Scattering amplitudes in (certain) gauged supergravity theories

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based on work in progress with
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$\mathcal{N}$ = 8 SG has 28 vector fields; truncate to $\mathcal{N}$ < 8 SGs with n < 28 v. m'plets

Carrasco, Chiodaroli, Gunaydin, RR;
Damgaard, Huang, Sondergaard, Zhang

Factorized $\mathcal{N}$=4 theories

3	4	$\mathcal{N}=4\times\mathcal{N}=0\vert_{\mathbf{Z}_2}$	$\frac{SO(6,6)\times SU(1,1)}{SO(6)\times SO(6)\times U(1)}$	$\mathcal{N}=4$ sugra, 6 vector multiplets
4	4	$\mathcal{N}=4\times\mathcal{N}=0\vert_{\tilde{\mathbf{Z}}_4}$	$\frac{SO(6,4)\times SU(1,1)}{U(4)\times SO(4)}$	$\mathcal{N}=4$ sugra, 4 vector multiplets
5	4	$\mathcal{N}=4\times\mathcal{N}=0\vert_{\mathbf{Z}_2\times\mathbf{Z}_2}$	$\frac{SO(6,2)\times SU(1,1)}{SO(6)\times SO(2)\times U(1)}$	$\mathcal{N}=4$ sugra, 2 vector multiplets
6	4	$\mathcal{N}=2\vert_{\mathbf{Z}_2}\times\mathcal{N}=2\vert_{\mathbf{Z}_2}$	$\frac{SO(6,2)\times SU(1,1)}{SO(6)\times SO(2)\times U(1)}$	$\mathcal{N}=4$ sugra, 2 vector multiplets
7	4	$\mathcal{N}=4\times\mathcal{N}=0\vert_{\mathbf{Z}_4}$	$\frac{SU(1,1)}{U(1)}$	pure $\mathcal{N}=4$ sugra

Factorized $\mathcal{N}$=2 theories

9	2	$\mathcal{N}=2\vert_{\mathbf{Z}_2}\times\mathcal{N}=0\vert_{\mathbf{Z}_2}$	$\frac{SO(6,2)\times SU(1,1)}{SO(6)\times SO(2)\times U(1)}$	$\mathcal{N}=2$ sugra, 7 vector multiplets
10	2	$\mathcal{N}=2\vert_{\mathbf{Z}_2}\times\mathcal{N}=0\vert_{\tilde{\mathbf{Z}}_4}$	$\frac{SO(4,2)\times SU(1,1)}{SO(4)\times SO(2)\times U(1)}$	$\mathcal{N}=2$ sugra, 5 vector multiplets
11	2	$\mathcal{N}=2\vert_{\mathbf{Z}_2}\times\mathcal{N}=0\vert_{\mathbf{Z}_2\times\mathbf{Z}_2}$	$\frac{SU(1,1)\times SU(1,1)\times SU(1,1)}{U(1)\times U(1)\times U(1)}$	$\mathcal{N}=2$ sugra, 3 vector multiplets
12	2	$\mathcal{N}=2\vert_{\mathbf{Z}_2}\times\mathcal{N}=0\vert_{\mathbf{Z}_4}$	$\frac{SU(1,1)}{U(1)}$	$\mathcal{N}=2$ sugra, 1 vector multiplet

Focusing on $\mathcal{N}$=2 theories…

- Members of an infinite family of SGs coupled to n vector multiplets known as the generic Jordan family — Gunaydin, Sierra, Townsend
- Uniquely specified by a 3-index tensor on the space of scalar fields

$$\mathcal{M} = \frac{SO(n,2)\times SU(1,1)}{SO(n)\times SO(2)\times U(1)}$$

- U-duality symmetry of 4d theory: $SO(n,2)\times SU(1,1)$
- Can be lifted to 5 dimensions; U-duality symmetry: $SO(n-1,1)\times SO(1,1)$

$$\mathcal{M} = \frac{SO(n-1,1)\times SO(1,1)}{SO(n-1)}$$

Simplest to describe in D=5: — Gunaydin, Sierra, Townsend

$$e^{-1}\mathcal{L} = -\frac{R}{2} - \frac{1}{4}\mathring{a}_{IJ}F^I_{\mu\nu}F^{J\mu\nu} - \frac{1}{2}g_{xy}\partial_\mu\varphi^x\partial^\mu\varphi^y + \frac{e^{-1}}{6\sqrt{6}}C_{IJK}\epsilon^{\mu\nu\rho\sigma\lambda}F^I_{\mu\nu}F^J_{\rho\sigma}A^K_\lambda$$

graviphoton + matter

functions of φ

constants

► The generic Jordan family of MESGT

Maxwell/Einstein 5d supergravity theories

Gunaydin, Sierra, Townsend

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$$C^{IJK}C_{J(MN}C_{PQ)K} = \delta^I_{(M}C_{NPQ)}$$ (whenever C related to Jordan algebra)

Describe $\mathcal{M} = \frac{SO(n-1,1)}{SO(n-1)}\times SO(1,1)$ as hypersurface in ambient space

Everything is determined by a prepotential

$$N(\xi) = \left(\frac{2}{3}\right)^{3/2}C_{IJK}\xi^I\xi^J\xi^K \qquad \xi = (\phi^x, \rho) \qquad a_{IJ} = -\frac{1}{2}\partial_I\partial_J \ln N(\xi)$$

$$\mathring{a}_{IJ}(\varphi) = a_{IJ}\big|_{N(\xi)=1} \qquad g_{xy}(\varphi) = \mathring{a}_{IJ}\partial_x\xi^I\partial_y\xi^J$$

$\mathring{a}_{IJ}$ in terms of vielbeine on the scalar manifold:

$$\mathring{a}_{IJ} = h_I h_J + h^a_I h^a_J \qquad h^I h^J \mathring{a}_{IJ} = 1 \qquad h^I_a h^J_b \mathring{a}_{IJ} = \delta_{ab} \qquad h^I h_{Ia} = h_I h^{Ia} = 0$$

- canonical basis for C : $C_{000} = 1 \quad C_{00i} = 0 \quad C_{0ij} = -\frac{1}{2}\delta_{ij} \quad i,j = 1,\ldots,n$
  all other entries C_{ijk} arbitrary

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Gunaydin, Sierra, Townsend

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- Generic Jordan family: (natural basis) $\quad N(\xi) = \sqrt{2}\xi^0\Big((\xi^1)^2 - (\xi^2)^2 - \ldots - (\xi^n)^2\Big)$

Dimensional reduction to four dimensions

Gunaydin, Sierra, Townsend

- one extra vector multiplet

$$\begin{aligned} e^{-1}\mathcal{L}^{(4)} \;=\; & -\frac{1}{2}R - \frac{1}{2}e^{3\sigma}W_{\mu\nu}W^{\mu\nu} - \frac{3}{4}\partial_\mu\sigma\partial^\mu\sigma \\ & -\frac{1}{4}e^{\sigma}\mathring{a}_{IJ}(F^I_{\mu\nu} + 2W_{\mu\nu}A^I)(F^{J\mu\nu} + 2W^{\mu\nu}A^J) \\ & -\frac{1}{2}e^{-2\sigma}\mathring{a}_{IJ}\partial_\mu A^I\partial^\mu A^J - \frac{3}{4}\mathring{a}_{IJ}\partial_\mu h^I\partial^\mu h^J \\ & +\frac{e^{-1}}{2\sqrt{6}}C_{IJK}\epsilon^{\mu\nu\rho\sigma}\Big\{F^I_{\mu\nu}F^J_{\rho\sigma}A^K + 2F^I_{\mu\nu}W_{\rho\sigma}A^JA^K + \frac{4}{3}W_{\mu\nu}W_{\rho\sigma}A^IA^JA^K\Big\} \end{aligned}$$

- Fermion terms are known
- all vectors $F^A = \{W, F^I\}$ and their duals $\tilde{F}^A$ in rep. of $SO(n,2)$
- $\begin{pmatrix} F^A \\ \tilde{F}^A \end{pmatrix}$ doublet of $SU(1,1)$
- dilaton/axion parametrize $SU(1,1)/U(1)$
- Particular values of n: heterotic string on smooth $\mathrm{K3} \times T^2$

What does it take to do momentum space perturbation theory (5d)?

Vector kinetic term must be positive definite

⟶ C_{IJK} is such that there is a value of scalars $\varphi_x = c_x$ such that $\mathring{a}_{IJ}(c) = \delta_{IJ}$

⟶ Find that point...

In natural basis: $c^I = (\frac{1}{\sqrt{2}}, 1, 0\ldots, 0)$ $\quad C_{011} = \frac{\sqrt{3}}{2}$ $\quad C_{0rs} = -\frac{\sqrt{3}}{2}\delta_{rs}$ $\quad r, s = 2, \ldots, n$

....and expand around it, *e.g.*

$$\begin{aligned}
\mathring{a}_{00} &= 1 + 4\varphi^1 + 6(\varphi^1)^2 - 2(\varphi^2)^2 - \ldots - 2(\varphi^n)^2 , \\
\mathring{a}_{11} &= 1 - 2\varphi^1 + 3(\varphi^1)^2 + 3(\varphi^2)^2 + \ldots + 3(\varphi^n)^2 , \\
\mathring{a}_{1r} &= -2\varphi^r + 6\varphi^1\varphi^r , \\
\mathring{a}_{rs} &= (1 - 2\varphi^1 + 3(\varphi^1)^2 + (\varphi^2)^2 + \ldots + (\varphi^n)^2)\delta_{rs} + 2\varphi^r\varphi^s
\end{aligned}$$

...find Lagrangian (*e.g.* cubic terms)

$$\begin{aligned}
e^{-1}\mathcal{L}_3 = &\frac{1}{\sqrt{3}}\varphi^1(\partial_\mu\varphi^1)^2 + \frac{4}{\sqrt{3}}\varphi^r\partial_\mu\varphi^1\partial^\mu\varphi^r + \frac{1}{\sqrt{3}}\varphi^1(\partial_\mu\varphi^r)^2 - \frac{4}{\sqrt{3}}\varphi^1\partial_{[\mu}A^0_{\nu]}\partial^\mu A^{0\nu} \\
&+\frac{2}{\sqrt{3}}\varphi^1\partial_{[\mu}A^1_{\nu]}\partial^\mu A^{1\nu} + 4\varphi^r\partial_{[\mu}A^1_{\nu]}\partial^\mu A^{r\nu} + \frac{2}{\sqrt{3}}\varphi^1\partial_{[\mu}A^r_{\nu]}\partial^\mu A^{r\nu} \\
&+\sqrt{2}e^{-1}\epsilon^{\mu\nu\rho\sigma\lambda}\Big(\partial_\mu A^0_\nu\partial_\rho A^1_\sigma A^1_\lambda - \partial_\mu A^0_\nu\partial_\rho A^r_\sigma A^r_\lambda\Big) ,
\end{aligned}$$

... compute amplitudes from Feynman graphs, *etc.*

Scattering amplitudes in the 4d generic Jordan family MESGT

$$\mathcal{L}_{\mathcal{N}=2} = \mathrm{Tr}\Big[- \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D^\mu\bar{\varphi})(D_\mu\varphi) - \frac{g^2}{4}[\varphi,\bar{\varphi}]^2 - i\bar{\lambda}D_\mu\bar{\sigma}^\mu\lambda + \sqrt{2}g\lambda^\alpha[\varphi,\lambda^\beta]\epsilon_{\alpha\beta} + \sqrt{2}g\bar{\lambda}^\alpha[\bar{\varphi},\bar{\lambda}^\beta]\epsilon_{\alpha\beta}\Big]$$

- On-shell multiplets: $\mathrm{G}_+ = A_{2+} + \eta_\alpha\lambda_{2+}^\alpha + \eta^2\varphi_2$ $\qquad \mathrm{G}_- = \bar{\varphi}_2 + \eta_\alpha\lambda_{2-}^\alpha + \eta^2 A_{2-}$

$$\mathcal{L}_{\mathcal{N}=0} = \mathrm{Tr}\Big[- \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(D_\mu\phi^{\hat{A}})(D^\mu\phi^{\hat{B}})\delta_{\hat{A}\hat{B}} + \frac{g^2}{4}[\phi^{\hat{B}},\phi^{\hat{C}}][\phi^{\hat{B}'},\phi^{\hat{C}'}]\delta_{\hat{B}\hat{B}'}\delta_{\hat{C}\hat{C}'}\Big]$$

- Double-copy spectrum... and... on-shell multiplets

spin = 2 :	$h_{++} = A_{1+}\otimes A_{2+}$	$h_{--} = A_{1-}\otimes A_{2-}$
spin = 3/2 :	$\psi_+^\alpha = A_{1+}\otimes\lambda_{2+}^\alpha$	$\psi_-^\alpha = A_{1-}\otimes\lambda_{2-}^\alpha$
spin = 1 :	$V_+ = A_{1+}\otimes\varphi_2$	$\tilde{V}_- = A_{1-}\otimes\varphi_2$
	$\tilde{V}_+ = A_{1+}\otimes\bar{\varphi}_2$	$V_- = A_{1-}\otimes\bar{\varphi}_2$
	$V_+^{\hat{A}} = \phi_1^{\hat{A}}\otimes A_{2+}$	$V_-^{\hat{A}} = \phi_1^{\hat{A}}\otimes A_{2-}$
spin = 1/2 :	$\tilde{\zeta}_+^\alpha = A_{1+}\otimes\lambda_{2-}^\alpha$	$\tilde{\zeta}_-^\alpha = A_{1-}\otimes\lambda_{2+}^\alpha$
	$\zeta_+^{\hat{A}\alpha} = \phi_1^{\hat{A}}\otimes\lambda_{2+}^\alpha$	$\zeta_-^{\hat{A}\alpha} = \phi_1^{\hat{A}}\otimes\lambda_{2-}^\alpha$
spin = 0 :	$S_{+-} = A_{1+}\otimes A_{2-}$	$S_{-+} = A_{1-}\otimes A_{2+}$
	$S^{\hat{A}} = \phi_1^{\hat{A}}\otimes\varphi_2$	$\bar{S}^{\hat{A}} = \phi_1^{\hat{A}}\otimes\bar{\varphi}_2$

$$\begin{aligned}
\mathrm{H}_+ &= h_{++} + \eta_\alpha\psi_+^\alpha + \eta^2 V_+ \\
\mathrm{H}_- &= V_- + \eta_\alpha\psi_-^\alpha + \eta^2 h_{--} \\
\widetilde{\mathrm{V}}_+ &= \tilde{V}_+ + \eta_\alpha\tilde{\zeta}_+^\alpha + \eta^2 S_{+-} \\
\widetilde{\mathrm{V}}_- &= S_{-+} + \eta_\alpha\tilde{\zeta}_-^\alpha + \eta^2\tilde{V}_- \\
\mathrm{V}_+^{\hat{A}} &= V_+^{\hat{A}} + \eta_\alpha\zeta_+^{\hat{A}\alpha} + \eta^2 S^{\hat{A}} \\
\mathrm{V}_-^{\hat{A}} &= \bar{S}^{\hat{A}} + \eta_\alpha\zeta_-^{\hat{A}\alpha} + \eta^2 V_-^{\hat{A}}
\end{aligned}$$

- More efficient on-shell multiplets:

$\mathcal{H}_+ = \mathrm{H}_+ + \eta^3\eta^4\widetilde{\mathrm{V}}_+$ $\quad \mathcal{H}_- = \widetilde{\mathrm{V}}_- + \eta^3\eta^4\mathrm{H}_-$ $\quad \mathcal{V}^{\hat{A}} = \mathrm{V}_+^{\hat{A}} + \eta^3\eta^4\mathrm{V}_-^{\hat{A}}$ $\quad \mathcal{G} = \mathrm{G}_+ + \eta^3\eta^4\mathrm{G}_-$

$\mathcal{N}$=2 SG w/ vector multiplets: spectrum does not fix theory uniquely; interactions can be different

- If scalar manifold is symmetric space (incomplete summary)

  - $$\mathcal{M}_{gJf} = \frac{SO(n-1,1)}{SO(n-1)} \times SO(1,1)$$
    $$N_{gJf}(\xi) = \sqrt{2}\xi^0\Big((\xi^1)^2 - (\xi^2)^2 - \ldots - (\xi^n)^2\Big)$$
  - $$\mathcal{M}_{g-NJf} = \frac{SO(n,1)}{SO(n)}$$
    $$N_{g-NJf}(\xi) = \frac{3\sqrt{3}}{2\sqrt{2}}\Big(\sqrt{2}\xi^0(\xi^1)^2 - \xi^1\Big((\xi^2)^2 + \ldots + (\xi^n)^2\Big)\Big)$$
  - Magical cases for $n = 4, 8, 14, 26$
    $$\mathcal{M}_{\rm magical} \in \left\{\frac{SL(3,\mathbf{R})}{SO(3)},\ \frac{SL(3,\mathbf{C})}{SU(3)},\ \frac{SU^*(6)}{USp(6)},\ \frac{E_{6(-26)}}{F_4}\right\}$$

- Scalar manifolds need not be a symmetric spaces
- E.g. after reduction to d=4: $\mathcal{N}$=2 SG coupled to 15 vector multiplets truncation of $\mathcal{N}$=8 SG

$$\mathbf{Z}_2: \quad (1,1,-1,-1,-1,-1,-1,-1) \qquad \frac{SO^*(12)}{U(6)} \qquad h, 2\psi, 16F, 30\lambda, 30\phi$$

$\mathcal{N}=2$ SG with 15 vm's

Different on-shell symmetries ⟶ different S matrices

Global symmetries of MESGTs:

Off shell: $SU(2)_R \times$ symmetries of C_{IJK} realized in $\mathcal{L}$

On shell: symmetries of the S matrix

$SU(2)_R \times$ potential enhancement

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Question: can any of the global symmetries be gauged without introducing additional fields
- which ones?
- what happens?

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Question: can any of the global symmetries be gauged without introducing additional fields
- which ones?
- what happens?

When known, detailed answer can be given on case by case basis

Some general features:

1) gauge a subgroup of the manifest off-shell symmetry
2) 5d is advantageous: U-duality is a symmetry of Lagrangian
3) Extra couplings; non-trivial scalar potentials may appear
4) *R*-symmetry may or may not be gauged
   yes $\longrightarrow$ gen. Jordan family: Minkowski vacua but broken susy GST
   no $\longrightarrow$ YMESGT
5) Gauge group may be noncompact; unitarity is preserved

Focus on the generic Jordan family:

- can gauge in d=5 and dimensionally-reduce or directly in d=4
- avoid *R*-symmetry gauging (more later)

5d U-duality: $SO(n-1,1)\times SO(1,1)$ — Lagrangian sym.; compact/noncompact gauging; compact: $K\subset SO(n-1)$

4d U-duality: $SO(n,2)\times SU(1,1)$ — but only $SO(n)\times SO(2)$ in Lagrangian

Dimensional reduction ⟶ - compact gauge at most $K\subset SO(n-1)$ w/ $\dim(K)\leq n-1$
- first basis change; restore $SO(n)$; then gauge more

- The generic Jordan family MESGT Lagrangian:

$$e^{-1}\mathcal{L}=-\frac{R}{2}-\frac{1}{4}\mathring{a}_{IJ}F^I_{\mu\nu}F^{J\mu\nu}-\frac{1}{2}g_{xy}\partial_\mu\varphi^x\partial^\mu\varphi^y+\frac{e^{-1}}{6\sqrt{6}}C_{IJK}\epsilon^{\mu\nu\rho\sigma\lambda}F^I_{\mu\nu}F^J_{\rho\sigma}A^K_\lambda$$

$$N(\xi)=\sqrt{2}\xi^0\Big((\xi^1)^2-(\xi^2)^2-\ldots-(\xi^n)^2\Big)$$

Gauge group generators: $(M_r)_I{}^J$, $[M_r,M_s]=f_{rs}{}^tM_t$

$$(M_r)_{(I}{}^LC_{JK)L}=0$$

Gauging in d=5:

Gunaydin, Sierra, Townsent

- Covariantization:

$$\partial_\mu\varphi^x\to\partial_\mu\varphi^x+gA^s_\mu K^x_s$$

$$\nabla_\mu\lambda^{ia}\to\nabla_\mu\lambda^{ia}+gL^{ab}_tA^t_\mu\lambda^{ib}$$

$$F^I_{\mu\nu}\to\mathcal{F}^I_{\mu\nu}=\partial_\mu A^I_\nu-\partial_\nu A^I_\mu+gf^I{}_{JK}A^J_\mu A^K_\nu$$

Killing vector: $K^x_s=-\sqrt{\frac{3}{2}}(M_s)^J{}_Ih_Jh^{Ix}$

$L^{ab}_t=(M_t)^J{}_Ih^{[a|}_Jh^{I|b]}-\Omega^{ab}_xK^x_t$

Spin connection on $\mathcal{M}$

$f^I{}_{JK}$: Zero unless indices are in gauge group

- Modifications to the vector triple coupling:

$$\frac{e^{-1}}{6\sqrt{6}}C_{IJK}\epsilon^{\mu\nu\rho\sigma\lambda}F^I_{\mu\nu}F^J_{\rho\sigma}A^K_\lambda\longrightarrow$$

$$\frac{e^{-1}}{6\sqrt{6}}C_{IJK}\epsilon^{\mu\nu\rho\sigma\lambda}\Big\{F^I_{\mu\nu}F^J_{\rho\sigma}A^K_\lambda+\frac{3}{2}gf^K{}_{J'K'}F^I_{\mu\nu}A^J_\rho A^{J'}_\sigma A^{K'}_\lambda+\frac{3}{5}g^2A^I_\mu f^J{}_{I'J'}A^{I'}_\nu A^{J'}_\rho f^K{}_{K'L'}A^{K'}_\sigma A^{L'}_\lambda\Big\}$$

- No potential ⟶ there is a Minkowski ground state with unbroken susy
- Reduction to 4d: - potential from nonabelian field strength;
  - zero minimum energy ⟶ Minkowski ground state
  - choose symplectic frame (depending on purpose); part of the definition of the theory

- E.g. to quartic order:

$$e^{-1}\mathcal{L} = -\frac{1}{2}R - g_{I\bar{J}}\mathcal{D}_\mu z^I \mathcal{D}^\mu \bar{z}^J + \frac{1}{4}\mathrm{Im}\mathcal{N}_{AB}\mathcal{F}^A_{\mu\nu}\mathcal{F}^{B\mu\nu} - \frac{e^{-1}}{8}\epsilon^{\mu\nu\rho\sigma}\mathrm{Re}\mathcal{N}_{AB}\mathcal{F}^A_{\mu\nu}\mathcal{F}^B_{\rho\sigma} + g^2\mathcal{P}_4$$

$$\mathcal{P}_4 = -e^{\mathcal{K}} g_{rs} f^{rtu} f^{svw} z^t \bar{z}^u z^v \bar{z}^w$$

$$\mathcal{D}_\mu z^I \equiv \partial_\mu z^I + g A^J_\mu f^I_{JK} z^K$$

$$\mathcal{F}^I_{\mu\nu} \equiv 2\partial_{[\mu} A^I_{\nu]} + g f^I_{JK} A^J_\mu A^K_\nu$$

$$\mathcal{N}_{AB} = \begin{pmatrix} -i & 2z^J \\ 2z^I & -i + \frac{4}{\sqrt{3}}\tilde{C}_{IJK}\bar{z}^K \end{pmatrix} + \ldots$$

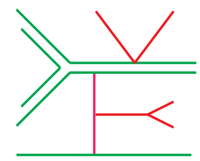
- with specific expression for *g=1+…*
- $Sp(2n+4)$ transf. for diag. supersymmetry and for $SO(n)$ symmetry

- Structure of amplitudes of YMESGTs

Field content: adjoint fields and gauge singlet fields

⟶ Amplitudes in structure constant basis: strings of f^{rst} and δ^{rs} "connected" by gauge-singlets

⟶ Amplitudes in trace basis: Multi-trace amplitudes present already at tree level



Designer's gauge theories

Modify the MESGT double-copy construction to include non-abelian couplings

- Minimal couplings with spin-0 and spin-1/2 fields

$$\mathcal{L}^{\mathrm{YMESGT}} \sim \ldots f_{rst} V^r_\mu (\phi^s \partial_\mu \phi^t - \phi^t \partial_\mu \phi^s) \oplus f_{rst} V^r_\mu \bar{\psi}^s \sigma^\mu \psi^t \ldots$$

⟶ standard 3-point S-matrix elements

$$[M_3] = 1$$

- Require that this can be factorized…

$$M_3 = A_3 A_3'$$

… and that A_3 and A_3' are Lorentz-invariant

$$[A_3] = 0 \quad \& \quad [A_3'] = 1$$

from dim 3 operator (4d counting) ↗ ↖ from standard dim 4 operator (4d counting)

⟶ Unique local option: trilinear scalar operator

- Repeat counting in generic dimension; conclusion is unchanged

What about other minimal couplings?

- Spin 3/2

Standard Lagrangian: $\mathcal{L}_3 \sim \cdots + \bar{\psi}_\mu \gamma^{\mu\nu\rho}(\partial_\nu + iV_\nu)\psi_\rho + \ldots$

2-gravitino matrix element: $M_3(1^{\psi_+}, 2^{V_+}, 3^{\psi_-}) \sim \frac{[12]^4}{[13][23]^2}$

Try to factorize as: $M_3 = A_3 A_3'$

With A_3 and A_3' for $s \leq 1$ fields:

$$\psi_+ = A_+ \otimes \lambda_+, \quad \psi_- = A_- \otimes \lambda_-, \quad V_+ = A_+ \otimes \phi \oplus \lambda_+ \otimes \lambda_+$$

⟶ No local dimension 3 operator can be constructed;
gauging R symmetry is more involved (story for another time)

$$\mathcal{L}_{\mathcal{N}=2} = \mathrm{Tr}\Big[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D^\mu\bar{\varphi})(D_\mu\varphi) - \frac{g^2}{4}[\varphi,\bar{\varphi}]^2 - i\bar{\lambda}D_\mu\bar{\sigma}^\mu\lambda + \sqrt{2}g\lambda^\alpha[\varphi,\lambda^\beta]\epsilon_{\alpha\beta} + \sqrt{2}g\bar{\lambda}^\alpha[\bar{\varphi},\bar{\lambda}^\beta]\epsilon_{\alpha\beta}\Big]$$

- On-shell multiplets: $\mathrm{G}_+ = A_{2+} + \eta_\alpha \lambda_{2+}^{\alpha} + \eta^2 \varphi_2$, $\mathrm{G}_- = \bar{\varphi}_2 + \eta_\alpha \lambda_{2-}^{\alpha} + \eta^2 A_{2-}$ $\qquad \mathcal{G} = \mathrm{G}_+ + \eta^3\eta^4 \mathrm{G}_-$

$$\mathcal{L}_{\mathcal{N}=0} = \mathrm{Tr}\Big[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(D_\mu\phi^{\hat{A}})(D^\mu\phi^{\hat{B}})\delta_{\hat{A}\hat{B}} + \frac{g^2}{4}[\phi^{\hat{B}},\phi^{\hat{C}}][\phi^{\hat{B}'},\phi^{\hat{C}'}]\delta_{\hat{B}\hat{B}'}\delta_{\hat{C}\hat{C}'} + \frac{gg'}{3!}F_{\hat{A}\hat{B}\hat{C}}\phi^{\hat{A}}[\phi^{\hat{B}},\phi^{\hat{C}}]\Big]$$

- Double-copy spectrum... and... on-shell multiplets

spin $= 2$:	$h_{++} = A_{1+} \otimes A_{2+}$	$h_{--} = A_{1-} \otimes A_{2-}$
spin $= 3/2$:	$\psi_+^\alpha = A_{1+} \otimes \lambda_{2+}^\alpha$	$\psi_-^\alpha = A_{1-} \otimes \lambda_{2-}^\alpha$
spin $= 1$:	$V_+ = A_{1+} \otimes \varphi_2$	$\tilde{V}_- = A_{1-} \otimes \varphi_2$
	$\tilde{V}_+ = A_{1+} \otimes \bar{\varphi}_2$	$V_- = A_{1-} \otimes \bar{\varphi}_2$
	$V_+^{\hat{A}} = \phi_1^{\hat{A}} \otimes A_{2+}$	$V_-^{\hat{A}} = \phi_1^{\hat{A}} \otimes A_{2-}$
spin $= 1/2$:	$\tilde{\zeta}_+^\alpha = A_{1+} \otimes \lambda_{2-}^\alpha$	$\tilde{\zeta}_-^\alpha = A_{1-} \otimes \lambda_{2+}^\alpha$
	$\zeta_+^{\hat{A}\alpha} = \phi_1^{\hat{A}} \otimes \lambda_{2+}^\alpha$	$\zeta_-^{\hat{A}\alpha} = \phi_1^{\hat{A}} \otimes \lambda_{2-}^\alpha$
spin $= 0$:	$S^{\hat{A}} = \phi_1^{\hat{A}} \otimes \varphi_2$	$\bar{S}^{\hat{A}} = \phi_1^{\hat{A}} \otimes \bar{\varphi}_2$
	$S_{+-} = A_{1+} \otimes A_{2-}$	$S_{-+} = A_{1-} \otimes A_{2+}$

$$\mathrm{H}_+ = h_{++} + \eta_\alpha \psi_+^\alpha + \eta^2 V_+$$
$$\mathrm{H}_- = V_- + \eta_\alpha \psi_-^\alpha + \eta^2 h_{--}$$
$$\widetilde{\mathrm{V}}_+ = \tilde{V}_+ + \eta_\alpha \tilde{\zeta}_+^\alpha + \eta^2 S_{+-}$$
$$\widetilde{\mathrm{V}}_- = S_{-+} + \eta_\alpha \tilde{\zeta}_-^\alpha + \eta^2 \tilde{V}_-$$
$$\mathrm{V}_+^{\hat{m}} = V_+^{\hat{m}} + \eta_\alpha \zeta_+^{\hat{m}\alpha} + \eta^2 S^{\hat{m}}$$
$$\mathrm{V}_-^{\hat{m}} = \bar{S}^{\hat{m}} + \eta_\alpha \zeta_-^{\hat{m}\alpha} + \eta^2 V_-^{\hat{m}}$$
$$\mathrm{V}_+^{r} = V_+^{r} + \eta_\alpha \zeta_+^{r\alpha} + \eta^2 S^{r}$$
$$\mathrm{V}_-^{r} = \bar{S}^{r} + \eta_\alpha \zeta_-^{r\alpha} + \eta^2 V_-^{r}$$

- More efficient on-shell multiplets:

$$\mathcal{H}_+ = \mathrm{H}_+ + \eta^3\eta^4 \widetilde{\mathrm{V}}_+ \quad \mathcal{H}_- = \widetilde{\mathrm{V}}_- + \eta^3\eta^4 \mathrm{H}_- \quad \mathcal{V}^{\hat{m}} = \mathrm{V}_+^{\hat{m}} + \eta^3\eta^4 \mathrm{V}_-^{\hat{m}} \quad \mathcal{V}^{r} = \mathrm{V}_+^{r} + \eta^3\eta^4 \mathrm{V}_-^{r}$$

Color/kinematics duality of the vector-scalar theory:
- should hold order by order in g'

4pt: Chiodaroli, Jin, RR; Chiodaroli, Gunaydin, Johansson, RR

- Highest power of g' : manifest duality if F obeys Jacobi identity

$$\mathcal{A}_5^{(0)}(1^{\phi^{\hat{A}_1}}2^{\phi^{\hat{A}_2}}3^{\phi^{\hat{A}_3}}4^{\phi^{\hat{A}_4}}5^{\phi^{\hat{A}_4}})\big|_{g'^3} = g^3 g'^3 \sum_{\sigma\in\mathcal{S}(2,3,4,5)} \frac{F^{a_1 a_{\sigma(3)} b} F^{b a_{\sigma(4)} c} F^{c a_{\sigma(5)} a_{\sigma(2)}}}{s_{1\sigma(3)} s_{\sigma(2)\sigma(5)}} \mathrm{f}^{a_1 a_{\sigma(3)} b} \mathrm{f}^{b a_{\sigma(4)} c} \mathrm{f}^{c a_{\sigma(5)} a_{\sigma(2)}}$$

- Lower powers of g' -- less trivial; e.g.

$$\begin{aligned}
&\mathcal{A}_5^{(0)}(1^{\phi^{\hat{A}_1}}2^{\phi^{\hat{A}_2}}3^{\phi^{\hat{A}_3}}4^{\phi^{\hat{A}_3}}5^{\phi^{\hat{A}_3}})\big|_{g'} \\
&\quad = \frac{1}{2} g^3 g' F^{\hat{A}_1\hat{A}_2\hat{A}_3}\Bigg[\Bigg(\frac{k_{12\bar{3}}\cdot k_{4\bar{5}}}{s_{12}s_{45}} \mathrm{f}^{a_1 a_2 b}\mathrm{f}^{b a_3 c} + \frac{k_{\bar{1}23}\cdot k_{4\bar{5}}}{s_{23}s_{45}} \mathrm{f}^{a_2 a_3 b}\mathrm{f}^{b a_1 c} \\
&\qquad\qquad + \frac{k_{1\bar{2}3}\cdot k_{4\bar{5}}}{s_{13}s_{45}} \mathrm{f}^{a_3 a_1 b}\mathrm{f}^{b a_2 c}\Bigg)\mathrm{f}^{c a_4 a_5} + (3\leftrightarrow 4) + (3\leftrightarrow 5)\Bigg] \\
&\quad + \frac{1}{2} g^3 g' F^{\hat{A}_1\hat{A}_2\hat{A}_3}\Bigg[-\Bigg(\frac{1}{s_{13}}+\frac{1}{s_{24}}\Bigg)\mathrm{f}^{a_1 a_3 b}\mathrm{f}^{b a_5 c}\mathrm{f}^{c a_2 a_4} - \Bigg(\frac{1}{s_{13}}+\frac{1}{s_{25}}\Bigg)\mathrm{f}^{a_1 a_3 b}\mathrm{f}^{b a_4 c}\mathrm{f}^{c a_2 a_5} \\
&\qquad\qquad + (3\leftrightarrow 4) + (3\leftrightarrow 5)\Bigg]
\end{aligned}$$

- Check BCJ amplitudes relations, e.g.

$$s_{24}A_5^{(0)}(1,2,4,3,5) = A_5^{(0)}(1,2,3,4,5)(s_{14}+s_{45}) + A_5^{(0)}(1,2,3,5,4)s_{14}$$

- Higher points check out as well ⟶ Double-copy into YMESGTs

Generic Jordan family YMESGTs form gauge theory

Vector-scalar theory

$$\mathcal{A}_3^{(0),\mathcal{N}=0}(1g_-^a, 2g_-^b, 3g_+^c) = ig\frac{\langle 12\rangle^3}{\langle 23\rangle\langle 31\rangle}\tilde{\mathrm{f}}^{abc}$$

$$\mathcal{A}_3^{(0),\mathcal{N}=0}(1\phi^{a\hat{A}}, 2\phi^{b\hat{B}}, 3g_+^c) = -ig\frac{[13][32]}{[12]}\tilde{\mathrm{f}}^{abc}\delta^{\hat{A}\hat{B}}$$

$$\mathcal{A}_3^{(0),\mathcal{N}=0}(1\phi^{a\hat{A}}, 2\phi^{b\hat{B}}, 3\phi^{c\hat{C}}) = \frac{i}{\sqrt{2}}gg'\tilde{\mathrm{f}}^{abc}F^{\hat{A}\hat{B}\hat{C}}$$

$\mathcal{N}$=2 sYM

$$\mathcal{A}_{3,\mathrm{MHV}}^{(0),\mathcal{N}=2}(1,2,3) = i\tilde{\mathrm{f}}^{abc}\ \mathcal{Q}_3^{34}\ \frac{\delta^{(4)}(\sum\eta_i^\alpha|i\rangle)}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}$$

$$\mathcal{A}_{3,\overline{\mathrm{MHV}}}^{(0),\mathcal{N}=2}(1,2,3) = i\tilde{\mathrm{f}}^{abc}\ \tilde{\mathcal{Q}}_3^{34}\ \frac{\delta^{(2)}(\frac{1}{2}\sum\epsilon_{ijk}[ij]\eta_k^\alpha)}{[12][23][31]}$$

$$\mathcal{Q}_n^{34} = \sum_{1\le i<j\le n}\langle ij\rangle^2(\eta_i^3\eta_i^4)(\eta_j^3\eta_j^4) \qquad \tilde{\mathcal{Q}}_3^{34} = \frac{1}{2}\sum_{i\neq j\neq k}^{3}[ij]^2(\eta_k^3\eta_k^4)$$

$$\mathcal{M}_3^{(0)}(1^{\mathcal{H}_-}, 2^{\mathcal{H}_-}, 3^{\mathcal{H}_+}) = -i\frac{\kappa}{2}\frac{\langle 12\rangle^2}{\langle 23\rangle^2\langle 31\rangle^2}\ \mathcal{Q}_3^{34}\ \delta^{(4)}(\sum\eta_i^\alpha|i\rangle)$$

$$\mathcal{M}_3^{(0)}(1^{\mathcal{V}^{\hat{A}}}, 2^{\mathcal{V}^{\hat{B}}}, 3^{\mathcal{V}^{\hat{C}}}) = -i\frac{\kappa}{2\sqrt{2}}\frac{g'F^{\hat{A}\hat{B}\hat{C}}}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}\ \mathcal{Q}_3^{34}\ \delta^{(4)}(\sum\eta_i^\alpha|i\rangle)$$

$$\mathcal{M}_3^{(0)}(1^{\mathcal{V}^{\hat{A}}}, 2^{\mathcal{V}^{\hat{B}}}, 3^{\mathcal{H}_-}) = \frac{\kappa}{2}\frac{\delta^{\hat{A}\hat{B}}}{\langle 12\rangle^2}\ \mathcal{Q}_3^{34}\ \delta^{(4)}(\sum\eta_i^\alpha|i\rangle)$$

Comparison with direct supergravity calculations:
- break up in components
- compare them one by one
- reassemble result into transformation of multiplets

Generic Jordan family YMESGTs form gauge theory

Vector-scalar theory

$$\mathcal{A}_3^{(0),\mathcal{N}=0}(1g_-^a, 2g_-^b, 3g_+^c) = ig\frac{\langle 12\rangle^3}{\langle 23\rangle\langle 31\rangle}\tilde{\mathrm{f}}^{abc}$$

$$\mathcal{A}_3^{(0),\mathcal{N}=0}(1\phi^{a\hat{A}}, 2\phi^{b\hat{B}}, 3g_+^c) = -ig\frac{[13][32]}{[12]}\tilde{\mathrm{f}}^{abc}\delta^{\hat{A}\hat{B}}$$

$$\mathcal{A}_3^{(0),\mathcal{N}=0}(1\phi^{a\hat{A}}, 2\phi^{b\hat{B}}, 3\phi^{c\hat{C}}) = \frac{i}{\sqrt{2}}gg'\tilde{\mathrm{f}}^{abc}F^{\hat{A}\hat{B}\hat{C}}$$

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$$\mathcal{A}_{3,\mathrm{MHV}}^{(0),\mathcal{N}=2}(1,2,3) = i\tilde{\mathrm{f}}^{abc}\;\mathcal{Q}_3^{34}\;\frac{\delta^{(4)}(\sum\eta_i^\alpha|i\rangle)}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}$$

$$\mathcal{A}_{3,\overline{\mathrm{MHV}}}^{(0),\mathcal{N}=2}(1,2,3) = i\tilde{\mathrm{f}}^{abc}\;\tilde{\mathcal{Q}}_3^{34}\;\frac{\delta^{(2)}(\frac{1}{2}\sum\epsilon_{ijk}[ij]\eta_k^\alpha)}{[12][23][31]}$$

$$\mathcal{Q}_n^{34} = \sum_{1\le i<j\le n}\langle ij\rangle^2(\eta_i^3\eta_i^4)(\eta_j^3\eta_j^4) \qquad \tilde{\mathcal{Q}}_3^{34} = \frac{1}{2}\sum_{i\neq j\neq k}^{3}[ij]^2(\eta_k^3\eta_k^4)$$

$$\mathcal{M}_3^{(0)}(1^{\mathcal{H}_-}, 2^{\mathcal{H}_-}, 3^{\mathcal{H}_+}) = -i\frac{\kappa}{2}\frac{\langle 12\rangle^2}{\langle 23\rangle^2\langle 31\rangle^2}\;\mathcal{Q}_3^{34}\;\delta^{(4)}(\sum\eta_i^\alpha|i\rangle)$$

$$\mathcal{M}_3^{(0)}(1^{\mathcal{V}^{\hat{A}}}, 2^{\mathcal{V}^{\hat{B}}}, 3^{\mathcal{V}^{\hat{C}}}) = -i\frac{\kappa}{2\sqrt{2}}\frac{g'F^{\hat{A}\hat{B}\hat{C}}}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}\;\mathcal{Q}_3^{34}\;\delta^{(4)}(\sum\eta_i^\alpha|i\rangle)$$

$$\mathcal{M}_3^{(0)}(1^{\mathcal{V}^{\hat{A}}}, 2^{\mathcal{V}^{\hat{B}}}, 3^{\mathcal{H}_-}) = \frac{\kappa}{2}\frac{\delta^{\hat{A}\hat{B}}}{\langle 12\rangle^2}\;\mathcal{Q}_3^{34}\;\delta^{(4)}(\sum\eta_i^\alpha|i\rangle)$$

An observation: because the 3-scalar matrix element is constant and thus are the same on shell and off shell all trilinear vertices involving $V_\mu^{\hat{A}}$, $\zeta^{\hat{A}\alpha}$ and $S^{\hat{A}}$ have an off-shell double-copy structure ⟶ loop level consequences

Comparison w/ Lagrangian

$$\mathcal{M}_3^{(0)}(1^{\mathcal{H}_-}, 2^{\mathcal{H}_-}, 3^{\mathcal{H}_+}) = -i\frac{\kappa}{2}\frac{\langle 12\rangle^2}{\langle 23\rangle^2\langle 31\rangle^2}\;\mathcal{Q}_3^{34}\;\delta^{(4)}(\sum\eta_i^\alpha|i\rangle)$$

$$\mathcal{M}_3^{(0)}(1^{\mathcal{V}^{\hat{A}}}, 2^{\mathcal{V}^{\hat{B}}}, 3^{\mathcal{V}^{\hat{C}}}) = -i\frac{\kappa}{2\sqrt{2}}\frac{g'F^{\hat{A}\hat{B}\hat{C}}}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}\;\mathcal{Q}_3^{34}\;\delta^{(4)}(\sum\eta_i^\alpha|i\rangle)$$

$$\mathcal{M}_3^{(0)}(1^{\mathcal{V}^{\hat{A}}}, 2^{\mathcal{V}^{\hat{B}}}, 3^{\mathcal{H}_-}) = \frac{\kappa}{2}\frac{\delta^{\hat{A}\hat{B}}}{\langle 12\rangle^2}\;\mathcal{Q}_3^{34}\;\delta^{(4)}(\sum\eta_i^\alpha|i\rangle)$$

$$F^{\hat{A}\hat{B}\hat{C}} = 2i\begin{cases} f^{rst} & \text{if}\quad \hat{A}=r\wedge\hat{B}=s\wedge\hat{C}=0 \\ 0 & \text{if}\quad \hat{A}=\hat{m}\vee\hat{B}=\hat{n}\vee\hat{C}=\hat{p}\end{cases}$$

- Double-copy vs. Lagrangian field identification

$$h_{\pm\pm} = h_{\pm\pm}^{\mathcal{L}} \qquad V_\pm = \pm ie^{-i\theta}V_\pm^{\mathcal{L}}$$

$$\tilde{V}_\pm = \pm ie^{i\theta}\tilde{V}_\pm^{\mathcal{L}} \qquad S_{+-} = -iS_{+-}^{\mathcal{L}} \qquad S_{-+} = +iS_{-+}^{\mathcal{L}}$$

$$V_\pm^{\hat{m}} = V_\pm^{\hat{m}\mathcal{L}} \qquad S^{\hat{m}} = e^{-i\theta}S^{\hat{m}\mathcal{L}} \qquad \bar{S}^{\hat{m}} = e^{i\theta}\bar{S}^{\hat{m}\mathcal{L}}$$

$$V_\pm^r = V_\pm^{r\mathcal{L}} \qquad S^r = e^{-i\theta}S^{r\mathcal{L}} \qquad \bar{S}^r = e^{i\theta}\bar{S}^{r\mathcal{L}}$$

$$\tau_{\mathcal{L}} = \sigma_{\mathcal{L}} + 4\pi ie^{\phi_{\mathcal{L}}}$$

vs.

$$\phi_{\mathrm{KLT}} \sim S_{+-} + S_{-+}$$

- Recombine in on-shell superfields:

$$\mathrm{H}_+ = \mathrm{H}_+^{\mathcal{L}} \qquad \mathrm{H}_- = \mathrm{H}_-^{\mathcal{L}}$$

$$\tilde{\mathrm{V}}_+ = \tilde{\mathrm{V}}_+^{\mathcal{L}} \qquad \tilde{\mathrm{V}}_- = \tilde{\mathrm{V}}_-^{\mathcal{L}}$$

$$\mathrm{V}_+^{\hat{m}} = \mathrm{V}_+^{\mathcal{L}\hat{m}} \qquad \mathrm{V}_-^{\hat{m}} = \mathrm{V}_-^{\mathcal{L}\hat{m}}$$

$$\mathrm{V}_+^r = \mathrm{V}_+^{\mathcal{L}r} \qquad \mathrm{V}_-^r = \mathrm{V}_-^{\mathcal{L}r}$$

Some 4-point examples

$\mathcal{N}$=4 sYM numerators w/ $\delta^{(8)}$ extracted

$$\mathcal{A}_4^{(0),\mathcal{N}=2}(1^{\mathcal{G}}2^{\mathcal{G}}3^{\mathcal{G}}4^{\mathcal{G}}) = \left(\frac{\hat{n}_s c_s}{s} + \frac{\hat{n}_t c_t}{t} + \frac{\hat{n}_u c_u}{u}\right) \mathcal{Q}_4^{34}\, \delta^{(4)}(\sum_i \eta_i |i\rangle)$$

$$c_s = \tilde{f}^{a_1a_2b}\tilde{f}^{a_3a_4b} \qquad c_t = \tilde{f}^{a_1a_4b}\tilde{f}^{a_2a_3b} \qquad c_u = \tilde{f}^{a_1a_3b}\tilde{f}^{a_4a_2b}$$

Use $\mathcal{O}(g')$ and $\mathcal{O}(g'^2)$ vector-scalar amplitudes

Choose $\hat{n}_t = 0$

$$\mathcal{A}_4^{(0)}(1^{\phi^{\hat{A}_1}}2^{\phi^{\hat{A}_2}}3^{\phi^{\hat{A}_3}}4^{\phi^{\hat{A}_4}})|_{g'^2} = g^2 g'^2 \left(\frac{1}{s}F^{\hat{A}_1\hat{A}_2\hat{B}}F^{\hat{A}_3\hat{A}_4\hat{B}}\mathfrak{f}^{a_1a_2b}\mathfrak{f}^{a_3a_4b} + \frac{1}{u}F^{\hat{A}_3\hat{A}_1\hat{B}}F^{\hat{A}_2\hat{A}_4\hat{B}}\mathfrak{f}^{a_3a_1b}\mathfrak{f}^{a_2a_4b} + \frac{1}{t}F^{\hat{A}_2\hat{A}_3\hat{B}}F^{\hat{A}_1\hat{A}_4\hat{B}}\mathfrak{f}^{a_2a_3b}\mathfrak{f}^{a_1a_4b}\right)$$

$$\hat{A}_1 \neq \hat{A}_2 \neq \hat{A}_3 \neq \hat{A}_4$$

$$M_4^{(0)}(1^{\mathcal{V}^{\hat{A}_1}}2^{\mathcal{V}^{\hat{A}_2}}3^{\mathcal{V}^{\hat{A}_3}}4^{\mathcal{V}^{\hat{A}_4}}) = i\left(\frac{\kappa}{2}\right)^2 g'^2 \left(\frac{\hat{n}_s}{s}\, F^{\hat{A}_1\hat{A}_2\hat{B}}F^{\hat{A}_3\hat{A}_4\hat{B}} + \frac{\hat{n}_u}{u}\, F^{\hat{A}_3\hat{A}_1\hat{B}}F^{\hat{A}_2\hat{A}_4\hat{B}}\right) \mathcal{Q}_4^{34}\delta^{(4)}(\sum_i \eta_i|i\rangle)$$

$$M_4^{(0)}(1^{\mathcal{V}^{\hat{A}_1}}2^{\mathcal{V}^{\hat{A}_2}}3^{\mathcal{V}^{\hat{A}_3}}4^{\mathcal{H}_+}) = -\frac{i}{\sqrt{2}}\left(\frac{\kappa}{2}\right)^2 g' F^{\hat{A}_1\hat{A}_2\hat{A}_3}\, \frac{1}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}\, \mathcal{Q}_4^{34}\delta^{(4)}(\sum_i \eta_i|i\rangle)$$

- Same field and parameter identification as for 3-point amp's reproduce Lagrangian calculations

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$$\mathcal{A}_4^{(0)}(1^{\phi^{\hat{A}_1}}2^{\phi^{\hat{A}_2}}3^{\phi^{\hat{A}_3}}4^{\phi^{\hat{A}_4}})|_{g'^2} = g^2 g'^2 \left(\frac{1}{s}F^{\hat{A}_1\hat{A}_2\hat{B}}F^{\hat{A}_3\hat{A}_4\hat{B}}\mathfrak{f}^{a_1a_2b}\mathfrak{f}^{a_3a_4b} + \frac{1}{u}F^{\hat{A}_3\hat{A}_1\hat{B}}F^{\hat{A}_2\hat{A}_4\hat{B}}\mathfrak{f}^{a_3a_1b}\mathfrak{f}^{a_2a_4b} + \frac{1}{t}F^{\hat{A}_2\hat{A}_3\hat{B}}F^{\hat{A}_1\hat{A}_4\hat{B}}\mathfrak{f}^{a_2a_3b}\mathfrak{f}^{a_1a_4b}\right)$$

$$\hat{A}_1 \neq \hat{A}_2 \neq \hat{A}_3 \neq \hat{A}_4$$

$$M_4^{(0)}(1^{\mathcal{V}^{\hat{A}_1}}2^{\mathcal{V}^{\hat{A}_2}}3^{\mathcal{V}^{\hat{A}_3}}4^{\mathcal{V}^{\hat{A}_4}}) = i\left(\frac{\kappa}{2}\right)^2 g'^2 \left(\frac{\hat{n}_s}{s}\, F^{\hat{A}_1\hat{A}_2\hat{B}}F^{\hat{A}_3\hat{A}_4\hat{B}} + \frac{\hat{n}_u}{u}\, F^{\hat{A}_3\hat{A}_1\hat{B}}F^{\hat{A}_2\hat{A}_4\hat{B}}\right) \mathcal{Q}_4^{34}\delta^{(4)}(\sum_i \eta_i|i\rangle)$$

$$M_4^{(0)}(1^{\mathcal{V}^{\hat{A}_1}}2^{\mathcal{V}^{\hat{A}_2}}3^{\mathcal{V}^{\hat{A}_3}}4^{\mathcal{H}_+}) = -\frac{i}{\sqrt{2}}\left(\frac{\kappa}{2}\right)^2 g' F^{\hat{A}_1\hat{A}_2\hat{A}_3}\, \frac{1}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}\, \mathcal{Q}_4^{34}\delta^{(4)}(\sum_i \eta_i|i\rangle)$$

- Same field and parameter identification as for 3-point amp's reproduce Lagrangian calculations

  - Color/kinematics duality for gravity amplitudes
  - General tree-level argument

- 5-point tree amplitudes:
  - Use c/k-satisfying representation of $\mathcal{N}$=2 sYM amplitude obtained by $\mathbf{Z}_2$ - truncating the $\mathcal{N}$=4 amplitudes of Broedel & Carrasco;
    - * all 15 color structures appear to be present

- 5-point tree amplitudes:
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    - * all 15 color structures appear to be present

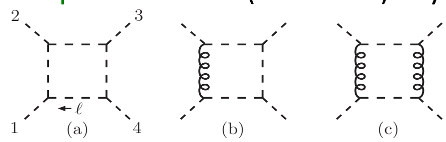
- Color-ordered (MHV) tree-level single-trace vector-graviton at higher-points
  - non-supersymmetric vector+gravity: perturbiner formalism — Selivanov
    (solutions of the classical equations of motion which are generating functions of tree-level form factors)
  - KLT-like formalism + analytic properties — Bern, de Freitas, Wong

$$M_n(1_g^-, 2_g^+, 3_g^+, \ldots, m_g^+, (m+1)_h^-, (m+2)_h^+, \ldots, n_h^+)$$
$$= i g^{m-2} \Big(-\frac{\kappa}{2}\Big)^{n-m} \frac{\langle 1\ m+1\rangle^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle \cdots \langle m\,1\rangle}\ S(1, m+1, \{h^+\}, \{g^+\})$$

$$S(i,j,\{h^+\},\{g^+\}) = \Big(\prod_{m\in\{h^+\}} \frac{d}{da_m}\Big)$$
$$\times \prod_{l\in\{g^+\}} \exp\Big[\sum_{n_1\in\{h^+\}} a_{n_1} \frac{\langle l\,i\rangle\,\langle l\,j\rangle\,[l\,n_1]}{\langle n_1\,i\rangle\,\langle n_1\,j\rangle\,\langle l\,n_1\rangle} \exp\Big[\sum_{\substack{n_2\in\{h^+\}\\ n_2\neq n_1}} a_{n_2} \frac{\langle n_1\,i\rangle\,\langle n_1\,j\rangle\,[n_1\,n_2]}{\langle n_2\,i\rangle\,\langle n_2\,j\rangle\,\langle n_1\,n_2\rangle} \exp\Big[\ldots\Big]\Big]\Big]\Big|_{a_j=0}.$$

On 1-loop amplitudes

- Grade amplitudes following dep. on supergravity gauge coupling
  - highest power = one g' in each vertex ⟶ all from minimal coupling
  - ► in SG Lagrangian: given by an $\mathcal{N}$=2 sYM terms ⟶ same from c/k
- An example: 4-vector amplitude
  - Vector-scalar amplitude: three (classes of) Feynman graphs



Color/kinematics-satisfying form: need 3 master integrals; pick boxes

$$n^{(a)}_{\text{box}}(1,2,3,4) = (g')^4 F^{\hat{B}\hat{A}_1\hat{C}} F^{\hat{C}\hat{A}_2\hat{D}} F^{\hat{d}\hat{A}_3\hat{E}} F^{\hat{E}\hat{A}_4\hat{B}}$$

$$n^{(b)}_{\text{box}}(1,2,3,4,\ell) = \frac{i}{2}(g')^2\big((\delta^{\hat{A}_1\hat{A}_2} F^{\hat{C}\hat{A}_3\hat{B}} F^{\hat{B}\hat{A}_4\hat{C}} - F^{\hat{A}_2\hat{A}_4\hat{B}} F^{\hat{B}\hat{A}_3\hat{A}_1}) s_{1\bar{\ell}} + F^{\hat{A}_1\hat{A}_4\hat{B}} F^{\hat{B}\hat{A}_3\hat{A}_2}(s_{12} - s_{2\bar{\ell}})\big) + \text{cyclic}$$

$$n^{(c)}_{\text{box}}(1,2,3,4,\ell) = (A\delta^{\hat{A}_1\hat{A}_2}\delta^{\hat{A}_3\hat{A}_4} + B\delta^{\hat{A}_1\hat{A}_3}\delta^{\hat{A}_2\hat{A}_4} + C\delta^{\hat{A}_1\hat{A}_4}\delta^{\hat{A}_2\hat{A}_3}) + \text{cyclic}$$

Triangle & bubble numerators:

$$n^{(x)}_{\text{tri.}}([1,2],3,4,\ell) = n^{(x)}_{\text{box}}(1,2,3,4,\ell) - n^{(x)}_{\text{box}}(2,1,3,4,\ell)$$

$$n^{(x)}_{\text{bub.}}([1,2],[3,4],\ell) = n^{(x)}_{\text{tri.}}([1,2],3,4,\ell) - n^{(x)}_{\text{tri.}}([1,2],4,3,\ell)$$

$$c_{\text{box}} = c^{(a)}_{\text{box}} = c^{(b)}_{\text{box}} = c^{(c)}_{\text{box}} = f^{ba_1c} f^{ca_2d} f^{da_3e} f^{ea_4b}$$

$$c_{\text{tri.}}([1,2],3,4) = c_{\text{box}}(1,2,3,4) - c_{\text{box}}(2,1,3,4)$$

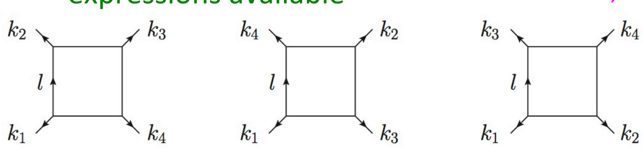
$$c_{\text{bub.}}([1,2],[3,4]) = c_{\text{tri.}}([1,2],3,4) - c_{\text{tri.}}([1,2],4,3)$$

On 1-loop amplitudes

- Grade amplitudes following dep. on supergravity gauge coupling
  - highest power = one g' in each vertex ⟶ all from minimal coupling
  - ► in SG Lagrangian: given by an $\mathcal{N}$=2 sYM terms ⟶ same from c/k
- An example: 4-vector amplitude

  - $\mathcal{N}$=2 sYM: color/kinematics-satisfying expressions available

Carrasco, Chiodaroli, Gunaydin, RR; Nohle; Ochirov, Tourkine; Johansson, Ochirov



$$n_i = st A^{(0)}_4(1234) N_i$$

$$N_i^{\mathcal{N}} = N_i^{\mathcal{N}=4} - (4-\mathcal{N}) N_i^{\text{chiral}}\ , \quad N_{1,2,3}^{\mathcal{N}=4} = 1\ , \quad \mathcal{A}^{(1)}_{n,\mathcal{N}=4} = \mathcal{A}^{(1)}_{n,\mathcal{N}=4} - (4-\mathcal{N})\mathcal{A}^{(1)}_{n,\text{chiral}}$$

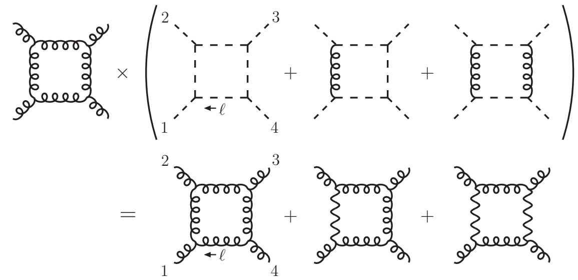
$$N_1^{\text{chiral}} = \frac{1}{s}(\tau_{l,k_1} - \tau_{l,k_2}) + \frac{2}{stu}(s\tau_{l,k_1}\tau_{l,k_2} - u\tau_{l,k_1}\tau_{l,k_3} - t\tau_{l,k_2}\tau_{l,k_3})$$

$$N_2^{\text{chiral}} = \frac{1}{s^2}(t\tau_{l,k_2} + s\tau_{l,k_3} - u\tau_{l,k_1}) + \frac{2}{stu}(s\tau_{l,k_1}\tau_{l,k_2} - u\tau_{l,k_1}\tau_{l,k_3} - t\tau_{l,k_2}\tau_{l,k_3}) - \frac{2i}{s^2}\varepsilon_{k_1,k_2,k_3,l}$$

$$N_3^{\text{chiral}} = -\frac{1}{u}(\tau_{l,k_3} + \tau_{l,k_1}) + \frac{2}{stu}(s\tau_{l,k_1}\tau_{l,k_2} - u\tau_{l,k_1}\tau_{l,k_3} - t\tau_{l,k_2}\tau_{l,k_3})$$

- The other numerators from Jacobi identity

1-loop 4-vector amplitude in MESGTs
in the generic Jordan family



$$\mathcal{M}_4^{(1)} = -\left(\frac{\kappa}{2}\right)^4 \sum_{\mathcal{S}_4} \sum_{i\in\{\text{box,tri.,bub.}\}} \int \frac{d^D\ell}{(2\pi)^D} \frac{1}{S_i} \frac{n_i\tilde{n}_i}{D_i}$$

$$D_{\text{box}} = \ell_1^2\ell_2^2\ell_3^2\ell_4^2\,, \quad D_{\text{tri.}} = s\ell_2^2\ell_3^2\ell_4^2\,, \quad D_{\text{bub.}} = s^2\ell_2^2\ell_4^2$$

$$\{S_{\text{box}}, S_{\text{tri.}}, S_{\text{bub.}}\} = \{8, 4, 16\}$$

- Can reduce to regular master integrals, etc.
- $(g')^4$ given by an $\mathcal{N}$=2 sYM expression, as implied by general argument

Summary and outlook

- Discussed gauging of non-abelian symmetries of MESGTs
  - $\mathcal{N}$ < 4 SG: field content does not specify the theory uniquely
- General characteristics of introducing minimal couplings through 2-copy
- Identified the 2-copy structure of the generic Jordan family of YMESGTs
- Constructed the map between the 2-copy and Lagrangian fields
- Showed how to compute tree- and loop-level amplitudes
- 2-copy constructions for other MESGTs and YMESGTs?
  - some in terms of gauge theories with bifundamental matter
- Scattering amplitudes in SGTs with gauged R symmetry?
- What to do when there is no Minkowski ground state?
  - Witten diagrams, embedding formalism, Mellin space, etc.
  - connection with large body of work in AdS/CFT context