Simple form factors in $\mathcal{N} = 4$ SYM

Amplitudes 2014

- IPhT CEA-Saclay -

based on:

hep-th/1402.1300 - BP, Spence, Travaglini, Wen
hep-th/1406.1443 - Brandhuber, BP, Travaglini, Wen

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10/06/14

Outline

1. Motivation
2. Half-BPS operators
3. MHV form factors of half-BPS operators
   - Tree level
   - One loop
   - Minimal form factors at two loops
4. Conclusions

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4. Conclusions
5. Lunch

Motivation I: Bridge between amplitudes and correlation functions

Amplitudes

$$A_n = \langle 1 \cdots n | 0 \rangle$$

on-shell

Corr. functions

$$C_n = \langle \mathcal{O}_1 \cdots \mathcal{O}_n \rangle$$

off-shell

- Recursion relations
- On-shell diagrams
- Grassmannian
- Amplituhedron

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Motivation I: Bridge between amplitudes and correlation functions

Form Factors

Amplitudes

$$A_n = \langle 1 \cdots n | 0 \rangle$$

Corr. functions

$$C_n = \langle \mathcal{O}_1 \cdots \mathcal{O}_n \rangle$$

on shell

off shell

$$F_{\mathcal{O},n} = \langle 1 \cdots n | \mathcal{O} | 0 \rangle$$

"A little bit off shell"

Definition of a form factor

- Gauge invariant local operator $\mathcal{O}(x)$
- n-particle state $\langle 1 \cdots n|$

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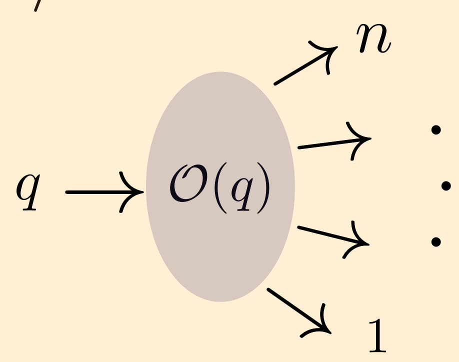
$$F_{\mathcal{O},n}(q) := \int d^4x \, e^{-iqx} \, \langle 1 \cdots n|\mathcal{O}(x)|0\rangle$$

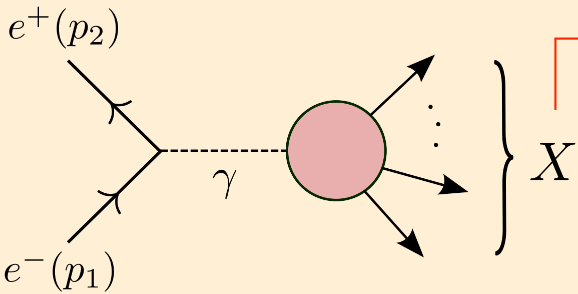
$$= \delta^{(4)}\Big(q - \sum_{i=1}^{n} p_i\Big)\langle 1 \cdots n|\mathcal{O}(0)|0\rangle$$



$q^2 \neq 0$

Motivation II: Form factors are interesting on their own...

Ex 1: E&M hadronic current



→ hadronic state

$$e\,\bar{\nu}(p_2)\gamma_\mu u(p_1)\,\frac{\eta^{\mu\nu}}{(p_1+p_2)^2}\,(-e)\langle X|J_\nu^{\rm em}(0)|0\rangle$$

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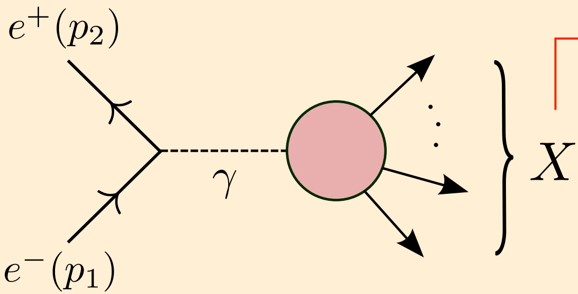

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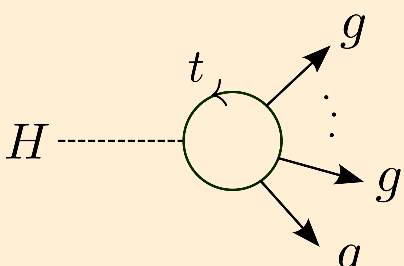
Ex 2: Higgs+multigluon amplitudes in QCD



$$\langle Hg\cdots g|\mathrm{Tr}F^2(0)|0\rangle$$

$$m_H < 2m_t \longrightarrow \mathcal{L}_{\text{eff}} \sim H\,\mathrm{Tr}F^2$$

Related via SUSY to $F_{\mathrm{Tr}\phi^2,n}$ in $\mathcal{N}=4$

[[Brandhuber, Gurdogan, Mooney, Travaglini, Yang (2011)]]

Half-BPS operators $\mathcal{O}_k := \mathrm{Tr}(\phi^k)$

$k = 2$ [[Brandhuber, Spence, Travaglini, Yang (2010)]]

- Part of the stress-tensor multiplet
- Related to Higgs+multigluon amps in QCD

$$F_{\mathrm{Tr}(\phi^2)}(1^\phi, 2^\phi; q) = \delta^{(4)}(q - p_1 - p_2)$$

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$$F_{\mathrm{Tr}(\phi^2)}(1^\phi, 2^\phi; q) \;=\; \delta^{(4)}(q - p_1 - p_2)$$

Adding arbirarily many g^+ → "MHV"

$$F_{\mathrm{Tr}(\phi^2)}(i^\phi, j^\phi, \{g^+\}; q) \;=\; \frac{\langle i\, j\rangle^2}{\langle 12\rangle \cdots \langle n1\rangle}\delta^{(4)}(q - P)$$

$$P \;=\; \sum_{i=1}^{n} p_i$$

Tree

Generalisation for any k

[[BP, Spence, Travaglini, Wen (2014)]]

* "Minimal" form factors: # legs = # fields

$$F_{\mathrm{Tr}(\phi^k)}(1^\phi, \ldots, k^\phi; q) \;=\; \delta^{(4)}(q-P)$$

* Adding g^+ → "MHV family"

$$F_{\mathrm{Tr}(\phi^k)}(i_1^\phi, \ldots, i_k^\phi, \{g^+\}; q) \;=\; \frac{\langle i_1\, i_2\rangle \cdots \langle i_k\, i_1\rangle}{\langle 12\rangle \cdots \langle n1\rangle}\delta^{(4)}(q-P)$$

Supersymmetric form factors

[[Brandhuber, Travaglini, Yang; + Gurdogan, Mooney; Bork, Kazakov, Vartanov;
BP, Spence, Travaglini, Wen; Brandhuber, BP, Travaglini, Wen]]

- External states ⟶ $\mathcal{N}=4$ Nair's Superwavefunction

$$\Phi(p,\eta) \quad := \quad g^{+}(p) + \eta_A\,\lambda^{A}(p) + \frac{\eta_A\eta_B}{2!}\,\phi^{AB}(p) + \epsilon^{ABCD}\frac{\eta_A\eta_B\eta_C}{3!}\,\bar{\lambda}_D(p) + \eta_1\eta_2\eta_3\eta_4\,g^{-}(p)$$

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- Operator ⟶ Harmonic projection [[Galperin, Ivanov, Ogievetsky, Sokatchev (2001)]]

$k=2$ ⟶ Chiral part of stress-tensor multiplet

$$\mathcal{T} \;:=\; \mathrm{Tr}(W^{++}W^{++}) = \mathrm{Tr}(\phi^{++}\phi^{++}) + \cdots + (\theta^{+})^4\mathcal{L}$$

W: Chiral vector superfield

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$$\mathcal{T}_k \;:=\; \mathrm{Tr}[(W^{++})^k] = \mathrm{Tr}[(\phi^{++})^k] + \cdots$$

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$\phi^{++} \rightarrow \phi_{12}$

MHV super form factors

[[Brandhuber, Gurdogan, Mooney, Travaglini, Yang (2011); Bork, Kazakov, Vartanov (2011)]]

Definition of form factors of $\mathcal{T}_k$:

$$\mathcal{F}_{\mathcal{T}_k,n}(q,\gamma_+) := \int d^4x\, d^4\theta^+ e^{-(iqx+i\theta^{+a}_{\alpha}\gamma^{\alpha}_{+a})}\langle 1\cdots n|\mathcal{T}_k(x,\theta^+)|0\rangle$$

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For $\mathcal{T}_2$ (Ward ids.):

$$\mathcal{F}_{\mathcal{T}_2,n} = \frac{\delta^{(4)}(q-P)\,\delta^{(4)}(\gamma_+ - \mathcal{Q}_+)\,\delta^{(4)}(\mathcal{Q}_-)}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}\,, \qquad \mathcal{Q}_\pm = \sum_{i=1}^{n}\lambda_i\eta_{\pm,i}$$

Compare with superamplitude:

$$\mathcal{A}^{\mathrm{MHV}} = \frac{\delta^{(4)}(P)\delta^{(4)}(\mathcal{Q})}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}, \qquad \mathcal{Q} = \sum_{i=1}^{n}\lambda_i\eta_i$$

MHV super form factors of $\mathcal{T}_3$

[[BP, Spence, Travaglini, Wen (2014)]]

$$\mathcal{F}_{\mathcal{T}_2,n} = \frac{\delta^{(4)}(q-P)\,\delta^{(4)}(\gamma_+ - \mathcal{Q}_+)\,\delta^{(4)}(\mathcal{Q}_-)}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$$

$$\mathcal{F}_{\mathcal{T}_3,n} \;=\; \mathcal{F}_{\mathcal{T}_2,n}\,\delta\Big(\sum_{i<j=1}^{n}\langle i\,j\rangle\,\eta_{-,i}\cdot\eta_{-,j}\Big)$$

Extra R-charge

$$\eta_{-,i}\cdot\eta_{-,j} \;:=\; \tfrac{1}{2}\,\eta_{-a,i}\eta_{-b,j}\,\epsilon^{ab}$$

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Extra R-charge

- Adjacent BCFW shifts ⟶ Boundary term $\propto \mathcal{F}_{\mathcal{T}_2,n-1}$

  ⟶ On-shell recursion relation

$$\mathcal{F}_{\mathcal{T}_k,n} = \frac{\langle n-1\,1\rangle}{\langle n-1\,n\rangle\langle n\,1\rangle}\mathcal{F}_{\mathcal{T}_k,n-1} + (\eta_{-,n})^2\mathcal{F}_{\mathcal{T}_{k-1},n-1}$$

Soft-factor

Boundary

MHV super form factors of $\mathcal{T}_k$

[[BP, Spence, Travaglini, Wen (2014)]]

- Solution of recursion relation for all k (conjecture)

Ex: $k = 4$

$$\mathcal{F}_{\mathcal{T}_4,n} = \mathcal{F}_{\mathcal{T}_2,n} \sum_{1\leq i\leq j}^{n-3} \sum_{j<k\leq l}^{n-2} (2-\delta_{ij})(2-\delta_{kl}) \frac{\langle n\, i\rangle \langle j\, k\rangle \langle l\, n-1\rangle}{\langle n-1\, n\rangle} (\eta_{-,i}\cdot\eta_{-,j})(\eta_{-,k}\cdot\eta_{-,l})$$

Consistency checks:

Cyclicity:

$k = 4$
$\forall n$
analytically

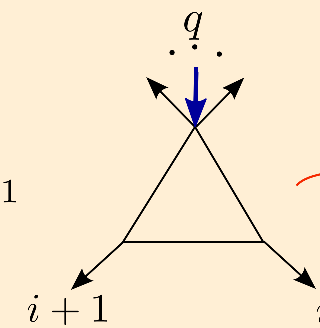
$k \leq 6$
$n \leq 7$
numerically

1 loop

MHV form factors of $\mathcal{T}_k$ at one loop

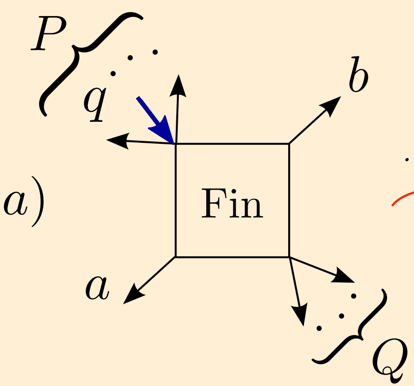
[[BP, Spence, Travaglini, Wen (2014)]]

$$\mathcal{F}^{\mathrm{MHV}(1)}_{\mathcal{T}_k,n} = -\mathcal{F}^{\mathrm{MHV}(0)}_{\mathcal{T}_k,n} \sum_{i=1}^{n} s_{i\,i+1}$$



IR divergences

$$+\mathcal{F}^{\mathrm{MHV}(0)}_{\mathcal{T}_2,n} \sum_{a,b} f_{\mathcal{T}_k}(a+1,\ldots,b-1,b,a)$$



Gen. unitarity

$$\mathcal{F}_{\mathcal{T}_k,n} = \mathcal{F}_{\mathcal{T}_2,n}\, f_{\mathcal{T}_k}$$

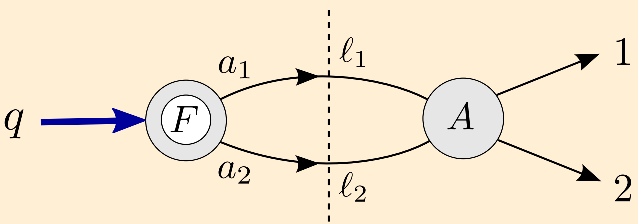
2 loops

Minimal form factors

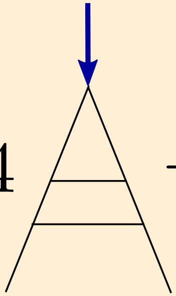
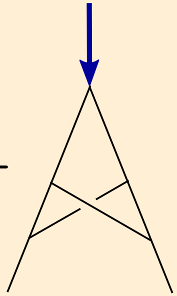
- As many particles as fields

$$F_k \; := \; \langle 1^{\phi} \cdots k^{\phi} | \mathrm{Tr}\phi^k(0) | 0 \rangle$$

- Ex: F_2 at two loops [[van Neerven (1976); Gehrmann, Henn, Huber (2012); Brandhuber, Travaglini, Yang (2012)]]



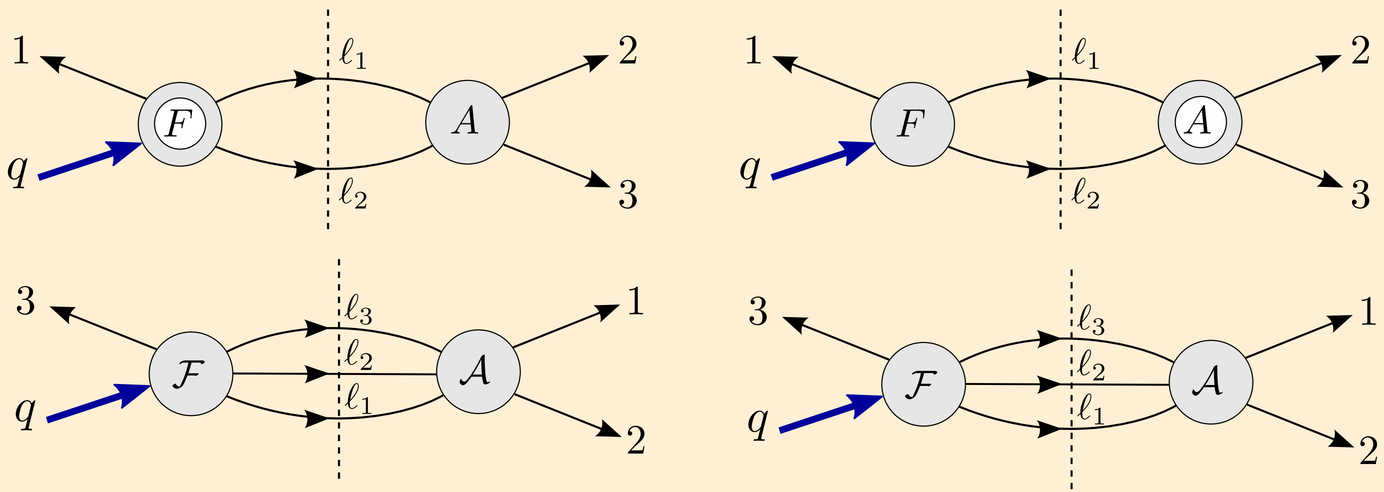
- $F^{a_1 a_2} \propto \delta^{a_1 a_2}$ → Non-planar contributions from 1-loop amplitude

$F_2^{(2)}(q^2) = 4$  $+$ 

$F_3^{(2)}$ at two loops

[[Brandhuber, BP, Travaglini, Wen (2014)]]

- Generalised unitarity

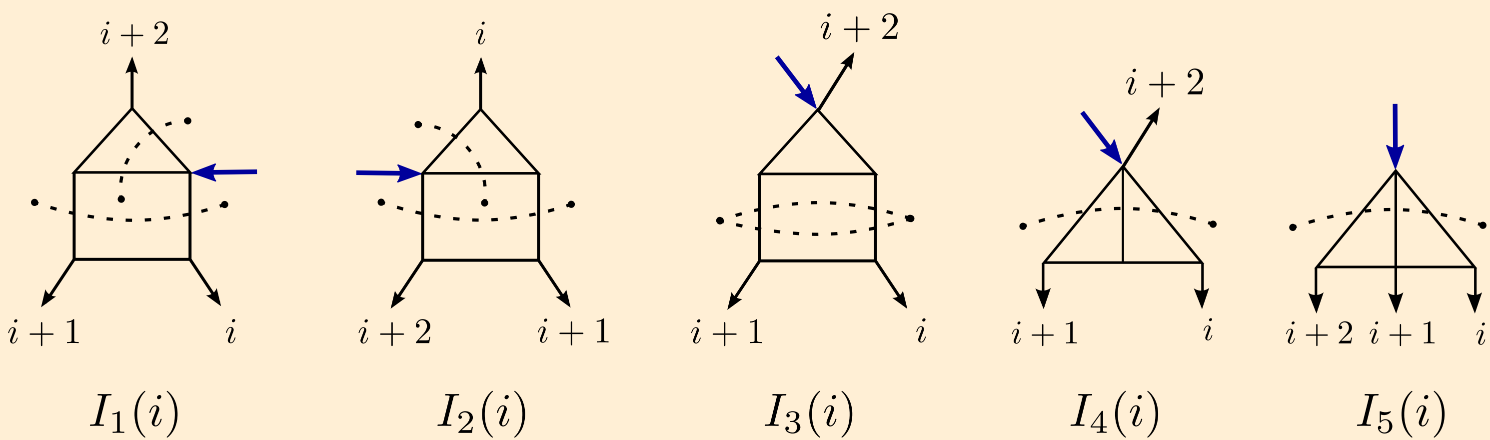


~~$\delta^{a_1 a_2}$~~

No non-planar integrals ✓

$F_3^{(2)}$ at two loops

- Result: $$F_3^{(2)} = \sum_{i=1}^{3} \Big[I_1(i) + I_2(i) + I_3(i) + I_4(i) - I_5(i) \Big]$$



Decompose in terms of master integrals

[FIRE]

Analytical expressions for all integrals ✓

[[Gehrmann, Remiddi ('00 + '01)]]

$F_3^{(2)}$ at two loops

- Uniform transcendentality four
- Many Goncharov polylogs with numerical coefficients
- Variables:

$$u = \frac{s_{12}}{q^2}, \quad v = \frac{s_{23}}{q^2}, \quad w = \frac{s_{31}}{q^2}$$

$$q = p_1 + p_2 + p_3 \quad \Rightarrow \quad u + v + w = 1$$

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Can we simplify this?

Form factor remainder

$$R_3^{(2)} := F_3^{(2)}(\epsilon) - \overbrace{\frac{1}{2}\left[F_3^{(1)}(\epsilon)\right]^2 - f^{(2)}F_3^{(1)}(2\epsilon) - C^{(2)}}^{\text{ABDK/BDS ansatz}} + \mathcal{O}(\epsilon)$$

$$f^{(2)} = -f_0^{(2)} - f_1^{(2)}\epsilon - f_2^{(2)}\epsilon^2$$

Properties:

- Finite
- Symmetric under permutations of (u, v, w)

Form factor remainder

ABDK/BDS ansatz

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$$f^{(2)} = -f_0^{(2)} - f_1^{(2)}\epsilon - f_2^{(2)}\epsilon^2$$

Finiteness

$$f_1^{(2)} = 2\zeta_3 \sim \Gamma_{\text{coll}}^{(2)}$$

$$f_0^{(2)} = 2\zeta_2 \sim \Gamma_{\text{cusp}}^{(2)}$$

Properties:

- Finite
- Symmetric under permutations of (u, v, w)

Also: $C^{(2)} = 0,\ f_2^{(2)} = 2\zeta_4$

Simplifying the remainder

$R_3^{(2)}$ → Very messy

$R_3^{(2)}$ → Non-classical polylogs

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↓

Symbol → Transc. k function $\longrightarrow$ k -fold $\bigotimes$ product

[[Goncharov, Spradlin, Volovich, Vergu (2010)]]

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[[Goncharov, Spradlin, Volovich, Vergu (2010)]]

↓

$$\mathcal{S}_3^{(2)}(u,v,w) = u \otimes v \otimes \left[\frac{u}{w} \otimes_S \frac{v}{w}\right] + \frac{1}{2} u \otimes \frac{u}{(1-u)^3} \otimes \frac{v}{w} \otimes \frac{v}{w}$$
$$+ \ \text{perms}(u,v,w)$$

(notice unusual 2nd and last entry conditions)

Simplifying the remainder

$$\mathcal{S}_3^{(2)}$$

→ Very simple

→ Goncharov's condition

$$\left\{ \begin{array}{c} \mathcal{S}_{abcd} - \mathcal{S}_{bacd} - \mathcal{S}_{abdc} + \mathcal{S}_{badc} \\ - (a \leftrightarrow c\,,\, b \leftrightarrow d) = 0 \\ \delta\left(\mathcal{S}_3^{(2)}\right)\Big|_{\Lambda^2 B_2} = 0 \end{array} \right.$$

↓

Can be integrated back to classical polylogs

Simplifying the remainder

$\mathcal{S}_3^{(2)}$ → Very simple → Goncharov's condition

$$\begin{cases} \mathcal{S}_{abcd} - \mathcal{S}_{bacd} - \mathcal{S}_{abdc} + \mathcal{S}_{badc} \\ \quad - (a \leftrightarrow c\,,\, b \leftrightarrow d) = 0 \\ \delta\left(\mathcal{S}_3^{(2)}\right)\Big|_{\Lambda^2 B_2} = 0 \end{cases}$$

↓

Can be integrated back to classical polylogs

Strategy:

1) Decompose the symbol into $\mathrm{A}\otimes\mathrm{A}$, $\mathrm{S}\otimes\mathrm{A}$, $\mathrm{A}\otimes\mathrm{S}$, $\mathrm{S}\otimes\mathrm{S}$

2) Compare with

Function	$\mathrm{A}\otimes\mathrm{A}$	$\mathrm{S}\otimes\mathrm{A}$	$\mathrm{A}\otimes\mathrm{S}$	$\mathrm{S}\otimes\mathrm{S}$
$\mathrm{Li}_4(z_1)$	×	×	✓	✓
$\mathrm{Li}_3(z_1)\log(z_2)$	×	×	✓	✓
$\mathrm{Li}_2(z_1)\mathrm{Li}_2(z_2)$	✓	✓	✓	✓
$\mathrm{Li}_2(z_1)\log(z_2)\log(z_3)$	×	✓	✓	✓
$\log(z_1)\log(z_2)\log(z_3)\log(z_4)$	×	×	×	✓

We find $\mathrm{A}\otimes\mathrm{A}=0,\ \mathrm{S}\otimes\mathrm{A}=0$ → ~~Li_2~~

Simplifying the remainder

$\mathcal{S}_3^{(2)}$ → Very simple

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$$\begin{cases} \mathcal{S}_{abcd} - \mathcal{S}_{bacd} - \mathcal{S}_{abdc} + \mathcal{S}_{badc} \\ \quad - (a \leftrightarrow c\,,\, b \leftrightarrow d) = 0 \\ \delta\left(\mathcal{S}_3^{(2)}\right)\Big|_{\Lambda^2 B_2} = 0 \end{cases}$$

↓

Can be integrated back to classical polylogs

Strategy:

3) Choose a list of entries

$$\left\{u, v, w, 1-u, 1-v, 1-w, -\frac{u}{v}, -\frac{u}{w}, -\frac{v}{u}, -\frac{v}{w}, -\frac{w}{u}, -\frac{w}{v}, -\frac{uv}{w}, -\frac{uw}{v}, -\frac{vw}{u}\right\}$$

4) Fix "beyond the symbol" terms (π, ζ_i) using numerics

Simplifying the remainder

$\mathcal{S}_3^{(2)}$ → Very simple

→ Goncharov's condition

$$\begin{cases} \mathcal{S}_{abcd} - \mathcal{S}_{bacd} - \mathcal{S}_{abdc} + \mathcal{S}_{badc} \\ \quad - (a \leftrightarrow c\,,\, b \leftrightarrow d) = 0 \\ \delta\left(\mathcal{S}_3^{(2)}\right)\Big|_{\Lambda^2 B_2} = 0 \end{cases}$$

↓

Can be integrated back to classical polylogs

$$\mathcal{R}_3^{(2)} := -\frac{3}{2}\,\mathrm{Li}_4(u) + \frac{3}{4}\,\mathrm{Li}_4\left(-\frac{uv}{w}\right) - \frac{3}{2}\log(w)\mathrm{Li}_3\left(-\frac{u}{v}\right) + \frac{\log^4(u)}{32}$$
$$+\frac{\log^2(u)}{16}\left[\log^2(v) - 2\log(v)\log(w) + 10\zeta_2\right]$$
$$-\frac{\log(u)}{4}\left[\zeta_2\log(v) - 2\zeta_3\right] + \frac{7}{16}\zeta_4 + \mathrm{perms}(u,v,w)$$

Simplifying the remainder

$\mathcal{S}_3^{(2)}$ → Very simple

→ Goncharov's condition

$$\left\{ \begin{array}{c} \mathcal{S}_{abcd} - \mathcal{S}_{bacd} - \mathcal{S}_{abdc} + \mathcal{S}_{badc} \\ - (a \leftrightarrow c\,,\, b \leftrightarrow d) = 0 \\ \delta\left(\mathcal{S}_3^{(2)}\right)\Big|_{\Lambda^2 B_2} = 0 \end{array} \right.$$

↓

Can be integrated back to classical polylogs

Also from $\delta(\mathcal{S}_3^{(2)})\Big|_{B_3 \otimes \mathbb{C}}$

$$\mathcal{R}_3^{(2)} := -\frac{3}{2}\,\mathrm{Li}_4(u) + \frac{3}{4}\,\mathrm{Li}_4\left(-\frac{uv}{w}\right) - \frac{3}{2}\log(w)\mathrm{Li}_3\left(-\frac{u}{v}\right) + \frac{\log^4(u)}{32}$$

$$+\frac{\log^2(u)}{16}\left[\log^2(v) - 2\log(v)\log(w) + 10\zeta_2\right]$$

$$-\frac{\log(u)}{4}\left[\zeta_2\log(v) - 2\zeta_3\right] + \frac{7}{16}\zeta_4 + \mathrm{perms}(u,v,w)$$

Comments

Some disagreements with previous work

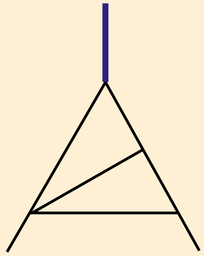
[[Bork, Kazakov, Vartanov hep-th/1011.2440]]

1) $\Gamma^{(2)}_{\text{coll}}$ $\quad f^{(2)} = -f_0^{(2)} - f_1^{(2)}\epsilon - f_2^{(2)}\epsilon^2$

Ours: $f_1^{(2)} = 2\zeta_3$ → Same for amplitudes and FFs of $\text{Tr}\phi^2$

Theirs: $f_1^{(2)} = 14\zeta_3$

2) Appearance of the integral G_3



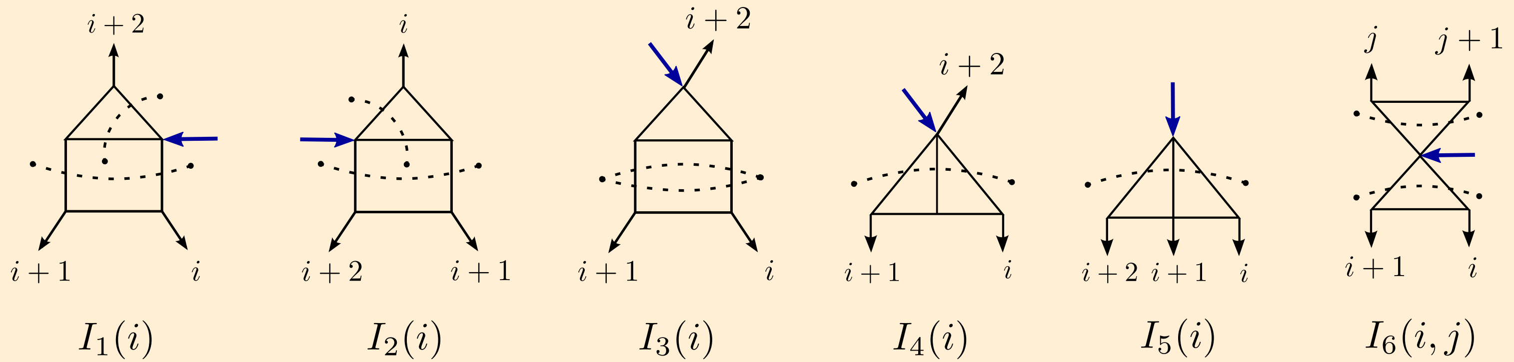
At most one triangle connected to the form factor

[[Boels, Kniehl, Tarasov, Yang (2013)]]

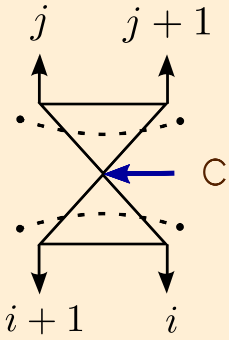
Generalisation to all $R_k^{(2)}$

- Same strategy as for $R_3^{(2)}$

$$F_k^{(2)} = \sum_{i=1}^{n} \left[I_1(i) + I_2(i) + I_3(i) + I_4(i) - I_5(i) + \frac{1}{2} \sum_{j=i+2}^{i-2} I_6(i,j) \right]$$



Generalisation to all $R_k^{(2)}$



cancels cross-terms of $\left[F_k^{(1)}\right]^2$ in the definition of $R_k^{(2)}$ ✓

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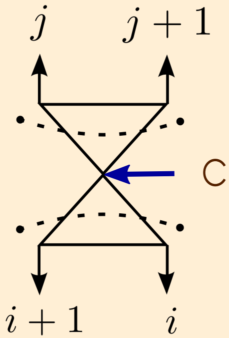

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↓

Everything can be decomposed in 3-particle building blocks

↓ Symbol

$$\mathcal{S}_k^{(2)} = \sum_{i=1}^{k} s_i^{(2)}(u_i, v_i, w_i)$$

$$u_i = \frac{u_{i\,i+1}}{u_{i\,i+1\,i+2}},\ v_i = \frac{u_{i+1\,i+2}}{u_{i\,i+1\,i+2}},\ w_i = \frac{u_{i+2\,i}}{u_{i\,i+1\,i+2}}, \quad u_i + v_i + w_i = 1$$

Building block symbol $s_i^{(2)}$

$s_i^{(2)}$ $\longrightarrow$ Does not satisfy Goncharov's condition

- Identify non-classical part of the remainder:

$$s_i^{(2)} = s_{\mathrm{cl},i}^{(2)} + s_{\mathrm{nc},i}^{(2)}$$

$$\mathcal{S}[\underbrace{-G\left(\{1-v_i, 1-v_i, 1, 0\}, u_i\right) \; - \; (u_i \leftrightarrow v_i)}_{\equiv\, r_{\mathrm{nc},i}^{(2)}}]$$

- Integrate $s_{\mathrm{cl},i}^{(2)}$ and fix beyond the symbol terms

General result for $R_k^{(2)}$

$$
\begin{aligned}
r^{(2)}_{\mathrm{cl},i} = {} & \mathrm{Li}_4(1-u_i)+\mathrm{Li}_4(u_i)-\mathrm{Li}_4\left(\frac{u_i-1}{u_i}\right)+\log\left(\frac{1-u_i}{w_i}\right)\left[\mathrm{Li}_3\left(\frac{u_i-1}{u_i}\right)\right. \\
& \left.-\mathrm{Li}_3\left(1-u_i\right)\right] + \log\left(u_i\right)\left[\mathrm{Li}_3\left(\frac{v_i}{1-u_i}\right) + \mathrm{Li}_3\left(-\frac{w_i}{v_i}\right) + \mathrm{Li}_3\left(\frac{v_i-1}{v_i}\right)\right. \\
& \left.-\frac{1}{3}\log^3\left(v_i\right) - \frac{1}{3}\log^3\left(1-u_i\right)\right] + \mathrm{Li}_2\left(\frac{u_i-1}{u_i}\right)\mathrm{Li}_2\left(\frac{v_i}{1-u_i}\right) \\
& -\mathrm{Li}_2\left(u_i\right)\left[\log\left(\frac{1-u_i}{w_i}\right)\log\left(v_i\right) + \frac{1}{2}\log^2\left(\frac{1-u_i}{w_i}\right)\right] - \frac{1}{24}\log^4\left(u_i\right) \\
& +\frac{1}{8}\log^2\left(u_i\right)\log^2\left(v_i\right)+\frac{1}{2}\log^2\left(1-u_i\right)\log\left(u_i\right)\log\left(\frac{w_i}{v_i}\right) + \frac{1}{6}\log^3\left(u_i\right)\log\left(w_i\right) \\
& +\frac{1}{2}\log\left(1-u_i\right)\log^2\left(u_i\right)\log\left(v_i\right) \quad + \quad \left(u_i \leftrightarrow v_i\right)
\end{aligned}
$$

General result for $R_k^{(2)}$

$$r^{(2)}_{\text{bts},i} = \zeta_2 \left[\log(u_i) \log\left(\frac{1-v_i}{v_i}\right) + \frac{1}{2}\log^2\left(\frac{1-u_i}{w_i}\right) - \tfrac{1}{2}\log^2(u_i) \right] - \zeta_3 \log(u_i) - \frac{\zeta_4}{2} + \ (u_i \leftrightarrow v_i)$$

$$r^{(2)}_{\text{nc},i} \ = -G\left(\{1-v_i, 1-v_i, 1, 0\}, u_i\right) \ - \ (u_i \leftrightarrow v_i)$$

$$\mathcal{R}_k^{(2)} \ = \ \sum_{i=1}^{k} \left[r^{(2)}_{\text{nc},i} + r^{(2)}_{\text{cl},i} + r^{(2)}_{\text{bts},i} \right]$$

Summary + Outlook

- On-shell recursion relation for tree-level MHV form factors
- MHV form factors of half-BPS operators in $\mathcal{N} = 4$ SYM

  all n ⟨ Tree level / One loop

- Finite two-loop remainder functions for minimal form factors

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Thank you

Soft and Collinear Limits

- Collinear limit $p_1 \,||\, p_2$
  (edges)

$$u \to 0, \;\; u + v \to 1$$

$$\mathcal{R}_3^{(2)} \sim \log(u)^2$$

- Soft limit $p_1 \to 0$
  (corners)

$$u = x\,\delta, \;\; v = 1 - \delta, \;\; w = y\,\delta$$

$$x + y = 1, \;\; \delta \to 0$$

$$\mathcal{R}_3^{(2)} \sim \log(\delta)^4$$