Soft behaviors in general gauge and gravity theories

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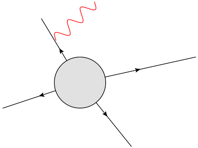
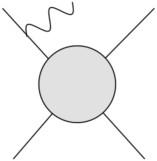
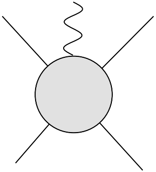
19th Itzykson Conference



Low (1958): subleading soft behavior from Ward identity



(a)



(b)

$$(a): \quad \sum_i e_i \frac{\epsilon_i \cdot k_i}{(\delta s \cdot k_i)} A_{n-1}(\cdots, k_i + \delta s, \cdots)$$

The subleading term is given as an operator acting on A_{n-1}: it is universal

QED: Low

$$A_{n+1}\Big|_{k_s \to \delta k_s} = \frac{1}{\delta} S^{(0)} A_{n-1} + S^{(1)} A_{n-1} + \mathcal{O}(\delta^1)$$

Gravity: Gross-Jackiw

$$M_{n+1}\Big|_{k_s \to \delta k_s} = \frac{1}{\delta} S^{(0)} M_{n-1} + S^{(1)} M_{n-1} + \delta S^{(2)} M_{n-1} + \mathcal{O}(\delta^2)$$

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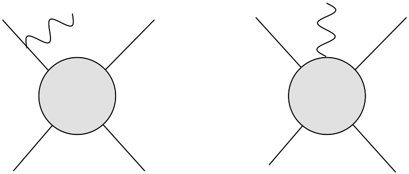

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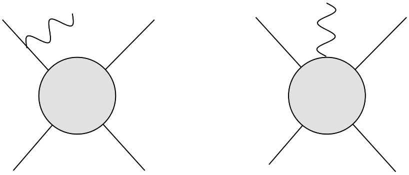

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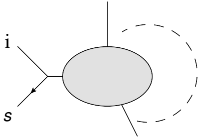
Tree-level soft via recurssion: Strominger-Cachazo, Casali:

$$\lambda_s \to \delta\lambda_s + z\lambda_n$$



$$+ \quad \cdots\cdots = \sum_{1\leq i<n} M_3(\hat{s}^+, i, -\hat{K}_{is})\frac{1}{K_{i,s}^2}M_n(\hat{K}_{is}, \ldots, \hat{n}) + R$$

$$\mathcal{O}(\delta^{-1}): \quad M_3(\hat{s}^+, i, \hat{K}_{is})\frac{1}{K_{is}^2}M_n(-\hat{K}_{is}, \cdots, \hat{n}) = \frac{1}{\delta^3}S_{s,i}^0 M_n\left(\tilde{\lambda}_i + \delta\frac{\langle s\, n\rangle}{\langle i\, n\rangle}\tilde{\lambda}_s, \tilde{\lambda}_n + \delta\frac{\langle s\, i\rangle}{\langle n\, i\rangle}\tilde{\lambda}_s\right),$$

$$S_{s,i}^{(0)} = \frac{\langle ni\rangle^2}{\langle ns\rangle^2}\frac{[is]}{\langle is\rangle}$$

S.He, C. Wen, Y-t:

$$\mathcal{O}(\delta^{-1}): \quad M_3(\hat{s}^+, i, \hat{K}_{is})\frac{1}{K_{is}^2}M_n(-\hat{K}_{is}, \cdots, \hat{n}) = \frac{1}{\delta^3}\, S^{(0)}_{s,i}\, e^{\delta\left(\frac{\langle s\, n\rangle}{\langle i\, n\rangle}\tilde{\lambda}_s\cdot\frac{\partial}{\partial\tilde{\lambda}_i}+\frac{\langle s\, i\rangle}{\langle n\, i\rangle}\tilde{\lambda}_s\cdot\frac{\partial}{\partial\tilde{\lambda}_n}\right)}\, M_n$$

$$M_{n+1} = \left(\frac{1}{\delta^3}S_G^{(0)} + \frac{1}{\delta^2}S_G^{(1)} + \frac{1}{\delta}S_G^{(2)}\right)M_n + \mathcal{O}(\delta^0)$$

$$A_{n+1} = \left(\frac{1}{\delta^2}S_{YM}^{(0)} + \frac{1}{\delta}S_{YM}^{(1)}\right)A_n + \mathcal{O}(\delta^0)$$

- Infinite soft theorem for MHV
- Double Copy

$$S_G^{(k)} = \sum_i K_{is}[S_{YM}^{(0)}(i)e^{J/2}]^2$$

- Supersymmetrized:

$$\mathcal{J} = \left[\frac{\langle sn\rangle}{\langle in\rangle}\left(\tilde{\lambda}_s\cdot\frac{\partial}{\partial\tilde{\lambda}_i}+\eta_s\cdot\frac{\partial}{\partial\eta_i}\right)+\frac{\langle si\rangle}{\langle ni\rangle}\left(\tilde{\lambda}_s\cdot\frac{\partial}{\partial\tilde{\lambda}_n}+\eta_s\cdot\frac{\partial}{\partial\eta_n}\right)\right]$$

-

$$S_{YM}^{(1)} = \frac{p_s^{\alpha\dot{\alpha}}}{\langle ns\rangle\langle s1\rangle[1n]}\frac{1}{2}\left[p_{n\alpha}{}^{\dot{\beta}}(M_{1\dot{\beta}\dot{\alpha}} - \epsilon_{\dot{\beta}\dot{\alpha}}(D_1 - h_1)) - n\leftrightarrow 1\right],$$

S.He, C. Wen, Y-t:

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We can ask....

1. What happens at loop level ?

- IR-divergences Bern, Davies, Nohle
- IR-finite rational terms Bern, Davies, Nohle, S. He, C. Wen, Y-t

$$\begin{aligned} A_{n+1}(+++\cdots+) &\to A_n(++\cdots+) \quad (\text{Exact}) \\ A_{n+1}(-++\cdots+) &\to A_n(++\cdots+) \quad (\text{Exact}) \\ A_{n+1}(-++\cdots+) &\to A_n(-+\cdots+) \\ \Delta S^{(1)} &= -S^{(0)} \frac{[n\, n{+}1]}{\langle n\, n{+}1\rangle} A_n^{(0)}(1^-, 2^+, \cdots, (n{-}1)^+, n^-) \end{aligned}$$

For gravity: $A_n \to M_n$ Soft operators are corrected at loop-level

- Soft theorems for integrand???: Cachazo, Yuan

$$\int d\ell^{4-2*\epsilon} \frac{1}{\cdots(\ell+\delta s_{si})^2\cdots}$$

2. How universal is this ? (Gravity coupled to matter, anomalies, string theory at finite α')

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A_{n+1}(-++\cdots{\color{red}+}) &\rightarrow A_n(-+\cdots+) \\
\Delta S^{(1)} &= -S^{(0)}\frac{[n\,n+1]}{\langle n\,n+1\rangle}A_n^{(0)}(1^-,2^+,\cdots,(n-1)^+,n^-)
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Kinematics: $(\tilde{\lambda}_s \to \delta\tilde{\lambda}_s)$

$$A_{n+1} = \left(S^{(0)}_{YM} + \delta S^{(1)}_{YM}\right) A_n + \mathcal{O}(\delta^2)$$

$$S^{(1)}_{YM} = S^{(0)}_{YM}\left[\frac{\langle sn\rangle}{\langle in\rangle}\tilde{\lambda}_s \cdot \frac{\partial}{\partial\tilde{\lambda}_1} + \frac{\langle s1\rangle}{\langle n1\rangle}\tilde{\lambda}_s \cdot \frac{\partial}{\partial\tilde{\lambda}_n}\right]$$

Solve $\sum_i p_i = 0$ in the following way:

$$|1] = -\sum_{i=2}^{n-1}\frac{\langle ni\rangle[i|}{\langle n1\rangle}, \quad |n] = -\sum_{i=2}^{n-1}\frac{\langle 1i\rangle[i|}{\langle 1n\rangle},$$

Subleading term vanishes $S^{(1)}_{YM}\, A_n = 0$

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$$A_{n+1} = S^{(0)}_{YM} A_n + \mathcal{O}(\delta^2)$$

Manifest soft integrands

Is there a manifest representation?

$$I_{n+1} = S^{(0)}_{YM} I_n + \mathcal{O}(\delta^2)$$

Drummond, Henn

$$Z_n \to \alpha Z_1 + \beta Z_{n-1} + \delta Z_s$$

Since $\tilde{\lambda}_i \to (Z_{i-1}, Z_i, Z_{i+1})$

$$\tilde{\lambda}_n = \delta \, \frac{\langle n{-}1 1\rangle \mu_s + \langle 1 s\rangle \mu_{n-1} + \langle s n{-}1\rangle \mu_1}{\langle 1 n{-}1\rangle^2 \alpha\beta}$$

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$$A_{n+1} = S^{(0)}_{YM} A_n + \mathcal{O}(\delta^2)$$

Arkani-Hamed, Bourjaily, Cachazo, Caron-Huot, Trnka

$$(\mathcal{N} = 4SYM): \quad R^{(L)}_{n,k} = R^{(L)}_{n-1,k} + \sum R^{(L')}(1, \cdots, i-1, I_i)[1, i-1, i, n-1, n] R^{(L-L')}(I_i, i, \cdots, \hat{n}_i) + \int_{GL(2)} [1, A, B, n-1, n] R^{(L-1)}_{n+2,k+1}(1, \cdots, \hat{n}, A, \hat{B})$$

$$\hat{n}_i = (n-1n) \cap (1i-1i), \quad I_i = (i-1i) \cap (1n-1n)$$

$$\hat{n} = (n-1n) \cap (1AB), \quad \hat{B} = (AB) \cap (1n-1n)$$

$$[a, b, c, d, e] \equiv \frac{\delta^{0|4}(\chi_a \langle bcde \rangle + \text{cyc})}{\langle abcd \rangle \langle bcde \rangle \langle cdea \rangle \langle deab \rangle \langle eabc \rangle}.$$

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$$\begin{aligned}
(\mathcal{N} = 4\mathrm{SYM}): \quad \mathrm{R}^{(\mathrm{L})}_{\mathrm{n,k}} &= \mathrm{R}^{(\mathrm{L})}_{\mathrm{n-1,k}} \\
&+ \sum \mathrm{R}^{(\mathrm{L}')}(1,\cdots,\mathrm{i-1},\mathrm{I_i})[1,\mathrm{i-1},\mathrm{i},\mathrm{n-1},\mathrm{n}]\mathrm{R}^{(\mathrm{L-L}')}(\mathrm{I_i},\mathrm{i},\cdots,\hat{\mathrm{n}}_\mathrm{i}) \\
&+ \int_{\mathrm{GL(2)}} [1,\mathrm{A},\mathrm{B},\mathrm{n-1},\mathrm{n}]\mathrm{R}^{(\mathrm{L-1})}_{\mathrm{n+2,k+1}}(1,\cdots,\hat{\mathrm{n}},\mathrm{A},\hat{\mathrm{B}})
\end{aligned}$$

$$\hat{\mathrm{n}}_\mathrm{i} = (\mathrm{n-1n})\cap(\mathrm{1i-1i}), \quad \mathrm{I_i} = (\mathrm{i-1i})\cap(\mathrm{1n-1n})$$

$$\hat{\mathrm{n}} = (\mathrm{n-1n})\cap(\mathrm{1AB}), \quad \hat{\mathrm{B}} = (\mathrm{AB})\cap(\mathrm{1n-1n})$$

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$$\begin{aligned}
(\mathcal{N}=4\mathrm{SYM}): \quad R^{(L)}_{n,k} &= R^{(L)}_{n-1,k} \\
&+ \sum R^{(L')}(1,\cdots,i-1,I_i)[1,i-1,i,n-1,n]R^{(L-L')}(I_i,i,\cdots,\hat{n}_i) \\
&\quad I_i = \delta(i-1i)\cap(1n-1s), \quad \mathcal{Z}_{\hat{n}_i} = \mathcal{Z}_1 + \mathcal{O}(\delta) \\
&+ \int_{GL(2)} [1,A,B,n-1,n]R^{(L-1)}_{n+2,k+1}(1,\cdots,\hat{n},A,\hat{B})
\end{aligned}$$

$$[1,i-1,i,n-1,n] \to \frac{\delta^4 \times \delta^{0|4}(\chi_{[i-1}\langle i],n-1,s,1\rangle + \chi_s\langle 1,i-1,i,n\rangle)}{\alpha\,\beta\,\delta^2\langle 1,i-1,i,n-1\rangle^3\langle n-1,s,1,i-1\rangle\langle n-1,s,1,i\rangle} + \mathcal{O}(\delta^3)$$

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$$[1,A,B,n-1,n]\to\frac{\delta^2}{\alpha\beta}\frac{\delta^{0|4}(\chi_{[A}\langle B],n-1,s,1\rangle+\chi_s\langle 1,A,B,n\rangle)}{\langle 1,A,B,n-1\rangle\langle A,B,n-1,1\rangle\langle 1,A,n-1,s\rangle\langle 1,B,n-1,s\rangle\langle n-1,1,A,B\rangle}$$

Manifest soft integrands

To all-loop order $\mathcal{N} = 4$ SYM

$$\mathrm{R}^{(L)}_{n,k} \to \mathrm{R}^{(L)}_{n-1,k} + \mathcal{O}(\delta^2)$$

$$\mathrm{I}_{n+1} = \left(\mathrm{S}^{(0)}_{\mathrm{YM}} + \delta \mathrm{S}^{(1)}_{\mathrm{YM}}\right) \mathrm{I}_n + \mathcal{O}(\delta^2)$$

Manifest soft integrands

$\mathcal{N} < 4$ SYM?

$$\mathcal{N} = 4 \text{ SYM} - \mathcal{N} \text{ Chiral}$$

CSW Brandhuber, Spence, Travaglini

$$\mathrm{R}^{(1),\mathrm{chiral}}_{\mathrm{n},2} = \mathrm{R}^{(1),\mathrm{chiral}}_{\mathrm{n}-1,2}$$

$$+ \quad \frac{\langle \mathrm{a}\hat{\mathrm{B}}\rangle\langle \mathrm{b}\hat{\mathrm{B}}\rangle}{\langle \mathrm{AB}\rangle^2\langle \mathrm{AB1n}-1\rangle\langle \mathrm{ABn}-1\mathrm{n}\rangle\langle \mathrm{AB1n}\rangle}\sum_{\mathrm{a}<\mathrm{i}\leq \mathrm{b}}\frac{\langle \mathrm{aI_i}\rangle\langle \mathrm{bI_i}\rangle}{\langle \mathrm{AB1i}-1\rangle\langle \mathrm{ABi}-1\mathrm{i}\rangle\langle \mathrm{AB1i}\rangle}$$

$$\mathcal{Z}_{\hat{B}} = \delta(AB) \cap (1n{-}1s)$$

$$\mathrm{R}^{(1)}_{\mathrm{n},2} \to \mathrm{R}^{(1)}_{\mathrm{n}-1,2} + \mathcal{O}(\delta^2)$$

Soft theorems are manifest for D planar integrand

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$$+ \quad \frac{\langle a\hat{B}\rangle\langle b\hat{B}\rangle}{\langle AB\rangle^2\langle AB1n{-}1\rangle\langle ABn{-}1n\rangle\langle AB1n\rangle}\sum_{a<i\leq b}\frac{\langle aI_i\rangle\langle bI_i\rangle}{\langle AB1i{-}1\rangle\langle ABi{-}1i\rangle\langle AB1i\rangle}$$

$$\mathcal{Z}_{\hat{B}} = \delta(AB)\cap(1n{-}1s)$$

$$R^{(1)}_{n,2} \rightarrow R^{(1)}_{n-1,2} + \mathcal{O}(\delta^2)$$

Soft theorems are manifest for D planar integrand

Manifest soft integrands

$\mathcal{N} < 4$ SYM?

$$\mathcal{N} = 4 \text{ SYM} - \mathcal{N} \text{ Chiral}$$

CSW Brandhuber, Spence, Travaglini

$$\mathrm{R}^{(1),\mathrm{chiral}}_{\mathrm{n},2} = \mathrm{R}^{(1),\mathrm{chiral}}_{\mathrm{n}-1,2}$$

$$+ \quad \frac{\langle \mathrm{a}\hat{\mathrm{B}}\rangle\langle \mathrm{b}\hat{\mathrm{B}}\rangle}{\langle \mathrm{AB}\rangle^2\langle \mathrm{AB1n{-}1}\rangle\langle \mathrm{ABn{-}1n}\rangle\langle \mathrm{AB1n}\rangle}\sum_{\mathrm{a<i\leq b}}\frac{\langle \mathrm{aI_i}\rangle\langle \mathrm{bI_i}\rangle}{\langle \mathrm{AB1i{-}1}\rangle\langle \mathrm{ABi{-}1i}\rangle\langle \mathrm{AB1i}\rangle}$$

$$\mathcal{Z}_{\hat{B}} = \delta(AB) \cap (1n{-}1s)$$

$$\mathrm{R}^{(1)}_{\mathrm{n},2} \to \mathrm{R}^{(1)}_{\mathrm{n}-1,2} + \mathcal{O}(\delta^2)$$

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- Integrands with manifest soft behavior have interesting properties
- But we don’t have guidance for how to find them

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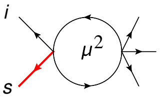
All Plus Rational terms

Why does the two-limit commute for all-plus? Bern, Morgan

$$A_5(+,+,+,+,+) = \frac{2}{\prod_{i=1}^{5}\langle i i+1\rangle}\left(-\frac{1}{2}\left[\frac{\mu^4 s_{12}s_{23}}{d_1d_2d_3d_5}+\text{cyclic}\right]+\frac{4i\mu^6\epsilon(1234)}{d_1d_2d_3d_4d_5}\right)$$

$$I_m[\mu^{2r}] = -\epsilon(1-\epsilon)\cdots(r-1-\epsilon)(4\pi)^r I_m^{D=4+2r-2\epsilon}\,,$$



Dimension shifting: Bern, Dixon, Perelstein, Rozowsky

$$A_n(+,\cdots,+) = \mu^4(\mathcal{N}=4\text{SYM})$$

$$M_n(+,\cdots,+) = \mu^8(\mathcal{N}=8\text{Sugra})$$

Non-commutativity stems from scalar bubbles

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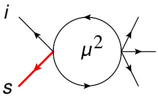

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Soft corrections from anomalies

$$\mathfrak{K}A_{n+1} = 0$$

$$A_{n+1}(\{\lambda_s, \tilde{\lambda}_s\}, \delta\lambda_s, \tilde{\lambda}_s) = \frac{1}{\delta^2} S^{(0)} A_n + \frac{1}{\delta} S^{(1)} A_n$$

$$\mathfrak{K} = \sum_{i=1}^{n} \frac{\partial^2}{\partial\lambda_i \partial\tilde{\lambda}_i} + \frac{\partial^2}{\delta\partial\lambda_s\partial\tilde{\lambda}_s} = \mathfrak{K}_0 + \frac{1}{\delta}\mathfrak{K}_s$$

Conformal invariance implies a differential equation for soft functions Larkoski

$$S^{(0)} = \frac{\langle 1n\rangle}{\langle 1s\rangle\langle sn\rangle}$$

$$\mathcal{O}(\delta^{-2}): \quad \mathfrak{K}_0 S^{(0)} A_n + \mathfrak{K}_s S^{(1)} A_n = \left(\frac{-\lambda_n}{\langle ns\rangle^2}\frac{\partial}{\partial\tilde{\lambda}_n} + \frac{-\lambda_1}{\langle 1s\rangle^2}\frac{\partial}{\partial\tilde{\lambda}_1}\right) A_n + (\mathfrak{K}_s S^{(1)}) A_n = 0$$

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- The tree-level soft functions are homogenous solutions to CDE
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- The all-plus amplitude $\mathrm{A_n}(+,+,\cdots,+)$ is conformal invariant (Self-dual)
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$$\mathrm{A_{n+1}}(-++\cdots+) \quad \to \quad a^0_{n+1} + \mathcal{O}(\delta)$$

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Are soft theorems universal ?

- Effective field theories with F^3, R^3 (Forbidden by susy)
- Effective field theories with $R^2\phi$ (appears in SUGRA due to U(1) anomalies)
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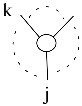
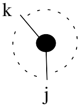
F^3 amplitudes

CSW representation Broedel, Dixon

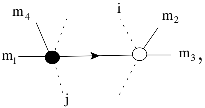


$$: \quad \frac{\langle jk\rangle^2\langle kl\rangle^2\langle lj\rangle^2}{\prod_{i=1}^{n}\langle i i+1\rangle}$$



$$: \quad \frac{\langle jk\rangle^4}{\prod_{i=1}^{n}\langle i i+1\rangle}$$

Here we will consider diagrams with only white vertex

(a) :


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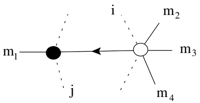
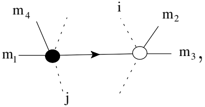
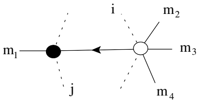


F^3 amplitudes

(*a*) : 

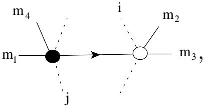
(*b*) : 

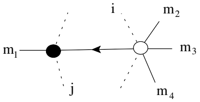
$$(a): \quad \frac{1}{\prod_{l=1}^{n}\langle l\, l+1\rangle}\langle m_1 m_4\rangle^4\ [i\, i{-}1\, j\, j{-}1*]\ \langle m_2 m_3\rangle^2\langle m_3\widehat{i{-}1}\rangle^2\langle\widehat{i{-}1}m_2\rangle^2$$

$$(b): \quad \frac{1}{\prod_{l=1}^{n}\langle l\, l+1\rangle}\langle m_1\widehat{i{-}1}\rangle^4\ [i\, i{-}1\, j\, j{-}1*]\ \langle m_2 m_3\rangle^2\langle m_3 m_4\rangle^2\langle m_4 m_2\rangle^2,$$

$$R^{F^3}_{1,\,n} = [i\,i{-}1\,j\,j{-}1*]\left(\langle m_1 m_4\rangle^4\langle m_2 m_3\rangle^2\langle m_3\widehat{i{-}1}\rangle^2\langle\widehat{i{-}1}m_2\rangle^2+\cdots\right)$$

F^3 amplitudes

(a) : 

(b) : 

$$(a): \quad \frac{1}{\prod_{l=1}^{n}\langle l\, l{+}1\rangle}\langle m_1 m_4\rangle^4\,[i\, i{-}1\, j\, j{-}1 *]\,\langle m_2 m_3\rangle^2\langle m_3 \widehat{i{-}1}\rangle^2\langle \widehat{i{-}1} m_2\rangle^2$$

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$$\boxed{R^{F^3}_{k,\,n} = R^{F^3}_{k,\,n-1} + \sum_j [n{-}1, n, 1, j{-}1, j] R^{F^3}_{k',\,j} R^{F^2}_{k-1-k',\,n+2-j} + (F^3 \leftrightarrow F^2)}$$

$$R^{F^3}_{k,\,n} = R^{F^3}_{k,\,n-1} + \mathcal{O}(\delta^2)$$

$$A_{n+1} = S^{(0)}_{YM} A_n + \mathcal{O}(\delta^2)$$

As expected, the presence of F^3 do not spoil $S^{(1)}$.

$R^3/R^2\phi$ amplitudes

$$M(1^-,2^-,3^-,4^-,5^+) = i s_{12}s_{34}A^{F^3}(1^-,2^-,3^-,4^-,5^+)A^{F^3}(2^-,1^-,4^-,3^-,5^+) + \mathcal{P}(2,3)$$

There is a mixing of operators (α'^2) R^3, (α'^1) ϕR^2

$$M_{n+1} \to \frac{1}{\delta^3}S^{(0)}M_n + \frac{1}{\delta^2}S^{(1)}M_n + \frac{1}{\delta}S^{(2)}M_n + \Delta^{(2)} + \mathcal{O}(\delta^0)$$

$$\Delta^{(2)} = \sum_j -2\frac{\langle 1j\rangle^3}{[1j]}M_n(\phi, i_1^-, i_2^-, \cdots, i_{n-2}^-, n^+)$$

For $\mathcal{N} \leq 4$ there are U(1) anomalies: $R^2\tau$ Carrasco, Kallosh, Roiban, Tseytlin

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Finite α' string amplitude

String amplitudes at finite α' Mafra, Schlotterer, Stieberger

$$A_n = \sum_{\sigma \in S_{n-3}} F^{(2_\sigma, \ldots, (n-2)_\sigma)} A_{YM}(1, 2_\sigma, \ldots, (n-2)_\sigma, n-1, n)$$

$$A_4 = F^{(2)} A_{YM} = s_{12} \int_0^1 dz_2\, z_2^{s_{12}-1} (1-z_2)^{s_{23}} A_{YM}$$

Consider the 5-pt amplitude

$$A_5 = F^{(2,3)} A_{YM}(1,2,3,4,5) + F^{(3,2)} A_{YM}(1,3,2,4,5)$$

$$\begin{aligned} F^{(2,3)} &= s_{12}s_{34} \int_0^1 dz_2 \int_{z_2}^1 dz_3 z_2^{s_{12}-1} z_3^{s_{13}} z_{32}^{s_{23}} (1-z_2)^{s_{24}} (1-z_3)^{s_{34}-1}, \\ F^{(3,2)} &= s_{13}s_{24} \int_0^1 dz_2 \int_{z_2}^1 dz_3 z_2^{s_{12}} z_3^{s_{13}-1} z_{32}^{s_{23}} (1-z_2)^{s_{24}-1} (1-z_3)^{s_{34}} \end{aligned}$$

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Integrate z_3 out and keep terms up to the subleading order we obtain,

$$F^{(2,3)} \rightarrow s_{12} \int_0^1 dz_2\, z_2^{s_{12}-1} (1-z_2)^{s'_{24}} \left[1 + \delta(s_{23} + s'_{34} + k_2 \cdot p_4) \log(1-z_2)\right].$$

Finite α' string amplitude

$$A_4(1,2,4,5) = F^{(2)} A_{YM} = s_{12} \int_0^1 dz_2\, z_2^{s_{12}-1} (1-z_2)^{s'_{24}} A_{YM}(1,2,4,5)$$

$$\begin{aligned} k'_4 &= \frac{|4\rangle\langle 5|(1+2)}{\langle 45\rangle}, \quad p_4 = \frac{|4\rangle\langle 5|3}{\langle 45\rangle}, \\ k'_5 &= \frac{|5\rangle\langle 4|(1+2)}{\langle 54\rangle}, \quad p_5 = \frac{|5\rangle\langle 4|3}{\langle 54\rangle}. \end{aligned}$$

$$\begin{aligned} F^{(2,3)}_{S^{(1)}} &= \frac{\langle 34\rangle\langle 51\rangle[31]}{\langle 45\rangle} s_{12} \int_0^1 dz_2\, z_2^{s_{12}-1} (1-z_2)^{s'_{24}} \log(1-z_2) \\ F^{(3,2)}_{S^{(1)}} &= -s_{13} s'_{24} \int_0^1 dz_2 z_2^{s_{12}} (1-z_2)^{s'_{24}-1} \log(z_2) \end{aligned}$$

$$S^{(1)}_{YM}(234) A(1,2,4,5) = \left(\frac{\tilde\lambda_3}{\langle 23\rangle} \cdot \frac{\partial}{\partial \tilde\lambda_2} + \frac{\tilde\lambda_3}{\langle 34\rangle} \cdot \frac{\partial}{\partial \tilde\lambda_4} \right) A(1,2,4,5)$$

Soft expansion is valid for finite α'

Finite α' string amplitude

$$A_4(1,2,4,5) = F^{(2)} A_{YM} = s_{12} \int_0^1 dz_2\, z_2^{s_{12}-1} (1-z_2)^{s'_{24}} A_{YM}(1,2,4,5)$$

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$$\begin{aligned} M_n &= \mathcal{A}_n(1,2,\ldots,n) \sum_{\{i\},\{j\}} f(i_1,\ldots,i_{\lfloor\frac{n}{2}\rfloor-1})\bar{f}(j_1,\ldots,j_{\lfloor\frac{n}{2}\rfloor-2})]\mathcal{A}_n(\{i\},1,n-1,\{j\},n) \\ &+ \mathrm{Perm}(2,\ldots,n-2) \end{aligned}$$

$$\begin{aligned} f(i_1,\ldots,i_m) &= \sin(\pi s_{1,i_m}) \prod_{k=1}^{m-1} \sin\pi\left(s_{1,i_k} + \sum_{l=k+1}^{m} g(i_k,i_l)\right), \\ \bar{f}(j_1,\ldots,j_m) &= \sin(\pi s_{j_1,n-1}) \prod_{k=2}^{m} \sin\pi\left(s_{j_k,n-1} + \sum_{l=1}^{k-1} g(j_l,j_k)\right), \end{aligned}$$

finite α' string amplitude

String amplitudes at finite α'

$$A(1,2,\ldots,n) = i g_s^{n-2} \int_{0=z_1, z_{n-1}=1, z_n=\infty} dz_2 \cdots dz_{n-2} \langle cV(1)V(2)\ldots cV(n-1)cV(n)\rangle$$

Take two vertices in the $q=-1$ picture and the remaining $n-2$ in the $q=0$ picture:

$$V_A^{(-1)} = (\epsilon\cdot\psi)e^{-\varphi}e^{ikX}, \quad V_A^{(0)} = (\epsilon\cdot\partial X + ik\cdot\psi\epsilon\cdot\psi)e^{ikX}$$

$$V_A^{(0)}(z_s)V_A^{(-1)}(z_{s\pm1}) \approx |z_s - z_{s\pm1}|^{2k_s k_{s\pm1}-1} e^{-\varphi(z_{s\pm1})} e^{i(k_{s\pm1})X(z_{s\pm1})} \times$$
$$\{\epsilon_s\cdot k_{s\pm1}\epsilon_{s\pm1}\cdot\psi + \epsilon_{s\pm1}\cdot k_s\epsilon_s\cdot\psi - \epsilon_{s\pm1}\cdot\epsilon_s\cdot k_s\psi + \epsilon_s\cdot k_{s\pm1}k_s\cdot X_s\epsilon_{s\pm1}\cdot\psi\}(z_{s\pm1}) + \cdots$$

Name of the game: write all soft expansion terms as BRST invariant operator acting on vertex operators

finite α' string amplitude

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$$(\epsilon_{s\pm1}\cdot k_s\epsilon_s\cdot\psi - \epsilon_{s\pm1}\epsilon_s\cdot k_s\cdot\psi)e^{-\varphi}e^{i(k_{s\pm1})X} = (k_s^\mu\epsilon_s^\nu - k_s^\nu\epsilon_s^\mu)\tilde\epsilon_\mu\frac{\partial}{\partial\tilde\epsilon^\nu}V_{\tilde A}^{(-1)}(k_{s\pm1})$$

$$(k_s^\mu\epsilon_s^\nu - k_s^\nu\epsilon_s^\mu)\tilde k_\mu\frac{\partial}{\partial\tilde k^\nu}V_{\tilde A}^{(-1)}(k_{s\pm1})$$

$$S^{(1)} = (k_s^\mu\epsilon_s^\nu - k_s^\nu\epsilon_s^\mu)[J_{\mu\nu}, V_{\tilde A}(k_{s\pm1})]$$

Conclusions and outlook

Conclusions

- Leading and subleading soft functions are universal.
- Subsubleading corrections are sensitive to scalar couplings $F^2\phi$ $R^2\phi$
- Manifest soft integrands exists for planar $\mathcal{N} \leq 4$ SYM
- Loop corrections are intimately tied to symmetries and their anomalies
- String theory at finite α' enjoy the same soft behavior as EFT.

Outlook

- IR-div corrections are known, Bern, Davies, Nhole, IR-finite?
- The tree-level YM soft functions are homogenous solution to conformal symmetry. Gravity?
- String theory soft theorems simply stems from OPE. Higher genus?

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