Progress in 3D Higher Spin Gravity

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Work with Ammon, Gutperle, Hijano, Perlmutter, Ugajin

Introduction

- Higher spin gravity is an (apparently) consistent theory that sits "midway" between low energy field theory and string theory (Fronsdal, Fradkin, Vasiliev, …)
  - infinite towers of fields
  - huge enlargement of gauge symmetry
  - nonlocal dynamics

- Extra symmetry provides soluble examples of AdS/CFT correspondence (Klebanov/Polyakov; Sezgin/Sundell Gaberdiel/Gopakumar; …)
  - can we gain insight into the big problems of quantum gravity?

Vasiliev equations

$$dW = W \wedge \star W$$

$$dB = W \star B - B \star W$$

$$dS_\alpha = W \star S_\alpha - S_\alpha \star W$$

$$S_\alpha \star S^\alpha = -2i(1 + B \star K)$$

$$S_\alpha \star B = B \star S_\alpha$$

- Quantities are functions of spacetime coordinates and auxiliary spinors y and z

W = metric and higher spin gauge fields
B = propagating scalar matter field
S = auxiliary field

- equations give self-consistent interacting generalization of sensible linearized theory

- Fairly good understanding of:
  - asymptotic symmetries of AdS solutions
  - classical solutions involving higher spin fields: AdS, black holes, RG flows, solitonic “defects”
  - linearized fluctuations of matter and higher spin fields around these backgrounds
- Primitive understanding of:
  - solutions including backreaction (in principle, described by Vasiliev equations)
  - quantum effects (lack of action and nonlocal equations must be confronted)

Duality

- Gaberdiel and Gopakumar conjecture duality between bulk HS theory and W_N minimal model CFT

$$\frac{SU(N)_k \oplus SU(N)_1}{SU(N)_{k+1}}$$ CFT has W_N symmetry

- central charge: $c = (N-1)\frac{k(2N+1+k)}{(N+k)(N+k+1)}$
- Need a large c limit to compare with classical bulk
- Option 1: ‘t Hooft limit: $k, N \to \infty\ , \quad \lambda = \frac{N}{k+N}$ fixed

$c \sim N(1-\lambda^2)$ puzzling appearance of light states

- Option 2: $k \to -(N+1) \quad N$ fixed

non-unitary (Perlmutter, Prochazka, Raeymaekers)

More on CFT

- Primary operators are labeled by two SL(N) irreps

(Λ_+, Λ_-) known formula for conformal dimensions

- focus on two special cases, with k=-(N+1)

$$(\Lambda, 0) \quad \Rightarrow \quad h_{(\Lambda,0)} = -\Lambda \cdot \rho = -\sum_{j=1}^{N} \frac{j(N-j)}{2}\Lambda_j$$ light

$$(0, \Lambda) \quad \Rightarrow h_{(0,\Lambda)} = -\frac{c}{N(N^2-1)} C_2(\Lambda)$$ heavy

- CFT correlation functions of these operators can all be computed in principle
- In ‘t Hooft limit have: $h_{(f,0)} = \frac{1+\lambda}{2}$

HS sector

- Upon setting $S_\alpha = \rho\tilde{z}_\alpha, \quad B = 2\lambda - 1$ the Vasiliev system reduces to CS theory based on gauge group hs[λ] x hs[λ]

hs[λ] is natural continuation of SL(N)

Get generalization of SL(2) x SL(2) formulation of 3D AdS gravity (Achucarro/Townsend ; Witten)

$$A = (\omega^a + \tfrac{1}{l}e^a)J_a\ , \quad \overline{A} = (\omega^a - \tfrac{1}{l}e^a)J_a$$

$$R_{\mu\nu} = \tfrac{1}{l^2}g_{\mu\nu} \iff \begin{aligned} dA + A\wedge A &= 0 \\ d\overline{A} + \overline{A}\wedge\overline{A} &= 0 \end{aligned}$$

- asymptotic symmetry: $W_\infty(\lambda)$ (Henneaux, Rey; Campoleoni et. al.; Hartman, Gaberdiel)

- Ordinary 3D gravity is a consistent subsector
  - example: SL(3) describes ordinary gravity coupled to a massless spin-3 field (Campoleoni et. al.)

$$g_{\mu\nu} \sim \mathrm{Tr}(\mathrm{e}_\mu \mathrm{e}_\nu) \ , \qquad \varphi_{\alpha\beta\gamma} \sim \mathrm{Tr}(\mathrm{e}_\alpha \mathrm{e}_\beta \mathrm{e}_\gamma)$$

$$e \sim A - \overline{A}$$

Gauge symmetry includes coord. transformations under which $g_{\mu\nu}$ and $\varphi_{\alpha\beta\gamma}$ transform as tensors, as well as spin-3 gauge transformations under which $g_{\mu\nu}$ transforms in novel way

e.g. Ricci scalar not gauge invariant

Matter in 3D HS gravity

- We study linearized scalar matter via equation

$$dC + AC - C\overline{A} = 0$$

  - When background is AdS, components of C obey the Klein-Gordon equation with a mass fixed by the gauge group and representation. With higher spin fields present, find higher order wave equation

- use gauge invariance to generate solutions:
  - Set $A = \overline{A} = 0$ and let C = c = constant
  - transform to $A = g^{-1}dg, \quad \overline{A} = \overline{g}^{-1}d\overline{g}, \quad C = g^{-1}c\overline{g}$
  - physical scalar: $\Phi = \mathrm{Tr}\ C$

Conical defects

(Castro, Gopakumar, Gutperle, Raeymaekers)

- Now consider CFT primaries (0,Λ) with

$$h_{(0,\Lambda)} = -\frac{c}{N(N^2-1)} C_2(\Lambda) \qquad k = -(N+1)$$

- Since these are heavy, natural to relate them to classical solutions in the bulk
- Consider pure SL(N,C) CS theory (Euc. signature), and look for smooth solutions that are topologically equivalent to global AdS
- Since angular circle in AdS is contractible, demand trivial gauge holonomy around this cycle

- Demand angular holonomy is in center:

$$H = e^{\oint A} \in Z_N$$

- For constant (in ϕ) connections, this imposes conditions on eigenvalues of $\omega = \oint A$

$$\mathrm{eig}(\omega) = 2\pi i (n_1, n_2, \ldots, n_N)$$

$$n_j = r_j + \tfrac{N+1}{2} - \tfrac{B}{N} - j \qquad B = \textstyle\sum_j r_j$$

- Integers r_j can be identified with Young tableau data

$$r_j \;=\; \text{number of boxes in jth row}$$

- Energy of these solutions matches dimension of CFT primary (0,Λ) in this rep.

Correlation functions in bulk

- Bulk and boundary share same symmetry, so pure current correlators will match
- Next step is to consider three-point functions

$$\langle \mathcal{O}\mathcal{O}J^{(s)} \rangle$$

  - also determined by symmetry in terms two-point function, but it is very instructive to verify agreement
- In bulk, we can either compute linearized gauge field sourced by scalar at quadratic order, or solve for scalar in the presence of gauge field.
- We use second approach (Ammon, PK, Perlmutter)

- Need to solve $dC + AC - C\overline{A} = 0$ where gauge fields go to delta function on the boundary
- Start from same C=c as gave scalar bulk-bndy propagator in pure AdS
- Find gauge transformation that generates gauge field with prescribed boundary conditions
- Compute correlator via

$$\langle \mathcal{O}_\pm(z_1)\overline{\mathcal{O}}_\pm(z_2)J^{(s)}(z_3)\rangle = \lim_{\rho=\infty} e^{2h\rho}\mathrm{Tr}\left[g^{-1}c\overline{g}\right]$$

$$= \frac{(-1)^{s-1}}{2\pi}\frac{\Gamma(s)^2}{\Gamma(2s-1)}\frac{\Gamma(s\pm\lambda)}{\Gamma(1\pm\lambda)}\left(\frac{z_{12}}{z_{13}z_{23}}\right)^s\langle \mathcal{O}_\pm(z_1)\overline{\mathcal{O}}_\pm(z_2)\rangle$$

$\mathcal{O}_- \sim$ standard quantization

$\mathcal{O}_+ \sim$ alternate quantization

(Ammon, P.K., Perlmutter)

matches CFT

Four-point functions (Hijano, PK, Perlmutter)

- CFT correlators of four primaries can be computed, e.g. via Coulomb gas formalism (Papadodimas, Raju; Chang, Yin; Hijano, PK, Perlmutter)
- At present, we know how to compute one (large) class of bulk four-point functions

$$\langle \mathcal{O}_{\Lambda_+}(z_1)\overline{\mathcal{O}}_{\Lambda_+}(z_2)D_{\Lambda_-}(z_3)\overline{D}_{\Lambda_-}(z_4)\rangle$$

$$\mathcal{O}_{\Lambda_+} \sim (\Lambda_+, 0) \sim \text{perturbative scalar}$$

$$D_{\Lambda_-} \sim (0, \Lambda_-) \sim \text{defect}$$

- This correlator is found by computing scalar two-point function in defect background
  - We employ our usual gauge transformation method, which just involves matrix manipulation

- Example: defining rep scalar, arbitrary defect

$$\langle \mathcal{O}_{\Lambda_+}(z)\overline{\mathcal{O}}_{\Lambda_+}(0)D_{\Lambda_-}(-\infty)\overline{D}_{\Lambda_-}(\infty)\rangle = \left|\sum_{j=1}^{N}\frac{e^{-in_j z}}{\prod_{l\neq j}(n_l-n_j)}\right|^2$$

  - this is result on cylinder
  - n_j fixed by Young tableau data of defect rep.

- General CFT result expressed in terms of hypergeometric functions and sum over conformal blocks
- Remarkably, for k=-(N+1) these collapse down to the expression above
- Partial results available for other scalar reps

Black Holes

- Black hole solutions carrying higher spin charge have been constructed (Gutperle, P.K; P.K., Perlmutter, …)

- Solutions contribute to generalized partition functions $Z(\beta, \mu_i) = \mathrm{Tr}\left[e^{-\beta(H+\sum_i \mu_i Q_i)}\right]$

- Partition function extracted from bulk matches that of dual CFT in high temperature regime

  (P.K., Perlmutter; Gaberdiel, Hartman, Jin)

- Main subtlety involves interpretation of spacetime solution. Non-obvious causal structure due to higher spin gauge invariance

Building HS Black Holes

- BTZ:

$$A = (e^{\rho}L_1 - \tfrac{2\pi}{k}e^{-\rho}\mathcal{L}L_{-1})dx^+ + L_0 d\rho$$
$$\overline{A} = -(e^{\rho}L_{-1} - \tfrac{2\pi}{k}e^{-\rho}\overline{\mathcal{L}}L_1)dx^- - L_0 d\rho$$

$\Rightarrow$ BTZ $\quad \mathcal{L} = \frac{M-J}{4\pi} \quad \overline{\mathcal{L}} = \frac{M+J}{4\pi}$

- Now add in spin-3 chemical potential. Ward identity analysis establishes:

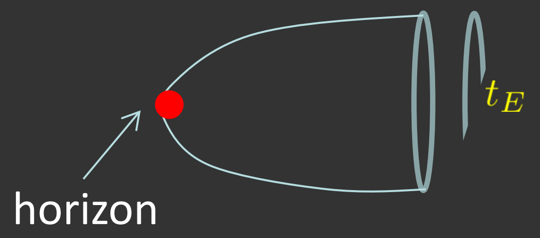
$$A_- \sim \mu e^{2\rho}W_2 + \cdots$$

chiral spin-3 chemical potential → μ; spin-3 generator → W_2

- Expect this to induce spin-3 charge: $A_+ \sim e^{-2\rho}\mathcal{W}W_{-2}$
  - In hs(λ) case expect infinite number of charges to be induced, due to nonlinear symmetry algebra

Smoothness conditions

- ordinary gravity: relation between (M,Q) and their conjugate potentials (T,μ) fixed by smoothness at Euclidean horizon



- Inapplicable for HS black holes, since even existence of event horizon is a gauge dependent statement.
- Instead: demand trivial holonomy around thermal circle

Thermodynamics

$$Z(\tau,\alpha) = \mathrm{Tr}\left[e^{4\pi^2 i(\tau\mathcal{L}+\alpha\mathcal{W})}\right]$$

- holonomy condition implies integrability:

$$\frac{\partial\mathcal{L}}{\partial\alpha} = \frac{\partial\mathcal{W}}{\partial\tau}$$

$$\ln Z(\tau,\alpha) = \frac{i\pi k}{2\tau}\left[1 - \frac{4}{3}\frac{\alpha^2}{\tau^4} + \frac{400}{27}\frac{\lambda^2-7}{\lambda^2-4}\frac{\alpha^4}{\tau^8} - \frac{1600}{27}\frac{5\lambda^4-85\lambda^2+377}{(\lambda^2-4)^2}\frac{\alpha^6}{\tau^{12}} + \cdots\right]$$

valid for: $\tau \to 0, \quad \alpha \to 0, \quad \frac{\alpha}{\tau^2} \quad \text{fixed}$

- first few terms found to agree with CFT computation based on modular properties of partition function

(Gaberdiel, Hartman, Jin)

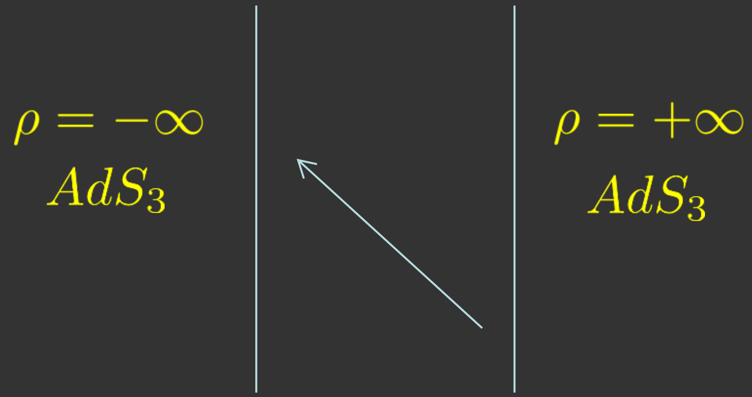
Causal structure

- Metric for non-rotating case takes form

$$ds^2 = d\rho^2 - F(\rho)dt^2 + G(\rho)d\phi^2$$

$F(\rho),\ G(\rho)\ >\ 0$ no event horizon!

traversable wormhole:

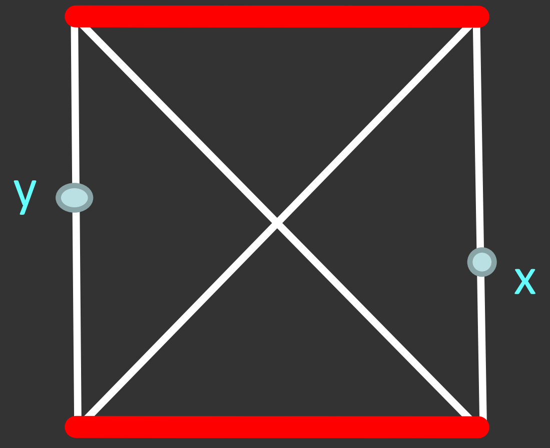


- But when holonomy conditions are obeyed, one can find a true black hole metric somewhere on this gauge orbit (Ammon, Gutperle, P.K., Perlmutter)

Probing causual structure

- Our "black hole" metrics either look like traversable wormholes or black holes, depending on choice of gauge

- To map out physical causal structure we can compute AdS/CFT two-point functions of probe scalars, and look for lightcone singularities



- Black hole causal structure: $G(x,y)$ nonsingular

propagator for HS black hole

(PK, Perlmutter)

- Black holes of hs(λ) theory only known perturbatively in hs chemical potential
- Similarly, propagator must be worked out perturbatively. Divergences would indicate change in causal structure

first order correction:

$$\frac{i\alpha e^{\rho/2}}{16\tau^2}\Big[\cosh^2(2\overline{Z})\left(-4(Z+\overline{Z})(\cosh(4Z)-2)-\sinh(4Z)\right)$$
$$+4e^{2\rho}\tau\overline{\tau}\sinh(4\overline{Z})\left(-4(Z+\overline{Z})\sinh(4Z)+2(\cosh(4Z)-1)\right)$$
$$-(4e^{2\rho}\tau\overline{\tau})^2\sinh^2(2\overline{Z})\left(4(Z+\overline{Z})(\cosh(4Z)+2)-3\sinh(4Z)\right)\Big]$$
$$\times\left(\cosh(2Z)\cosh(2\overline{Z})+4e^{2\rho}\tau\overline{\tau}\sinh(2Z)\sinh(2\overline{Z})\right)^{-5/2},$$

$$Z=\frac{iz}{4\tau}$$

- only singularities are on light cone for single sided correlator.

General entropy formula

- By adapting the conical singularity approach to CS theory one finds

$$S = 2\pi i k \mathrm{Tr}\left[\mathrm{A_z}(\tau \mathrm{A_z} + \overline{\tau} \mathrm{A_{\overline{z}}}\right]$$

(P.K., Ugajin;
See also de Boer, Jottar)

Easily seen to exhibit correct first law variation

- Closest we have to area law type expression, but note explicit dependence on tau
- Other authors have obtained different results for entropy (Campoleoni et. All; Perez, Troncoso, Tempo; de Boer, Jottar)
  - disagreement with CFT, so violates 2^{nd} law?

Generalized Entropy

- Can consider entropy of more generalized density matrices (see Lewkowycz, Maldacena)

$$Z = \mathrm{Tr}[Pe^{-\int H(t)dt}]$$

- Example in HS gravity involves varying spin-3 chemical potential. Described by black hole with inhomogeneous horizon
- Can again show agreement between bulk and CFT

$$Z \sim Z_0 + \frac{1}{\tau^5} \int d\phi \left(\alpha^2 + (\partial\alpha)^2 + (\partial^2\alpha)^2\right) + \ldots$$

(work in progress)

Conclusion

- Steady progress in understanding 3D HS gravity, and AdS/CFT duality

- Many open questions:
  - Better understanding of scalar interactions
  - Quantum corrections
  - Light states and phase structure
  - subleading corrections to black hole entropy