M2-branes in Background Fields

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Introduction

Once mysterious, Multiple M2-brane actions are now almost common.

There is a complete and convincing proposal for the effective Lagrangian of n M2-branes in $\mathbb{R}^8/\mathbb{Z}_k$ for arbitrary k and n

- Lagrangians are new types of highly supersymmetric Chern-Simons matter theories in $D = 3$.
  - Constructed from a triple product rather than a Lie-bracket
- Hopefully this will lead to a big increase in our understanding of M-theory beyond supergravity
  - M2-brane CFT's 'define' M-theory in asymptotically AdS_4 backgrounds

Introduction

PLAN:

Here I will describe the highly supersymmetric Chern-Simons Gauge Theory in three-dimensions.

- Concentrate on the $\mathcal{N} = 8$ theory.
- Construction via 3-algebras
- M-theory Interpretation
- Coupling to Background C-fields
  - background fluxes lead to a massive deformation
  - Puzzle: what is the spacetime origin of the $(\mathit{flux})^2$ terms
- ABJM

$\mathcal{N} = 8$

To construct a theory of multiple M2-branes what do we want?

- 3D field theory with 16 susys ($\mathcal{N} = 8$)
- 8 dynamical scalars with an $SO(8)$ R-symmetry
- no dynamical gauge field
- Parity invariant
- Conformal invariance

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So just start from scratch:

A stack of M2-branes has 8 scalars X^I and their fermionic superpartners Ψ, $\Gamma_{012}\Psi = -\Psi$.

- We assume that these take values in some vector space

$\mathcal{N} = 8$

A natural guess for the susy algebra is, ignoring gauge symmetries, [Bagger, NL]

$$\begin{aligned} \delta X^I &= i\bar{\epsilon}\Gamma^I\Psi \\ \delta\Psi &= \partial_\mu X^I \Gamma^\mu \Gamma^I \epsilon + [X^I, X^J, X^K]\Gamma^{IJK}\epsilon, \end{aligned}$$

where $[A, B, C]$ is totally anti-symmetric triple product.

- So our vector space needs a triple product: 3-algebra

$\mathcal{N} = 8$

Closure of the algebra implies a gauge symmetry:

$$[\delta_1, \delta_2]X^I = 2i\bar{\epsilon}_1\Gamma^\mu\epsilon_2\partial_\mu X^I + 2i\bar{\epsilon}_1\Gamma^{JK}\epsilon_2[X^I, X^J, X^K]$$

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This must be dealt with to realize the full superalgebra
We will proceed by introducing a basis T^a for $\mathcal{A}$.

$$[T^a, T^b, T^c] = f^{abc}{}_d T^d \ , \qquad f^{abc}{}_d = f^{[abc]}{}_d$$

so

$$\delta X^I_d = \Lambda_{ab} f^{cab}{}_d X^I_c$$

and introduce the covariant derivative:

$$D_\mu X^I_c = \partial_\mu X^I_c - \tilde{A}_\mu{}^c{}_d X^I_c$$

$\mathcal{N} = 8$

Full superalgebra takes the form [Bagger, NL]

$$
\begin{aligned}
\delta X^I_d &= i\bar{\epsilon}\Gamma^I \Psi_d \\
\delta \Psi_d &= D_\mu X^I_d \Gamma^\mu \Gamma^I \epsilon - \frac{1}{6} X^I_a X^J_b X^K_c f^{abc}{}_d \Gamma^{IJK}\epsilon \\
\delta \tilde{A}_\mu{}^c{}_d &= i\bar{\epsilon}\Gamma_\mu \Gamma_I X^I_a \Psi_b f^{abc}{}_d ,
\end{aligned}
$$

Indeed this closes (on-shell) if f^{abcd} satisfies the fundamental identity:

$$
f^{efg}{}_b f^{cba}{}_d + f^{fea}{}_b f^{cbg}{}_d + f^{gaf}{}_b f^{ceb}{}_d + f^{age}{}_b f^{cfb}{}_d = 0.
$$

This ensures that the gauge symmetries $\delta_\Lambda X^I_d = \Lambda_{ab} f^{cab}{}_d X^I_c$ generated by the triple product are those of a Lie-algebra with matrix representatives $\tilde{\Lambda}^c{}_d = \Lambda_{ab} f^{cab}{}_d$ acting on X^I_d.

- **N.B.** Closure was obtained first by [Gustavsson] using, but equivalent algebraic approach that gives closure

$\mathcal{N} = 8$

The invariant Lagrangian is a Chern-Simons theory [Bagger, NL]:

$$\begin{aligned}\mathcal{L} \;=\; & -\frac{1}{2}(D_\mu X^{aI})(D^\mu X^I_a) + \frac{i}{2}\bar{\Psi}^a\Gamma^\mu D_\mu \Psi_a + \frac{i}{4}\bar{\Psi}_b\Gamma_{IJ}X^I_c X^J_d \Psi_a f^{abcd} \\ & + \frac{1}{2}\varepsilon^{\mu\nu\lambda}(f^{abcd}A_{\mu ab}\partial_\nu A_{\lambda cd} + \frac{2}{3}f^{cda}{}_g f^{efgb}A_{\mu ab}A_{\nu cd}A_{\lambda ef}) \\ & - \frac{1}{12}\mathrm{Tr}([X^I, X^J, X^K])^2\end{aligned}$$

- Tr is an invariant trace (inner-product) on $\mathcal{A}$
- gauge invariance implies $f^{abcd} = f^{[abcd]}$
- $\tilde{A}_\mu{}^c{}_d = f^{abc}{}_d A_{\mu ab}$
- Chern-Simons term implies $f^{abc}{}_d$ is quantized

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Has all the expected symmetries of multiple M2-branes: 16 susys, $SO(8)$ R-symmetry, Parity (f^{abcd} is parity odd).

No continuous free parameter but weakly coupled as $f^{abc}{}_d \to 0$

$\mathcal{N} = 8$

If Tr is positive definite then there is only one finite-dimensional possibility [Nagy],[Gauntlett, Gutowski],[Papadopoulos]:

$$f^{abcd} = \frac{2\pi}{k}\varepsilon^{abcd}$$

Although examples with an infinite dimensional 3-algebra arise from the Nambu bracket.

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Although examples with an infinite dimensional 3-algebra arise from the Nambu bracket.

In this case the Lagrangian is that of an $SU(2) \times SU(2)$ Chern-Simons theory coupled to matter in the bi-fundamental.

$$\begin{aligned} \mathcal{L}_{CS} &= \frac{k}{4\pi}\mathrm{tr}(\tilde{A}^+ \wedge d\tilde{A}^+ + \frac{2}{3}\tilde{A}^+ \wedge \tilde{A}^+ \wedge \tilde{A}^+) \\ &\quad -\frac{k}{4\pi}\mathrm{tr}(\tilde{A}^- \wedge d\tilde{A}^- + \frac{2}{3}\tilde{A}^- \wedge \tilde{A}^- \wedge \tilde{A}^-) \end{aligned}$$

- quantization condition implies $k \in \mathbf{Z}$
- $f^{abcd} \leftrightarrow -f^{abcd}$ corresponds to switching the two $SU(2)$'s

$\mathcal{N} = 8$

What is the multiple M2-brane interpretation?

Look at the Vacuum moduli space [NL, Tong], [Distler, Mukhi, Papageorgakis, van Raamsdonk]

$$\mathcal{M}_k = \mathbb{R}^{16}/D_{2k}$$

- D_{2k} - dihedral group
- $\mathcal{M}_1 = \mathbb{R}^8/\mathbb{Z}_2 \times \mathbb{R}^8/\mathbb{Z}_2$
  - vacuum moduli space of an $SO(4)$ gauge theory
- $\mathcal{M}_2 = \left(\mathbb{R}^8/\mathbb{Z}_2 \times \mathbb{R}^8/\mathbb{Z}_2\right)/\mathbb{Z}_2$
  - vacuum moduli space of an $SO(5)$ gauge theory

Two 2-branes on $\mathbb{R}^8/\mathbb{Z}_2$

No clear picture for $k > 2$ (although for $k = 3$ one finds the vacuum moduli space of a G_2 gauge theory).

$\mathcal{N} = 8$

What does one expect for two M2-branes on orbifold $\mathbb{R}^8/\mathbb{Z}_2$?

- $\mathcal{N} = 8$, $SO(8)$ R-symmetry and parity
- two possible orbifolds depending on the value of discrete torsion [Sethi],[Berkooz,Kapustin]:
  - $O(4)$ gauge group
  - $SO(5)$ or $O(4)$ gauge group
- The $k = 2$ agrees (although not clear if it is for $O(4)$ or $SO(5)$.

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So the bottom line is that it's not clear what the interpretation is

- for $k = 2$ there is full agreement with M-theory.

Coupling to Background Fields

Turning on a background 4-form flux should lead to a 'Myers' effect and induce mass terms preserving all supersymmetry.

- Vacua correspond to M2's 'blown-up' into M5's on fuzzy S^3 [Benna],[Bagger,NL]

There is a deformation of the $\mathcal{N} = 8$ theory that breaks $SO(8) \to SO(4) \times SO(4)$ [Gomis, Salim, Passerini], [Hosomichi,Lee and Lee]

$$\begin{aligned}\mathcal{L}_{mass} &= \mathcal{L} - \frac{1}{2}\mu^2 \mathrm{Tr}(X^I, X^J)\delta_{IJ} \\ &\quad -\mu \mathrm{Tr}(X^A, [X^B, X^C, X^D])\epsilon_{ABCD} \\ &\quad -\mu \mathrm{Tr}(X^{A'}, [X^{B'}, X^{C'}, X^{D'}])\epsilon_{A'B'C'D'} \\ &\quad + \mathit{Fermion\ mass\ terms}\end{aligned}$$

We'd like to understand how all these terms arise in a more general background (see also papers by [Li,Wang], [Kim, Kwon, Nakajima, Tolla])

Coupling to Background Fields

First try to construct the most general 'Myers'-like terms for M2-branes

- Gauge invariant quantities are $\mathrm{Tr}(X^I, X^J)$, $\mathrm{Tr}(X^I, [X^J, X^K, X^L])$ ...

$$
\begin{aligned}
S_{flux} \;=\; & \frac{T_{M2}}{3!}\int d^3x\,\epsilon^{\mu\nu\lambda}\Big(NC_{\mu\nu\lambda} + 3BT_{M2}^{-1}C_{\mu IJ}\mathrm{Tr}(D_\nu X^I, D_\lambda X^J) \\
& + 3CT_{M2}^{-1}C_{\mu\nu IJKL}\mathrm{Tr}(D_\lambda X^I, [X^J, X^K, X^L]) \\
& + ET_{M2}^{-2}C_{IJKLMN}\mathrm{Tr}([D_\nu X^I, D_\nu X^J, D_\lambda X^K], [X^L, X^M, X^N])\Big)
\end{aligned}
$$

- Consistent with $\mathbf{Z}_2$ orbifold.
- The spacetime coordinates are $x^I = T_{M2}^{-\frac{1}{2}} X^I$

Coupling to Background Fields

Taking the decoupling limit $T_{M2} \to \infty$ and only consider Lorentz invariant terms:

$$S_{flux} = \int d^3x N T_{M2} \epsilon^{\mu\nu\lambda} G_{\mu\nu\lambda} + C \tilde{G}_{IJKL} \mathrm{Tr}(X^I, [X^J, X^K, X^L])$$

- first term is just a constant

We can supersymmetize this second term:

$$\mathcal{L} = \mathcal{L}_0 - \frac{m^2}{2} \mathrm{Tr}(X^I, X^I) - i \frac{C}{16} \mathrm{Tr}(\bar{\Psi}, \Gamma^{IJKL} \Psi) \tilde{G}_{IJKL} + C \tilde{G}_{IJKL} \mathrm{Tr}(X^I, [X^J, X^K, X^L])$$

Coupling to Background Fields

$\mathcal{N} = 8$ is preserved if we shift the Fermion supervariation to

$$\delta\Psi_a = \delta_0\Psi_a + \frac{C}{8}\Gamma^{IJKL}\Gamma^M\epsilon X_a^M \tilde{G}_{IJKL}$$

Provided that $\tilde{G}_{IJKL}$ is self-dual (in the transverse space) and

$$(\Gamma^{IJKL}\tilde{G}_{IJKL})^2 = \frac{32m^2}{C^2}(1 + \Gamma^{345678910})$$

For example $G = \mu(dx^3 \wedge dx^4 \wedge dx^5 \wedge dx^6 + dx^7 \wedge dx^8 \wedge dx^9 \wedge dx^{10})$ gives the mass deformation of [Gomis, Salim, Passerini], [Hosomichi,Lee and Lee]

Coupling to Background Fields

What is the origin of the Bosonic mass which is quadratic in the flux?

- Doesn't depend triple product - exists even for a single M2.

Supersymmetry is a consequence of κ-symmetry

- κ-symmetry requires that the background satisfies the equations of motion of eleven-dimensional supergravity [Bergshoeff, Sezgin, Townsend]
- Fluxes curve background geometry

So we should find the quardratic flux term by looking at a single M2-brane moving in a curved background

Coupling to Background Fields

Consider the effective action

$$S = -T_{M2}\int d^3x\sqrt{-\det(\partial_\mu x^M \partial_\nu x^N g_{MN})}$$

To lowest order in the fluxes $g_{MN} = \eta_{MN}$. To next order we try

$$g_{MN} = \begin{pmatrix} e^{2\omega}\eta_{\mu\nu} & 0 \\ 0 & g_{IJ} \end{pmatrix}$$

where $\omega = 1 + T_{M2}^{-1}\omega_{IJ}X^I X^J + \ldots$ and $g_{IJ} = \delta_{IJ} + \ldots$

To lowest order in T_{M2} the action, in static gauge, is

$$S = -\int d^3x\left(T_{M2} + 3\omega_{IJ}X^I X^J + \frac{1}{2}\partial_\mu X^I \partial^\mu X^J \delta_{IJ} + \ldots\right)$$

so we find a quadratic mass-term if $\omega \neq 1$.

Coupling to Background Fields

Solving the linearized Einstein equation in the presence of the flux gives

$$\omega = 1 + \frac{1}{16 \cdot 4!\, T_{M2}} G^2 \delta_{IJ} X^I X^J$$

and so we find the mass term

$$m^2_{IJ} = \frac{3}{64 \cdot 4!} G^2 \delta_{IJ}$$

- $SO(8)$-invariant mass-squared term for the Bosons.

Comparing with the supersymmetric Lagrangian we can determine that $C = 2$ *i.e.*

$$S_{flux} = \int d^3x \epsilon^{\mu\nu\lambda} C_{\mu\nu IJKL} \mathrm{Tr}(D_\lambda X^I, [X^J, X^K, X^L])$$

$\mathcal{N} = 6$ & ABJM

To generalize to arbitrary number of M2's it turns out that we should consider an orbifold with 12 supersymmetries:

There is an $\mathbb{R}^8/\mathbb{Z}_k$ orbifold that preserves 12 susys ($\mathcal{N} = 6$)

$$\begin{pmatrix} Z^1 \\ Z^2 \\ Z^3 \\ Z^4 \end{pmatrix} \sim \begin{pmatrix} \omega & & & \\ & \omega & & \\ & & \omega^{-1} & \\ & & & \omega^{-1} \end{pmatrix} \begin{pmatrix} Z^1 \\ Z^2 \\ Z^3 \\ Z^4 \end{pmatrix} \qquad \omega = e^{2\pi i/k}$$

- $SO(8) \to SU(4) \times U(1)$

$\mathcal{N} = 6$ & ABJM

This construction is by [Aharony, Bergman, Jafferis and Maldacena]

- Only impose $\mathcal{N} = 6$ and an $SU(4) \times U(1)$ R-symmetry in the Lagrangian.
- Constructed $U(n) \times U(n)$ Chern-Simons Matter theories at level $(k, -k)$
- Vacuum moduli space is $\mathrm{Sym}^n(\mathbb{R}^8/\mathbb{Z}_k)$
- Describes n M2-branes in this $\mathbb{R}^8/\mathbb{Z}_k$ orbifold.

This construction can be further generalized to include discrete torsion $H_4(\mathbb{R}^8/\mathbb{Z}_k) = \mathbb{Z}^k$ [Aharony, Bergman, Jafferis]:

- $U(m) \times U(n)$ CS theory with level $(k, -k)$ coupled to bi-fundamental matter
- conjectured that $|m - n| \leq k$
  -*e.g.* $n = m$, $n = m + 1$,...,$n = m + k - 1$
  -$n = m + k$ is equivalent to $n = m$
  -always strongly coupled

$\mathcal{N} = 6$ & ABJM

Following the logic of the $\mathcal{N} = 8$ construction let us construct the most general Lagrangian with $\mathcal{N} = 6$ susy, conformal invariance and an $SU(4) \times U(1)$ R-symmetry.

- scalars $Z^A_a \in \mathbf{4}_1$ of $SU(4) \times U(1)$
  -complex conjugates $\bar{Z}_{A\bar{a}} \in \bar{\mathbf{4}}_{-1}$
- fermions $\psi_{Aa} \in \bar{\mathbf{4}}_1$ of $SU(4) \times U(1)$
- susys $\epsilon_{AB} \in \mathbf{6}_0$ of $SU(4) \times U(1)$
  - $(\epsilon_{AB})^* = \epsilon^{AB} = \frac{1}{2}\varepsilon^{ABCD}\epsilon_{CD}$
- complex conjugation raises/lowers and A-index and flips the $U(1)$ charge
- Trace form $h^{\bar{a}b} = \mathrm{Tr}(T^{\bar{a}}, T^b)$

$\mathcal{N} = 6$ & ABJM

Starting from the most general form for the susy's one finds [Bagger, NL]

$$\begin{aligned}
\delta Z^A_d &= i\bar{\epsilon}^{AB}\psi_{Bd} \\
\delta\psi_{Bd} &= \gamma^\mu D_\mu Z^A_d \epsilon_{AB} + f^{ab\bar{c}}{}_d Z^C_a Z^A_b \bar{Z}_{C\bar{c}}\epsilon_{AB} + f^{ab\bar{c}}{}_d Z^C_a Z^D_b \bar{Z}_{B\bar{c}}\epsilon_{CD} \\
\delta\tilde{A}_\mu{}^c{}_d &= -i\bar{\epsilon}_{AB}\gamma_\mu Z^A_a \psi^B_{\bar{b}} f^{ca\bar{b}}{}_d + i\bar{\epsilon}^{AB}\gamma_\mu \bar{Z}_{A\bar{b}}\psi_{Ba} f^{ca\bar{b}}{}_d
\end{aligned}$$

Provided that

$$f^{ab\bar{c}}{}_e f^{ef\bar{g}}{}_d = f^{af\bar{g}}{}_e f^{eb\bar{c}}{}_d + f^{bf\bar{g}}{}_e f^{ae\bar{c}}{}_d - f_{\bar{e}}{}^{f\bar{g}\bar{c}} f^{ab\bar{e}}{}_d$$

and

$$f^{ab\bar{c}\bar{d}} = -f^{ba\bar{c}\bar{d}}, \qquad f^{*\bar{c}\bar{d}ab} = f^{ab\bar{c}\bar{d}}.$$

N.B Recover the $\mathcal{N} = 8$ theory when f^{abcd} is real and totally anti-symmetric

$\mathcal{N} = 6$ & ABJM

The Lagrangian has a similar form to the $\mathcal{N} = 8$ case [Bagger, NL]:

$$
\begin{aligned}
\mathcal{L} \;=\; & -D^\mu \bar{Z}^a_A D_\mu Z^A_a - i\bar{\psi}^{Aa}\gamma^\mu D_\mu \psi_{Aa} - \Upsilon^{CD}_{Bd}\bar{\Upsilon}^{Bd}_{CD} + \mathcal{L}_{CS} \\
& - if^{ab\bar{c}\bar{d}}\bar{\psi}^A_{\bar{d}}\psi_{Aa}Z^B_b\bar{Z}_{B\bar{c}} + 2if^{ab\bar{c}\bar{d}}\bar{\psi}^A_{\bar{d}}\psi_{Ba}Z^B_b\bar{Z}_{A\bar{c}} \\
& + \frac{i}{2}\varepsilon_{ABCD}f^{ab\bar{c}\bar{d}}\bar{\psi}^A_{\bar{d}}\psi^B_{\bar{c}}Z^C_a Z^D_b - \frac{i}{2}\varepsilon^{ABCD}f^{cd\bar{a}\bar{b}}\bar{\psi}_{Ac}\psi_{Bd}\bar{Z}_{C\bar{a}}\bar{Z}_{D\bar{b}}
\end{aligned}
$$

where

$$
\Upsilon^{CD}_{Bd} = f^{ab\bar{c}}{}_d Z^C_a Z^D_b \bar{Z}_{B\bar{c}} - \frac{1}{2}\delta^C_B f^{ab\bar{c}}{}_d Z^E_a Z^D_b \bar{Z}_{E\bar{c}} + \frac{1}{2}\delta^D_B f^{ab\bar{c}}{}_d Z^E_a Z^C_b \bar{Z}_{E\bar{c}}.
$$

and

$$
\mathcal{L}_{CS} = \frac{1}{2}\varepsilon^{\mu\nu\lambda}\left(f^{ab\bar{c}\bar{d}}A_{\mu\bar{c}b}\partial_\nu A_{\lambda\bar{d}a} + \frac{2}{3}f^{ac\bar{d}}{}_g f^{ge\bar{f}\bar{b}}A_{\mu\bar{b}a}A_{\nu\bar{d}c}A_{\lambda\bar{f}e}\right).
$$

$\mathcal{N} = 6$ & ABJM

As before $f^{ab\bar{c}}{}_d$ also defines a triple product:

$$[X, Y; \bar{Z}]_d = f^{ab\bar{c}}{}_d X_a Y_b \bar{Z}_{\bar{c}}$$

- gauge symmetry $\delta_\Lambda Z_d^A = \Lambda_{\bar{c}b} f^{ab\bar{c}}{}_d Z_a^A$ acts as a derivation

One natural class of solutions: Let X, Y, Z be complex $n \times m$ matrices

$$[X, Y; \bar{Z}] = \frac{2\pi}{k}(XZ^\dagger Y - YZ^\dagger X)$$

- Gauge symmetry is $\delta X = MX - XN$ with $M \in u(m)$ and $N \in u(n)$
  - $SU(m) \times SU(n)$ Chern-Simons with matter in bi-fundamental
- gives the [ABJM] and [ABJ] models by gauging the $U(1)$ global symmetry

$\mathcal{N} = 6$ & ABJM

There is a known mass deformation of this theory too [Hosomichi,Lee,Lee,Lee,Park], [Gomis,Rodriguez-Gomez,van Raamsdonk, Verlinde].

So what about the 'Myers'-terms?

$$
\begin{aligned}
S_{flux} \;=\; & \frac{T_{M2}}{3!}\int d^3x\epsilon^{\mu\nu\lambda}\Big(NC_{\mu\nu\lambda} + 3BT_{M2}^{-1}C_{\mu A}{}^{B}\mathrm{Tr}(D_\nu Z^A, D_\lambda \bar{Z}_B) \\
& +3CT_{M2}^{-1}C_{\mu\nu A}{}^{B}{}_{C}{}^{D}\mathrm{Tr}(D_\lambda Z^A, [\bar{Z}_B, \bar{Z}_D, Z^C]) \\
& +ET_{M2}^{-2}C_{A}{}^{B}{}_{C}{}^{D}{}_{E}{}^{F}\mathrm{Tr}([D_\nu Z^A, D_\nu Z^B, D_\lambda \bar{Z}_C], [Z^D, Z^E, \bar{Z}_F])\Big) \\
& +c.c.
\end{aligned}
$$

$\mathcal{N} = 6$ & ABJM

Same story:

- Supersymmetry gives Fermionic terms and a quadratic mass
  - Bosonic mass is also $SO(8)$ invariant and Abelian
- Same background calculations gives the origin of the flux-squared terms.

Gauge invariance preserves $U(1)$ symmetry and hence is consistent with $\mathbf{Z}_k$ orbifold

- Although for $k = 1$ there is no orbifold and hence there should be additional Myers terms.

Conclusion

Conclusions: We have reviewed the Lagrangians to describe n M2's in $\mathbb{R}^8/\mathbb{Z}_k$ for any n, k.

- focused on Lagrangians with $\mathcal{N} = 8$ and $SO(8)$ R-symmetry.

We have also discussed the coupling of these Lagangians to 'Myers' Terms

- Supersymmetrized the flux terms
- Quadratic flux terms arise from back reaction on the bulk metric