Amplitudes in D=6 N=(1,1) SYM Theory

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JHEP 1404 (2014) 121, arXiv:1402.1024 [hep-th]
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Amplitudes 2014, June 13

Motivation

Maximal SYM

D=4 N=4
D=6 N=2
D=10 N=1

- **Partial or total cancellation of UV divergences (all bubble and triangle diagrams cancel)**
- **First UV divergent diagrams at D=4+6/L**
- **Conformal or dual conformal symmetry**
- **Common structure of the integrands**

Bern, Dixon &Co 10
Drummond, Henn, Korchemsky, Sokatchev
Arkani-Hammed 12

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$$\mathcal{M}_n \equiv \frac{A_n}{A_n^{tree}} = 1 + \sum_{L=1}^{\infty} \left(\frac{g^2 N_c}{16\pi^2}\right)^L M_n^{(L)}(\epsilon) = \exp\left[\sum_{l=1}^{\infty} \left(\frac{g^2 N_c}{16\pi^2}\right)^l \left(f^{(l)}(\epsilon) M_n^{(1)}(l\epsilon) + C^{(l)} + E_n^{(l)}(\epsilon)\right)\right]$$

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$$\mathcal{M}_n(\epsilon) = \exp\left[-\frac{1}{8}\sum_{l=1}^{\infty}\left(\frac{g^2 N_c}{16\pi^2}\right)^l \left(\frac{\gamma_{cusp}^{(l)}}{(l\epsilon)^2} + \frac{2G_0^{(l)}}{l\epsilon}\right)\sum_{i=1}^{n}\left(\frac{\mu^2}{-s_{i,i+1}}\right)^{l\epsilon} + \frac{1}{4}\sum_{l=1}^{\infty}\left(\frac{g^2 N_c}{16\pi^2}\right)^l \gamma_{cusp}^{(l)} F_n^{(1)}(0) + C(g)\right]$$

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$$\mathcal{M}_n(\epsilon) \quad = \quad \exp\left[-\frac{1}{8}\sum_{l=1}^{\infty}\left(\frac{g^2N_c}{16\pi^2}\right)^l\left(\frac{\gamma_{cusp}^{(l)}}{(l\epsilon)^2}+\frac{2G_0^{(l)}}{l\epsilon}\right)\sum_{i=1}^{n}\left(\frac{\mu^2}{-s_{i,i+1}}\right)^{l\epsilon}+\frac{1}{4}\sum_{l=1}^{\infty}\left(\frac{g^2N_c}{16\pi^2}\right)^l\gamma_{cusp}^{(l)}F_n^{(1)}(0)+C(g)\right]$$

$$M_4^{(1-loop)}(\epsilon) = A_4^{(1-loop)}/A_4^{(tree)} \quad = \frac{\Gamma(1-\epsilon)^2}{\Gamma(1-2\epsilon)}\left[\frac{1}{\epsilon^2}\left((\frac{\mu^2}{s})^{\epsilon}+(\frac{\mu^2}{-t})^{\epsilon}\right)-\frac{1}{2}\log^2\left(\frac{s}{-t}\right)-\frac{\pi^2}{3}\right]+\mathcal{O}(\epsilon)$$

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D=6

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D=6 $[g^2] \sim \frac{1}{M^2}$

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D=6 $[g^2] \sim \dfrac{1}{M^2}$ **Toy model for gravity**

Color decomposition & Spinor helicity formalism

Color ordered amplitude

$$\mathcal{A}_n^{a_1\ldots a_n}(p_1^{\lambda_1}\ldots p_n^{\lambda_n}) = \sum_{\sigma\in S_n/Z_n} Tr[\sigma(T^{a_1}\ldots T^{a_n})]A_n(\sigma(p_1^{\lambda_1}\ldots p_n^{\lambda_n})) + \mathcal{O}(1/N_c)$$

Planar Limit $N_c \to \infty,\ g_{YM}^2 \to 0$ and $g_{YM}^2 N_c$ - fixed

Cheung, O'Connell 09, Bern&Co 10

Spinor helicity formalism

Momentum $p^\mu,\ p^2 = 0,\ \mu = 0, ..., 5$ $\quad SO(5,1)$

$SU^*(4) \rightarrow \lambda^{Aa}$

$$p_{AB} = p_\mu(\sigma^\mu)_{AB},\ p^{AB} = p^\mu(\bar{\sigma}_\mu)^{AB}$$

$$p^{AB} = \lambda^{Aa}\lambda_a^B,\ p_{AB} = \tilde{\lambda}_A^{\dot{a}}\tilde{\lambda}_{B\dot{a}}$$

Little group in D=6: $SO(4) \simeq SU(2) \times SU(2)$

Lorentz invariant structures: $\quad \lambda(i)^{Aa}\tilde{\lambda}(j)_A^{\dot{a}} \doteq \langle i_a|j_{\dot{a}}] = [j_{\dot{a}}|i_a\rangle$

$$\langle 1_a 2_b 3_c 4_d\rangle \doteq \epsilon_{ABCD}\lambda_1^{Aa}\lambda_2^{Bb}\lambda_3^{Cc}\lambda_4^{Dd} \qquad [1_{\dot{a}}2_{\dot{b}}3_{\dot{c}}4_{\dot{d}}] \doteq \epsilon^{ABCD}\tilde{\lambda}_{A,1}^{\dot{a}}\tilde{\lambda}_{B,2}^{\dot{b}}\tilde{\lambda}_{C,3}^{\dot{c}}\tilde{\lambda}_{D,4}^{\dot{d}}$$

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Spinor helicity formalism

Helicity is no longer conserved in D=6!

Momentum $\quad p^\mu,\ p^2 = 0,\ \mu = 0, ..., 5 \qquad SO(5,1)$

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Superfield formalism in D=6

$\mathcal{N}=(1,1)$ D=6 on-shell superspace $=\{\lambda_a^A,\tilde{\lambda}_A^{\dot{a}},\eta_a^I,\overline{\eta}_{I'\dot{a}}\}$

N=(1,1) on-shell states

$\Phi^{--},\Phi^{-+},\Phi^{+-},\Phi^{++},$
$\Psi^{-a},\Psi^{+a},\bar{\Psi}^{-\dot{a}},\bar{\Psi}^{+\dot{a}},$
$A^{a\dot{a}}$

Harmonic superspace
(*Dennen, Huang, Siegel 10*)

$$\frac{SU(2)_R}{U(1)}\times\frac{SU(2)_R}{U(1)}$$

$$u_I^{\mp}\ \text{and}\ \overline{u}^{\pm I'}$$

$$q^{\mp A}=u_I^{\mp}q^{AI},\ \overline{q}_A^{\pm}=u^{\pm I'}\overline{q}_{AI'},$$
$$\eta_a^{\mp}=u_I^{\mp}\eta_a^I,\ \overline{\eta}_{\dot{a}}^{\pm}=u^{\pm I'}\overline{\eta}_{I'\dot{a}},$$

$$\{q^{AI},q^{BJ}\}=p^{AB}\epsilon^{IJ}$$
$$\{\bar{q}_{AI'},\bar{q}_{BJ'}\}=p_{AB}\epsilon_{I'J'}$$

$$p^{AB}=\sum_i^n\lambda_i^{Aa}\lambda_{a,i}^B,\quad q^A=\sum_i^n\lambda_a^{A,i}\eta_i^a,\quad \overline{q}_A=\sum_i^n\tilde{\lambda}_{A,i}^{\dot{a}}\overline{\eta}_{\dot{a},i}$$

$$\{\lambda_a^A,\tilde{\lambda}_A^{\dot{a}},\eta_a^-,\overline{\eta}_{\dot{a}}^+\}$$

The full amplitude

$$A_n(\{\lambda_a^A,\tilde{\lambda}_A^{\dot{a}},\eta_a,\overline{\eta}_{\dot{a}}\})=\delta^6(p^{AB})\delta^4(q^A)\delta^4(\overline{q}_A)\mathcal{P}_n(\{\lambda_a^A,\tilde{\lambda}_A^{\dot{a}},\eta_a,\overline{\eta}_{\dot{a}}\})$$

The delta function always factorizes!

Polynomial of degree
2n-8 in Grassmannian variables

Grassmannian delta function is defined as:

$$\delta^4(q^A)=\frac{1}{4!}\epsilon_{ABCD}\delta(\sum_i^n q_i^A)\delta(\sum_k^n q_k^A)\delta(\sum_i^n q_l^A)\delta(\sum_k^n q_p^A)$$

Tree level amplitude:

n=4 $$A_4^{(0)}=-ig_{YM}^2\delta^6(p^{AB})\frac{\delta^4(q^A)\delta^4(\overline{q}_A)}{st}\qquad \mathcal{P}_4=-i/st$$

In components

$$A_4^{(0)}(1_{a\dot{a}}2_{b\dot{b}}3_{c\dot{c}}4_{d\dot{d}})=-ig_{YM}^2\frac{\langle 1_a2_b3_c4_d\rangle[1_{\dot{a}}2_{\dot{b}}3_{\dot{c}}4_{\dot{d}}]}{st}$$

Perturbation Expansion for the Amplitudes

A_4/A_4^{tree}

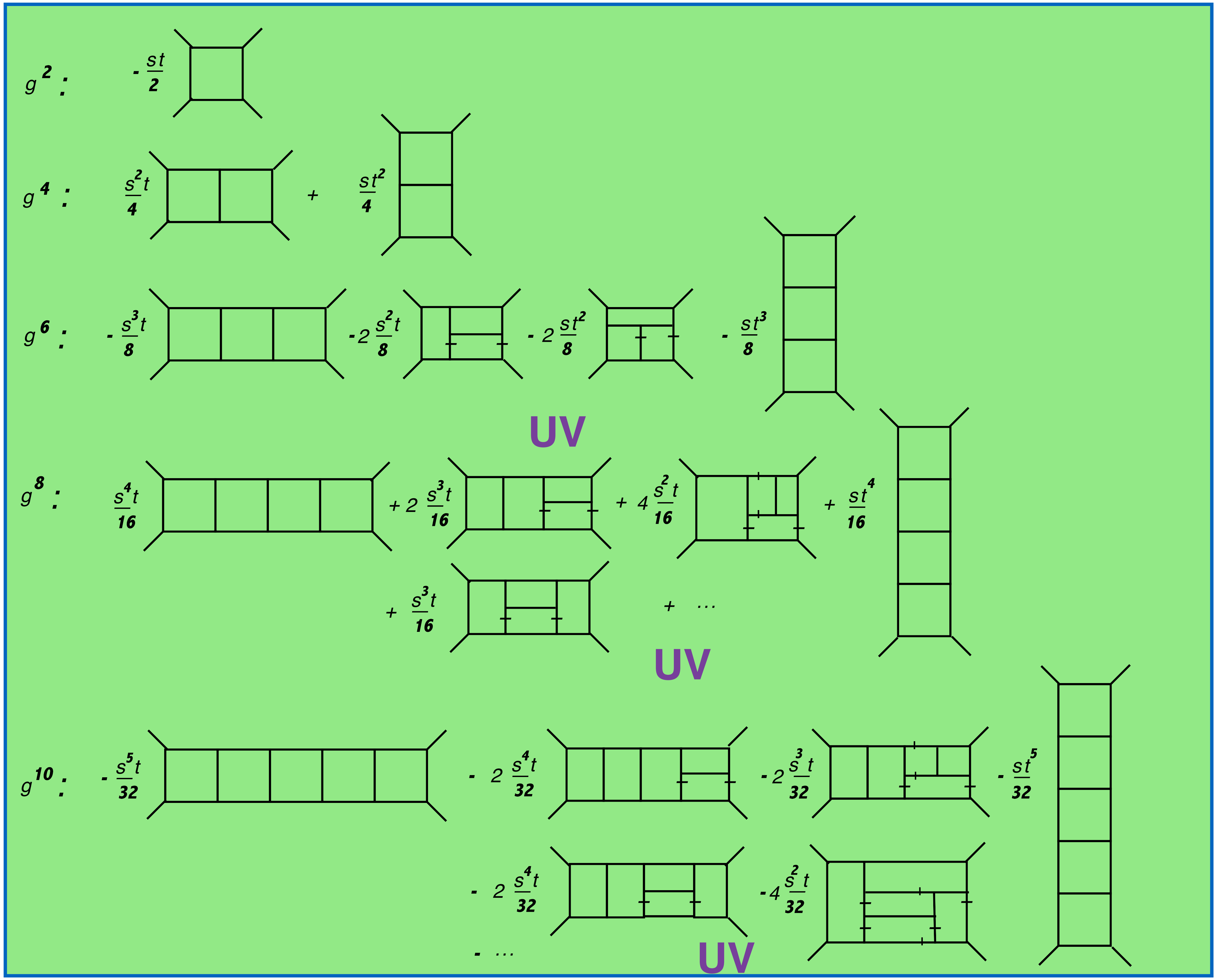
No bubbles
No Triangles

First UV div at three loops

D=4+6/L

$$[g^2] \sim \frac{1}{M^2}$$

IR finite

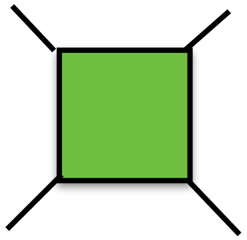


1
2
4
15
60

Universal expansion for any D in maximal SYM

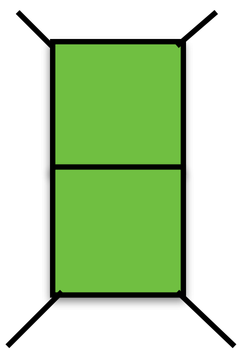
Perturbation Expansion for the Amplitudes

Exact calculation



$$p_i^2 = 0, \;\; m = 0$$

$$B_1(s,t) = \frac{\pi^3}{(2\pi)^6}\,\frac{b_2(x)}{s+t}, \quad b_2(x) = \frac{L^2(x)+\pi^2}{2}, \quad L(x) \doteq \log(x), \quad x = \frac{t}{s}$$



$$B_2(s,t) = \left(\frac{\pi^3}{(2\pi)^6}\right)^2 \left(\frac{b_4(x)}{t} + \frac{b_3(x)}{s+t}\right)$$

Anastasiou, Tausk, Tejeda-Yeomans , 00
Bork,Kazakov,Vlasenko, 13

$$\begin{aligned} b_4(x) \;\; &= \;\; \left(2\zeta_3 - 2Li_3(-x) - \frac{\pi^2}{3}L(x)\right)L(1+x) + \left(\frac{1}{2}L(x) + \frac{\pi^2}{2}\right)L^2(1+x) \\ &+ \;\; \left(2L(x)L(1+x) - \frac{\pi^2}{3}\right)Li_2(-x) + 2L(x)S_{1,2}(-x) - 2S_{2,2}(-x) \end{aligned}$$

$$b_3(x) = -2\zeta_3 + \frac{\pi^2}{3}L(x) - \left(L(x)+\pi^2\right)L(1+x) - 2L(x)Li_2(-x) + 2Li_3(-x)$$

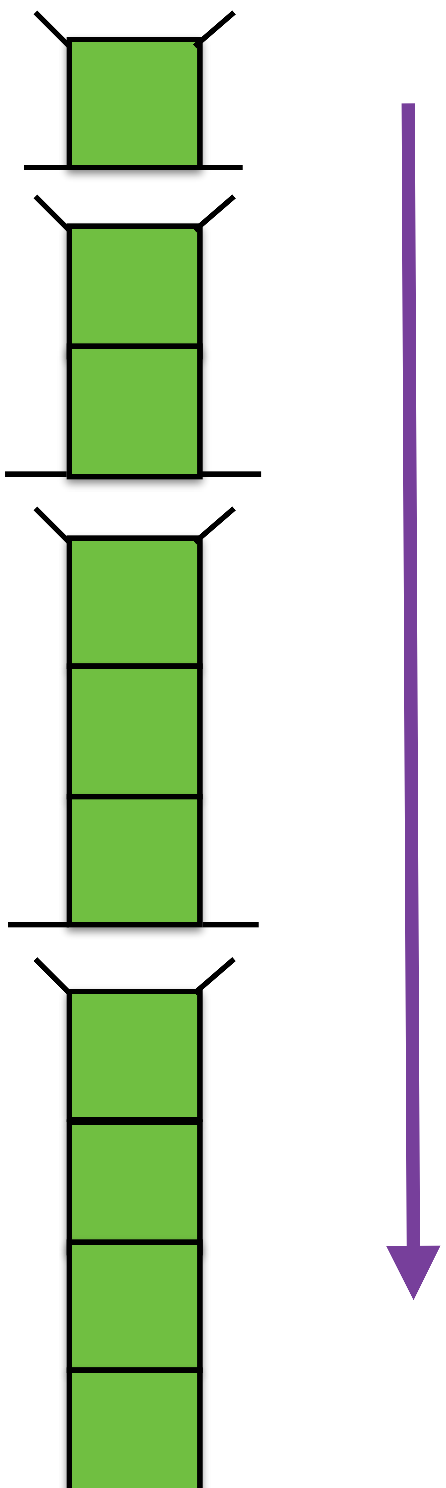
Regge Limit $s \to \infty, \;\; t < 0, \;\; fixed$

$$B_1(s,t) \sim \frac{1}{2}L^2(x) \qquad\qquad B_2(s,t) \sim \frac{1}{12}L^4(x)$$

Perturbation Expansion for the Amplitudes

Leading Logarithms **UV finite**

Regge Limit $s \to \infty, \quad t < 0, \ fixed$



$$B_n(t,s) \simeq \frac{1}{s}\frac{L^{2n}(x)}{n!(n+1)!}, \qquad L \equiv \log(s/t)$$

Bork,Kazakov,Vlasenko, 13

$$\left.\frac{A_4}{A_4^{(0)}}\right|_{L.L.} = \sum_{n=0}^{\infty} \frac{(-g^2 t/2)^n L^{2n}(x)}{n!(n+1)!}, \quad \text{where} \quad g^2 \equiv \frac{g_{YM}^2 N_c}{64\pi^3}.$$

$$\sum_{n=0}^{\infty} \frac{(-g^2 t/2)^n L^{2n}(x)}{n!(n+1)!} = \frac{I_1(2y)}{y}, \qquad y \equiv \sqrt{g^2|t|/2}\ L(x)$$

$$\left.\frac{A_4}{A_4^{(0)}}\right|_{L.L.} \sim \left(\frac{s}{t}\right)^{\alpha(t)-1}$$

!

Regge behaviour

Exact for $N_c \to \infty$

$$\alpha(t) = 1 + 2\sqrt{g^2|t|/2} = 1 + \sqrt{\frac{g_{YM}^2 N_c |t|}{32\pi^3}}$$

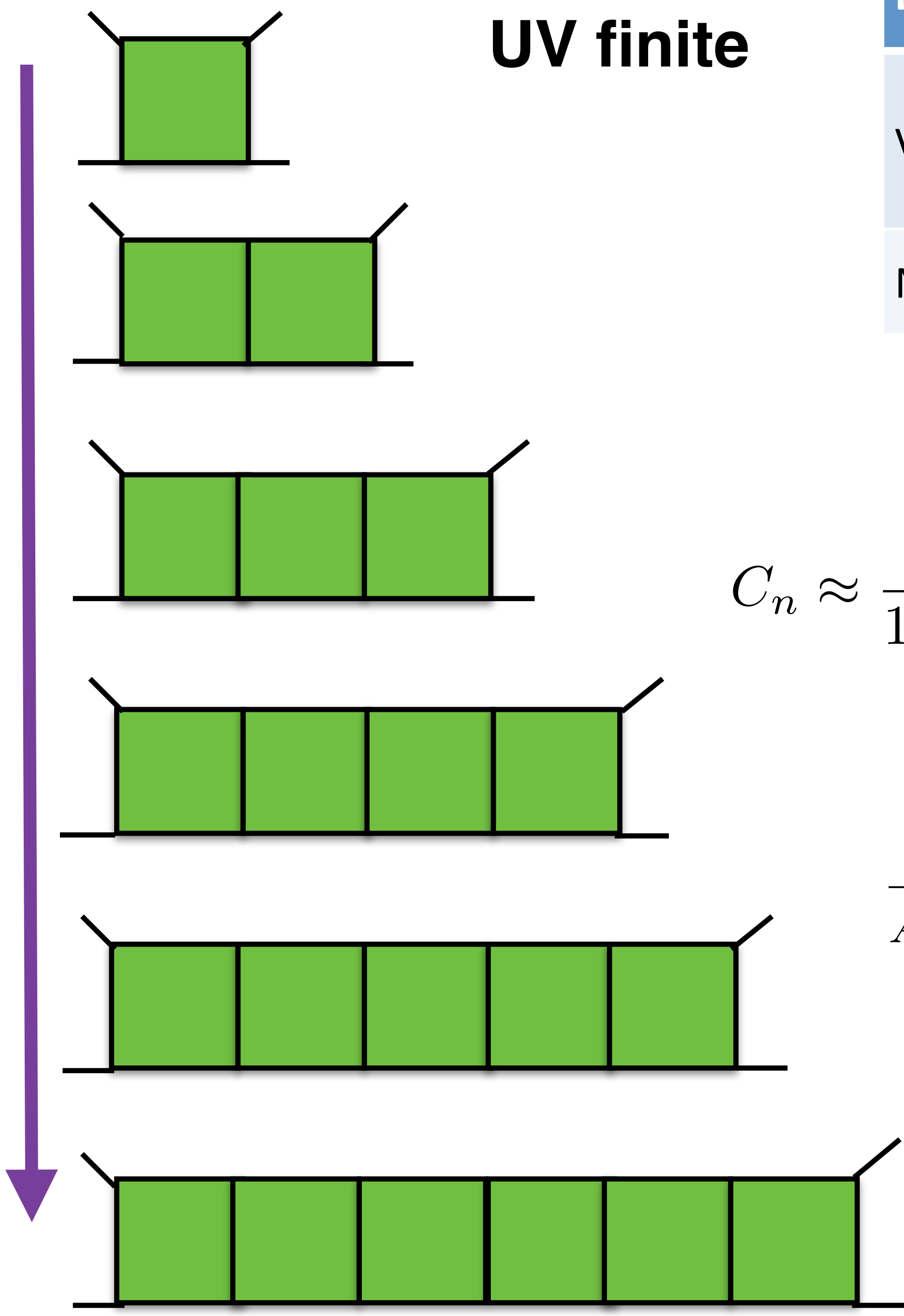
Perturbation Expansion for the Amplitudes

$$B_n(s,t) = \frac{1}{s}\left(C_n + O(t/s)\right), \quad n \geq 2$$

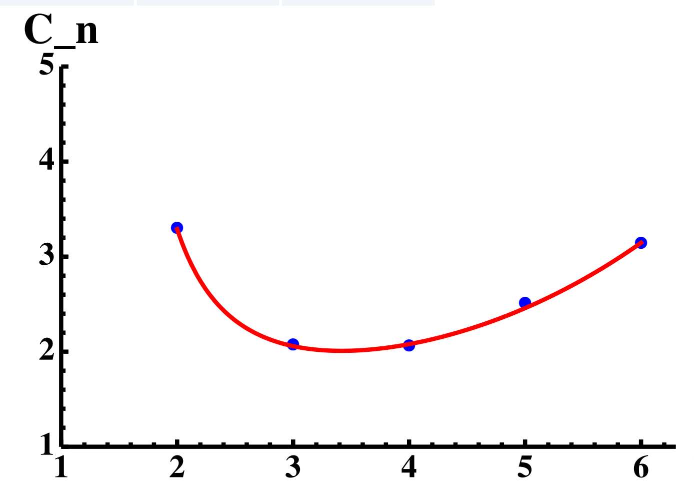
Kazakov, 14

Leading Powers

UV finite



Loops	1	2	3	4	5	6
Values						
Numerics						



Interpolation

$$C_n \approx \frac{1.63^n}{1.31n - 1.80} \approx 0.76\frac{(\pi^2/6)^n}{n - 4/3}, \quad n \geq 2$$

$$\left.\frac{A_4}{A_4^{(0)}}\right|_{L.P.} \approx -\frac{g^2 t}{2}\left[\frac{\pi^2}{2} - \sum_{n=2}^{\infty} 0.76\frac{(-g^2 s/2)^{n-1}(\pi^2/6)^n}{n-4/3}\right]$$

$$= -\frac{g^2 t}{2}\frac{\pi^2}{2}\left[1 - 0.76\frac{g^2 s \pi^2}{24}\,{}_2F_1(1,\frac{2}{3},\frac{5}{3},-\frac{g^2 s\pi^2}{12})\right] \approx -\frac{g^2 t}{2}\frac{\pi^2}{2}\left[1 - \left(\frac{g^2 s}{2}\right)^{1/3}\right].$$

$$\left.\frac{A_4}{A_4^{(0)}}\right|_{L.P.} \approx g^2 t \frac{\pi^2}{4}\left(\frac{g^2 s}{2}\right)^{1/3}$$

!

Perturbation Expansion for the Amplitudes

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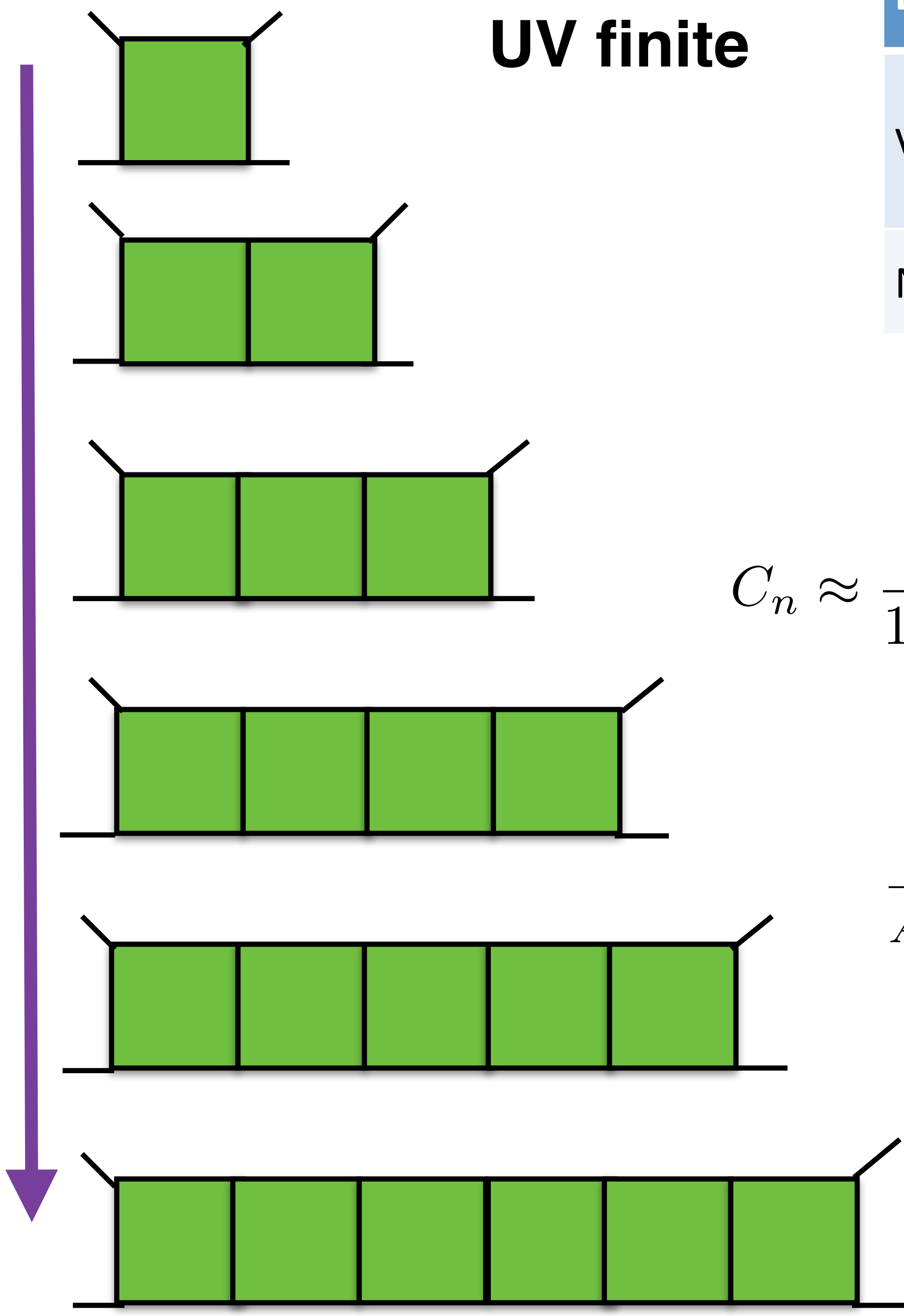
Kazakov, 14

Leading Powers



UV finite

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Values	$\frac{\pi^2}{2}$					
Numerics						

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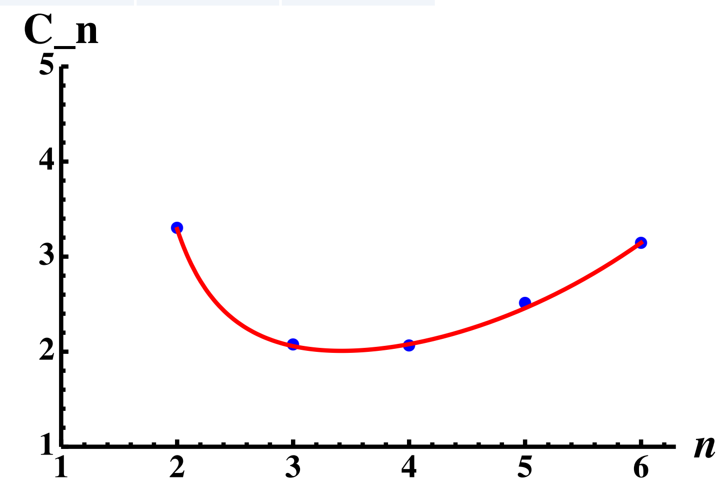

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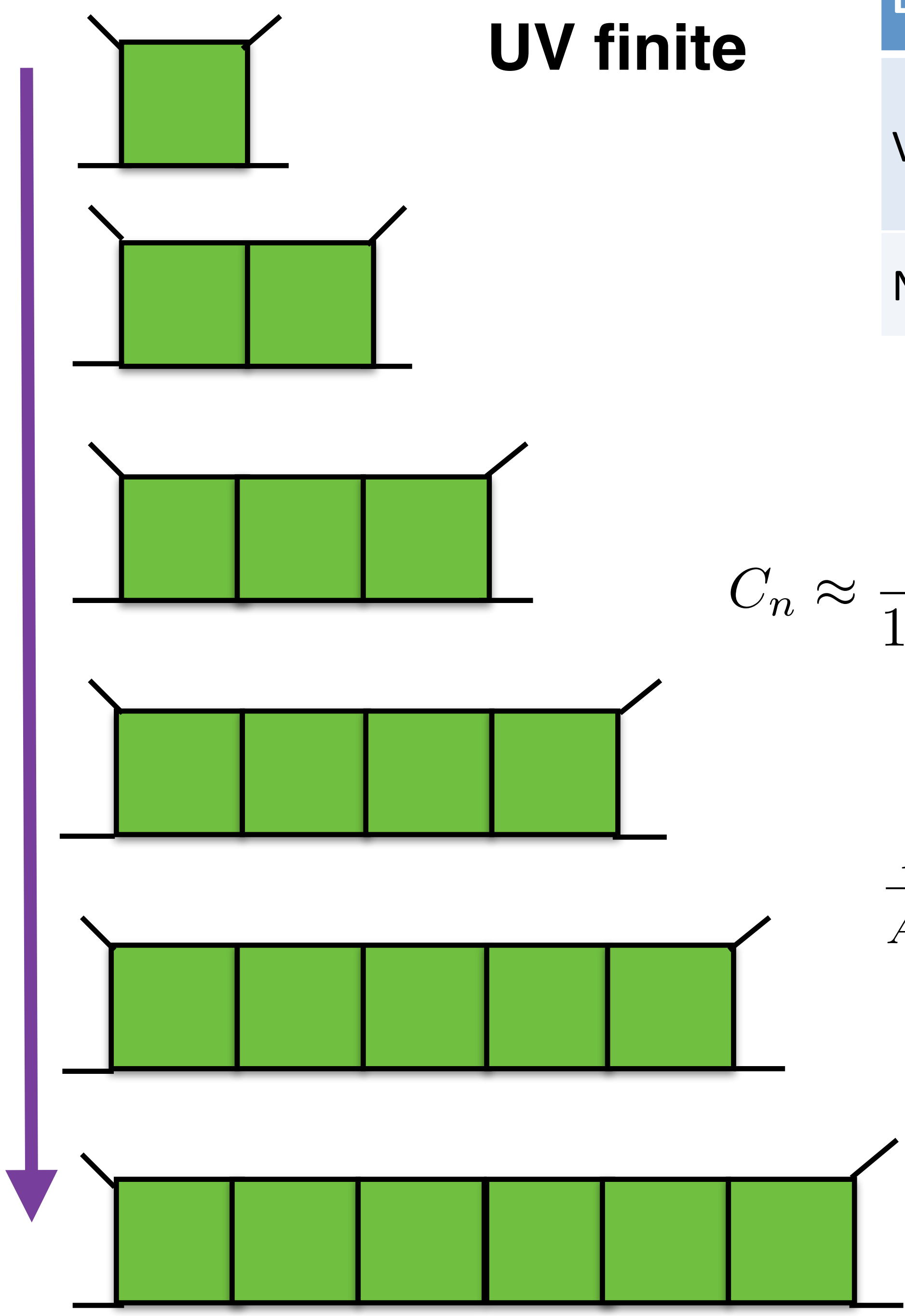
Perturbation Expansion for the Amplitudes

$$B_n(s,t) = \frac{1}{s}\left(C_n + O(t/s)\right), \quad n \geq 2$$

Kazakov, 14

Leading Powers

UV finite



Loops	1	2	3	4	5	6
Values	$\frac{\pi^2}{2}$	$\frac{\pi^2}{3}$				
Numerics						

Interpolation

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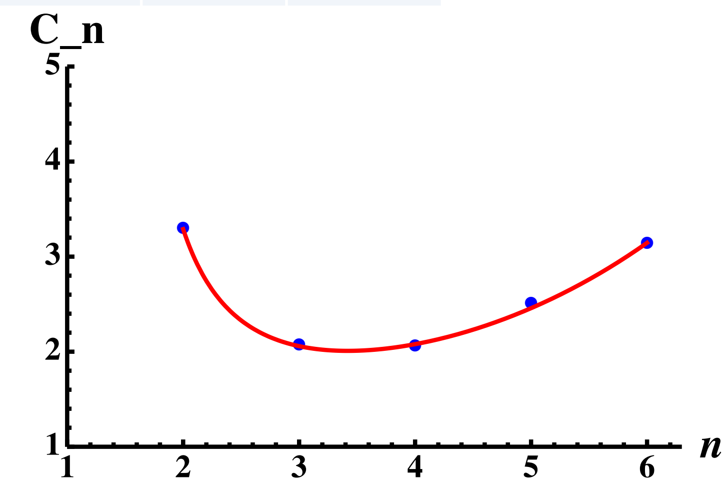

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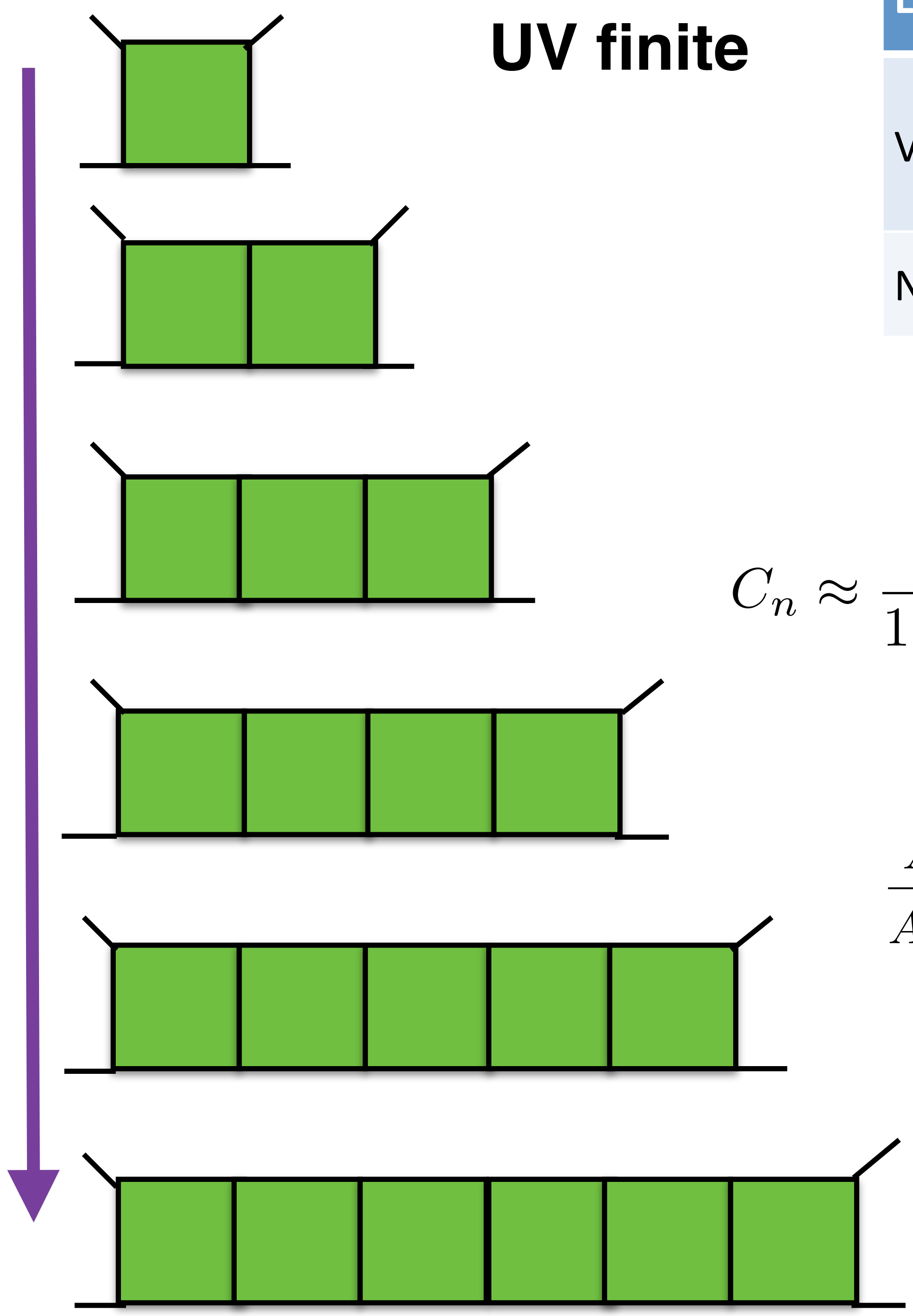
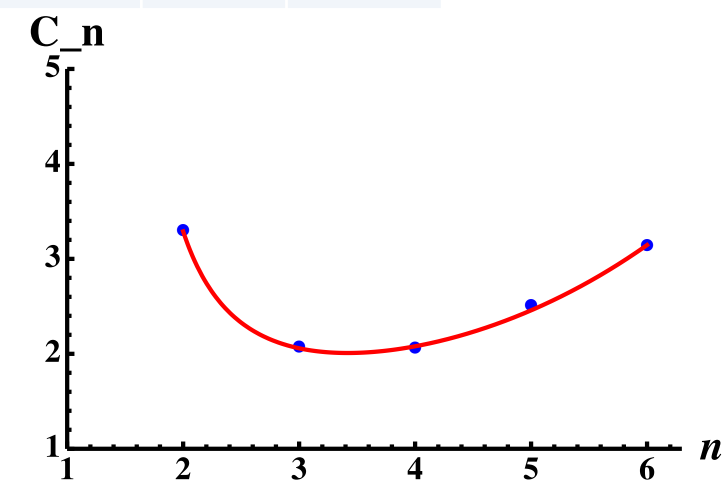


Loops	1	2	3	4	5	6
Values	$\frac{\pi^2}{2}$	$\frac{\pi^2}{3}$	$-\pi^2 + \frac{31\pi^6}{1890}$ $-8\zeta_3 + 4\zeta_3^2$			
Numerics						

Interpolation



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Kazakov, 14

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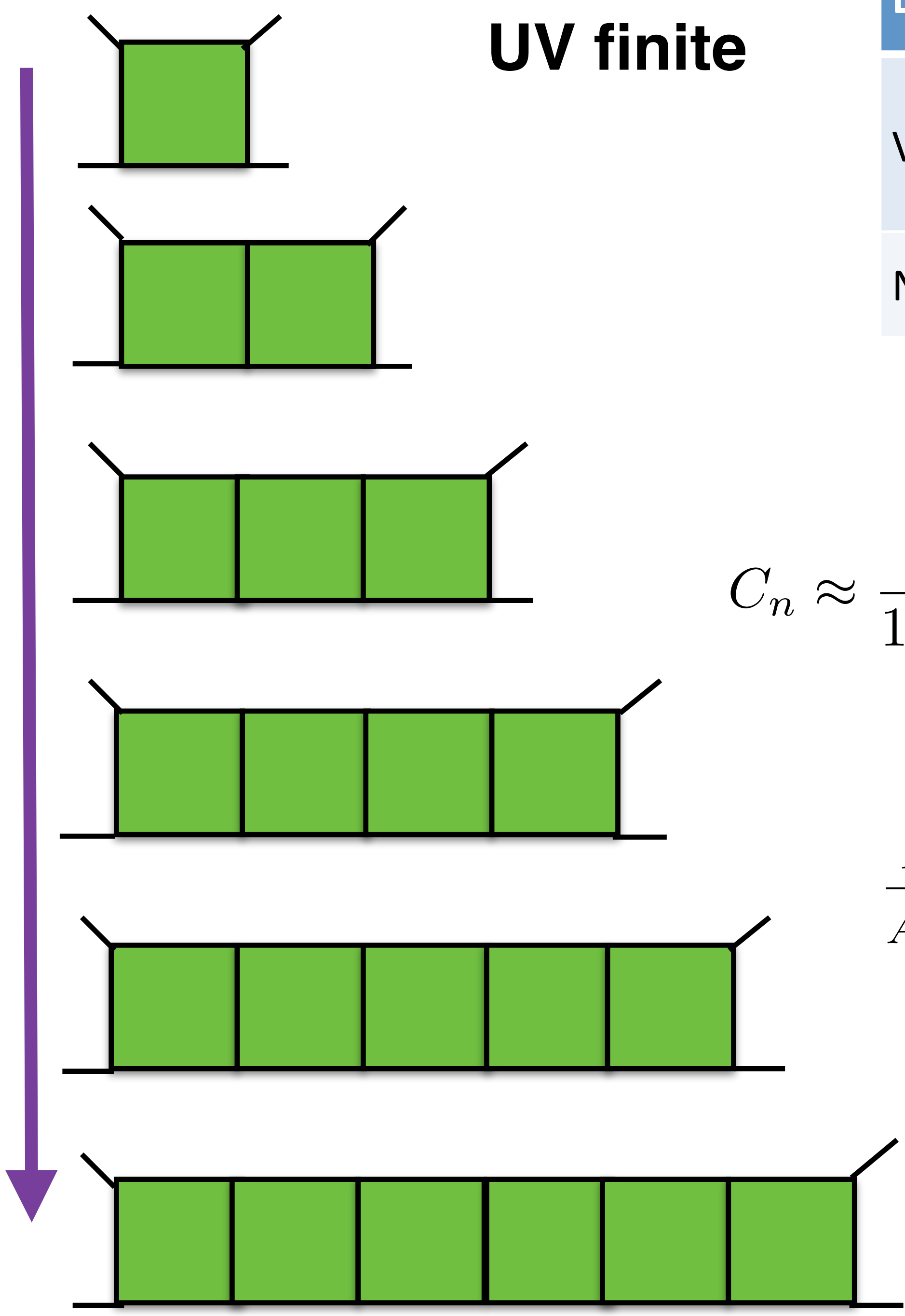

UV finite

Loops	1	2	3	4	5	6
Values	$\frac{\pi^2}{2}$	$\frac{\pi^2}{3}$	$-\pi^2 + \frac{31\pi^6}{1890} - 8\zeta_3 + 4\zeta_3^2$			
Numerics	4.93					

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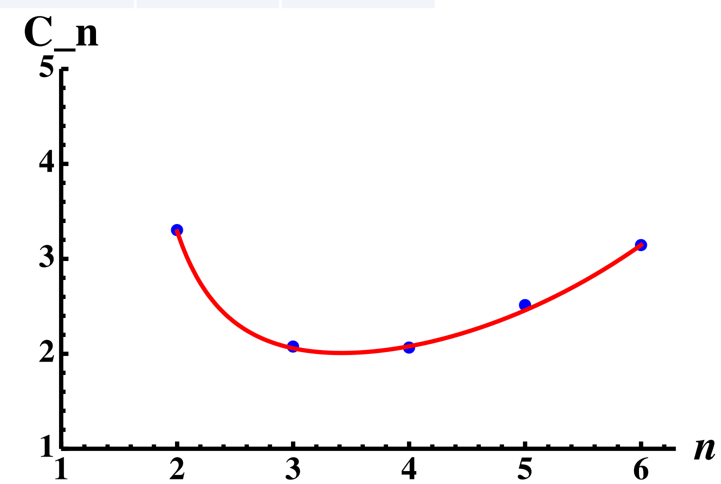

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Perturbation Expansion for the Amplitudes

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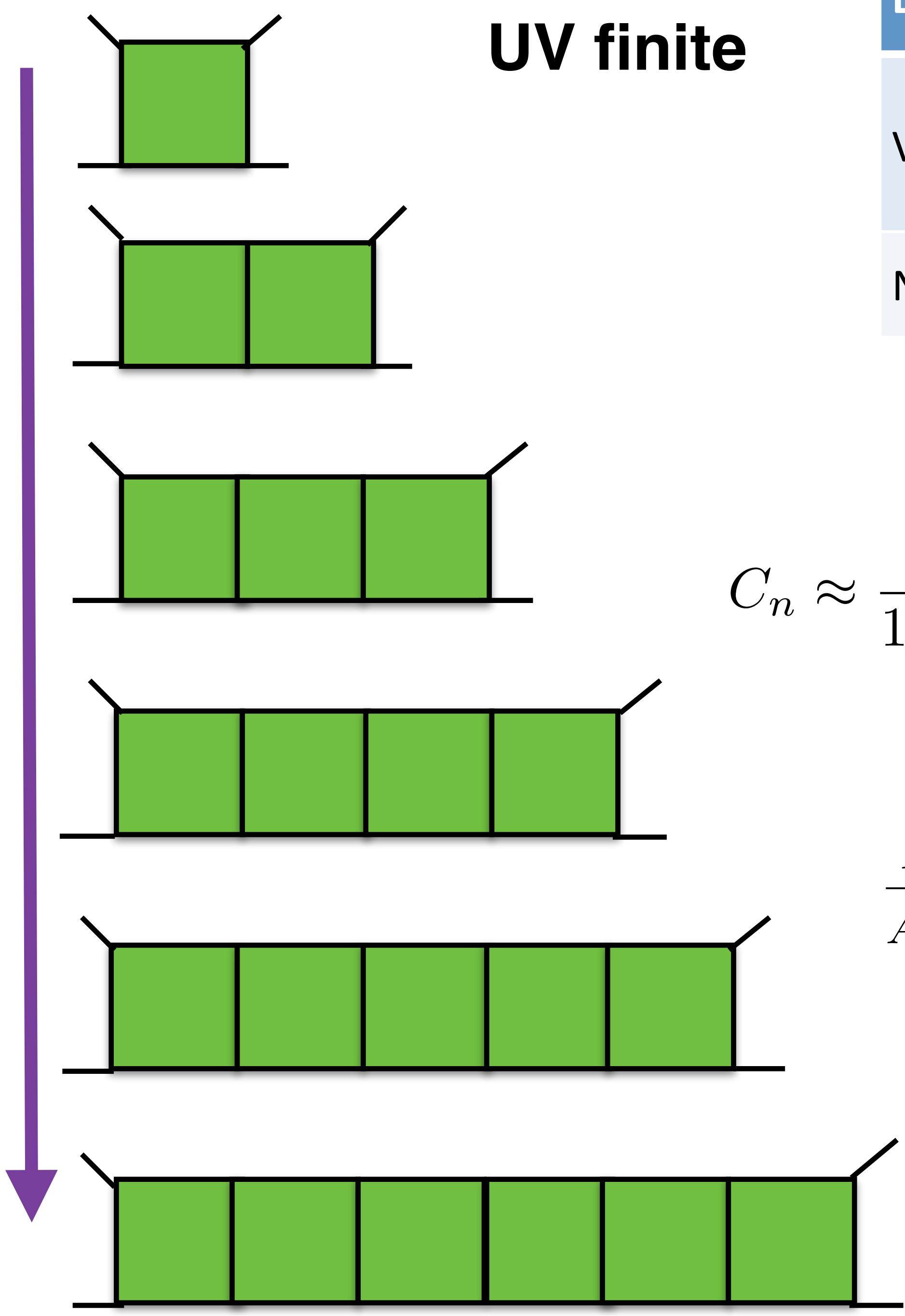

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Kazakov, 14

Loops	1	2	3	4	5	6
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Numerics	4.93	3.29				

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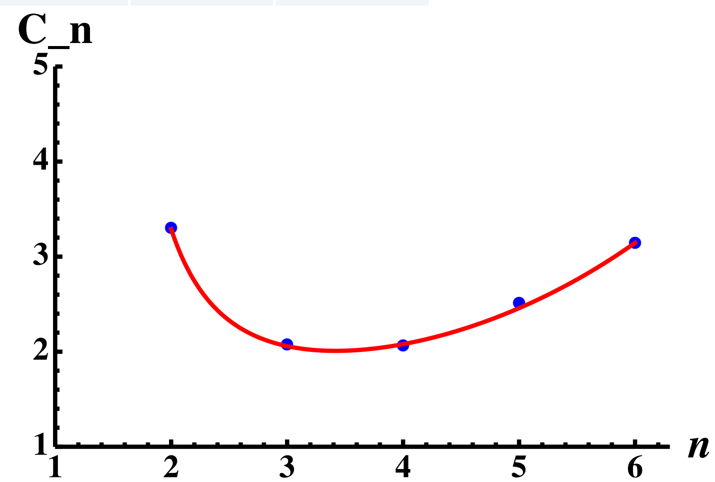

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Kazakov, 14

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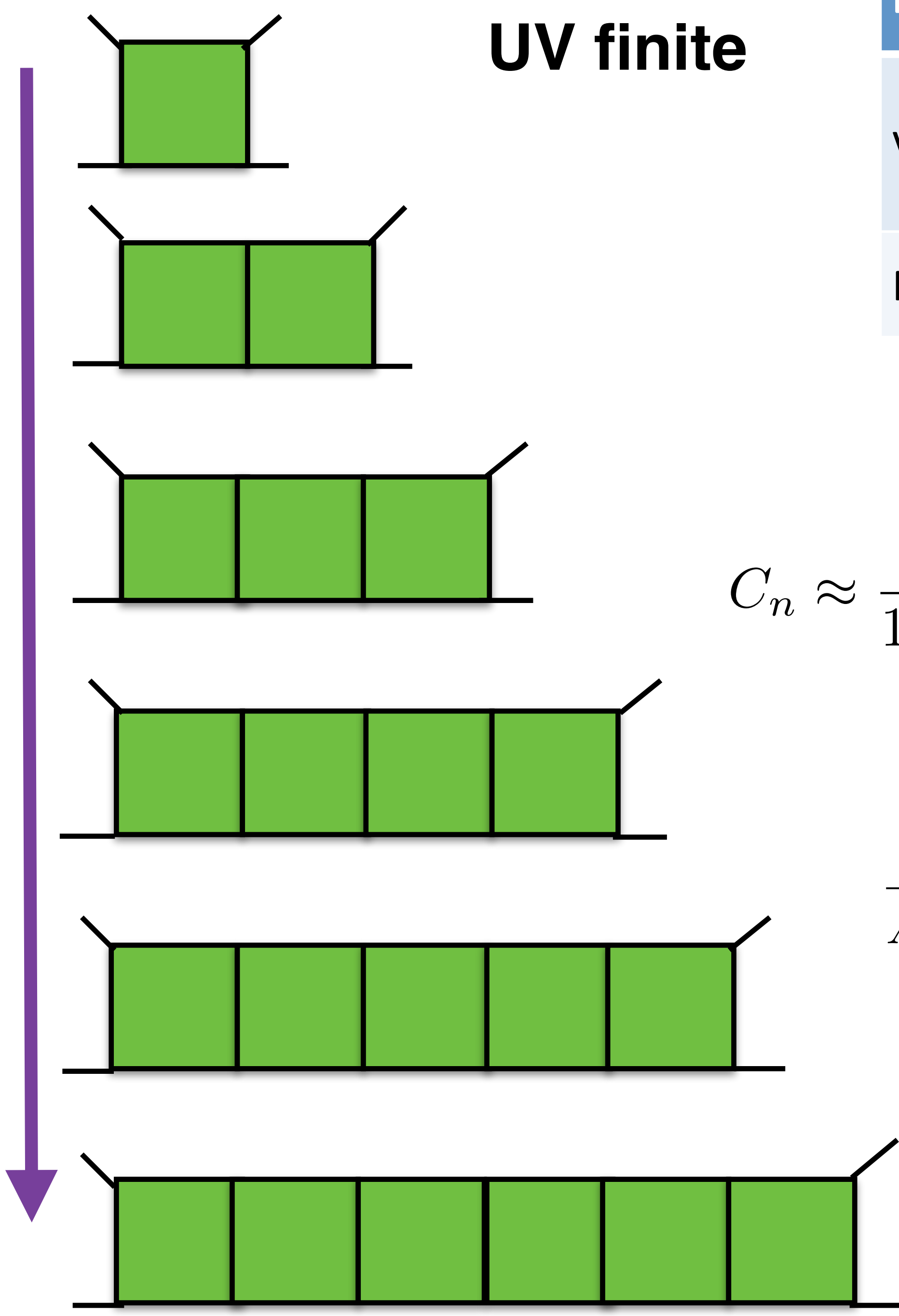

Loops	1	2	3	4	5	6
Values	$\frac{\pi^2}{2}$	$\frac{\pi^2}{3}$	$-\pi^2 + \frac{31\pi^6}{1890}$ $-8\zeta_3 + 4\zeta_3^2$			
Numerics	4.93	3.29	2.06			

Interpolation

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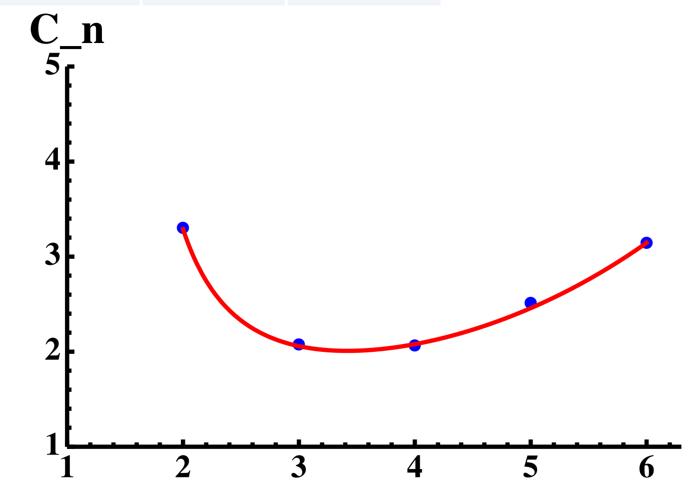

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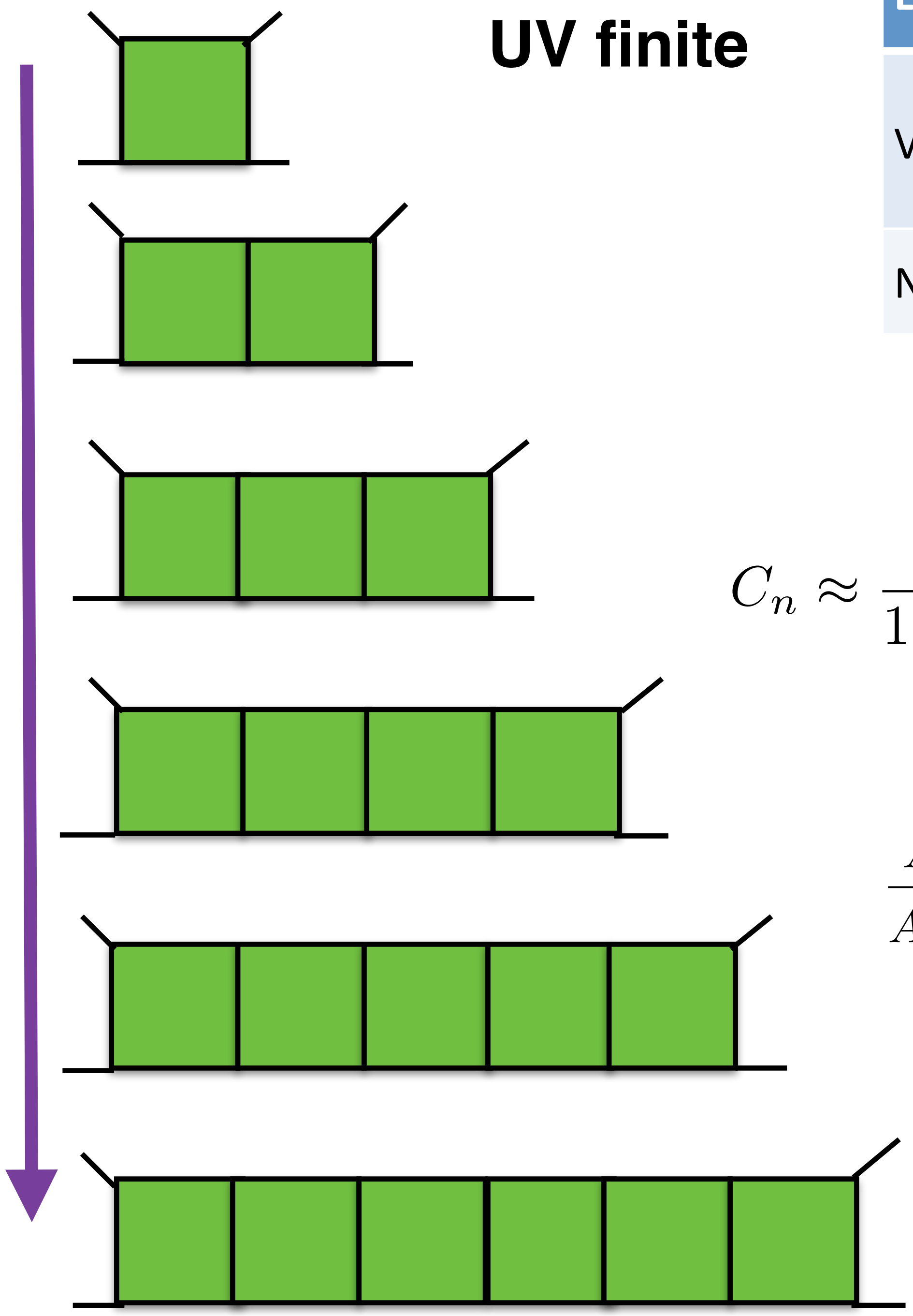
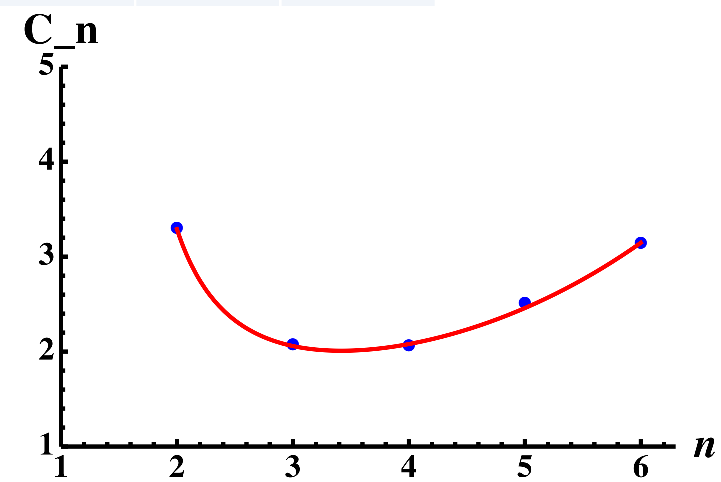
Kazakov, 14

Leading Powers



UV finite

Loops	1	2	3	4	5	6
Values	$\frac{\pi^2}{2}$	$\frac{\pi^2}{3}$	$-\pi^2 + \frac{31\pi^6}{1890} - 8\zeta_3 + 4\zeta_3^2$			
Numerics	4.93	3.29	2.06	2.05		



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Kazakov, 14

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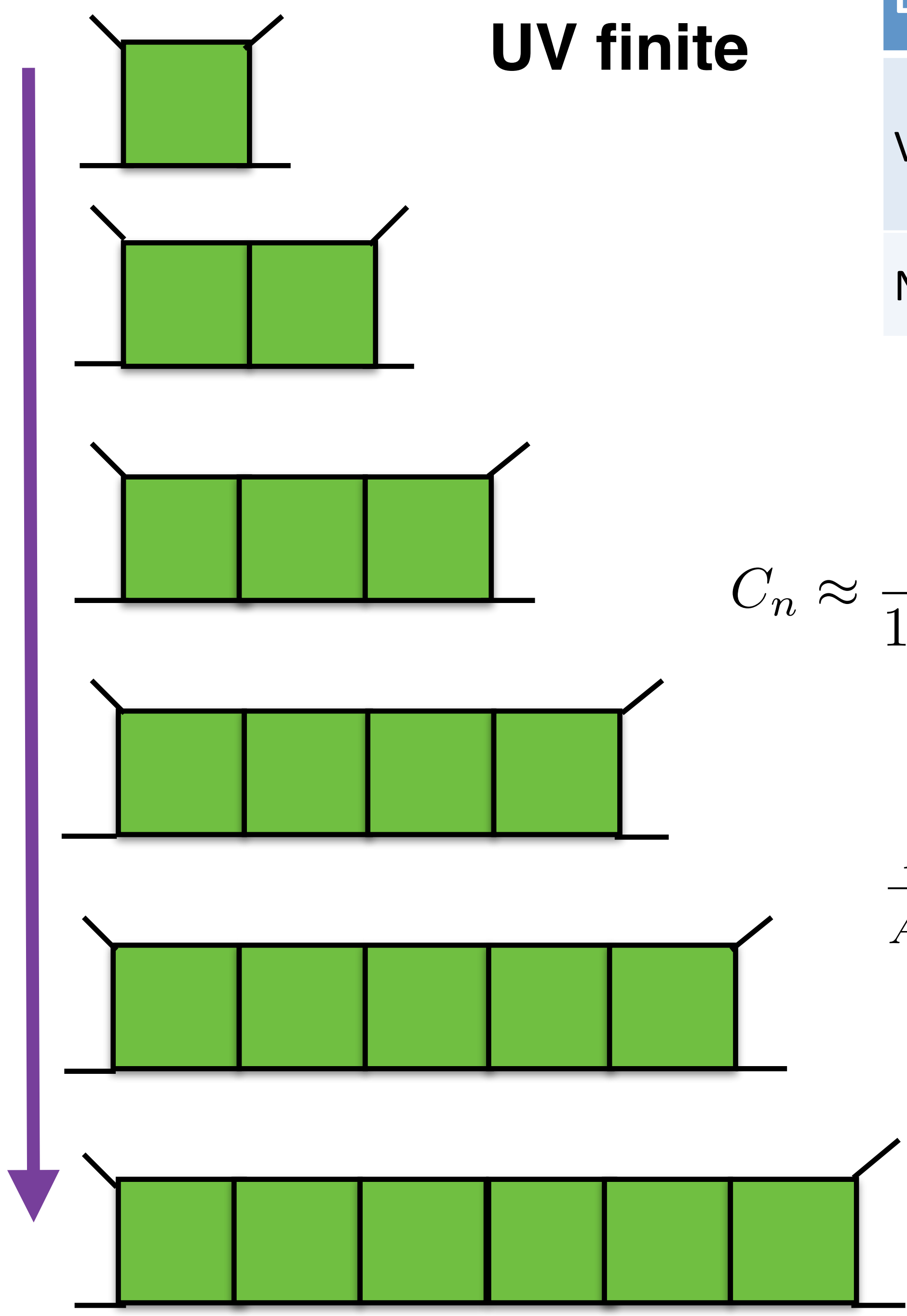

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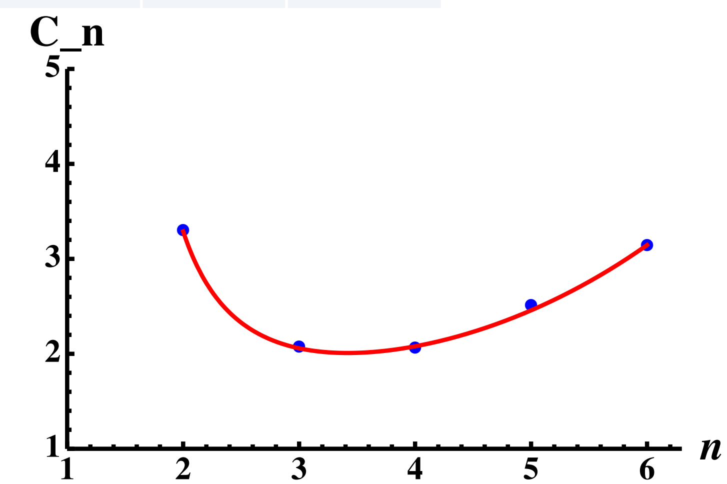

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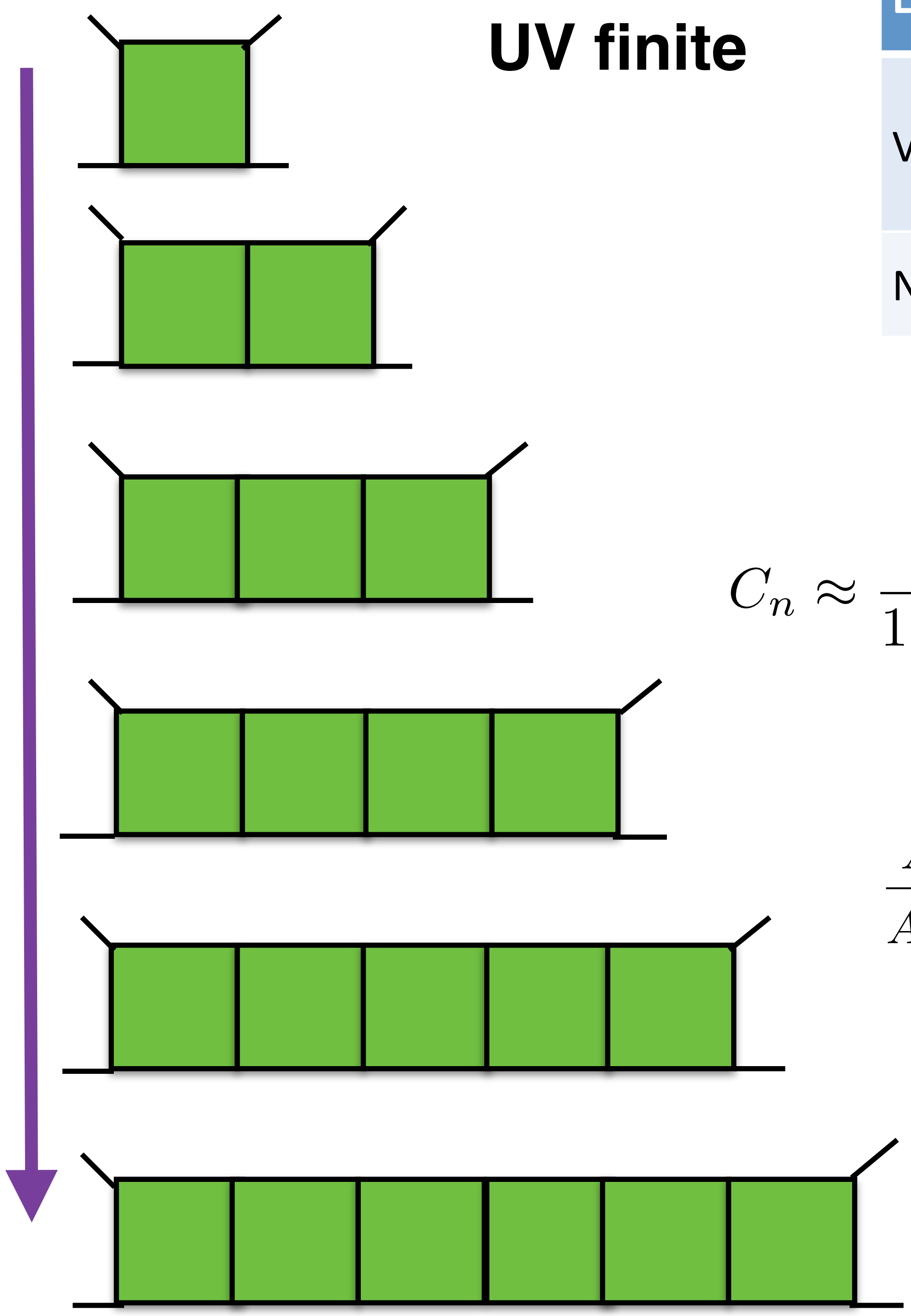

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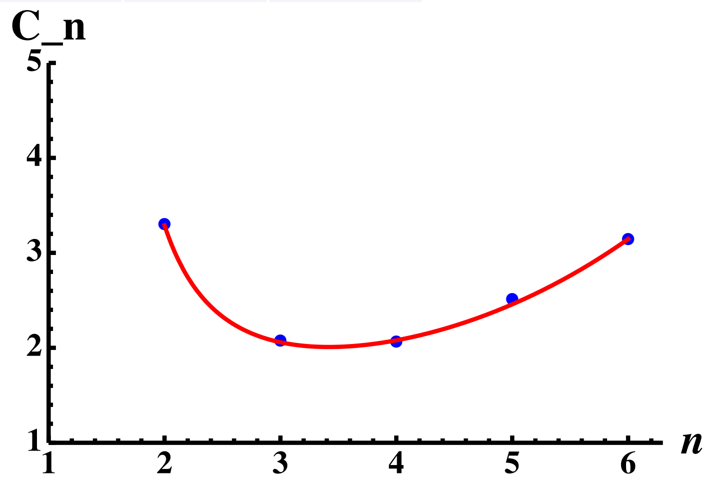

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Perturbation Expansion for the Amplitudes

Leading Divergences

Loops	Combinatorics	Divergence
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Geom progression !?

$$\frac{A_4}{A_4^{(0)}}\bigg|_{Leading\ \ Div.} = 2\frac{t}{s}\sum_{n=1}^{\infty}\left(-\frac{g^2s}{2}\right)^{n+2}\left(\frac{1}{6\epsilon}\right)^n = 2\frac{t}{s}\left(-\frac{g^2s}{2}\right)^2\frac{\frac{-g^2s}{12\epsilon}}{1+\frac{g^2s}{12\epsilon}}$$

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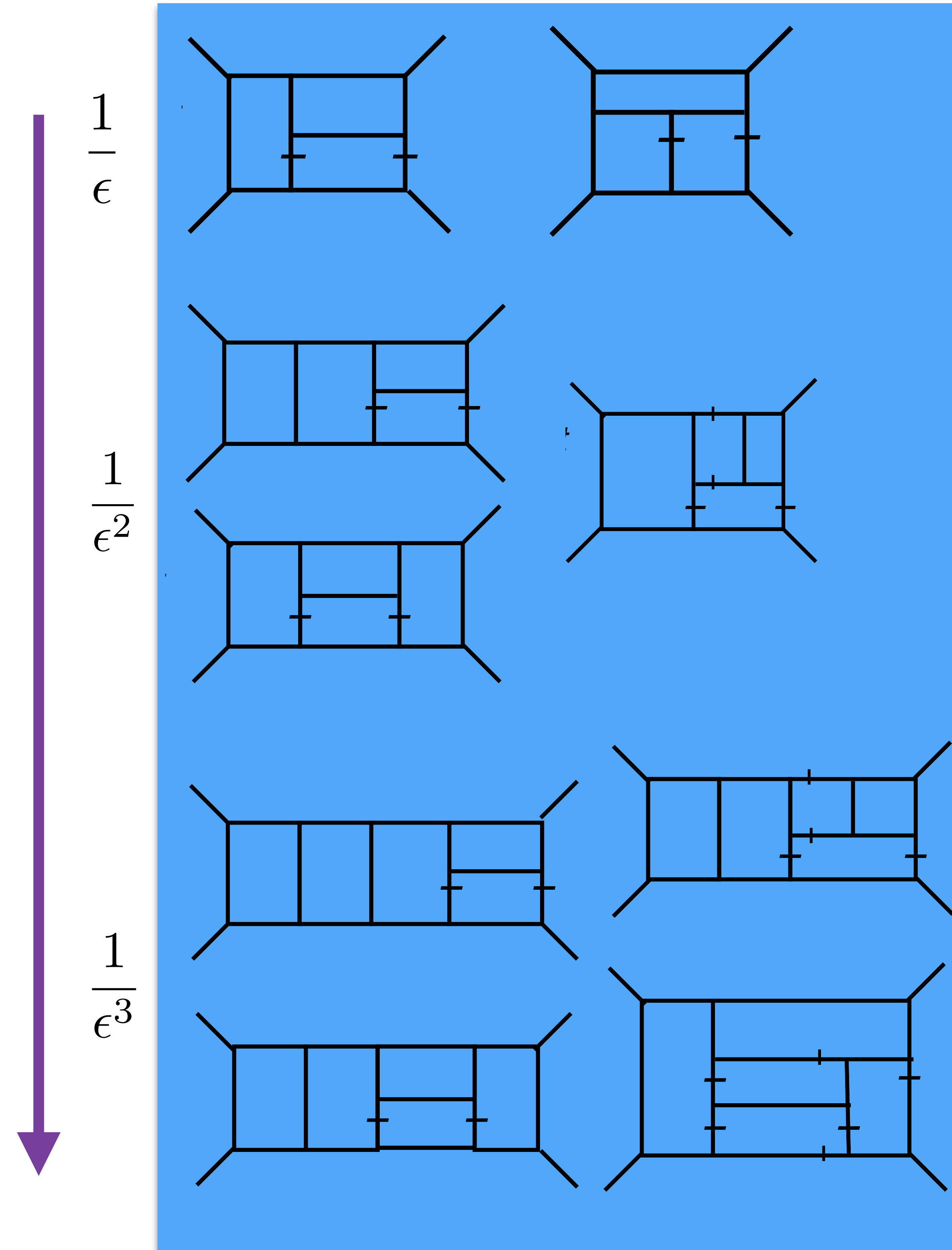

Perturbation Expansion for the Amplitudes

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$\frac{1}{\epsilon}$

$\frac{1}{\epsilon^2}$

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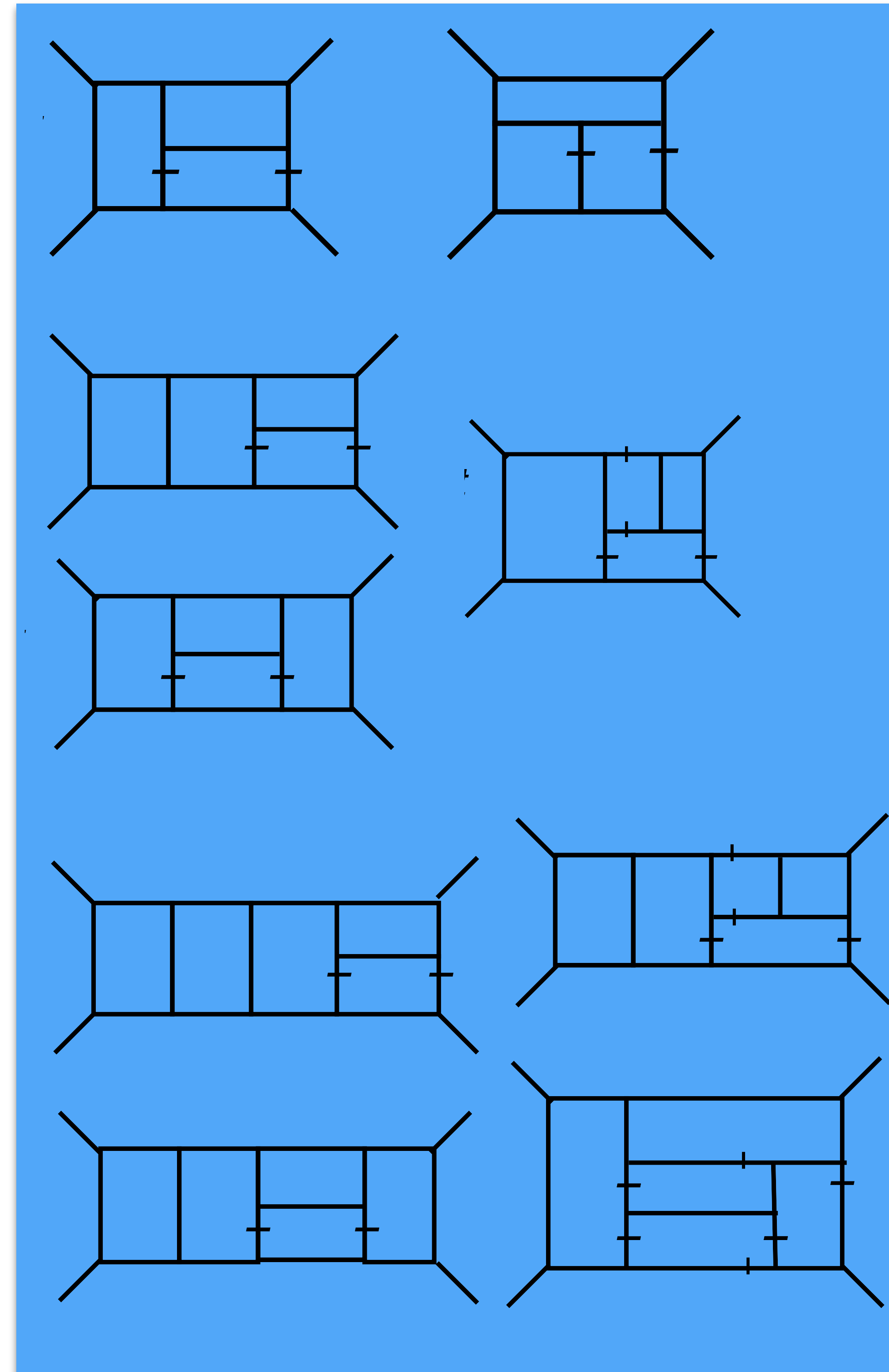

In the limit ε->0 the full expression is FINITE !

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- That would imply that severe UV divergences present in any given order of PT are actually artifacts of the weak coupling expansion.

Discussion

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- In order to understand the nonrenormalizable theories one has to find an alternative description.

Discussion

- **If this is true, one may try to apply the same arguments to quantum gravity. This would mean that one should not be confused by nonrenormalizability of PT in quantum gravity.**
- **It may well be that the full theory is meaningful, PT is just not applicable here.**

- **In order to understand the nonrenormalizable theories one has to find an alternative description.**
- **The result of an alternative approach might be quite different from the PT one.**