$\times\Phi_{1p_1}(\nu_1,n_1)\left(\prod_{\sigma=1}^{m-1}\Phi_{p_\sigma p_{\sigma+1}}(\nu_\sigma,n_\sigma|\nu_{\sigma+1},n_{\sigma+1})\right)\Phi_{p_m 1}(\nu_m,n_m)$$

Impact factors



$\phi_{2,3}$

production vertices



$\phi_{2,2}$

Results on Building Blocks ω, ϕ

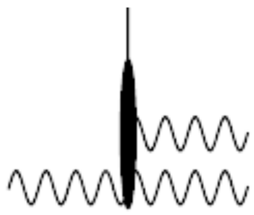
$\omega_p(\nu,n)$ ~ lowest Eigenvalue of BFKL/BKP Hamiltonian

on open SL(2) spin chain of length p in SL(2) rep (ν,n)

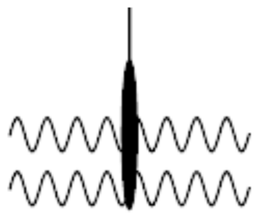
ω_2 up to N^2LLA [Fadin,Lipatov] . [Dixon,Drummond,Duhr,Pennington]

Hamiltonian known in NLLA for $p > 2$ [Bartels,Lipatov,Fadin,Vacca]

Simplest impact factors & production vertices in LLA



[Bartels,Lipatov, Sabio-Vera]
N^3:[Dixon et al.]



[Bartels,Kormilitzin, Lipatov, Prygarin]

Sufficient to construct MRL of R in all regions, n=6,7

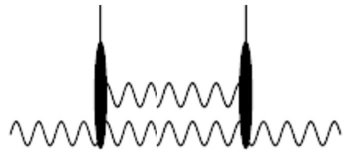
Building the Remainder

Cut (and pole) contributions used to build Remainder

e.g.

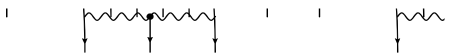
$$\left(e^{R+i\pi\delta}\right)^{\mathrm{MRL}} = \cos\pi\omega_0 + i\frac{a}{2}\sum_{n=-\infty}^{\infty}(-1)^n\left(\frac{w}{w^*}\right)^{\frac{n}{2}}\int\frac{d\nu}{\nu^2+\frac{n^2}{4}}|w|^{2i\nu}\Phi(\nu,n)\left((u_1-1)\frac{|w|}{|1+w|^2}\right)^{-\omega(\nu,n)}$$

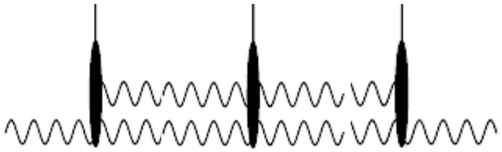
[Bartels,Lipatov,Sabio Vera] [Fadin,Lipatov]



↔[Caron-Huot]

Similarly, for n=7:





[Bartels,Kormilitzin,

Lipatov, Prygarin]

$\varrho_3 = (-++-)$

P = (2,2)

In general several cuts *P* may contribute to one region ρ

need to construct case by case

Consistency of amplitude imposes strong constraints

New (almost Feynman free) computational technique [Bartels ..]


Strong Coupling

Strong Coupling TBA

[Alday,Gaiotto, Maldacena][Alday,Maldacena,Sever,Vieira]

R = free energy of 1D quantum system involving 3n-15

particles [m_A,C_A] with integrable interaction [$K_{AB} \leftrightarrow S_{AB}$]

complex masses ↓ chemical potentials ↓ rapidity ↓

1 $$\log Y_A(\theta) = p_A(\theta) + \sum_B \int d\theta' K_{AB}(\theta-\theta') \log\left(1+Y_B(\theta')\right)$$

$(a,s)\ \ a=1,2,3;\ \ s=1,\ldots,n-5$ `particle densities'

2 Compute $A_{\text{free}}(m,C) = \sum_A \int d\theta\, |m_A| \cosh\theta \log\left(1+Y_A(\theta)\right)$

3 Solve $u_{\alpha\sigma} = \left.\frac{Y_{A_{\alpha\sigma}}}{1+Y_{A_{\alpha\sigma}}}\right|_{\theta=\theta_{\alpha\sigma}}$ for $(m(u),C(u))$ and determine

$$R_n^\infty = A_{\text{free}}(u) + \ldots$$

MRL = Large Mass Limit

gauge theory 1D quantum system

Multi-Regge regime reached when [Bartels, VS, Sprenger]

$$m_s e^{i(s-1)\frac{\pi}{4}} \to \infty$$

4D MRL = 2D IR

while keeping C_s and $v_s = \mathrm{Im}\left(m_s e^{i(s-1)\frac{\pi}{4}}\right)$ fixed

Rough idea: Cross ratios assume MRL values $u_1 \to 1; u_2, u_3 \to 0$

$\leftrightarrow$ $Y_A \to 0$ using that $u_{\alpha\sigma} = Y_{A_{\alpha\sigma}}(1+Y_{A_{\alpha\sigma}})^{-1}\big|_{\theta=\theta_{\alpha\sigma}}$

$\ln Y_A =$ $p_A(\theta) = -m_A \cosh(\theta) + C_A$

Remainder function vanishes in MRL $R^{\infty,\mathrm{MRL}}_{n,\varrho_e} = 0$

vacuum energy vanishes for infinite masses

Cut contributions = excitation energies

gauge theory 1D quantum system

Upon analytic continuation of system parameters, one may create excitations ← sols. of $Y(\theta) = -1$ crossing contour

compare ↕ [Dorey,Tateo]

Upon analytic continuation of the kinematic invariants, one may pick up cut contributions ← s. above

In MRL (1D low energy limit), excitations contribute a sum of bare energies to remainder *R* (↔ free energy)

Multi-Regge Bethe Ansatz

In MRL: TBA → BA for rapidities θ of bare excitations

$\theta_\mu^{(A)}, \mu = 1, \ldots, N_A$ $\kappa = -\mathrm{sign}(\mathrm{Im}\theta)$

1 $$e^{p_A(\theta_\mu^{(A)})} = \prod_B \prod_{\nu=1}^{N_B} S_{AB}(\theta_\mu^{(A)} - \theta_\nu^{(B)})^{\kappa_\nu^{(B)}}$$

From solution of BA equations compute the energy

2 $$A_{\mathrm{free}} \sim \sum_A \sum_{\nu=1}^{N_A} \tilde{\kappa}_\nu^{(A)} |m_s| \sinh(\theta_\nu^{(A)})$$

3 **Solve** $u_{\alpha\sigma} = \left.\frac{Y_{A_{\alpha\sigma}}}{1+Y_{A_{\alpha\sigma}}}\right|_{\theta=\theta_{\alpha\sigma}}$ **for** $(m(u), C(u))$ **& <u>compute *R*</u>**

with $\log Y_A(\theta) = p_A(\theta) - \sum_B \sum_{\nu=1}^{N_B} \kappa_\nu^{(B)} \log S_{AB}(\theta - \theta_\nu^{(B)})$

Example: The n = 6 solution

Relevant BA solution for region ρ = (- + -)

has $N^{(1)} = 0, N^{(2)} = 0, N^{(3)} = 2$ Input from numerics

Symmetry => $\theta_* = \theta^3_{\nu=1} = -\theta^{(3)}_{\nu=2}$

$$S_1(\theta) = i\frac{1 - ie^\theta}{1 + ie^\theta}$$

BA for θ_* $e^{p_3(\theta_*)} = \frac{1}{S_1(-2\theta_*)} = S_1(2\theta_*)$ => $\theta^{(3)}_\nu = \pm i\frac{\pi}{4}$

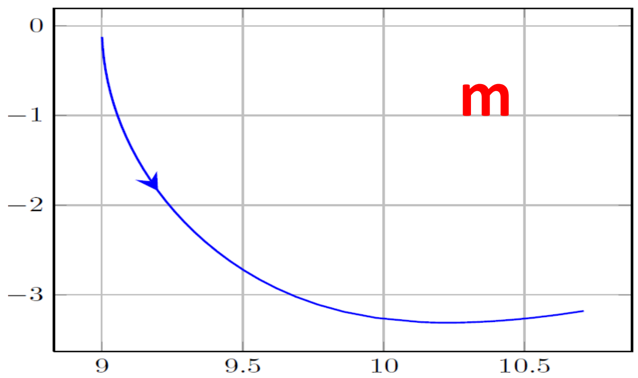
$$\mathcal{R}^{\infty,\text{MRL}}_{(-+-)}(u_1, w) \sim \frac{\sqrt{\lambda}}{2\pi} e_2 \left(\ln(1 - u_1) + \frac{1}{2}\frac{\ln|w|^2}{\ln|1 + w|^4} \right)$$

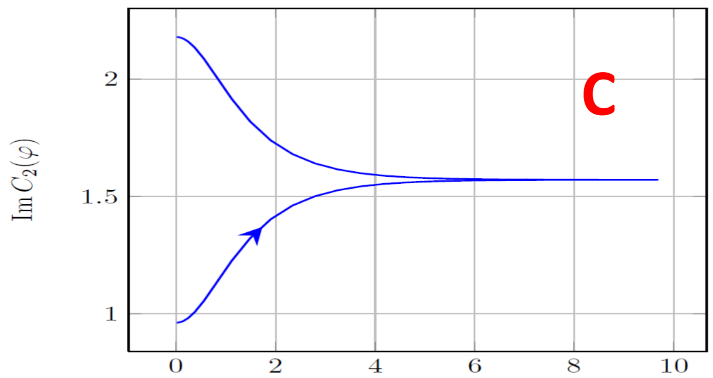
$$e_2 = -\sqrt{2} + \frac{1}{2}\ln(3 + 2\sqrt{2}) \sim -.53$$

Note: Difference with Amplitudes 2013

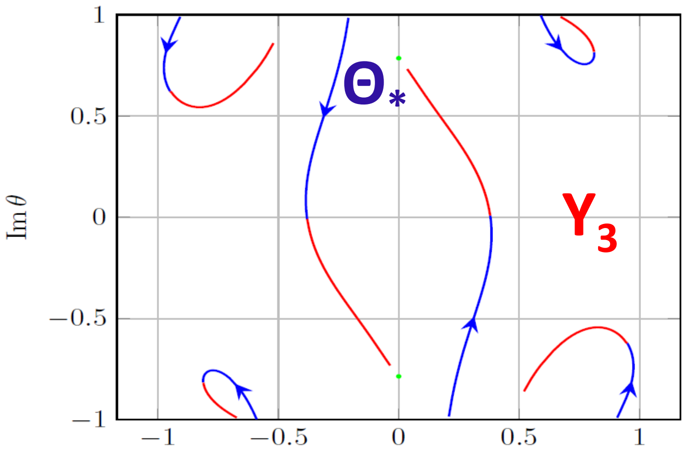
Remarks on Numerics n = 6

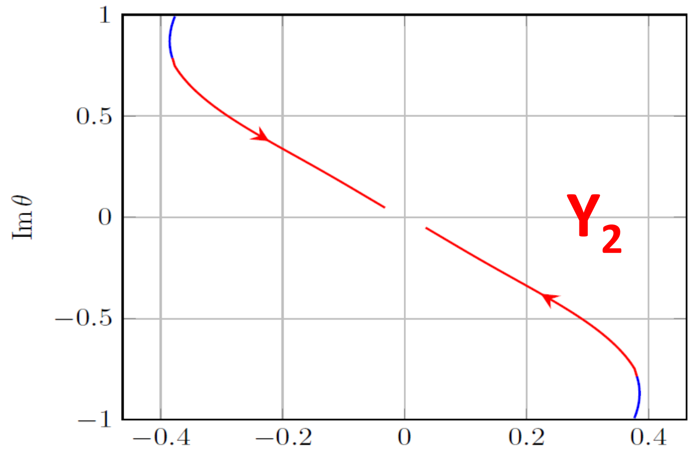
Variation of system parameters m,C to reach ρ = (- + -)





Movement of solutions for $Y_3(\theta)$= -1 and $Y_2(\theta)$ = -1 along path





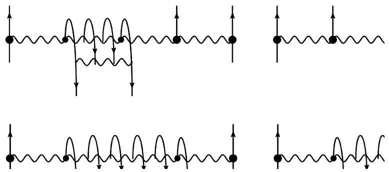
The n = 7 Remainder Function

Now computed for three of four non-trivial regions

expressed through

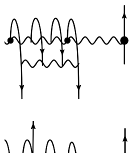
same as in n=6

$$\mathcal{R}^{\infty}(u,w) = \frac{\sqrt{\lambda}}{2\pi} e_2 \left(\ln(1-u_1) + \frac{1}{2} \frac{\ln|w|^2}{\ln|1+w|^4} \right)$$



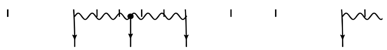
$\varrho_1 = (-+-+)$

$$R_{7,\varrho_1}^{\infty\,\mathrm{MRL}} + i\pi\delta_{7,\varrho_1} = \mathcal{R}^{\infty}(u_{a1})$$



$\varrho_2 = (+-+-)$

$$R_{7,\varrho_2}^{\infty\,\mathrm{MRL}} + i\pi\delta_{7,\varrho_2} = \mathcal{R}^{\infty}(u_{a2})$$



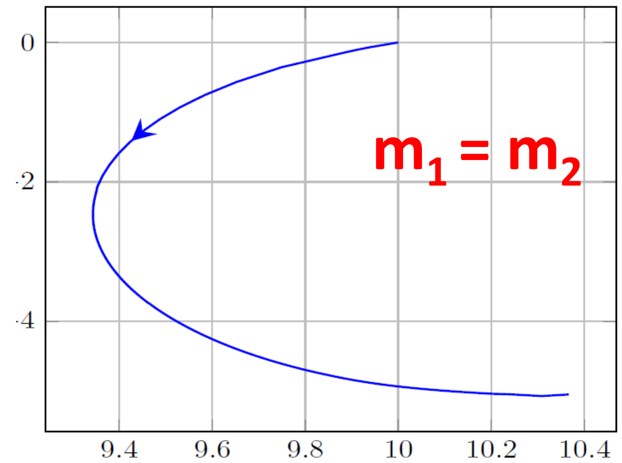
$$R_{7,\varrho_3}^{\infty\,\mathrm{MRL}} + i\pi\delta_{7,\varrho_3} = \mathcal{R}^{\infty}(u_{a1}) + \mathcal{R}^{\infty}(u_{a2})$$

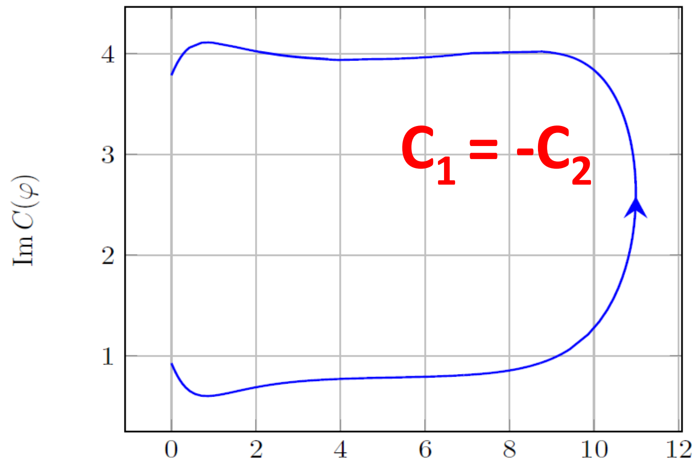
$\varrho_3 = (-++-)$

Consistent with factorization in N=4 SYM!

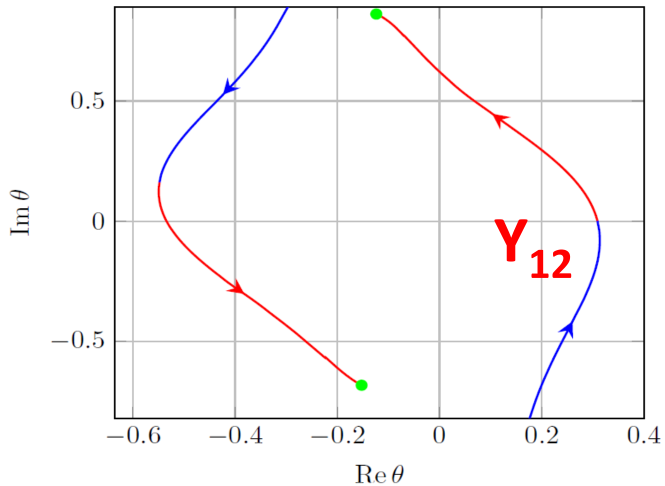
Remarks on Numerics n = 7

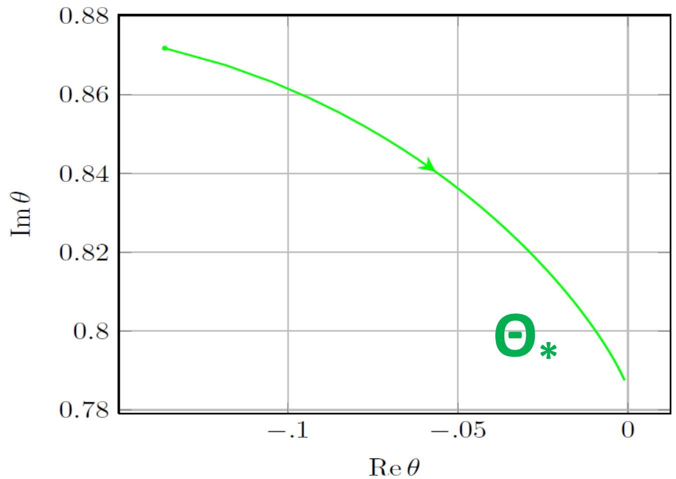
Variation of system parameters m,C to reach ρ = (- ++ -)





Movement of solutions for $Y_{12}(\theta) = -1$ along path (sim for Y_{31})





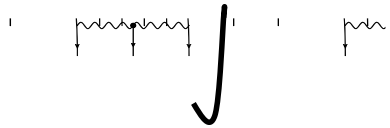
Conclusions and Outlook

R_n simplifies in MRL at weak and at strong coupling

Strong coupling simplifications possess nice physics

interpretation: MRL = IR limit of TBA

Fourth region for n = 7 ? Several cuts *P* contribute ?



Case n = 8 ? Weak coupling: *work in progress* [Bartels et al.]

At strong coupling: New number e_3 should appear! ↔ ω_3

Interpolation between weak and strong coupling ? [Basso , Caron-Huot.Ssver]

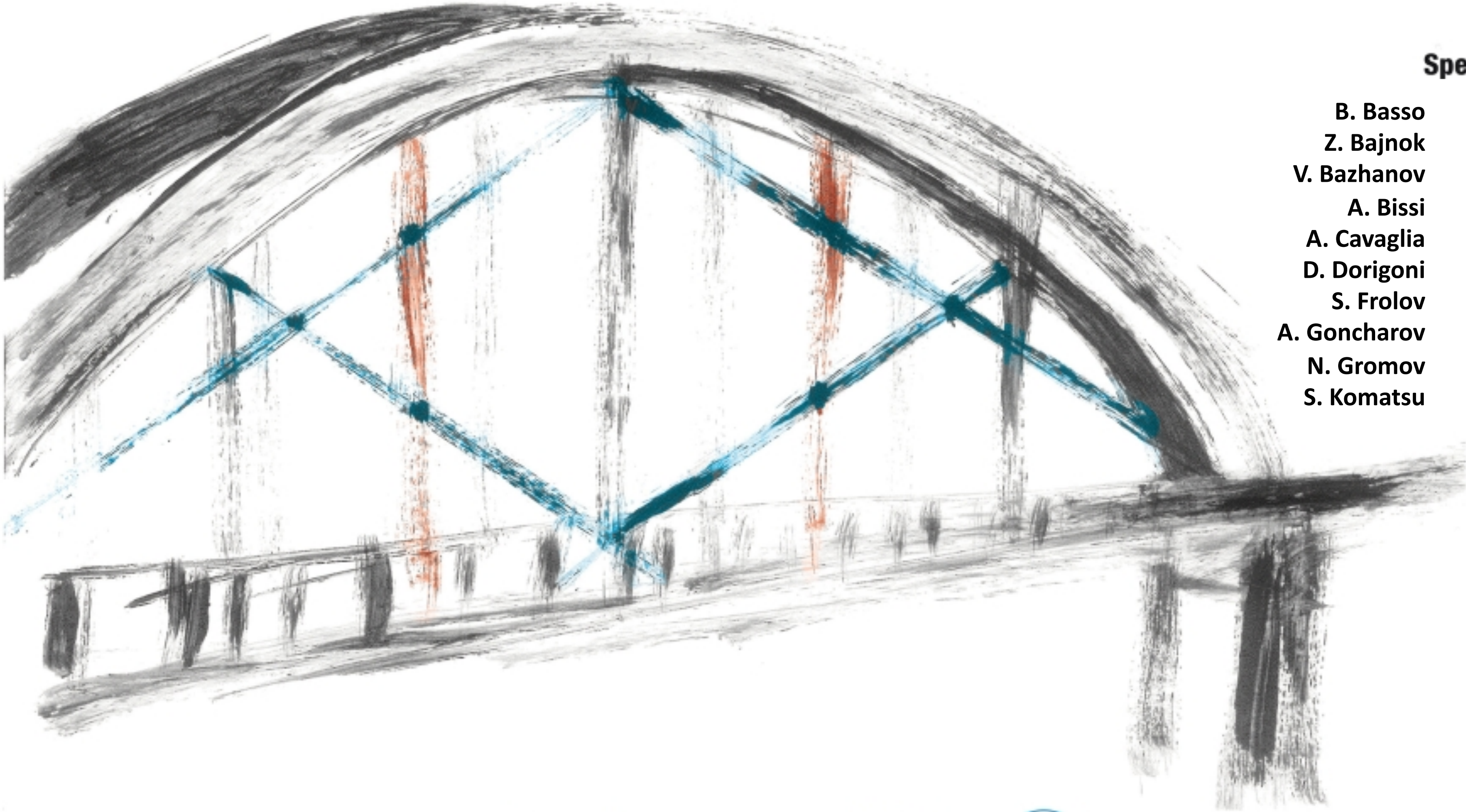
Relation between MRL and OPE & collinear limit ? [Hatsuda]

→ Basso's talk

Integrability in Gauge & String Theory

14 - 18 July 2014

DESY, Hamburg



Speakers include:

B. Basso
Z. Bajnok
V. Bazhanov
A. Bissi
A. Cavaglia
D. Dorigoni
S. Frolov
A. Goncharov
N. Gromov
S. Komatsu
G. Korchemsky
L. Lipatov
J.-M. Maillet
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