General Rotating Black Holes, Untwisted

(Extracting Conformal Symmetry from General Black)

Mirjam Cvetič

Progress to extract from geometry (mesoscopic approach)
an underlying conformal symmetry &
promoting it to two-dimensional conformal field theory
→
governing microscopic structure of four and five dimensional
asymptotically flat non-extreme rotating charged black holes

Recent efforts: w/ Finn Larsen 1106.3341 & 1112.4856
w/ Gary Gibbons 1201.0601
w/Monica Guica & Zain Saleem 1302.7032

[Earlier work: w/ Donam Youm '94-'96:multi-charged rotating asympt.Mink. BH's
w/ Finn Larsen '97-'99,'10: greybody factors; special(BPS) microsc.
w/ Chong, Lü & Pope '06-'08: (AdS) rotating black hole solutions
w/ Chow, Lü & Pope '09: special (Kerr/CFT) microscopics
w/Gibbons & Pope'11;w/Lü & Pope'13: products of horizon entropies]

Key Issue in Black Hole Physics:

How to relate

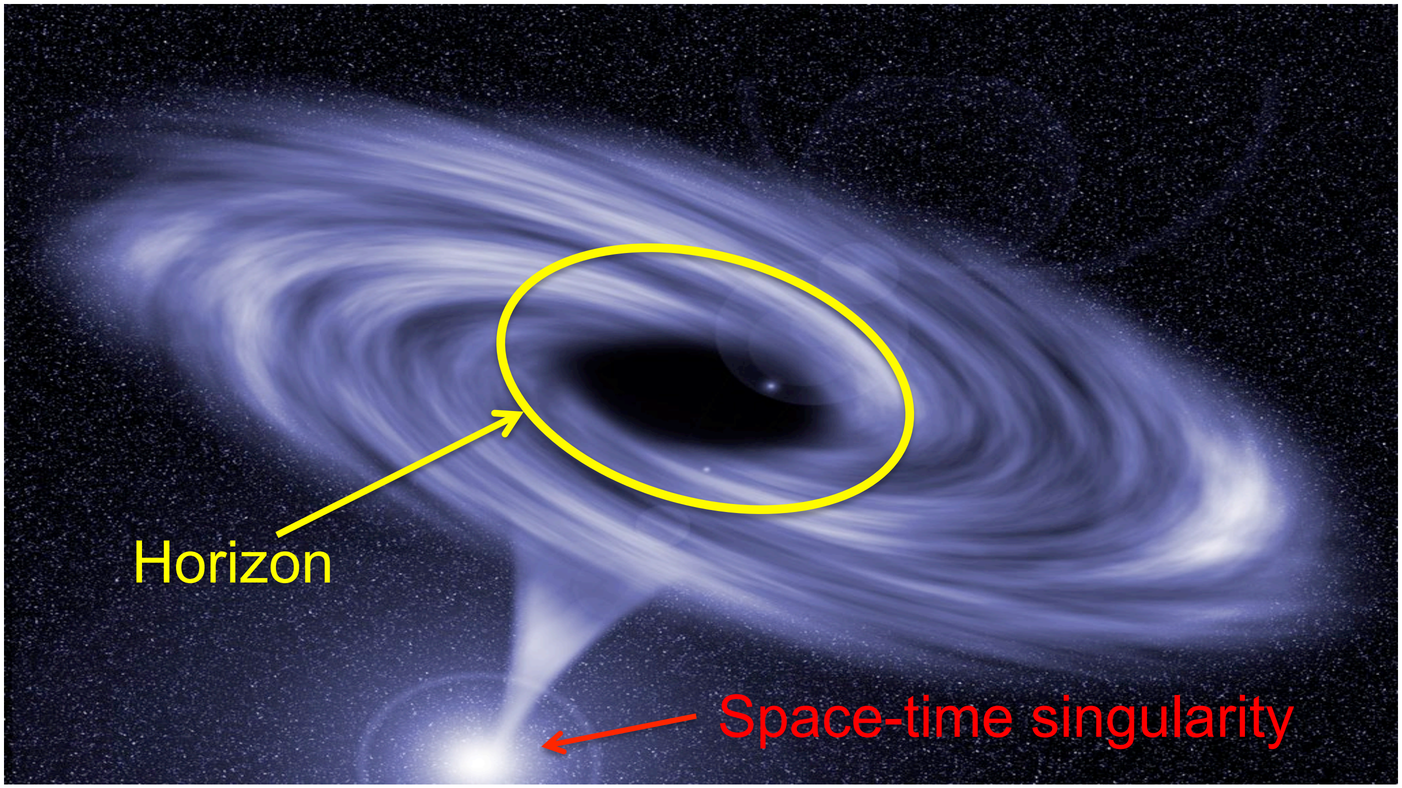
Bekenstein-Hawking - thermodynamic entropy: $S_{thermo}=\frac{1}{4} A_{hor}$

(A_{hor}= area of the black hole horizon; $c=\hbar=1$)

to

Statistical entropy: $S_{stat} = \log N_i$?

Where do black hole microscopic degrees N_i come from?



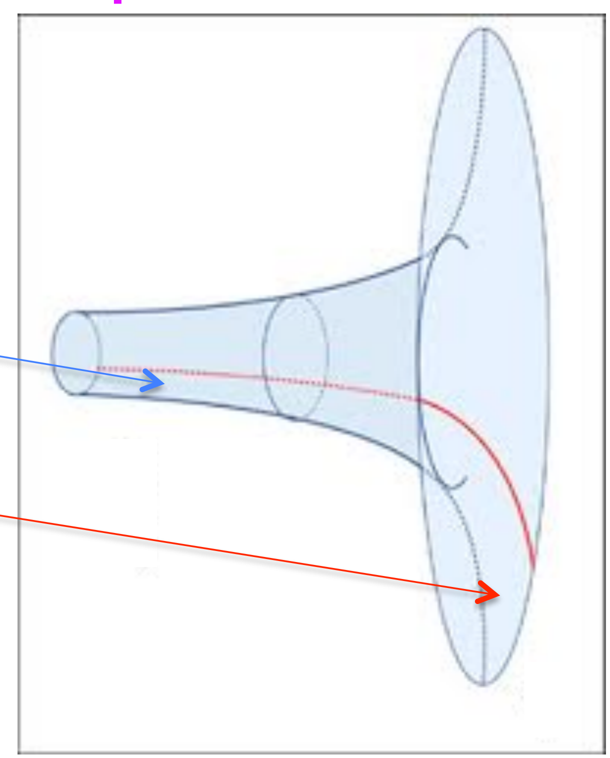
Microscopic origin of entropy for supersymmetric (BPS) multi-charged black holes w/(schematic)

$$M = Q_i + P_i$$

M-mass, Q_i-electric charges, P_i-magnetic charges

Systematic study of microscopic degrees quantified via:
AdS/CFT (Gravity/Field Theory) correspondence

[A string theory on a
specific Curved Space-Time (in D-dimensions)
related to
specific Field Theory (in (D-1)- dimensions)
on its boundary]



Maldacena'97

Specific microscopic studies of black holes in string theory, in particular relation to 2d-dim CFT via AdS_3/CFT_2 correspondence extensively explored:

- BPS (supersymmetric) limit $(m \rightarrow 0)$ [M=Q]

Strominger &Vafa'96

- near-BPS limit $(m << 1)$ . . .Maldacena &Strominger'97

- near-BPS multi-charged rotating black holes M.C. & Larsen'98

Further developments:

- (near-)extreme rotating black holes $(m-l<<l)$

  Kerr/CFT correspondence Guica,Hartman,Song & Strominger 0809.4266…

- extreme AdS charged rotating black holes in diverse dim.

. . . . .M.C., Chow, Lü & Pope 0812.2918

Another approach: internal structure of black holes
via probes such as scalar wave equation
in the black hole background (greybody factors)

If certain terms in the wave equation omitted →
$SL(2,R)^2$ symmetry & radial solution hypergeometric functions

Omission justified for special backgrounds:
- near-BPS limit $(m<<1)$ Maldacena & Strominger'97
- near-extreme Kerr limit $(m-1<<1)$ M.C. & Larsen'97
- low-energy probes $(\omega<<1)$ Das & Mathur'96…

Also super-radiant limit $(\omega-n\Omega<<1)$
D=4 Kerr Bredberg,Hartman,Song & Strominger 0907.3477
D=4,5 multi-charged rotating M.C. & Larsen 0908.1136

On the other hand for general (nonextreme) black hole backgrounds there is NO $SL(2,R)^2$ symmetry

This would seem to doom a CFT interpret. of the general BH's

Related proposal dubbed "hidden conformal symmetry"

Castro, Maloney &Strominger 1004.0096

asserts conformal symmetry suggested by certain terms of the massless wave equation is there, just that it is spontaneously broken (w/ $\omega \to 0$ restoring it)

Extensive follow up…

In this talk a different perspective:

Program to quantify ``conventional wisdom'' that also non-extreme (asymptotically flat) black holes might have microscopic explanation in terms of 2D CFT

M.C. & Larsen '97-'99

But such black holes have typically negative specific heat $c_p < 0$ due to the coupling between the internal structure of the black hole and modes that escape to infinity

Should focus on the black hole "by itself" → one must necessarily enclose the black hole in a box, thus creating an equilibrium system

[Must be taken into account in any precise discussion of black hole microscopics]

→ The box leads to a ``mildly'' modified geometry, dubbed

Subtracted Geometry

The rest of the talk:

I. Quantify subtracted geometry of a black hole in a box
- via conformal symmetry of a wave equation
- its lift on $S^1 \rightarrow AdS_3 \times$ Sphere

M.C.& Larsen 1106.3341 & 1112.4856

II. Identify gauge and scalar field sources supporting subtracted geometry:
- as a scaling limit of certain BH's
- as an ``infinite boost'' Harrison transf. on the original BH

M.C. & Gibbons 1201.0601

III. Interpolating geometries between the original black hole and their subtracted geometries:
- via solution generating technics (reducing on time)
- in lifted geometry via T-dualities and Melvin twists

M.C., Guica & Saleem 1302.7032

For the case study we choose: most general black holes of D=5 N=4 (or N=8) un-gauged supergravity, actually its generating solution

N=4 (N=8) supersymmetric ungauged SG in D=5 can be obtained as a toroidal reduction of Heterotic String (Type IIA String) on $T^{(10-D)}$ (D=5).
The relevant subsector for generating solutions can also be viewed as D=5 N=2 SG coupled to three vector super-multiplets:

$$e^{-1}\mathcal{L} = R - \tfrac{1}{2}\partial\vec{\varphi}^2 - \tfrac{1}{4}\sum_{i=1}^{3} X_i^{-2}(F^i)^2 \cdot + \tfrac{1}{24}|\epsilon_{ijk}|\,\epsilon^{\mu\nu\rho\sigma\lambda} F^i_{\mu\nu} F^j_{\rho\sigma} A^k_\lambda$$

$$X_1 = e^{-\frac{1}{\sqrt{6}}\varphi_1 - \frac{1}{\sqrt{2}}\varphi_2}, \qquad X_2 = e^{-\frac{1}{\sqrt{6}}\varphi_1 + \frac{1}{\sqrt{2}}\varphi_2}, \qquad X_3 = e^{\frac{2}{\sqrt{6}}\varphi_1}$$

Gravity with two scalar fields & three U(1)-gauge fields
[special case: when U(1) gauge fields identified→ Maxwell-Einstein Theory in D=5]

Such three charge rotating solutions were obtained by employing solution generating techniques c.f., Ehlers,… Gibbons, Sen

a) Reduce D=5 stationary solution-
Kerr BH (with mass m and two angular momenta l_1 and l_2)
to D=3 on time-like and one angular Killing vectors

b) D=3 Largrangian has O(3,3) symmetry

c) Acting with an $SO(1,1)^3$ subgroup of O(3,3) transformations
[preserving asymptotic Minkowski space-time]
on the dimensionally reduced solution to generate new solutions with three parameters δ_i

$$H = \begin{pmatrix} \cosh\delta_i & \sinh\delta_i \\ \sinh\delta_i & \cosh\delta_i \end{pmatrix}$$

d) Upon lifting back to D = 5, arrive at spinning solutions
with two angular momenta & three charges parameterised by the three δ_i

M.C. & Youm hep-th/9603100

D=5 Kerr Solution:

$$
\begin{aligned}
ds^2 = &-\frac{r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta-2m}{r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta}dt^2+\frac{r^2(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta)}{(r^2+l_1^2)(r^2+l_2^2)-2mr^2}dr^2\\
&+(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta)d\theta^2+\frac{4ml_1l_2\sin^2\theta\cos^2\theta}{r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta}d\phi d\psi\\
&+\frac{\sin^2\theta}{r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta}[(r^2+l_1^2)(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta)+2ml_1^2\sin^2\theta]d\phi^2\\
&+\frac{\cos^2\theta}{r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta}[(r^2+l_2^2)(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta)+2ml_2^2\cos^2\theta]d\psi^2\\
&-\frac{4ml_1\sin^2\theta}{r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta}dtd\phi-\frac{4ml_2\cos^2\theta}{r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta}dtd\psi.
\end{aligned}
$$

m - mass; l_{12} - two angular momenta

Myers&Perry'86

Metric:

$$
\begin{aligned}
ds_E^2 = \bar{\Delta}^{\frac{1}{3}} \Bigg[& -\frac{(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta)(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta - 2m)}{\bar{\Delta}} dt^2 \\
& + \frac{r^2}{(r^2 + l_1^2)(r^2 + l_2^2) - 2mr^2} dr^2 + d\theta^2 + \frac{4m\cos^2\theta\sin^2\theta}{\bar{\Delta}} [l_1 l_2 \{(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta) \\
& - 2m(\sinh^2\delta_{e1}\sinh^2\delta_{e2} + \sinh^2\delta_e\sinh^2\delta_{e1} + \sinh^2\delta_e\sinh^2\delta_{e2})\} + 2m\{(l_1^2 + l_2^2) \\
& \times \cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e - 2l_1 l_2\sinh^2\delta_{e1}\sinh^2\delta_{e2}\sinh^2\delta_e\}] d\phi d\psi \\
& - \frac{4m\sin^2\theta}{\bar{\Delta}} [(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta)(l_1\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e - l_2\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e) \\
& + 2ml_2\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e] d\phi dt - \frac{4m\cos^2\theta}{\bar{\Delta}} [(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta) \\
& \times (l_2\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e - l_1\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e) + 2ml_1\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e] d\psi d \\
& + \frac{\sin^2\theta}{\bar{\Delta}} [(r^2 + 2m\sinh^2\delta_e + l_1^2)(r^2 + 2m\sinh^2\delta_{e1} + l_1^2\cos^2\theta + l_2^2\sin^2\theta)(r^2 + 2m\sinh^2\delta_e \\
& + l_1^2\cos^2\theta + l_2^2\sin^2\theta) + 2m\sin^2\theta\{(l_1^2\cosh^2\delta_m - l_2^2\sinh^2\delta_m)(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta) \\
& + 4ml_1 l_2\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e - 2m\sinh^2\delta_{e1}\sinh^2\delta_{e2} \\
& \times (l_1^2\cosh^2\delta_e + l_2^2\sinh^2\delta_e) - 2ml_2^2\sinh^2\delta_e(\sinh^2\delta_{e1} + \sinh^2\delta_{e2})\}] d\phi^2 \\
& + \frac{\cos^2\theta}{\bar{\Delta}} [(r^2 + 2m\sinh^2\delta_e + l_2^2)(r^2 + 2m\sinh^2\delta_{e1} + l_1^2\cos^2\theta + l_2^2\sin^2\theta)(r^2 + 2m\sinh^2\delta_e \\
& + l_1^2\cos^2\theta + l_2^2\sin^2\theta) + 2m\cos^2\theta\{(l_2^2\cosh^2\delta_e - l_1^2\sinh^2\delta_e)(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta) \\
& + 4ml_1 l_2\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e - 2m\sinh^2\delta_{e1}\sinh^2\delta_{e2} \\
& \times (l_1^2\sinh^2\delta_e + l_2^2\cosh^2\delta_e) - 2ml_1^2\sinh^2\delta_e(\sinh^2\delta_{e1} + \sinh^2\delta_{e2})\}] d\psi^2 \Bigg],
\end{aligned}
$$

where

$$
\begin{aligned}
\bar{\Delta} \equiv\ & (r^2 + 2m\sinh^2\delta_{e1} + l_1^2\cos^2\theta + l_2^2\sin^2\theta)(r^2 + 2m\sinh^2\delta_{e2} + l_1^2\cos^2\theta + l_2^2\sin^2\theta) \\
& \times (r^2 + 2m\sinh^2\delta_e + l_1^2\cos^2\theta + l_2^2\sin^2\theta),
\end{aligned}
$$

Scalar and gauge fields:

$$g_{11} = \frac{r^2 + 2m\mathrm{sinh}^2\delta_{e1} + l_1^2\mathrm{cos}^2\theta + l_2^2\mathrm{sin}^2\theta}{r^2 + 2m\mathrm{sinh}^2\delta_{e2} + l_1^2\mathrm{cos}^2\theta + l_2^2\mathrm{sin}^2\theta},$$

$$e^{2\varphi} = \frac{(r^2 + 2m\mathrm{sinh}^2\delta_e + l_1^2\mathrm{cos}^2\theta + l_2^2\mathrm{sin}^2\theta)^2}{(r^2 + 2m\mathrm{sinh}^2\delta_{e2} + l_1^2\mathrm{cos}^2\theta + l_2^2\mathrm{sin}^2\theta)(r^2 + 2m\mathrm{sinh}^2\delta_{e1} + l_1^2\mathrm{cos}^2\theta + l_2^2\mathrm{sin}^2\theta)},$$

$$A^{(1)}_{t\,1} = \frac{m\mathrm{cosh}\delta_{e1}\mathrm{sinh}\delta_{e1}}{r^2 + 2m\mathrm{sinh}^2\delta_{e1} + l_1^2\mathrm{cos}^2\theta + l_2^2\mathrm{sin}^2\theta},$$

$$A^{(1)}_{\phi\,1} = m\mathrm{sin}^2\theta\frac{l_1\mathrm{sinh}\delta_{e1}\mathrm{sinh}\delta_{e2}\mathrm{cosh}\delta_e - l_2\mathrm{cosh}\delta_{e1}\mathrm{cosh}\delta_{e2}\mathrm{sinh}\delta_e}{r^2 + 2m\mathrm{sinh}^2\delta_{e1} + l_1^2\mathrm{cos}^2\theta + l_2^2\mathrm{sin}^2\theta},$$

$$A^{(1)}_{\psi\,1} = m\mathrm{cos}^2\theta\frac{l_1\mathrm{cosh}\delta_{e1}\mathrm{sinh}\delta_{e2}\mathrm{sinh}\delta_e - l_2\mathrm{sinh}\delta_{e1}\mathrm{cosh}\delta_{e2}\mathrm{cosh}\delta_e}{r^2 + 2m\mathrm{sinh}^2\delta_{e1} + l_1^2\mathrm{cos}^2\theta + l_2^2\mathrm{sin}^2\theta},$$

$$A^{(2)}_{t\,1} = \frac{m\mathrm{cosh}\delta_{e2}\mathrm{sinh}\delta_{e2}}{r^2 + 2m\mathrm{sinh}^2\delta_{e2} + l_1^2\mathrm{cos}^2\theta + l_2^2\mathrm{sin}^2\theta},$$

$$A^{(2)}_{\phi\,1} = m\mathrm{sin}^2\theta\frac{l_1\mathrm{cosh}\delta_{e1}\mathrm{sinh}\delta_{e2}\mathrm{cosh}\delta_e - l_2\mathrm{sinh}\delta_{e1}\mathrm{cosh}\delta_{e2}\mathrm{sinh}\delta_e}{r^2 + 2m\mathrm{sinh}^2\delta_{e2} + l_1^2\mathrm{cos}^2\theta + l_2^2\mathrm{sin}^2\theta},$$

$$A^{(2)}_{\psi\,1} = m\mathrm{cos}^2\theta\frac{l_1\mathrm{sinh}\delta_{e1}\mathrm{cosh}\delta_{e2}\mathrm{sinh}\delta_e - l_2\mathrm{cosh}\delta_{e1}\mathrm{sinh}\delta_{e2}\mathrm{cosh}\delta_e}{r^2 + 2m\mathrm{sinh}^2\delta_{e2} + l_1^2\mathrm{cos}^2\theta + l_2^2\mathrm{sin}^2\theta},$$

$$\begin{aligned} B_{t\phi} = & -2m\sin^2\theta(l_1\sinh\delta_{e1}\sinh\delta_{e2}\cosh\delta_e - l_2\cosh\delta_{e1}\cosh\delta_{e2}\sinh\delta_e)(r^2 + l_1^2\cos^2\theta \\ & + l_2^2\sin^2\theta + m\sinh^2\delta_{e1} + m\sinh^2\delta_{e2})/[(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta + 2m\sinh^2\delta_{e1}) \\ & \times(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta + 2m\sinh^2\delta_{e2})], \end{aligned}$$

$$\begin{aligned} B_{t\psi} = & -2m\cos^2\theta(l_2\sinh\delta_{e1}\sinh\delta_{e2}\cosh\delta_e - l_1\cosh\delta_{e1}\cosh\delta_{e2}\sinh\delta_e)(r^2 + l_1^2\cos^2\theta \\ & + l_2^2\sin^2\theta + m\sinh^2\delta_{e1} + m\sinh^2\delta_{e2})/[(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta + 2m\sinh^2\delta_{e1}) \\ & \times(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta + 2m\sinh^2\delta_{e2})], \end{aligned}$$

$$B_{\phi\psi} = \frac{2m\cosh\delta_e\sinh\delta_e\cos^2\theta\sin^2\theta(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta + m\sinh^2\delta_{e1} + m\sinh^2\delta_{e2})}{(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta + 2m\sinh^2\delta_{e1})(r^2 + l_1^2\cos^2\theta + l_2^2\sin^2\theta + 2m\sinh^2\delta_{e2})},$$

Solution specified by mass, three charges and two angular momenta:

$$
\begin{aligned}
M &= 2m(\cosh^2\delta_{e1} + \cosh^2\delta_{e2} + \cosh^2\delta_e) - 3m, \\
Q_1^{(1)} &= 2m\cosh\delta_{e1}\sinh\delta_{e1}, \quad Q_1^{(2)} = 2m\cosh\delta_{e2}\sinh\delta_{e2}, \quad Q = 2m\cosh\delta_e\sinh\delta_e \\
J_\phi &= 4m(l_1\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e - l_2\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e), \\
J_\psi &= 4m(l_2\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e - l_1\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e).
\end{aligned}
$$

Inner and outer horizon:

$$r_\pm^2 = m - \frac{1}{2}l_1^2 - \frac{1}{2}l_2^2 \pm \frac{1}{2}\sqrt{(l_1^2 - l_2^2)^2 + 4m(m - l_1^2 - l_2^2)},$$

Special cases: all δ_i equal & $l_1 = l_2 = 0$ — Reissner-Nordström BH in D=5

$m \rightarrow 0$ $\delta_i \rightarrow \infty$ w/ Q_i finite — Supersymmetric (BPS) limit

$(l_1^2 - l_2^2)^2 + 4m(m - l_1^2 - l_2^2) = 0$ — Extreme -Kerr limit

We shall employ a bit more compact form w/ a warp factor Δ_o (as U(1) fibration over 4d base):

M. C., Chong, Lü & Pope: hep-th/06006213

Metric:
$$ds_5^2 = -\Delta_0^{-2/3} G(dt + \mathcal{A})^2 + \Delta_0^{1/3} ds_4^2 \ ,$$
$$ds_4^2 = \frac{dx^2}{4X} + \frac{dy^2}{4Y} + \frac{U}{G}(d\chi - \frac{Z}{U}d\sigma)^2 + \frac{XY}{U}d\sigma^2$$

$$\Delta_0 = (x+y)^3 H_1 H_2 H_3 \qquad H_i = 1 + \frac{\mu \sinh^2 \delta_i}{x+y} \ , \ (i = 1,2,3)$$

$$X = (x+a^2)(x+b^2) - \mu x \ ,$$

Two Horizons (X=0)

$l_1 \rightarrow a$, $l_2 \rightarrow b$

$m \rightarrow \mu$

$$Y = -(a^2 - y)(b^2 - y) \ ,$$
$$U = yX - xY \ ,$$
$$Z = ab(X+Y)$$
$$G = (x+y)(x+y-\mu) \ ,$$

Ergosphere G=0

$$x = r^2 \ ,$$
$$y = a^2 \cos^2\theta + b^2 \sin^2\theta$$
$$\sigma = \frac{1}{a^2 - b^2}(a\phi - b\psi)$$
$$\chi = \frac{1}{a^2 - b^2}(b\phi - a\psi)$$

$$\mathcal{A} = \frac{\mu \Pi_c}{x+y-\mu}[(a^2+b^2-y)d\sigma - abd\chi] - \frac{\mu \Pi_s}{x+y}(abd\sigma - yd\chi)$$

$$\Pi_c \equiv \prod_{i=1}^{3} = \cosh\delta_i, \quad \Pi_s \equiv \prod_{i=1}^{3} \sinh\delta_i$$

Sources:

two scalars: $X_i = H_i^{-1}(H_1 H_2 H_3)^{1/3}$ i=1,2,3 w/ $X_1X_2X_3=1$

three gauge potentials:
$$\begin{aligned} A^1 = & \frac{2m}{(x+y)H_1}\{\sinh\delta_1 \cosh\delta_1 dt \\ + & \sinh\delta_1 \cosh\delta_2 \cosh\delta_3 [ab d\chi + (y - a^2 - b^2) d\sigma] \\ + & \cosh\delta_1 \sinh\delta_2 \sinh\delta_3 (ab d\sigma - y d\chi)\} \end{aligned}$$

A^2, A^3 via cyclic permutations

Thermodynamics - Suggestive of weakly interacting 2-dim CFT w/ ``left-'' & ``right-moving'' excitations [noted already, M.C. & Youm'96]

Area of outer horizon $S_+ = S_L + S_R$

[Area of inner horizon $S_- = S_L - S_R$]

$$S_L = \pi\mu\sqrt{\mu-(b-a)^2}\,(\Pi_c+\Pi_s)$$

$$S_R = \pi\mu\sqrt{\mu-(b+a)^2}\,(\Pi_c-\Pi_s)$$

Surface gravity (inverse temperature) of

outer horizon $\beta_H = ½\,(\beta_L + \beta_R)$

[inner horizon $\beta_- = ½\,(\beta_L - \beta_R)$]

$$\beta_R = \frac{2\pi\mu}{\sqrt{\mu-(b+a)^2}}\,(\Pi_c+\Pi_s)$$

$$\beta_L = \frac{2\pi\mu}{\sqrt{\mu-(b-a)^2}}\,(\Pi_c-\Pi_s)$$

Two angular velocities:

$$\beta_H\Omega_R = \frac{2\pi(b+a)}{\sqrt{\mu-(b+a)^2}}$$

$$\beta_H\Omega_L = \frac{2\pi(b-a)}{\sqrt{\mu-(b-a)^2}}$$

$$\Pi_c \equiv \prod_{i=1}^{3}\cosh\delta_i\ , \quad \Pi_s \equiv \prod_{i=1}^{3}\sinh\delta_i$$

Shown recently, all independent of the warp factor Δ_o !

M.C.& Larsen 1106.3341

Subtracted geometry obtained by changing only the warp factor $\Delta_o \rightarrow \Delta$ such that the scalar wave eq. preserves precisely SL(2,R)^2

Wave eq. written for a metric with an implicit warp factor Δ:

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\Phi) = 0$$

Equation separable: $\Phi \sim e^{-i\omega t + im_R(\phi+\psi) + im_L(\phi-\psi)}\ \eta(x)\ \zeta(y)$

$$\Bigg[4\partial_x X\partial_x + \frac{x_+ - x_-}{x - x_+}\left(\frac{\beta_R\omega}{4\pi} - m_R\frac{\beta_H\Omega_R}{2\pi} + \frac{\beta_L\omega}{4\pi} - m_L\frac{\beta_H\Omega_L}{2\pi}\right)^2$$
$$- \frac{x_+ - x_-}{x - x_-}\left(\frac{\beta_R\omega}{4\pi} - m_R\frac{\beta_H\Omega_R}{2\pi} - \frac{\beta_L\omega}{4\pi} + m_L\frac{\beta_H\Omega_L}{2\pi}\right)^2 + \boxed{\mu\omega^2(1 + \sum_i \sinh^2\delta_i) + x\omega^2 + y\omega^2 + \frac{\Delta - \Delta_0}{G}\omega^2}\Bigg]\Phi$$
$$= j(j+2)\Phi$$

Adjust Δ to cancel → SL(2,R)^2 restored!

S^3 Laplacian eigenvalues

$$\Delta_0 = \prod_{i=1}^{3}(x + y + \mu\sinh^2\delta_i)$$

Original warp factor

$$\Delta = \mu^2\left[(x+y)(\Pi_c^2 - \Pi_s^2) + \mu\Pi_s^2\right]$$

Subtracted geometry warp factor

$$\Pi_c \equiv \prod_{i=1}^{3}\cosh\delta_i\ , \quad \Pi_s \equiv \prod_{i=1}^{3}\sinh\delta_i$$

Remarks:

I. Subtracted geometry does not satisfy Einstein's equation with original sources → additional gauge & scalar field sources

II. Subtraction that results in exact conformal symmetry ↔

Black hole in a box, which has to be supported by additional sources (return to them later)

Asymptotic geometry of a Lifshitz-type w/ a deficit angle

$$ds_5^2 = -\left(\frac{R}{R_0}\right)^8 dt^2 + 12 dR^2 + R^2 d\Omega_3^2$$

→ black hole in an ``asymptotically conical box'' (w/ deficit angle)

→ the box is confining (``softer'' than AdS)

Lift on a circle to 6-dimensions:

$$
\begin{aligned}
ds_6^2 &= \Delta(\frac{1}{\mu}d\alpha + \mathcal{B})^2 + \Delta^{-1/3}ds_5^2 \\
&= \Delta(\frac{1}{\mu}d\alpha + \mathcal{B})^2 - \Delta^{-1}G(dt + \mathcal{A})^2 + ds_4^2
\end{aligned}
$$

where the KK-field along α is

$$
\mathcal{B} = \frac{1}{\Delta}\left[\mu((a^2+b^2-y)\Pi_s - ab\Pi_c)d\sigma + \mu(y\Pi_c - ab\Pi_s)d\chi - \frac{\Pi_s\Pi_c}{\Pi_c^2 - \Pi_s^2}dt\right]
$$

Geometry factorizes: locally AdS_3 x S^3

globally S^3 (trivially) fibered over BTZ black hole

$$8G_3M_3 = \frac{\ell^2}{\mu^2(\Pi_c^2 - \Pi_s^2)^2}\left[(\mu - a^2 - b^2)(\Pi_c^2 + \Pi_s^2) + 4ab\Pi_c\Pi_s\right] \ ,$$
$$4G_3J_3 = \frac{\ell^3}{\mu^2(\Pi_c^2 - \Pi_s^2)^2}\left[ab(\Pi_c^2 + \Pi_s^2) - (a^2 + b^2 - \mu)\Pi_s\Pi_c\right] \ .$$

w/ geometry [SL(2,R) x SL(2,R)]/Z_2 x SO(4)

→ conformal symmetry of AdS_3 can be promoted to Virasoro algebra
& standard microscopic calculation (via AdS_3/CFT_2) w/Cardy formula
à la Brown-Hennaux

w/ central charge, $c = \frac{3\ell_A}{2G_3}$ and conformal weights $h_+ = \frac{M_3\ell_A + J_3}{2}$ $h_- = \frac{M_3\ell_A - J_3}{2}$

guarantees identification of statistical and BH entropy

[long spinning string interpretation]

M.C. & /Gibbons 1201.0601

Sources supporting subtracted geometry were originally obtained as a scaling limit of another black hole (denoted w/ ``tilded'' variables) w/ two equal infinite boosts $\tilde\delta_1 = \tilde\delta_2 \equiv \tilde\delta$. & one $\tilde\delta_3$ finite one (formally near horizon region for BH w/ two-charges → ∞, one → 0):

$\epsilon \to 0$

$$\tilde x = x\epsilon, \quad \tilde t = t\epsilon^{-1}, \quad \tilde y = y\epsilon, \quad \tilde\sigma = \sigma\epsilon^{-1/2}, \quad \tilde\chi = \chi\epsilon^{-1/2},$$
$$\tilde m = m\epsilon, \quad \tilde a^2 = a^2\epsilon, \quad \tilde b^2 = b^2\epsilon,$$
$$2\tilde m \sinh^2\tilde\delta \equiv Q = 2m\epsilon^{-1/2}(\Pi_c^2 - \Pi_s^2)^{1/2}, \quad \sinh^2\tilde\delta_3 = \frac{\Pi_s^2}{\Pi_c^2 - \Pi_s^2}$$

``Untilded'' variables are those of the subtracted geometry metric of non-extreme black hole with w/ three charges $\delta_1,\delta_2,\delta_3$ and subtracted warp factor

$$\Delta = (2m)^2(x+y)(\Pi_c^2 - \Pi_s^2) + (2m)^3\Pi_s^2$$

Fully determined sources: Scalars: $X_1 = X_2 = X_3^{-\frac{1}{2}} = \frac{\Delta^{\frac{1}{3}}}{2m}$;

Gauge potentials: $A^1 = A^2 = -\frac{x+y}{2m}\,dt + y\Pi_c\,d\sigma - y\Pi_s\,d\chi\,,$

$$A^3 = \frac{(2m)^4\Pi_s\Pi_c}{(\Pi_c^2 - \Pi_s^2)\Delta}\,dt + \frac{\Pi_s}{\Delta}[ab\,d\chi + (y - a^2 - b^2)d\sigma] + \frac{\Pi_c}{\Delta}(ab\,d\sigma - y\,d\chi)$$

Comments:

a) Scaling limit of BH (with ``tilded'' parameters), resulting in subtracted geometry of non-extreme BH (with ``untilded'' parameters) is
reminiscent of near-BPS limit (``dilute gas'') in the near horizon limit,
w/ two (equal) charges $\rightarrow \infty$ & third one $\rightarrow 0$

b) Infinite charges can be gauged away (by rescaling the scalars).
However, the asymptotic metric is of Lifshitz type
(``softer'' than AdS)

c) In retrospect the lift to D=6 as $AdS_3 \times S^3$ expected (due to
BPS-like nature of the scaling limit for BH's with tilded parameters)

Further Remarks: D=4 Black Holes

Non-extreme Rotating Asymptotically Minkowski BH's in D=4, parameterized by mass, angular momentum and (four-)charges

Metric of D=4 rotating four-charge black holes

(generating solutions of N=4,8 ungauged supergravity)

M.C. & Youm '96, Chong, M.C., Lü & Pope'04

$$ds_4^2 = -\Delta_0^{-1/2} G(dt + \mathcal{A})^2 + \Delta_0^{1/2}\left(\frac{dr^2}{X} + d\theta^2 + \frac{X}{G}\sin^2\theta d\phi^2\right)$$

$$\begin{aligned}
X &= r^2 - 2mr + a^2 \ , \\
G &= r^2 - 2mr + a^2\cos^2\theta \ , \\
\mathcal{A} &= \frac{2ma\sin^2\theta}{G}\left[(\Pi_c - \Pi_s)r + 2m\Pi_s\right] d\phi \ , \\
\Delta_0 &= \prod_{I=0}^{3}(r + 2m\sinh^2\delta_I) + 2a^2\cos^2\theta[r^2 + mr\sum_{I=0}^{3}\sinh^2\delta_I + 4m^2(\Pi_c - \Pi_s)\Pi_s \\
&\quad - 2m^2\sum_{I<J<K}\sinh^2\delta_I\sinh^2\delta_J\sinh^2\delta_K] + a^4\cos^4\theta \ .
\end{aligned}$$

$$\Pi_c \equiv \prod_{I=0}^{3}\cosh\delta_I \ , \quad \Pi_s \equiv \prod_{I=0}^{3}\sinh\delta_I \ .$$

$$G_4 M = \frac{1}{4}m\sum_{I=0}^{3}\cosh 2\delta_I \ ,$$ Mass

$$G_4 Q_I = \frac{1}{4}m\sinh 2\delta_I \ , \ (I = 0,1,2,3)$$ Four charges

$$G_4 J = ma(\Pi_c - \Pi_s) \ ,$$ Angular momentum

Special cases:

$\delta_I = \delta$ Kerr-Newman

& a = 0 Reisner-Nordström

$\delta_I = 0$ Kerr

& a = 0 Schwarzschild

Further Remarks:

Rotating Asymptotically Minkowski BH's in D=4,
parameterized by mass, angular momentum and (four-)charges

Subtracted geometry prescription works in D=4 for general (four-)charge rotating black holes, too! M.C. & Larsen 1112.4856

Subtracted geometry for D=4 rotating four-charge rotating black holes:

$$ds_4^2 = -\Delta_0^{-1/2} G (dt + \mathcal{A})^2 + \Delta_0^{1/2} \left(\frac{dr^2}{X} + d\theta^2 + \frac{X}{G} \sin^2\theta d\phi^2 \right)$$

$$X = r^2 - 2mr + a^2 \,,$$

$$G = r^2 - 2mr + a^2 \cos^2\theta \,,$$

$$\Pi_c \equiv \prod_{I=0}^{3} \cosh\delta_I \,, \quad \Pi_s \equiv \prod_{I=0}^{3} \sinh\delta_I$$

$$\mathcal{A} = \frac{2ma\sin^2\theta}{G} \left[(\Pi_c - \Pi_s) r + 2m\Pi_s \right] d\phi \,,$$

$$\Delta_0 = \prod_{I=0}^{3} (r + 2m\sinh^2\delta_I) + 2a^2\cos^2\theta [r^2 + mr \sum_{I=0}^{3} \sinh^2\delta_I + 4m^2(\Pi_c - \Pi_s)\Pi_s$$
$$- 2m^2 \sum_{I<J<K} \sinh^2\delta_I \sinh^2\delta_J \sinh^2\delta_K] + a^4\cos^4\theta \,.$$

$$\boxed{\Delta_0 \to \Delta = (2m)^3 r (\Pi_c^2 - \Pi_s^2) + (2m)^4 \Pi_s^2 - (2m)^2 (\Pi_c - \Pi_s)^2 a^2 \cos^2\theta}$$

Further Remarks:

Rotating Asymptotically Minkowski BH's in D=4, parameterized by mass, angular momentum and (four-)charges

Subtracted geometry prescription works in D=4 for general (four-) charge rotating black holes, too! M.C. & Larsen 1112.4856

Metric written with a warp factor; termodynamics independent of a warp factor

Allows for restoration of $SL(2,R)^2$ in the wave eq.

Lift to D=5: locally 4 $AdS_3 \times S^2$; globally S^2 fibered over BTZ

Quantitative microscopics again à la Brown-Henneaux

Further Insights into the origin of subtracted geometry:

These geometries can be obtained as an infinite boost Harrison transformations on the original solution, i.e.
SO(1,1) transformations [change aymptotics]: $H \sim \begin{pmatrix} 1 & 0 \\ \beta & 1 \end{pmatrix}$ $\beta \to 1$

acting on the original solution reduced on a time-like Killing vector.

Explicitly shown for a D=4 Schwarzschild BH
& conjectured that it works for general D=4 multi-charged BH's

M.C. & Gibbons 1201.0601

Subsequently confirmed

Virmani 1203.5088
Sahay & Virmani 1305.2800

Interpolating geometries studied for:

Static BH's; CFT interpretation as (2,2) irrelevant deformations

Baggio, de Boer, Jottar & Mayerson1210.7695

→ General rotating BH's

Highlights for interpolating geometries for general rotating BH's

M. C., Guica & Saleem 1302.7036

a) Harrison transformations on D=4 BH's -- specific $SO(1,1)^4$ boosts of O(4,4) symmetry [change asymptotics] for time-like Killing vector reduced action

D=4 BH $\longrightarrow$ D=4 subtracted geometry

$$H_i \sim \begin{pmatrix} 1 & 0 \\ \beta_i & 1 \end{pmatrix} \qquad \beta_i \to 1 \ (i=1,2,3) \qquad \beta_4 \to \text{finite}$$

b) Harrison transformations ←→ on S^1 lifted geometry: coordinate transf. $t+\beta\, y$ (Melvin) twists

[related to spectral flow, e.g., Bena, Bobev & Warner'08]

c) Interpolating solutions obtained in lifted geometry via sets of T-duality transf. followed by Melvin twists

d) Deformations from subtracted geometry $(\beta_i\text{-}1 \ll 1)$ in dual CFT via (2,2) & (1,2) (irrelevant) operators

Digression:

How about Harrison transf. on angular Killing vector reduced action $\leftarrow\rightarrow$

S^1 lifted geometry should correspond to $\phi+\alpha\, y$ coordinate transf.
(``standard'' Melvin twist)

$\rightarrow$ New geometrical insights?

c.f., Maxwell-Einstein Gravity: Gibbons, Mujtaba& Pope 1301.3927

Multi-charged BH's

work in progress M.C.,Gibbons,Pope & Saleem

Final comments: Entropy Products

Without cosmological constant, the product of areas associated with two horizons quantised → indicative of 2D CFT

D=4 $$A_+A_- = 64\pi^2(\prod_{i=1} Q_i + J^2)$$

Larsen'97(J=0), M.C. & Larsen'97 (J≠0)

D=5 $$A_+A_- = 64\pi^2(\prod_{i=1}^{3} Q_i + J_R^2 - J_L^2)$$

Generalized to rings & strings
Castro & Rodriguez 1204.1284
KK dyons M.C., Lü & Pope1306.4522

With cosmological constant ($1/g^2$), more that two horizons, and yet the product of areas associated with all (analytically continued) horizons also quantised:

D=4 $$\prod_{\alpha=1}^{4} A_\alpha = 64\pi^2 \frac{1}{g^4}(\prod_{i=1}^{4} Q_i + J^2)$$

M.C,. Gibbons & Pope 1011.008 (PRL)

D=5 $$\prod_{\alpha=1}^{3} A_\alpha = 64\pi^2 \frac{1}{g^3}(\prod_{i=1}^{3} Q_i + J_R^2 - J_L^2)$$

Generalized to Wu's solution:
M.C,. Lü & Pope1306.4522

→ possible underlying (higher dim conformal) symmetry?

Higher derivative gravities → evidence for quantisation

D=4 Maxwell-Weyl gravity: $\Pi_I S_I = J^2$

M.C,. Lü & Pope1306.4522
[related: Castro,Dehmami,Giribet & Kastor 1304.1696]

→ FURTHER STUDY