The gluon-fusion cross section at N3LO in the soft limit

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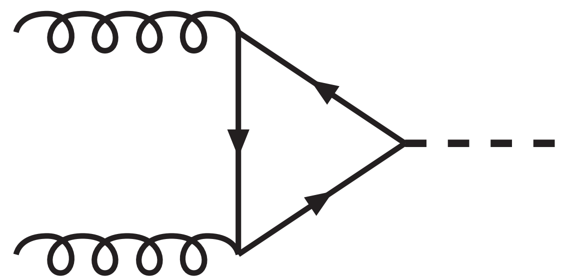
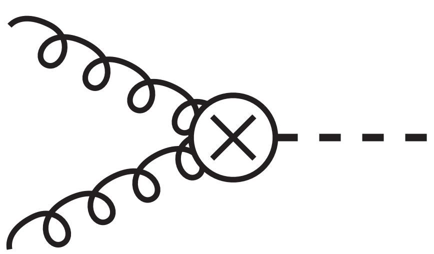
The gluon fusion cross section

- The dominant Higgs production mechanism at the LHC is gluon fusion.
  ➡ Loop-induced process.



- For a light Higgs boson, the dimension five operator describing a tree-level coupling of the gluons to the Higgs boson

$$\mathcal{L} = \mathcal{L}_{QCD,5} - \frac{1}{4v} C_1 \, H \, G^a_{\mu\nu} \, G_a^{\mu\nu}$$



- Top-mass corrections known at NNLO.

[Harlander, Ozeren; Pak, Rogal, Steinhauser; Ball, Del Duca, Marzani, Forte, Vicini; Harlander, Mantler, Marzani, Ozeren]

- In the rest of the talk, I will only concentrate on the effective theory.

The gluon fusion cross section

- The gluon fusion cross section is given in perturbation theory by

$$\sigma(p\,p \to H + X) = \tau \sum_{ij} \int_{\tau}^{1} dz\, \mathcal{L}_{ij}(z)\, \hat{\sigma}_{ij}(\tau/z)$$

- The (partonic) cross section depends up to an overall scale only on the ratio

$$\tau = \frac{m^2}{s} \qquad\qquad z = \frac{m^2}{\hat{s}}$$

- The partonic cross section known at NLO and NNLO.

[Dawson; Djouadi, Spira, Zerwas; Harlander, Kilgore; Anastasiou, Melnikov; Ravindran, Smith, van Neerven]

	σ [8 TeV]	$\delta\sigma$ [%]
LO	9.6 pb	~ 25%
NLO	16.7 pb	~ 20%
NNLO	19.6 pb	~ 7 - 9%
N3LO	???	~ 4 - 8%

[Fixed order only

The gluon fusion cross section

- So far no complete computation is available.
  - ➡ Scale variation at N3LO.
  - ➡ Approximate N3LO results exist.

[Moch, Vogt; Ball, Bonvini, Forte, Marzani, Ridolfi; Bühler, Lazopulos

The gluon fusion cross section

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- Can we push the state of the art one order higher?

- Challenge:
  performed so far...
  - ➡ Uncharted territory!
  - ➡ New conceptual challenges.

UV-Singularities
HopfAlgebras
ExpansionByRegions
MellinBarnes
MultiloopAmplitudes
ZetaValues
NestedSums
SoftCurrent
HypergeometricFunctions
Renormalisation
Symbols
Polylogarithms
IR-Singularities
MassFactorization
DimensionalShiftIdentities.
DifferentialEquations
IBPs
PhaseSpace
SectorDecomposition
Unitarity

Prime example of how new
developments from the
amplitude community can have
impact on phenomenology.

Outline

- The inclusive gluon-fusion cross section.
- Ingredients entering the cross section at threshold:
  - ➡ Soft triple-real emission.
  - ➡ Soft double-virtual-real emission.
  - ➡ Soft virtual-double-real emission.
- The gluon-fusion cross section in the soft limit.
- Conclusion & outlook.

The inclusive gluon-fusion cross section

The gluon fusion cross section

- A cross section computation requires two ingredients:

$$\hat{\sigma} = \int d\Phi \, |\mathcal{M}|^2$$

The gluon fusion cross section

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Matrix element

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Phase space integration

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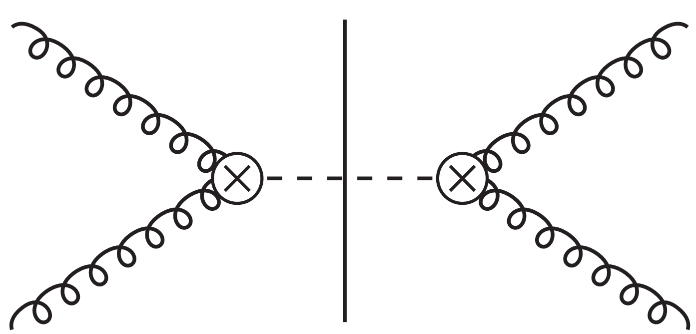
- Example:



The gluon fusion cross section

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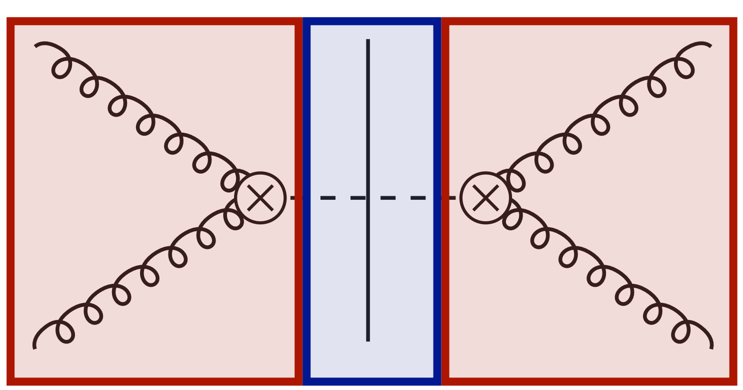
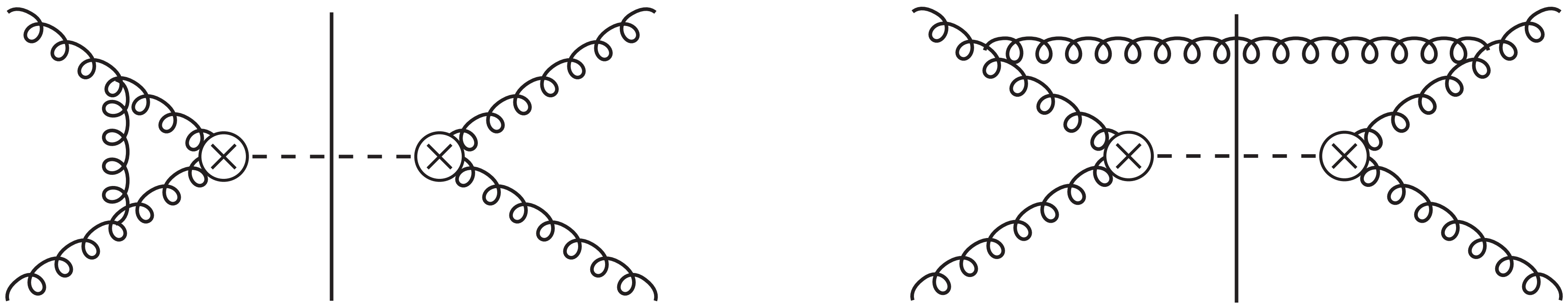


$$\int d\Phi_1 \, \mathcal{M}^{(0)} \, \mathcal{M}^{(0)*}$$

The gluon fusion cross section

- At

[Dawson; Djouadi, Spira, Zerwas]

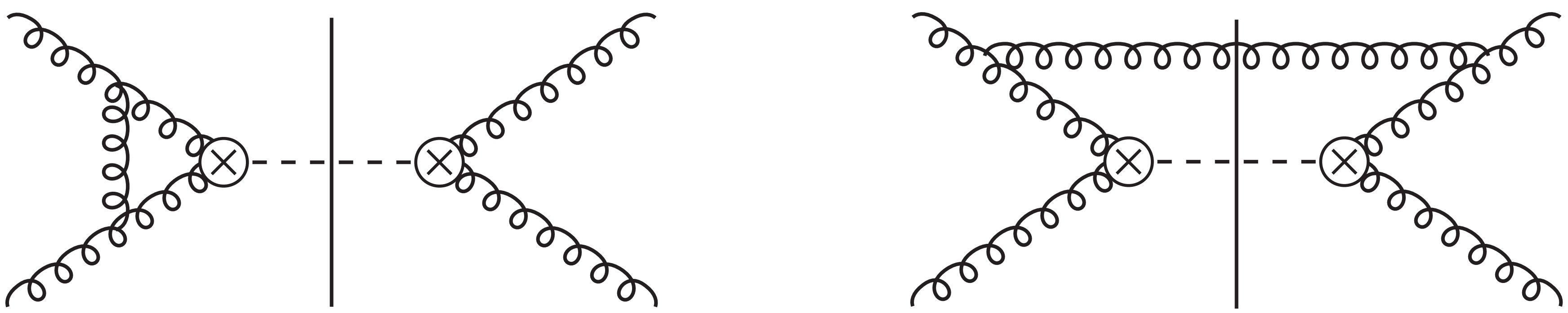


Virtual corrections ('loops') Real emission

The gluon fusion cross section

- At

[Dawson; Djouadi, Spira, Zerwas]



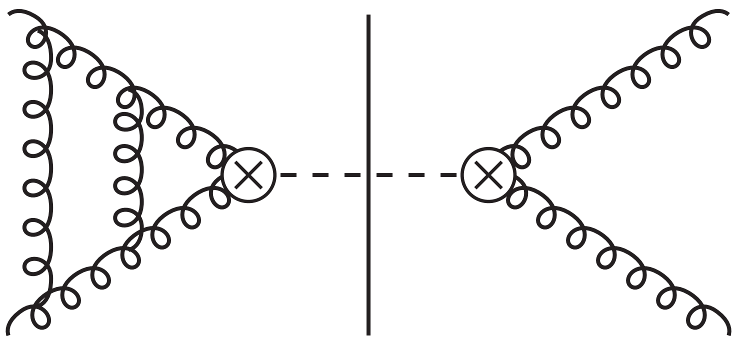
Virtual corrections ('loops') Real emission

- Both contributions are individually divergent:
  - ➡ UV divergences are handled by renormalization.
  - ➡ IR divergences cancelled by PDF counterterms.

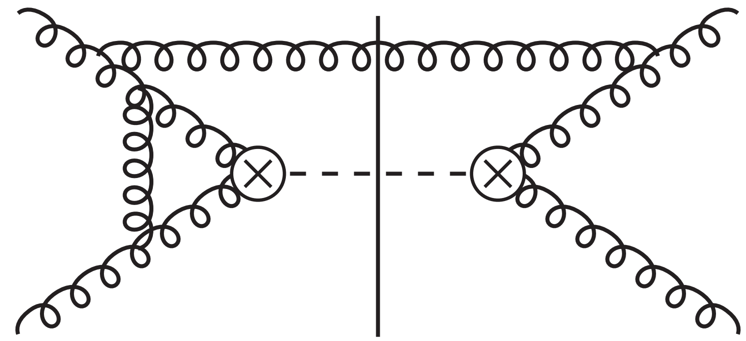
The gluon fusion cross section

- At

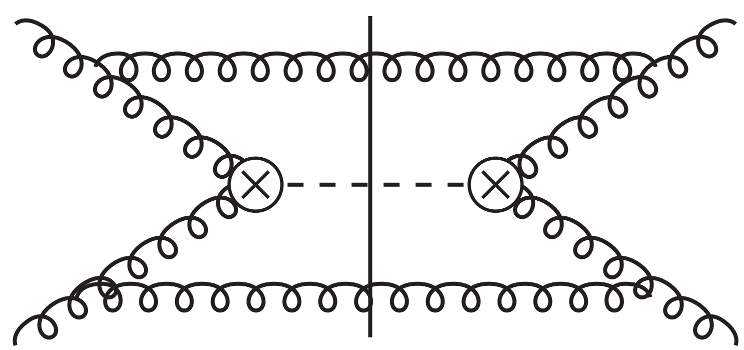
[Harlander, Kilgore; Anastasiou, Melnikov; Ravindran, Smith, van Neerven]



Double virtual



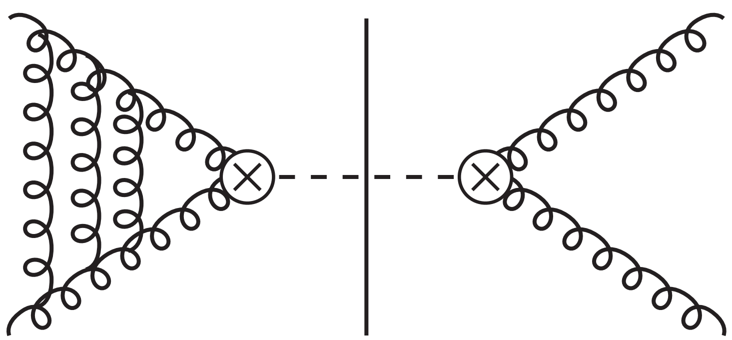
Real-virtual



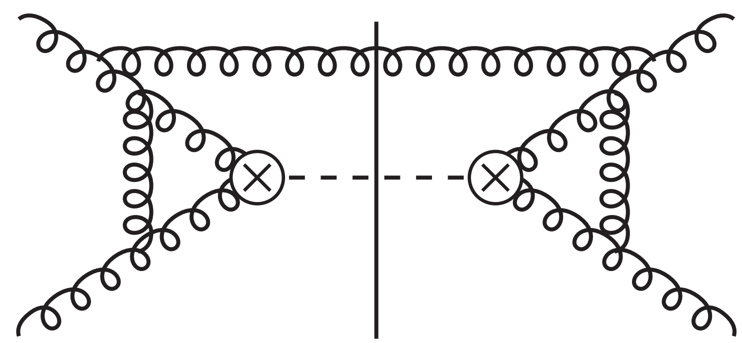
Double real

The gluon fusion cross section

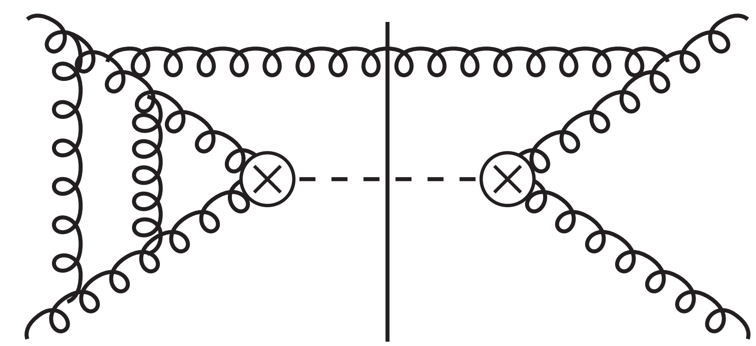
- At



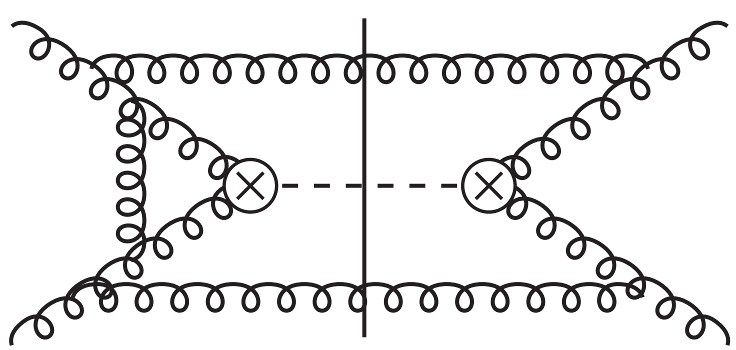
Triple virtual



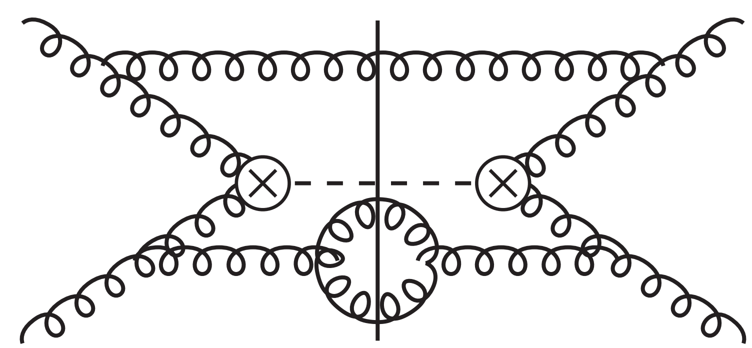
Real-virtual squared



Double virtual real



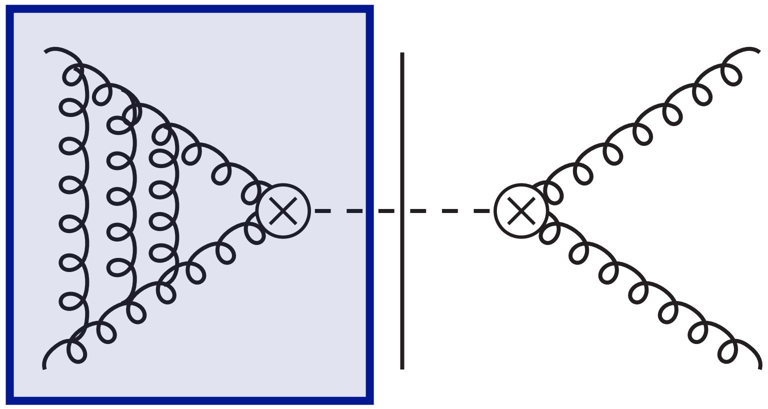
Double real virtual



Triple real

Triple virtual corrections

- The triple virtual corrections are directly related to the QCD form factor

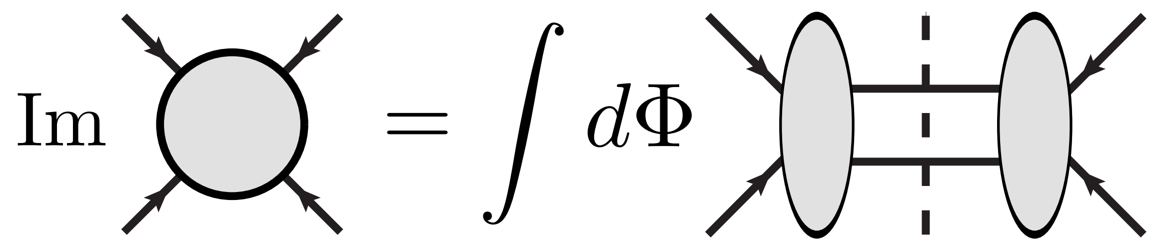


- The QCD form factor is known
  - ➡ at one loop.
  - ➡ at two loops. [Gonsalves; Kramer, Lampe; Gehrmann, Huber, Maître]
  - ➡ at three loops. [Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser; Gehrmann, Glover, Huber, Ikizlerli, Studerus]

- It is not the loops that are the problem!

Unitarity

- Optical theorem:



➡ Discontinuities of loop amplitudes are phase space integrals.

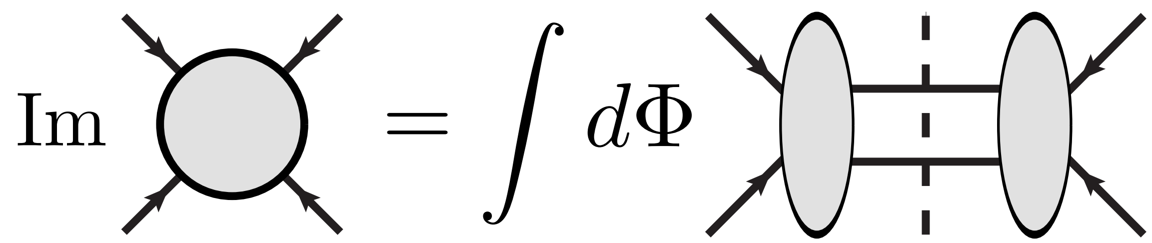
- Discontinuities of loop integrals are given by rule

$$\frac{1}{p^2 - m^2 + i\varepsilon} \to \delta_+(p^2 - m^2) = \delta(p^2 - m^2)\,\theta(p^0)$$

➡ See Ruth Britto's talk.

Reverse-unitarity

- Optical theorem:



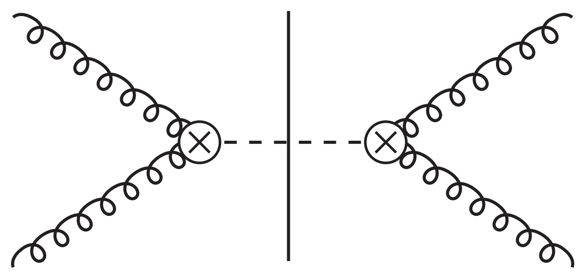
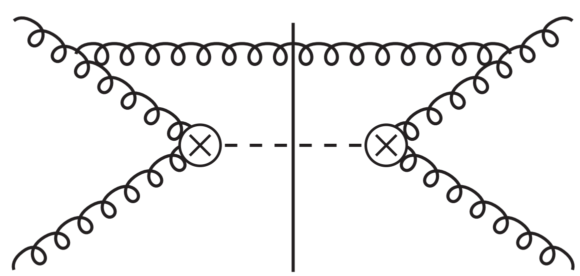
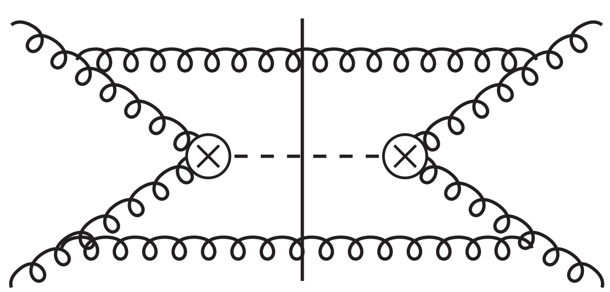
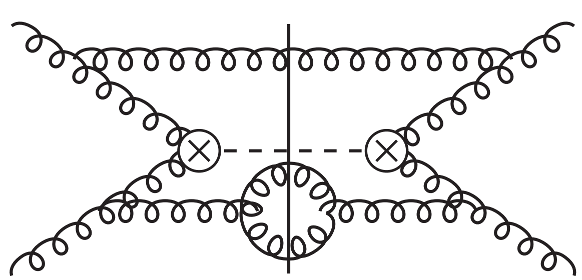
- We can read the optical theorem 'backwards' and write inclusive phase space integrals as unitarity cuts of loop integrals. [Anastasiou, Melnikov; Anastasiou, Dixon, Melnikov, Petriello]

  ➡ Rather than computing phase-space integrals, we can compute loop integrals with cuts!

  ➡ Makes inclusive phase space integrals accessible to all the technology developed for multi-loop computations!
    - Integration-by-parts & differential equations.

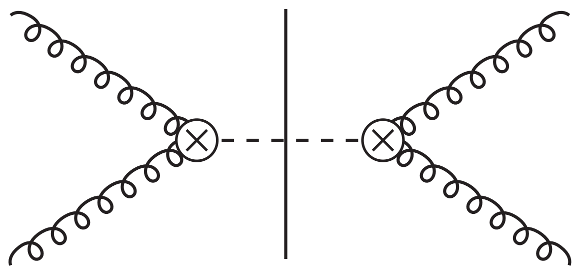
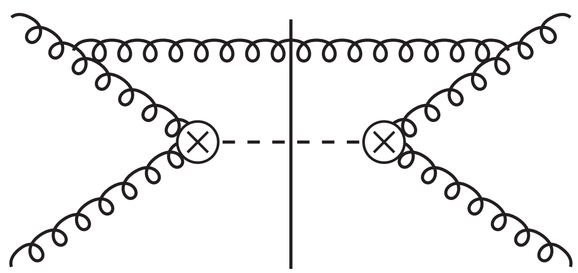
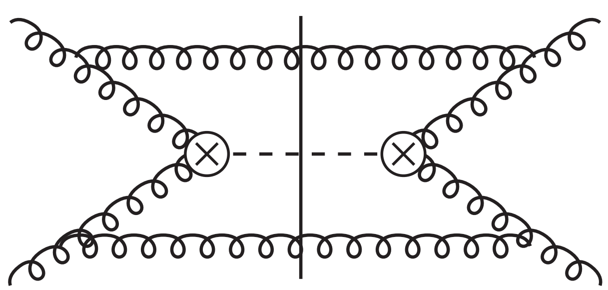
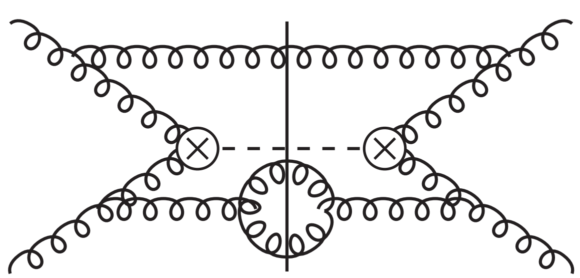
Reverse-unitarity @ N3LO

Growth in complexity for real emission

LO		1 diagram	1 integral
NLO			
NNLO			
N3LO			

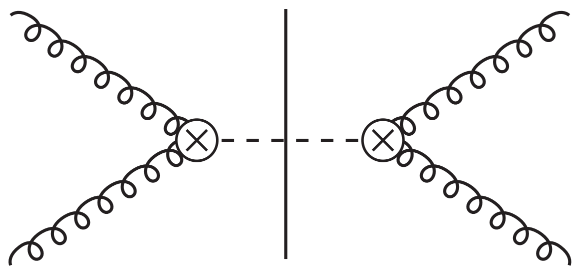
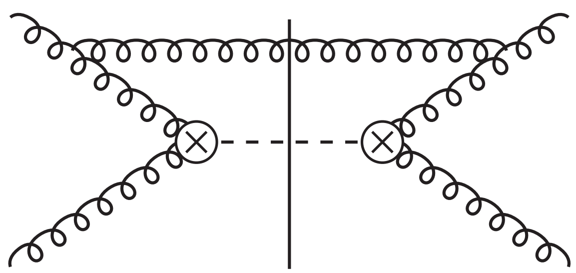
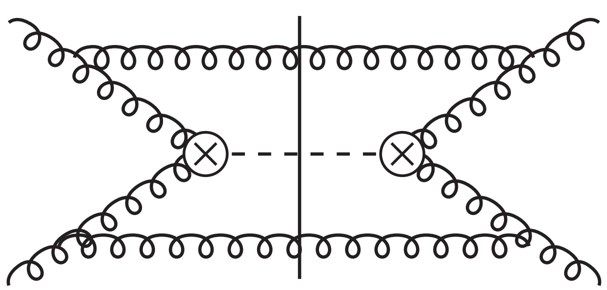
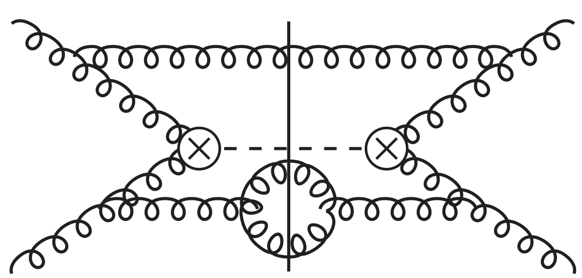
Reverse-unitarity @ N3LO

Growth in complexity for real emission

LO		1 diagram	1 integral
NLO		10 diagrams	1 integral
NNLO			
N3LO			

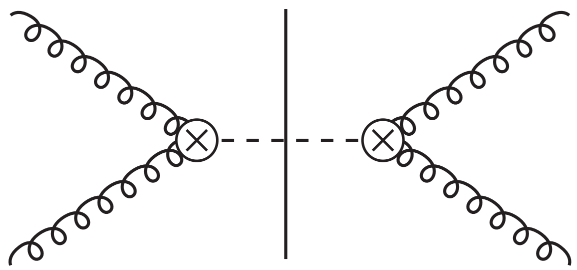
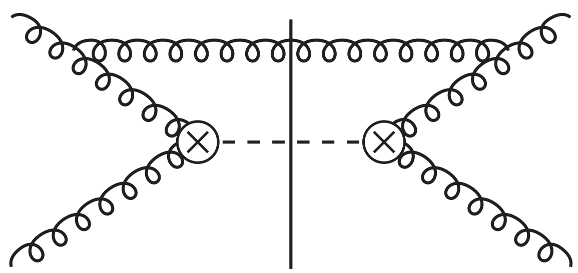
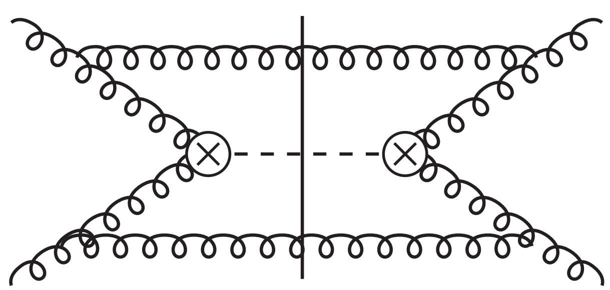
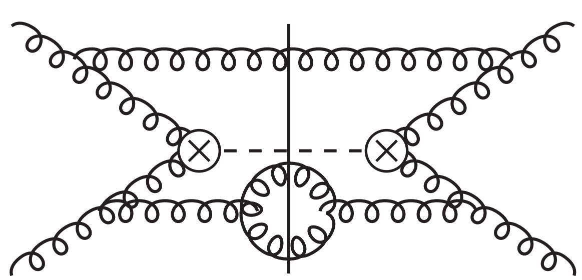
Reverse-unitarity @ N3LO

Growth in complexity for real emission

LO		1 diagram	1 integral
NLO		10 diagrams	1 integral
NNLO		381 diagrams	18 integrals
N3LO			

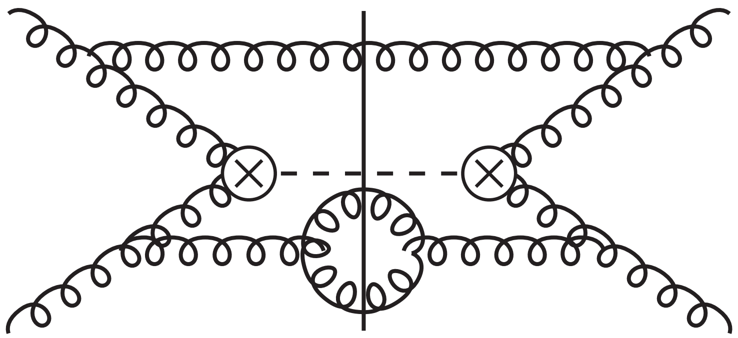
Reverse-unitarity @ N3LO

Growth in complexity for real emission

LO		1 diagram	1 integral
NLO		10 diagrams	1 integral
NNLO		381 diagrams	18 integrals
N3LO		26565 diagrams	~500 integrals

The threshold expansion

- ~ 500 master integrals only for triple real double real NNLO).
  - ➡ Tough nut to crack!
- Concentrate on some approximation first!



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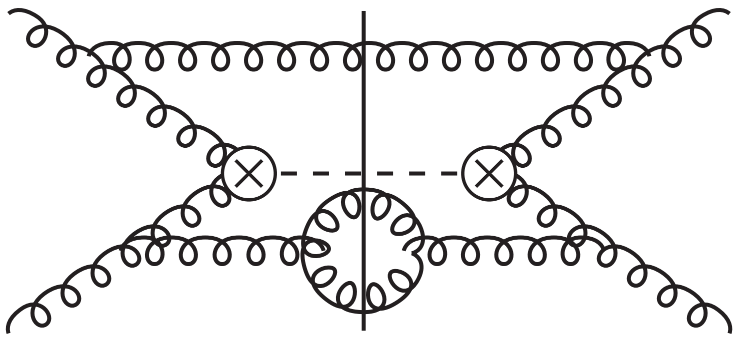


- Concentrate on some approximation first!
- The gluon fusion cross section depends on one single parameter:

$$z = \frac{m^2}{s} \qquad \bar{z} = 1 - z$$

- Close to threshold ($z \sim 1$), we can approximate the triple real cross section by a power series:

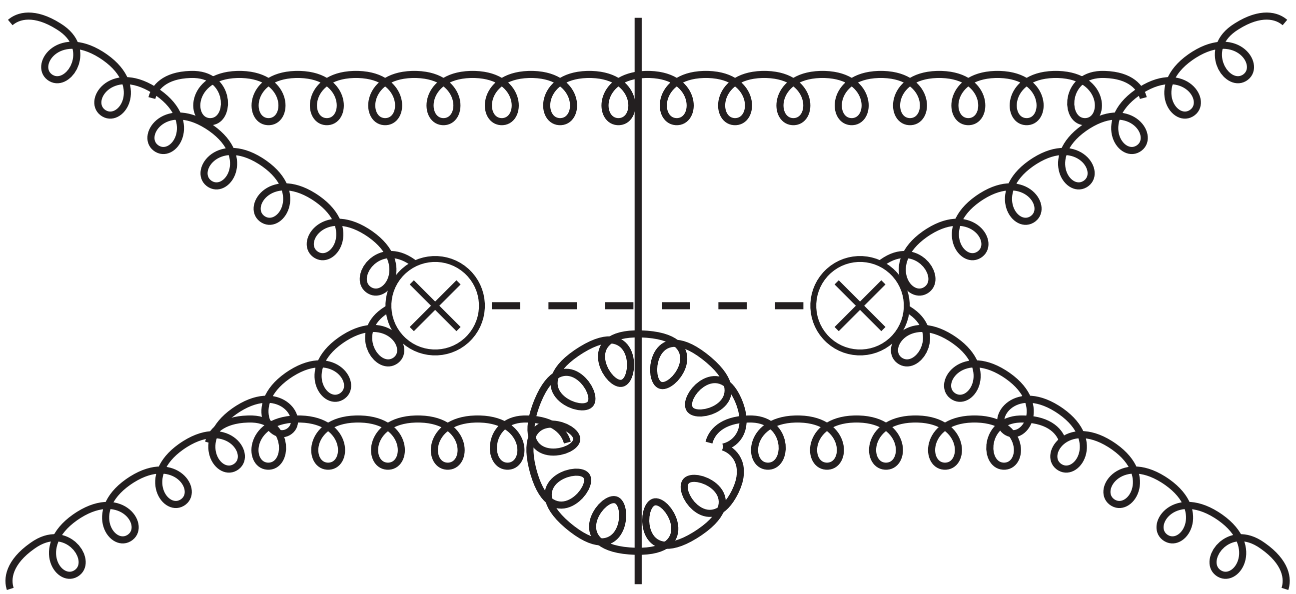
$$\hat{\sigma}(z) = \sigma_{-1} + \sigma_0 + (1-z)\,\sigma_1 + \mathcal{O}(1-z)^2$$

- Goal:

The threshold expansion

- Formally, this expansion corresponds to 'expansion by regions'. [Beneke, Smirnov
  - ➡ In the limit:
    momenta are, e.g., 'hard', 'soft' or 'collinear'.
  - ➡ Extend this to inclusive phase space.
  - ➡ Advantage:
    themselves.
- Higgs production at threshold (soft-virtual):
  - ➡ Every real gluon is soft.
  - ➡ Every virtual gluon is either hard or soft.
- N.B.:
  virtual and/or real gluon!
  - ➡ Universality of soft emissions!

Soft t emissions



Triple real emission

- ~500 master integrals.
- Subprocesses:

$g\,g \to H\,g\,g\,g$	$g\,q \to H\,g\,g\,q$	$q\,\bar{q} \to H\,g\,g\,g$	$q\,q \to H\,g\,q\,q$
$g\,g \to H\,g\,q\,\bar{q}$	$g\,q \to H\,q\,\bar{q}\,q$	$q\,\bar{q} \to H\,g\,q\,\bar{q}$	$q\,\bar{Q} \to H\,g\,q\,\bar{Q}$
	$g\,q \to H\,Q\,\bar{Q}\,q$	$q\,\bar{q} \to H\,g\,Q\,\bar{Q}$	$q\,Q \to H\,g\,q\,Q$
Soft-virtual	Next-to-soft-virtual		

- If we concentrate on the first two terms in the expansion, all ~500 master integrals can be reduced to only 10 integrals!

NNLO example

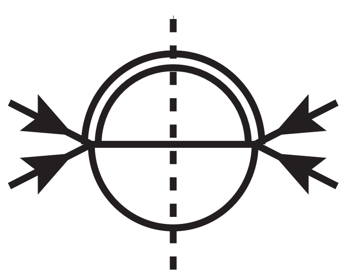
- NNLO integral:

$$
\begin{aligned}
\int d\Phi_3 &= \bar{z}^{3-4\epsilon}\, \Phi_3^S(\epsilon) \sum_{n=0}^{\infty} \frac{(1-\epsilon)_n (2-2\epsilon)_n}{(4-4\epsilon)_n}\, \bar{z}^n \\
&= \bar{z}^{3-4\epsilon}\, \Phi_3^S(\epsilon) \left[1 + \frac{1-\epsilon}{2}\, \bar{z} + \frac{(1-\epsilon)(2-\epsilon)(3-2\epsilon)}{4(5-4\epsilon)}\, \bar{z}^2 + \mathcal{O}(\bar{z}^3) \right]
\end{aligned}
$$

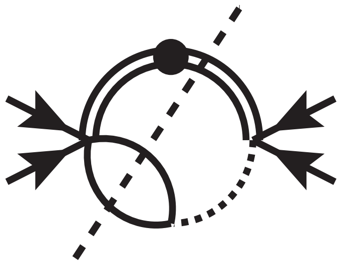
$$
\Phi_3^S(\epsilon) = \frac{1}{2(4\pi)^{3-2\epsilon}} \frac{\Gamma(1-\epsilon)^2}{\Gamma(4-4\epsilon)}
$$

- Diagrammatic expansion:

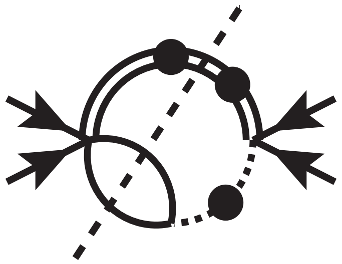
$$\int d\Phi_3 = \bar{z}^{3-4\epsilon} \Big[$$



$$- \bar{z}$$



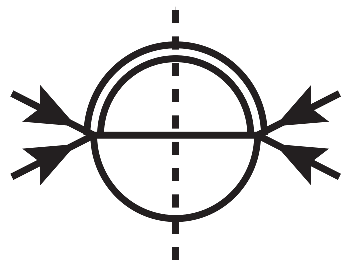
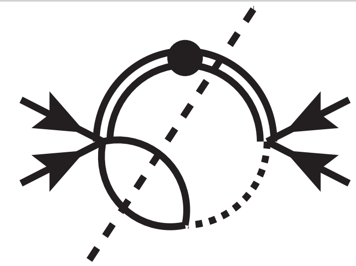
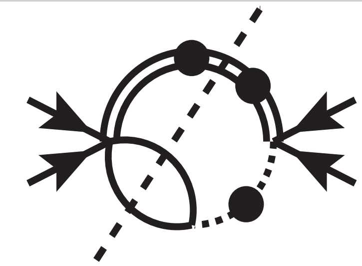
$$+ \bar{z}^2$$



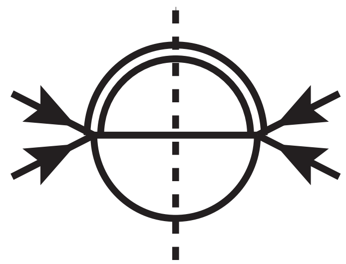
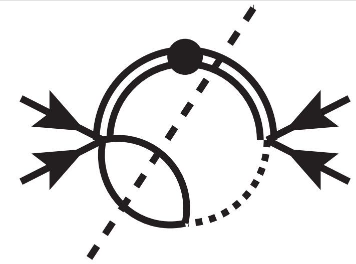
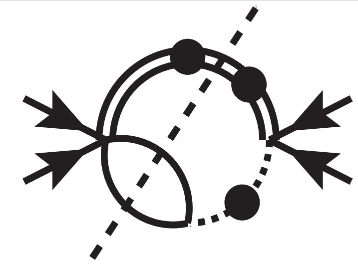
$$+ \mathcal{O}(\bar{z}^3) \Big]$$

➡ The coefficients themselves have a loop interpretation.

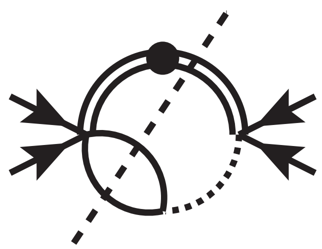
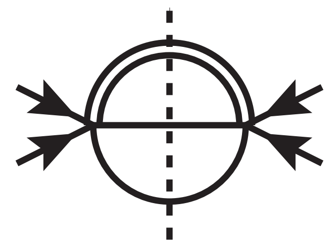
NNLO example

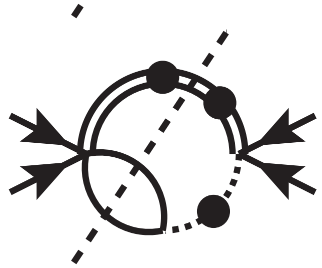
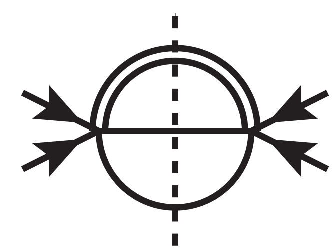
$\int d\Phi_3 = \bar{z}^{3-4\epsilon} \Big[$  $- \bar{z}$  $+ \bar{z}^2$  $+ \mathcal{O}(\bar{z}^3) \Big]$

NNLO example

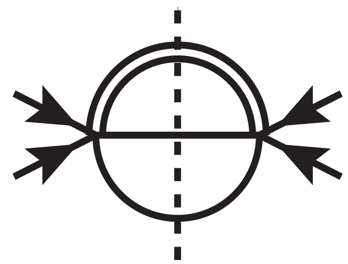
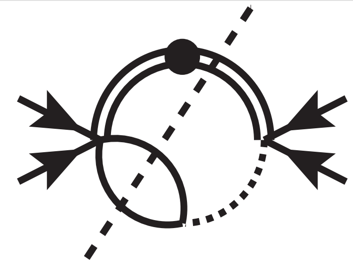
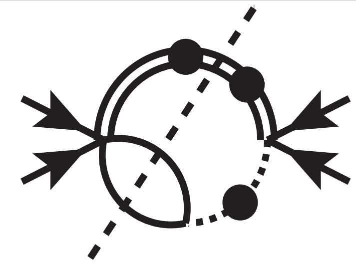
$$\int d\Phi_3 = \bar{z}^{3-4\epsilon} \Big[$$

$$- \bar{z}$$

$$+ \bar{z}^2$$

$$+ \mathcal{O}(\bar{z}^3) \Big]$$

- Using IBP identities:

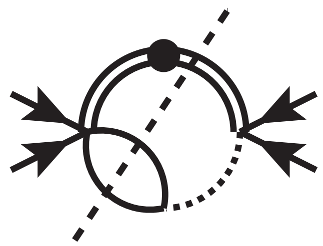
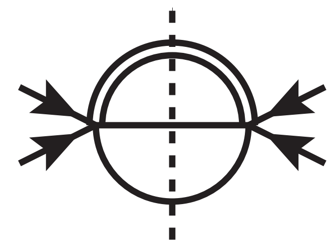

$$= -\frac{1-\epsilon}{2}$$

,

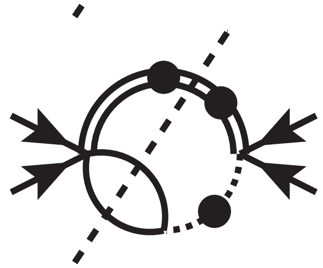
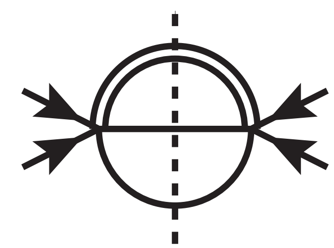

$$= \frac{(1-\epsilon)(2-\epsilon)(3-2\epsilon)}{4(5-4\epsilon)}$$


NNLO example

$$\int d\Phi_3 = \bar{z}^{3-4\epsilon}\Big[$$  $$-\,\bar{z}$$  $$+\,\bar{z}^2$$  $$+\,\mathcal{O}(\bar{z}^3)\Big]$$

- Using IBP identities:

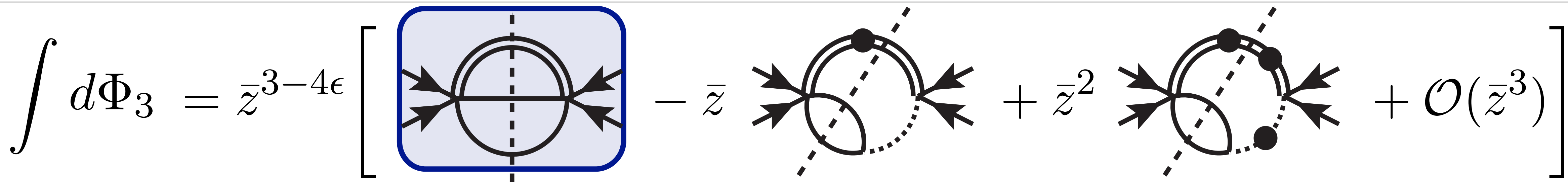
 $$= -\frac{1-\epsilon}{2}$$  ,

 $$= \frac{(1-\epsilon)(2-\epsilon)(3-2\epsilon)}{4(5-4\epsilon)}$$ 

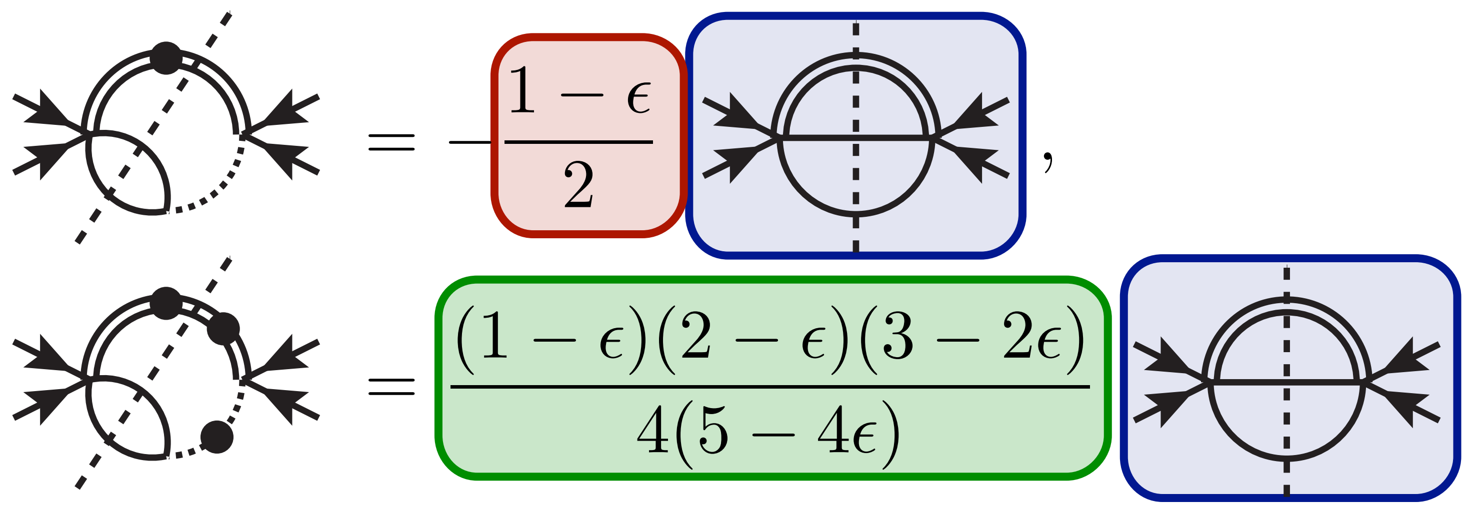
- To be compared with exact result:

$$\bar{z}^{3-4\epsilon}\,\Phi_3^S(\epsilon)\left[1+\frac{1-\epsilon}{2}\,\bar{z}+\frac{(1-\epsilon)(2-\epsilon)(3-2\epsilon)}{4(5-4\epsilon)}\,\bar{z}^2+\mathcal{O}(\bar{z}^3)\right]$$

NNLO example



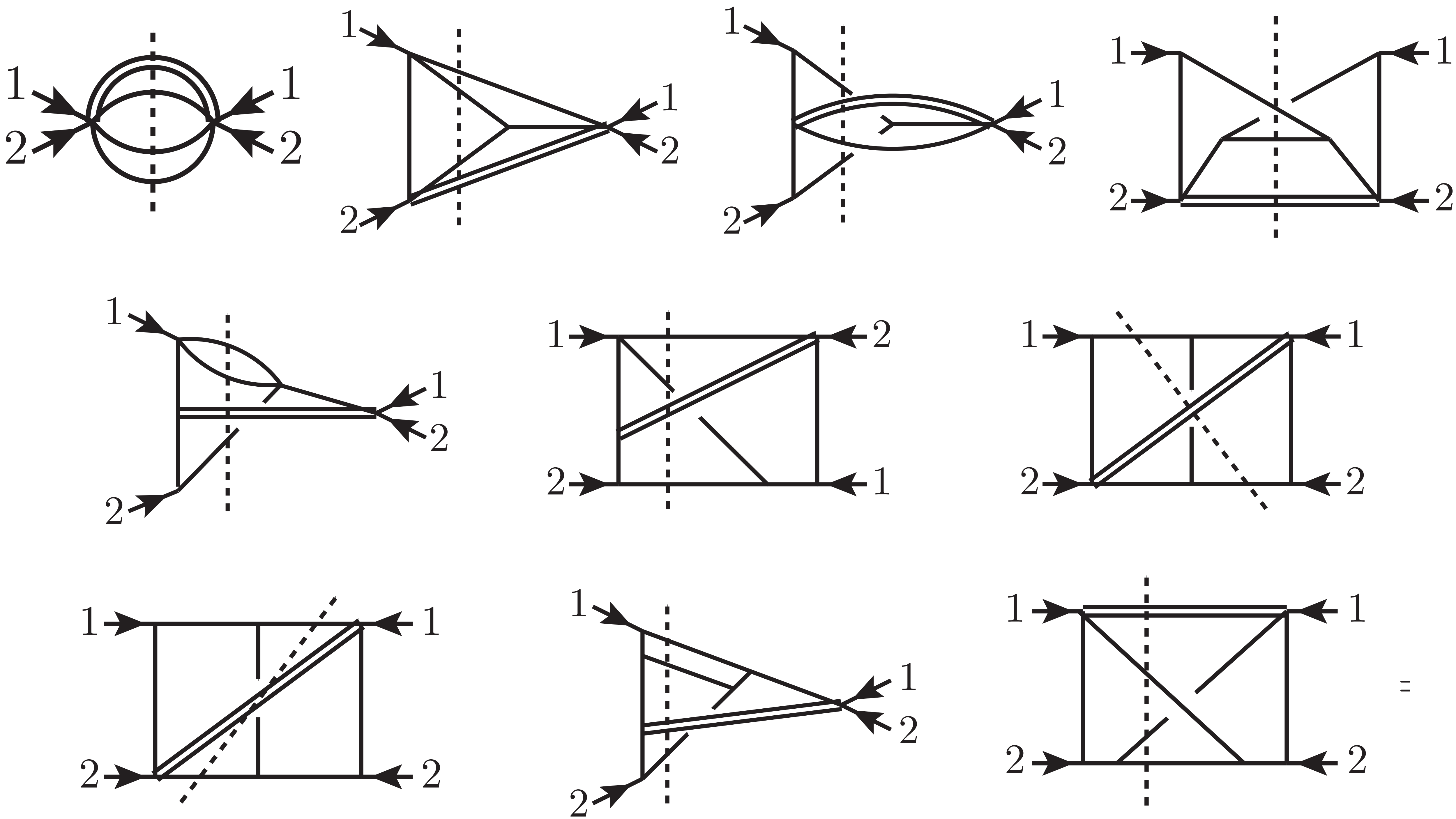
- Using IBP identities:



- To be compared with exact result:

$$\bar{z}^{3-4\epsilon}\,\Phi_3^S(\epsilon)\left[1 + \frac{1-\epsilon}{2}\,\bar{z} + \frac{(1-\epsilon)(2-\epsilon)(3-2\epsilon)}{4(5-4\epsilon)}\,\bar{z}^2 + \mathcal{O}(\bar{z}^3)\right]$$

Soft triple real emissions



[Anastasiou, Dulat, CD, Mistlberger]

Soft triple real emissions

- We were able to compute all the master integrals analytically.
- General strategy:
  - ➡ There is a canonical way to turn each of these integrals into a Mellin-Barnes integral.

$$\mathcal{F}_6(\epsilon) = \frac{\Gamma(6-6\epsilon)}{\epsilon\,\Gamma(1-6\epsilon)\Gamma(1-\epsilon)^2}\int_{-i\infty}^{+i\infty}\frac{dz_1dz_2}{(2\pi i)^2}\,\Gamma\left(-z_1\right)\Gamma\left(z_1+1\right)\Gamma\left(-z_2\right)\Gamma\left(z_2+1\right)$$
$$\times\,\frac{\Gamma\left(-\epsilon+z_1-z_2\right)\Gamma\left(z_2-\epsilon\right)\Gamma\left(-2\epsilon-z_1+z_2\right)\Gamma\left(-\epsilon-z_1+z_2\right)}{\Gamma\left(-\epsilon+z_2+1\right)\Gamma\left(-2\epsilon-z_1+z_2+1\right)}\,.$$

  - ➡ All of the integrals can be computed as a Laurent series in dimensional regularization.
  - ➡ One of the integrals required use of symbols and coproducts

Soft triple real emissions

$$\mathcal{F}_6(\epsilon) = \frac{10}{\epsilon^5} - \frac{137}{\epsilon^4} + \frac{1}{\epsilon^3}\Big(40\,\zeta_2 + 675\Big) + \frac{1}{\epsilon^2}\Big(320\,\zeta_3 - 548\,\zeta_2 - 1530\Big)$$
$$+ \frac{1}{\epsilon}\Big(1500\,\zeta_4 - 4384\,\zeta_3 + 2700\,\zeta_2 + 1620\Big) + 5160\,\zeta_5 + 320\,\zeta_2\,\zeta_3 - 20550\,\zeta_4$$
$$+ 21600\,\zeta_3 - 6120\,\zeta_2 - 648 + \epsilon\Big(18340\,\zeta_6 + 1280\,\zeta_3^2 - 70692\,\zeta_5 - 4384\,\zeta_2\,\zeta_3$$
$$+ 101250\,\zeta_4 - 48960\,\zeta_3 + 6480\,\zeta_2\Big) + \mathcal{O}(\epsilon^2)\,.$$

- Intriguing observation:
  - ➡ All
    all

- How can we be sure that we have obtained the correct results..?

Soft triple real emissions

- We can compute the Mellin-Barnes integrals numerically and compare to our analytic results.

Soft triple real emissions

- We can compute the Mellin-Barnes integrals numerically and compare to our analytic results.
- The integrals in four dimensions are related to the integrals in six dimensions:

$$\mathcal{F}_6(D+2)\mathcal{R} = \frac{(4256 - 6684D + 4224D^2 - 1345D^3 + 216D^4 - 14D^5)}{3(D-4)^2(D-3)(D-2)^2} + \frac{(D-4)(3D-10)}{9(D-2)^2(3D-7)}\mathcal{F}_2(D) - \frac{(D-4)^3}{24(D-2)(3D-11)(3D-7)}\mathcal{F}_6(D)\,,$$

  ➡ Similar to dimensional shift identities for loops. [Tarasov]

- The integrals are finite in six dimensions.

  ➡ Strong constraint!

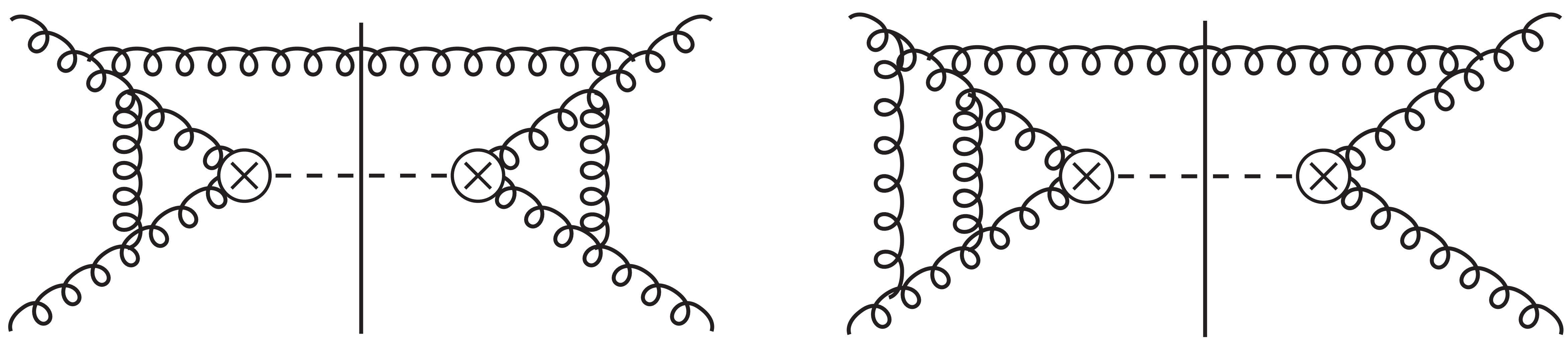
Soft triple real cross sections

- The integrals immediately allow us to write down the first two terms in the soft expansion of the cross section, e.g.,

$$
\begin{aligned}
\sigma^{S(0)}_{g\,g\to H+g\,q\,\bar q} &= \frac{2^5}{3^7}\frac{1}{8(N_c^2-1)^2}(4\pi\alpha_S)^3\Phi_4^S(\epsilon)C_AC_Fc_H^2N_f \\
\times\Bigg\{ & \frac{153090}{\epsilon^4}-\frac{1604043}{\epsilon^3}+\frac{1}{\epsilon^2}\Big(-29160\zeta_2+4903902\Big) \\
&+\frac{1}{\epsilon}\Big(-204120\zeta_3+321732\zeta_2-4833675\Big)-874800\zeta_4+2252124\zeta_3-911088\zeta_2 \\
&+203535+\epsilon\Big(-2711880\zeta_5-233280\zeta_2\zeta_3+9651960\zeta_4-6290136\zeta_3-492210\zeta_2 \\
&+1667109\Big)+\epsilon^2\Big(-9360360\zeta_6-816480\zeta_3^2+29921076\zeta_5+2573856\zeta_2\zeta_3 \\
&-26589060\zeta_4-4323186\zeta_3+4693212\zeta_2+1294731\Big) \\
&+2C_AC_F\Bigg[\frac{167670}{\epsilon^4}-\frac{1743039}{\epsilon^3}+\frac{1}{\epsilon^2}\Big(-29160\zeta_2+5267592\Big)+\frac{1}{\epsilon}\Big(-204120\zeta_3 \\
&+321732\zeta_2-5183163\Big)-874800\zeta_4+2252124\zeta_3-911088\zeta_2+337959 \\
&+\epsilon\Big(-2711880\zeta_5-233280\zeta_2\zeta_3+9651960\zeta_4-6290136\zeta_3-492210\zeta_2+1651749\Big) \\
&+\epsilon^2\Big(-9360360\zeta_6-816480\zeta_3^2+29921076\zeta_5+2573856\zeta_2\zeta_3-26589060\zeta_4 \\
&-4323186\zeta_3+4693212\zeta_2+1284491\Big)\Bigg]+\mathcal{O}(\epsilon^3)\Bigg\}.
\end{aligned}
$$

[Anastasiou, Dulat, CD, Mistlberger]

Soft double-virtual-real emissions



Double-virtual-real emission

- Subprocesses:

$$\underbrace{g\,g \to H\,g}_{\text{Soft-virtual}} \qquad \underbrace{g\,q \to H\,q}_{\text{Next-to-soft-virtual}} \qquad q\,\bar{q} \to H\,g$$

- The phase space is trivial (2-body phase space).

$$|\mathcal{M}^{(L)}(g\,g \to H\,g)\rangle = \sum_{k=0}^{L} \varepsilon^{\mu}\, J_{\mu}^{a(k)}\, |\mathcal{M}^{(L-k)}(g\,g \to H)\rangle$$

Double-virtual-real emission

- Subprocesses:

$$g\,g \to H\,g \qquad g\,q \to H\,q \qquad q\,\bar{q} \to H\,g$$

$\underbrace{}$ Soft-virtual

$\underbrace{}$ Next-to-soft-virtual

- The phase space is trivial (2-body phase space).
- If the final-state gluon is soft
  into the soft current:

$$|\mathcal{M}^{(L)}(g\,g \to H\,g)\rangle = \sum_{k=0}^{L} \varepsilon^{\mu}\, J_{\mu}^{a(k)}\, |\mathcal{M}^{(L-k)}(g\,g \to H)\rangle$$

Double-virtual-real emission

- Subprocesses:

$$g\,g \to H\,g \qquad\qquad g\,q \to H\,q \qquad\qquad q\,\bar{q} \to H\,g$$

Soft-virtual (for $g\,g \to H\,g$) — Next-to-soft-virtual (for $g\,q \to H\,q$)

- The phase space is trivial (2-body phase space).
- If the final-state gluon is soft
  into the soft current:

$$|\mathcal{M}^{(L)}(g\,g \to H\,g)\rangle = \sum_{k=0}^{L} \varepsilon^{\mu}\,J_{\mu}^{a(k)}\,|\mathcal{M}^{(L-k)}(g\,g \to H)\rangle$$

- The soft current had previously been computed
  - ➡ at one loop. [Catani, Grazzini
  - ➡ at two loops, up to finite terms. [Badger, Glover
- At N3LO, we need higher-order terms at two loops.

The two-loop soft current

- Two parallel computations of these higher-order terms (for the interference with the Born soft current):
  - ➡ Two-loop Wilson line computation up to weight 6. [Li, Zhu
  - ➡ Extraction from the two-loop matrix element for $\gamma^* \to q\,\bar{q}\,g$ to all orders in epsilon. [CD, Gehrmann

The two-loop soft current

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- Method:
  integrals as an expansion in the soft limit while keeping the coefficients exact in epsilon:

$$F_i(y,z;\epsilon) = \sum_{m,n=0}^{2} y^{-m\epsilon}\, z^{-n\epsilon}\, f_{i,mn}(y,z;\epsilon)\,,$$

$$f_{i,mn}(y,z;\epsilon) = \sum_{k=r_y}^{\infty} \sum_{l=r_z}^{\infty} c_{i,mn}^{kl}(\epsilon)\, y^k\, z^l$$

The two-loop soft current

- The two-loop soft current to all orders in dimensional regularization

$$r^{(2)}_{soft} = N\,N_f\,R_1(\epsilon) + N^2\,R_2(\epsilon)\,,$$

$$R_1(\epsilon) = \frac{2\,\Gamma(-2\epsilon)}{(1+\epsilon)\,\Gamma(4-2\epsilon)}\,\frac{\Gamma(1-2\epsilon)^2\,\Gamma(1+2\epsilon)^2}{\Gamma(1-\epsilon)^2\Gamma(1+\epsilon)^2}\left[3\,\frac{\Gamma(1-\epsilon)\Gamma(1-2\epsilon)}{\Gamma(1-3\epsilon)} - \frac{(1+\epsilon^3)}{\epsilon^2\,(1+\epsilon)}\,\frac{\Gamma(1-2\epsilon)^2}{\Gamma(1-4\epsilon)}\right],$$

$$\begin{aligned} R_2(\epsilon) = {} & \frac{\Gamma(1-2\epsilon)^3\Gamma(1+2\epsilon)^2}{6\,\epsilon^4\,\Gamma(1-\epsilon)\Gamma(1+\epsilon)^2\Gamma(1-3\epsilon)}\Bigg\{(1+4\epsilon)\,{}_4F_3(1,1,1-\epsilon,-4\epsilon;2,1-3\epsilon,1-2\epsilon;1) \\ & - 6\epsilon\left[\psi(1-3\epsilon)+\psi(1-2\epsilon)-\psi(1-\epsilon)-\psi(1+\epsilon)\right] + \frac{(14\epsilon^3+4\epsilon^2+5\epsilon-3)}{2(1+\epsilon)(3-2\epsilon)(1-2\epsilon)}\Bigg\} \\ & + \frac{(1+4\epsilon)}{3\,\epsilon^4\,(1+2\epsilon)}\,\frac{\Gamma(1-2\epsilon)^4\Gamma(1+2\epsilon)^2}{\Gamma(1-\epsilon)^2\Gamma(1+\epsilon)^2\Gamma(1-4\epsilon)}\Bigg\{2\,{}_3F_2(1,-2\epsilon,2\epsilon+1;1-\epsilon,2\epsilon+2;1) \\ & - \frac{\Gamma(1+\epsilon)\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\,{}_3F_2(-2\epsilon,\epsilon+1,2\epsilon+1;1-\epsilon,2\epsilon+2;1) + \frac{(1+2\epsilon)\,(6\epsilon^4+13\epsilon^3-16\epsilon^2-38\epsilon+3)}{4(1+4\epsilon)(1+\epsilon)(3-2\epsilon)(1-2\epsilon)}\Bigg\} \end{aligned}$$

- The soft-virtual RVV contribution to Higgs@N3LO is easily obtained from this by multiplying by the phase space.

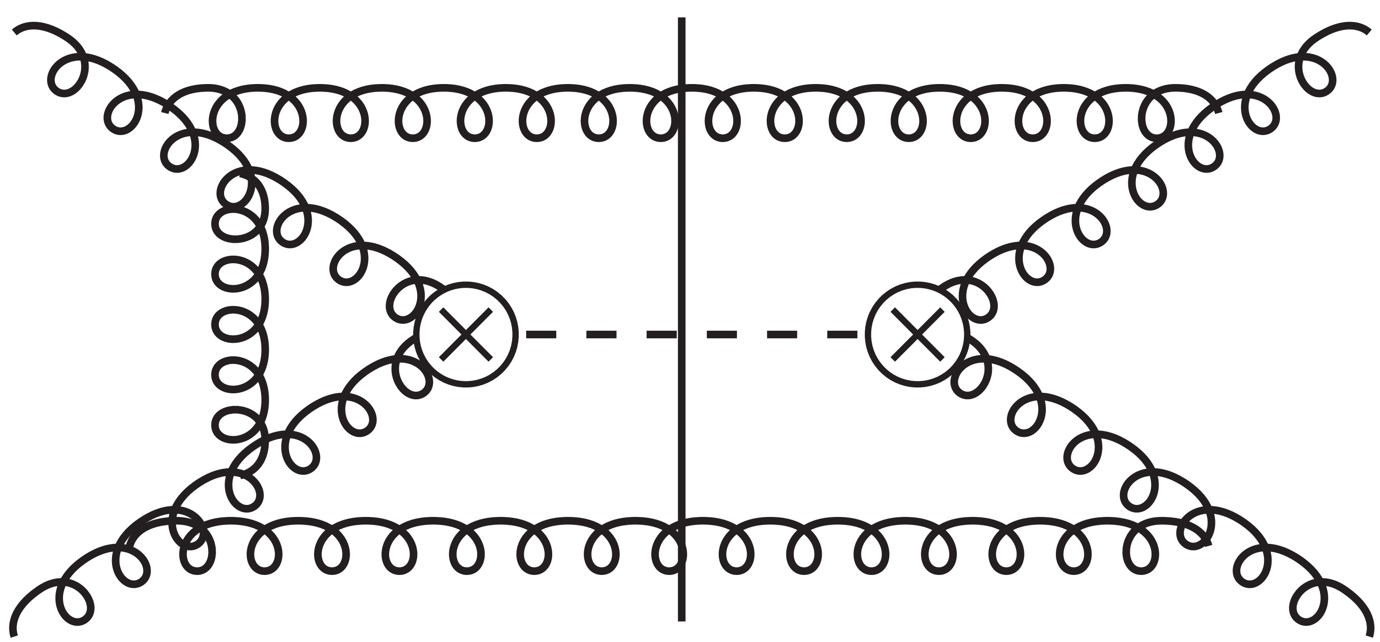
Real-virtual squared

- Contribution from one-loop-squared can easily be computed exactly.
- We did the computation in four different ways:
  - ➡ Threshold expansion by expanding hypergeometric functions.
  - ➡ Threshold expansion from expansion by regions.
  - ➡ Reverse-unitarity and differential equations.
  - ➡ Direct integration of the matrix element over phase space.

[Anastasiou, CD, Dulat, Herzog, Mistlberger]

- Confirmed by independent computation. [Kilgore]
- Full two-loop matrix element is also known. [Glover, Gehrmann, Jaquier, Koukoutsakis
  - ➡ Can be done in the same way, but need two-loop collinear counterterms.

Soft virtual-double-real emissions



Virtual-double-real emission

- Subprocesses:

$g\,g \to H\,g\,g$	$g\,q \to H\,g\,q$	$q\,\bar{q} \to H\,g\,g$	$q\,q \to H\,q\,q$
$g\,g \to H\,q\,\bar{q}$		$q\,\bar{q} \to H\,q\,\bar{q}$	$q\,Q \to H\,q\,Q$
		$q\,\bar{q} \to H\,Q\,\bar{Q}$	$q\,\bar{Q} \to H\,q\,\bar{Q}$

Virtual-double-real emission

- Subprocesses:

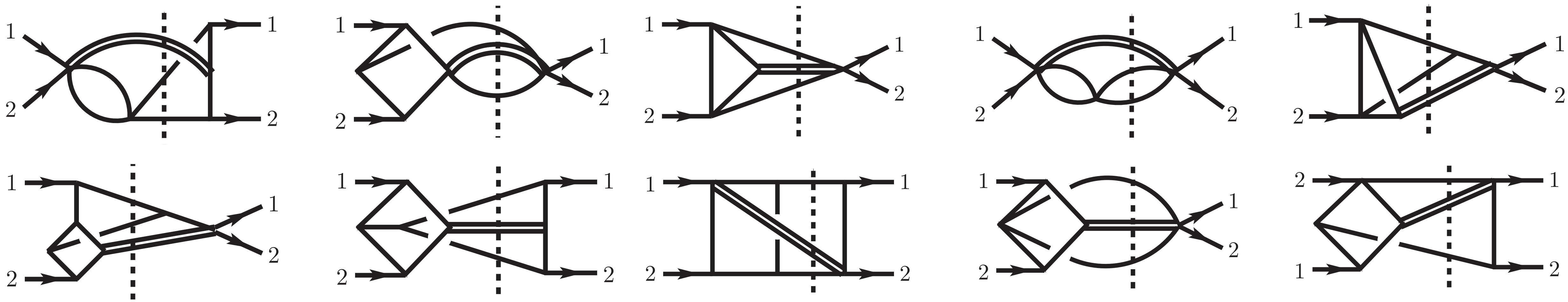
$$g\,g \to H\,g\,g \qquad g\,q \to H\,g\,q \qquad q\,\bar{q} \to H\,g\,g \qquad q\,q \to H\,q\,q$$

$$g\,g \to H\,q\,\bar{q} \qquad q\,\bar{q} \to H\,q\,\bar{q} \qquad q\,Q \to H\,q\,Q$$

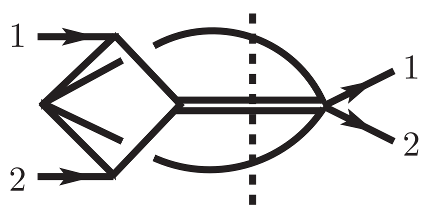
$$q\,\bar{q} \to H\,Q\,\bar{Q} \qquad q\,\bar{Q} \to H\,q\,\bar{Q}$$

- The soft-virtual term receives contributions from two regions:
  - ➡ The virtual gluon is hard.
  - ➡ The virtual gluon is soft.
- The hard region is trivial (tree-level emission of two soft gluons).
- The soft region can be dealt with in a way similar to the soft triple real emission.
  - ➡ IBP reduction in soft limit and soft master integrals.

VRR soft master integrals



- All integrals can be computed by combining the soft expansion for the virtuals with the phase space techniques developed for the triple real emission. [Anastasiou, CD, Dulat, Furlan, Herzog, Mistlberger



$$= -\frac{4\Gamma(4-4\epsilon)\Gamma(1-3\epsilon)}{\epsilon(1+\epsilon)(1-2\epsilon)\Gamma(3-6\epsilon)\Gamma(1-\epsilon)}\,{}_3F_2(1,1,1-\epsilon;2-3\epsilon,2+\epsilon;1)$$

$$= -\frac{12\zeta_2}{\epsilon} - 8\zeta_2 - 36\zeta_3 + (-112\zeta_2 - 24\zeta_3 + 33\zeta_4)\,\epsilon + (720\zeta_3\zeta_2 - 672\zeta_2$$

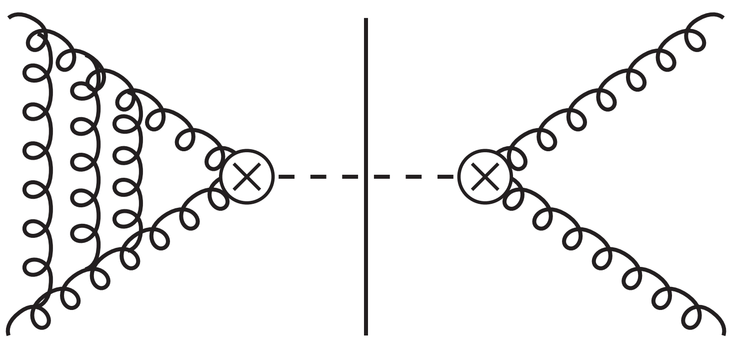
$$-336\zeta_3 + 22\zeta_4 - 450\zeta_5)\,\epsilon^2 + \big(1512\zeta_3^2 + 480\zeta_2\zeta_3 - 2016\zeta_3 - 4032\zeta_2 + 308\zeta_4$$

$$-300\zeta_5 + \frac{16881}{4}\zeta_6\Big)\,\epsilon^3 + \mathcal{O}(\epsilon^4)\,.$$

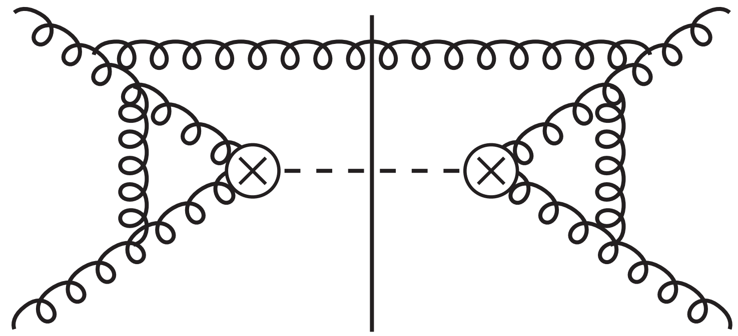
- Results recently confirmed by independent computation using Wilson lines. [Li, von Manteuffel, Schabinger, Zhu

The gluon-fusion cross section in the soft limit

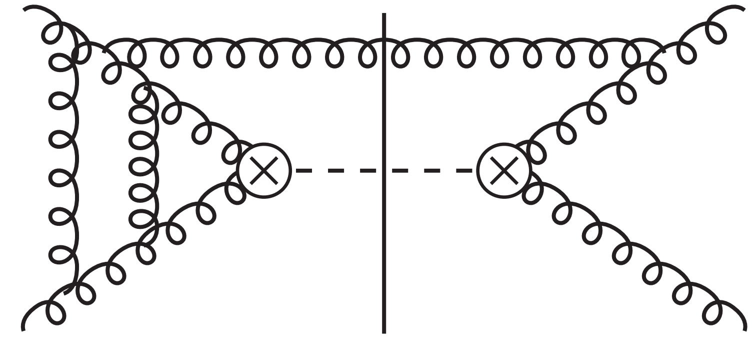
N3LO status: soft-virtual



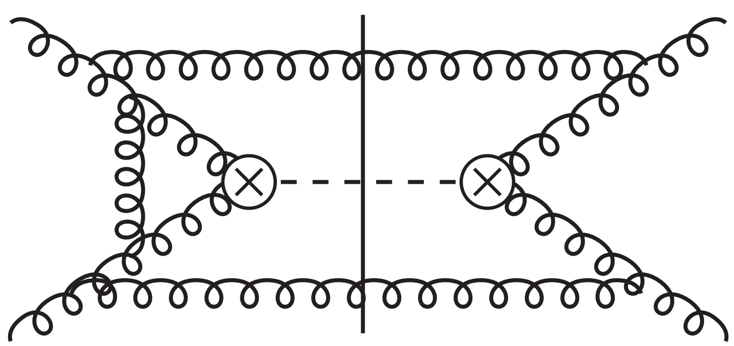
✓ Triple virtual



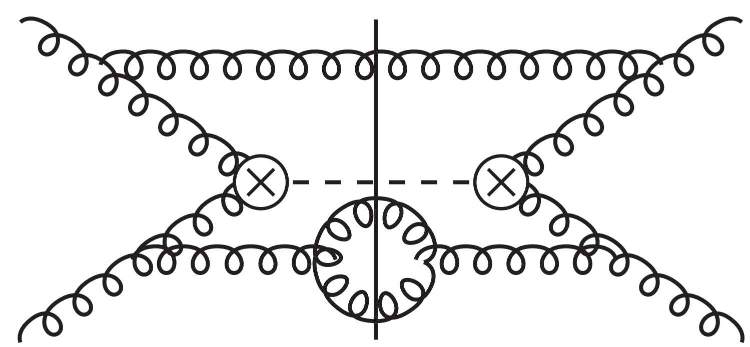
✓ Real-virtual squared



✓ Double virtual real



✓ Double real virtual



✓ Triple real

✓ +

The soft-virtual approximation

- The

$$\hat{\sigma}(z) = \boxed{\sigma_{-1}} + \sigma_0 + (1-z)\,\sigma_1 + \mathcal{O}(1-z)^2$$

- The soft-virtual term receives contributions from a 'pole' at $z \sim 1$:

$$(1-z)^{-1+n\epsilon} = \frac{\delta(1-z)}{n\,\epsilon} + \left[\frac{1}{1-z}\right]_+ + n\epsilon\left[\frac{\log(1-z)}{1-z}\right]_+ + \mathcal{O}(\epsilon^2)$$

- Plus-distribution terms already known. [Moch, Vogt

- Complete three-loop corrections are contained the delta function term.
  - ➡ The soft-virtual term contains the complete three-loop corrections plus the correction from the emission of up to three soft gluons.

The soft-virtual approximation

- At NLO and NNLO, the soft-virtual term reads ($\mu_R = \mu_F = m_H$)

$$\hat{\sigma}^{SV}_{gg}(z) = \frac{\pi\, C(\mu^2)^2}{v^2\,(N^2-1)^2} \sum_{k=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^k \hat{\eta}^{(k)}(z)$$

$$\hat{\eta}^{(0)}(z) = \delta(1-z) \qquad \hat{\eta}^{(1)}(z) = 2\,C_A\,\zeta_2\,\delta(1-z) + 4\,C_A \left[\frac{\log(1-z)}{1-z}\right]_+$$

$$\hat{\eta}^{(2)}(z) = \delta(1-z)\left\{C_A^2\left(\frac{67}{18}\zeta_2 - \frac{55}{12}\zeta_3 - \frac{1}{8}\zeta_4 + \frac{93}{16}\right) + N_F\left[C_F\left(\zeta_3 - \frac{67}{48}\right) - C_A\left(\frac{5}{9}\zeta_2 + \frac{1}{6}\zeta_3 + \frac{5}{3}\right)\right]\right\}$$
$$+ \left[\frac{1}{1-z}\right]_+ \left[C_A^2\left(\frac{11}{3}\zeta_2 + \frac{39}{2}\zeta_3 - \frac{101}{27}\right) + N_F\,C_A\left(\frac{14}{27} - \frac{2}{3}\zeta_2\right)\right]$$
$$+ \left[\frac{\log(1-z)}{1-z}\right]_+ \left[C_A^2\left(\frac{67}{9} - 10\,\zeta_2\right) - \frac{10}{9}\,C_A\,N_F\right]$$
$$+ \left[\frac{\log^2(1-z)}{1-z}\right]_+ \left(\frac{2}{3}\,C_A\,N_F - \frac{11}{3}\,C_A^2\right) + \left[\frac{\log^3(1-z)}{1-z}\right]_+ 8\,C_A^2\,.$$

The soft-virtual approximation

- All the integrals can be computed analytically!
  - ➡ 22 three-loop. [Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser; Gehrmann, Glover, Huber, Ikizlerli, Studerus]
  - ➡ 3 double-virtual-real. [CD, Gehrmann; Li, Zhu]
  - ➡ 7 real-virtual-squared. [Anastasiou, CD, Dulat, Herzog, Mistlberger; Kilgore
  - ➡ 10 virtual-double-real. [Anastasiou, CD, Dulat, Furlan, Herzog, Mistlberger; Li, von Manteuffel, Schabinger, Zhu
  - ➡ 8 triple real. [Anastasiou, CD, Dulat, Mistlberger
- In addition, one needs:
  - ➡ three-loop splitting functions. [Moch, Vogt, Vermaseren
  - ➡ three-loop beta function. [Tarasov, Vladimirov, Zharkov; Larin, Vermaseren
  - ➡ three-loop Wilson coefficient. [Chetyrkin, Kniehl, Steinhauser; Schroder, Steinhauser; Chetyrkin, Kuhn, Sturm

Higgs soft-virtual @ N3LO

$$
\begin{aligned}
\hat{\eta}^{(3)}(z) = \delta(1-z)\Bigg\{& C_A^3\left(-\frac{2003}{48}\zeta_6+\frac{413}{6}\zeta_3^2-\frac{7579}{144}\zeta_5+\frac{979}{24}\zeta_2\zeta_3-\frac{15257}{864}\zeta_4-\frac{819}{16}\zeta_3+\frac{16151}{1296}\zeta_2+\frac{215131}{5184}\right)\\
&+N_F\Bigg[C_A^2\left(\frac{869}{72}\zeta_5-\frac{125}{12}\zeta_3\zeta_2+\frac{2629}{432}\zeta_4+\frac{1231}{216}\zeta_3-\frac{70}{81}\zeta_2-\frac{98059}{5184}\right)\\
&\quad+C_A C_F\left(\frac{5}{2}\zeta_5+3\zeta_3\zeta_2+\frac{11}{72}\zeta_4+\frac{13}{2}\zeta_3-\frac{71}{36}\zeta_2-\frac{63991}{5184}\right)+C_F^2\left(-5\zeta_5+\frac{37}{12}\zeta_3+\frac{19}{18}\right)\Bigg]\\
&+N_F^2\left[C_A\left(-\frac{19}{36}\zeta_4+\frac{43}{108}\zeta_3-\frac{133}{324}\zeta_2+\frac{2515}{1728}\right)+C_F\left(-\frac{1}{36}\zeta_4-\frac{7}{6}\zeta_3-\frac{23}{72}\zeta_2+\frac{4481}{2592}\right)\right]\Bigg\}\\
+\left[\frac{1}{1-z}\right]_+\Bigg\{& C_A^3\left(186\zeta_5-\frac{725}{6}\zeta_3\zeta_2+\frac{253}{24}\zeta_4+\frac{8941}{108}\zeta_3+\frac{8563}{324}\zeta_2-\frac{297029}{23328}\right)+N_F^2 C_A\left(\frac{5}{27}\zeta_3+\frac{10}{27}\zeta_2-\frac{58}{729}\right)\\
&+N_F\left[C_A^2\left(-\frac{17}{12}\zeta_4-\frac{475}{36}\zeta_3-\frac{2173}{324}\zeta_2+\frac{31313}{11664}\right)+C_A C_F\left(-\frac{1}{2}\zeta_4-\frac{19}{18}\zeta_3-\frac{1}{2}\zeta_2+\frac{1711}{864}\right)\right]\Bigg\}\\
+\left[\frac{\log(1-z)}{1-z}\right]_+\Bigg\{& C_A^3\left(-77\zeta_4-\frac{352}{3}\zeta_3-\frac{152}{3}\zeta_2+\frac{30569}{648}\right)+N_F^2 C_A\left(-\frac{4}{9}\zeta_2+\frac{25}{81}\right)\\
&+N_F\left[C_A^2\left(\frac{46}{3}\zeta_3+\frac{94}{9}\zeta_2-\frac{4211}{324}\right)+C_A C_F\left(6\zeta_3-\frac{63}{8}\right)\right]\Bigg\}\\
+\left[\frac{\log^2(1-z)}{1-z}\right]_+\Bigg\{& C_A^3\left(181\zeta_3+\frac{187}{3}\zeta_2-\frac{1051}{27}\right)+N_F\left[C_A^2\left(-\frac{34}{3}\zeta_2+\frac{457}{54}\right)+\frac{1}{2}C_A C_F\right]-\frac{10}{27}N_F^2 C_A\Bigg\}\\
+\left[\frac{\log^3(1-z)}{1-z}\right]_+\Bigg\{& C_A^3\left(-56\zeta_2+\frac{925}{27}\right)-\frac{164}{27}N_F C_A^2+\frac{4}{27}N_F^2 C_A\Bigg\}\\
+\left[\frac{\log^4(1-z)}{1-z}\right]_+& \left(\frac{20}{9}N_F C_A^2-\frac{110}{9}C_A^3\right)+\left[\frac{\log^5(1-z)}{1-z}\right]_+ 8C_A^3\,.
\end{aligned}
$$

[Anastasiou, CD, Dulat, Furlan, Gehrmann, Herzog, Mistlberger

Conclusion & Outlook

- We have completed the first computation of the Higgs boson cross section at N3LO in the soft-virtual approximation.
- ‘Amplitude-technology’ played a crucial role in the computation of the integrals
- More terms in the expansion/full computation in progress.
- Soft term already allows to extract interesting results:
  - ➡ Extension to Drell-Yan @ N3LO in the soft limit.
    [Ahmed, Mahakhud, Mathews, Rana, Ravindran
  - ➡ Extension to rapidity distribution @ N3LO in soft limit.
    [Ahmed, Mandal, Rana, Ravindran
  - ➡ Soft-gluon resummation at N3LL.
    [Bonvini, Marzani; Catani, Cieri, de Florian, Ferrara, Grazzini

Home Speakers and Participants Schedule Registration Travel Info Contact

Conference “Amplitudes 2015”

Dates & Venue

15 – 19 June 2015

ETH/Uni Zürich
Zurich, Switzerland

Organisers

- Babis Anastasiou (ETH Zürich)
- Niklas Beisert (ETH Zürich)
- Johannes Brödel (ETH Zürich)
- Thomas Gehrmann (Universität Zürich)
- Claude Duhr (Durham University)

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- Swiss National Science Foundation
- ETH Zürich
- Pauli Center for Theoretical Studies

Back up

Higgs soft-virtual @ N3LO

- How can we be sure that we got it right?
  - ➡ Plus-distribution terms agree with Moch & Vogt.
  - ➡ All master integrals were computed analytically and cross checked numerically.
  - ➡ Independent computations of matrix elements and integrals.
    - All but the triple-real contribution have been confirmed by other groups.

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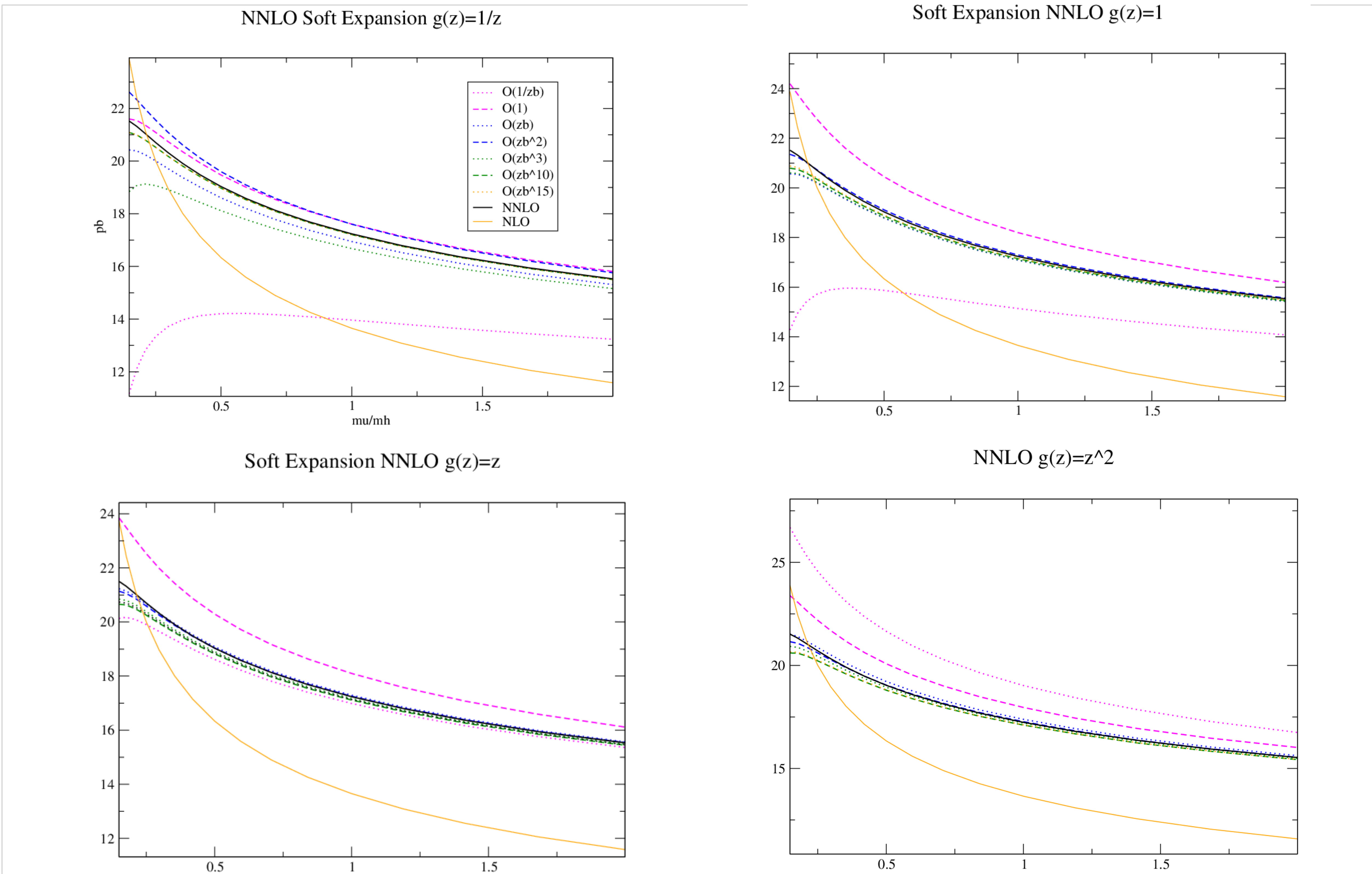
- Caveat emptor!

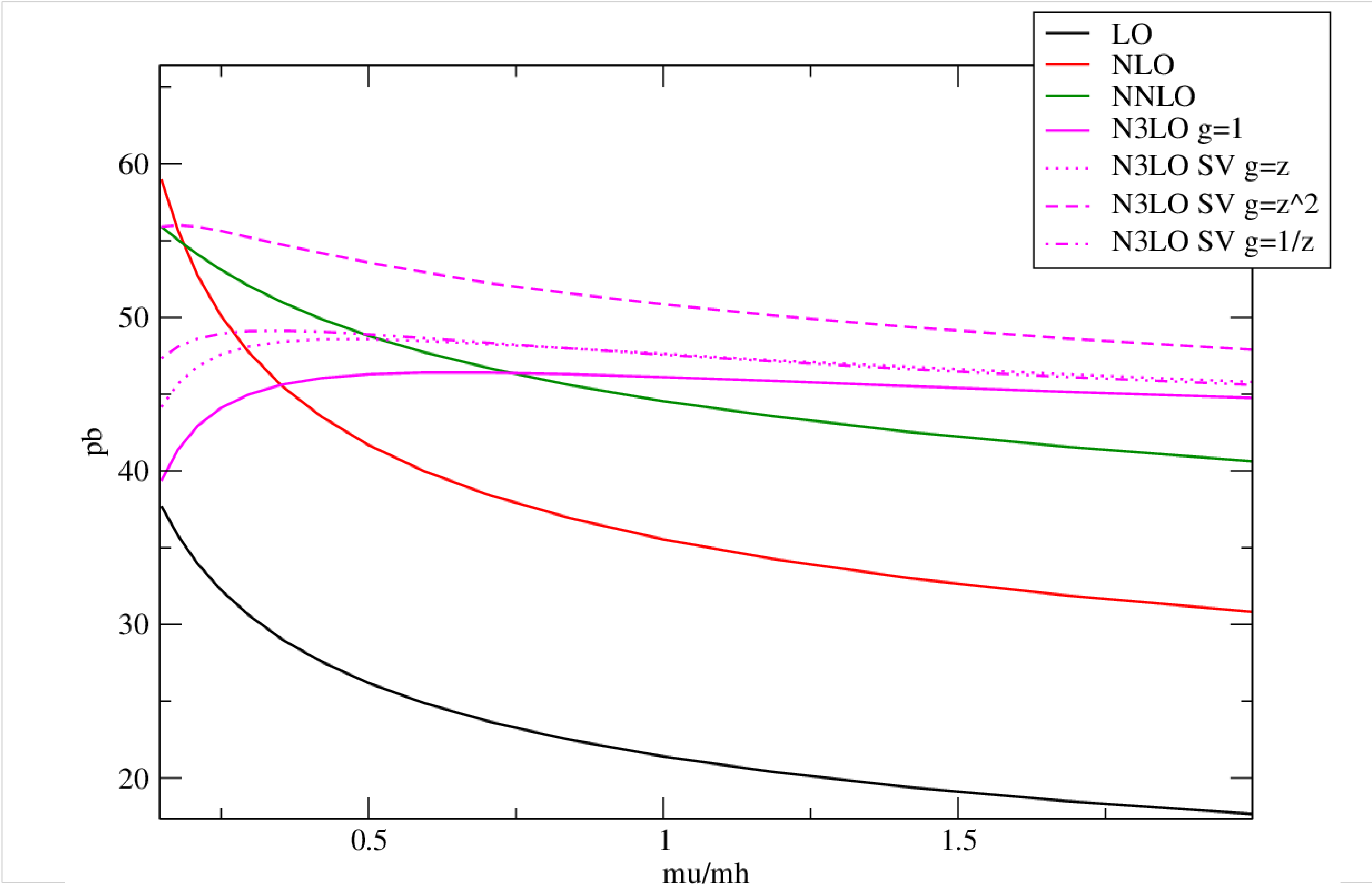
$$\int dx_1\, dx_2\, [f_i(x_1)\, f_j(x_2) z g(z)] \left[\frac{\hat{\sigma}_{ij}(s,z)}{z g(z)}\right]_{\text{threshold}} \qquad \lim_{z\to 1} g(z) = 1$$

  - ➡ Formally all these choices are equivalent!

Higgs soft-virtual @ N3LO



Higgs soft-virtual @ N3LO



Higgs soft-virtual @ N3LO