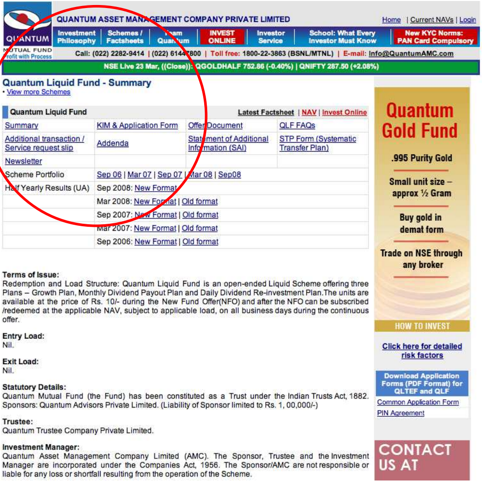
Holographic Descriptions of Quantum Liquids

A review talk about applications of string theory to strongly-coupled-field-theory phenomena *at finite density*

what’s a ‘quantum liquid’ ?



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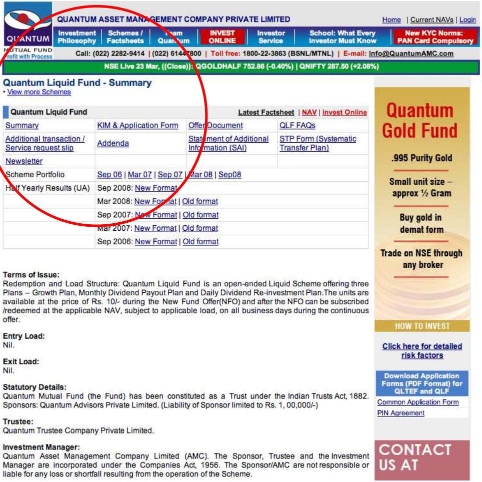
what's a 'quantum liquid'?



what's a 'quantum liquid'?

Liquid:

A phase of matter where

• translational symmetry isn't broken

(not a solid)

• interactions are important, $\exists$ collective motion, density is correlated

(not a gas)

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Quantum Liquid:
quantum correlations are important.

Quantum liquids

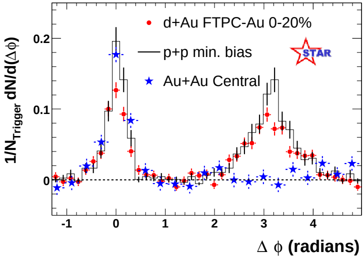
- electrons in a metal
- He_3, He_4
- quark-gluon plasma (QGP),
- ultra-cold atoms (definitions later)
- electrons in cuprates, heavy-fermion materials, organics...

For some of these, our questions are answered by weakly-coupled bosons or Landau's Fermi liquid theory. (more later)

In most cases: (QCD, cold atoms at unitarity, all of cond mat)
we know the microscopic description, but don't know how to use it, because the microscopic constituents are strongly interacting.
(and hence not the right variables.)
To see that the QGP at RHIC is not so amenable to perturbative QCD...

Quark-gluon plasma is strongly coupled

QGP is strongly coupled: a liquid, not a gas. [RHIC]

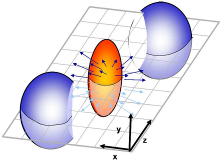


1. It is opaque:

2. It exhibits rapid thermalization,

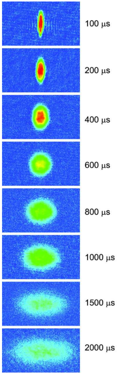
rapid hydro-ization to a fluid with very low viscosity.

It exhibits collective motion ('elliptic flow'):



$\longrightarrow$

[O'Hara et al]



Nearly-perfect fluids

Better (less viscous) fluids are more strongly coupled:

e.g. Baym-Pethick, Arnold-Moore-Yaffe]

$$\left(\frac{\eta}{s}\right)_{perturbative} \sim \frac{1}{g^4 \ln\left(\frac{1}{g^2}\right)}$$

$s =$ entropy density.

The two most perfect fluids that exist:

quark-gluon plasma
length scale: $<$ fermi
temperature/energy: 300 MeV
size of experiment: km
$\frac{\eta}{s} < 0.3$

lithium atoms at unitarity
length scale: micrometer
temp./energy: nanokelvin
size of experiment: meter
$\frac{\eta}{s} \sim 0.4$

What do we want to know?

- thermodynamics, phase diagram: (What's the $T = 0$ ground state? What's the equation of motion at finite T? What are the right low-energy degrees of freedom?)
- hydrodynamics: (transport coefficients, nonlinear flows)
- finite-wavenumber probes: (finite-k linear response (e.g. ARPES), hard probes, non-hydrodynamic excitations)

The AdS/CFT correspondence can be used to study strongly-coupled many-body systems.

Gravity limit, when valid, gives an answer to what degrees of freedom to use, and lets us study the above observables.

Compare to other approaches

Perturbation theory: even in asymptotically free theories, the liquid phase is not the right regime.

Lattice calculations good for static things. (but see [H. Meyer])

Also, $\exists$ sign problem at finite density of fermions.

Important disclaimer: so far we don't have a gravity dual for a QFT which exists in nature.

Immediate progress relies on 'universality'.

Practical note: In a relativistic QFT, the vacuum is also interesting, and can also maybe be studied using the stringy dual.

Vacuum is hard, finite density configurations exhibit more universal behavior.

The QGP is a deconfined phase, and hence it doesn't matter that one is studying (the dual of) a gauge theory that never confines at any scale.

One can hope that there is some universal physics of deconfined gauge theory plasma, or perhaps even strongly-coupled QFT stuff.

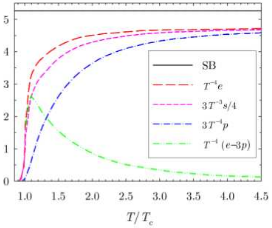
Plan for this talk

1. introduction (over)
2. relativistic CFT liquid (towards QGP)
3. Galilean CFT liquid (towards cold atoms at unitarity)
4. relativistic CFT at finite density of some conserved charge
   (towards non-BCS superconductors and non-Fermi liquids)
5. grandiose conclusions

Relativistic CFT plasma (towards QGP)

focus on results (not mine!) and lessons about where this approach can succeed.

approximate QGP as CFT plasma

lattice: 

[Boyd et al]

stefan-boltzmann: energy $=$ 3 pressure $= cT^4$: scale invariance
E/T^4 from lattice QCD $\sim 0.8 E/T^4$ of free QCD [Boyd et al]

try to approximate as CFT with a gravity dual.
E/T^4 from AdS/CFT $= 0.75 E/T^4$ of free SYM [Klebanov et al]
lesson 1 from AdS/CFT: thermodynamics not a good measure of strong coupling

Transport is very different at strong coupling

[Son,Starinets,Policastro,Kovtun]

$\frac{\eta}{s} = \infty$ from free gauge theory. $\frac{\eta}{s} = \frac{1}{4\pi}$ from AdS/CFT.

Kubo Formula: $\eta = \lim_{\omega \to 0} \lim_{k \to 0} \frac{1}{\omega} \mathrm{Im} \langle T^x_y T^x_y \rangle$

T^x_y couples to $\phi \equiv h^y_x$ in the bulk: $\Box \phi = 0$

Idea: Graviton absorption cross section $\propto$ area of the horizon.

CFT wasn't the crucial assumption for the viscosity result:

$$\text{let } \xi \equiv \frac{\pi}{\phi} \stackrel{r \to \infty}{\to} G$$

($\pi \equiv \frac{\partial L}{\partial(\partial_r \phi)}$, canonical momentum) [Iqbal-Liu]

$$\Box \phi = 0 \quad \Rightarrow \quad \partial_r \xi|_{k=0} = 0 \quad \Rightarrow \quad G = \xi(\text{horizon})$$

Membrane paradigm is correct for some things: those associated with massless bulk fields.

$\frac{\eta}{s} = \frac{1}{4\pi}$ for *Einstein* gravity.

Universality

It's not a lower bound:

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 - 4\lambda_{GB}\right)$$

[Brigante-Liu-Myers-Shenker-Yaida, Kats-Petrov, Buchel-Myers-Sinha]

For $\lambda_{GB} > \frac{9}{100}$, the dual CFT violates microcausality.

This coincides with a bound on central charges $\frac{a}{c}$ for $\mathcal{N} = 1$ SCFTs.

[Hofman-Maldacena]

currently viable lower bound:

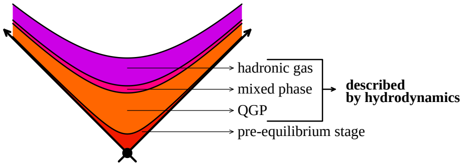
$$\frac{\eta}{s} \geq \frac{1}{4\pi}\frac{16}{25}$$

lesson 2 from AdS/CFT:

$\frac{1}{4\pi}$ is a reasonable value for $\frac{\eta}{s}$ in a strongly-coupled liquid.

Flowing quantum liquids

Expansion of the plasma is important for what comes out at RHIC:



From [Heller-Janik-Peschanski review: 0811.3113]

[Janik-Peschanski]: approximate boost-invariant flow

infinite in space, translation-invariant in $x^{1,2}$, expands with $v^3 \propto x^3$

A derivation of the statement that hydro is a good description of the late stages of this plasma: the hydrodynamic behavior is an outcome of a bulk calculation.

Solution: a black brane with a time-dependent horizon radius: the movement of the horizon into the bulk of AdS implements the cooling of the expanding gauge theory plasma.

Demanding regularity of solution reproduces the same values for E/P, $\frac{\eta}{s}$ ... previously found for small perturbations around *static* fluid.

Flowing quantum liquids

More generally: [Bhattacharyya, Hubeny, Minwalla, Rangamani, et seq]

non-static black hole horizon in AdS

$\leftrightarrow$ fluid configuration in gauge theory

Both sides: long-wavelength expansion in terms of variables adapted to conserved quantities.

$$\text{static BH}: \; ds^2 = -2dtdr - r^2\left(1 - \frac{T^4}{\pi^4 r^4}\right)dt^2 + r^2\eta_{ij}dx^i dx^j \; .$$

Slowly-varying boost $u(x)$:

$$ds^2 = -2u_\mu dx^\mu dr - r^2\left(1 - \frac{T^4}{\pi^4 r^4}\right)u_\mu u_\nu dx^\mu dx^\nu + r^2(\eta_{\mu\nu} + u_\mu u_\nu)dx^\mu dx^\nu$$

einstein equations for $u(x)$ $\leftrightarrow$ Navier-Stokes equations

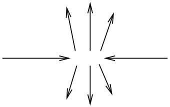
There are people who might be disappointed by this result: you might have thought that gravity might conceivably average over (turbulent) flows, like it averages over microstates. directly produce a picture of the ensemble of flows, Kolmogorov 5/3 law....

Approach to equilibrium

These results show that *eventually* hydro is a good description of such a state of matter.

Important question for interpreting RHIC data: how long does it take before hydro sets in?

initially in gold-gold collision: momentum-space distribution is very anisotropic:



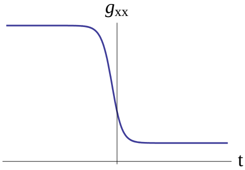
after time τ: locally thermal distribution and hydrodynamics.

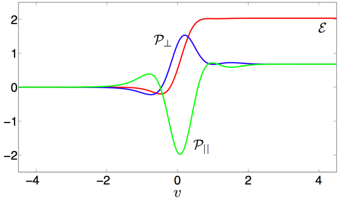
τ at RHIC: much smaller than indicated by perturbation theory.
(τ affects measurement of viscosity:
good elliptic flow requires both low η and early applicability of hydro)

Approach to equilibrium

bulk picture: dynamics of gravitational collapse, ring-down

1. quasinormal modes of a small BH [Freiss et al, hep-th/0611005] $\tau \sim \frac{1}{8T_{peak}}$.
2. far-from equilibrium processes: [Chesler-Yaffe, 0812.2053] (PDEs!)

input: 

output: 

black hole forms from vacuum initial conditions.

brutally brief summary: all relaxation timescales $\tau \simeq T^{-1}$.

3. model pancaked nuclei by colliding shock-waves in AdS

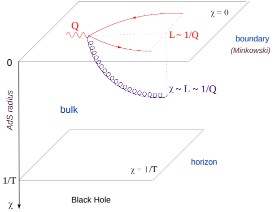
[Janik-Peschanski, ..., Grumiller-Romatschke, Kovchegov et al]

ambitious idea: estimate of number of particles produced in collision from area of resulting horizon: [Gubser-Pufu-Yarom, 0805.1511]

$N \sim \frac{1}{7.5} S_{BH}$

Hard probes of gauge theory plasma

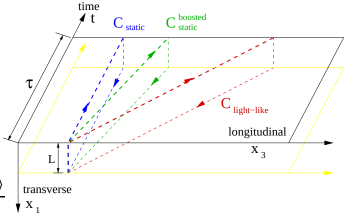
Can we test the opacity of the plasma?
$\exists$ many possible hard probes of QGP:
heavy quarks, light quarks, R-currents, mesons, baryons, glueballs...
Calculations involving probes with energies $\gg T$ show that a strongly-coupled CFT doesn't make jets. [Polchinski-Strassler, Hatta-Iancu-Mueller, Hofman-Maldacena]
Universal picture of energy loss of $\omega \gg T$ probes through 'democratic' parton cascade [Hatta, Iancu, Mueller]:



Hard probes of gauge theory plasma: light quarks

The fact that jets get made at RHIC depends on asymptotic freedom, but we can put them in and study their evolution through the medium.

[BDMPS, Kovner-Wiedemann] can rewrite amplitude for energy loss of an ultra-relativistic parton in terms of partially-lightlike wilson loop:



'jet-quenching parameter': $\hat{q} \equiv \frac{d\langle (p_\perp)^2 \rangle}{dx}$

$$\langle W[C_{light-like}] \rangle = e^{-\frac{1}{4\sqrt{2}} \hat{q} L^- L^2}$$

evaluate loop using AdS/CFT [Liu-Rajagopal-Wiedemann]

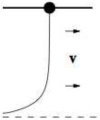
$\hat{q} \sim 5 - 15\ \mathrm{GeV}^2/\mathrm{fm}$ to reproduce RHIC data. $\hat{q}_{pert\ th}$ too small.

$$\hat{q}_{AdS} = \frac{\pi^{3/2}\Gamma(3/4)}{\Gamma(5/4)} \sqrt{\lambda} T^3 \sim 5$$

not proportional to s, or number of scatterers.

Hard probes of gauge theory plasma: heavy quarks

Drag an external quark through the plasma at constant velocity:



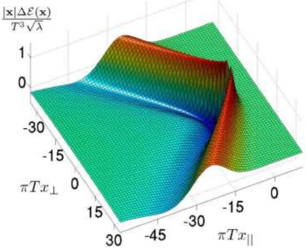
drag coefficient is momentum-independent.

$$\frac{dE}{dx} = -\frac{\pi}{2}\sqrt{\lambda}T^2 \frac{v}{\sqrt{1-v^2}}$$ [Karch et al, Gubser et al, Casalderrey-Solana-Teaney...]

diffusion of flavor: $D = \frac{2}{\pi T \sqrt{\lambda}}$ (Contrast with $D_{transverse\ momentum} \sim \frac{1}{4\pi T}$.)

where does the energy go? evaluate $\langle T_{\mu\nu} \rangle$ [Chesler-Yaffe, Gubser-Pufu-Yarom]

mach cone of trailing string:



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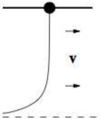

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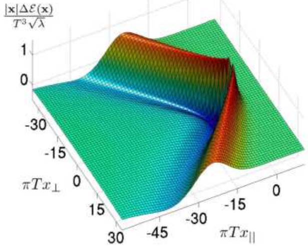
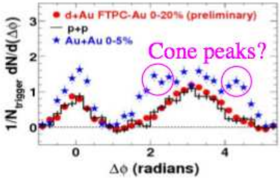


$\overset{?}{\Longrightarrow}$



from [Torrieri et al] $\Longrightarrow$ bumps in two-jet correlations?

but:

heavy-ion colliders are unwieldy.

The QGP lasts for a time of order a few light-crossing times of a nucleus.

Wouldn't it be nice if we could do a quantum gravity experiment on a table top...

Galilean CFT Liquid (towards cold atoms at unitarity)

Note restriction to Gal.-invariance $\partial_t - \vec{\nabla}^2$

distinct from: Lifshitz-like fixed points $\partial_t^2 - (\vec{\nabla}^2)^2$

are not relativistic, but have antiparticles.

gravity duals of those: S. Kachru, X. Liu, M. Mulligan, 0808.1725

Cold atoms at unitarity

Strongly-coupled Galilean-invariant CFTs exist, and people can make them in relatively small laboratories [Zwierlein et al, Hulet et al, Thomas et al]

The symmetries are those of the free schrodinger equation:

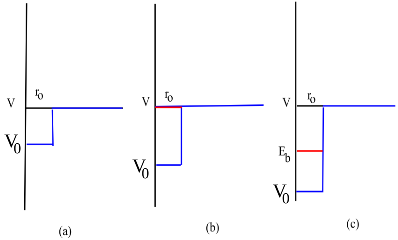
$$i\partial_t\psi = \partial_x^2\psi$$

$$\vec{x} \to \lambda\vec{x}, \quad t \to \lambda^z t, \quad z = 2$$

This is a *free* 'NRCFT'.

towards interacting NRCFT

Consider nonrelativistic fermionic particles ('atoms') interacting via some short-range attractive two-body potential $V(r)$, *e.g.*:



a) $V_0 < 1/mr_0^2$: No bound state
b) $V_0 = 1/mr_0^2$: Bound state with zero energy
c) $V_0 > 1/mr_0^2$: At least one bound state with non-zero energy.
scattering length $|a| \sim$ size of bound-state wavefunction.
case b corresponds to infinite scattering length.

unitarity limit, $a \to \infty$

When atoms collide, they spend a long time considering whether or not to bind. σ saturates bound on scattering cross section from (s-wave) unitarity (*i.e.* $a \to \infty$ is the strongest possible coupling).

For physics at wavelengths $\gg r_0$, there is no scale in the problem.

dilatations: $a \to \lambda a, r_0 \to \lambda r_0$. $a = \infty, r_0 = 0$ is a fixed point.

In this limit, the details of the potential are irrelevant, can choose $V = \delta^d(r)$:

$$\mathcal{L} \sim \bar{\psi}_\alpha i\partial_t \psi^\alpha - \bar{\psi}_\alpha \frac{\vec{\nabla}^2}{2M}\psi^\alpha + g\bar{\psi}_\uparrow\psi_\uparrow\bar{\psi}_\downarrow\psi_\downarrow$$

g has a fixed point where $a \gg \text{interparticle dist} \gg r_0$

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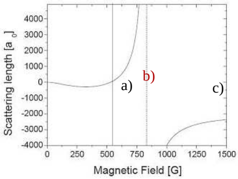

Lithium atoms have a boundstate with a different magnetic moment

Zeeman effect $\Longrightarrow$ scattering length can be controlled using an external magnetic field.

Strongly-coupled NRCFT

The fixed-point theory ("fermions at unitarity") is a strongly-coupled nonrelativistic CFT [Nishida-Son].

universality: it also describes neutron-neutron scattering [Mehen-Stewart-Wise]

Two-body physics is completely solved. Many body physics is mysterious.

Experiments: very low viscosity, $\frac{\eta}{s} \sim \frac{5}{4\pi}$ [Thomas, Schafer]

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AdS/CFT? Clearly we can't approximate it as a *relativistic* CFT.

Different hydro: conserved particle number.

A holographic description?

Method of the missing box [Coleman]

AdS : relativistic CFT

A holographic description?

Method of the missing box [Coleman]

AdS : relativistic CFT

$\boxed{\text{“}\textit{schrodinger spacetime}\text{”}}$: galilean-invariant CFT

A metric whose isometry group is the schrödinger group:

$$L^{-2} ds^2_{\mathrm{Schr}^z_d} = \frac{2d\xi dt + d\vec{x}^2 + dr^2}{r^2} - 2\beta^2 \frac{dt^2}{r^{2z}}$$

‘schrödinger space’ [Son; Balasubramanian, JM]

$z = 2$ has Schrödinger symmetry

Discrete spectrum of $\hat{N} = i\partial_\xi$ requires compact $\xi \simeq \xi + L_\xi$

$\frac{\beta}{L_\xi}$ is an invariant parameter $\sim M$.

Supported by a massive gauge field $A = r^{-z} dt, \quad m_A^2 = -d(d+z)$.

Can extend GKPW prescription.

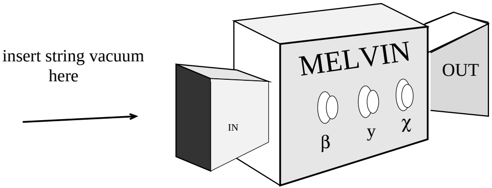
But: the vacuum of a galilean-invariant field theory is extremely boring: no antiparticles! no stuff!

How to add stuff?

A holographic description of more than zero atoms?

A black hole (BH) in schrodinger spacetime. [A. Adams, K. Balasubramanian, JM; Maldacena et al; Rangamani et al]

Here, string theory was useful: A solution-generating machine named Melvin [Ganor et al]



IN: $AdS_5 \times S^5$ OUT: schrodinger $\times S^5$

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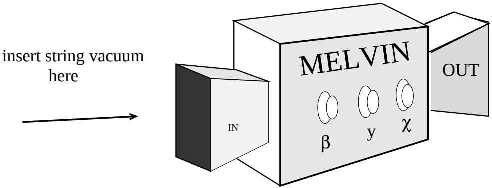


IN: $AdS_5 \times S^5$ OUT: schrodinger $\times S^5$

IN: AdS_5 BH $\times S^5$ OUT: schrodinger BH $\times$ squashed S^5

This black hole gives the thermo and hydro of some NRCFT ('dipole theory' [Ganor et al]).

$$\text{Einstein gravity} \implies \frac{\eta}{s} = \frac{1}{4\pi}.$$

Satisfies laws of thermodynamics, correct scaling laws, correct kubo relations

[Rangamani-Ross-Son, McEntee-JM-Nickel]

But it's a different class from unitary fermions:

$$F \sim \frac{T^4}{\mu^2}, \quad \mu < 0$$

This black hole gives the thermo and hydro of some NRCFT ('dipole theory' [Ganor et al]).

$$\text{Einstein gravity} \implies \frac{\eta}{s} = \frac{1}{4\pi}.$$

Satisfies laws of thermodynamics, correct scaling laws, correct kubo relations

[Rangamani-Ross-Son, McEntee-JM-Nickel]

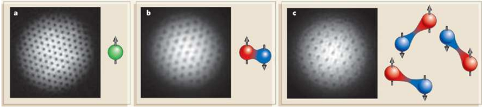
But it's a different class from unitary fermions:

$$F \sim \frac{T^4}{\mu^2}, \quad \mu < 0$$

Hope for future:

This is because of DLCQ Clear from *e.g.* [Barbon-Fuertes]

Unnecessary assumption: All of Schrod must be realized geometrically $\implies \exists\ \xi$-direction, spectrum of $\hat{N} = \mathbb{Z}$.

Superfluid? 

Should break ξ-isometry (like Gregory-Laflamme, Klebanov-Strassler), cut off IR geometry.

An easier way to choose a rest frame (and break conformal invariance):

Relativistic CFT at finite density

(towards non-BCS superconductors and non-Fermi liquids)

Charged black hole in AdS

Consider any relativistic CFT_d with

- an Einstein gravity dual
- a conserved $U(1)$ current ($\to$ gauge field in the bulk).

An ensemble with finite chemical potential for that current is described by the AdS Reissner-Nordstrom black hole:

$$ds^2 = \frac{r^2}{R^2}\left(-fdt^2 + d\vec{x}^2\right) + R^2 \frac{dr^2}{r^2 f}, \qquad A = \mu\left(1 - \left(\frac{r_0}{r}\right)^{d-2}\right) dt$$

$$f(r) = 1 + \frac{Q^2}{r^{2d-2}} - \frac{M}{r^d}, \;\; f(r_0) = 0, \;\; \mu = \frac{g_F Q}{c_d R^2 r_0^{d-1}}, \;\; c_d \equiv \sqrt{\frac{2(d-2)}{d-1}}$$

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Consider any relativistic CFT$_d$ with

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Thermodynamics at low temperature: $\frac{\mu}{T}$ is a dimensionless parameter.

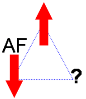
$$T = \frac{d r_0}{4\pi R^2}\left(1 - \frac{(d-2)Q^2}{d r_0^{2d-2}}\right)$$

$$s(T=0) = 2\pi e_d \rho \;\neq 0, \quad e_d \equiv \frac{g_F}{\sqrt{2d(d-1)}}$$

Emergent quantum criticality

Such a large low-energy density of states is reminiscent of systems

with 'frustration':



[from Wikipedia]

many approximate groundstates, with energies split by a small amount ($\ll \mu$).

The degrees of freedom describing these groundstates are encoded by the near-horizon region, which is $AdS_2 \times \mathbb{R}^{d-1}$:

$$ds^2 = \frac{R_2^2}{\sigma^2}\left(-d\tau^2 + d\sigma^2\right) + \frac{r_0^2}{R^2} d\vec{x}^2, \qquad A_\tau = \frac{e_d}{\sigma}.$$

The scale invariance is **emergent**. μ broke the UV scale invariance.

Some dual CFT describes the many possible groundstates.

e.g. [Lu-Mei-Pope-Vasquez-Poritz]

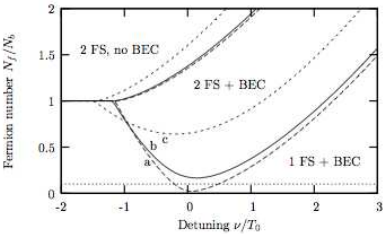
Note: Fragmentation issues [Maldacena-Michelson-Strominger] ameliorated by infinite volume of $\mathbb{R}^{d-1}$.

Bose-fermi mixtures

In general, the charge will be carried by both bosons and fermions in the boundary theory.

∃ other examples where only fermions are charged: in 3d [Emparan], with branes [Sakai-Sugimoto, Kulaxizi-Parnachev]

$\Longrightarrow$ Bose-Fermi mixture.



from [Powell-Sachdev-Buchler]

Probes of this groundstate

One-body green's functions of charged (composite) operators:

$$G_R(t, \vec{x}) = i\theta(t)\langle[\mathcal{O}_q^\dagger(t, \vec{x}), \mathcal{O}_q(0, 0)]_\pm\rangle$$

($\pm$ for $\mathcal{O}$ fermionic or bosonic.)

To study ground-state properties, look for poles near zero-frequency:

Can rewrite Laplace and Dirac equations as schodinger problems.

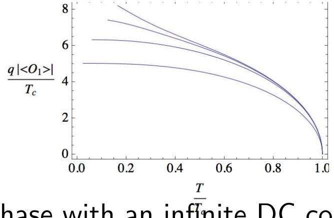
Poles of $G \leftrightarrow$ *normalizible* $E = 0$ boundstates.

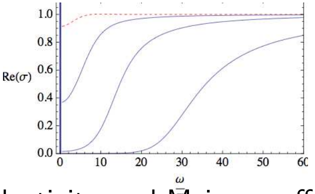
spinor: poles in LHP always [Faulkner-Liu-JM-Vegh, to appear]

scalar: $\exists$ poles in UHP $\implies$ growing modes of charged operator: holographic superconductor [Gubser, Hartnoll-Herzog-Horowitz...]

Holographic superconductors

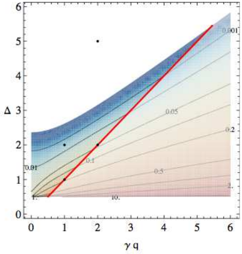
Second-order phase transition (with mean-field exponents):





to a phase with an infinite DC conductivity, and Meissner effect.

• A description of a superfluid (or weakly-gauged superconductor) without quasiparticles.



[Denef-Hartnoll]

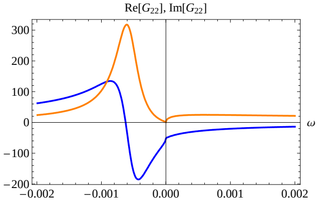
• Sometimes $\langle \mathcal{O} \rangle = 0$!

(vs: a weakly-coupled charged boson at $\mu \neq 0$ will condense.)

More on holographic superconductors from Sean.

Fermi surfaces from holography

A surface of $\omega = 0$ poles of the spinor G_R at $|\vec{k}| = k_F$:



[Hong Liu, JM, David Vegh, 0903.2477
Tom Faulkner, HL, JM, DV, 0906.abcd
TF, Nabil Iqbal, HL, JM, DV, in progress
also: Sung-Sik Lee, 0809.3402

Cubrovic-Zaanen-Schalm, 0904.1933]

first: context.

Ubiquity of Landau fermi liquid

Basic question: what is the ground state of a nonzero density of interacting fermions? ($\exists$ sign problem)

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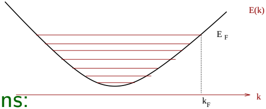
Lore: it's always a fermi liquid [Landau 50s].

Recall: if we had *free* fermions, we would fill single-particle energy levels $E(k)$



until we ran out of fermions: Low-energy excitations: remove or add electrons near the fermi surface E_F, k_F.

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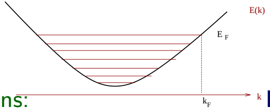
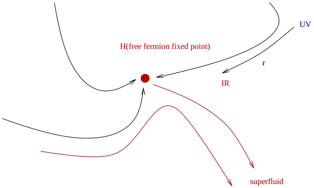


until we ran out of fermions: Low-energy excitations: remove or add electrons near the fermi surface E_F, k_F.

Claim [Landau]: The low-energy excitations of the interacting theory are still weakly-interacting fermionic, charged 'quasiparticles'

(the electrons that filled the fermi sea, with some dressing)

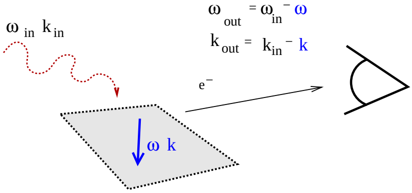


why: [Polchinski, Shankar, Benfatto-Gallivotti 92]

expectations from Landau

observable of interest: 'spectral density' $\mathrm{Im}\, G(\omega, k)$
(density of states which couples to the operator at ω, k)

measure by 'ARPES':



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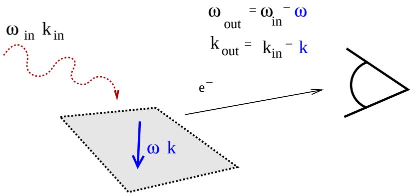

landau fermi liquid: a *pole* in the electron propagator G at $\omega = \omega_\star(k)$

• zero energy modes $\omega_\star = 0$ at $k = k_F$
• with linear dispersion $\omega_\star(k) \sim v_F(k - k_F)$
• a width that goes like $\omega_\star^2 / E_F$

momentum-space occupation $n(k) = \int d\omega f(\omega) \mathrm{Im}\, G(\omega, k)$
jumps by Z (the residue of the pole) at k_F.

non-fermi liquids exist in nature

In fact, other phases are possible

and the most robust ones are strongly coupled:

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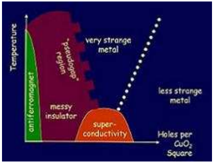
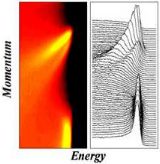
and the most robust ones are strongly coupled: 

For example: the 'normal' phase of high-T_c cuprate superconductors is a 2+1-dimensional 'strange metal'.

[ZX Shen]



There is a fermi surface,
but its properties are *not* Landauesque
(residue of pole at $\omega = 0$ is zero, width is linear in $\omega_\star$, resistivity is linear in T, Fermi 'arcs')

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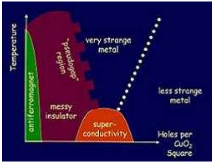

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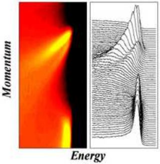

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$\exists$ perturbative phases [Holstein et al, Baym et al, Polchinski, Halperin-Lee-Read, Altshuler et al, Nayak-Wilczek, Schafer-Schwenzer...] but at parametrically low temperatures.

A nonperturbative description of such a phase would be valuable.

non-fermi liquids from holography

Consider a (2+1)-d rel. CFT w/ conserved $U(1)$ a gravity dual, and a charged fermionic operator [SS Lee; H Liu, D Vegh, JM]

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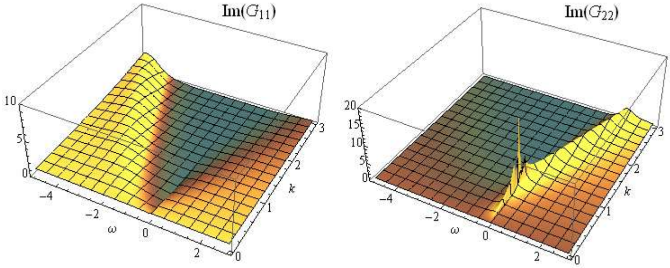

residue of pole at $\omega = 0$ is zero, width $\propto \omega_\star$

1. Particle hole asymmetry: no sharp peak for $k > k_F$!
2. At small k, G exhibits discrete scale invariance ($\sim$ Efimov effect).
3. *Emergent* scale invariance near k_F [Senthil]: non-relativistic CFT!

The location of the Fermi surface ($\omega = 0$) is determined by short-distance physics (analogous to band structure)
but the behavior near the FS is universal:
determined by IR CFT associated with AdS_2 region: $\mathcal{G} = c_k \omega^{2\nu}$
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determined by IR CFT associated with AdS_2 region: $\mathcal{G} = c_k \omega^{2\nu}$

$\nu \equiv R_2 \sqrt{m^2 + k^2 - q^2 e_d^2}$. From a 'matching' calculation (both in the ODE sense and in the RG sense!) [Faulkner, Liu, JM, Vegh]:

$$G_R(\omega, k) = \frac{h}{k_\perp + \frac{1}{v_F}\omega + c_k \omega^{2\nu}}$$

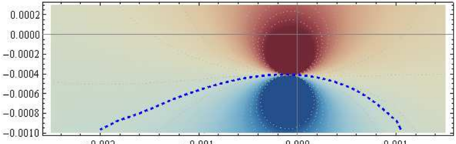
(if $2\nu_{k_F} \in \mathbb{Z}$: $\omega^{2\nu} \to \omega^{2\nu} \log \omega$)

Of the form found by coupling a FS perturbatively to massless boson [Holstein et al, Baym et al, Polchinski, Halperin-Lee-Read, Altshuler et al, Nayak-Wilczek, Schafer-Schwenzer...].

Here: scaling region is not parametrically small (the range of frequencies and temperatures over which it holds is $\gg e^{-137}$)

Spinor pole is always in LHP by properties of c_k:

$\arg c_k = \arg\left(e^{2\pi i \nu} \pm e^{-2\pi q e_d}\right)$



Two concluding remarks

What's special about Einstein gravity?

a) It describes our universe well and we've studied it a lot.
The bending-of-light-by-the-sun experiments say nothing about the gravity theory describing the possible dual of some condensed matter.
b) It's what comes from perturbative string theory in flat space at leading order in the α' and g_s expansions.
c) The einstein term is the leading irrelevant operator (besides the cosmological constant) by which we can couple metric fluctuations.
RG flows in the bulk correspond to some kind of uber-motion in the space of field theories.

Holographic universality: many *classes* of RG fixed points (univ. classes!) may be described by the same gravity dual
(near a fixed point in the space of bulk gravity theories.).
(perhaps they are all very special (large N))
(eg: spectral density calculation only depended on *quadratic terms* in bulk action.)

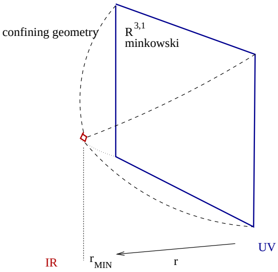
Are there new strongly-coupled phases of matter?

Ab initio prediction of liquid phase? [Weisskopf]

An old strongly-coupled phase of matter from holography

If we didn't happen to be made from the excitations of a confining gauge theory (QCD), [H Liu]
we would have predicted color confinement using AdS/CFT.

A cartoon by which we would have discovered confinement:



(hologram:

if IR region is missing, no low-energy excitations, mass gap.)

Are there new strongly-coupled phases of matter?

Ab initio prediction of liquid phase? [Weisskopf]

of confinement, of superconductivity, of fractional quantum Hall states...

Our ability to imagine possibilities for phases of matter so far has been limited by weak coupling descriptions

and by our ability to build things.

What other phases of matter may still be hidden?

The end. Thanks for listening.