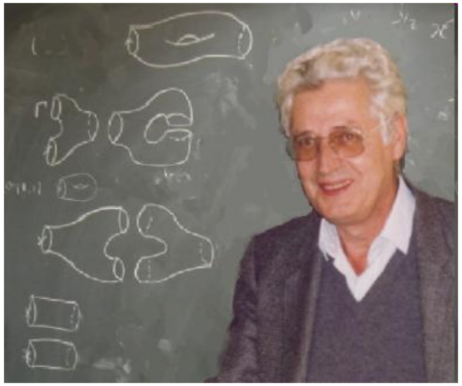
Quantum Features of Black Holes



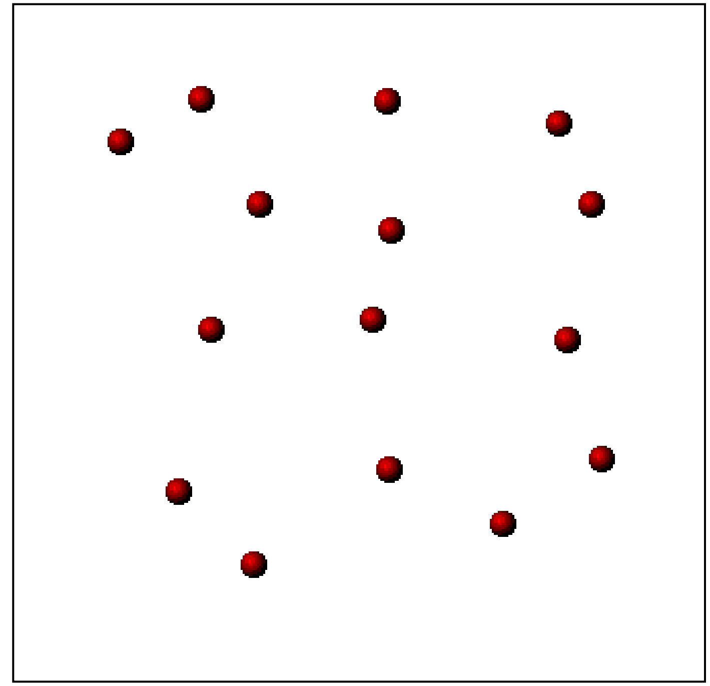
Jan de Boer, Amsterdam

Paris, June 18, 2009

Based on:

arXiv:0802.2257 - JdB, Frederik Denef, Sheer El-Showk, Ilies Messamah, Dieter van den Bleeken
arXiv:0807.4556 - JdB, Sheer El-Showk, Ilies Messamah, Dieter van den Bleeken
arXiv:0811.0263 - Vijay Balasubramanian, JdB, Sheer El-Showk, Ilies Messamah
arXiv:0906.0011 - JdB, Sheer El-Showk, Ilies Messamah, Dieter van den Bleeken.
arXiv:0906.3272 - Vijay Balasubramanian, JdB, S. Sheikh-Jabbari, J.Simon

Black hole thermodynamics



One can associate an entropy $S = \frac{A}{4G}$ and a temperature to a black hole in such a way that it seems to obey the laws of thermodynamics, just like a gas in a box. Is this just an analogy? If not, what are the atoms that underlie black holes?

These atoms cannot be gravitons or something like $S \sim V$ that there would $S \sim A$ give rise to an entropy that scales as rather than .

The AdS/CFT correspondence has provided an indirect answer: these atoms are the degrees of freedom of a holographically dual field theory that lives in one dimension less.

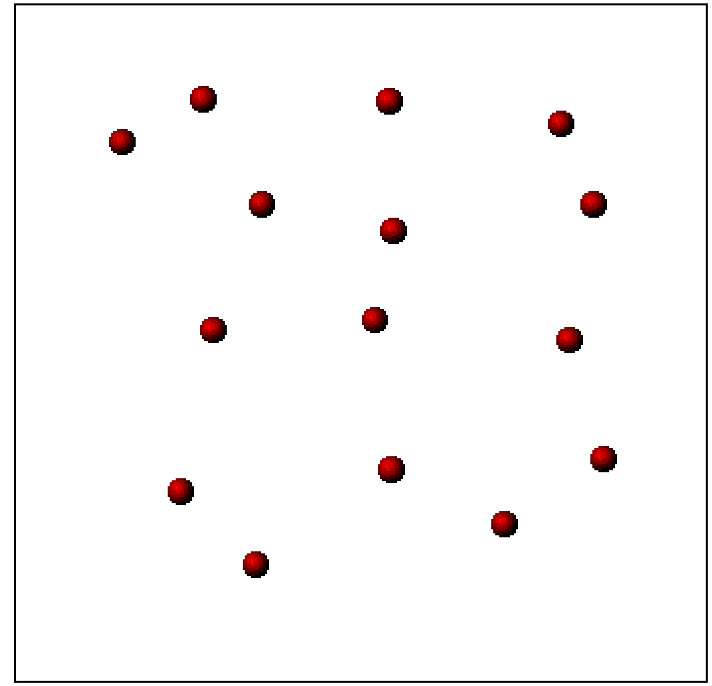
Though this resolves the issue in principle, it is not so easy to apply it to practical questions about black holes.

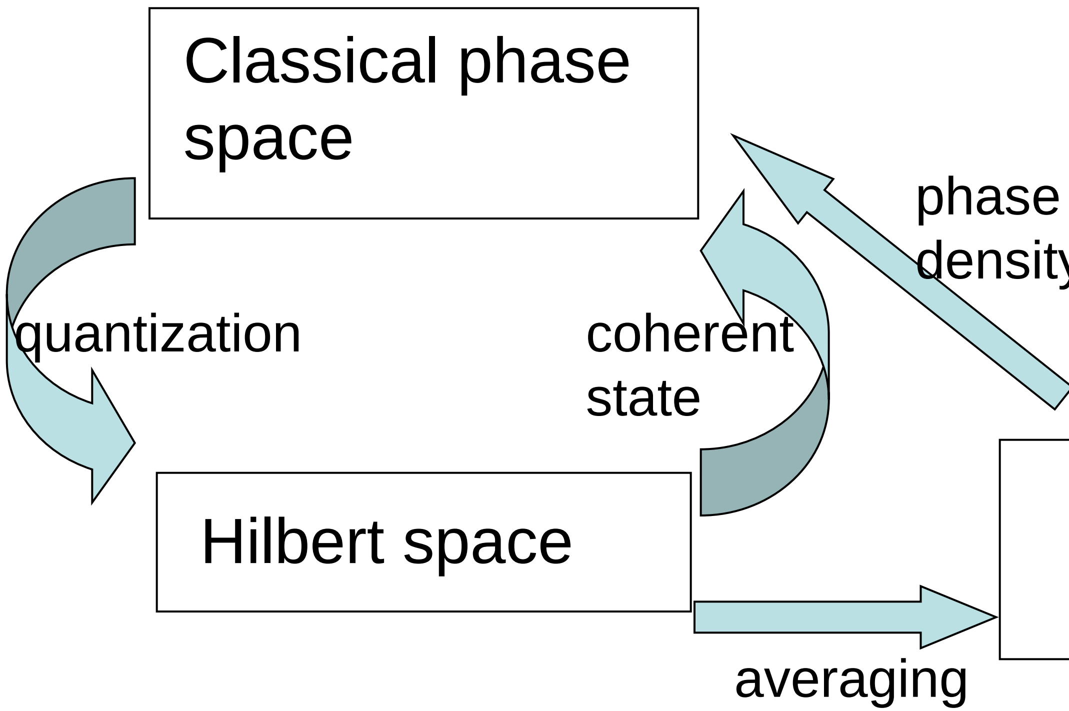
One practical question it does answer:

For many black holes, the dual field theory description has been successfully used to reproduce the entropy of the black hole by counting the number of degrees of freedom in the field theory.

Individual degrees of freedom for

Atoms in a box





Idea: an analogous story applies to black holes as well. This would explain why, when collapsing different pure states to form black holes, they are difficult to distinguish from each other: no hair theorem.

It would also allow us to compute, in principle, how information about the initial state is encoded in the resulting Hawking radiation.

In certain cases it has been shown that for black holes the classical phase space of the atoms can equivalently be described by spaces of smooth solutions to the (super)gravity equations of motion. (pioneered by Mathur: fuzzballs).

Smoothness here is crucial: singularities arise after averaging (or coarse graining) over degrees of freedom. The smoothness requirement is also what makes this idea compatible with holography.

Pure states	$\Longleftrightarrow$	smooth geometries	$\Longleftrightarrow$	$S = 0$
Mixed states	$\Longleftrightarrow$	singular geometries	$\Longleftrightarrow$	$S \neq 0$

Many caveats:

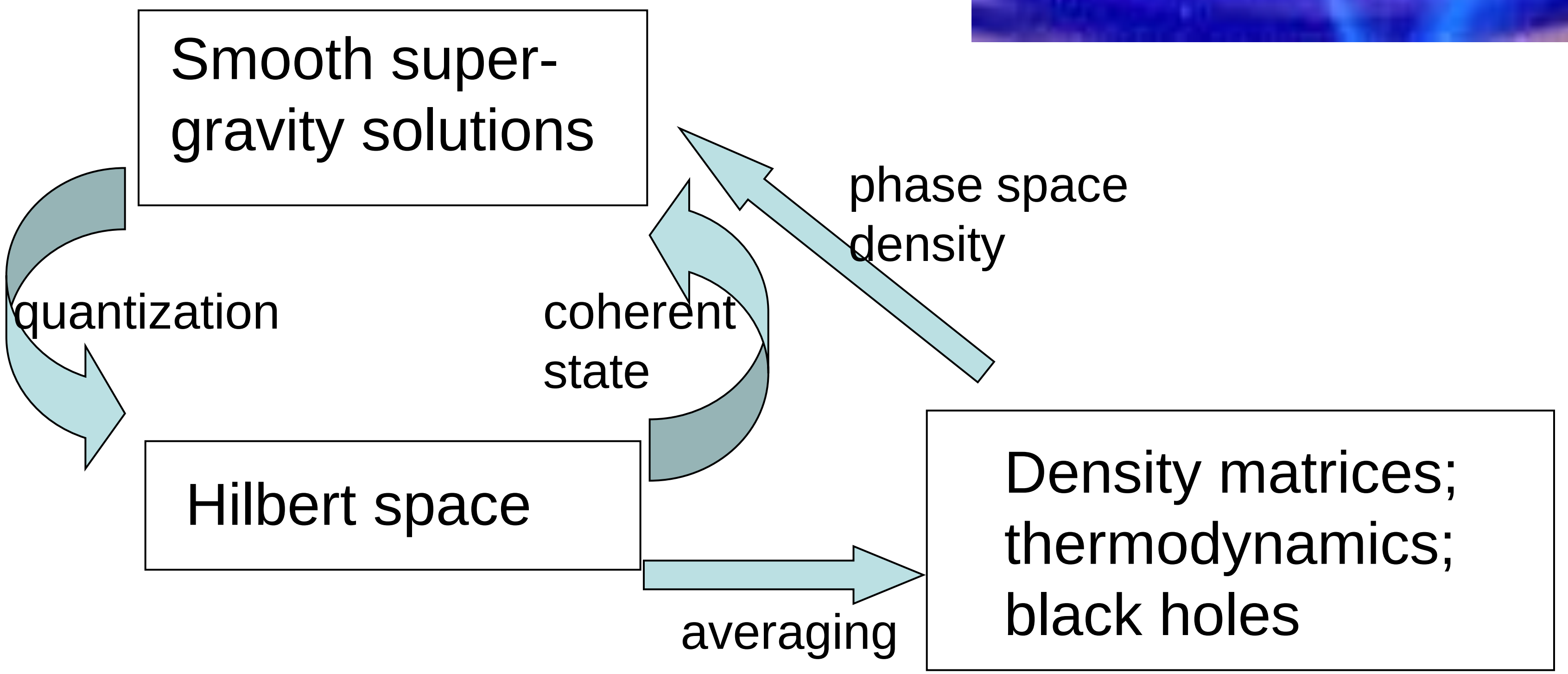
-need to include smooth solutions with Planck-size curvature.

-not clear what happens when higher derivative corrections are included.

-only works for extremal supersymmetric black holes.

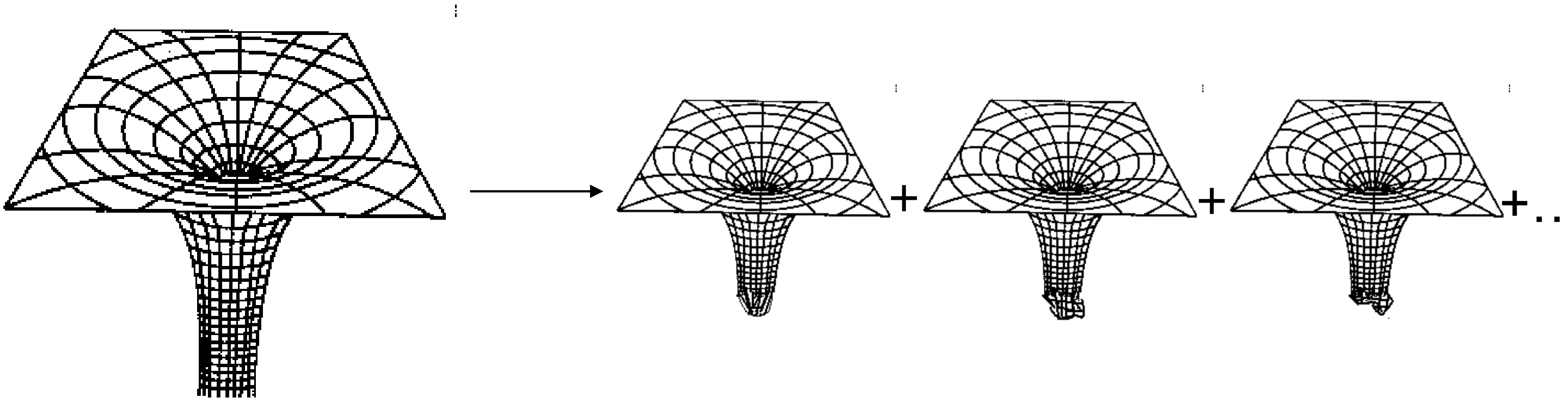
-so far no complete description for any macroscopic black hole: strong evidence that stringy degrees of freedom are always necessary (see later)

Black holes



Smooth super-gravity solutions
quantization
Hilbert space
coherent state
averaging
Density matrices; thermodynamics; black holes
phase space density

This picture has been developed in great detail for "small" black holes. One can make sense of the notion of "adding" geometries, and show that



Of course, we would like to generalize this picture to large, macroscopic black holes.

Adding geometries in AdS/CFT:

An asymptotic AdS geometry is dual to a state. From asymptotics can read off the one-point functions of operators in the field theory.

$$\langle \psi_1 | \mathcal{O}_k | \psi_1 \rangle = a_k$$

Geometry 1

$$\langle \psi_2 | \mathcal{O}_k | \psi_2 \rangle = b_k$$

Geometry 2

$$\rho = \frac{1}{2}\left(|\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2| \right)$$

Shortcut for BPS

$$\mathrm{Tr}(\rho \mathcal{O}_k) = (a_k + b_k)/2$$

Σ(Geometries)

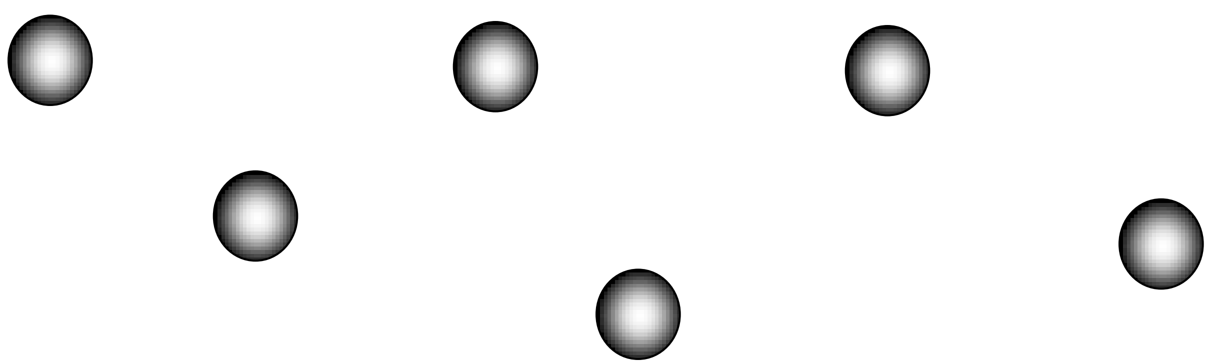
Solve field equations with new boundary conditions

Large supersymmetric black holes carrying electric charge Q and magnetic charge P exist in four dimensions. (P and Q can be vectors with many components).

There exists however a much larger set of solutions of the gravitational field equations, which includes bound states of black holes, and also many smooth solutions.

Lopes Cardoso, de Wit, Kappeli, Mohaupt; Denef; Bates, Denef

Put black holes with charges $\Gamma_i = (P_i, Q_i)$ at locations $\vec{x}_i \in \mathbb{R}^3$



There are corresponding solutions of the field equations only if (necessary, not sufficient)

$$\langle h, \Gamma_i \rangle + \sum_{j \neq i} \frac{\langle \Gamma_j, \Gamma_i \rangle}{|\vec{x}_j - \vec{x}_i|} = 0$$

Here, $\langle \Gamma_1, \Gamma_2 \rangle = P_1 \cdot Q_2 - P_2 \cdot Q_1$ is the electric-magnetic duality invariant pairing between charge vectors. The constant vector h determines the asymptotics of the solution.

Solutions are stationary with angular momentum

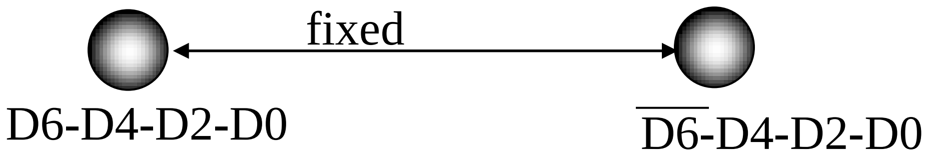
$$\vec{J} = \frac{1}{4} \sum_{i \neq j} \langle \Gamma_i, \Gamma_j \rangle \frac{\vec{x}_i - \vec{x}_j}{|\vec{x}_i - \vec{x}_j|}$$

Typical setup: type IIA on CY

Magnetic charges: D6,D4

Electric charges: D0,D2

Example:



- Whenever the total D6-brane charge of a solution vanishes, one can take a decoupling limit so that the geometry (after uplifting to d=5) becomes asymptotic to $AdS_3 x S^2 x CY$. (dual=MSW (0,4) CFT) Maldacena, Strominger, Witten

- When the centers correspond to pure branes with only a world-volume gauge field, the 5d uplift is a smooth geometry. The space of all such solutions will be our candidate phase space.

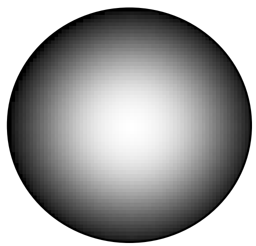
Uplift of a D4-D2-D0 black hole yields the BTZ black hole, and can apply Cardy.

When the two centers correspond to pure fluxed D6-branes, i.e. they correspond to D6-branes with a non-trivial gauge field configuration there is a coordinate change which maps the solution into global AdS_3.

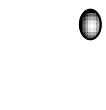
Denef, Gaiotto, Strominger, vdBleeken, Yin

This coordinate transformation correspond to spectral flow in the CFT.

The BMPV black hole does not admit a decoupling limit to AdS_3. Cannot use CFT methods to compute its entropy. But



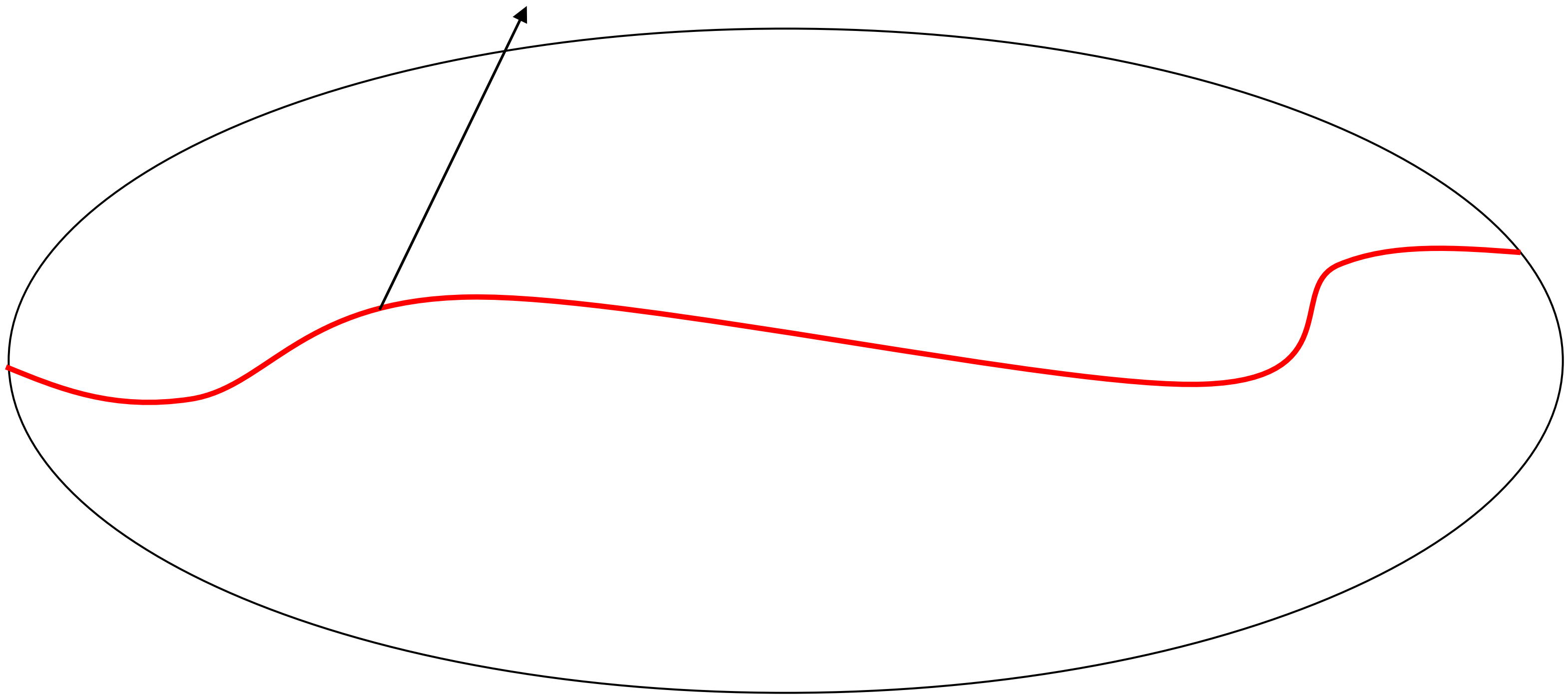
BMPV



D6 + flux

can be put in AdS_3. Dual to a sector of the CFT (cf Sen's talk) which we do not know how to characterize. In Cardy regime single centered black hole dominates entropy, but numerical evidence suggests that for $E_0 < c/24$ the above configuration dominates (entropy enigma). May in principle be able to microscopically determine BMPV entropy in this way.

Set of smooth solutions



Full phase space=set of all solutions of the equations of motion.

$$\omega \sim \int d\Sigma_\mu \left(\delta \frac{\delta \mathcal{L}}{\delta(\partial_\mu \phi)} \wedge \delta\phi \right)$$

Result:

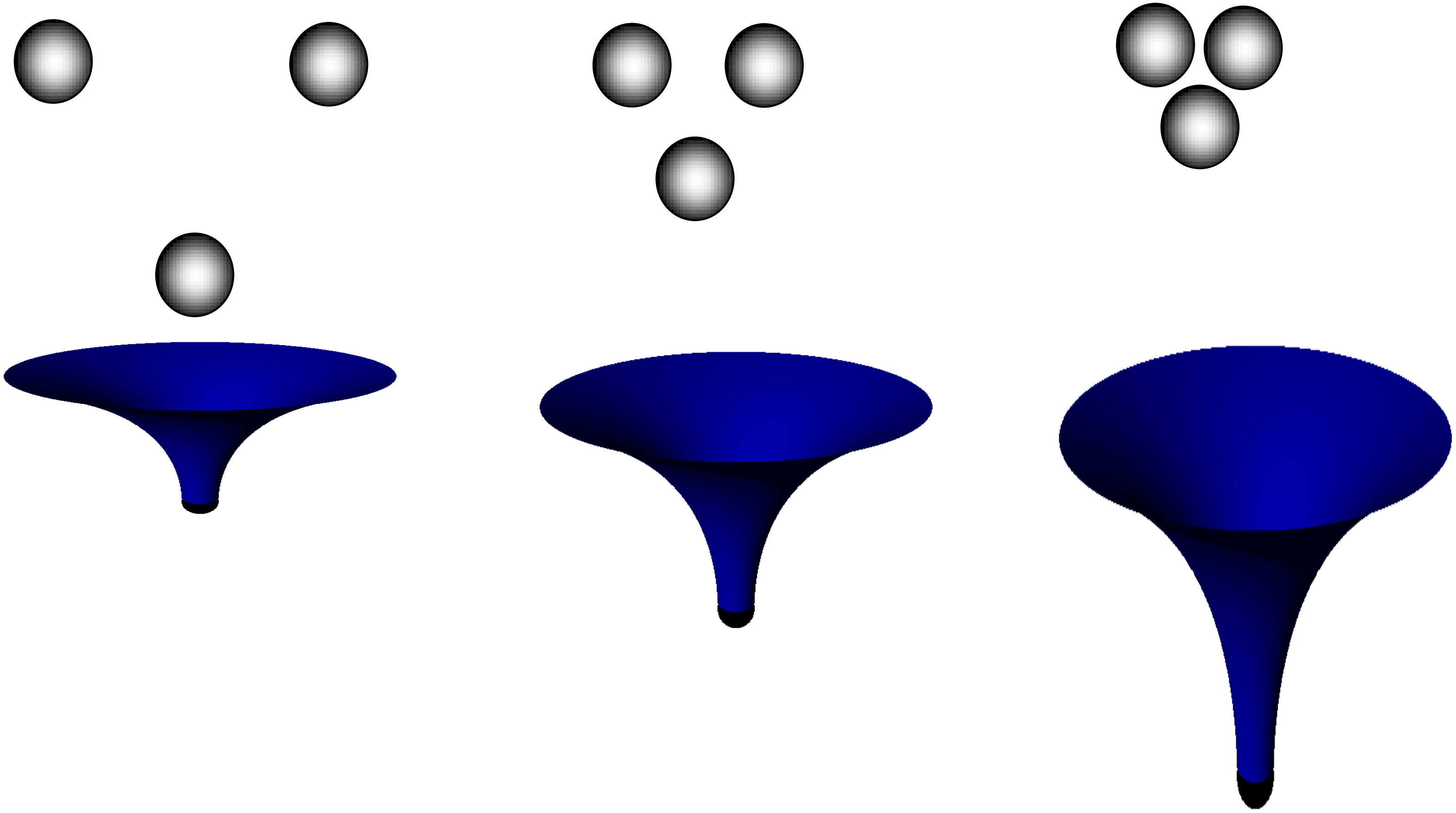
$$\omega = \frac{1}{4} \sum_{p \neq q} \langle \Gamma_i, \Gamma_j \rangle \frac{\epsilon_{ijk} (\delta(x_p - x_q)^i \wedge \delta(x_p - x_q)^j)\,(x_p - x_q)^k}{|\mathbf{x}_p - \mathbf{x}_q|^3}$$

Can now use various methods to quantize the phase space, e.g. geometric quantization. Can explicitly find wavefunctions for various cases.

In particular, one can use this to reproduce and extend a mathematical result known as the wall-crossing formula.

Bena, Wang, Warner; Denef, Moore

Of particular interest: scaling solutions: solutions where the constituents can approach each other arbitrarily closely.



In space-time, a very deep throat develops, which approximates the geometry outside a black hole ever more closely.

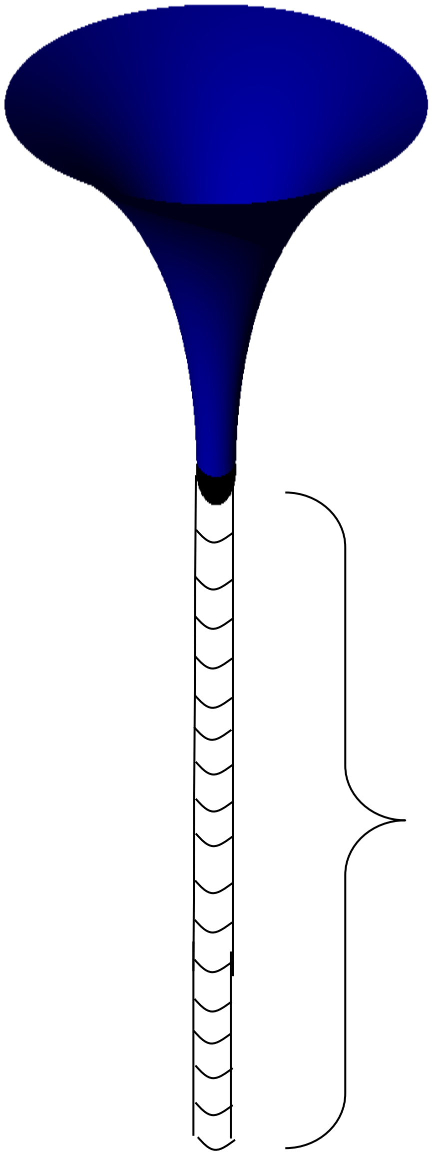
None of these geometries has large curvature: they should all be reliably described by general relativity.

However, this conclusion is incorrect!

The symplectic volume of this set of solutions is finite. Throats that are deeper than a certain critical depth are all part of the same $\hbar$-size cell in phase space: wave-functions cannot be localized on such geometries.

Quantum effects become highly macroscopic and make the physics of very deep throats nonlocal.

This is an entirely new breakdown of effective field theory.



Wave functions have support
on all these geometries

As a further consistency check of this picture, it also resolves an apparent inconsistency that emerges when embedding these geometries in AdS/CFT.

This is related to the fact that very deep throats seem to support a continuum of states as seen by an observer at infinity, while the field theories dual to AdS usually have a gap in the spectrum.

Bena, Wang, Warner

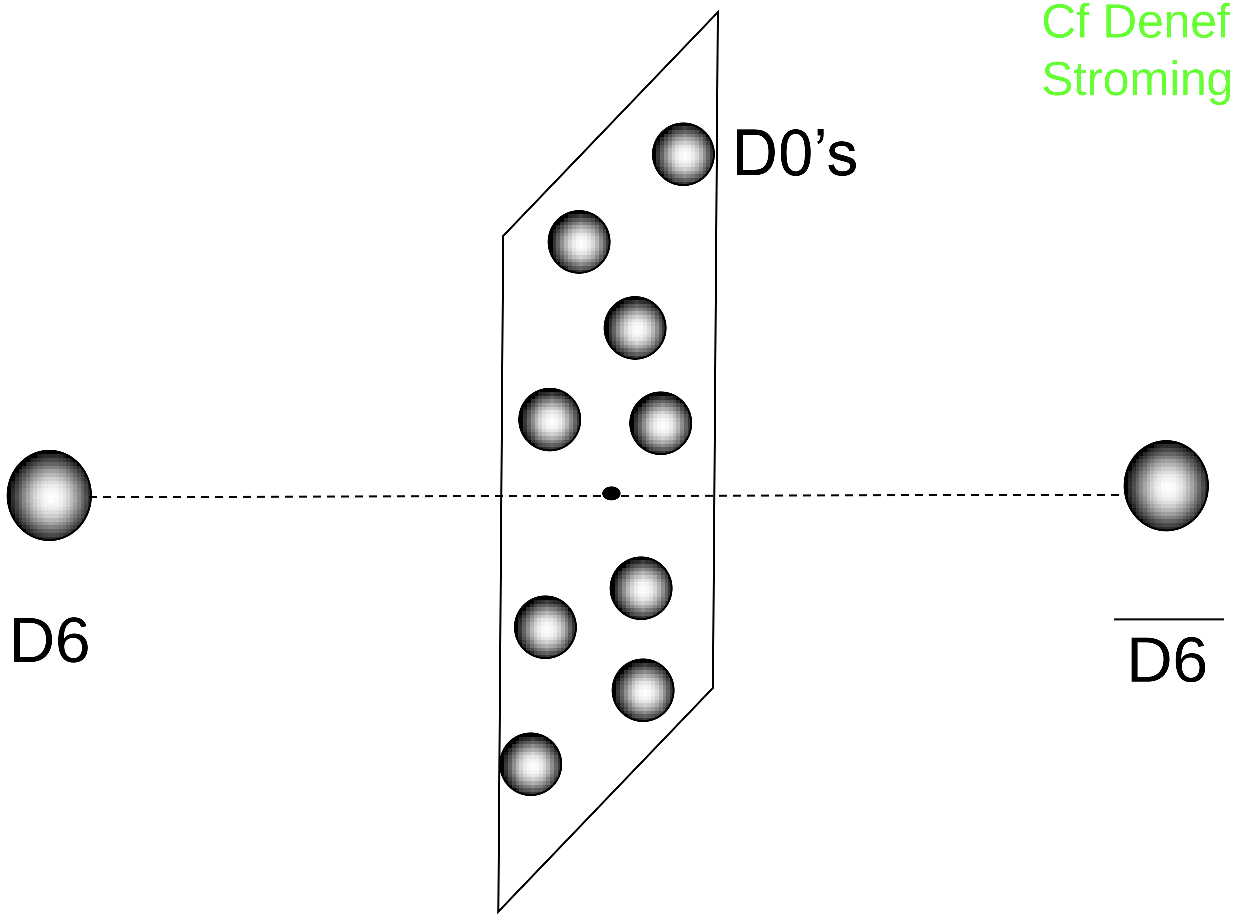
The gap one obtains agrees with the expected gap 1/c in the dual field theory (the dual 2d field theory appears after lifting the solutions to five dimensions and taking a decoupling limit).

Are there enough smooth supergravity solutions to account for the black hole entropy?

This is not a prediction of AdS/CFT.

Largest set we have been able to find:

Cf Denef, Gaiotto, Strominger, vdBleeken, Yin



In terms of standard 2d CFT quantum numbers we find the following number of states:

$$\left(\frac{3}{16}\zeta(3)L_0^2\right)^{1/3} \qquad L_0 \leq c/6$$

$$\left(\frac{3}{2}c\zeta(3)\left(L_0 - \frac{c}{12}\right)\right)^{1/3} \qquad L_0 \geq c/6$$

This is less than the black hole entropy, which scales as

$$S \sim 2\pi\left(\frac{c}{6}L_0\right)^{1/2}$$

Perhaps we are simply missing many solutions?

Try to find upper bound: count the number of states in a gas of BPS supergravitons. Result:

$$\left(\frac{3}{16}\zeta(3)L_0^2\right)^{1/3}$$

Clearly backreaction will be important. Difficult to deal with, but can impose one dynamical feature: stringy exclusion principle.

Maldacena, Strominger

We find precisely the same result as before:

$$\left(\frac{3}{16}\zeta(3)L_0^2\right)^{1/3} \qquad L_0 \leq c/6$$

$$\left(\frac{3}{2}c\zeta(3)\left(L_0 - \frac{c}{12}\right)\right)^{1/3} \qquad L_0 \geq c/6$$

- Strongly suggests supergravity is not sufficient to account for the entropy.
- Stringy exclusion principle is visible in classical supergravity (and not so stringy).

Summary:

- Gravitational entropy arises from coarse graining microstates
- For almost all states ψ, $\langle\psi|g_{\mu\nu}|\psi\rangle$ looks like a black hole geometry to great accuracy.
- For small black holes, can realize all states in terms of smooth supergravity solutions.
- For large black holes, need both smooth supergravity solutions as well as stringy degrees of freedom.
- Required non-locality arises because the fluctuations a $\langle\psi|\delta g^2_{\mu\nu}|\psi\rangle$ in the metric are much larger than naively expected near the horizon.
- Low energy effective field theory breaks down in a non-local way due to the same quantum effects

Towards more realistic black holes? Try to repeat the arguments e.g. for extremal Kerr. Dual to a CFT?

Guica, Hartman, Song, Strominger

The near-horizon limit of extremal Kerr is Bardeen, Horowitz

$$ds^2 = 2G_4 J \,\Box(\theta)^2 \left[-r^2 dt^2 + \frac{dr^2}{r^2} + d\theta^2 + \Lambda(\theta)^2 \,(d\varphi + r\,dt)^2 \right] ,$$

with

$$\Box(\theta)^2 = \frac{1+\cos^2\theta}{2}, \qquad \Lambda(\theta) = \frac{2\sin\theta}{1+\cos^2\theta} .$$

Diffeomorphisms of the form:

$$\zeta_\lambda = \lambda(\varphi)\partial_\varphi - r\lambda(\varphi)'\partial_r$$

Generate a Virasoro algebra → “chiral CFT”

Cardy reproduces entropy of extremal Kerr.

Best candidate dual is the DLCQ of a 2d CFT.

Can be made more precise in other cases. for example, the near horizon limit of extremal BTZ looks like

$$ds^2 = \frac{\ell^2}{4}\left(-y^2\, dt^2 + \frac{dy^2}{y^2}\right) + \ell^2 \left(d\phi - \frac{1}{2}\, y\, dt\right)^2$$

$$y \to \infty$$

This is AdS_2 with an electric field. Finite y slices, with precisely implement the definition of Seiberg for taking the DLCQ limit. The boundary indeed has a null circle. More precisely, the above metric is dual to

$$|\mathrm{thermal}\rangle_L \otimes |c/24\rangle_R$$

Features:

- The DLCQ operation freezes the right movers. The AdS_2 isometries are part of the right movers, and therefore all physical excitations are constant on AdS_2. All physical excitations involve φ.
- In the bulk this follows from an AdS-fragmentation argument.
- Presence of c/24 in the right-movers explains why Cardy still works (at least with susy: use elliptic genus). Just having a Virasoro is not enough.
- For near-horizon of Kerr, the AdS_2 part can also not be excited.

Amsel, Horowitz, Marolf, Roberts;
Diaz, Reall, Santor

The near-horizon geometry of extremal BTZ and extremal Kerr even share some dynamics:

$$ds^2 = L^2 \square^2 \left[-\partial_r \beta(t, r) \left(-dt^2 + dr^2 \right) + d\theta^2 + \Lambda^2 \left(d\varphi + \beta(t, r) dt \right)^2 \right]$$

4d Einstein eqns

Same equation of motion

3d Einstein eqn + cosm const

$$ds^2 = \frac{\ell^2}{4} \left[-\partial_r \beta(t, r) \left(-dt^2 + dr^2 \right) + \left(d\varphi + \beta(t, r) dt \right)^2 \right]$$

Central charge of CFT dual of BTZ also agrees with that of Kerr/CFT.

All this strongly suggests that the near-horizon geometry of extremal Kerr is dual to the DLCQ of some 2d CFT.

It would be very interesting to find the holographic dual of this 2d CFT.

For now will simply assume it exists and assume the mass gap is 1/c.

This puts the radius for quantum fluctuations in

$$ds^2 = 2G_4 J \Box(\theta)^2 \left[-r^2 dt^2 + \frac{dr^2}{r^2} + d\theta^2 + \Lambda(\theta)^2 \ (d\varphi + r dt)^2\right] \ ,$$

at r of order $1/J$. For GRS 1915+105, $J \sim 2 \times 10^{79}.$

This is a very small distance and seems related to quantization of angular momenta….

OUTLOOK:

- Several naïve black hole expectations have been made precise in extremal supersymmetric situations. (coarse graining microstates, typicality,….)

- Extend to other (cosmological) singularities? New interpretation of the Hartle-Hawking no-boundary proposal? Entropy of cosmological horizon is sum over smooth cosmologies?

- Extend to generic Schwarzschild black holes: AdS/CFT may allow us to make some progress in this direction.

- Can we understand anything about the stringy degrees of freedom that we need to account for the entropy of a large black hole?

- What happens when you fall into a black hole? Fluctuations in the metric are larger than you would naively expect and just enough for information to come out. Eventually classical geometry will cease to exist and you will thermalize……

- Explore the open string picture in more detail (this involves some quantum mechanical gauge theory and interesting connections between the Coulomb and Higgs branch)

- Finally, try to address more complicated dynamical black hole questions (see e.g. Erik Verlinde’s talk next week on holographic neutron stars – joint work with K. Papadodimas)