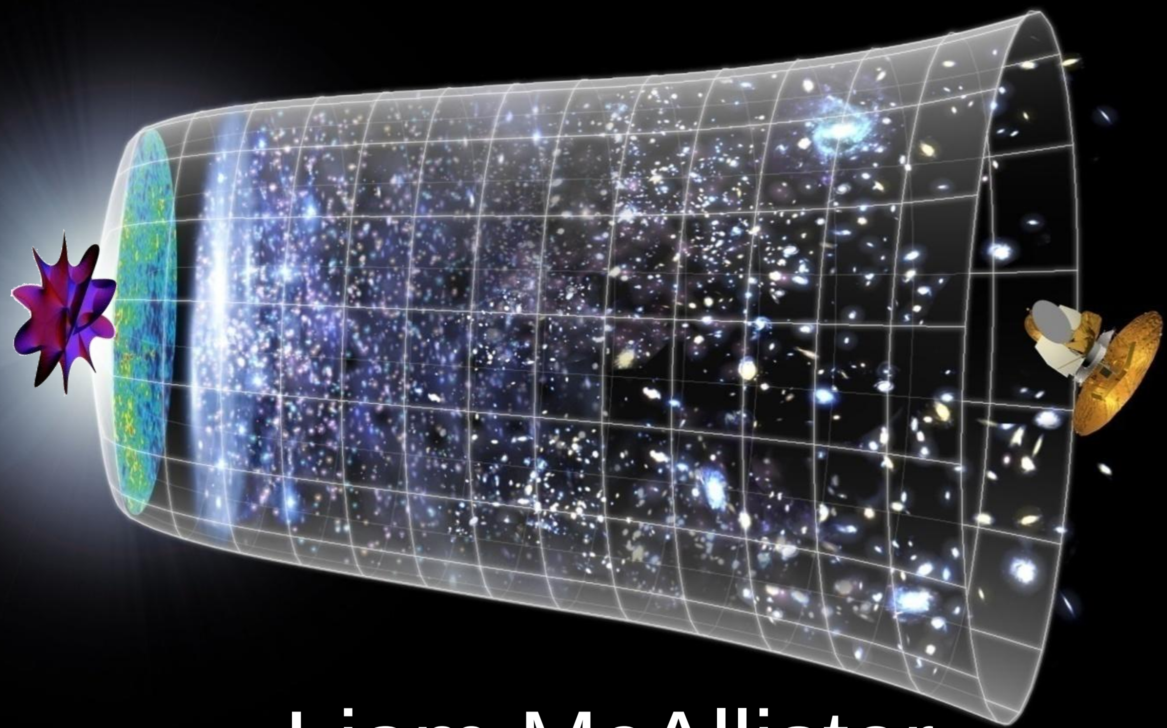
Inflation in String Theory

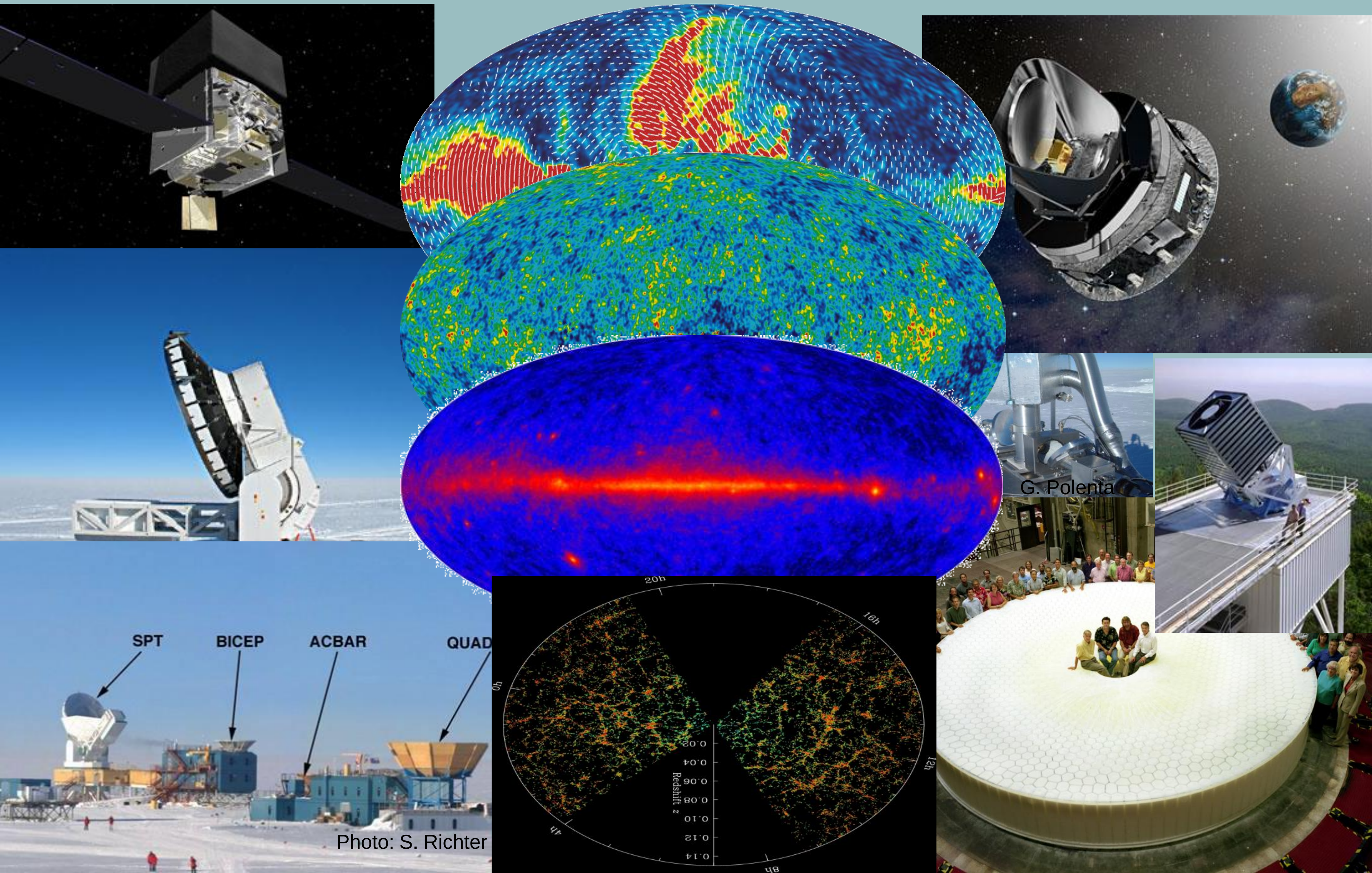


Liam McAllister
Cornell

Itzykson Meeting, Saclay

June 17, 2009

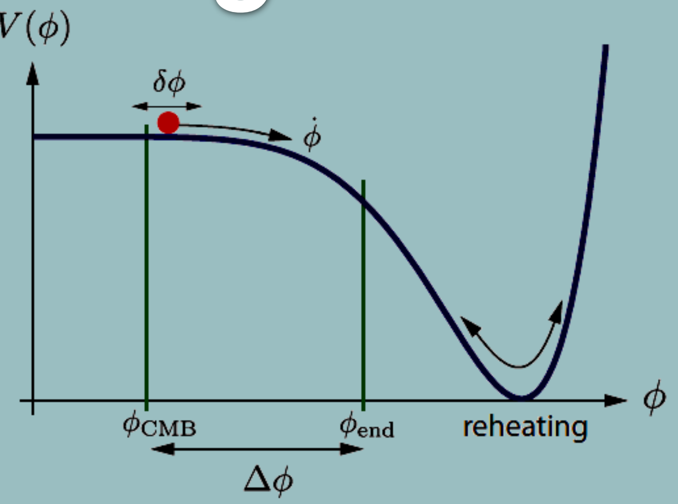
A golden age for cosmology



Physics of inflation: background

- Scalar field φ with a potential,

$$\boxed{\mathcal{L} = \tfrac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - V(\varphi)}$$



- Potential drives acceleration.

$$\boxed{ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2 \qquad H \approx const.}$$

- Acceleration is prolonged if V(φ) is flat in Planck units:

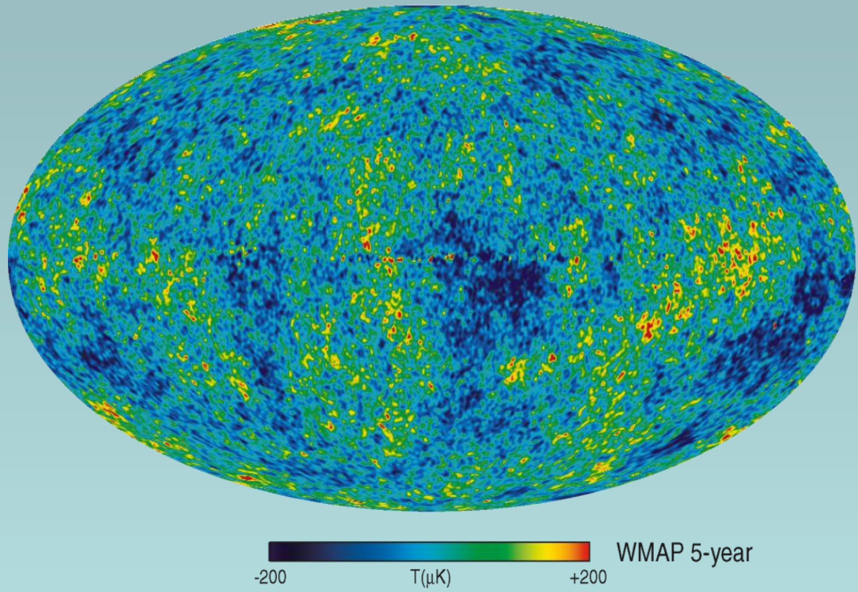
$$\eta \equiv M_{pl}^2 \frac{V''}{V} \ll 1 \quad \text{and} \quad \varepsilon \equiv \frac{1}{2} M_{pl}^2 \left(\frac{V'}{V}\right)^2 \ll 1$$

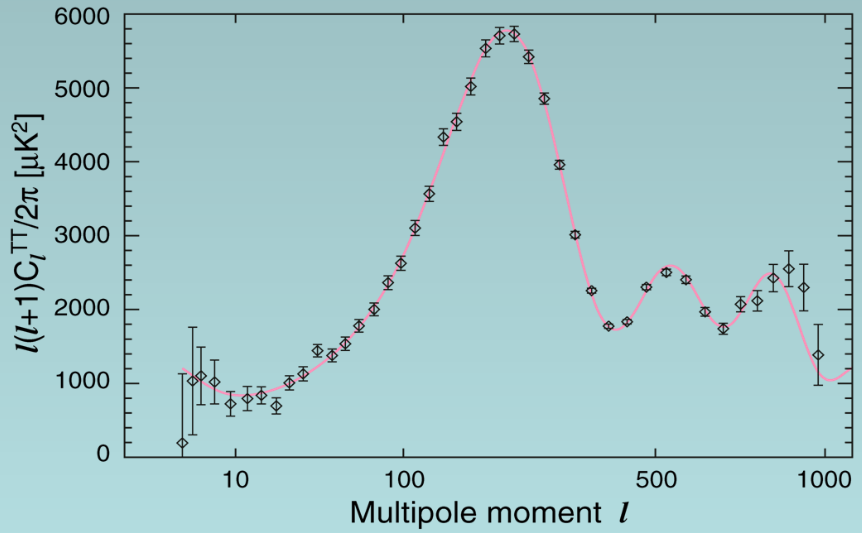
Physics of inflation: scalar perturbations

$$g_{ij} = a^2(t)[1 + 2\zeta]\delta_{ij}$$

$$\langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}'} \rangle = (2\pi)^3 \, \delta(\mathbf{k} + \mathbf{k}') \, \frac{2\pi^2}{k^3} P_s(k)$$

$$P_s(k) = A_s(k_\star) \left(\frac{k}{k_\star} \right)^{n_s(k_\star) - 1}$$



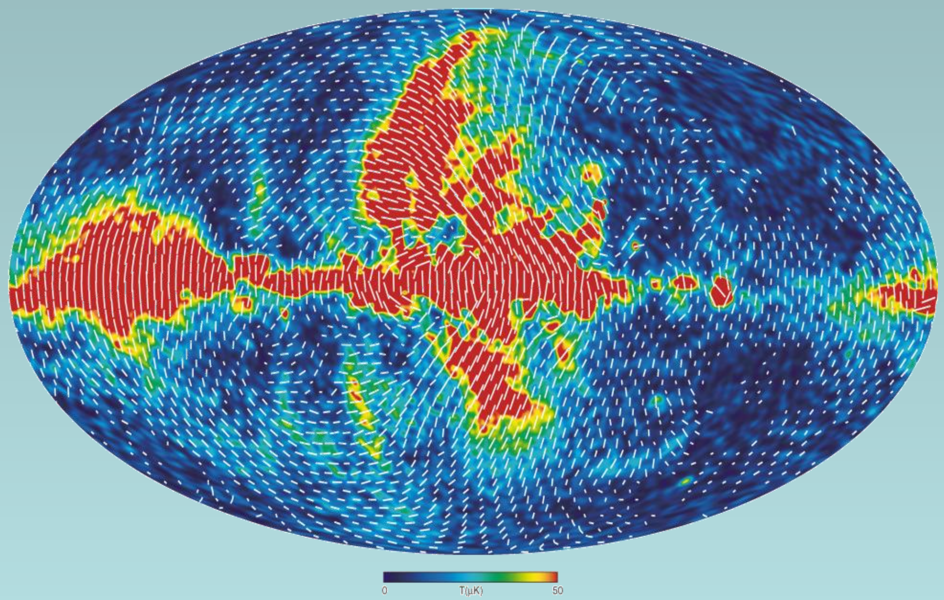


Physics of inflation: tensor perturbations

$$g_{ij} = a^2(t)[\delta_{ij} + h_{ij}]$$

$$\langle h_{\mathbf{k}} h_{\mathbf{k}'} \rangle = (2\pi)^3 \, \delta(\mathbf{k} + \mathbf{k}') \, \frac{2\pi^2}{k^3} P_t(k)$$

$$P_t(k) = A_t(k_\star) \left(\frac{k}{k_\star} \right)^{n_t(k_\star)}$$



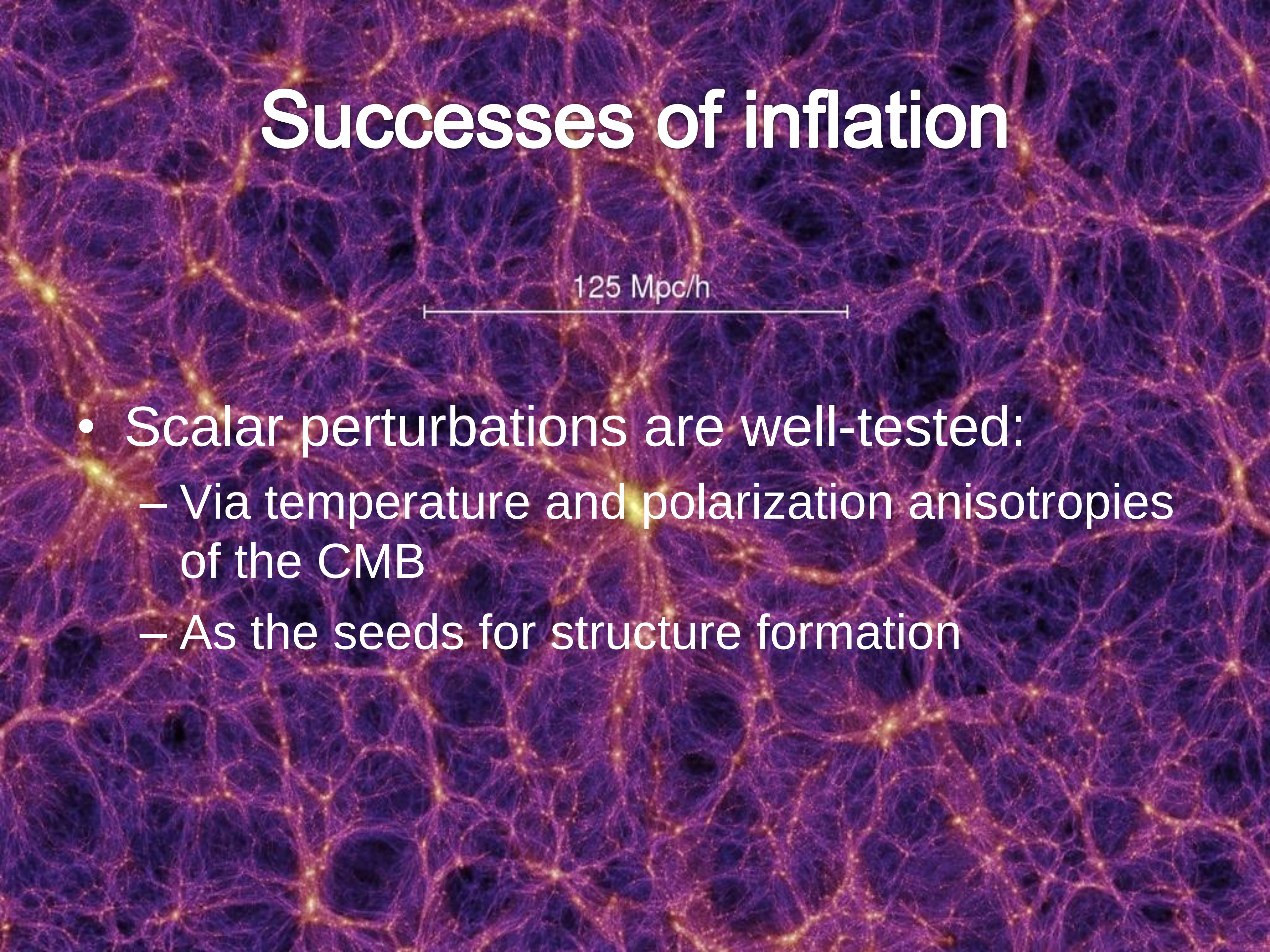
$$r \equiv \frac{P_t}{P_s}$$

tensor-to-scalar ratio, a measure of the primordial tensor signal

Successes of inflation

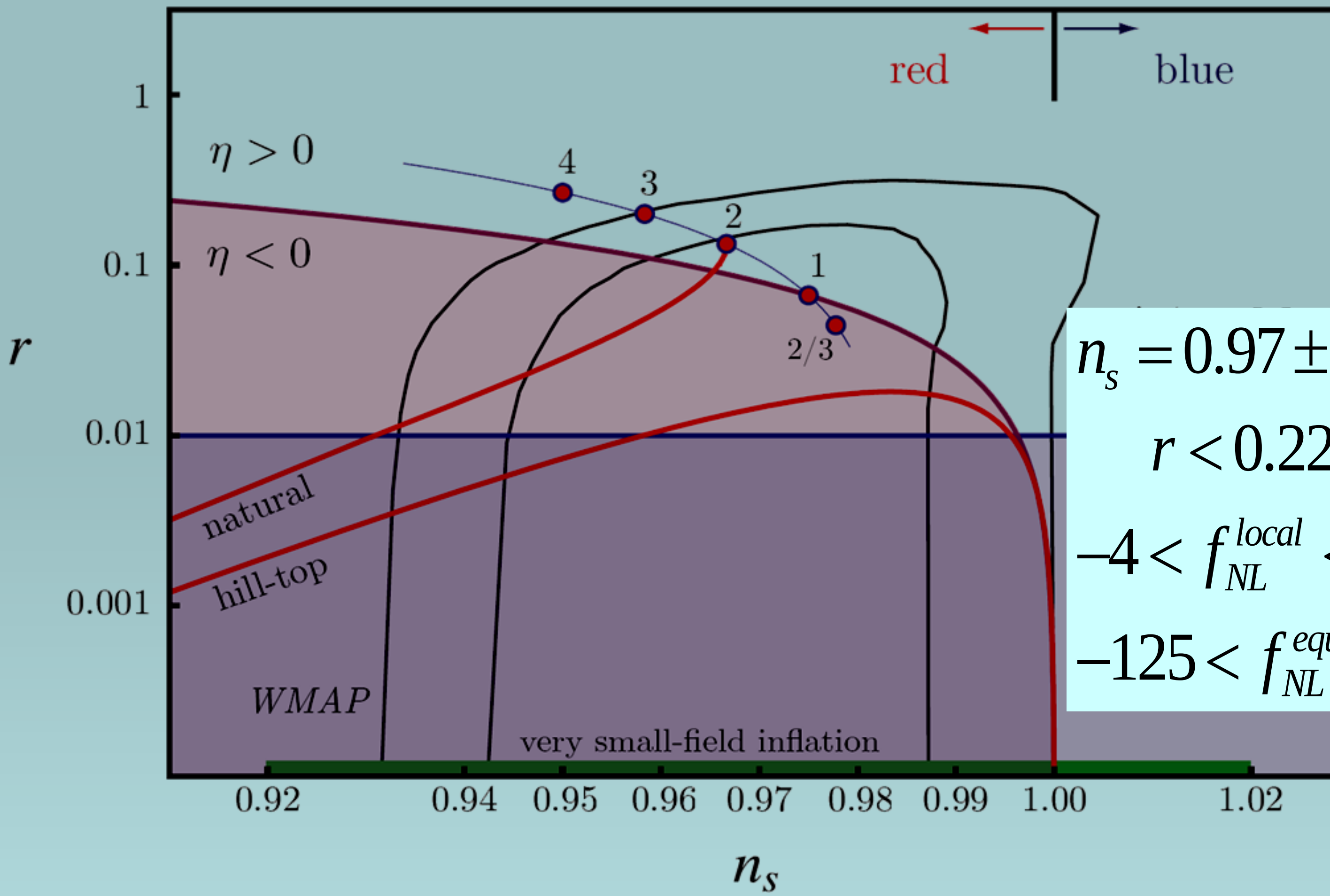
- Ameliorates horizon problem.
- Successful generic predictions:
  - Flatness (Boomerang + DASI + Maxima)
  - Spectrum of scalar perturbations that are:
    - Nearly scale-invariant (COBE)
    - Adiabatic (WMAP+ACBAR+CBI)
    - Nearly Gaussian (WMAP)
    - Correlated on super-horizon scales (WMAP)

Successes of inflation



- Scalar perturbations are well-tested:
  – Via temperature and polarization anisotropies of the CMB
  – As the seeds for structure formation

Current Constraints



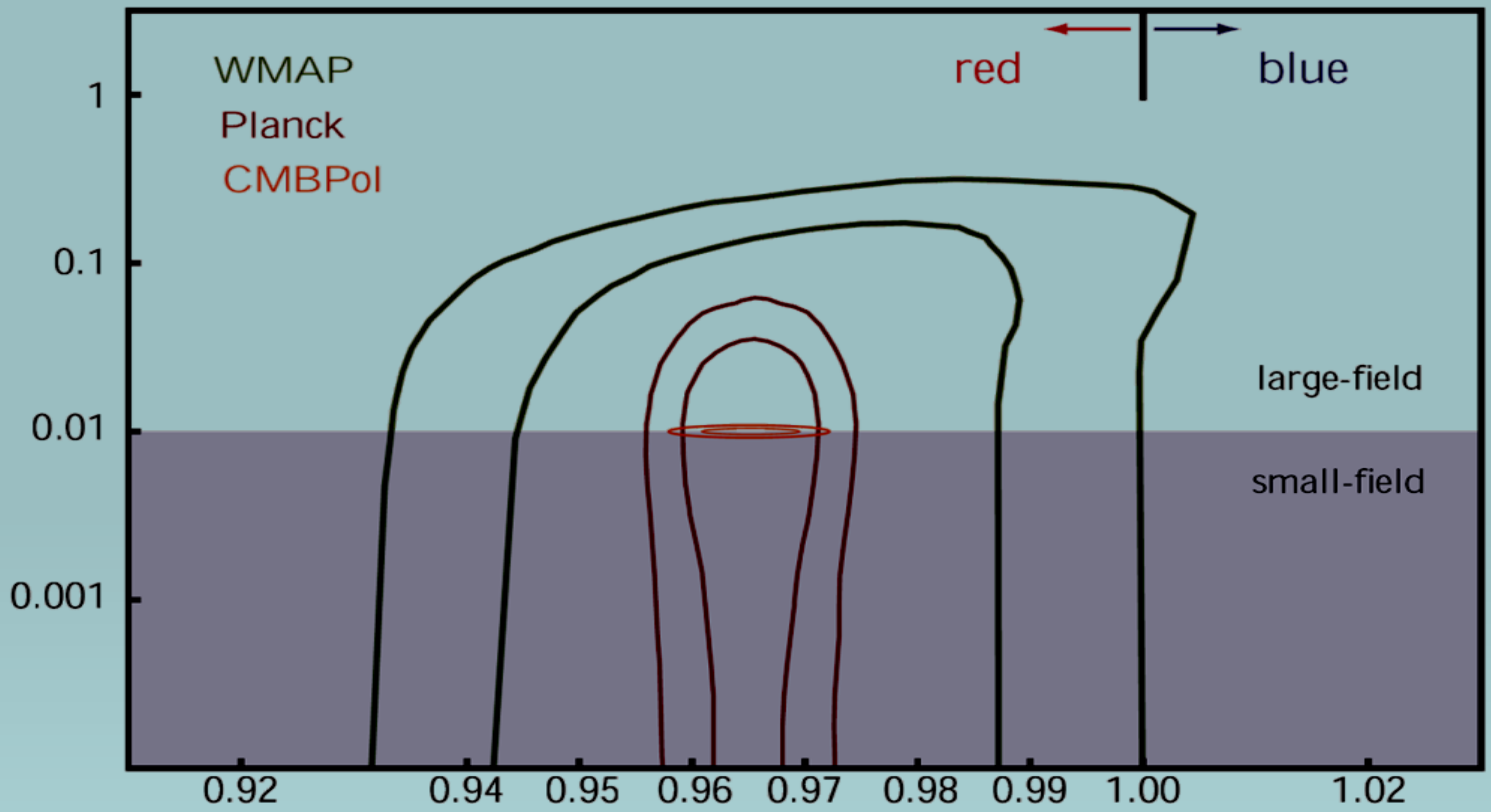
$$n_s = 0.97 \pm .015 \quad (r \neq 0)$$

$$r < 0.22 \quad (95\%)$$

$$-4 < f_{NL}^{local} < 80 \quad (95\%)$$

$$-125 < f_{NL}^{equil} < 435 \quad (95\%)$$

Possible Future Constraints



Current Sub-Orbital and Ground-Based Experiments

US Balloons	US Ground-based			European Ground-based
EBEX	ABS	ACT	BICEP2	BRAIN
SPIDER	Keck Array	MBI	Poincare	QUIJOTE
PIPER	PolarBeaR	QUIET	SPT	

Many things yet to be seen!

Label	Definition	Physical Origin	Current Status
A_s	Scalar Amplitude	V, V'	$(2.445 \pm 0.096) \times 10^{-9}$
n_s	Scalar Index	V', V''	0.960 ± 0.013
α_s	Scalar Running	V', V'', V'''	only upper limits
A_t	Tensor Amplitude	V (Energy Scale)	only upper limits
n_t	Tensor Index	V'	only upper limits
r	Tensor-to-Scalar Ratio	V'	only upper limits
Ω_k	Curvature	Initial Conditions	only upper limits
$f_{\rm NL}$	Non-Gaussianity	Non-Slow-Roll, Multi-Field	only upper limits
S	Isocurvature	Multi-Field	only upper limits
$G\mu$	Topological Defects	End of Inflation	only upper limits

Next decade: may anticipate observation of deviations from the generic predictions:

- Spectrum of tensor perturbations
- Primordial non-Gaussianity (e.g., three-point function)

Goal:

Develop theoretical models of the early universe suitable for an era of precision cosmology.

Plan of the talk

- I will first carefully explain why string theory can be useful for inflation.
- I will then explain what one needs to do to produce a useful model of inflation in string theory.
- I will then review two well-motivated scenarios, sketching the mechanisms and pointing out their strengths and weaknesses.

I.
Inflation and Planck-scale physics

Inflation is sensitive to Planck-scale physics.

- Inflationary Lagrangian generically receives critical contributions from Δ ˆ 6 Planck-suppressed operators.
  - Very generally, we expect contributions from integrating out massive degrees of freedom to which the inflaton couples.
  - The key point is that for inflation, even Planck-mass degrees of freedom are important (for O(1) couplings).
  - Moreover, we know that some new degrees of freedom must appear at, or well below, the Planck scale.
- In this sense, inflation is sensitive to the ultraviolet completion of gravity.
  - This is a remarkable opportunity for string theory.

Planck-suppressed corrections

$$\boxed{V \to V + \frac{\varphi^2}{M_P^2} V \Rightarrow \delta m_\varphi^2 = 6H^2 \Rightarrow \delta\eta = 2} \qquad \eta \equiv \frac{m_\varphi^2}{3H^2} \ll 1$$

- Another statement of the problem: without a symmetry, inflaton mass can run up to the cutoff Λ, and Λ>H.
- Note that this even affects low-scale models with small inflaton excursions.
- For models with large inflaton excursions, $\Delta\phi \gtrsim M_p$, the situation is more dramatic: since $\Delta\phi \gtrsim M_p \geq \Lambda$, one must understand why the inflaton couples so weakly to the massive d.o.f. that its potential can remain flat over a range $\gg \Lambda$.
  - i.e., one needs to understand why the inflaton enjoys a symmetry that controls its couplings to Planck-mass d.o.f.

String theory can help us understand couplings to Planck-scale physics.

- Since the UV completion of gravity matters for inflation, we are well advised to study inflation in quantum gravity.
- String theory provides toy models of inflation in quantum gravity in which we can study, e.g.,
  - Couplings to Planck-mass degrees of freedom
  - Symmetries preventing such couplings

…but this requires detailed moduli stabilization

- In string inflation, the Planck-suppressed contributions take various forms (string loop and α' corrections, both perturbative and nonperturbative; Euclidean D-brane contributions; backreaction effects; …)
- In practice, most of these contributions may be understood as arising from integrating out massive moduli.
- Knowing (and controlling) the inflaton potential therefore requires detailed information about moduli stabilization, i.e. about the effective action in a stabilized vacuum.

Example: the eta problem from an F-term potential

$$\Delta V = \frac{\varphi^2}{M_P^2} V_0$$

$$V \approx V_F = e^{K/M_{pl}^2} \left(K^{A\bar{B}} D_A W D_{\bar{B}} \overline{W} - \frac{3}{M_{pl}^2} |W|^2 \right)$$

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$$K = K(0) + K_{,\phi\bar{\phi}}(0)\phi\bar{\phi} + \ldots$$

$$\mathcal{L} \approx K_{,\phi\bar{\phi}}\partial\phi\partial\bar{\phi} -$$

$$- e^{K(0)/M_{pl}^2}\left(1 + \frac{1}{M_{pl}^2}K_{,\phi\bar{\phi}}\phi\bar{\phi}\right)\left(K^{A\bar{B}} D_A W D_{\bar{B}} \overline{W} - \frac{3}{M_{pl}^2}|W|^2\right) + \ldots .$$

the canonically-normalized inflaton φ obeys $\partial\varphi\partial\bar{\varphi} \approx K_{,\phi\bar{\phi}}(0)\partial\phi\partial\bar{\phi}$,

$$\Delta m_\varphi^2 \approx V_F(0)/M_{pl}^2 = 3H^2$$

Options for dealing with the sensitivity to Planck-scale physics.

I. Invoke a symmetry strong enough to forbid all such contributions.
   - i.e., forbid the inflaton from coupling to massive d.o.f.

Freese, Frieman, Olinto 90;
Arkani-Hamed, Cheng, Creminelli, Randall 03;
Kallosh, Hsu, Prokushkin 04;
Dimopoulos, Kachru, McGreevy, Wacker 05;
Conlon & Quevedo 05;
L.M., Silverstein, Westphal 08

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L.M., Silverstein, Westphal 08

II. Enumerate all relevant contributions and determine whether fine-tuned inflation can occur.
   - i.e., arrange for cancellations.

Baumann, Dymarsky, Klebanov, L.M., 07;
Haack, Kallosh, Krause, Linde, Lüst, Zagermann, 08;
Baumann, Dymarsky, Kachru, Klebanov, L.M., 08

II.
The task for the model-builder

In an ideal world…

- We would begin with a theory of INITIAL conditions valid at the string scale.
- We would understand the dynamics connecting the INITIAL state to a configuration with lower energy and six (metastably) compact dimensions.
- We would predict a suitable initial configuration for the period of inflation that produced the CMB.
- The resulting inflationary epoch would have signatures that could not arise in any effective field theory model.
- I would personally predict specific, decisive signatures that would be observed a few months later.

In an ideal world…

- We would begin with a theory of INITIAL conditions valid at the string scale.
  - Much thought, not much progress.
- We would understand the dynamics connecting the INITIAL state to a configuration with lower energy and six (metastably) compact dimensions.
  - Still waiting for a reliable realization of something like Brandenberger-Vafa.
- We would predict a suitable initial configuration for the period of inflation that produced the CMB.
  - Limited work, not much success.
- The resulting inflationary epoch would have signatures that could not arise in any effective field theory model.
  - No evidence this is even possible.
- I would personally predict specific, decisive signatures that would be observed a few months later.

In the real world…

- We begin our analysis with six (metastably) compact dimensions. Moduli stabilization is incorporated, but as an assumption, not a dynamical output.
- We typically assume a smooth initial patch of size > H^{-1}.
- We typically assume suitable initial conditions for the inflaton's homogeneous evolution, e.g. small kinetic energy.
- All analyses undertaken in effective field theory derived by dimensional reduction from a specified compactification.
- Specification of 10D data is usually incomplete, given the lack of explicit metrics and fully explicit methods of moduli stabilization.

These limitations reflect the state of the field. Few scenarios overcome any of them.

There are many other flaws that we *have* learned to overcome in the past decade!

Progress in the past decade

- MODULI STABILIZATION.
  - Solid qualitative understanding in several classes of compactifications.
  - Has crucial consequences for the inflation action. Properly incorporated in the handful of most advanced models.

- MECHANISMS.
  - Nontrivial kinetic terms (e.g. DBI).
  - Warping.
  - Symmetries of string compactifications.

- SIGNATURES.
  - Non-Gaussianity.
  - Cosmic strings.
  - Constraints among observables (in given classes of models).

What governs our choices?

- Theory side:
  - Computability. One must carefully choose a configuration that admits enough approximations to be tractable (local approximation; toroidal orientifold…)
  - Naturalness. Often search in vain for models in which the inflaton potential is naturally flat.
- Observations side:
  - Signatures:
    - Tensor modes require super-Planckian range
    - Non-Gaussianity requires deviation from single-field slow roll
      - E.g., nontrivial kinetic terms, multiple fields, broken slow roll
- As usual, one tries to do things that are doable, elegant, and interesting.

CHECKLIST for INFLATION IN STRING THEORY

- ❑ Are the moduli stabilized?
- ❑ Has the inflaton action been computed correctly?
  - ❑ Effects of massive moduli? Adiabatic approximation?
  - ❑ String loop corrections? α' corrections? Failures of other approximations (probe, noncompact, specific geometry)? Nonperturbative effects?
- ❑ Has it been established that inflation can occur?
  - ➢ If it is claimed that inflation works naturally without fine tuning, the claim is almost certainly wrong.
- ❑ Special cases:
  - ❑ If the field range is super-Planckian, is this under control?
  - ❑ If nontrivial kinetic terms matter, are they under control?
  - ❑ If the number of fields is large, are their quantum effects included?
- ❑ Is the phenomenology acceptable?
  - ❑ CMB constraints, reheating, defects…
  - ➢ If firm predictions are made, ask how this was accomplished.

III.
Selected scenarios
for
inflation in string theory

A partial classification

- INFLATON
  - Open string (e.g., moving D3-brane)
  - Closed string (e.g., Kahler modulus)
- FIELD RANGE
  - Small field
  - Large field
- MECHANISM
  - Slow roll
  - DBI
  - Repeated events (e.g. chain inflation, trapped inflation)
- SYMMETRY
  - No symmetry
  - Discrete symmetry forbidding inflaton mass
  - Shift symmetry
- ACCURACY/COMPLETENESS
  - Obviously incomplete
  - “Nearly all terms computed”
  - Limited only by explicitness of moduli stabilization

Example 1: D3-brane inflation

- INFLATON
  - Open string (e.g., moving D3-brane)
  - Closed string (e.g., Kahler modulus)
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Example 2: axion monodromy inflation

- INFLATON
  - Open string (e.g., moving D3-brane)
  - Closed string (e.g., Kahler modulus)
- FIELD RANGE
  - Small field
  - Large field
- MECHANISM
  - Slow roll
  - DBI
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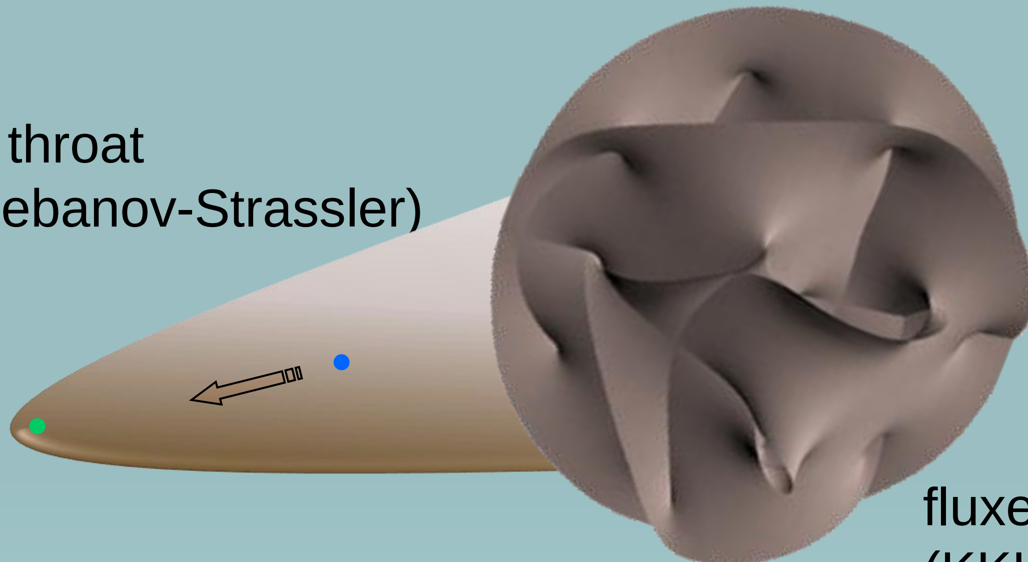
III.
Example 1:
D3-brane inflation

Dvali&Tye 1998
Dvali,Shafi,Solganik 2001
Burgess,Majumdar,Nolte,Quevedo,Rajesh,Zhang 2001
Kachru, Kallosh, Linde, Maldacena, L.M., Trivedi, 2003

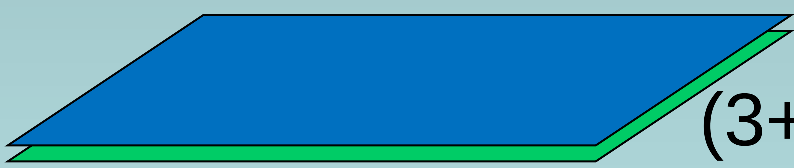
Warped D-brane inflation

warped throat
(e.g. Klebanov-Strassler)



CY orientifold, with
fluxes and nonperturbative W
(KKLT 2003)

D3-brane



(3+1)d

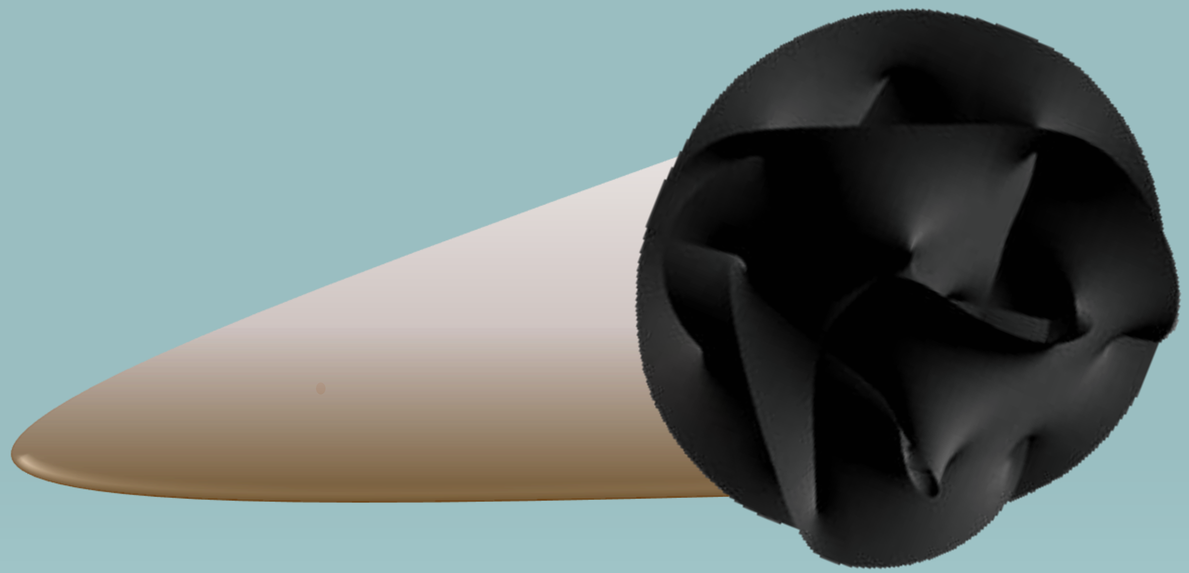
anti-D3-brane

warped throat gives:

weak Coulomb potential
control of energy scales
explicit local geometry
dual CFT

Kachru, Kallosh, Linde, Maldacena, L.M., Trivedi, 2003

What is the D3-brane potential?



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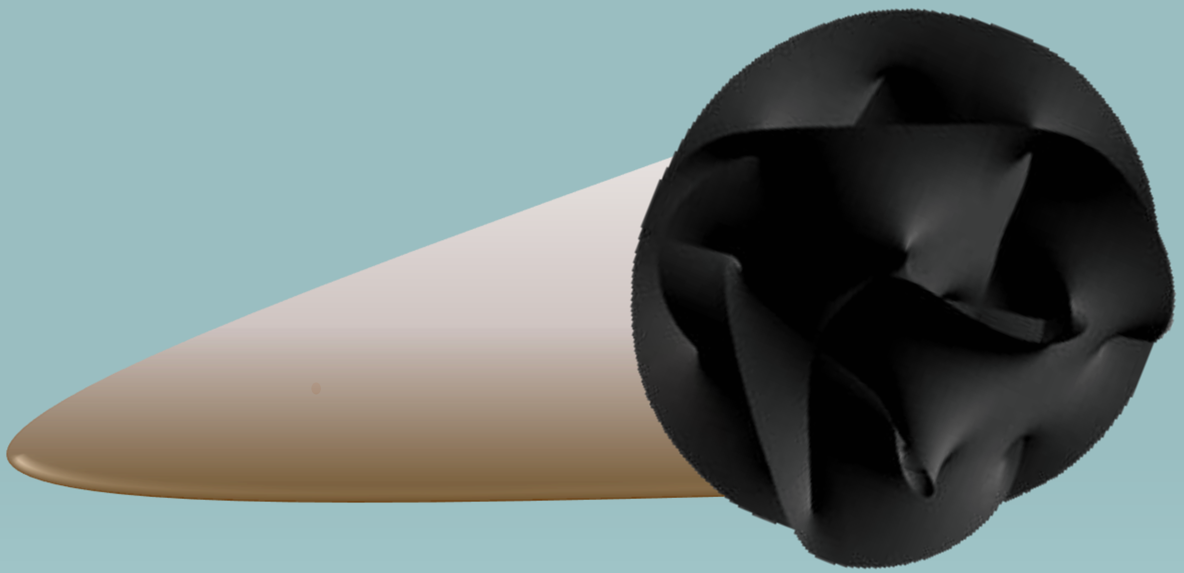

Specifically, what is the effect of moduli stabilization on the potential for a D3-brane in a throat?

D3-branes in flux compactifications

$$ds^2 = e^{2A(y)} g_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + e^{-2A(y)} \tilde{g}_{mn}(y) \mathrm{d}y^m \mathrm{d}y^n$$

$$\tilde{F}_5 = (1 + \star_{10}) \left[\mathrm{d}\alpha(y) \wedge \mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3 \right]$$

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$$V = T_3 \Phi_-$$

$$\Phi_\pm \equiv e^{4A} \pm \alpha$$

$$G_\pm \equiv (i \pm \star_6) G_3$$

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ISD solutions:

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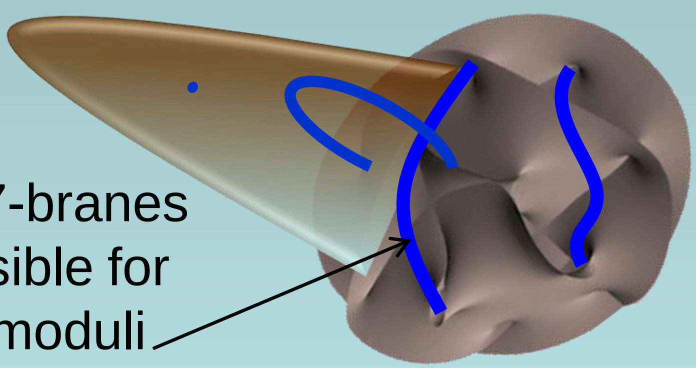
D3-branes feel no potential in ISD solutions (‘no-scale’),
but
nonperturbative stabilization of Kähler moduli will spoil this.

D3-branes in KKLT compactifications

$$W = \int G_3 \wedge \Omega + A(y) e^{-a\rho}\,, \qquad \mathcal{K} = -3\,\log\Big(\rho + \bar{\rho} - k(y,\bar{y})\Big)$$



ED3/D7-branes responsible for Kähler moduli stabilization

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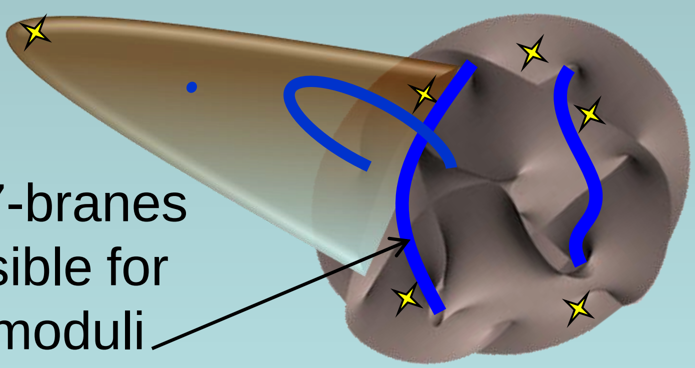
For generic A(y), solutions to $D_\rho W = D_y W = 0$

i.e., supersymmetric D3-brane vacua, are **isolated**. But where are they, and what is the potential in between?



ED3/D7-branes responsible for Kahler moduli stabilization

DeWolfe, L.M., Shiu, & Underwood, hep-th/0703088.

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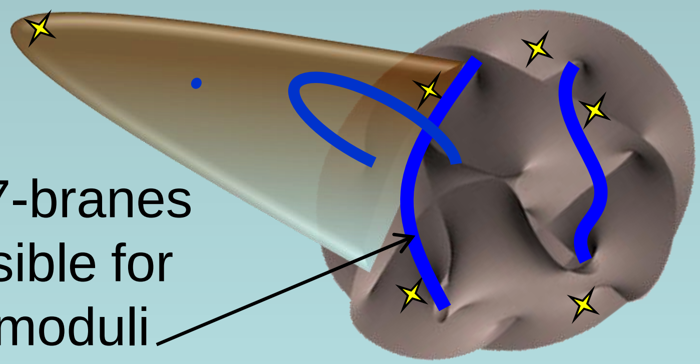


We need to know A(y).

DeWolfe, L.M., Shiu, & Underwood, hep-th/0703088.

Options:

- Compute A(y) in a special case.

  Berg, Haack, Kors, hep-th/0404087

  Baumann, Dymarsky, Klebanov, Maldacena, L.M., & Murugan, hep-th/0607050.

- Characterize the structure of the potential more generally.

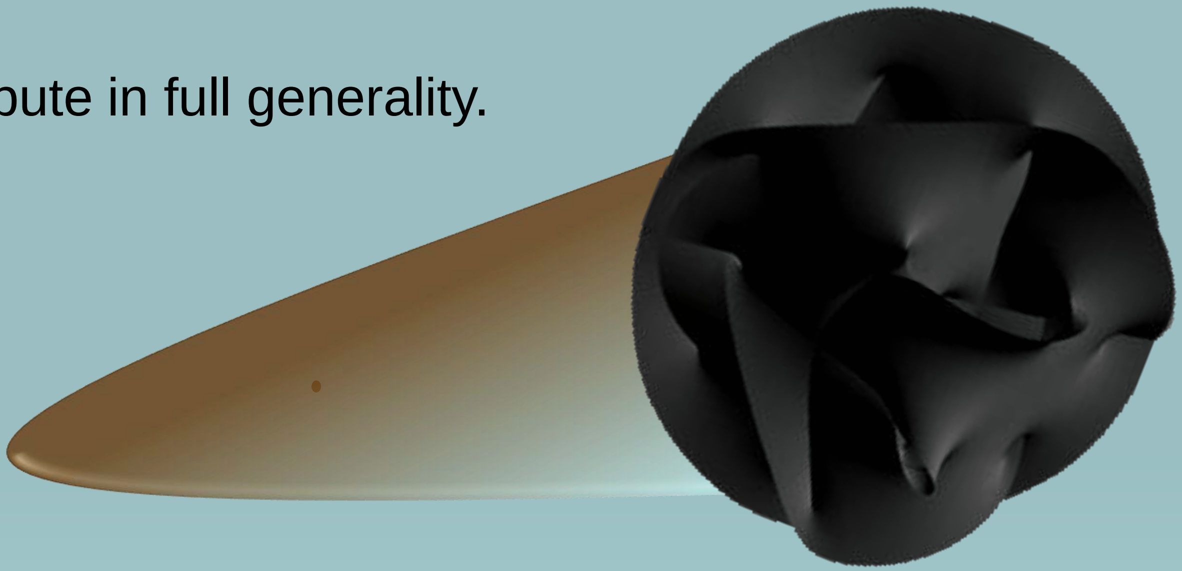
  Baumann, Dymarsky, Kachru, Klebanov, & L.M., 0808.2811.

  Baumann, Dymarsky, Kachru, Klebanov, & L.M., in preparation.

Today: characterize the general structure.

General structure of the D3-brane potential?

Clearly hard to compute in full generality.



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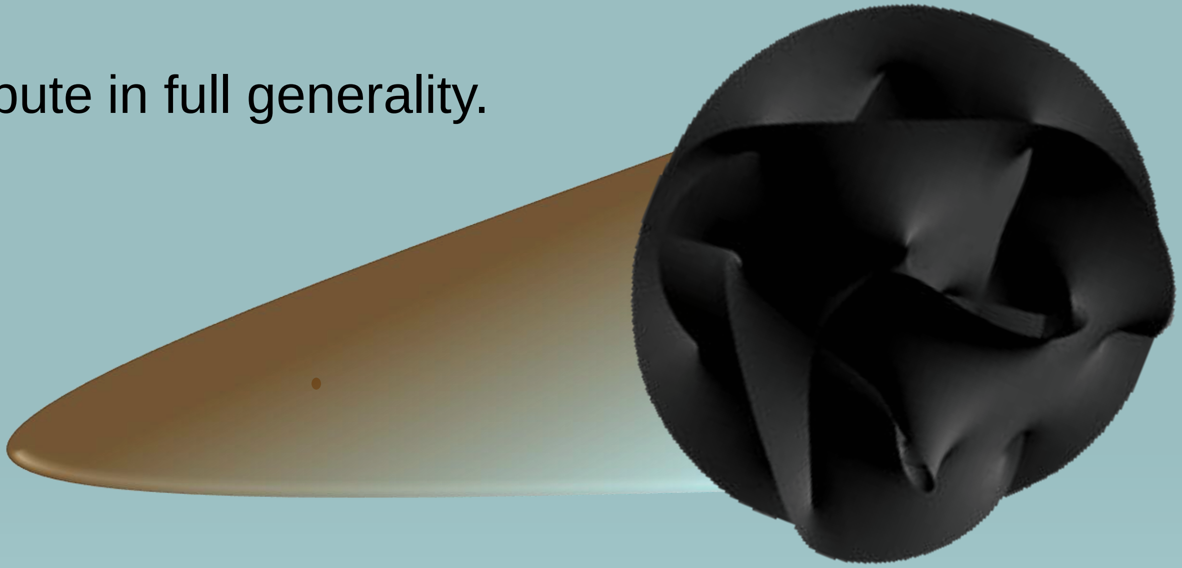

Goal: systematic coupling of compact space to a local model.

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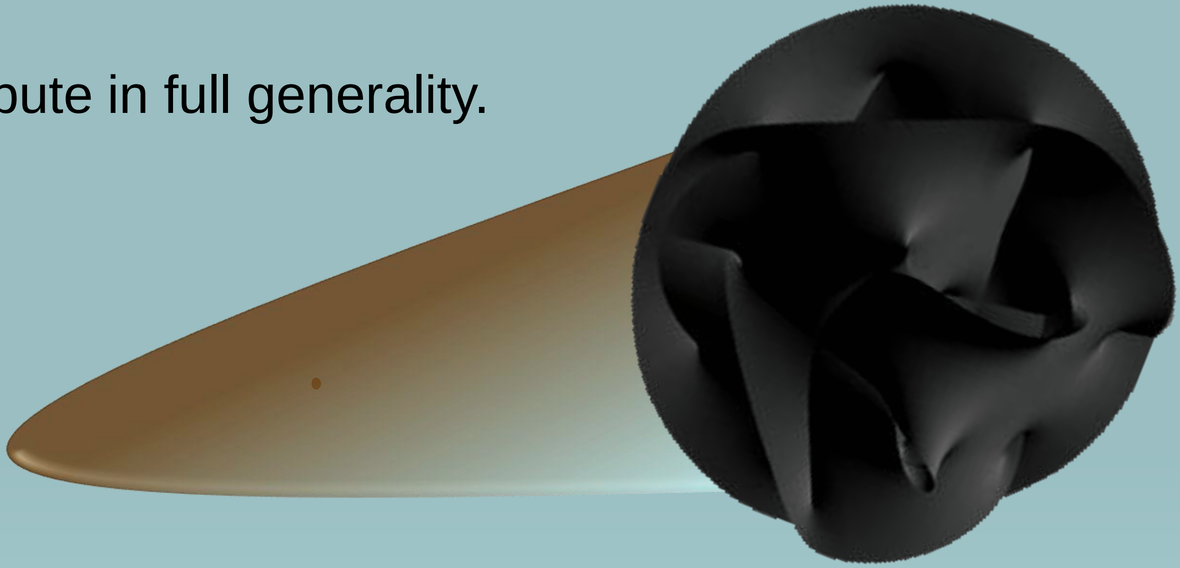

Goal: systematic coupling of compact space to a local model.

Idea: for a D3-brane well inside a warped throat, *leading* effects captured by structure of throat + *some* information about boundary conditions in UV.

Concretely, I will compute the potential for a D3-brane in a Klebanov-Strassler throat attached to a general bulk whose Kähler moduli are stabilized nonperturbatively. In practice, will use Klebanov-Witten SCFT.

A Simple Idea

The D3-brane potential comes from Φ_- alone. So we are interested in the profile of Φ_-.

$$V = T_3\,\Phi_-$$

$$\Phi_\pm \equiv e^{4A} \pm \alpha$$

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The D3-brane potential comes from Φ_- alone. So we are interested in the profile of Φ_-.

Arbitrary compactification effects can be represented by specifying boundary conditions for Φ_- in the UV of the throat, i.e. by allowing arbitrary non-normalizable Φ_- profiles.

A Simple Idea

The D3-brane potential comes from Φ_- alone. So we are interested in the profile of Φ_-.

Arbitrary compactification effects can be represented by specifying boundary conditions for Φ_- in the UV of the throat, i.e. by allowing arbitrary non-normalizable Φ_- profiles.

Filtering in the throat

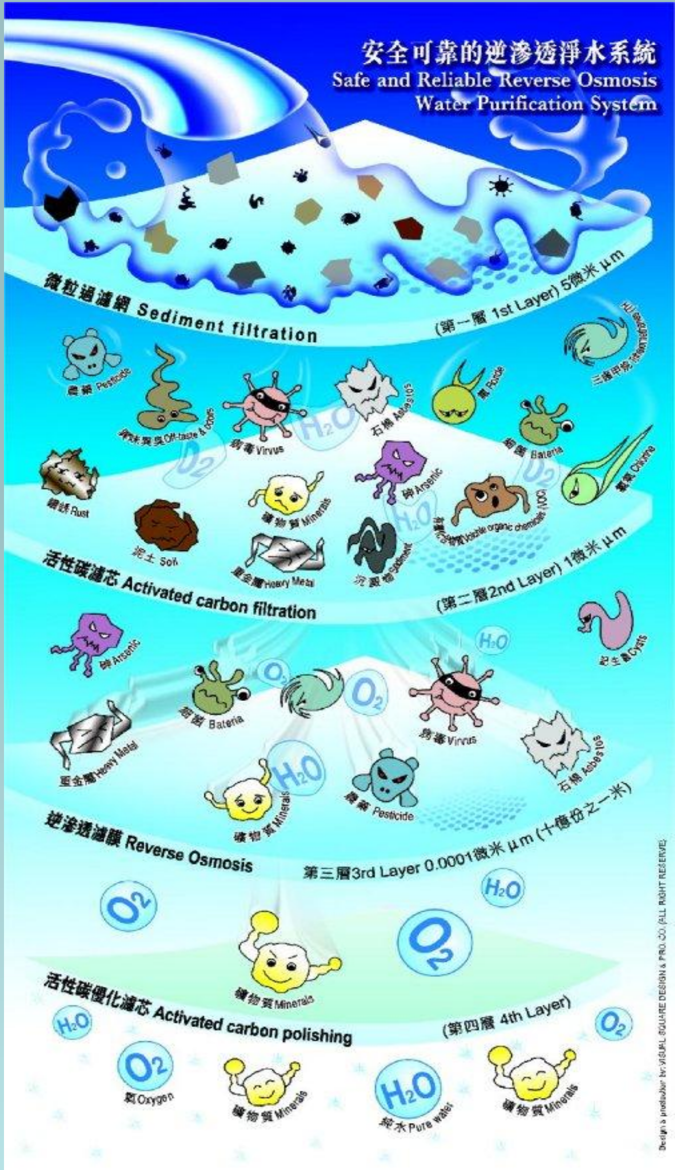
The warped geometry filters the compactification effects; in gauge theory variables,

$$V(\Lambda) \propto \sum_i c_i \left(\frac{\Lambda}{\Lambda_{UV}}\right)^{\Delta_i}$$

The leading contributions are those that diminish least rapidly towards the IR, i.e. the most relevant operators in the gauge theory.

By determining the spectrum of dimensions Δ_i we can extract the leading terms in the potential.

$$\boxed{V(r) = \sum_i c_i r^{\Delta_i} f_i(\Psi)}$$



Concrete example, gravity side

Consider linearized Φ_- perturbations around a finite-length KS throat, which we approximate by $AdS_5 \times T^{1,1}$.

$$ds^2 = h^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + h^{1/2}\left(dr^2 + r^2 ds^2_{T^{1,1}}\right)$$

$$h(r) = \frac{27\pi g_s}{4r^4}\alpha'^2 N$$

In general, many other modes are turned on, but at the *linear* level they do not couple to D3-branes.

EOM linearized around ISD compactifications

$$ds^2 = e^{2A(y)} g_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + e^{-2A(y)} \tilde{g}_{mn}(y) \mathrm{d}y^m \mathrm{d}y^n$$

$$\tilde{F}_5 = (1 + \star_{10}) \left[\mathrm{d}\alpha(y) \wedge \mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3 \right]$$

$$G_\pm \equiv (i \pm \star_6) G_3$$

$$V = T_3 \, \Phi_-$$

$$\Phi_\pm \equiv e^{4A} \pm \alpha$$

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GKP

$$\widetilde{\nabla}^2 \Phi_\pm = \frac{e^{8A+\phi}}{24} |\widetilde{G_\pm}|^2 + e^{-4A} |\tilde{\nabla} \Phi_\pm|^2 + \mathrm{local}$$

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Linearity + absence of sources.

The potential from IASD flux

We should solve $\tilde{\nabla}^2 \Phi_\pm = \frac{e^{8A+\phi}}{24} |\widetilde{G}_\pm|^2$ incorporating a source.

To do this, one solves the G e.o.m, turns on a general G background, and extracts the leading terms in Φ_- that result.

$$\boxed{G_- = \sum_\lambda c_\lambda r^\lambda h_\lambda(\Psi)}$$

Answer:

$$\boxed{\Phi_-(r) = c_1 r^1 f_1(\Psi) + c_{3/2} r^{3/2} f_{3/2}(\Psi) + c_2 r^2 f_2(\Psi) \ldots}$$

D. Baumann, A. Dymarsky, S. Kachru, I. Klebanov, & L.M., work in progress.

The leading terms, gravity side

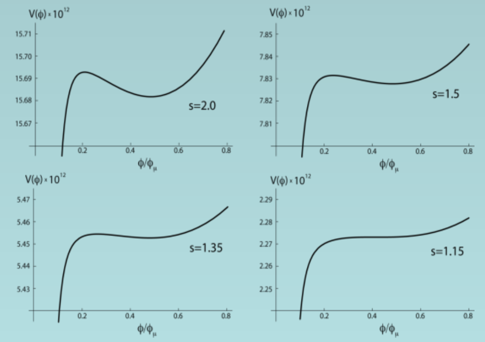
Recap: after solving $\widetilde{\nabla}^2 \Phi_\pm = \frac{e^{8A+\phi}}{24} |\widetilde{G_\pm}|^2$

with a general G background, the D3-brane potential is

$$\boxed{V(r) = c_1 r^1 + c_{3/2} r^{3/2} + c_2 r^2 + \ldots}$$

supported by $G_- \neq 0$ from homogeneous solution

So the D3-brane potential originates in UV perturbations of G_- and Φ_- .



For a better understanding, let’s try another perspective.

Perturbations of the dual gauge theory

Normalizable perturbations in supergravity correspond to perturbations of the state of the dual CFT. These IR contributions typically decouple from the compactification, and hence are easily included.

$$\varphi(r) = \alpha r^{-\Delta} + \beta r^{\Delta-4}$$

Non-normalizable perturbations in supergravity correspond to perturbations of the Lagrangian of the dual CFT. These UV contributions originate in the compact region.

Maldacena, **1997**
Gubser, Klebanov, & Polyakov, **1998**
Witten, **1998**

Gauge theory version

Arbitrary compactification effects can be represented by incorporating arbitrary perturbations of the CFT Lagrangian, including coupling it to 4D gravity and to hidden sector degrees of freedom.

(For anti-D3, cf. Aharony, Antebi, Berkooz.)

Gauge theory version

Arbitrary compactification effects can be represented by incorporating arbitrary perturbations of the CFT Lagrangian, including coupling it to 4D gravity and to hidden sector degrees of freedom.

$$\mathcal{L}_0 + \delta\mathcal{L} = \int d^2\theta d^2\bar{\theta}\,(K_0 + \delta K) + \int d^2\theta\,(W_0 + \delta W) + h.c.$$

$$\delta K = \sum c_i \mathcal{O}_{\Delta_i} \qquad \delta W = \sum d_i \mathcal{O}^{chiral}_{\Delta_i}$$

By following this prescription for the Klebanov-Witten SCFT dual to AdS_5 x $T^{1,1}$, one precisely reproduces the gravity-side potential.

D. Baumann, A. Dymarsky, S. Kachru, I. Klebanov, & L.M., 0808.2811.

Gauge theory version

Moreover, the leading (r^1) term comes from a superpotential perturbation by the lowest-dimension gauge invariant operator in the Klebanov-Witten SCFT,

$$\delta W = Tr(AB)$$

Klebanov-Witten SCFT:
SU(N) x SU(N) gauge group
SU(2) x SU(2) x $U(1)_R$ global symmetry
bifundamentals A_i, B_i

exactly as one would expect.

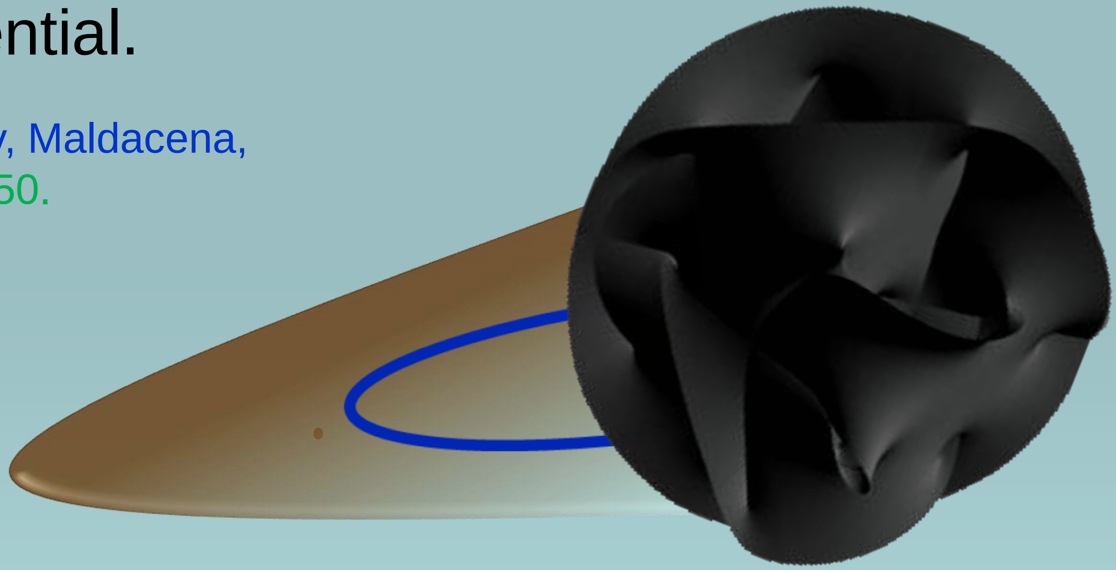
(relation to G-flux: cf. Graña & Polchinski 0009211.)

General structure attested in examples

Special case is explicitly computable: assume the moduli-stabilizing D7-branes hang into the throat region

Can compute superpotential.

Baumann, Dymarsky, Klebanov, Maldacena, L.M., & Murugan, hep-th/0607050.



Resulting potential:

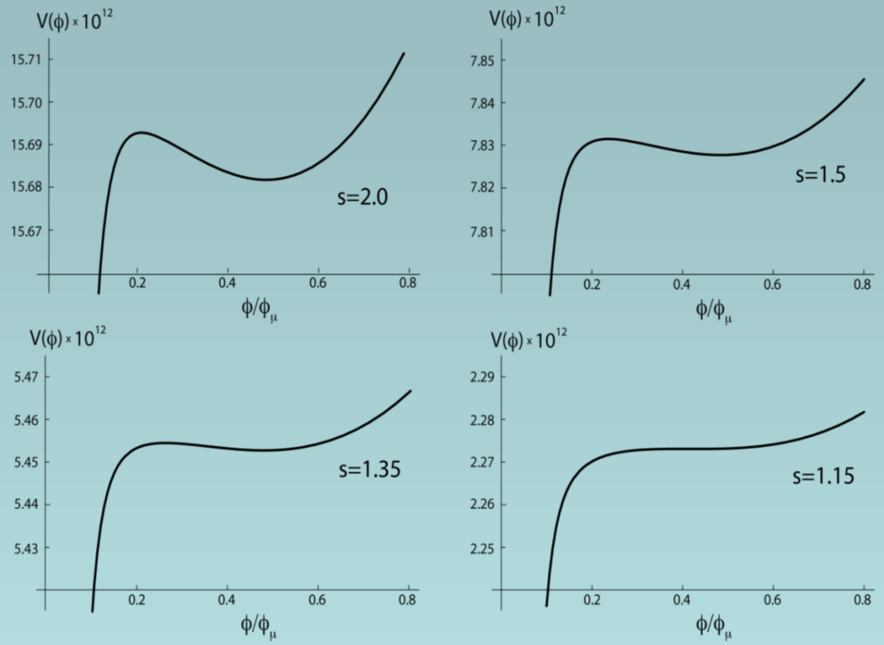
$$\boxed{V(r) = c_1 r^1 f_1(\Psi) + c_{3/2} r^{3/2} f_{3/2}(\Psi) + c_2 r^2 f_2(\Psi) \ldots}$$

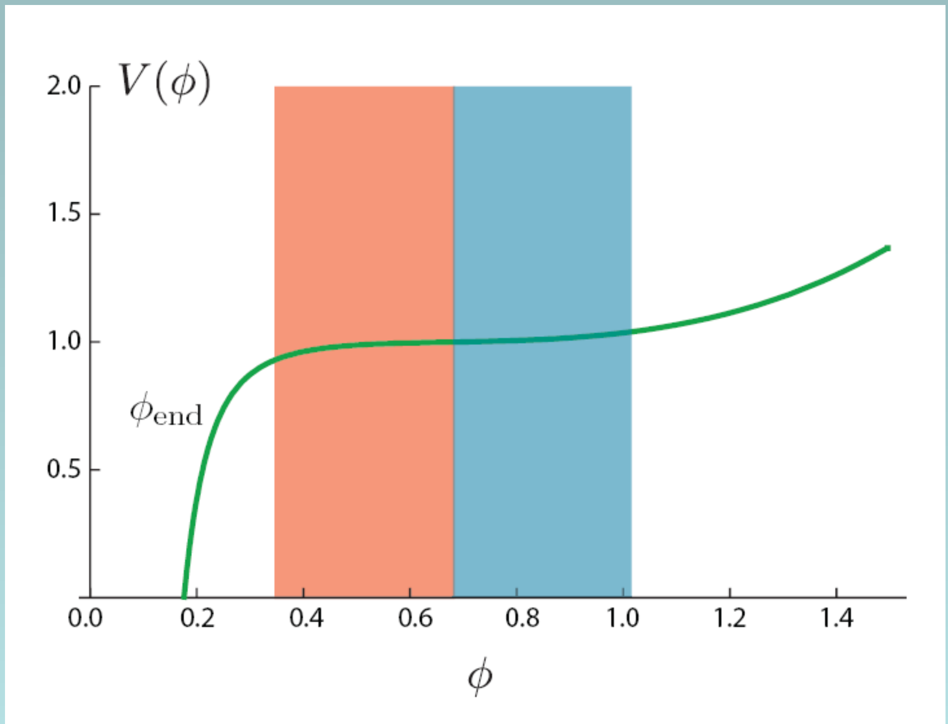
Identical structure!

Phenomenology:

Inflection point inflation

$$V(r) = c_1 r^1 + c_{3/2} r^{3/2} + c_2 r^2 + \dots$$





III.
Example 2:
Axion Monodromy Inflation

L.M., Silverstein,& Westphal, 0808.0706
Flauger, L.M., Pajer, Silverstein, Westphal, Xu 0906.nnnn

see also:

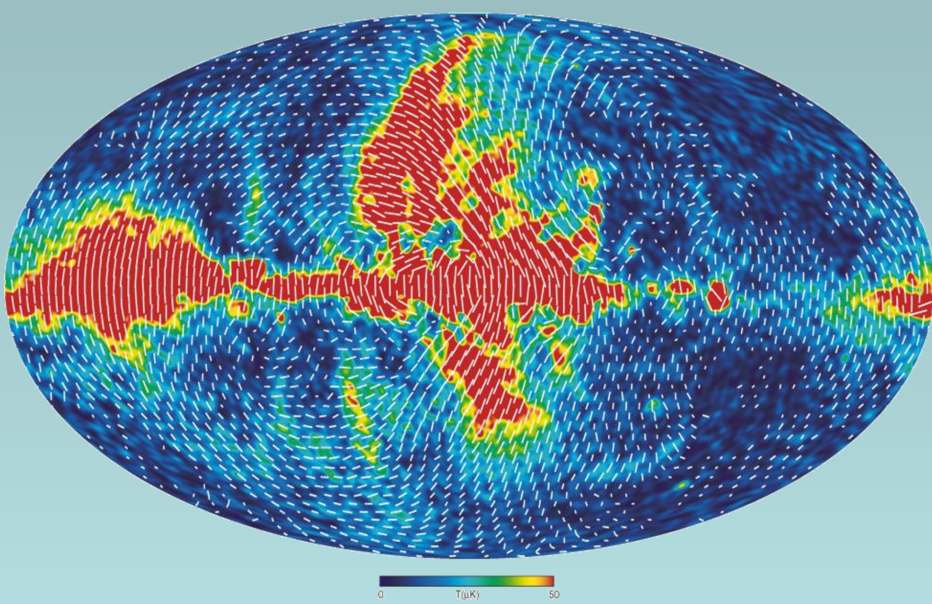
Silverstein & Westphal, 0803.3085.

Physics of inflation: tensor perturbations

$$g_{ij} = a^2(t)[\delta_{ij} + h_{ij}]$$

$$\langle h_{\mathbf{k}} h_{\mathbf{k}'} \rangle = (2\pi)^3 \, \delta(\mathbf{k} + \mathbf{k}') \, \frac{2\pi^2}{k^3} P_t(k)$$

$$P_t(k) = A_t(k_\star) \left(\frac{k}{k_\star} \right)^{n_t(k_\star)}$$

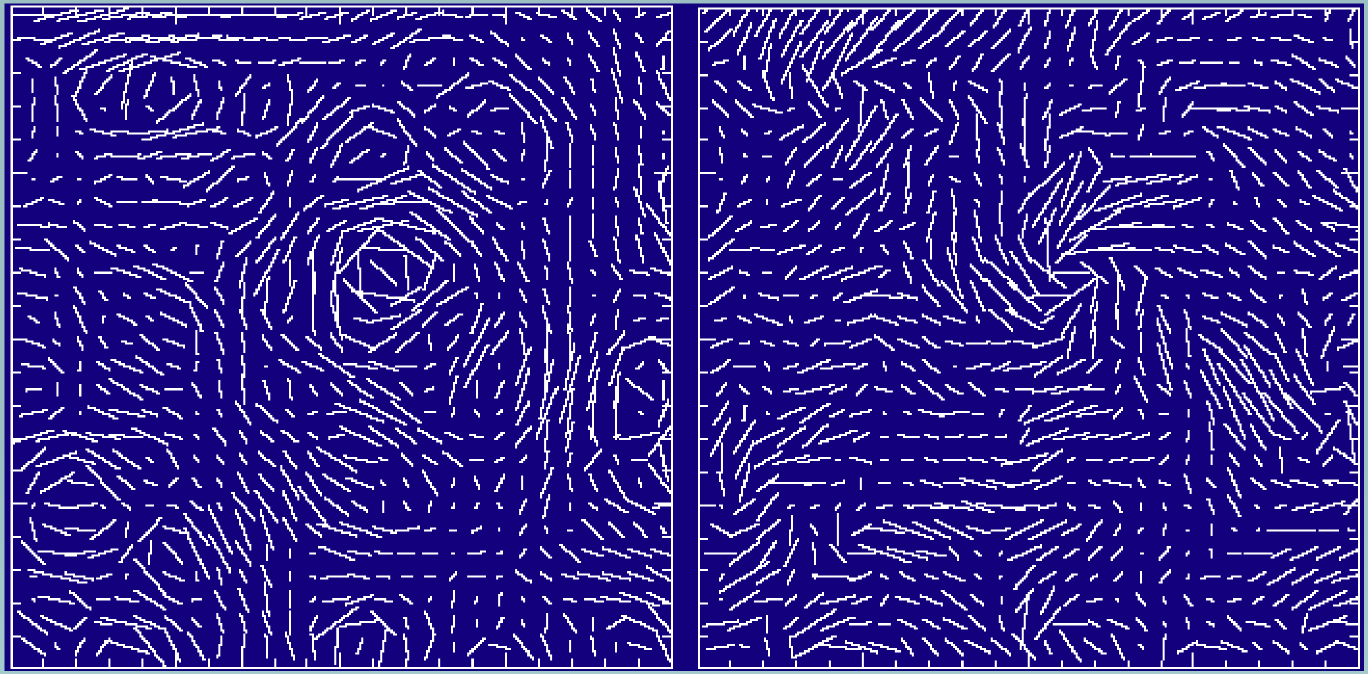


$$r \equiv \frac{P_t}{P_s}$$

tensor-to-scalar ratio, a measure of the primordial tensor signal

E-mode vs. B-mode

(curl-free) (curl)



Primordial tensors induce curl of CMB photons' polarization (B-mode), and hence may be visible: Planck, SPIDER, QUIET, BICEP, EBEX, PolarBEAR, CMBPol?

Image: Seljak and Zaldarriaga

Lyth Bound

D.H. Lyth, 1996

$$\frac{\Delta\phi}{M_{\rm pl}} \gtrsim 1.06 \times \left(\frac{r_\star}{0.01}\right)^{1/2}$$

Threshold for detection:

$r < 0.22$	WMAP+
$r \sim 0.05$	Planck
$r \;\square\; 0.01$	next decade
$r \sim 0.001?$	ultimate?

Lyth Bound

D.H. Lyth, 1996

$$\frac{\Delta\phi}{M_{\rm pl}} \gtrsim 1.06 \times \left(\frac{r_\star}{0.01}\right)^{1/2}$$

Observable tensors require trans-Planckian field variation, which requires ultraviolet input.

Large field inflation in string theory?

- Need:
  - Large range
  - Flat potential (e.g., symmetry)

Field range for any scenario involving a D3-brane in a throat:

$$\boxed{\left(\frac{\Delta\varphi}{M_P}\right)^2 < \frac{4}{\square}}$$

N = the number of colors in the dual gauge theory

$$\boxed{r < .009 \times \frac{4}{\square} \;\square\; .01}$$

D. Baumann and L.M., hep-th/0610285.

Natural Inflation in string theory?

Freese, Frieman, & Olinto, 1990: $\boxed{V = \Lambda^4 \cos(\varphi / f)}$ $f > M_P$

Axion shift symmetry protects inflaton potential.

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Dimopoulos, Kachru, McGreevy, & Wacker, hep-th/0507205

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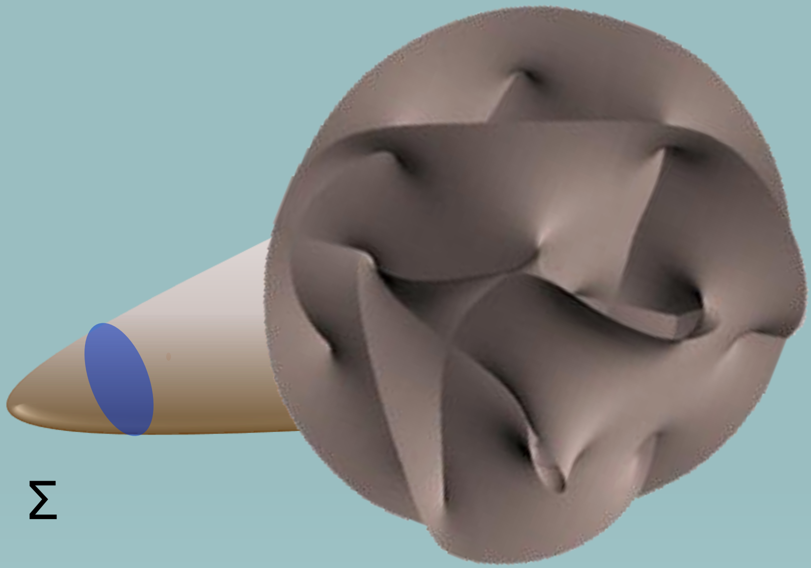
Our idea: recycle a single axion N times.

"Axion monodromy"

Axion Inflation from Wrapped Fivebranes



D5-brane/NS5-brane

$$\int_\Sigma B_2 \qquad \int_\Sigma C_2$$

$$V = \int_\Sigma \frac{d^2\xi\, e^{-\Phi}}{(2\pi)^5 \alpha'^3} \sqrt{\det(G+B)} \approx \frac{1}{(2\pi)^5 g_s \alpha'^3} \sqrt{L^4 + b^2} \approx \mu^3 \varphi_b$$

Axion monodromy from wrapped fivebranes

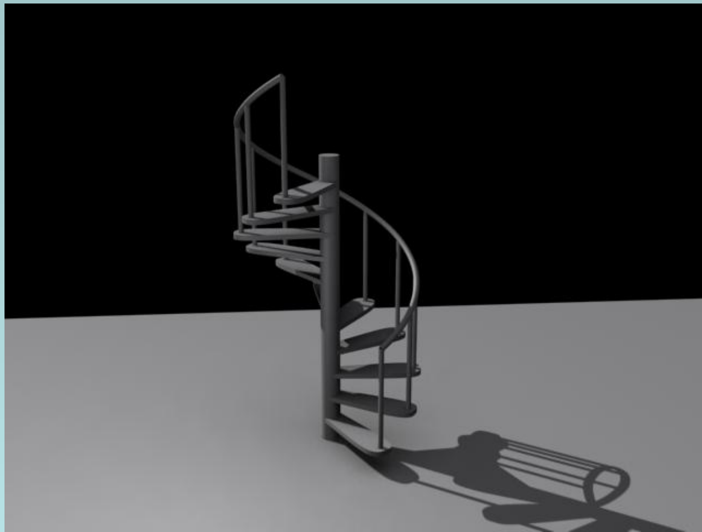
$$V = \int_\Sigma \frac{d^2\xi\, e^{-\Phi}}{(2\pi)^5 \alpha'^3} \sqrt{\det\left(G+B\right)} \approx \frac{1}{(2\pi)^5 g_s \alpha'^3} \sqrt{L^4 + b^2} \approx \mu^3 \varphi_b$$

- Fivebrane contribution not periodic: as axion shifts by a period, potential undergoes a monodromy

  cf. inflation from D-brane monodromy Silverstein&Westphal, 0803.3085.

- This unwraps the axion circle and provides a linear potential over an *a priori* unlimited field range.

- In practice, controllable over large ($\gg M_P$) but finite range.



Nonperturbative corrections

- e.g., ED3/ED1 in K

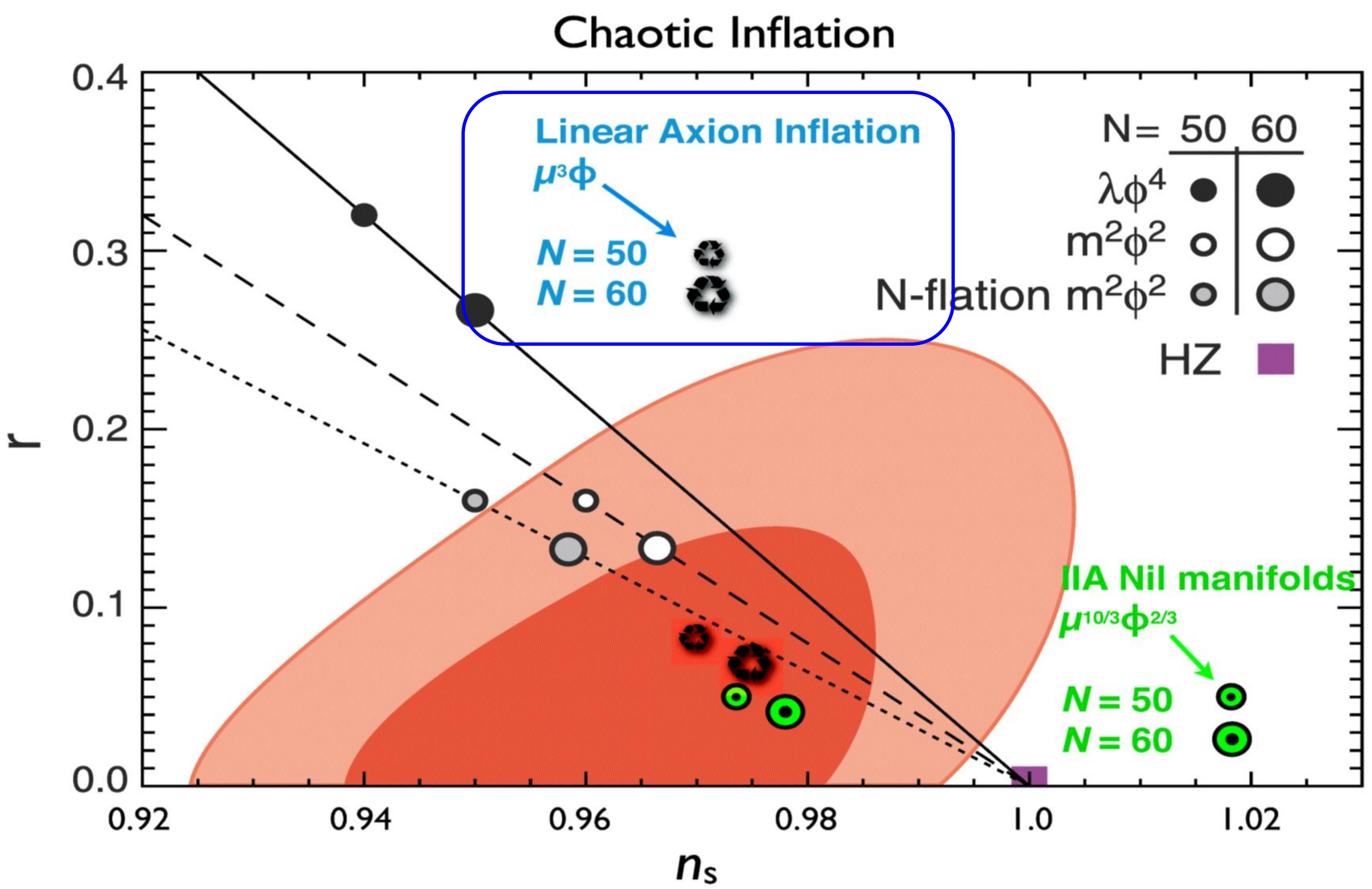
$$V(\phi) = \mu^3\phi + \overbrace{b\mu^3 f}^{\Lambda^4}\cos\left(\frac{\phi}{f}\right)$$

- $b < 1 \Rightarrow$ monotonic potential

Ripples in the CMB?

$$P_s(k) = A_s \left(\frac{k}{k_*}\right)^{n_s-1} \left[1 + \delta n_s \cos\left(\frac{\phi_k}{f}\right)\right]$$

$$\delta n_s = -12b\sqrt{\frac{\frac{\pi}{8}\coth\left(\frac{\pi}{2f\phi_{in}}\right) f\phi_{in}}{(1+(3f\phi_{in})^2)}} \sim b\sqrt{f}$$


Chaotic Inflation
0.4
0.3
0.2
0.1
0.0
r
0.92
0.94
0.96
0.98
1.0
1.02
n_s
Linear Axion Inflation
$\mu^3\phi$
$N = 50$
$N = 60$
N= 50 60
$\lambda\phi^4$
$m^2\phi^2$
N-flation $m^2\phi^2$
HZ
IIA Nil manifolds
$\mu^{10/3}\phi^{2/3}$
$N = 50$
$N = 60$

Conclusions

- Inflation in string theory is strongly motivated by the sensitivity of inflation to Planck-scale physics.
- Progress in recent years:
  - a few reasonably complete models, thanks to advances in moduli stabilization
  - various mechanisms (e.g. DBI, warping, monodromy)
  - some interesting signatures (e.g. equilateral NG, cosmic superstrings, features in spectrum and bispectrum)
- In the future:
  - need much more comprehensive understanding of the space of possible models
  - can readily make progress given better compactification technology
  - near-future experiments will surely constrain, and may observe, distinctive signatures of inflation!

$$< \zeta_{k_1}\zeta_{k_2}\zeta_{k_3} > \;=\; (2\pi)^7\delta^3(K)\frac{P^2}{(k_1k_2k_3)^2}f_{res}\sin\left(\frac{2\log(K)}{\phi f}\right)$$

$$\begin{aligned} f_{res} &\simeq \frac{9}{4}\frac{b}{(f\phi)^{3/2}} \\ &= \frac{9}{4}b\left(\frac{\omega}{H}\right)^{3/2} \end{aligned}$$