Holography from CFT

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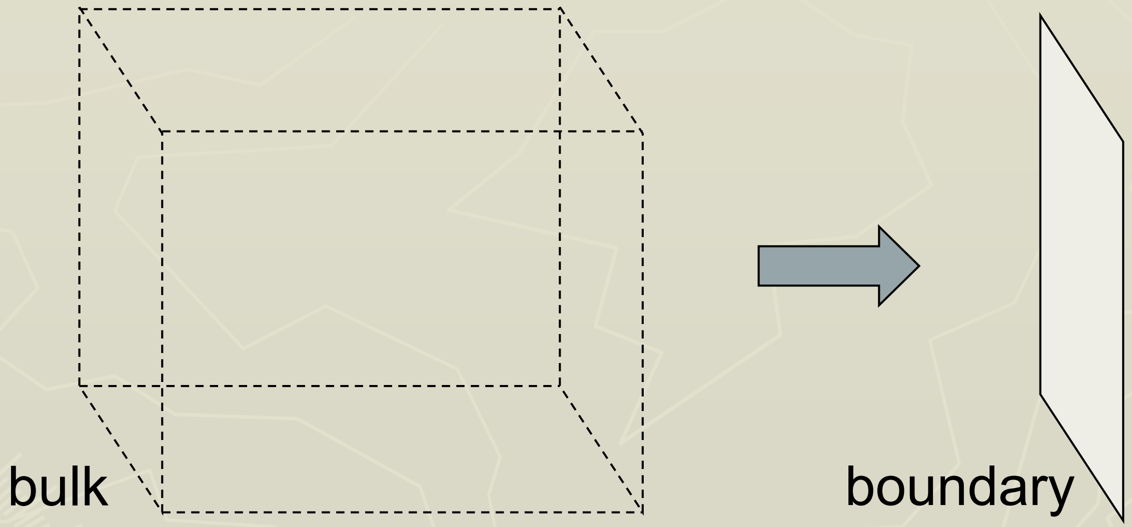
Holography maps a higher dimensional bulk into a lower dimensional boundary:



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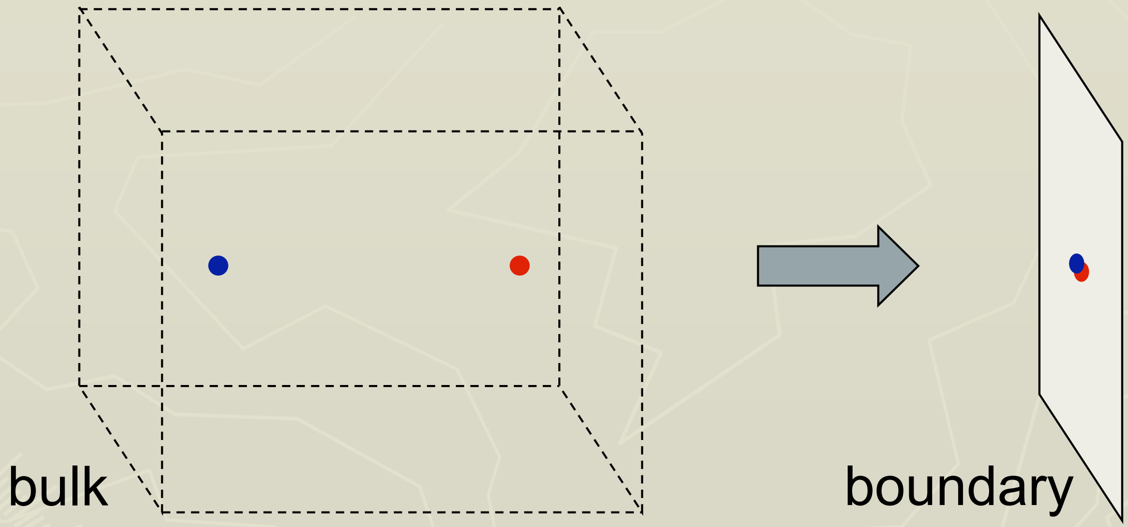
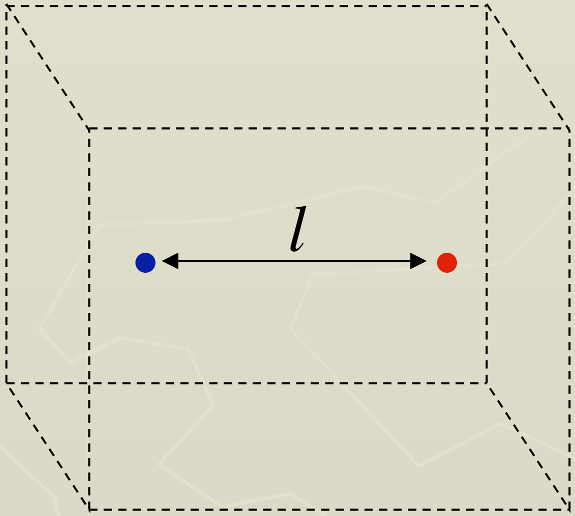
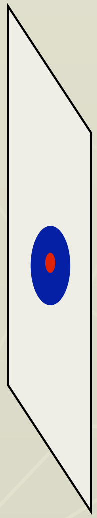


Excitations that are coincident in the boundary may be far apart in the bulk, and so could not interact directly. How can we see this from the point of view of the boundary CFT?

Part of the answer:



Holography maps $E \sim r/L^2_{\mathrm{AdS}}$, so locality in E implies locality in r:



diameter $\sim 1/E$

Such locality shows up in RG, color transparency, BFKL, but is only approximate, $\delta E/E \sim \delta r/r \sim O(1)$. In the metric $ds^2 = L^2_{\mathrm{AdS}} dr^2/r^2$ this is $l \sim L_{\mathrm{AdS}}$. However, we expect bulk physics to be local down to a fixed physical scale l_{string}, while $L_{\mathrm{AdS}} \sim \lambda^{1/4} l_{\mathrm{string}}$ can be parametrically larger.

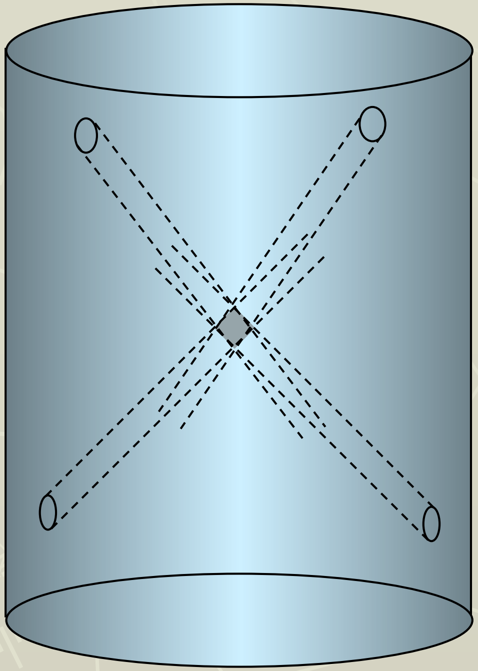
We are interested in this "sub-horizon holography (SHH)", as opposed to the weaker "horizon holography (HH)".

- What conditions does SHH imply on the CFT?

- How well is SHH understood? Do we even know that it is true?

Study locality via scattering in global AdS (JP, hep-th/0901076, Susskind hep-th/0901079). Focus by folding 4-pt function into localized sources Ψ_i:

$$\mathcal{A} = \left(\prod_{i=1}^{4} \int dt_i \, d^3\theta_i \, \Psi_i(t_i, \theta_i) \right) \left\langle \prod_{j=1}^{4} \mathcal{O}(t_j, \theta_j) \right\rangle$$



If we change slightly the Ψ_i, the wavepackets will miss and the amplitude will fall rapidly. From the point of view of the CFT, one is making only a small change, and the large effect is puzzling.

Some further developments:

I. Giddings, hep-th/9907129: if the tails of the Ψ_i overlap, there is a divergence from coincident operators. No problem: take finite range sources.

II. Gary, Giddings & Penedones, 0903.4437: express locality condition compactly in terms of a particular singularity in the 4-pt function.

III. Gary & Giddings, 0904.3544: find problems if orders of limits are taken differently than in JP/ Susskind.

We'll say more about II later, though will formulate locality in another way.

Tests of AdS/CFT:

- BPS states and amplitudes
- Symmetry breaking and RG flows
- Long strings, BMN, integrability
- Numerical: light-cone and Monte Carlo
- Comparison with experiment (RHIC)

These test 2-pt, 3-pt amplitudes, and horizon-scale holography, but not SHH.

Perhaps SHH is not true, and $\mathcal{N} = 4$ Yang-Mills is only dual to gravity/string theory smeared on the horizon scale?

A necessary condition for SSH is $l_{\text{string}} << L_{\text{AdS}}$. This implies a gap in the spectrum of operator dimensions:

$$\Delta(\Delta-4) = L^2_{\text{AdS}}\, m^2 : \quad \begin{array}{ll} \text{KK states:} & m \sim 1/L_{\text{AdS}},\ \ \Delta \sim 1 \\ \text{string states:} & m \sim 1/l_{\text{string}},\ \ \Delta \sim \lambda^{1/4} \end{array}$$

It is therefore a necessary condition that most* operators get large anomalous dimensions. This is a very striking property of the CFT.

Conjecture: it is also a sufficient condition, with \`most' = all single-trace operators of spin > 2.

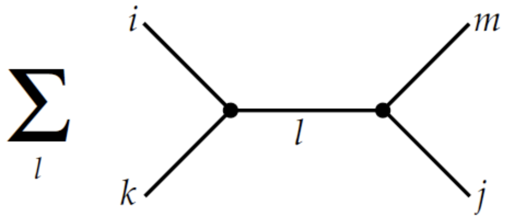
If true, this relates this mysterious property of the 4-pt function to a more intuitive property of the 2-pt function.

Constraints: OPE, conformal invariance, crossing, plus assumption about dimensions. Modular invariance won't be useful: what is it on $S^3 \times S^1$?

OPE: $$\mathcal{O}_i(x)\mathcal{O}_j(0) = \sum_k x^{\Delta_k - \Delta_i - \Delta_j} c^k_{ij} \mathcal{O}_k(0)$$

This converges within distance to nearest other operator. Within r_{conv}, the total contribution of high-dim operators is suppressed by $(x/r_{conv})^{\Delta_{large}}$. Going to the parametric limit $\Delta_{large} = \infty$, the OPE of low-dim operators closes on itself.

Using the OPE twice, can express the four-point function twice, can express it in terms of OPE coefficients:



$$\mathcal{A}_{ijkm} = \sum_l c^l{}_{ik} c_{ljm}$$

Conformal invariance: The operators in the four-point function can be brought to a 2-d plane and then to standard positions $0, 1, \infty, z$. Further, the contributions of conformal descendants can be related to those of their primary, so the sum reduces to the primaries.

The simplest CFT would contain only $T_{\mu\nu}$ (corresponding to a bulk with only gravity*). We will take an even simpler model, with only a scalar O of dimension Δ. (We expect soon to extend to scalar correlators in a CFT of $O + T_{\mu\nu}$). The low dimension primaries include the double-trace operators,

$$\mathcal{O}_{n,l} \equiv \mathcal{O}\overleftrightarrow{\partial}_{\mu_1} \ldots \overleftrightarrow{\partial}_{\mu_l} (\overleftrightarrow{\partial}_\nu \overleftrightarrow{\partial}^\nu)^n \mathcal{O} - \text{traces} \qquad \overleftrightarrow{\partial} = \overrightarrow{\partial} - \overleftarrow{\partial}$$

(traceless on μ). Spin = l (must be even),

$$\Delta(n,l) = 2\Delta + 2n + l + O(1/N^2).$$

We will work to first nontrivial order in $1/N^2$, so higher-trace operators do not enter.

$$\langle \mathcal{O}(0)\mathcal{O}(z,\overline{z})\mathcal{O}(1)\mathcal{O}(\infty)\rangle \equiv \mathcal{A}(z,\overline{z})$$

$$= \frac{1}{(z\overline{z})^{\Delta}} + \sum_{n=0}^{\infty}\sum_{l=0}^{\infty} p(n,l)\,\frac{g_{\Delta(n,l),l}(z,\overline{z})}{(z\overline{z})^{\Delta}}$$

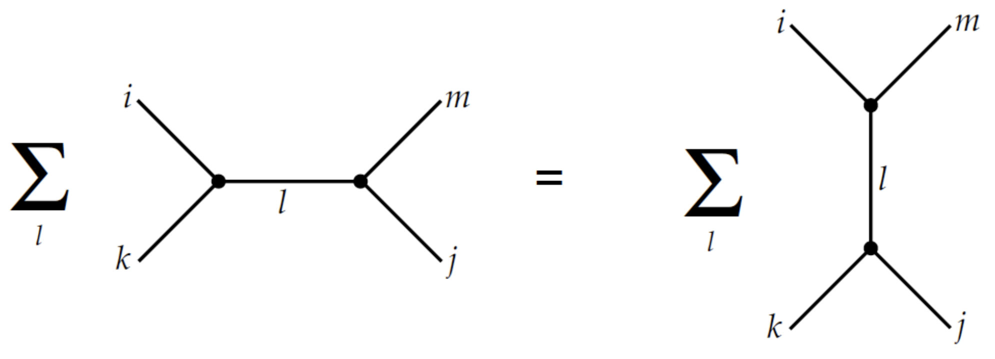
$$g_{E,l}(z,\overline{z}) = \frac{z\overline{z}}{z-\overline{z}}\left[k(E+l,z)\,k(E-l-2,\overline{z}) - k(E+l,\overline{z})\,k(E-l-2,z)\right]$$

$$k(\beta,z) = z^{\frac{\beta}{2}} F_{\beta/2}(z)\ , \quad F_{\beta/2}(z) = F\left(\frac{\beta}{2},\frac{\beta}{2},\beta,z\right)$$

We impose Z_2 symmetry $O \to -O$, so O does not appear in the sum.

$p(n,l)$, $\Delta(n,l)$ are unknowns to be determined by crossing.

Crossing:



$$\mathcal{A}(z, \bar{z}) = \mathcal{A}(1-z, 1-\bar{z}).$$

How constraining is this?* The equation is function of one complex number or, by separate analytic continuation in Re(z) and Im(z), a holomorphic function of two variables. By expanding in suitable complete sets this becomes a equation indexed by two integers, the same as the unknowns $p(n,l)$, $\Delta(n,l)$. Could be 1 solution, or 0, or ∞!

*cf. Rattazzi, Rychkov, Tonni, Vichi, arXiv:0807.0004

We will solve perturbative in $1/N^2$ (implicit already),

$$p(n,l) = p_0(n,l) + p_1(n,l)/N^2 + \dots$$
$$\Delta(n,l) = 2\Delta + 2n + l + \gamma_1(n,l)/N^2 + \dots$$

$p_0(n,l)$ is obtained from large-N factorization, and $1/N^2$ terms give the leading interaction.

There is an infinite number of solutions to the crossing equation at $1/N^2$: any local quartic Lagrangian in the bulk defines, via the AdS/CFT dictionary, CFT correlators that satisfy all axioms. The conjecture to be tested is that *all* solutions are obtained in this way.

Counting: if we restrict the growth of the interaction at high energy to a given power, there are a finite number of local bulk Lagrangians. Thus we can just count the number of solutions to crossing with the corresponding restriction, and see if they always match.

Similar, but simpler, is to restrict the spin l appearing in the intermediate state. E.g. ϕ^4 can create/destroy only spin 0; $\phi^2\,\partial_\mu\partial_\nu\phi\,\partial^\mu\partial^\nu\phi$ and $\phi^2\,\partial_\mu\partial_\nu\partial_\rho\phi\,\partial^\mu\partial^\nu\partial^\rho\phi$ can create/destroy spin 0, 2, etc. We can correspondingly restrict l appearing in CFT sum to the same maximum value *L.*

Expanding in a complete set, the crossing equation becomes

$$\sum_{\substack{l=0 \\ \text{even}}}^{L} \frac{\gamma_1(p,l)J(p+l,q)+\gamma_1(p-l,l)J(p-l,q)}{1+\delta_{l,0}} = (p \leftrightarrow q)$$

$J(p,q)$ is an overlap integral of hypergeometric functions. There is a similar equation for $p_1(n,l)$.

E.g. for L = 0, $\gamma_1(p,0)J(p,q) = \gamma_1(q,0)J(q,p)$

For $q = 0$ this implies

$$\gamma_1(p,0) = \gamma_1(0,0)\frac{J(0,p)}{J(p,0)}$$

so there is at most one solution normalized by $\gamma_1(0,0)$. Higher q give additional constraints, so it is surprising that there are any solutions at all, but there must be at least one coming from bulk interaction ϕ^4. This implies true, but non-obvious, relations among the $J(p,q)$.

Strategy: for each L, we get a lower bound on the number of solutions by counting bulk interactions, and an upper bound from crossing.

Crossing:

$$\sum_{\substack{l=0\\ \text{even}}}^{L} \frac{\gamma_1(p,l)J(p+l,q)+\gamma_1(p-l,l)J(p-l,q)}{1+\delta_{l,0}} = (p \leftrightarrow q)$$

Fixing p and letting $q = 0,1,\ldots, p-1$, we can solve for $\gamma_1(p,0)$, $\gamma_1(p,2)$, …, $\gamma_1(p,l_{\max})$, where $l_{\max}$ = smaller of $p-1$ and L. This leaves *at most* $(L+2)(L+4)/8$ independent parameters.

Local bulk interactions: up to field redefinition and integration by parts, independent terms correspond to flat space S-matrices

$$s^a t^a u^c + s^c t^a u^a + s^c t^a u^a,$$
$$a = 0, 1, \ldots, L/2 \quad c = 0, 1, \ldots, a.$$

($s + t + u = 4m^2$ allows to set largest exponents equal).

The number is again $(L+2)(L+4)/8$, so the conjecture is *true* in this model: all solutions to crossing are local in the bulk.

Further results

• Can give closed form solutions in many cases.

• Equations become degenerate in special cases where extra marginal operators exist (e.g. O^4 with $\Delta = 4$ in $d = 2$.

• $L = \infty$ solutions can all be obtained as limits of finite L solutions.

• Apparent counterexample: infinite sum of local interactions = non-local interaction. True, but expect the range to be string scale, $1/\Delta_{\text{large}}$ --- necessary for absence of divergences at $O(1/N^4)$.

Future directions, soon:

- Multiple scalars
- Elimination of Z_2 symmetry
- Inclusion of $T_{\mu\nu}$ in intermediate state $\rightarrow$ full-fledged CFT.

Future directions, longer term:

- External $T_{\mu\nu}$ operators
- Confirmation of conjecture in its general form
- Correlators of stringy operators
- Black hole states
- Nonconformal theories

Lessons:

• Closes potential loophole in AdS/CFT: mysterious property of four-point function follows from simple property of operator dimensions

• Allows to derive AdS/CFT without a stringy construction, assuming only planarity and a restricted spectrum of low dimension operators

• Extension of AdS/CFT to cosmology, e.g. dS/CFT…