Black Hole Information and Topological Entanglement

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18th Claude Itzykson Meeting

Frontiers of String Theory

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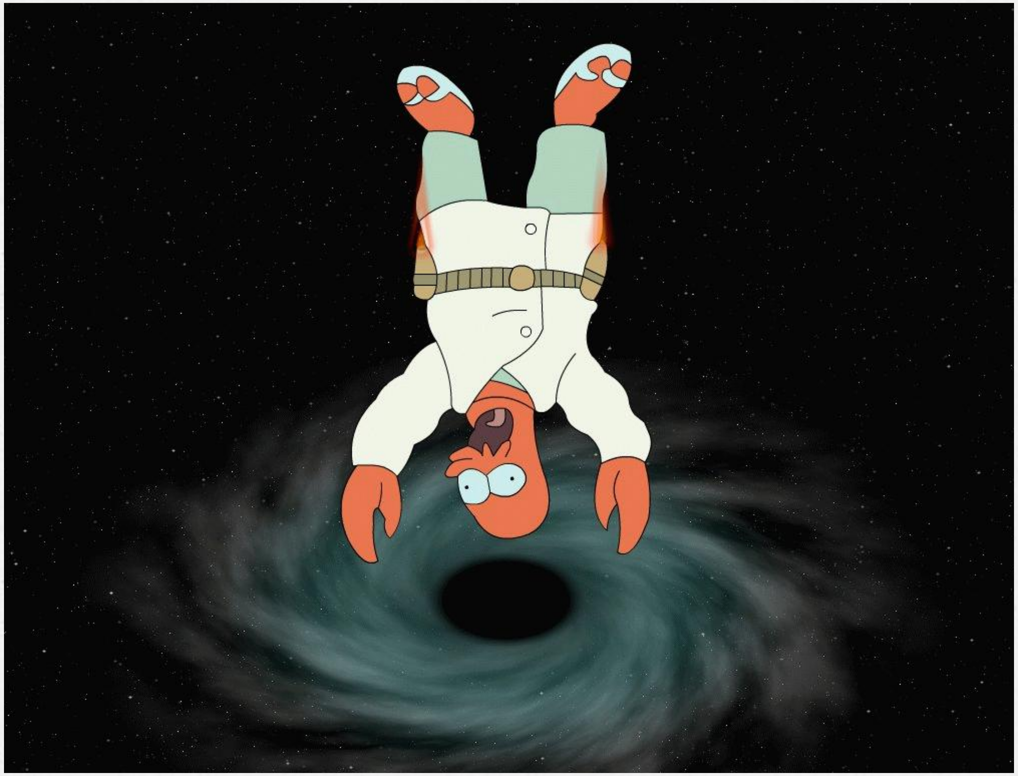
PCTS

How to fall into a black hole

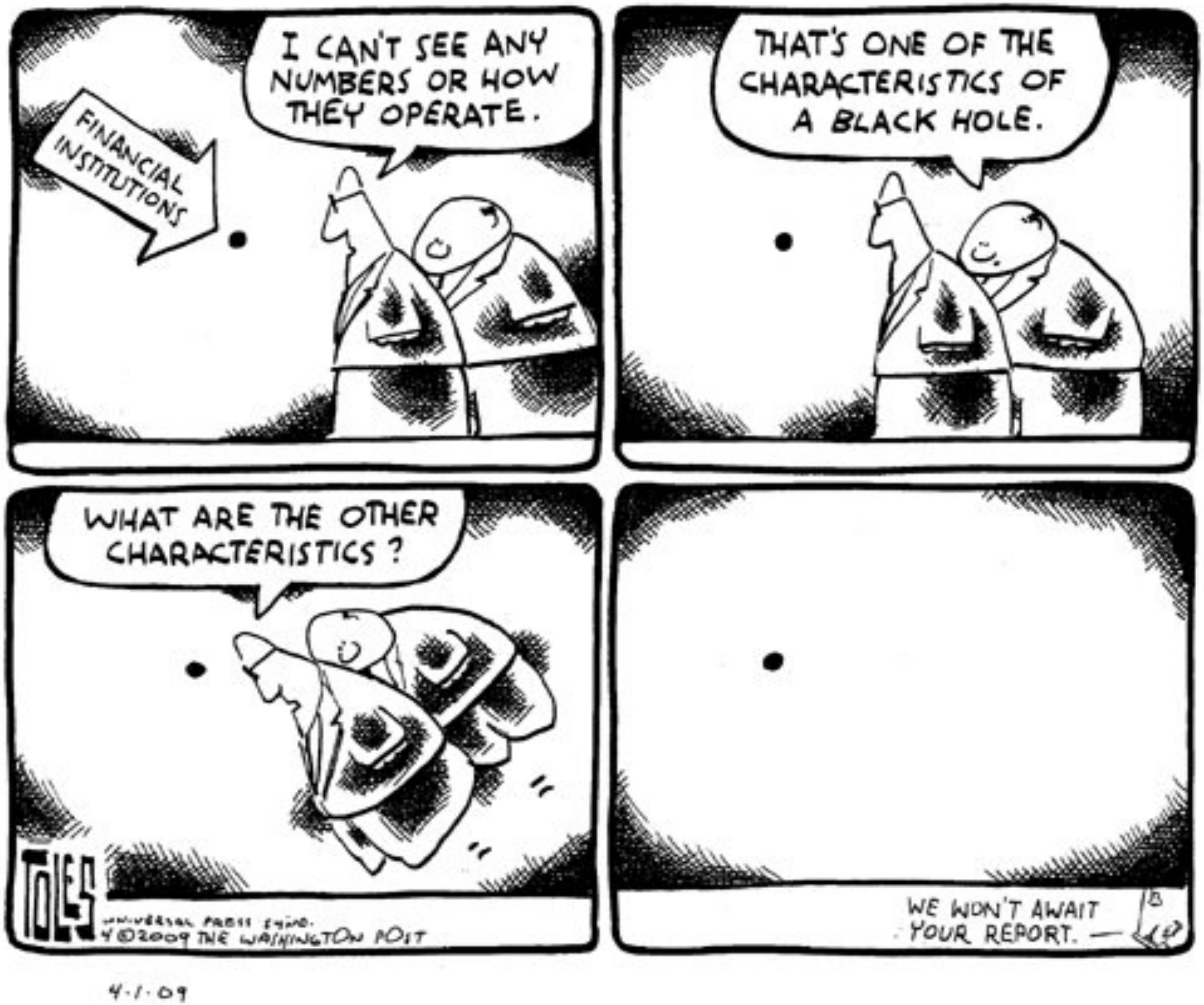


Why is empty space smooth?

Outline

- Balanced Holography
- Black Hole Entropy =

  Topological Entanglement
- Passing through the Firewall


FINANCIAL INSTITUTIONS
I CAN'T SEE ANY NUMBERS OR HOW THEY OPERATE.
THAT'S ONE OF THE CHARACTERISTICS OF A BLACK HOLE.
WHAT ARE THE OTHER CHARACTERISTICS?
TOLES
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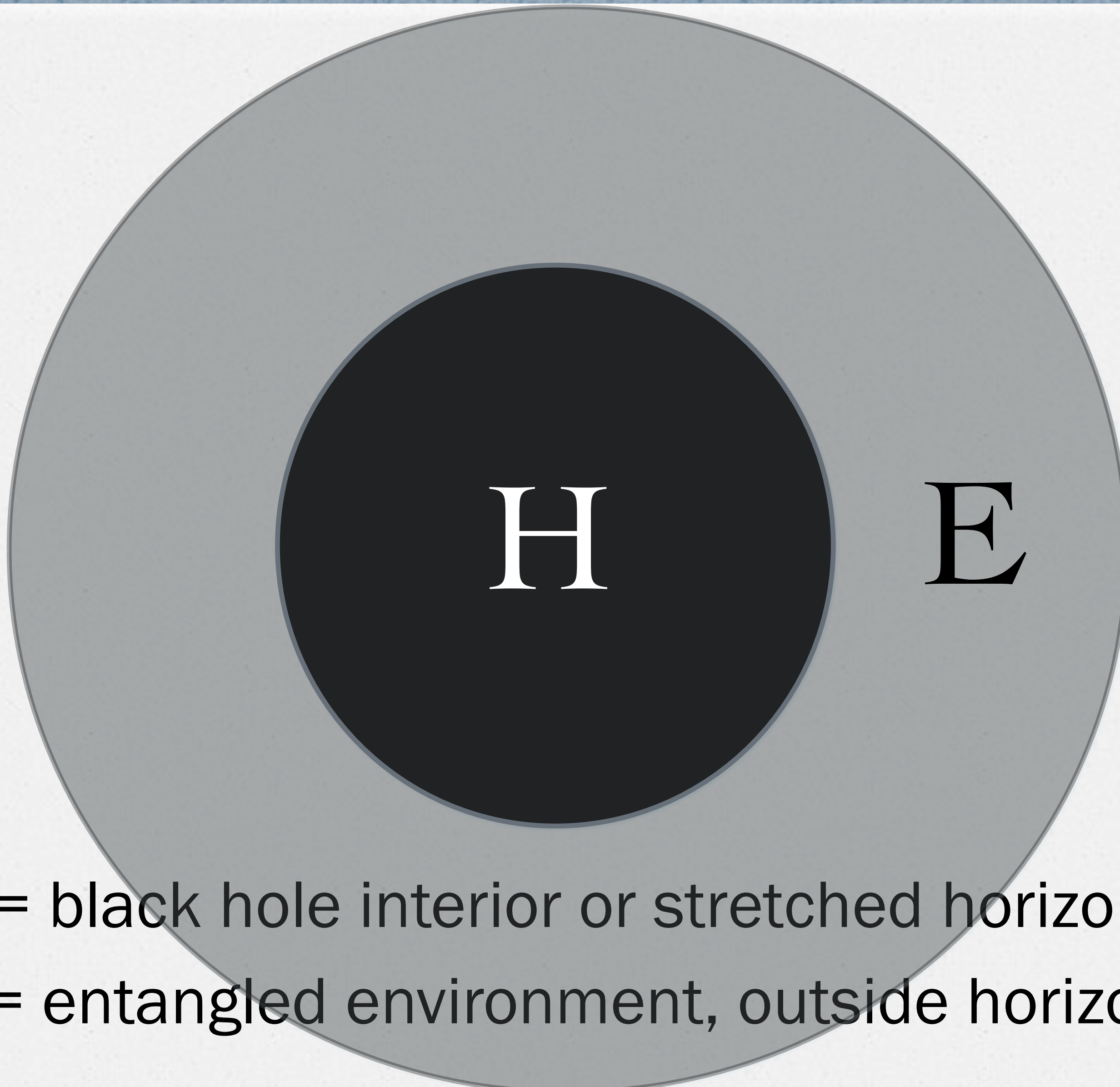
Three New Principles of Black Hole Information:

- Maximal Entanglement
- Balanced Holography
- Topological Protection

Maximal Entanglement:

- *A typical black hole H is maximally entangled with its environment E.*
- *The entanglement entropy saturates the Bekenstein-Hawking bound:*

$$S_{EE} = S_{B\text{-}H}$$



H = black hole interior or stretched horizon

E = entangled environment, outside horizon

Balanced Holography:

- *Not all maximally entangled BH states are equal: some are more equal than others:*
- *The number of entangled BH vacuum states saturates the B-H bound:*

$$S_{BH} = S_{B\text{-}H}$$

A general entangled state:

$$\left|\Psi\right\rangle = \sum_{i,j} \alpha_{ij} \left|i\right\rangle_{\mathrm{H}} \left|j\right\rangle_{\mathrm{E}}$$

A balanced entangled state:

$$\left|\Psi\right\rangle = \sum_{i} \alpha_{i} \left|i\right\rangle_{\mathrm{H}} \left|i\right\rangle_{\mathrm{E}} .$$

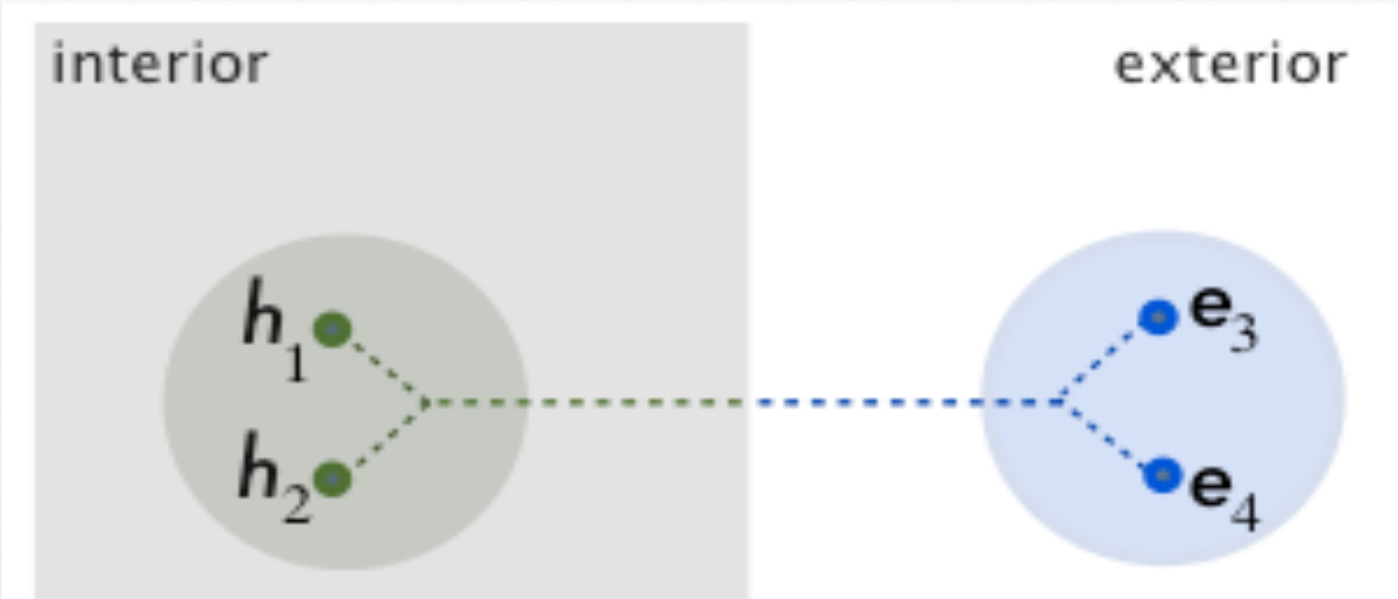
Topological Protection:

Black hole information is:

- *carried by non-local correlations between internal and external degrees of freedom*
- *‘topologically’ protected from local sources of decoherence*

Topological qubit

$$\left|\Psi\right\rangle = \alpha_0 \left|0\right\rangle_{\mathrm{H}} \left|0\right\rangle_{\mathrm{E}} + \alpha_1 \left|1\right\rangle_{\mathrm{H}} \left|1\right\rangle_{\mathrm{E}}$$



Majorana quartet

Topological entanglement entropy

$$S_A = -\mathrm{tr}(\rho_A \log \rho_A) \qquad\qquad \rho_A = \mathrm{tr}_B(|\psi\rangle\langle\psi|)$$

$$S_A = \frac{L}{\epsilon} + S_{\mathrm{top}}$$

measures topological long range order

Topological entanglement entropy

$$S_{\text{top}} = \log D \qquad D = \sqrt{\sum_a d_a^2}$$

$$d_a d_b = \sum_c N_{ab}{}^c d_c$$

$$D = S_0^0 \qquad d_a = \frac{S_0^a}{S_0^0}$$

d_a = quantum dimensions of anyon excitations

S_b^a = modular S-matrix of CFT conformal blocks

Topological entanglement entropy

$$S_{top} = \mathrm{Log}\, S_o^a$$

a = SL(2,R) x SL(2,R) representation that characterizes the BTZ black hole state

It is known that there are two types of representations in bosonic Liouville theory: continuous and identity representation. Their character formulas and their S-transformation are given by ($q \equiv e^{2\pi i\tau}$)

continuous representations; $\quad p > 0$

$$\chi_p(\tau) = \frac{q^{h-\frac{c}{24}}}{\prod_{n=1}(1-q^n)} = \frac{q^{\frac{p^2}{2}}}{\eta(\tau)}, \qquad h = \frac{p^2}{2} + \frac{\mathcal{Q}^2}{8}$$

$$\chi_p(-\frac{1}{\tau}) = 2\int_0^\infty dp' \cos(2\pi pp')\chi_{p'}(\tau), \tag{1.4}$$

identity representation; $\quad h = 0$

$$c = 1 + 3\mathcal{Q}^2$$

$$\mathcal{Q} = \sqrt{2}(b + 1/b)$$

$$\chi_{h=0}(\tau) = \frac{q^{-\frac{\mathcal{Q}^2}{8}}(1-q)}{\eta(\tau)},$$

$$\chi_{h=0}(-\frac{1}{\tau}) = 4\int_0^\infty dp \sinh(2\pi bp)\sinh(\frac{2\pi p}{b})\chi_p(\tau) \tag{1.5}$$

Eguchi, Sugawara, Taormina, arxiv:0803.0377

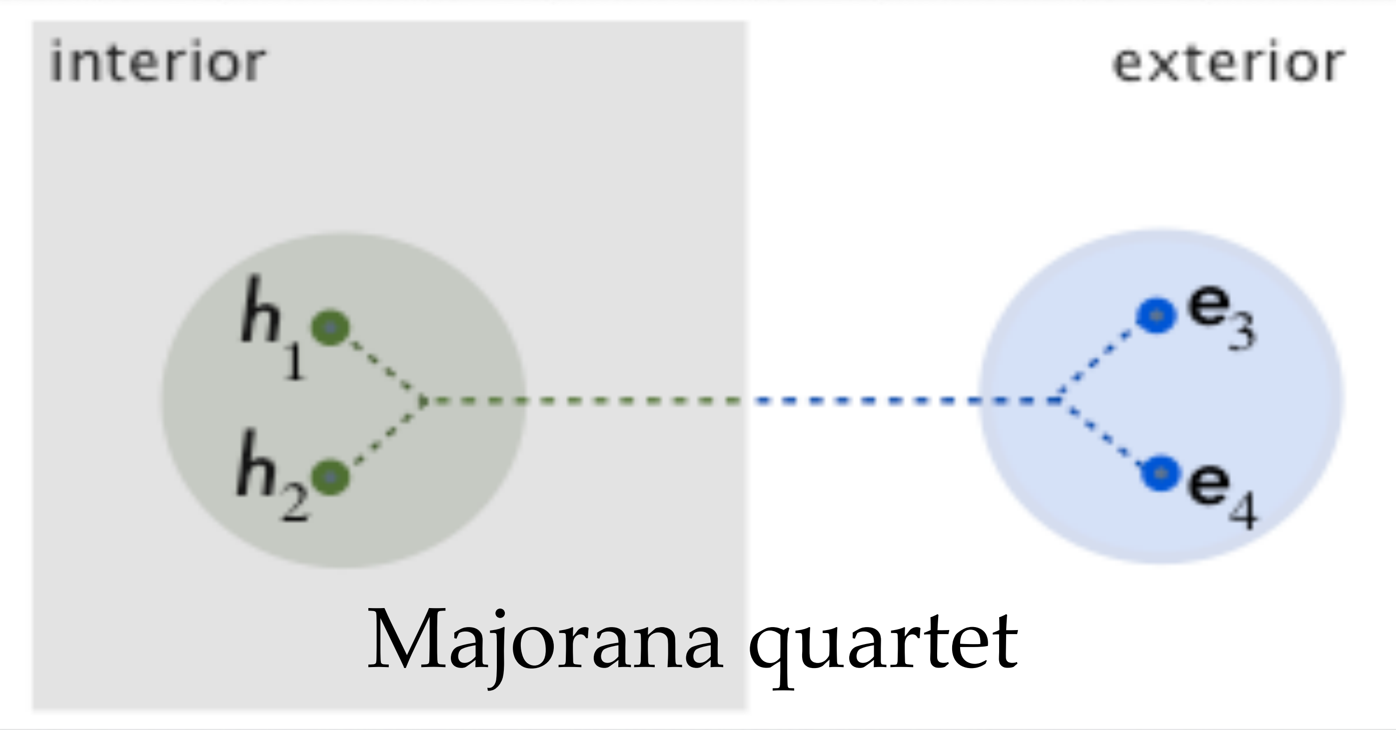
Topological entanglement entropy

$$S_{top} = \mathrm{Log}\, S_o^a \;=\; S_{B\text{-}H} \;\; !$$

a = SL(2,R) x SL(2,R) representation that characterizes the BTZ black hole state

Topological qubit

$$|\Psi\rangle = \alpha_0 |0\rangle_{\mathrm{H}} |0\rangle_{\mathrm{E}} + \alpha_1 |1\rangle_{\mathrm{H}} |1\rangle_{\mathrm{E}}$$



Majorana quartet

$$\left|\Psi\right\rangle = \alpha_0\left|0\right\rangle_{\mathrm{H}}\left|0\right\rangle_{\mathrm{E}} + \alpha_1\left|1\right\rangle_{\mathrm{H}}\left|1\right\rangle_{\mathrm{E}}$$

$$\mathbf{a}\left|\Psi\right\rangle = 0 \quad ?$$

$$\left|\Psi\right\rangle = \alpha_0\left|0\right\rangle_{\mathrm{H}}\left|0\right\rangle_{\mathrm{E}} + \alpha_1\left|1\right\rangle_{\mathrm{H}}\left|1\right\rangle_{\mathrm{E}}$$

$$\mathbf{a}\left|\Psi\right\rangle = 0 \quad ?$$

$$\mathbf{U}_{\mathrm{CNOT}}\left|\Psi\right\rangle = \left|0\right\rangle_{\mathrm{H}}\left(\alpha_0\left|0\right\rangle_{\mathrm{E}} + \alpha_1\left|1\right\rangle_{\mathrm{E}}\right)$$

$$\mathbf{h}\left|0\right\rangle_{\mathrm{H}} = 0, \qquad \mathbf{h}^\dagger\left|0\right\rangle_{\mathrm{H}} = \left|1\right\rangle_{\mathrm{H}}$$

$$\left|\Psi\right\rangle = \alpha_0\left|0\right\rangle_{\mathrm{H}}\left|0\right\rangle_{\mathrm{E}} + \alpha_1\left|1\right\rangle_{\mathrm{H}}\left|1\right\rangle_{\mathrm{E}}$$

$$\mathbf{a}\left|\Psi\right\rangle = 0 \quad \mathbf{!}$$

$$\mathbf{a} = \mathbf{U}_{\mathrm{CNOT}}^{-1}\mathbf{h}\mathbf{U}_{\mathrm{CNOT}}$$

$$\mathbf{h}\left|0\right\rangle_{\mathrm{H}} = 0, \qquad \mathbf{h}^{\dagger}\left|0\right\rangle_{\mathrm{H}} = \left|1\right\rangle_{\mathrm{H}}$$

- Black hole information is carried by logical qubits
- Vacuum entanglement is carried by virtual qubits

Topological qubit = virtual qubit + logical qubit

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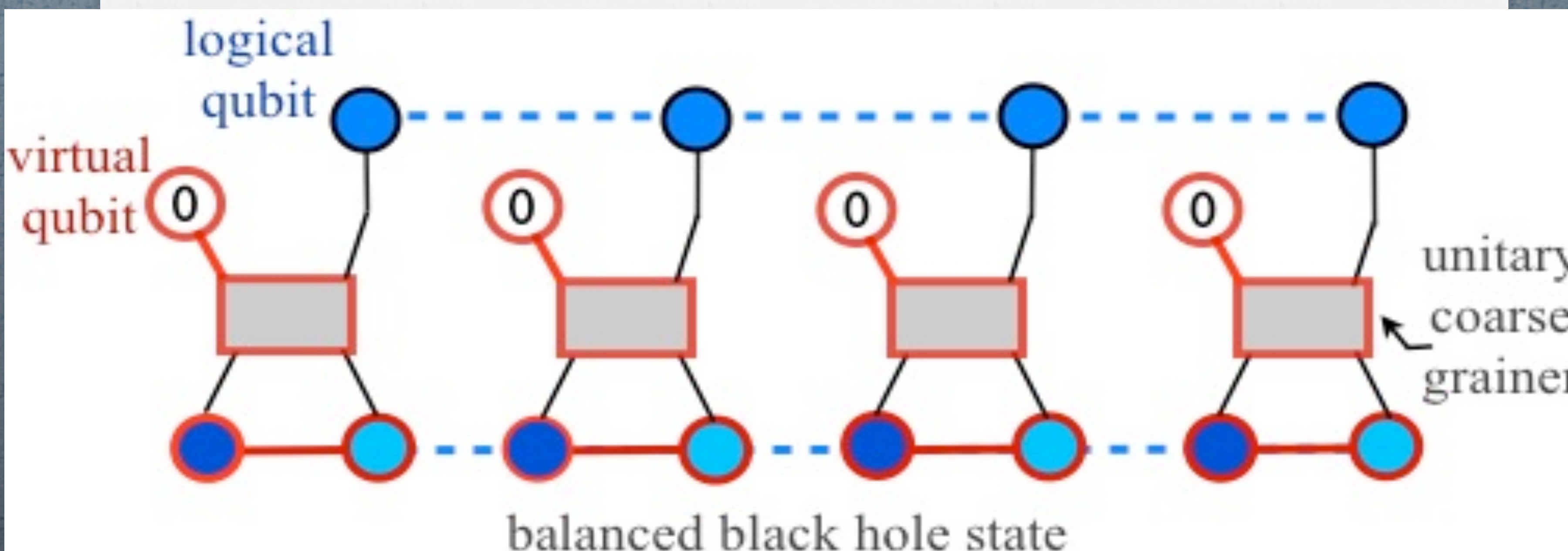

Black hole information must involve hidden long range quantum correlations, coded in the structure of space itself:

Space represents a highly entangled quantum state. Uniformity and locality are emergent properties rather than fundamental principles of the microphysics.