Lessons from the information paradox

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work in collaboration with
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Plan of the talk

(A) What exactly is the information paradox ?

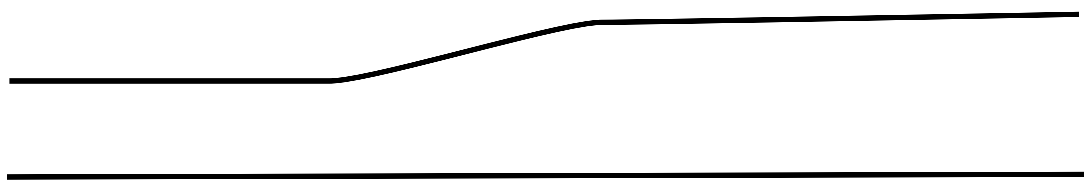
(B) Fuzzballs: a summary

(C) Conjectures: What happens to a collapsing shell ?
The role of large phase space volumes

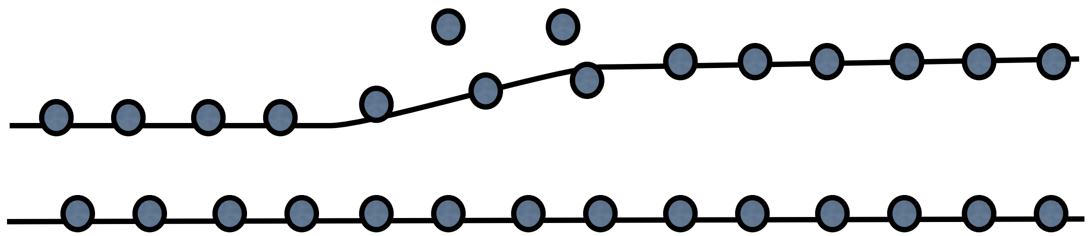
(D) Cosmology: What is the equation of state
of the very early Universe?

The black hole information paradox

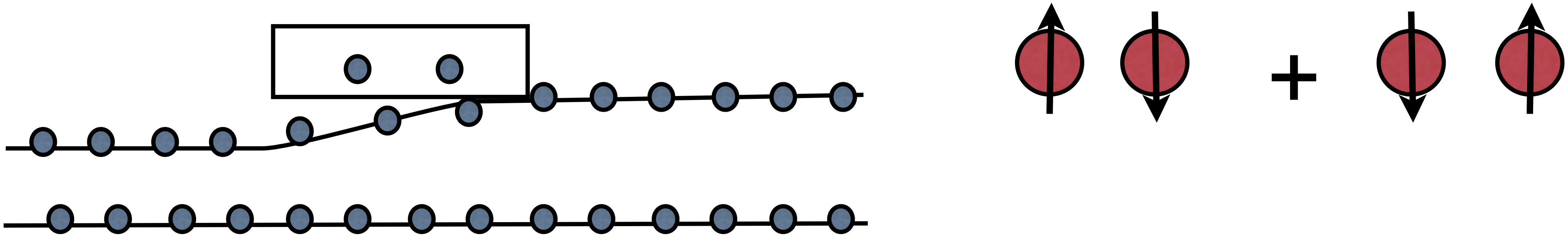
(a) Spacelike slices have a changing geometry in general



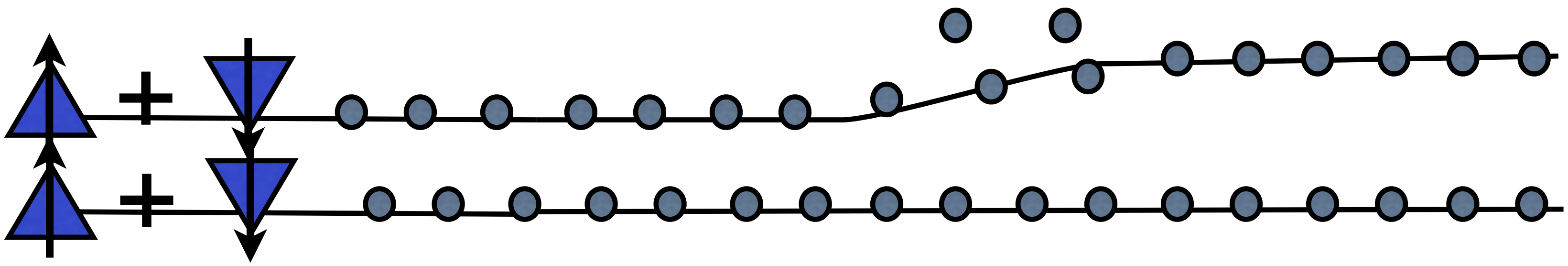
(b) The vacuum for quantum fields depends on the geometry of the slice, so particle pairs are created when the slice evolves



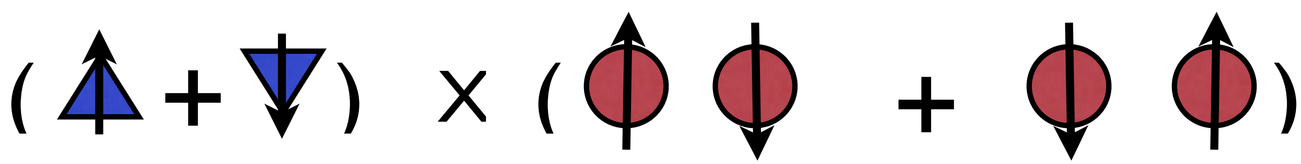
(c) These state of these pairs is correlated :



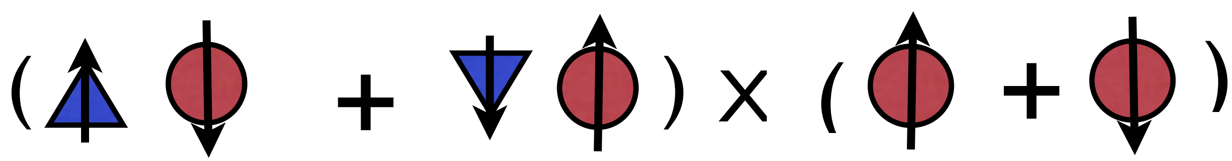
(d) There is no correlation with quanta far away: LOCALITY



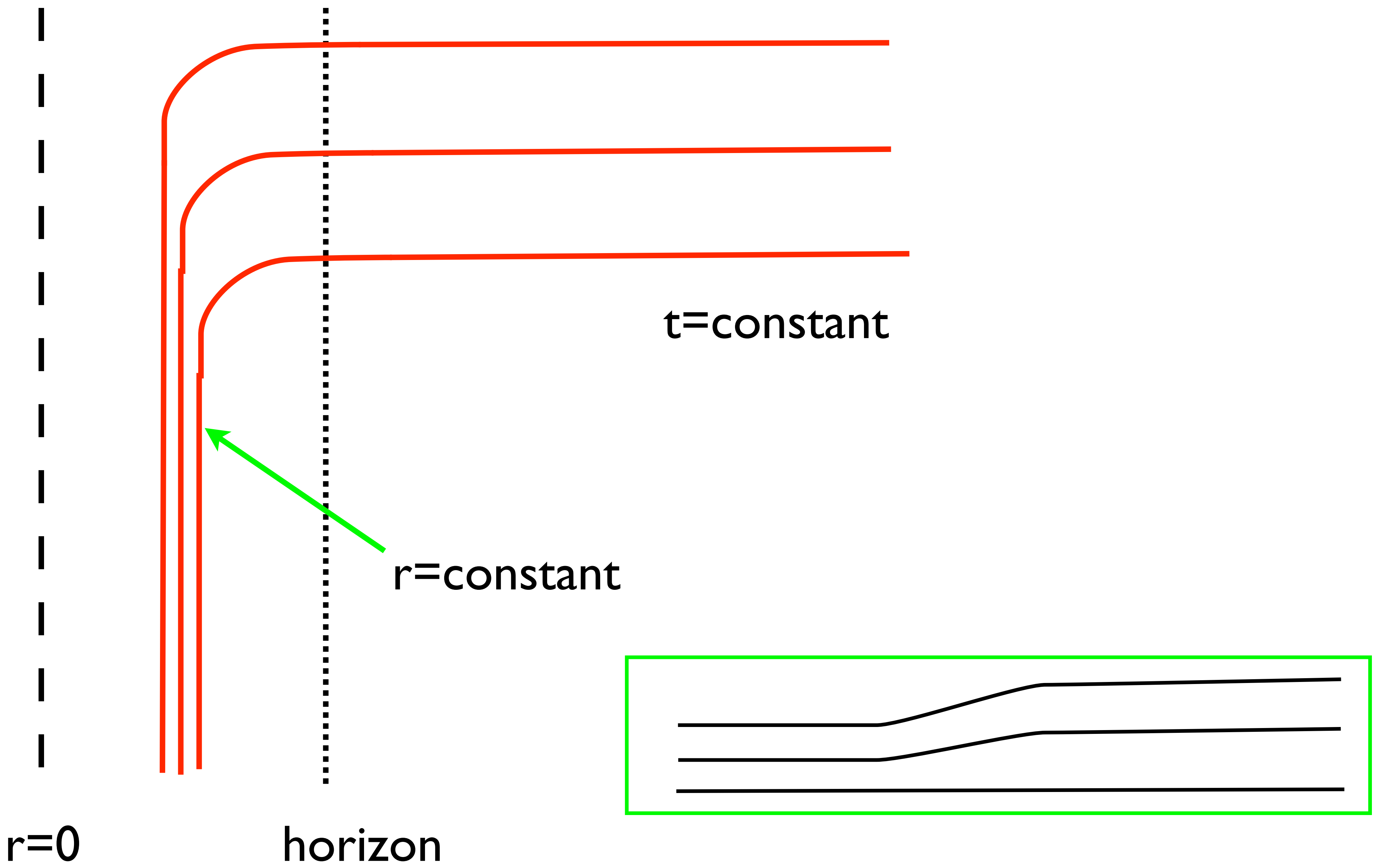
We have

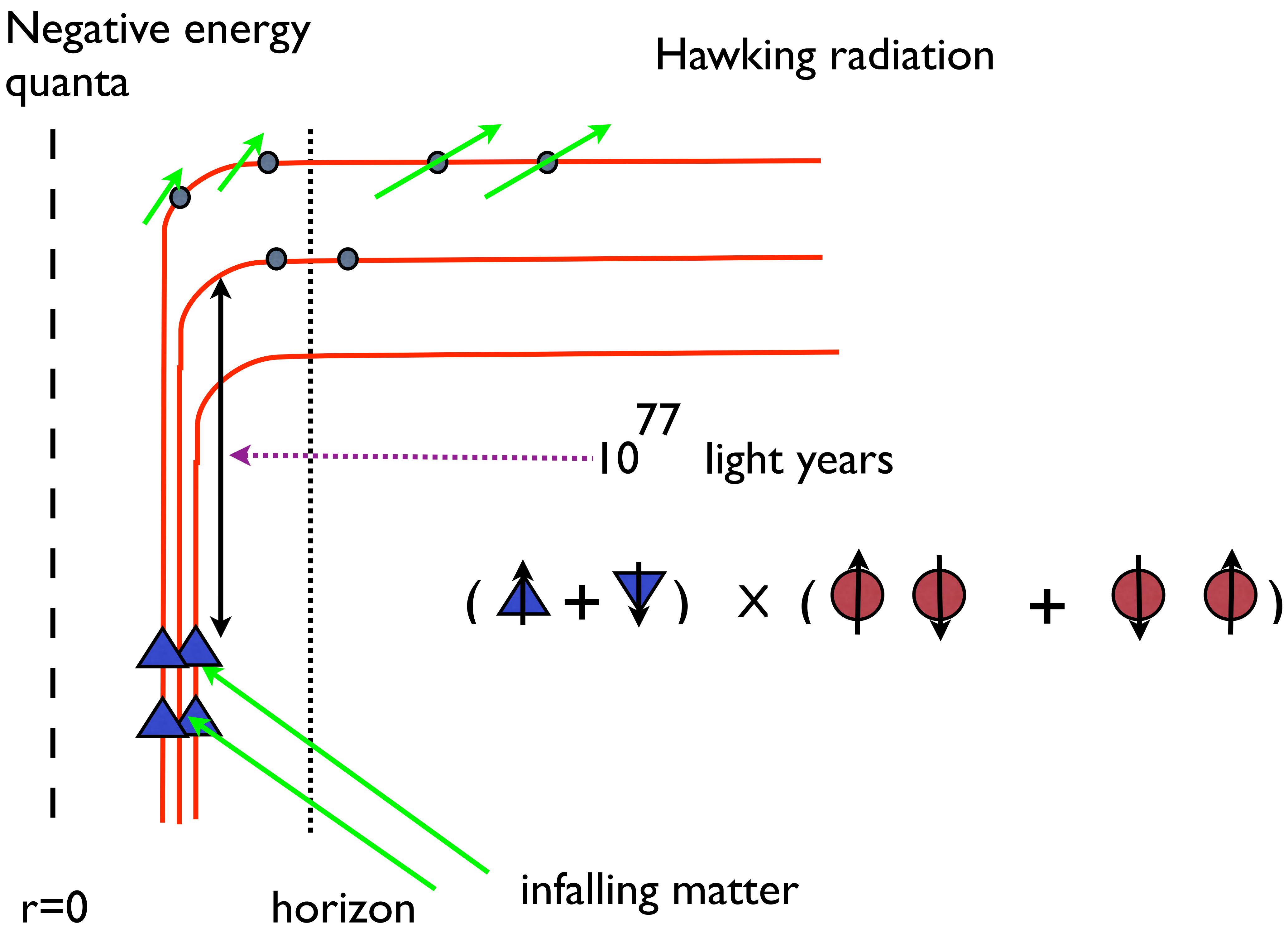


NOT

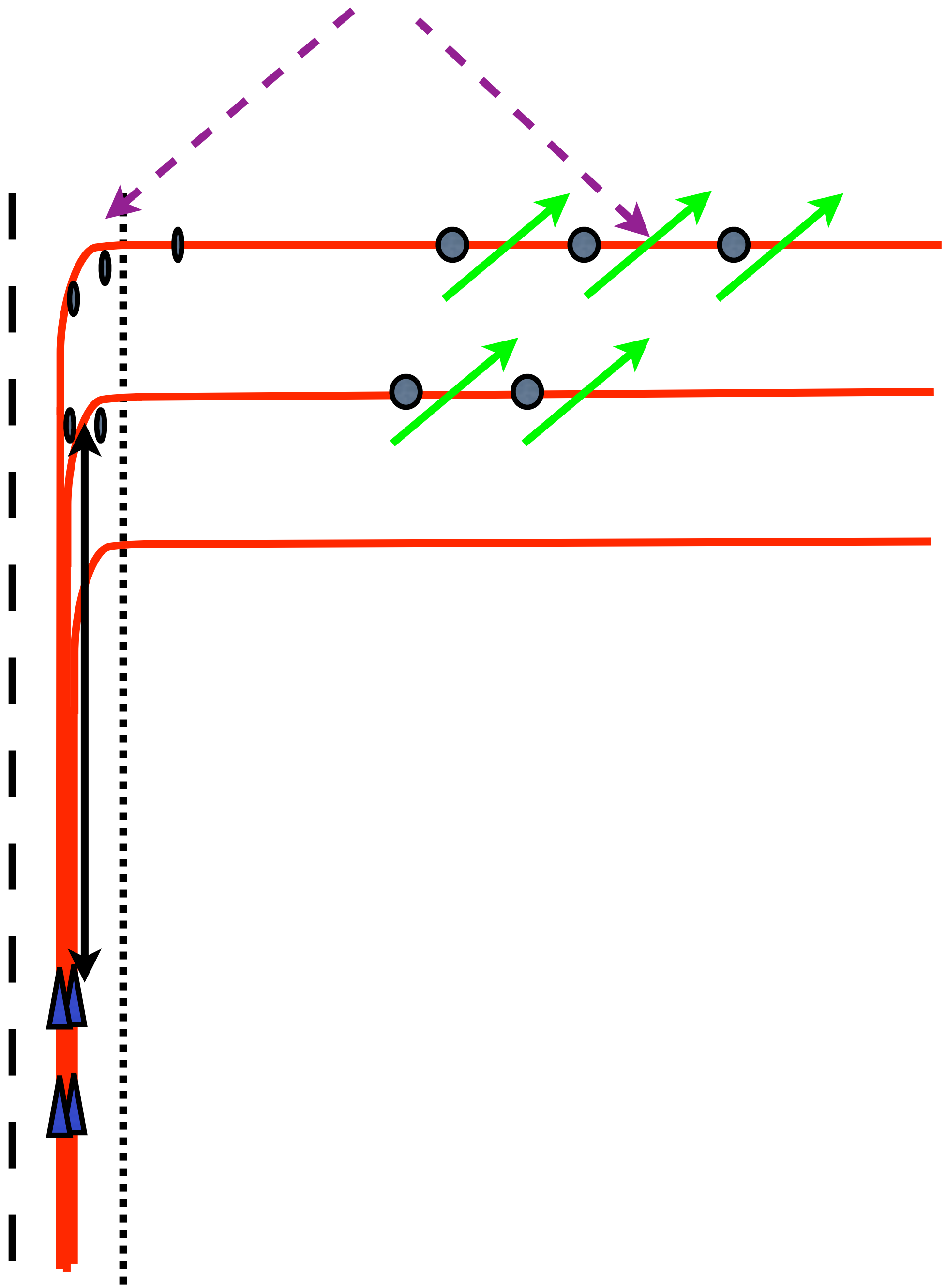


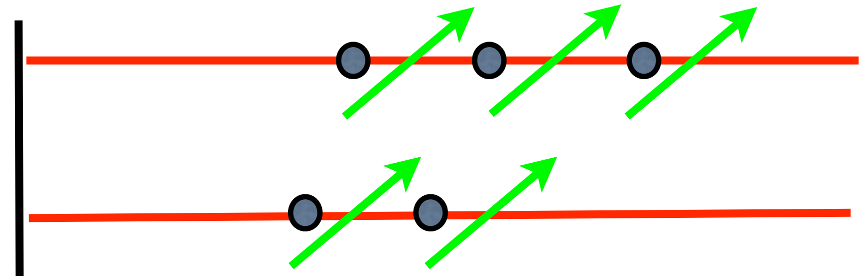
In the black hole we can make very long spacelike slices




Negative energy quanta
Hawking radiation
10^{77} light years
(+) x (+)
r=0
horizon
infalling matter

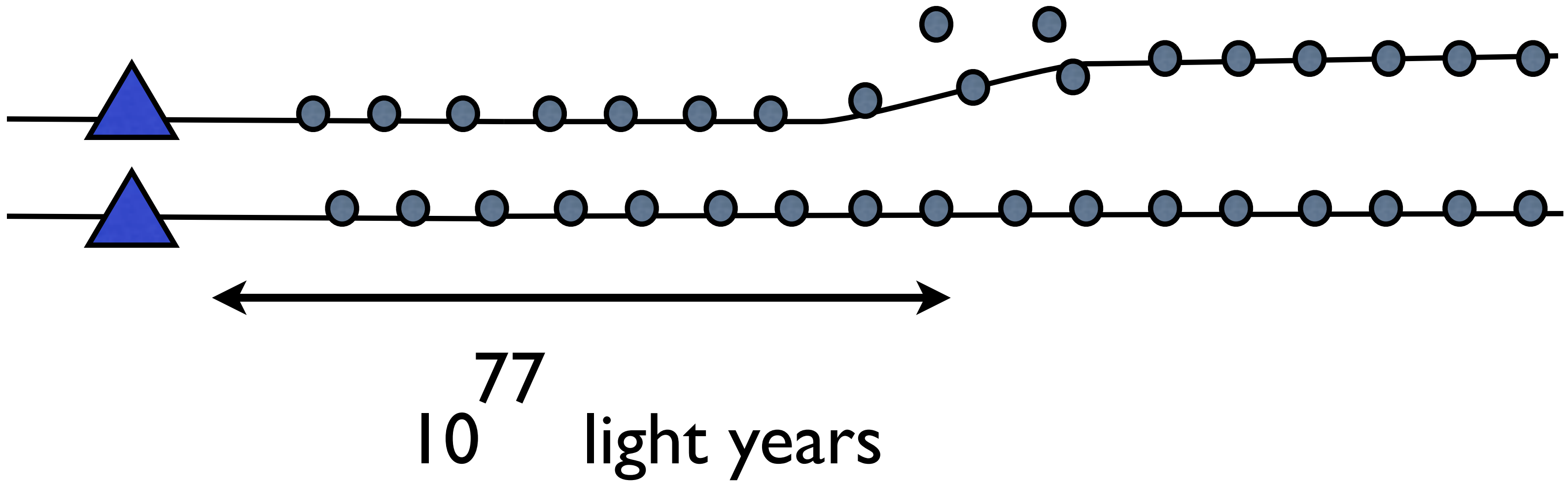
Entangled state



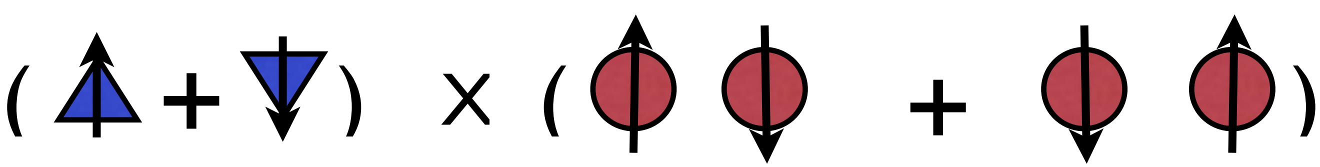


If the black hole evaporates away, we are left in a configuration which cannot be described by a pure state

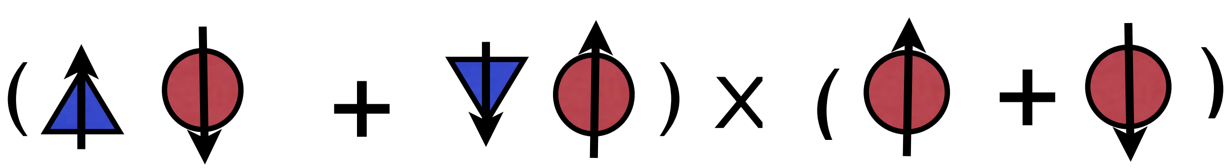
(Radiation quanta are entangled, but there is nothing that they are entangled with)



What we have:

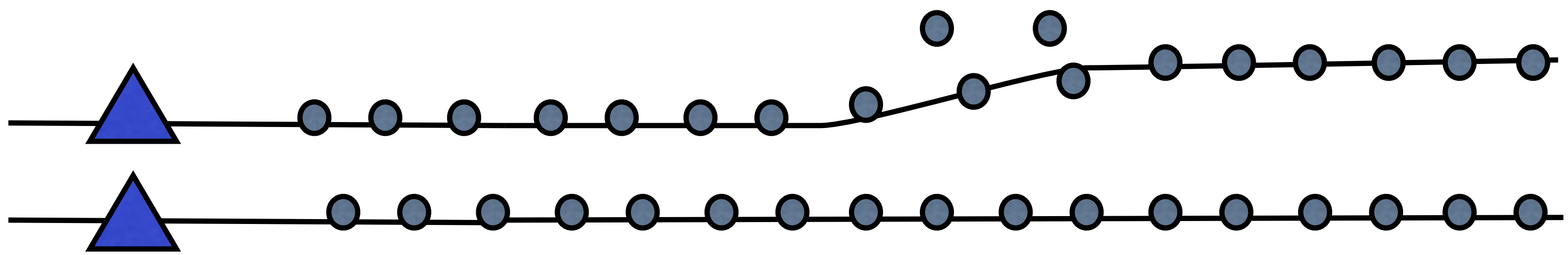


What we need: something like



How can this happen ?

Note that small quantum gravity effects cannot solve the problem



$$\Psi = \Psi_M \otimes \prod_{i=1}^{N} [\ \frac{1}{\sqrt{2}}(\uparrow \downarrow) + \frac{1}{\sqrt{2}}(\downarrow \uparrow)\ ]_i$$

$$\boxed{S_{entanglement} = N \ \ln 2}$$

$$\Psi = \Psi_M \otimes \prod_{i=1}^{N} [\ (\frac{1}{\sqrt{2}} + \epsilon_i)(\uparrow \downarrow) + (\frac{1}{\sqrt{2}} - \epsilon_i)(\downarrow \uparrow)\ ]_i$$

$$\boxed{|\epsilon_i| < \epsilon}$$

$$\boxed{S_{entanglement} > (1 - \epsilon^2)\ N \ \ln 2}$$

We need ORDER UNITY corrections to the evolution of low energy quanta at the horizon

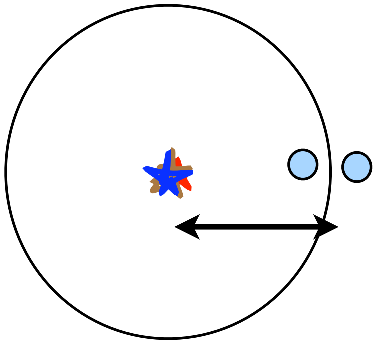
The Hawking 'theorem' can be made completely rigorous

If we are given that

(a) All quantum gravity effects are confined
to within a bounded distance like planck length or string length

and

(b) The vacuum of the theory is unique



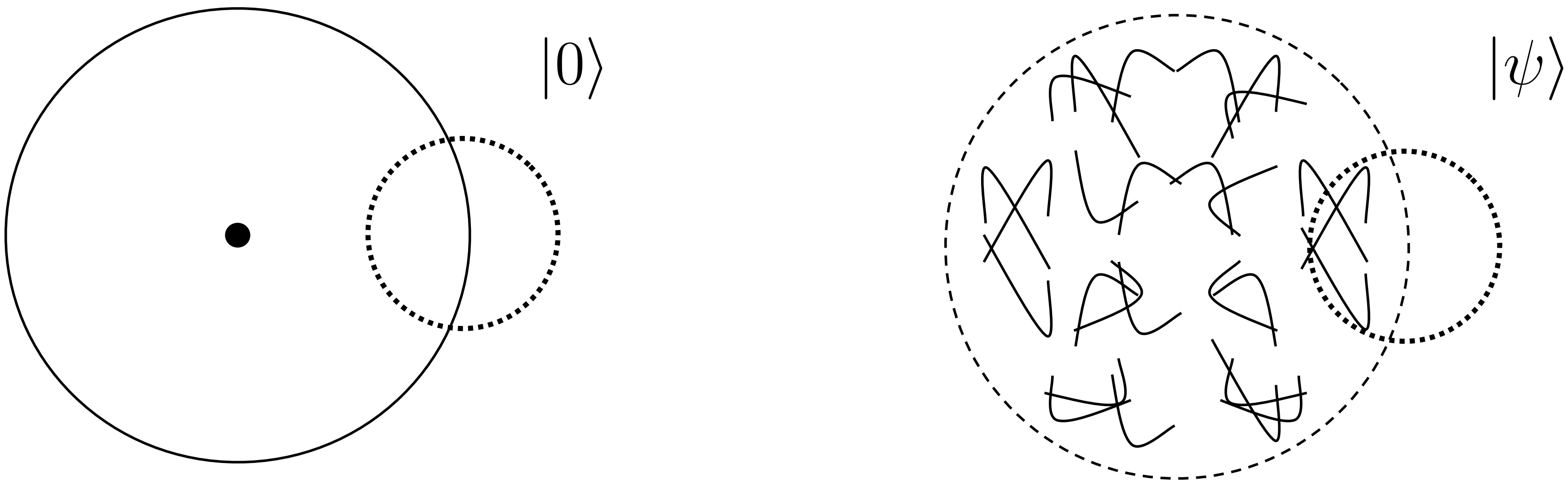
Then there WILL be information loss

Note: The information paradox should be distinguished from the
'Infall problem': What does an infalling observer feel ?

Infall problem: Heavy objects (E >> kT) over 'crossing time'

Hawking radiation: E ~ kT quanta over Hawking evaporation time

It is possible to avoid the paradox if the following happens ...



$$\langle 0|\psi\rangle \approx 0$$

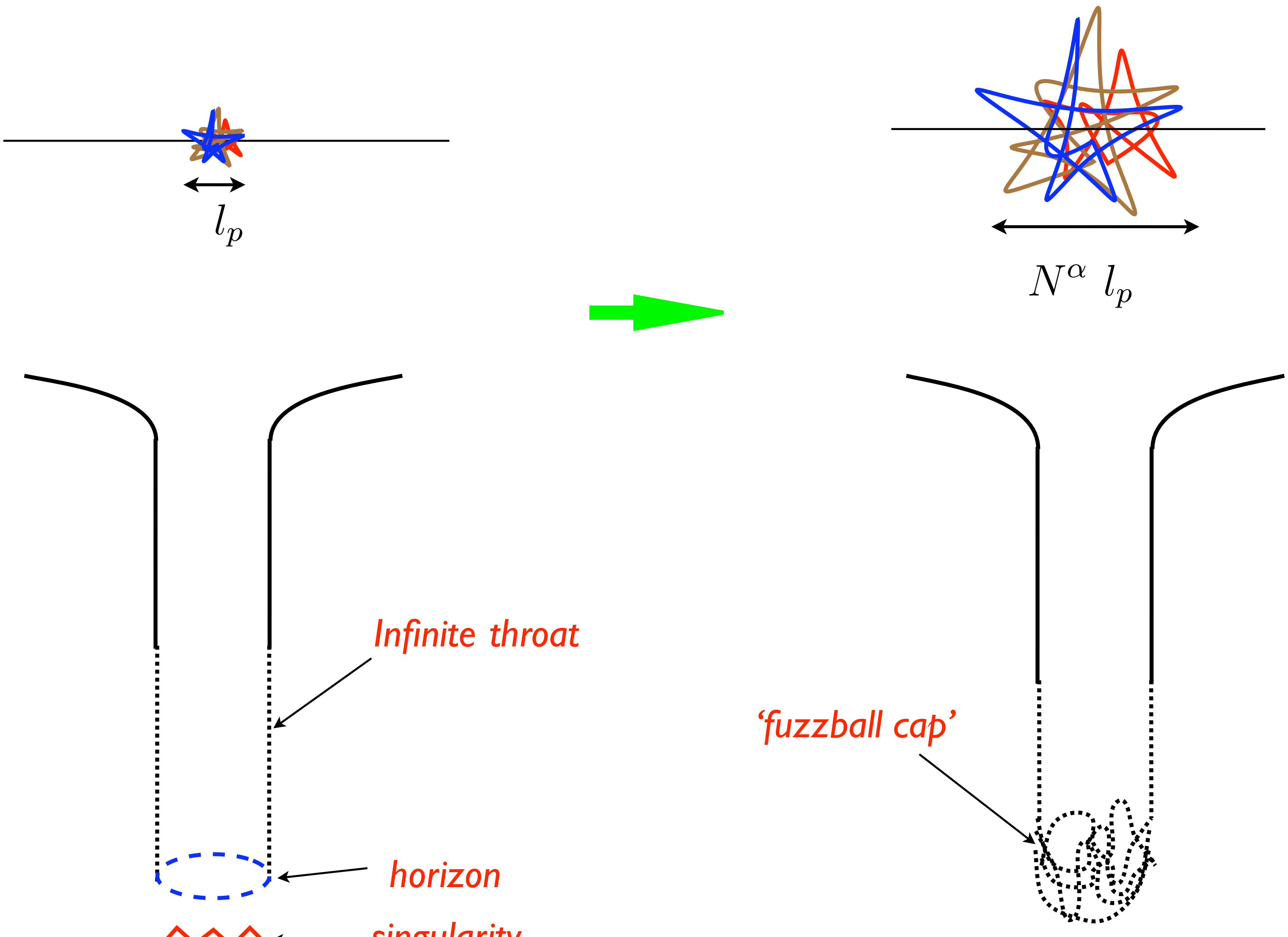
The fuzzball picture

In the traditional black hole, quantum gravity effects are assumed to stretch only over distances $\sim l_p$, and so the state near the horizon is the vacuum.

But a black hole is made of a large number of quanta N, so we must ask if the relevant length scales are $\sim l_p$ or $\sim N^{\alpha}\, l_p$

In string theory, it is easier to start with extremal holes:

A supersymmetric brane state in string theory: Mass = Charge

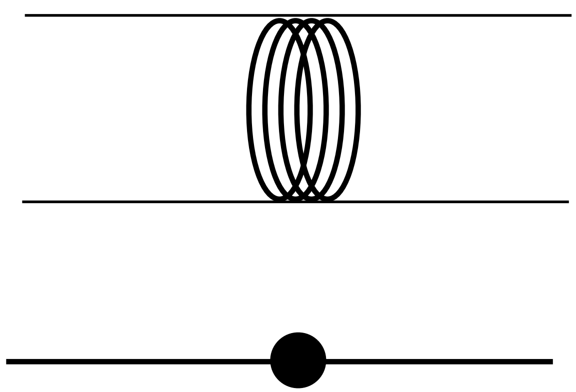


(SDM 97)

Making extremal black holes in string theory:

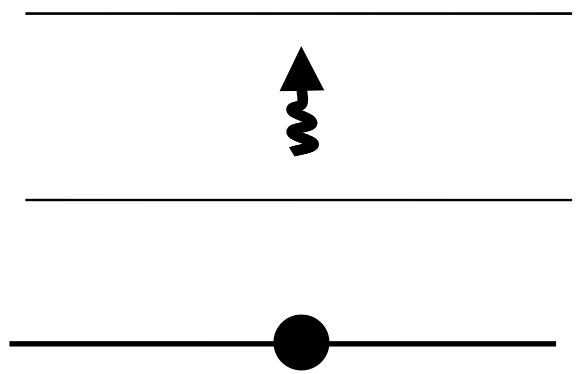
Wrap strings, branes etc on compact directions

This gives mass and charge at a given location from the viewpoint of noncompact directions ....



Winding charge n_1

Mass = Charge



Momentum charge n_p

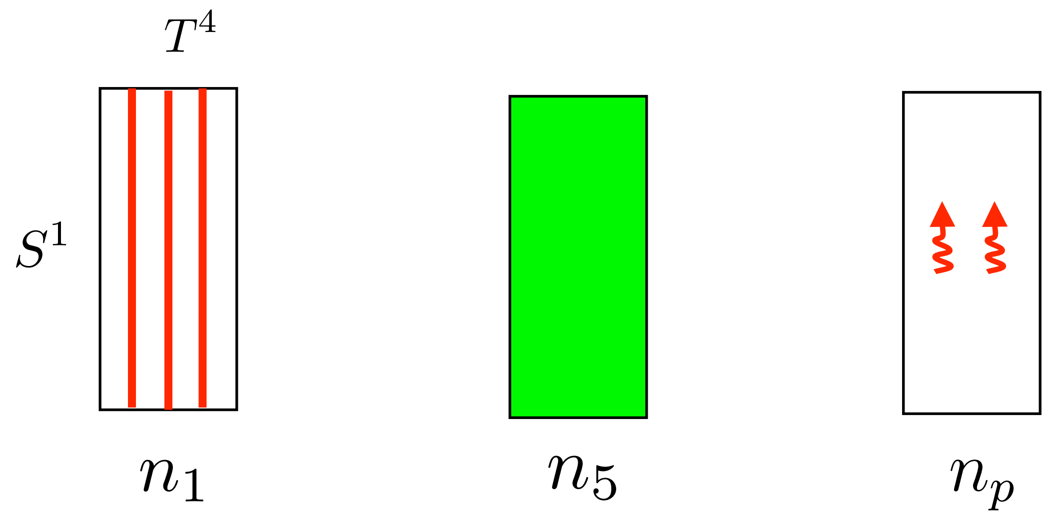
Mass = Charge

The setting: IIB string theory

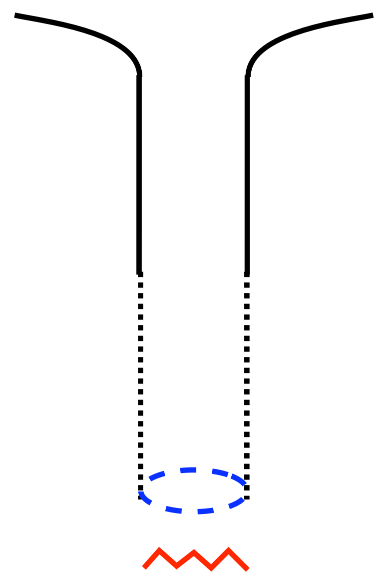
Compactification to 4+1 noncompact dimensions:

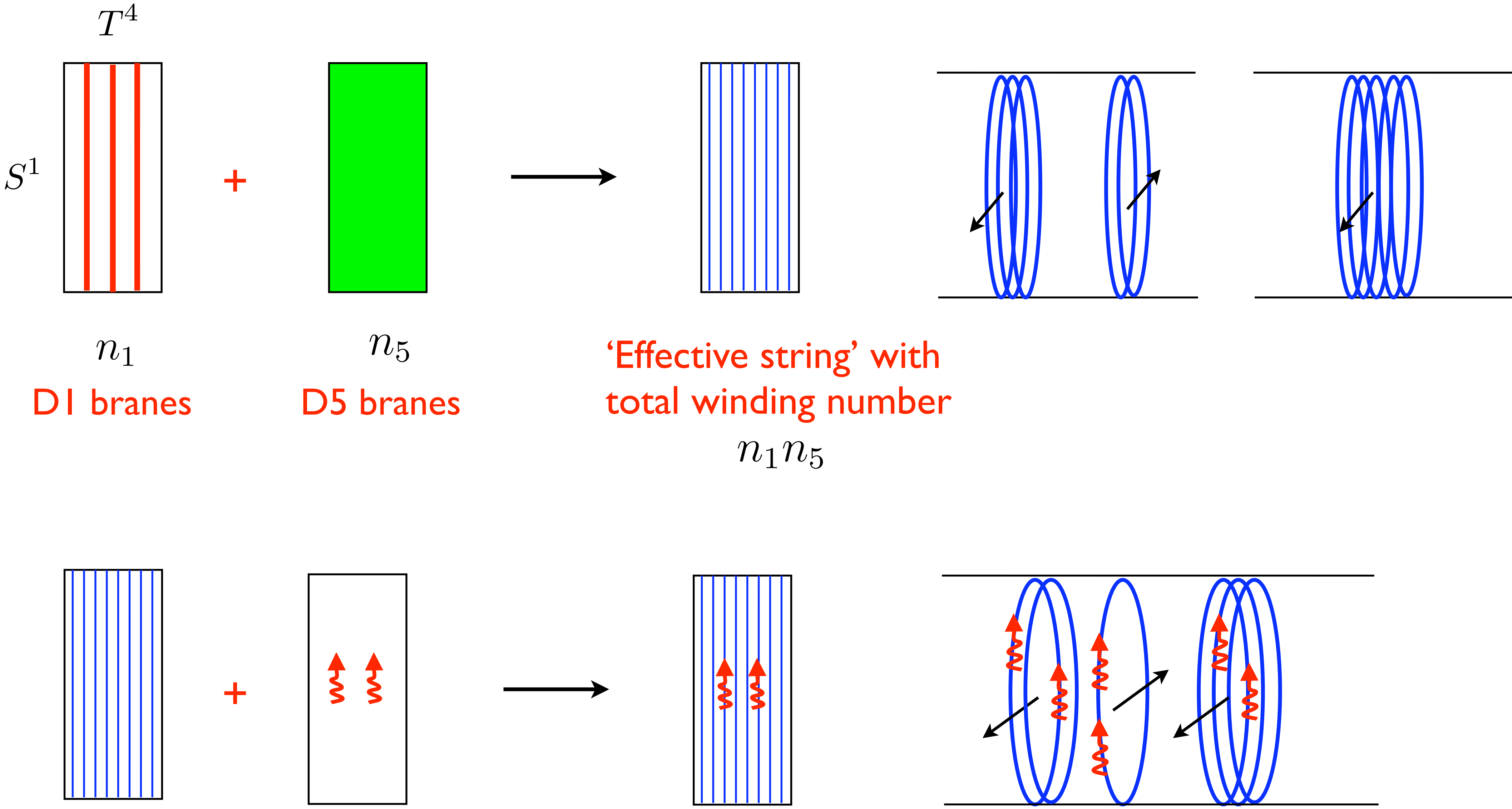
$$M_{9,1} \to M_{4,1} \times T^4 \times S^1$$

Charges:
D1, D5, P



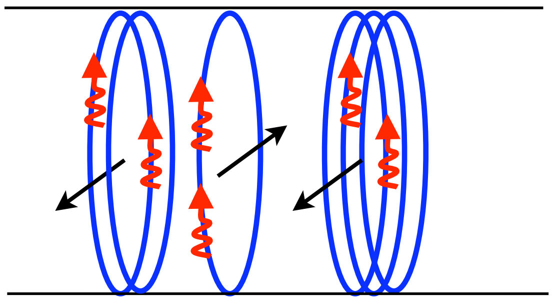
Naive expectation: The bound state of these charges, placed at the origin, will give rise to an extremal Riessner-Nordstrom black hole

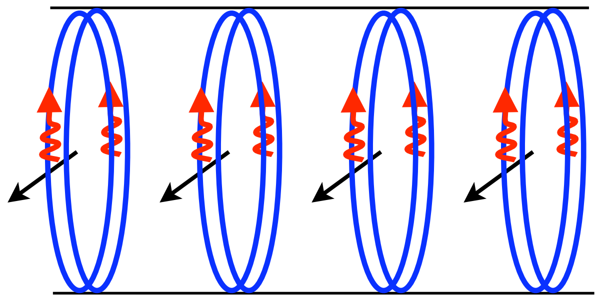




Count of microstates agrees with the Area entropy of black hole

(Strominger Vafa 96)

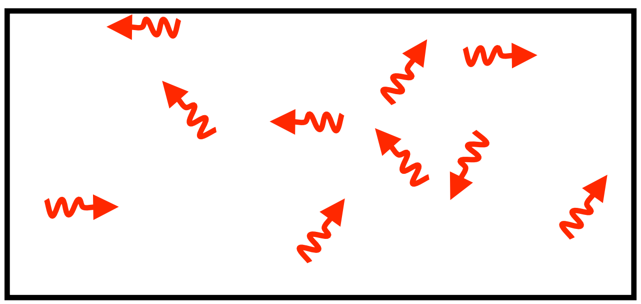


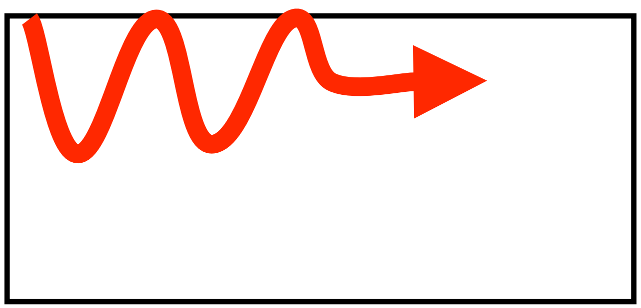


Generic D1D5P CFT state

Simple states: all components the same, excitations fermionic, spin aligned

Close analogy:





Black body radiation, many quanta, only a few in each harmonic

Very 'quantum' state

Special state: put all quanta into same harmonic, laser beam

State well described by classical E,B fields

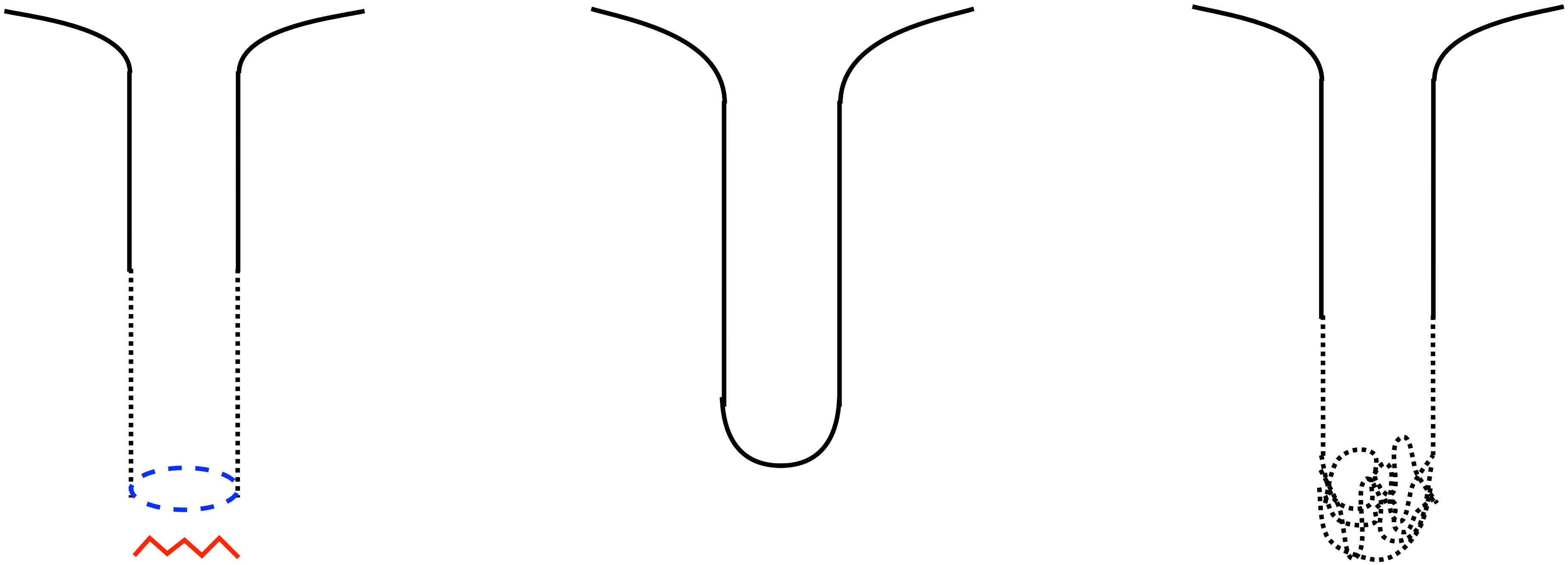
$$
\begin{aligned}
ds^2 &= -\frac{1}{h}(dt^2 - dy^2) + \frac{Q_p}{hf}(dt-dy)^2 + hf\left(\frac{dr_N^2}{r_N^2 + a^2\eta} + d\theta^2\right) \\
&+ h\left(r_N^2 - na^2\eta + \frac{(2n+1)a^2\eta Q_1 Q_5 \cos^2\theta}{h^2 f^2}\right)\cos^2\theta d\psi^2 \\
&+ h\left(r_N^2 + (n+1)a^2\eta - \frac{(2n+1)a^2\eta Q_1 Q_5 \sin^2\theta}{h^2 f^2}\right)\sin^2\theta d\phi^2 \\
&+ \frac{a^2\eta^2 Q_p}{hf}\left(\cos^2\theta d\psi + \sin^2\theta d\phi\right)^2 \\
&+ \frac{2a\sqrt{Q_1 Q_5}}{hf}\left[n\cos^2\theta d\psi - (n+1)\sin^2\theta d\phi\right](dt - dy) \\
&- \frac{2a\eta\sqrt{Q_1 Q_5}}{hf}\left[\cos^2\theta d\psi + \sin^2\theta d\phi\right]dy + \sqrt{\frac{H_1}{H_5}}\sum_{i=1}^{4} dz_i^2
\end{aligned}
$$

$$
\begin{aligned}
f &= r_N^2 - a^2\eta\, n\sin^2\theta + a^2\eta\,(n+1)\cos^2\theta \\
h &= \sqrt{H_1 H_5},\ \ H_1 = 1 + \frac{Q_1}{f},\ \ H_5 = 1 + \frac{Q_5}{f}
\end{aligned}
$$

$$
\eta \equiv \frac{Q_1 Q_5}{Q_1 Q_5 + Q_1 Q_p + Q_5 Q_p}
$$

(Giusto SDM Saxena 04)

This metric has no horizons, no closed timelike curves, no singularities



Naive geometry for the hole

Geometry for special state

Expect generic state has quantum, stringy 'cap'

With such a structure, there is no information paradox ....

Not all states have been made for all holes, but with these examples, the 'boot is on the other leg': if someone wants to argue there is a paradox, he needs to show that there *do* exists states with an 'information free horizon'

Work by many authors ....

Cvetic-Youm, Balasubramanian-Keski-Vakkuri-deBoer-Ross, Maldacena-Maoz ...

Lunin-SDM, Lunin-Maldacena-Maoz, SDM-Saxena-Srivastava, Giusto-SDM-Saxena

Taylor, Skenderis-Taylor, ...

Bena, Bena-Kruas, Bena-Warner, Bena-Warner + Wang, Ruef, Agata, Giusto

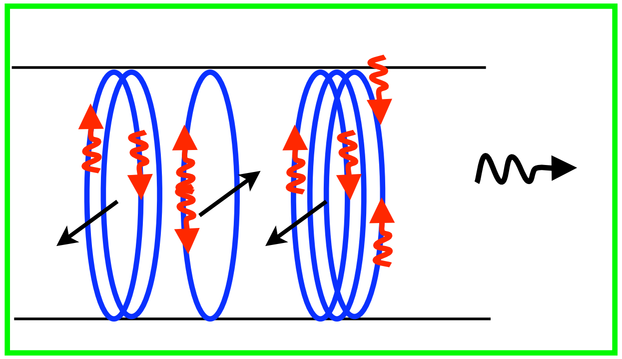
Balasubramanian-Gimon-Levi, Berglund-Gimon-Levi, Gimon-Levi, Saxena-Giusto-Potvin-Peet ...

deBoer-El-Showk-Messamah-Van den Bleeken, Balasubramanian-de Boer-El-Showk-Messamah

Jejjala-Madden-Ross-Titchner, Giusto-Ross-Saxena ...

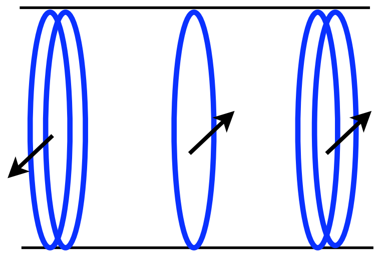
Chowdhury-SDM ...

Related work: Denef, Emparan, deBoer ...

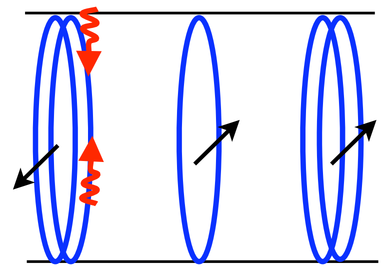


Most general state

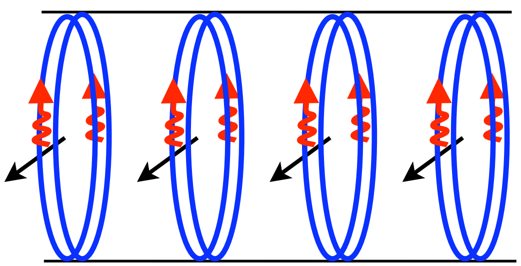
Known states:



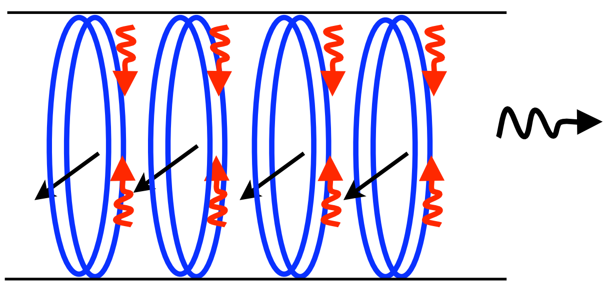
2-charge extremal

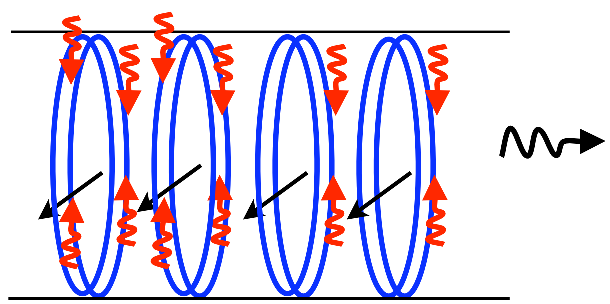


2-charge extremal + excitation



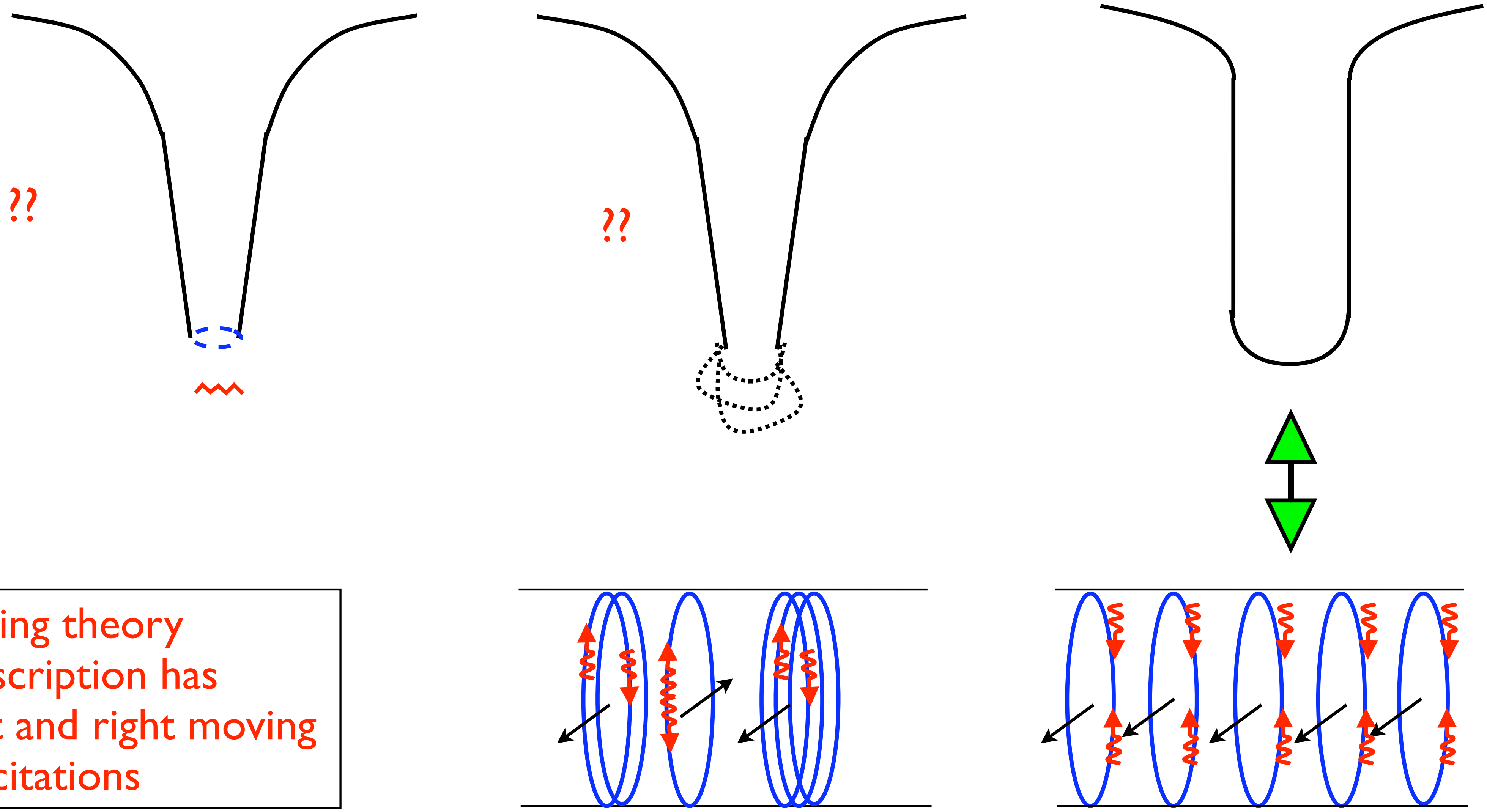
3-charge extremal: Large classes also known with CFT state not yet identified





Nonextremal: Some families known, radiation agrees

The Non-Extremal Hole :



String theory description has left and right moving excitations

$$
\begin{aligned}
\mathrm{d}s^2 \quad = \quad & -\frac{f}{\sqrt{\tilde{H}_1\tilde{H}_5}}(\mathrm{d}t^2 - \mathrm{d}y^2) + \frac{M}{\sqrt{\tilde{H}_1\tilde{H}_5}}(s_p\mathrm{d}y - c_p\mathrm{d}t)^2 \\
& + \sqrt{\tilde{H}_1\tilde{H}_5}\left(\frac{r^2\mathrm{d}r^2}{(r^2+a_1^2)(r^2+a_2^2) - Mr^2} + \mathrm{d}\theta^2\right) \\
& + \left(\sqrt{\tilde{H}_1\tilde{H}_5} - (a_2^2 - a_1^2)\frac{(\tilde{H}_1 + \tilde{H}_5 - f)\cos^2\theta}{\sqrt{\tilde{H}_1\tilde{H}_5}}\right)\cos^2\theta\,\mathrm{d}\psi^2 \\
& + \left(\sqrt{\tilde{H}_1\tilde{H}_5} + (a_2^2 - a_1^2)\frac{(\tilde{H}_1 + \tilde{H}_5 - f)\sin^2\theta}{\sqrt{\tilde{H}_1\tilde{H}_5}}\right)\sin^2\theta\,\mathrm{d}\phi^2 \\
& + \frac{M}{\sqrt{\tilde{H}_1\tilde{H}_5}}(a_1\cos^2\theta\,\mathrm{d}\psi + a_2\sin^2\theta\,\mathrm{d}\phi)^2 \\
& + \frac{2M\cos^2\theta}{\sqrt{\tilde{H}_1\tilde{H}_5}}[(a_1c_1c_5c_p - a_2s_1s_5s_p)\mathrm{d}t + (a_2s_1s_5c_p - a_1c_1c_5s_p)\mathrm{d}y]\mathrm{d}\psi \\
& + \frac{2M\sin^2\theta}{\sqrt{\tilde{H}_1\tilde{H}_5}}[(a_2c_1c_5c_p - a_1s_1s_5s_p)\mathrm{d}t + (a_1s_1s_5c_p - a_2c_1c_5s_p)\mathrm{d}y]\mathrm{d}\phi \\
& + \sqrt{\frac{\tilde{H}_1}{\tilde{H}_5}}\sum_{i=1}^{4}\mathrm{d}z_i^2
\end{aligned}
$$

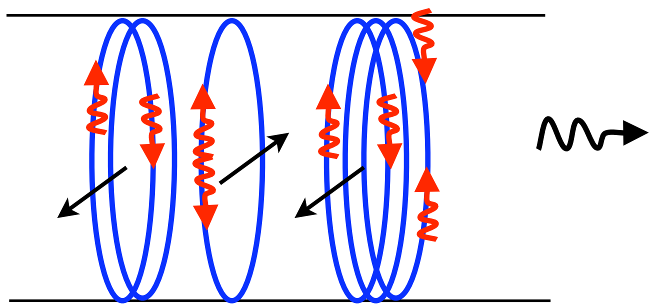
$$
\begin{aligned}
Q_1 &= \frac{g\alpha'^3}{V}n_1 \\
Q_5 &= g\alpha' n_5 \\
Q_p &= \frac{g^2\alpha'^4}{VR^2}n_p
\end{aligned}
$$

(Jejalla, Madden, Ross Titchener '05)

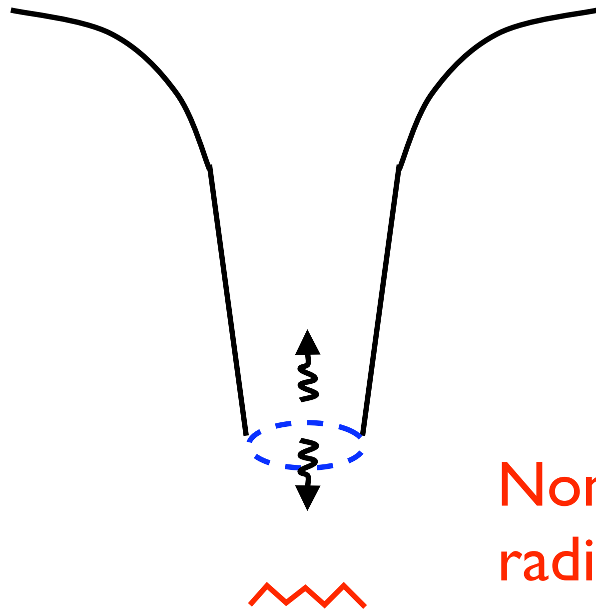
$$
\tilde{H}_i = f + M\sinh^2\delta_i, \quad f = r^2 + a_1^2\sin^2\theta + a_2^2\cos^2\theta
$$

$$
Q_1 = M\sinh\delta_1\cosh\delta_1, \quad Q_5 = M\sinh\delta_5\cosh\delta_5, \quad Q_p = M\sinh\delta_p\cosh\delta_p
$$

Hawking radiation



Unitary radiation process in CFT

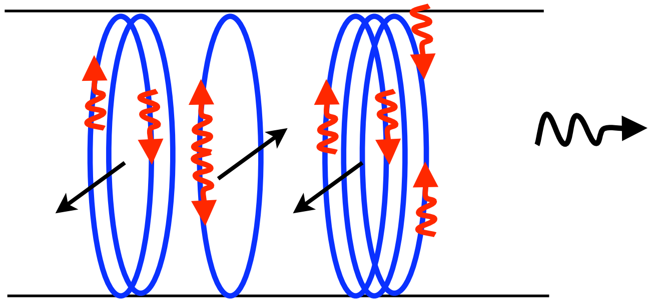


Non-Unitary radiation from semiclassical gravity

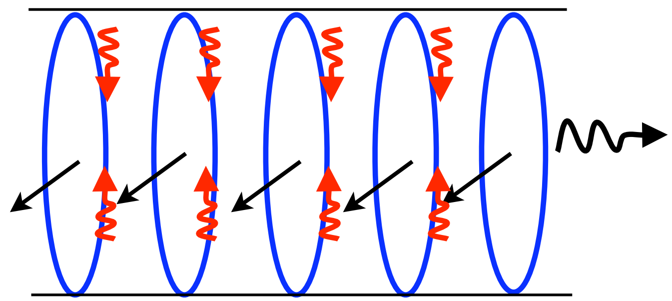
Radiation rates agree (Spins, greybody factors ...)

(Callan-Maldacena 96, Dhar-Mandal-Wadia 96, Das-Mathur 96, Maldacena-Strominger 96)

Can we get UNITARY radiation (information carrying) in the GRAVITY description ??



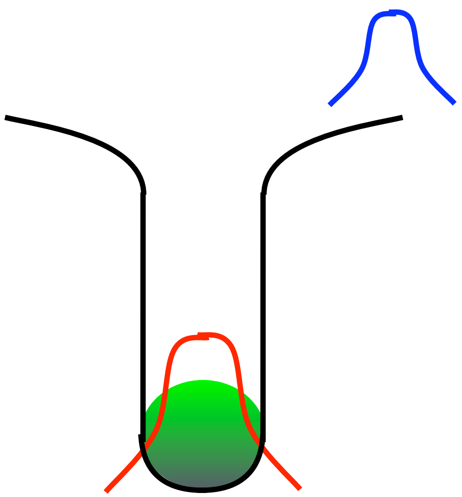
Rate agrees with Hawking radiation



Compute for particular state

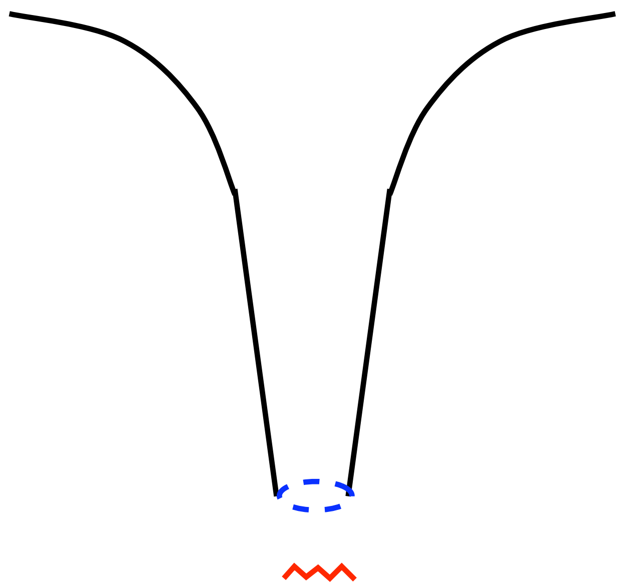
This gravity solution has no horizon, no singularity , but it has an ***ergoregion***, which has ergoregion emission (Cardoso, Dias, Jordan, Hobvedo, Myers 06)

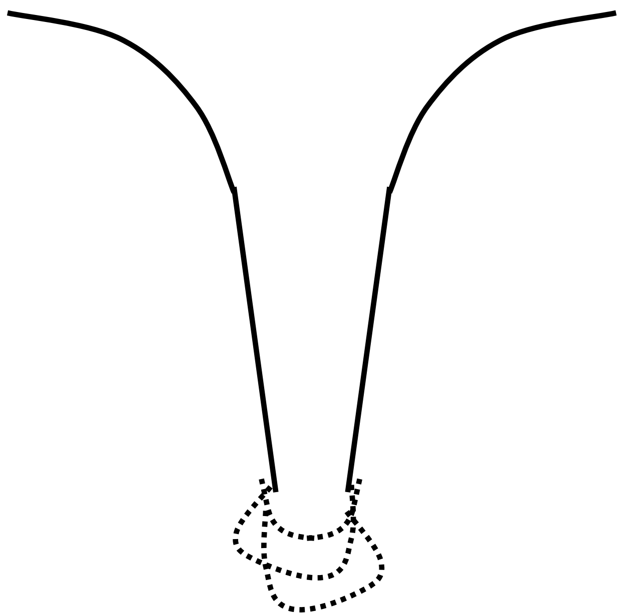
Find exact agreement of 'ergoregion emission rate with Hawking radiation rate (Chowdhury+SDM 06)



Thus we get information carrying Hawking emission from this particular microstate

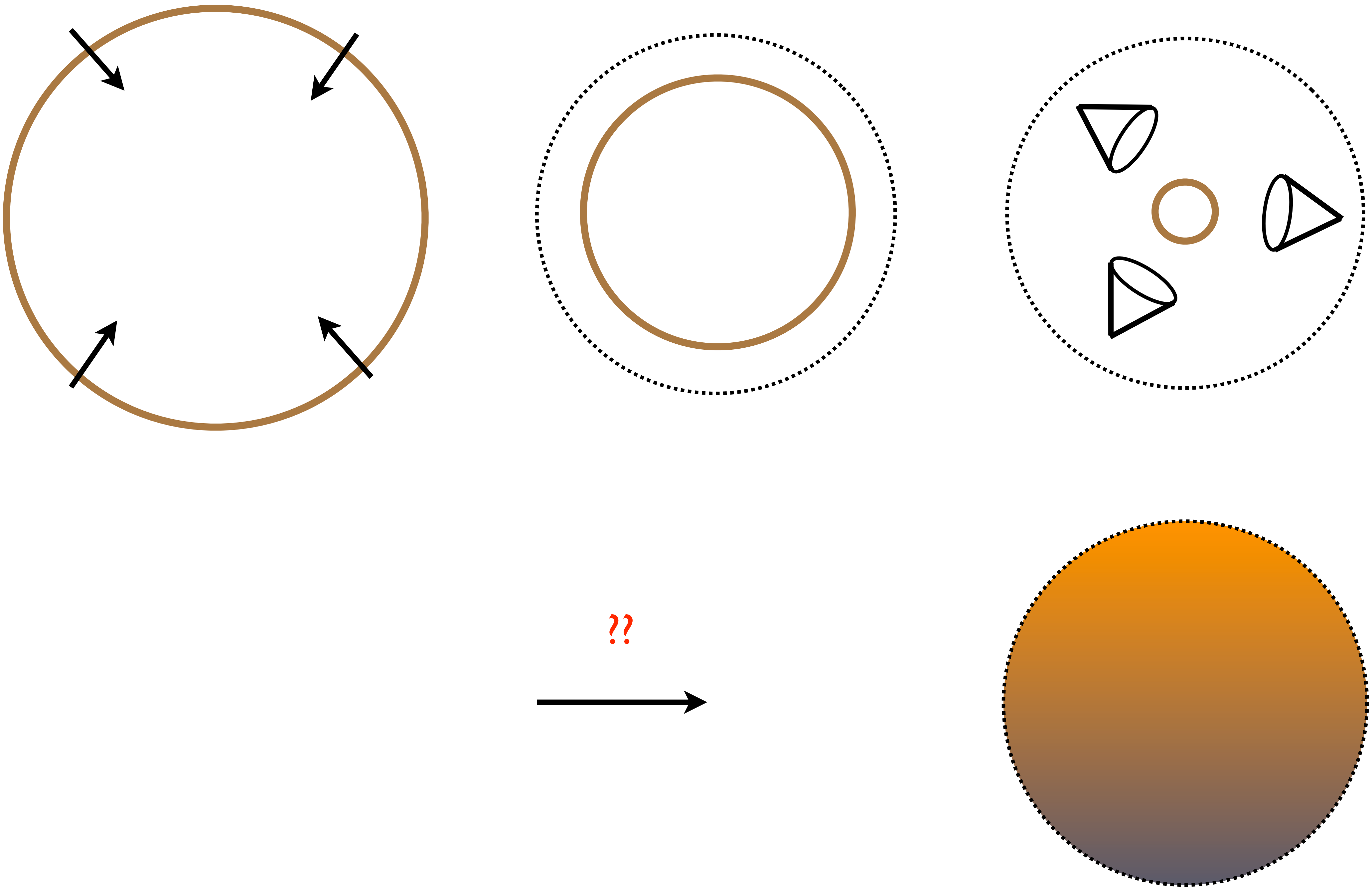
We have changed the interior of a classical sized horizon region ....



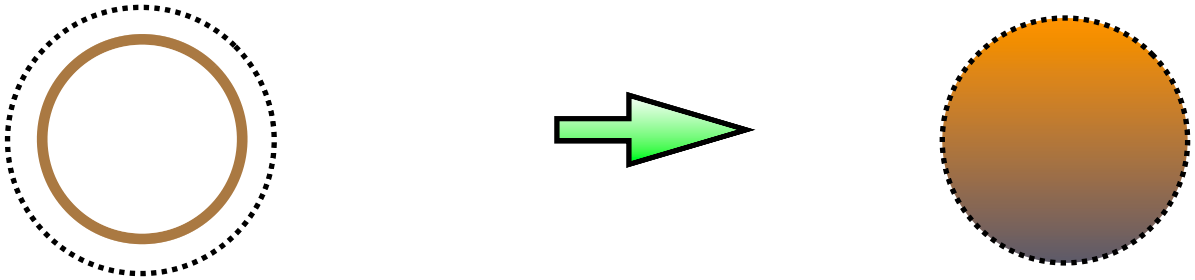


How can classical intuition fail in black holes ? : Conjectures

How could our classical intuition have gone wrong ?



Consider the amplitude for the shell to tunnel to a fuzzball state



$$S_{tunnel} \sim \frac{1}{G}\int R d^4x \sim \frac{1}{G}\frac{1}{(GM)^2}(GM)^4 \sim GM^2$$

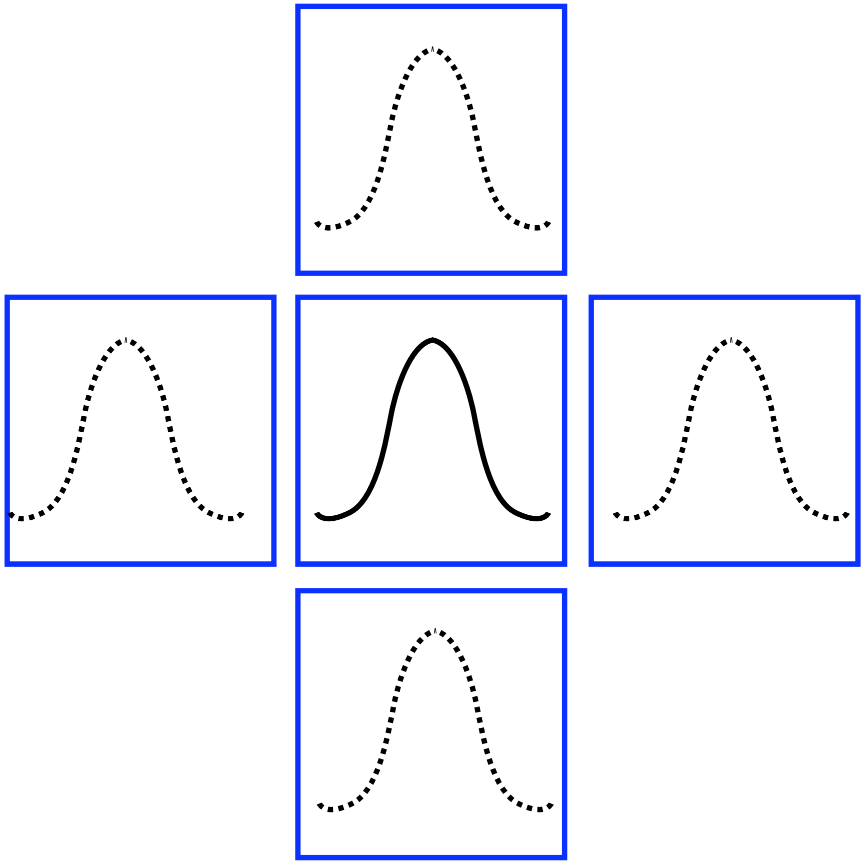
$$\mathcal{A} \sim e^{-S_{tunnel}}$$

Amplitude to tunnel is very small

$$\mathcal{N} \sim e^{S_{bek}} \sim e^{GM^2}$$

But the number of states that one can tunnel to is very large !

Toy model: Small amplitude to tunnel to a neighboring well, but there are a correspondingly large number of adjacent wells

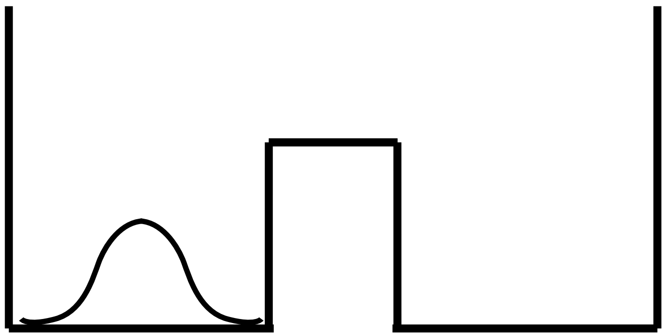


In a time of order unity, the wavefunction in the central well becomes a linear combination of states in all wells

How long does this tunneling process take ?

If it takes longer than Hawking evaporation time then it does not help ...

Tunneling in the double well:



$$\psi = e^{-iE_S t}\psi_S \; + \; e^{-iE_A t}\psi_A$$

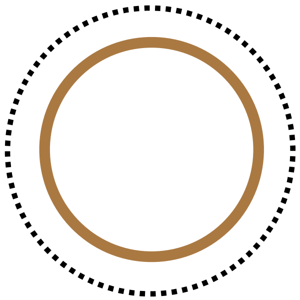
The wavefunction tunnels to the other well in a time

$$\Delta t = \frac{\pi}{\Delta E}$$

where

$$\Delta E = E_A - E_S$$

For the collapsing shell ...



$$\boxed{|\psi\rangle = \sum_k c_k |E_k\rangle}$$

$$E \sim \frac{P^2}{2M}$$

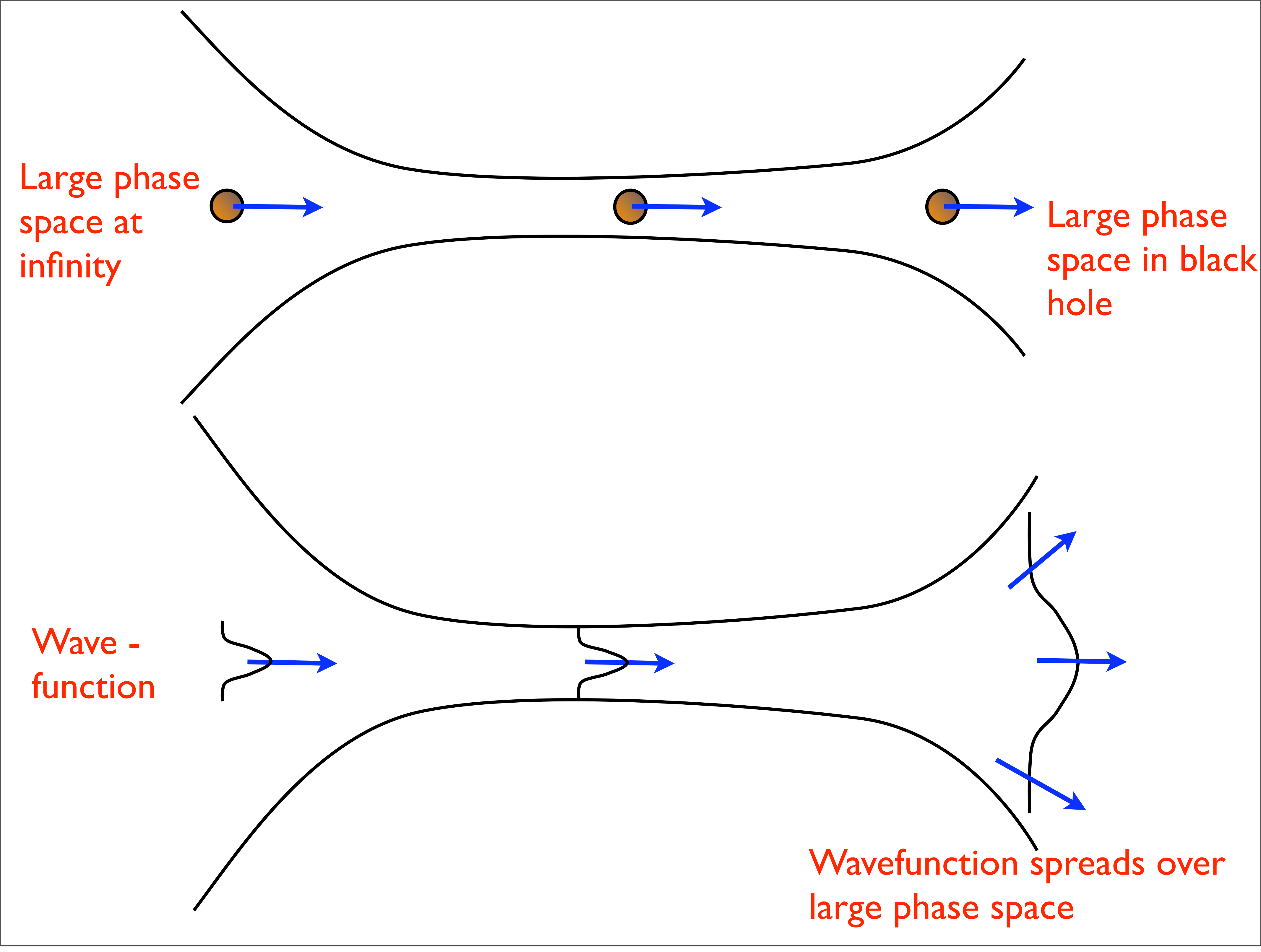
$$\Delta P \gg \frac{1}{R}$$

$$\boxed{\Delta E \sim \frac{P\Delta P}{M} \gg \frac{(\Delta P)^2}{M} \gg \frac{1}{MR^2}}$$

$$t_{dephase} \sim \frac{1}{\Delta E} \ll MR^2 \qquad t_{evap} \sim MR^2$$

$$\boxed{t_{dephase} \ll t_{evap}}$$

Thus the collapsing shell turns into a linear combination of fuzzball states in a time short compared to Hawking evaporation time


Large phase space at infinity
Large phase space in black hole
Wave - function
Wavefunction spreads over large phase space

Cosmology

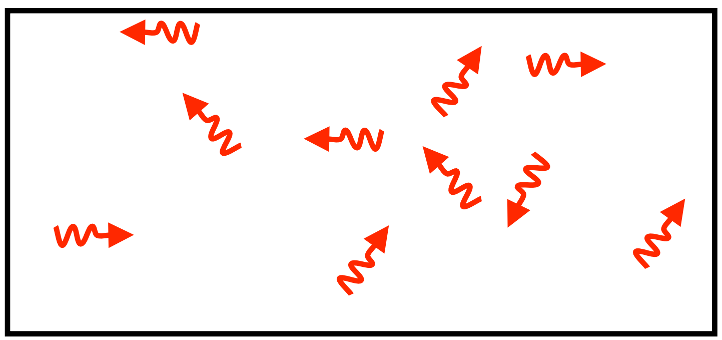
Start with a box of volume V

In the box put energy E

Take the limit of very large E

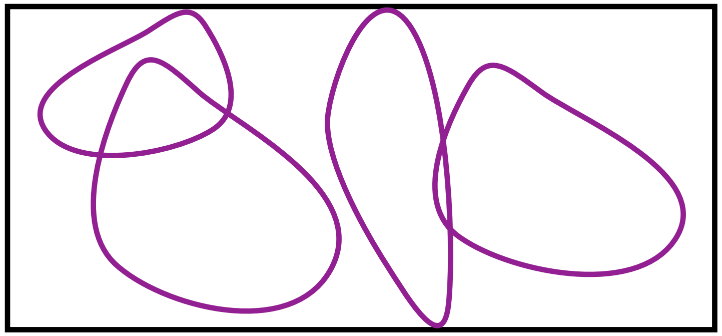
S(E) = ??

Question: What is the state of maximal entropy S, and how much is S(E) ?



Radiation

$$S \sim E^{\frac{D-1}{D}}$$

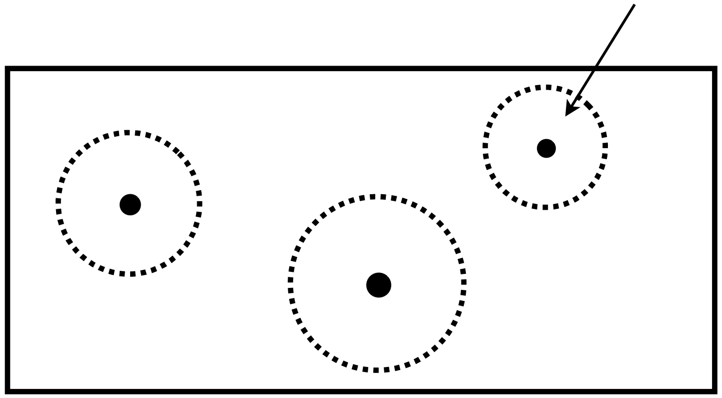


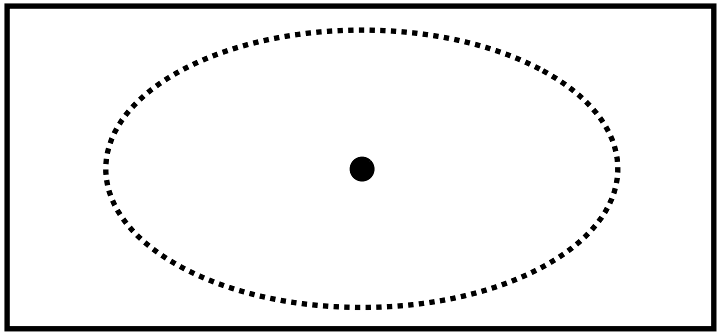
String gas
'Hagedorn phase'

$$\boxed{S \sim E \sim \sqrt{E}\sqrt{E}}$$

(Brandenberger+Vafa)

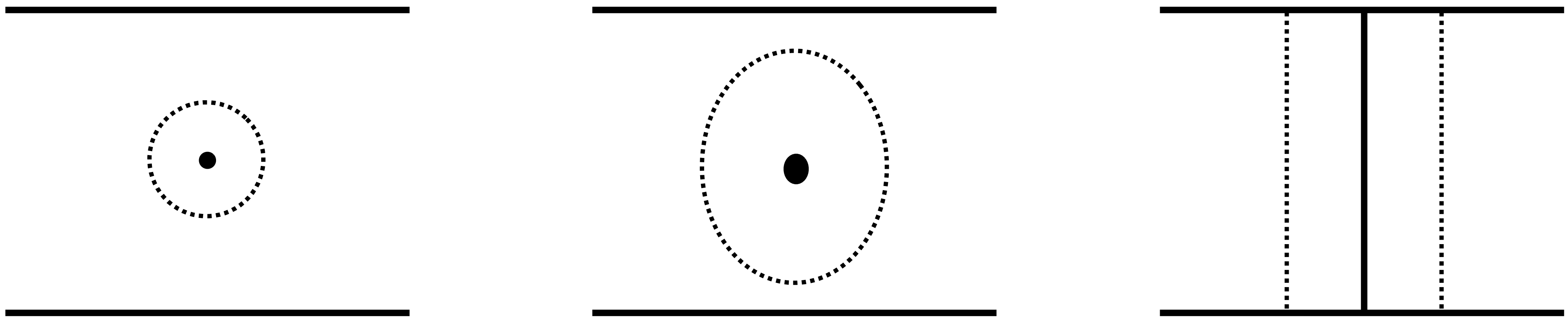
Black holes





What happens if we put even more energy ?

The Gregory Laflamme transition



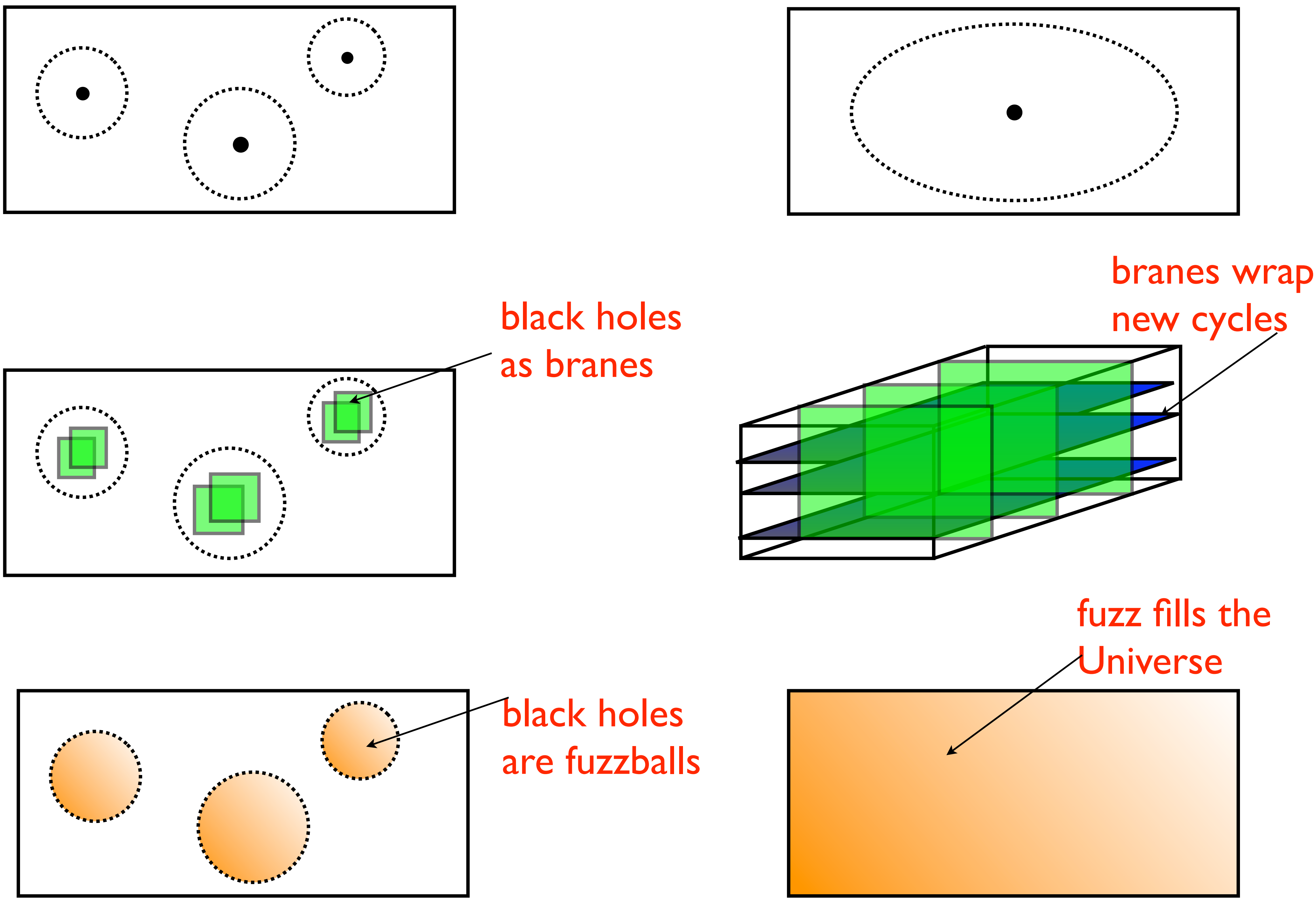
We now have a microscopic model for the Gregory Laflamme transition

(Chowdhury, Giusto, SDM 06)

The black hole is made of D-branes wrapped on cycles $(D1+D5+P\bar{P})$

The black string is made of these D-branes AND an additional set of branes wrapping the new cycle $(D1+D5+P\bar{P}+KK\,\overline{KK})$

The switch happens when the 4-charge system has more entropy than 3-charge



black holes
as branes
branes wrap
new cycles
black holes
are fuzzballs
fuzz fills the
Universe

Microscopic entropy formulae: Count of brane states agrees with Bekenstein Area entropy

2-charges $$S = 2\sqrt{2}\pi\sqrt{n_1 n_2}$$

3-charges $$S = 2\pi\sqrt{n_1 n_2 n_3}$$

4-charges $$S = 2\pi\sqrt{n_1 n_2 n_3 n_4}$$

Extremal holes

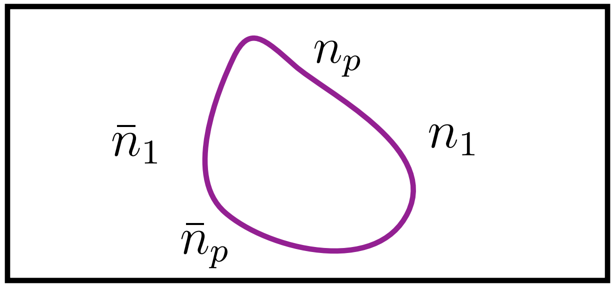
2 charges + nonextremality $$S = 2\pi\sqrt{n_1 n_2}(\sqrt{n_3} + \sqrt{\bar{n}_3})$$

3-charges + nonextremality $$S = 2\pi\sqrt{n_1 n_2 n_3}(\sqrt{n_4} + \sqrt{\bar{n}_4})$$

Near extremal holes

But simple extensions of these expressions also work exactly far from extremality (including the case of the neutral Schwarzschild hole) ...

(Horowitz Maldcena Strominger 96, Horowitz Lowe Maldacena 96)



Two charges

$$S = 2\pi\sqrt{2}(\sqrt{n_1} + \sqrt{\bar{n}_1})(\sqrt{n_p} + \sqrt{\bar{n}_p}) \sim \sqrt{E}\sqrt{E} \sim E$$

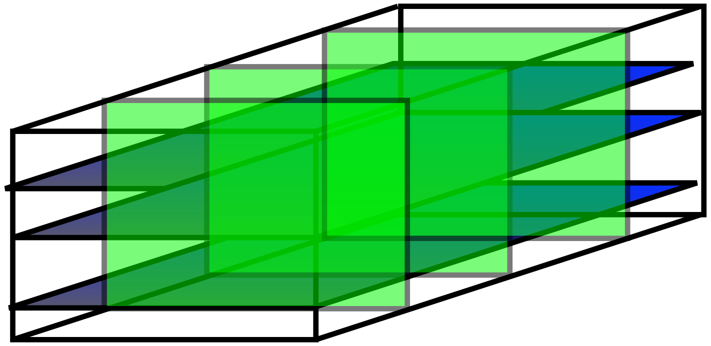
Three charges (4+1 d black hole)

$$S = 2\pi(\sqrt{n_1} + \sqrt{\bar{n}_1})(\sqrt{n_5} + \sqrt{\bar{n}_5})(\sqrt{n_p} + \sqrt{\bar{n}_p}) \sim E^{\frac{3}{2}}$$

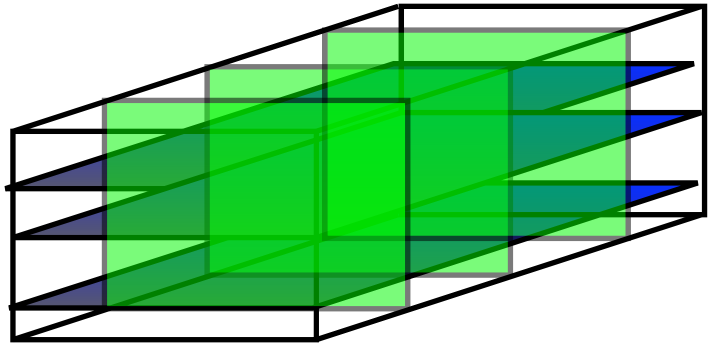
Four charges (3+1 d black hole)

$$S = 2\pi(\sqrt{n_1} + \sqrt{\bar{n}_1})(\sqrt{n_2} + \sqrt{\bar{n}_2})(\sqrt{n_3} + \sqrt{\bar{n}_3})(\sqrt{n_4} + \sqrt{\bar{n}_4}) \sim E^2$$

How do we get such large entropies ?



Degrees of freedom live at brane intersections, groupings of these degrees gives different states



'Fractional brane soup'

N charges, postulate .....

$$S = A_N \prod_{i=1}^{N} (\sqrt{n_i} + \sqrt{\bar{n}_i}) \sim E^{\frac{N}{2}}$$

M theory: Space is a 10-torus

Find that N can go upto 9

Black holes: Stress tensor is sum of brane antibrane stress tensors

Can solve expansion with this equation of state ....

(Chowdhury+Mathur 06)

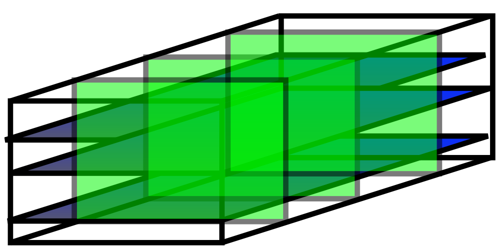
(A) There is no inflation, but nonlocal correlations stretch all across Universe because of nature of brane bound state ...

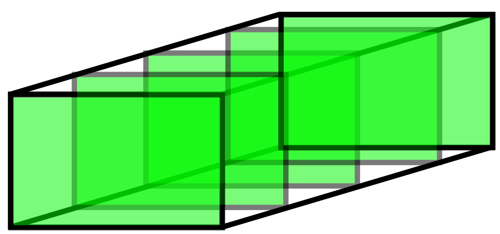
(B) At very early times, it is entropically favorable to have N=9 kinds of charges. As the Universe expands, we expect to go down to 8 charges, then 7, etc... till at the end we are left with radiation

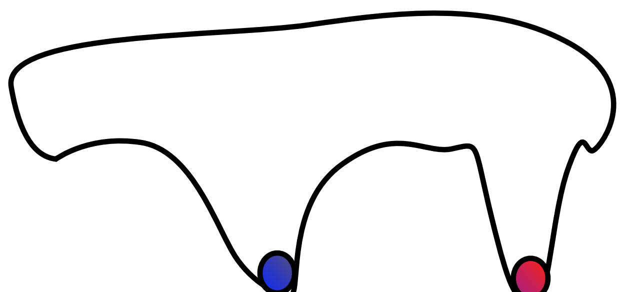
(C) At each `jump' we get a large entropy enhancement

(D) We should really ask the question: Why is this NOT the state of the very early Universe ? So far we have always taken the maximal entropy configuration ... should there be a different principle ?

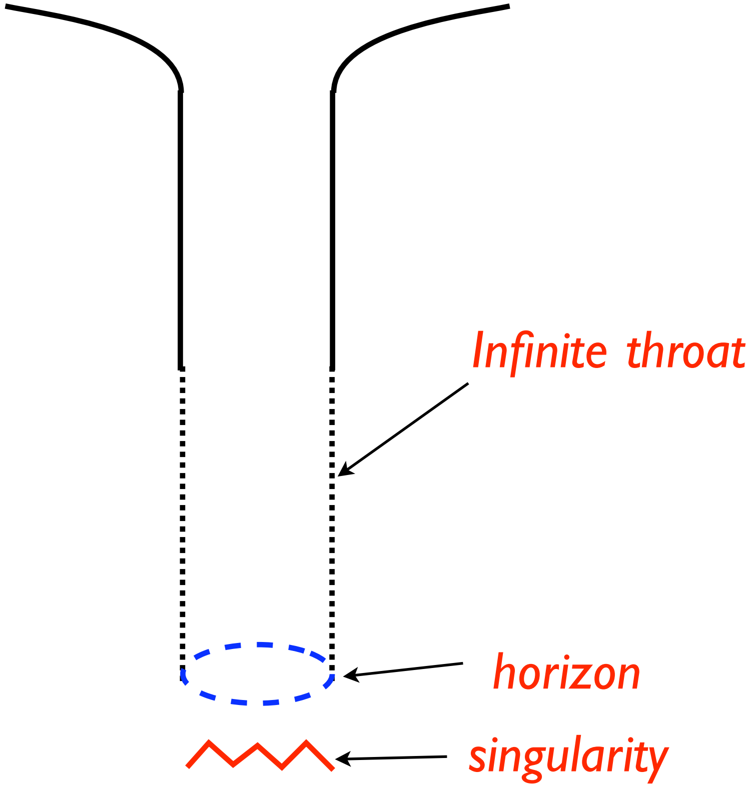
(E) Maybe left with branes/antibranes trapped as in KKLT ? Annihilation is slow: Hawking radiation rate ~ $\hbar$

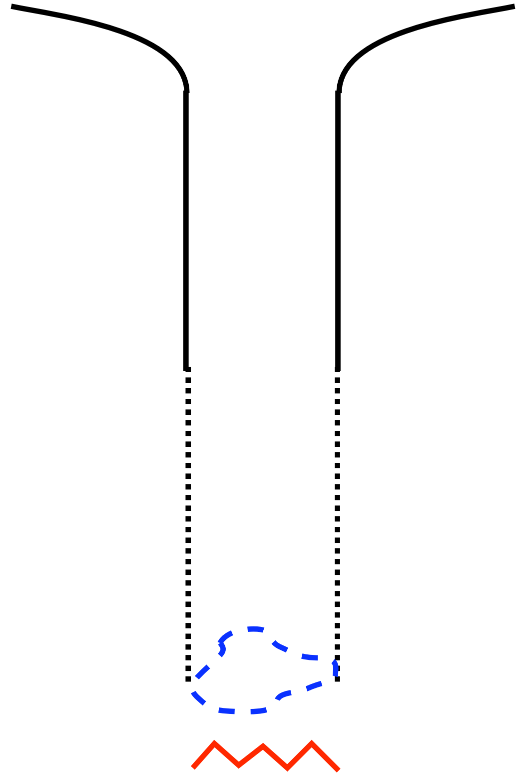






So what have we learnt ?





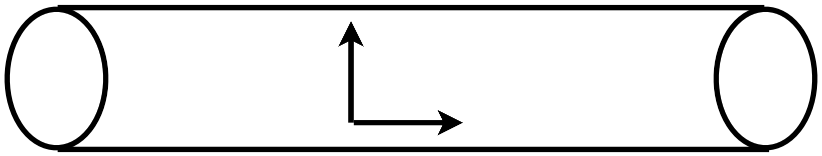
People looked for 'hair' at the horizon which would carry the information of the hole ...

But we dont find any ....

In string theory we have extra dimensions, which we will take to be compact circles

If we want a black hole in 3+1 d, then we have 6 compact directions

If we work PERTURBATIVELY, the extra directions give gauge fields and scalars ...

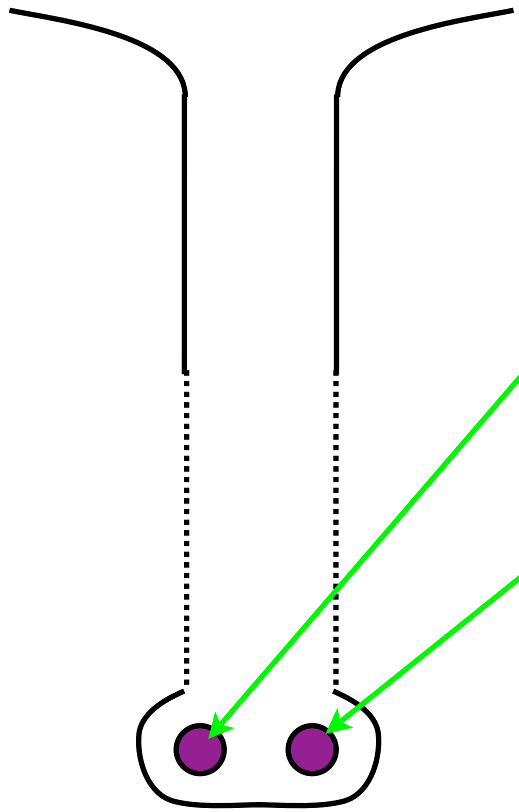


A_a

But we dont any hair from scalar or vector fields either ....

What turns out to happen:

Non-perturbative effects arising from these compact directions DO give hair



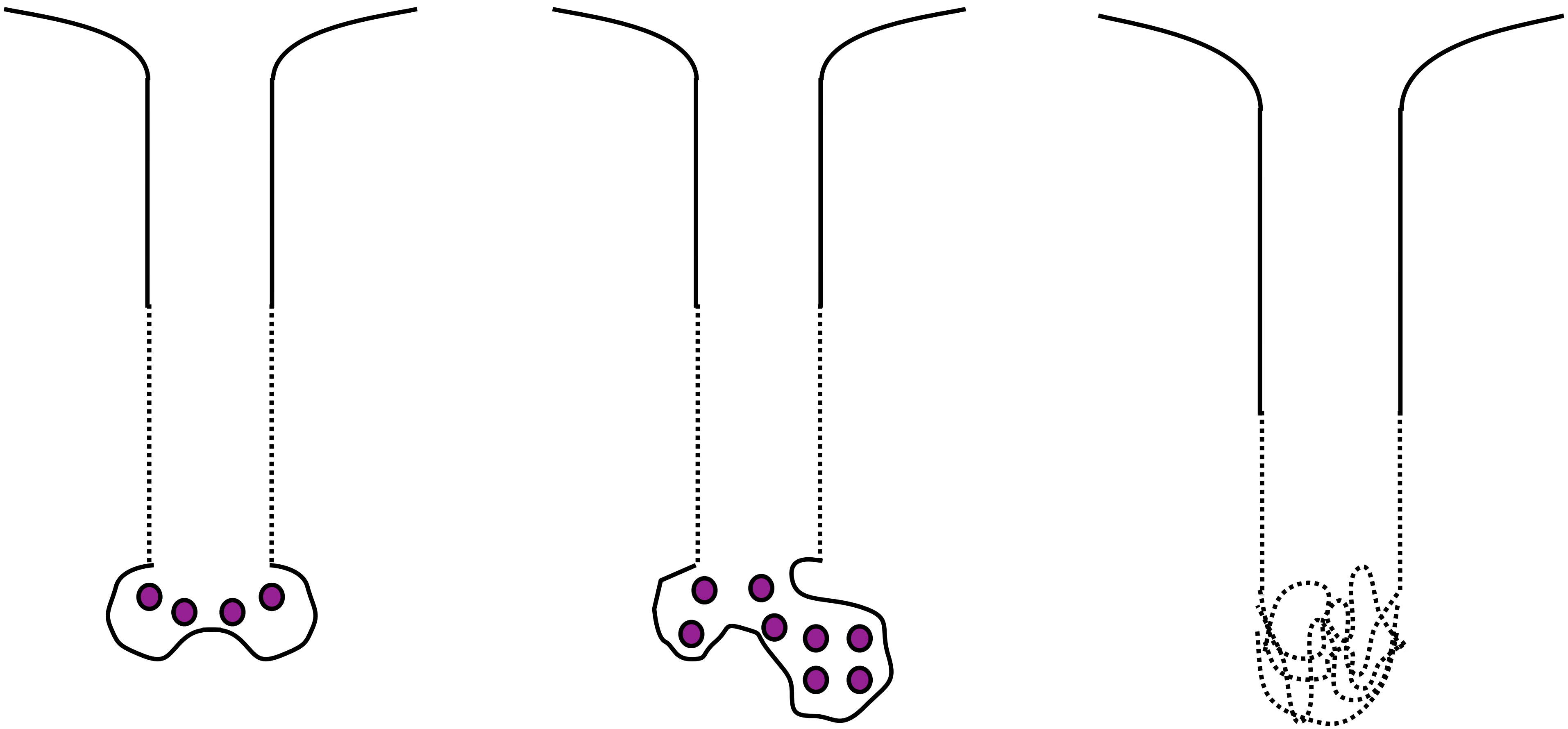
A compact circle can make a topological structure called a Kaluza-Klein mnopole

At some other point we get an anti-monopole

This gives one particular microstate of the black hole

It is a very special, nongeneric state

We can get a more complex state with more monopoles and anti-monopoles .....

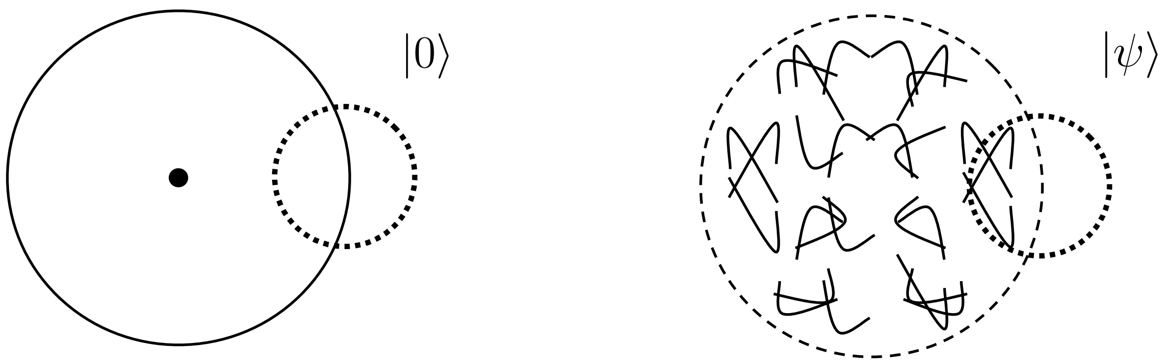


The generic state is complicated at short distances, and quantum fluctuations are large (a FUZZBALL). But we do not get the traditional geometry of the extremal hole ...

Summary:

String theory suggests that the black hole interior is a horizon sized 'fuzzball' ...

This will solve the black hole information puzzle ...



$$\langle 0|\psi\rangle \approx 0$$

Lesson: $\sim l_p \longrightarrow \sim N^{\alpha}\, l_p$

Underlying physics:

There are Exp[S] orthoginal wavefunctions for the different black hole

We cannot fit so many orthogonal wavefunctions in too small a region : horizon sized 'fuzzball'

More difficult questions:

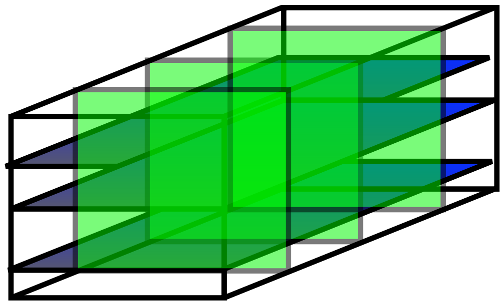
Can we construct all states for extremal holes? Nonextremal holes ?

How do we describe the structure of a generic state ?

How do we describe the infall of massive objects into generic states ?

Cosmology: In the early Universe we had a highly dense matter distribution

This may be similar to a matter crushed inside a black hole ...



$$S \sim E^{\frac{n}{2}}, \; n \leq 9 \qquad ??$$