Compactifications on Generalized Geometries

Jan Louis

Universität Hamburg

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Introduction

Phenomenological models in string theory:

- space-time background $M_{10} = M_4 \times \mathbf{Y_6}$
- $N = 1$ (spontaneously broken) supersymmetry

realized as

➪ **Heterotic string** with $\mathbf{Y_6}$: **Calabi-Yau threefold/orbifold**

➪ **Type II/I** with space-time filling D-branes and orientifold-planes

$\mathbf{Y_6}$: **generalized Calabi-Yau orientifold**

(**with background fluxes**)

Purpose of this talk:

review string compactifications on generalized geometries
(with $N = 2, 4$ supersymmetry)

Compactification:

Space-time background:	$\mathbf{M}_{10}$	$=$	$\mathbf{M_4} \times \mathbf{Y_6}$
Lorentz-group:	$\mathbf{SO(1,9)}$	$\rightarrow$	$\mathbf{SO(1,3)} \times \mathbf{SO(6)}$
10D Supercharge:	$\mathbf{16}$	$\rightarrow$	$(\mathbf{2},\mathbf{4}) \oplus (\bar{\mathbf{2}},\bar{\mathbf{4}})$

Impose:

1. existence of 4D supercharge(s) $\Rightarrow$ existence of global spinor(s) η

   $\Rightarrow$ $\mathbf{Y_6}$ has reduced structure group

$$\mathbf{SO(6)} \rightarrow \begin{cases} \mathbf{SU(3)} & \text{s.t.} \quad \mathbf{4} \rightarrow \mathbf{3}+\mathbf{1} \\ \mathbf{SU(2)} \times \mathbf{SU(2)} & \text{s.t.} \quad \mathbf{4} \rightarrow (\mathbf{2},\mathbf{1}) + (\mathbf{1},\mathbf{2}) \end{cases}$$

2. background preserves supersymmetry

$$\delta\Psi = \nabla\eta + (\gamma\cdot\mathbf{F})\,\eta + \dots \;=\; 0\;, \qquad \mathbf{F} = \text{background fluxes}$$

- $\nabla\eta \;=\; 0 \quad \Rightarrow$ $\mathbf{Y_6}$ is **Calabi-Yau manifold**
- here: study manifolds with SU(3)/SU(2)-structure ($\nabla\eta \neq \mathbf{0}$)

Manifolds with $SU(3)$ structure: [Gray,Hervella,Salamon,Chiossi,Hitchin,...]

characterized by two tensors $\mathbf{J}, \mathbf{\Omega}$ (follows from existence of η)

➪ $(1,1)$-form

$$\mathbf{J_{mn}} = \eta^\dagger \gamma_{[\mathbf{m}}\gamma_{\mathbf{n}]}\eta \ , \qquad \mathbf{dJ} \neq \mathbf{0}$$

➪ $(3,0)$-form

$$\mathbf{\Omega_{mnp}} = \eta^\dagger \gamma_{[\mathbf{m}}\gamma_{\mathbf{n}}\gamma_{\mathbf{p}]}\eta \ , \qquad \mathbf{d\Omega} \neq \mathbf{0}$$

Remarks:

- $\mathbf{dJ}, \mathbf{d\Omega}$ $\sim$ (intrinsic) torsion of Y_6
- **Calabi-Yau:** $\nabla\eta = \mathbf{0} \quad \Rightarrow \quad \mathbf{dJ} = \mathbf{d\Omega} = \mathbf{0}$

  $\Rightarrow$ torsion parameterizes the deviation from Calabi-Yau

Manifolds with $SU(3) \times SU(3)$ structure:

[Jescheck,Witt; Graña,Minasian,Petrini,Tomasiello; Graña,Waldram,JL; Bilal,Cassani; Kashani-Poor,Minasian, ...]

In type II string theory one can be slightly more general:

choose different spinors $\eta^{\mathbf{1}}, \eta^{\mathbf{2}}$ for the two gravitini $\Psi^{1,2}$

each η def. SU(3)-structure $\Rightarrow$ together: $\mathbf{SU(3)} \times \mathbf{SU(3)}$-structure
(characterized by pair $\mathbf{J^{1,2}}, \mathbf{\Omega^{1,2}}$)

Hitchin: embed in $\mathbf{SU(3)} \times \mathbf{SU(3)}$ in $\mathbf{O(6,6)}$ acting on $T \oplus T^*$

$\Rightarrow$ structure characterized by two pure spinors $\mathbf{\Phi}^+, \mathbf{\Phi}^-$ of $\mathbf{O(6,6)}$

$$\mathbf{\Phi}^+ = \mathbf{e^B} \eta_+^{\mathbf{1}} \otimes \bar{\eta}_+^{\mathbf{2}} \simeq \sum \mathbf{\Phi}^+_{\text{even}} \,, \qquad \mathbf{\Phi}^- = \mathbf{e^B} \eta_+^{\mathbf{1}} \otimes \bar{\eta}_-^{\mathbf{2}} \simeq \sum \mathbf{\Phi}^+_{\text{odd}} \,,$$

$\mathbf{SU(3)}$ structure ($\eta^{\mathbf{1}} = \eta^{\mathbf{2}}$): $\qquad \mathbf{\Phi}^+ = \mathbf{e^{B+iJ}} \,, \qquad \mathbf{\Phi}^- = \mathbf{e^B \Omega} \,,$

Low energy effective action:

$$\mathcal{S} = \int_{\mathbf{M_4}} \tfrac{\mathbf{1}}{\mathbf{2}}\mathbf{R} - \mathbf{g_{ab}}(\mathbf{z})\,\mathbf{D}_\mu \mathbf{z^a} \mathbf{D}^\mu \mathbf{z^b} - \mathbf{V}(\mathbf{z}) + \ldots$$

- Type II string theory: $\mathcal{S}$ is $N = 2$ gauged supergravity
- $\mathbf{z^a}$: coordinates of scalar manifold $\mathcal{M}$
  - correspond to deformations of $\mathbf{B}, \mathbf{J}, \mathbf{\Omega}$ or $\mathbf{\Phi}^+, \mathbf{\Phi}^-$
  - scalars from RR-sector
- $N = 2$ constraint: $\mathcal{M} = \mathcal{M}_{\text{SK}} \times \mathcal{M}_{\text{QK}}$

  $\text{IIA}: \quad \mathcal{M}_{\text{SK}} = \mathcal{M}_{\mathbf{\Phi}^+}\,, \quad \mathcal{M}_{\text{QK}} \supset \mathcal{M}_{\mathbf{\Phi}^-}\,, \qquad \text{IIB}: \quad \mathbf{\Phi}^+ \leftrightarrow \mathbf{\Phi}^-$

- Impose "standard $N = 2$" (no massive gravitino multiplets)

  $\Rightarrow$ $SU(3)$–structure without triplets $\mathbf{3}$: $dJ^2 = 0$ and $d\Omega^{3,1} = 0$

  $\Rightarrow$ $SU(3) \times SU(3)$–structure without $(\mathbf{3}, \mathbf{1}), (\mathbf{1}, \mathbf{3})$

Metric $\mathbf{g_{ab}}$:   special Kähler metric on $\mathcal{M} = \mathcal{M}_{\mathbf{\Phi}^+} \times \mathcal{M}_{\mathbf{\Phi}^-}$

[Hitchin; Graña,Gurrieri,Micu,Waldram,JL,...]

$$\mathbf{e}^{-\mathbf{K}_{\mathbf{\Phi}^+}} = \int_{\mathbf{Y}} \langle \mathbf{\Phi}^+, \overline{\mathbf{\Phi}}^+ \rangle$$

$$= \int_{\mathbf{Y}} \mathbf{J} \wedge \mathbf{J} \wedge \mathbf{J} \,, \qquad \text{for} \qquad \mathbf{\Phi}^+ = \mathbf{e}^{\mathbf{B}+\mathbf{i}\mathbf{J}} \,,$$

$$\mathbf{e}^{-\mathbf{K}_{\mathbf{\Phi}^-}} = \int_{\mathbf{Y}} \langle \mathbf{\Phi}^-, \overline{\mathbf{\Phi}}^- \rangle$$

$$= \int_{\mathbf{Y}} \mathbf{\Omega_3} \wedge \overline{\mathbf{\Omega}}_{\mathbf{3}} \,, \qquad \text{for} \qquad \mathbf{\Phi}^- = \mathbf{\Omega_3}$$

where $\langle \Phi^+, \overline{\Phi}^+ \rangle = \Phi_0^+ \wedge \overline{\Phi}_6^+ - \Phi_2^+ \wedge \overline{\Phi}_4^+ + \Phi_4^+ \wedge \overline{\Phi}_2^+ - \Phi_6^+ \wedge \overline{\Phi}_0^+$, etc.

- ➪ $e^{-\mathbf{K}}$ is quartic invariant of $O(6,6)$ (Hitchin functional)
- ➪ for $SU(3)$ same expression as in Calabi-Yau compactifications

Include RR-sector: geometry of $\mathcal{M}_{\rm QK}$ [Graña, Sim, Waldram, JL]

RR-scalars $C \neq 0 \quad \Rightarrow \quad \mathcal{M}_{\rm SK} \times \frac{\mathbf{SU(1,1)}}{\mathbf{U(1)}} \to \mathcal{M}_{\rm QK}$

$\mathbf{SO(6,6)} \times \mathbf{SU(1,1)} \to \mathbf{E_7}$ (U-duality group)

$\Rightarrow$ Exceptional Generalized Geometry [Hull; Pacheco,Waldram]

$\Rightarrow$ $N=2$ supergravity can be formulated in terms of $\mathbf{E_7}$ rep.

$$\mathbf{133} \to (\mathbf{1},\mathbf{3}) + (\mathbf{66},\mathbf{1}) + (\mathbf{32},\mathbf{2})$$

embed $\mathbf{\Phi}^-$ in $\mathbf{133^c}$: $\quad K_+ = K_1 + iK_2 = e^C(0,0,\mathbf{u^i}\mathbf{\Phi}^-)$

highest weight $SU(2)$ embedding $\quad [K_a, K_b] = 2\kappa\epsilon_{abc}K_c$ [Kobak,Swann]

Hyperkählercone: $\quad \mathcal{M}_{\rm QK} \to \mathcal{M}_{\rm HKC} \quad (= \frac{E_{7(7)}}{SO(12)^*} \times \mathbf{R}^+$ locally)

Kählerpotential: $\quad \chi \sim \sqrt{trK_+K_-} \sim e^{-\phi}\sqrt{i\langle \mathbf{\Phi}^-, \bar{\mathbf{\Phi}}^-\rangle}$

agrees with [Rocek,Vafa,Vandoren; Neitzke,Pioline,Vandoren]

Potential: is derived from Killing prepotential (or superpotential) $\vec{P}$

➪ IIA

$$\mathbf{P^1} + \mathbf{iP^2} \sim \int_{Y_6} \langle \mathbf{\Phi}^+, d\mathbf{\Phi}^- \rangle \ , \qquad \mathbf{P^3} \sim \int_{Y_6} \langle \mathbf{\Phi}^+, F \rangle$$

$$F \equiv \sum_{\text{RR-forms}} F^{\text{RR}}$$

➪ IIB: $\mathbf{\Phi}^+ \leftrightarrow \mathbf{\Phi}^-$

Note:

- as expected $\vec{P}$ depends on torsion and flux
- large volume mirror symmetry intact
- $\vec{P}$ can be given in terms of E_7 quantities $\longrightarrow$ [Graña, Sim, Waldram, JL]
- gauged $N = 2$ supergravity only with charged hypermultiplets

Non-perturbative dualities

Proposal: $\mathbf{Het.\ on\ K3 \times T^2\ with\ flux} \leftrightarrow \mathbf{IIA\ on\ SU(3) \times SU(3)}$

Problem: het. T^2-fluxes induce non-Abelian vector multiplets

(partial) resolution: [Aharony,Berkooz,Micu,JL]

consider **M-Theory on 7d-$SU(3)$-manifold:** $\mathbf{CY_6} \times_f S^1$

$$\text{with} \qquad \mathbf{d\omega_2^a = T^a_b\, \omega_2^b \wedge dz}\ , \qquad \omega_2^a|_{\mathbf{CY}} \in \mathbf{H^{(1,1)}(CY_6)}$$

$\Rightarrow$ charged vector multiplets with scalar derivatives: $D\vec{x} - \vec{k}_B A^B$

$$\text{Killing vectors obey}: \quad [k_a, k_b] = [k_0, k_0] = 0\ , \quad [k_a, k_0] = -T_a^b k_b$$

reason: het. $W^\pm$ masses heavy generically but light in M-theory regime

still not a perfect match

$\Rightarrow$ consider **het. compactifications on $SU(2)$-manifolds**

Manifolds with SU(2)-structure:

[Gauntlett,Martelli,Waldram; GMPT; Bovy,Lüst,Tsimpis; Triendl,JL, ...]

$$\mathbf{SU(3)} \times \mathbf{SU(3)} \supset \mathbf{SU(2)} \subset \mathbf{SO(6)}$$

2 cases:

$\eta^{\mathbf{1}} = \mathbf{c}\eta^{\mathbf{2}}$ along subspaces of Y_6 $\Rightarrow$ 2 supercharges globally defined

$\eta^{\mathbf{1}} \neq \mathbf{c}\eta^{\mathbf{2}}$ anywhere on Y_6 $\Rightarrow$ 4 supercharges globally defined

(global SU(2)-structure)

global SU(2)-structure:

- characterized by the pair $\mathbf{J^{1,2}}, \mathbf{\Omega^{1,2}}$
- or three 2-forms $\mathbf{J}_\alpha$ satisfying $\mathbf{J}_\alpha \wedge \mathbf{J}_\beta \sim \delta_{\alpha\beta}$

  and a complex 1-form $\mathbf{v}$ satisfying

$$\mathbf{v} \cdot \mathbf{v} = \mathbf{0}, \quad \bar{\mathbf{v}} \cdot \mathbf{v} = \mathbf{2}, \quad \iota_{\mathbf{v}} \mathbf{J}_\alpha = \mathbf{0}$$

properties of global SU(2)-structure:

- existence of $\mathbf{v}$ $\Rightarrow$ existence of almost product structure P satisfying $P^2 = 1$

  $\Rightarrow$ this splits the tangent space $TY = T_2Y \oplus T_4Y$

  $\Rightarrow$ and is a generalization of $K3 \times T^2$ where product structure is integrable.

- formulation in terms of (two) pure $O(6,6)$ spinors exists

- embedding into E_7 exits [Triendl,JL]

- $SU(2) \times SU(2)$ does not exist [Triendl,JL] (due to different chiralities in $d = 4$)

Low energy effective action:

$$\mathcal{S} = \int_{\mathbf{M_4}} \tfrac{\mathbf{1}}{\mathbf{2}}\mathbf{R} - \mathbf{g_{ab}}(\mathbf{z})\,\mathbf{D}_\mu \mathbf{z}^{\mathbf{a}} \mathbf{D}^\mu \mathbf{z}^{\mathbf{b}} - \mathbf{V}(\mathbf{z}) + \ldots$$

- Type II string theory: $\mathcal{S}$ is $N = 4$ gauged supergravity
- deformations of the NS-sector (metric, B, dilaton S) [Triendl,JL]

$$\mathcal{M}_{\mathrm{NS}} = \frac{\mathrm{SO}(4, 4+\mathrm{n})}{\mathrm{SO}(4) \times \mathrm{SO}(4+\mathrm{n})} \times \frac{\mathrm{SU}(1,1)_{\mathrm{S}}}{\mathrm{U}(1)} \times \frac{\mathrm{SU}(1,1)_{\mathrm{T}}}{\mathrm{U}(1)} \times \frac{\mathrm{SU}(1,1)_{\mathrm{U}}}{\mathrm{U}(1)}$$

- include RR-scalars [Triendl,JL]

$$\mathcal{M}_{\mathrm{IIA/B}} = \frac{\mathrm{SO}(6, 6+\mathrm{n})}{\mathrm{SO}(6) \times \mathrm{SO}(6+\mathrm{n})} \times \frac{\mathrm{SU}(1,1)_{\mathrm{T/U}}}{\mathrm{U}(1)}$$

- consistent with $N = 4$ constraint
- compute V $\longrightarrow$ [Reid-Edwards,Spanjaard & in preparation]

Heterotic on SU(2)-structure/$N=2$ [Martinez,Micu,JL]

➪ $N=2$ constraint: $\mathcal{M} = \mathcal{M}_{\rm SK} \times \mathcal{M}_{\rm QK}$

find:

$$\mathcal{M}_{\rm SK} = \frac{\mathbf{SO(2,2+m)}}{\mathbf{SO(2)\times SO(2+m)}} \times \frac{\mathbf{SU(1,1)_S}}{\mathbf{U(1)}}\ , \qquad \mathcal{M}_{\rm QK} \supset \frac{\mathbf{SO(4,4+n)}}{\mathbf{SO(4)\times SO(4+n)}}$$

➪ compute V for K3 fibered over T^2: $\mathbf{Y_6} = \mathbf{K3} \times_f T^2$

$$\mathbf{d}\omega_{\mathbf{2}}^{\mathbf{a}} \ = \ \mathbf{T^{a}_{ib}}\ \omega_{\mathbf{2}}^{\mathbf{b}} \wedge \mathbf{dz^{i}}\ , \qquad \omega_{\mathbf{2}}^{\mathbf{a}}|_{\mathbf{K3}} \in \mathbf{H^{(2)}(K3)}$$

KK reduction reveals charged hypermultiplets

$$\mathrm{P}_{\rm i}^{\alpha} \sim \epsilon_{\rm ij}\Big[\int_{\rm Y_6} \mathrm{dB}\wedge \mathrm{J}^{\alpha}\wedge \mathrm{dz}^{\rm j} - \tfrac{1}{2}\epsilon^{\alpha\beta\gamma}\int_{\rm Y_6} \mathrm{J}^{\beta}\wedge \mathrm{dJ}^{\gamma}\wedge \mathrm{dz}^{\rm j}\Big]\ .$$

- dual to 6d $SU(3)$ compactifications
- general $SU(2)$-structure compactifications $\longrightarrow$ [in preparation]

$\Rightarrow$ no complete picture yet

Type IIA orientifolds of SU(2)-structure/$N = 2$ [Danckaert, JL]

Orientifold projection: $N = 4 \rightarrow N = 2 \quad (\rightarrow N = 1)$

$$\sigma(\mathrm{v}) = -\bar{\mathrm{v}} \ , \quad \sigma(\mathrm{J}^3) = -\mathrm{J}^3 \ , \quad \sigma(\mathrm{J}^1 + \mathrm{i}\mathrm{J}^2) = -(\mathrm{J}^1 - \mathrm{i}\mathrm{J}^2) \ , \quad \sigma(\omega_2) = \pm\omega_2 \ ,$$

truncates spectrum and selects

$$\mathcal{M}_{\mathrm{SK}} \times \mathcal{M}_{\mathrm{SK}} \ \subset \ \mathcal{M}_{\mathrm{SK}} \times \mathcal{M}_{\mathrm{QK}} \ \subset \ \frac{\mathbf{SO(6, 6+n)}}{\mathbf{SO(6)} \times \mathbf{SO(6+n)}}$$

with

$$\mathcal{M}_{\mathrm{SK}} = \frac{\mathbf{SU(1,1)_U}}{\mathbf{U(1)}} \times \frac{\mathbf{SO(2, n_+)}}{\mathbf{SO(2)} \times \mathbf{SO(n_+)}} \ , \qquad \mathcal{M}_{\mathrm{QK}} = \frac{\mathbf{SO(4, n_-)}}{\mathbf{SO(4)} \times \mathbf{SO(n_-)}}$$

$\Rightarrow$ non-Abelian gauging with vectors and hypers charged

Killing vectors:

$$[k_0, k_1] \sim k_1 \ , \quad [k_0, k_a] \sim T_a^b k_b \ , \quad [k_1, k_a] = [k_a, k_b] = 0 \ .$$

cannot be obtained as $SU(3)$-structure compactifications.

Summary

discussed backgrounds with $SU(3)(\times SU(3))/SU(2)$ structure

- ➪ scalar manifold is independent of torsion
- ➪ NS-sector expressed in terms $O(6,6)$ quantities (generalized geometry)
- ➪ NS + RR-sector expressed in terms E_7 (exceptionally generalized geometry)
- ➪ (pre)potential depends on torsion and background fluxes
- ➪ generalized mirror symmetry intact (in the supergravity limit) non-perturbative dualities not quite yet